Dicke Hamiltonian, Reduced Basis

Jz, J+, J- Construction

In[38]:=
```
Jz[K_Integer] := Module[{J, dim, Jz},
   J = K / 2;
   dim = 2 J + 1;
   Jz = SparseArray[DiagonalMatrix[Reverse[Range[-J, J]]]];
   Return[Jz];
  ]

Jp[K_Integer] := Module[{J, dim, Jplus, mValues, i, j},
   J = K / 2;
   dim = 2 J + 1;
   Jplus = SparseArray[{}, {dim, dim}];
   mValues = Reverse[Range[-J, J]];
   For[i = 1, i < dim, i++,
    Jplus⟦i, i + 1⟧ = Sqrt[J * (J + 1) - mValues⟦i + 1⟧ * (mValues⟦i + 1⟧ + 1)];
   ];
   Return[SparseArray[Jplus]];
  ]

Jm[K_Integer] := Module[{J, dim, Jplus, mValues, i, j},
   J = K / 2;
   dim = 2 J + 1;
   Jplus = SparseArray[{}, {dim, dim}];
   mValues = Reverse[Range[-J, J]];
   For[i = 2, i ≤ dim, i++,
    Jplus⟦i, i - 1⟧ = Sqrt[J * (J + 1) - mValues⟦i⟧ * (mValues⟦i⟧ + 1)];
   ];
   Return[Jplus];
  ]
```

Complete Construction

In[107]:=

```
(*Constants*)
bsize = 70; ω0 = 1.0; ωc = 1.0; j = 0.07; K = 4;
(*Identity matrix for QHO*)
idHO = SparseArray[IdentityMatrix[bsize]];
idTLS = SparseArray[IdentityMatrix[K + 1]];


(*QHO Hamiltonian*)
H0HO = ωc * SparseArray[Band[{1, 1}] → Table[n + 1/2, {n, 0, bsize - 1}]];
(*Combined TLS Hamiltonian*)
HTLS = ω0 * Jz[K];
(*Annihilation operator definition*)
a = SparseArray[Band[{1, 2}] → Table[Sqrt[n], {n, 1, bsize - 1}], {bsize, bsize}];

Hindep = KroneckerProduct[HTLS, idHO] + KroneckerProduct[idTLS, H0HO];
Hcoup = j * (KroneckerProduct[Jp[K], a] + KroneckerProduct[Jm[K], a†] +
      KroneckerProduct[Jp[K], a†] + KroneckerProduct[Jm[K], a]);
Htot = Hindep + Hcoup;
```

Initial States, Observables Construction

Initial States

In[116]:=

```
(*QHO*)
ψ0HO = SparseArray[{1 → 1.0}, bsize];
(*TLS*)
ψ0TLS = SparseArray[{K - 1 → 1.0}, K + 1]; (*Second Excitation Manifold*)
Print[ψ0TLS // MatrixForm];
ψ0vec = KroneckerProduct[ψ0TLS, ψ0HO] // Flatten;
Print[Norm[ψ0vec]];
```

$$\begin{pmatrix} 0 \\ 0 \\ 1. \\ 0 \\ 0 \end{pmatrix}$$

1.

Observable Matrices

Oscillator Position

In[121]:=

```
xM = KroneckerProduct[IdentityMatrix[K + 1], 1/Sqrt[2] (a† + a)];
ConjugateTranspose[ψ0vec].xM.ψ0vec
```

Out[122]=

```
0.
```

Propagation

Calculating States

In[123]:=

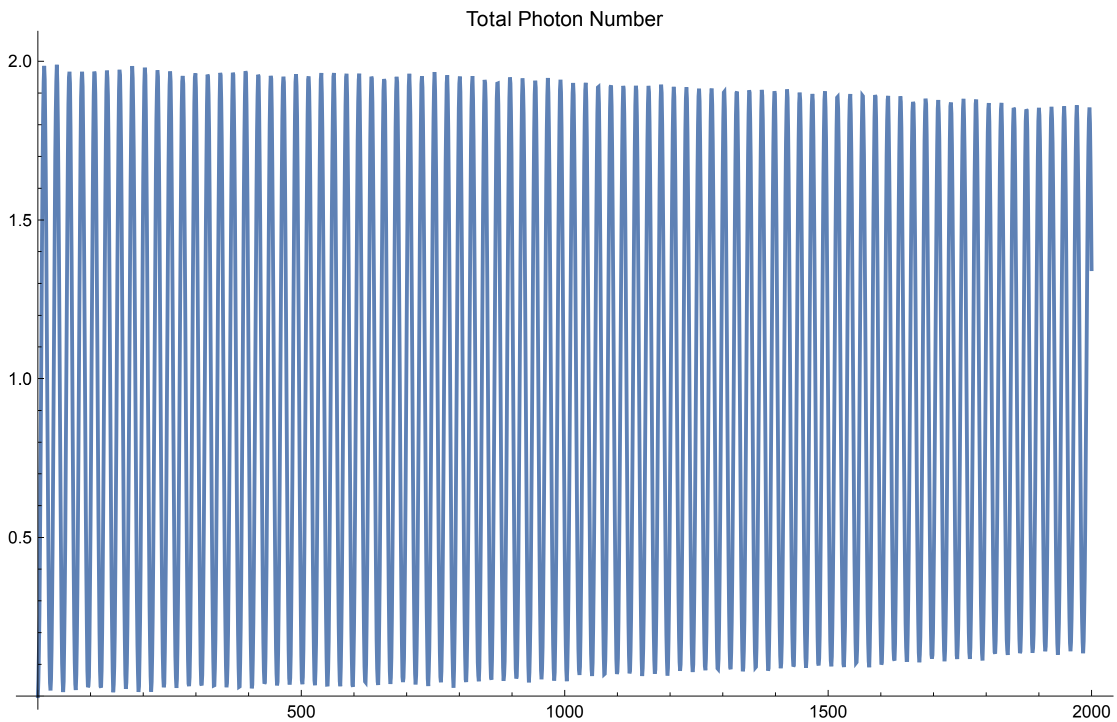
```
stateVector[t_] := MatrixExp[-I * Htot * t, ψ0vec];
tMax = 2000;
tRange = Range[0, tMax, 1];
ψs = ParallelTable[stateVector[t], {t, tRange}];
```

