$$K_k = (R + B^T P_{k+1} B)^{-1} (B^T P_{k+1} A)$$

Initial equations

$$d_k = (R + B^T P_{k+1} B)^{-1} (B^T p_{k+1} + r_k)$$

$$P_k = Q + K_k^T R K_k + (A - BK_k)^T P_{k+1} (A - BK_k)$$

$$p_k = q_k + (A - BK_k)^T (p_{k+1} - P_{k+1} B d_k) + K_k^T (R d_k - r_k)$$

$$K_k = K_{inf}$$

$$P_k = P_{inf}$$

$$d_k = \overbrace{(R + B^T P_{inf} B)^{-1}}^{c_1} (B^T p_{k+1} + r_k)$$

$$p_k = q_k + \overbrace{(A - B K_{inf})^T}^{c_2} (p_{k+1} - P_{inf} B d_k) + K_{inf}^T (R d_k - r_k)$$

$$C_1 = (R + B^T P_{inf} B)^{-1}$$

$$C_2 = (A - B K_{inf})^T$$

$$d_k = C_1 (B^T p_{k+1} + r_k)$$

$$p_k = q_k + \overbrace{(A - B K_{inf})^T p_{k+1} - (A - B K_{inf})^T P_{inf} B d_k}^{\text{expand}} + \overbrace{K_{inf}^T R d_k - K_{inf}^T r_k}^{\text{expand}}$$

rearrange terms and group d_k

$$p_k = q_k + C_2 p_{k+1} - K_{inf}^T r_k + \underbrace{(K_{inf}^T R - (A - B K_{inf})^T P_{inf} B)}_{} d_k$$

focus on these terms for a bit

$$K_{inf}^T R - (A - BK_{inf})^T P_{inf} B$$

$$= K_{inf}^T R - A^T P_{inf} B + K_{inf}^T B^T P_{inf} B$$

$$= K_{inf}^T (R + B^T P_{inf} B) - A^T P_{inf} B$$

From initial equations,

$$K_{inf} = (R + B^T P_{inf} B)^{-1} (B^T P_{inf} A)$$

So we have

$$= ((R + B^T P_{inf} B)^{-1} (B^T P_{inf} A))^T (R + B^T P_{inf} B) - A^T P_{inf} B$$

$$= (A^T P_{inf}^T B)((R + B^T P_{inf} B)^{-1})^T (R + B^T P_{inf} B) - A^T P_{inf} B$$

$P_{inf}^T = P_{inf}$, so $B^T P_{inf} B$ is symmetric $((B^T P_{inf} B)^T = B^T P_{inf} B)$

and $\overbrace{R^T = R}^{\text{assumption}}$, so $R + B^T P_{inf} B$ and its inverse are symmetric (assuming the inverse exists) and we get

$$= (A^T P_{inf}^T B)(R + B^T P_{inf} B)^{-1} (R + B^T P_{inf} B) - A^T P_{inf} B$$

$$= (A^T P_{inf} B) I - A^T P_{inf} B = 0$$

Plugging this back in to the equation for p_k, we get

$$p_k = q_k + C_2 p_{k+1} - K_{inf}^T r_k + (0) d_k$$

$$p_k = q_k + C_2 p_{k+1} - K_{inf}^T r_k$$