Dicke Hamiltonian, Reduced Basis

Jz, J+, J- Construction

In[422]:=

```
Jz[K_Integer] := Module[{J, dim, Jz},
  J = K / 2;
  dim = 2 J + 1;
  Jz = SparseArray[DiagonalMatrix[Reverse[Range[-J, J]]]];
  Return[Jz];
 ]

Jp[K_Integer] := Module[{J, dim, Jplus, mValues, i, j},
  J = K / 2;
  dim = 2 J + 1;
  Jplus = SparseArray[{}, {dim, dim}];
  mValues = Reverse[Range[-J, J]];
  For[i = 1, i < dim, i++,
   Jplus[[i, i + 1]] = Sqrt[J * (J + 1) - mValues[[i + 1]] * (mValues[[i + 1]] + 1)];
  ];
  Return[SparseArray[Jplus]];
 ]

Jm[K_Integer] := Module[{J, dim, Jplus, mValues, i, j},
  J = K / 2;
  dim = 2 J + 1;
  Jplus = SparseArray[{}, {dim, dim}];
  mValues = Reverse[Range[-J, J]];
  For[i = 2, i ≤ dim, i++,
   Jplus[[i, i - 1]] = Sqrt[J * (J + 1) - mValues[[i]] * (mValues[[i]] + 1)];
  ];
  Return[Jplus];
 ]
```

Complete Construction

In[425]:=

```
(*Constants*)
bsize = 60; ω0 = 1.0; ωc = 1.0; j = 0.07; K = 3;
(*Identity matrix for QHO*)
idHO = SparseArray[IdentityMatrix[bsize]];
idTLS = SparseArray[IdentityMatrix[K + 1]];


(*QHO Hamiltonian*)
H0HO = ωc * SparseArray[Band[{1, 1}] → Table[n + 1/2, {n, 0, bsize - 1}]];
(*Combined TLS Hamiltonian*)
HTLS = ω0 * Jz[K];
(*Annihilation operator definition*)
a = SparseArray[Band[{1, 2}] → Table[Sqrt[n], {n, 1, bsize - 1}], {bsize, bsize}];

Hindep = KroneckerProduct[HTLS, idHO] + KroneckerProduct[idTLS, H0HO];
Hcoup = j * (KroneckerProduct[Jp[K], a] + KroneckerProduct[Jm[K], a†] +
      KroneckerProduct[Jp[K], a†] + KroneckerProduct[Jm[K], a]);
Htot = Hindep + Hcoup;
```

Initial States, Observables Construction

Initial States

In[434]:=

```
(*QHO*)
ψ0HO = SparseArray[{1 → 1.0}, bsize];
(*TLS*)
ψ0TLS = SparseArray[{2 → 1.0}, K + 1];
Print[ψ0TLS // MatrixForm];
ψ0vec = KroneckerProduct[ψ0TLS, ψ0HO] // Flatten;
Print[Norm[ψ0vec]];
```

$$\begin{pmatrix} 0 \\ 1. \\ 0 \\ 0 \end{pmatrix}$$

1.

Observable Matrices

Oscillator Position

In[439]:=

```
xM = KroneckerProduct[IdentityMatrix[K + 1], 1/Sqrt[2] (a† + a)];
ConjugateTranspose[ψ0vec].xM.ψ0vec
```

Out[440]=

```
0.
```

Propagation

Calculating States

In[441]:=

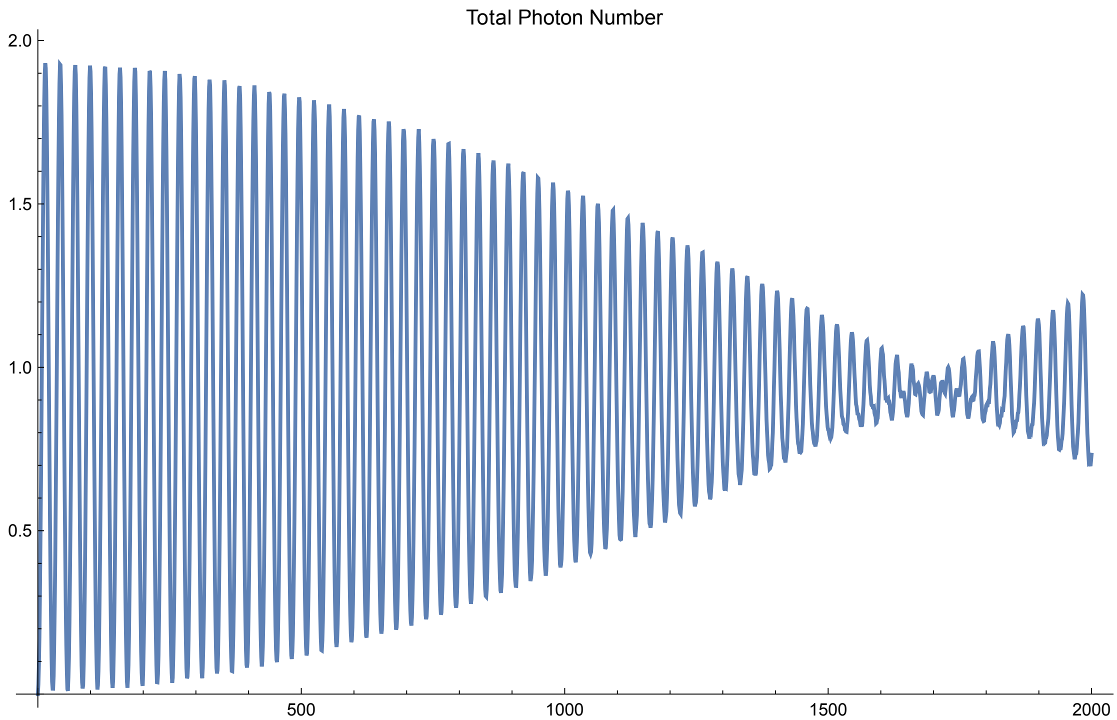
```
stateVector[t_] := MatrixExp[-I * Htot * t, ψ0vec];
tMax = 2000;
tRange = Range[0, tMax, 1];
ψs = ParallelTable[stateVector[t], {t, tRange}];
```