Photon Number Expectation in Cavity

In[127]:=

```
aDaggerA = KroneckerProduct[IdentityMatrix[K + 1], a†.a];
aDaggerAsr = aDaggerA.aDaggerA;
photons = Table[Conjugate[ψs⟦n⟧].aDaggerA.ψs⟦n⟧, {n, Length@tRange}];
ListLinePlot[{tRange, photons // Re} // Transpose,
 PlotRange → All, PlotLabel → "Total Photon Number", ImageSize → Large]
```

Out[130]=

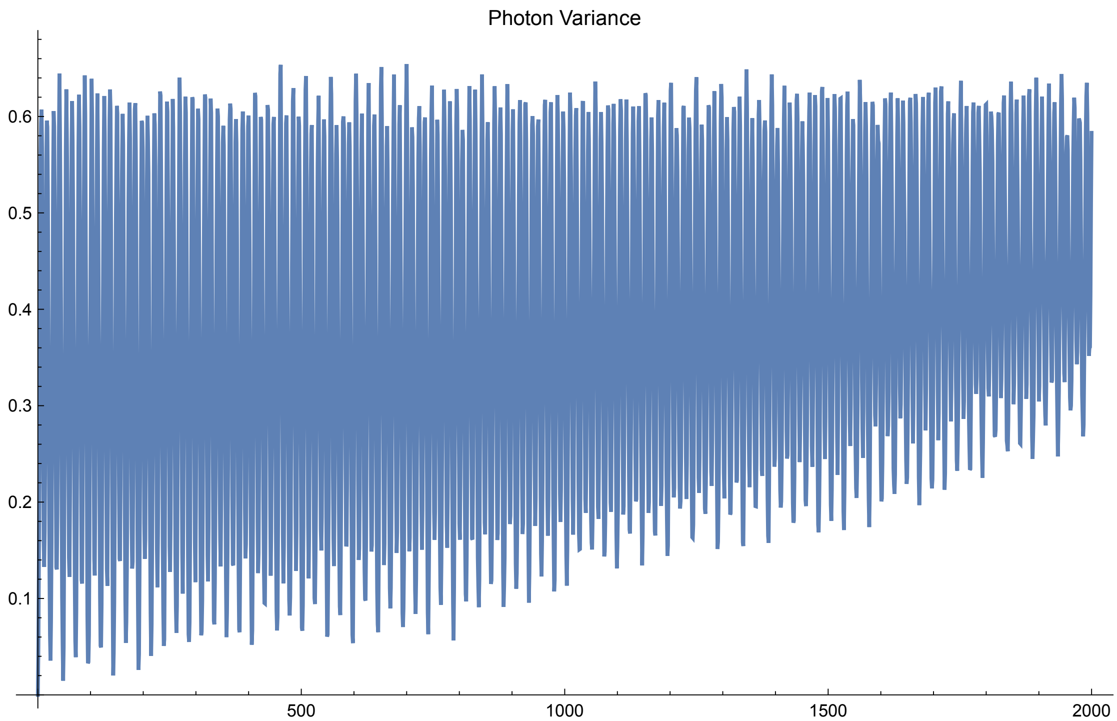


Photon Statistics, Variance

In[131]:=

```
newPhotons =
  Table[Conjugate[ψs[[n]]].aDaggerAsr.ψs[[n]], {n, Length@tRange}] - photons^2;
ListLinePlot[{tRange, newPhotons // Re} // Transpose,
 PlotRange → All, PlotLabel → "Photon Variance", ImageSize → Large]
```

Out[132]=



Excitation Spectrum

In[133]:=

```
eigv = Eigenvalues[N[Htot]];
ListLinePlot[{Sort[eigv]}, PlotRange → All, ImageSize → Large]
```

Eigenvalues : Because finding 350 out of the 350 eigenvalues and /or eigenvectors is likely to be faster with dense matrix methods, the sparse input matrix will be converted. If fewer eigenvalues and /or eigenvectors would be sufficient, consider restricting this number using the second argument to Eigenvalues.

Out[134]=