Equivalence of Density Matrix and Wavefunction Formalisms for Pure States

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Let $\rho = |\psi\rangle\langle\psi|$. Then, we have

$$i\hbar\frac{\partial}{\partial t}(|\psi\rangle\langle\psi|) \tag{1}$$

$$= i\hbar\left(\frac{\partial}{\partial t}|\psi\rangle\langle\psi| + |\psi\rangle\frac{\partial}{\partial t}\langle\psi|\right) \tag{2}$$

We note the following from Schrodinger's equation

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle \tag{3}$$

$$\frac{\partial}{\partial t}|\psi\rangle = \frac{1}{i\hbar}\hat{H}|\psi\rangle \tag{4}$$

$$\frac{\partial}{\partial t}\langle\psi| = -\frac{1}{i\hbar}\langle\psi|\hat{H} \tag{5}$$

Then, substituting back into equation (3), we get

$$= i\hbar\left(\frac{1}{i\hbar}\hat{H}|\psi\rangle\langle\psi| - \frac{1}{i\hbar}|\psi\rangle\langle\psi|\hat{H}\right) \tag{6}$$

$$= \hat{H}|\psi\rangle\langle\psi| - |\psi\rangle\langle\psi|\hat{H} \tag{7}$$

$$= [H, |\psi\rangle\langle\psi|] \tag{8}$$

$$= [H, \rho] \tag{9}$$

And we are done.