# Assignment -1: Initial Value Problem

Write computer programs to solve the following problems:

1. Consider the initial value problem:

$$\frac{dx}{dt} = -(1 + t + t^2) - (2t + 1)x - x^2$$

where, $0 \le t \le 3$ and $x(0) = -1/2$. The exact solution of this problem is $x(t) = -t - \frac{1}{e^t+1}$

(a) Compare the convergence using Euler's method with respect to the exact solution
(b) Repeat (a) by changing $x(0) = -1$. The exact solution in this case is $x(t) = -t - 1$

2. Consider the population model as given below

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{x^2}{1 + x^2}$$

The first term on the right hand side is known as the logistic growth term. This term results from the assumption that for small population levels, the population will grow at a rate proportional to the current level, while, for large population levels, limited resources will cause the growth rate to decrease and eventually become negative. The parameters $r$ and $k$ are called the natural growth rate of the population and the environmental carrying capacity, respectively. The second term on the right hand side represents the harvesting/predation of the species by some other species (ex, fish being caught by fishermen or insects being eaten by birds).

(a) For $r = 0.4$ and $k = 20$, use Euler's method to determine the eventual population level reached from an initial population of 2.44
(b) Repeat (a) if the initial condition is changed to 2.40

3. Suppose a plate of AISI 304 stainless steel is suspended in a furnace of temperature $T_F = 1500$K. The plate has a thickness of $d = 0.002$m, a density of $\rho = 7900$ kg/m$^3$ and initial temperature $T(0) = 300$K. The specific heat has a relation of $C_P = 0.162T + 446.47$ J/kg-K. The heating rate of this plate can be expressed as

$$\frac{dT}{dt} = \frac{2\sigma}{\rho d} \frac{T_F^4 - T^4}{C_P + T\frac{dC_P}{dT}}$$

Find how long does it take to reach thermal equilibrium. Use RK-4 method to solve this problem.

4. Consider the initial value problem:

$$\frac{dx}{dt} = 7t^2 - \frac{4x}{t}$$

Where, $1 \le t \le 6$ and $x(1) = 2$. The exact solution of this problem is $x(t) = t^3 + 1/t^4$. Compare the rate of convergence between RK-2 and RK-4 and report the error percentage at $t = 6$ from both these methods.

5. Consider the initial value problem

$$\frac{dx}{dt} = \frac{tx^2 - x}{t}$$

Where, $1 \leq t \leq 5$ and $x(1) = -\frac{1}{\ln(2)}$. The exact solution is $x(t) = -\frac{1}{t\ln(2t)}$. Compare the rate of convergence between RK-2 and Adam Bashforth Moulton 2nd order schemes.

6. Consider the initial value problem

$$\frac{dx}{dt} = \frac{4}{t}x + t^4 e^t$$

Where, $1 \leq t \leq 2$ and $x(1) = 0$. The exact solution is $x(t) = t^4(e^t - e)$. Compare the rate of convergence between RK-4 and Adam Bashforth Moulton 4th order schemes.

7. Consider the initial value problem of this 3rd order ODE

$$\frac{d^3x}{dt^3} + \frac{1}{2}x\frac{d^2x}{dt^2} = 0$$

$x(t)$ has a periodic nature. Simulate the first 4 cycles of this cycles by assuming a step size, $h = 0.25$. The initial conditions are, $x(0) = 0$, $x'(0) = 0$ and $x''(0) = 1$.