Solutions to the JC Model Assuming Photon Conservation

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We start with the JC Hamiltonian and a general state, assuming a constant number of photons in the system. The JC Hamiltonian is given by

$$\omega_c a^\dagger a + \omega_m \sigma_z + j(\sigma^+ a + \sigma^- a^\dagger) \tag{1}$$

And a general wavefunction in this TSS-cavity system with n photons is given by

$$|\psi\rangle = F(t) \cdot |e, n-1\rangle + G(t) \cdot |g, n\rangle \tag{2}$$

Where the coefficients represent the time evolution of the wavefunction. We apply the Hamiltonian to the general state, which yields, starting with the first superposition term

$$(\omega_c a^\dagger a + \omega_m \sigma_z + j(\sigma^+ a + \sigma^- a^\dagger)) \cdot F(t) \cdot |e, n-1\rangle \tag{3}$$

$$= F(t) \cdot \omega_c(n-1) |e, n-1\rangle + F(t) \cdot \omega_m |e, n-1\rangle + F(t) \cdot j\sqrt{n} |g, n\rangle \tag{4}$$

Similarly, for the second superposition term,

$$(\omega_c a^\dagger a + \omega_m \sigma_z + j(\sigma^+ a + \sigma^- a^\dagger)) \cdot G(t) \cdot |g, n\rangle \tag{5}$$

$$= G(t) \cdot \omega_c n |g, n\rangle - G(t) \cdot \omega_m |g, n\rangle + G(t) \cdot j\sqrt{n} |e, n-1\rangle \tag{6}$$

If we take the matrix representation of $|\psi\rangle$ to be

$$\begin{bmatrix} F(t) \\ G(t) \end{bmatrix} \tag{7}$$

Then we can rewrite the combined sum of the two bits from above as

$$\begin{bmatrix} \omega_c(n-1) + \omega_m & j\sqrt{n} \\ j\sqrt{n} & \omega_c n - \omega_m \end{bmatrix} \begin{bmatrix} F(t) \\ G(t) \end{bmatrix} \tag{8}$$

And so the matrix representation of the Hamiltonian is simply

$$\begin{bmatrix} \omega_c(n-1) + \omega_m & j\sqrt{n} \\ j\sqrt{n} & \omega_c n - \omega_m \end{bmatrix} \tag{9}$$