Photon Number Expectation in Cavity

In[445]:=

```
aDaggerA = KroneckerProduct[IdentityMatrix[K + 1], a†.a];
aDaggerAsr = aDaggerA.aDaggerA;
photons = Table[Conjugate[ψs[[n]]].aDaggerA.ψs[[n]], {n, Length@tRange}];
ListLinePlot[{tRange, photons // Re} // Transpose,
 PlotRange → All, PlotLabel → "Total Photon Number", ImageSize → Large]
```

Out[448]=

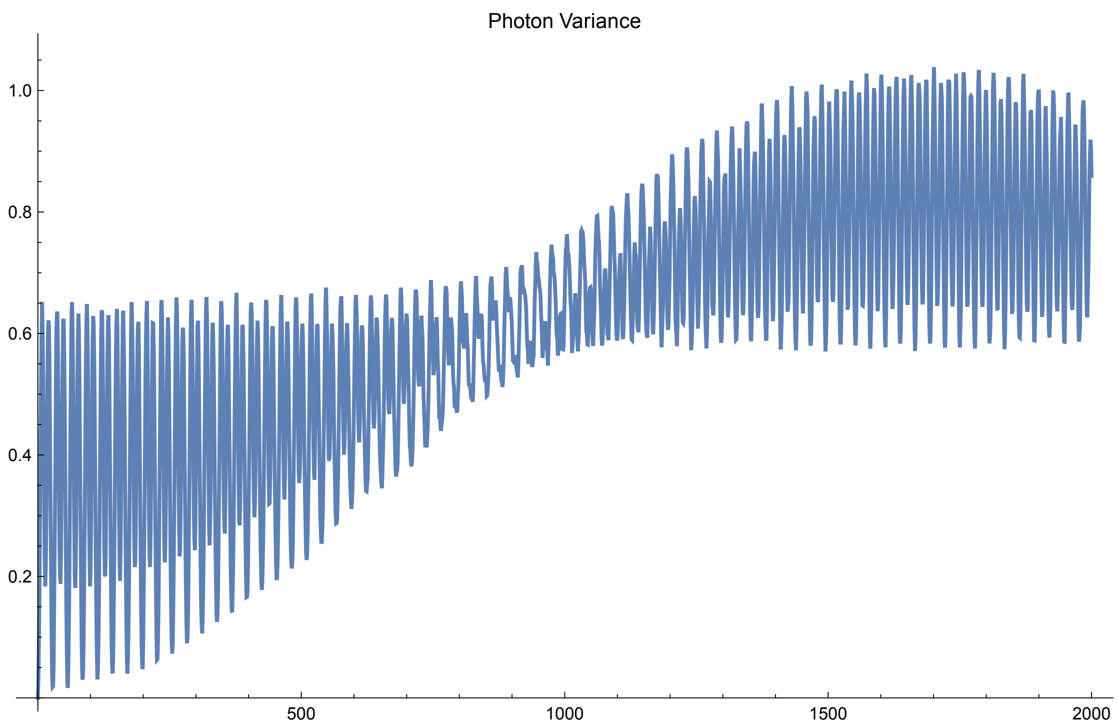


Photon Statistics, Variance

In[449]:=

```
newPhotons =
  Table[Conjugate[ψs[[n]]].aDaggerAsr.ψs[[n]], {n, Length@tRange}] - photons^2;
ListLinePlot[{tRange, newPhotons // Re} // Transpose,
 PlotRange → All, PlotLabel → "Photon Variance", ImageSize → Large]
```

Out[450]=



Excitation Spectrum

In[451]:=

```
eigv = Eigenvalues[N[Htot]];
ListLinePlot[{Sort[eigv]}, PlotRange → All, ImageSize → Large]
```

Eigenvalues : Because finding 240 out of the 240 eigenvalues and /or eigenvectors is likely to be faster with dense matrix methods, the sparse input matrix will be converted. If fewer eigenvalues and /or eigenvectors would be sufficient, consider restricting this number using the second argument to Eigenvalues.

Out[452]=