Conditional Risk Classification

For deciding class i

$$R(\alpha_i|x) = \sum_j \lambda(\alpha_i|\omega_j) P(\omega_j|x)$$

$$= 0 \cdot P(\omega_i|x) + \sum_{j \neq i} \lambda_s \cdot P(\omega_j|x)$$

$$= \lambda_s [1 - P(\omega_i|x)]$$

For Rejection

$$R(\alpha_{c+1}|x) = \sum_j \lambda_r P(\omega_j|x) = \lambda_r$$

Minimum Risk Decision

Choose class i if

- $R(\alpha_i|x) < R(\alpha_j|x)$ for all $j \neq i$
- $R(\alpha_i|x) < R(\alpha_{c+1}|x)$

Condition 1: From $R(\alpha_i|x) < R(\alpha_j|x)$

$$\lambda_s [1 - P(\omega_i|x)] < \lambda_s [1 - P(\omega_j|x)]$$

$\Rightarrow P(\omega_i|x) > P(\omega_j|x)$ for all $j \neq i$

Condition 2: From $R(a_i|x) < R(a_{c+1}|x)$

$$\lambda_s[1 - P(w_i|x)] < \lambda_r$$
$$\lambda_s - \lambda_s P(w_i|x) < \lambda_r$$
$$\lambda_s P(w_i|x) > \lambda_s - \lambda_r$$

$\Rightarrow P(w_i|x) > 1 - \lambda_r / \lambda_s$

Decision Rule

Decide class i if

1. $P(w_i|x) \geq P(w_j|x)$ for all j (i has max posterior)
2. $P(w_i|x) \geq 1 - \lambda_r/\lambda_s$ Posterior exceeds threshold

Reject otherwise (when max posterior $<$ threshold)

Special Cases

Case 1: $\lambda_r = 0$ (Rejection is free)

Threshold: $P(w_i|x) \geq 1 - 0/\lambda_s = 1$

Only classify if $P(w_i|x) = 1$, else reject

Case 2: $\lambda_R > \lambda_s$ (Rejection costs more than error)

Threshold: $P(w_i|x) \geq 1 - \lambda_r/\lambda_s < 0$

Since probabilities are always non-negative, this condition always satisfied

Result: Never reject