A quick note to explain how these values I'm calculating are relevant

1. Building Blocks: d_1 and d_2

First, we combine our inputs—current stock price S, strike K, time to expiry T, risk-free rate r, and implied volatility σ—into two intermediate terms:

$$d_1 = \frac{\ln(S/K) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}.$$

Intuitively, d_1 and d_2 measure how far "in or out of the money" the option is, accounting for time and volatility.

2. Delta (Δ)

- **Definition:** Change in option price per \$1 move in the underlying.
- **Formula:**

  $$\Delta_C = \Phi(d_1), \qquad \Delta_P = \Phi(d_1) - 1,$$

  where Φ is the standard normal CDF.
- **Interpretation:** A call delta of 0.6 means if the stock rises \$1, the option price rises by about \$0.60.

3. Gamma (Γ)

- **Definition:** Change in delta per \$1 move in the underlying.
- **Formula:**

  $$\Gamma = \frac{\phi(d_1)}{S\,\sigma\sqrt{T}},$$

  where ϕ is the standard normal PDF.
- **Interpretation:** High gamma means delta is very sensitive to underlying moves, requiring frequent hedge adjustments.

4. Vega

- **Definition:** Change in option price per 1% change in implied volatility.
- **Formula:**

  $$\text{Vega} = S\,\phi(d_1)\,\sqrt{T}.$$

- **Interpretation:** Vega of 0.10 means a 1% increase in volatility raises the option price by \$0.10.

5. Theta (Θ)

- **Definition:** Change in option price per unit time decay (typically one day).
- **Formula (call):**

$$\Theta_C = -\frac{S\,\phi(d_1)\,\sigma}{2\sqrt{T}} - r\,K\,e^{-rT}\,\Phi(d_2).$$

- **Interpretation:** A theta of $-\$0.05$ per day means the option loses \$0.05 in value each day if nothing else changes.

PDF: Probability Density Function

- A PDF, $f(x)$, describes the relative likelihood that a continuous random variable X is near the value x.
- Key property:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

- Probability that X lies in $[a, b]$:

$$P(a \le X \le b) \;=\; \int_a^b f(x)\,dx.$$

- *Example:* Standard normal PDF $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

CDF: Cumulative Distribution Function

- A CDF, $F(x)$, gives the probability that X is at most x:

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt.$$

- By construction, $F(x)$ increases from 0 (as $x \to -\infty$) to 1 (as $x \to +\infty$).
- *Example:* Standard normal CDF, denoted $\Phi(x)$, is the area under the normal curve up to x.

Relation to Options

- In Black–Scholes formulas, $\phi(d)$ (the PDF) appears in gamma and vega.
- $\Phi(d)$ (the CDF) appears in delta and the option price itself.