

Engin 150 Project 4: Modelling and Simulation of 3D Food Printing

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1 Introduction

In this project, the system being modelled is the extrusion of a two-phase food slurry. One phase is interstitial fluid that serves as the base, while the other phase comprises small functionalising particles that are suspended in the fluid. As we note in Figure 1 below, this slurry is suspended in a binder. Depending on the volume concentrations and intrinsic properties of the materials themselves, these food slurries can be highly viscous, which can make the printing process difficult, especially when a small nozzle is being used. This project analyses how printing the slurry with different processing parameters impacts the pressure gradient and nozzle wear rate in our system.

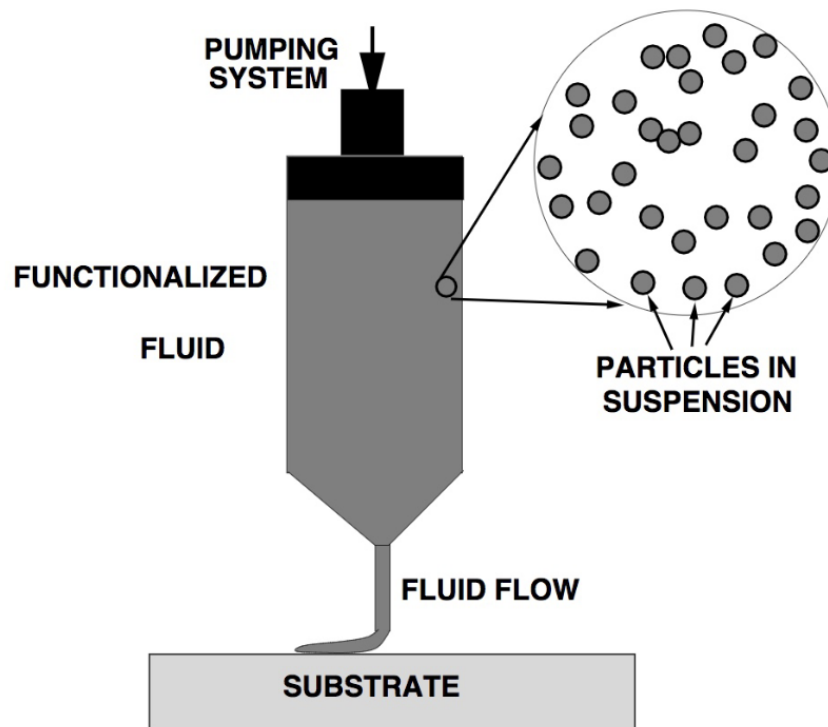


Figure 1: Printing Schematic. Source: Project Spec

2 Background and Theory

Before proceeding further, relevant modelling assumptions, equations, and terminology must be clarified. First and foremost, we use a quasi-1D flow model to model the flow of the food slurry. A quasi-static model is equivalent to saying that the acceleration is neglected ie. we neglect or leave out the inertial forces required to get the food slurry flowing, and instead focus on the viscous forces that must be counteracted and accounted for in order for the flow to continue. To this end, we begin by deriving a model for the pressure gradient to maintain a given flow rate in a channel. This pressure gradient is expressed in terms of the volume fraction of particles in the slurry v_p and the temperature of the slurry θ . The idealised channel can be thought of as a cylinder, having a circular cross-section $A = \pi R^2$, with radius R . For radial coordinate from channel center line r , the channel has velocity profile $v(r) = v_{max}(1 - (\frac{r}{R})^q)$, where v_{max} is the center line velocity. q is a parameter related to how turbulent the flow is. For fully developed laminar flow, $q = 2$, while higher values correspond to more turbulent flow. While flow rate Q itself is a function of time, total flow rate Q_0 is treated as a known constant.

$$Q = \int_A v dA = Q_0$$

Note: For this project, $Q_0 = 10^{-6} m^3/s$

2.1 Effective Properties

$$\begin{aligned}\mu(f) &= \mu_{f_0} e^{-k_1 \frac{\theta(t) - \theta_0}{\theta_0}} \\ \sigma(f) &= \sigma_{f_0} e^{-k_2 \frac{\theta(t) - \theta_0}{\theta_0}} \\ \sigma(p) &= \sigma_{p_0} e^{-k_3 \frac{\theta(t) - \theta_0}{\theta_0}}\end{aligned}$$

where the k_i are constants, and a subset of p means particle and a subset of f means fluid. Note that $v_f = 1 - v_p$

Effective density $\rho^* = v_f \rho_f + v_p \rho_p$ Effective heat capacity $C^* = C_f \rho_f + C_p \rho_p$ Effective viscosity $\mu^* = \mu_f (1 + 2.5 \frac{v_p}{1 + v_p})$ This equation holds as bulk and shear moduli of the particles tend to inf Effective conductivity σ^* is approximated by $0.5\sigma^{*, -} + 0.5\sigma^{*, +}$ in this project. Refer to Hashin-Strikman bounds for further info

2.2 Shear Stress

The pressure exerted by the flow of the slurry creates shear stress in the fluid, given by

$$\tau(r) = \mu^* \frac{\partial v}{\partial r} = -\frac{\mu^* v_{max} q}{R} \left(\frac{r}{R}\right)^{q-1}$$

where μ^* is effective viscosity, which is a function of v_p . We also note that $v_{max} = \frac{Q_0(q+2)}{Aq}$. Substituting this in our given equation, we get $\tau_w = -\tau(r=R) = \frac{\mu^* Q_0(q+2)}{\pi R^3}$. By inspecting our formula for τ_w , we see that it is proportional to q (the more turbulent the flow, the greater the wall shear stress), linearly proportional to Q_0 and μ^* (doubling the total total flow rate OR effective viscosity would also double the wall shear stress) and has an inverse cubic relationship with the radius R (halving the radius would multiply wall shear stress by a factor of 8). Thus we note that while a variety of different factors affect wall shear stress, the most important is R

2.3 Pressure Gradient

To simplify the remainder of the analysis, further assumptions are made: the particles are mixed uniformly throughout the slurry and are not elongated, thereby allowing us to calculate viscosity using equations that show up later in this report. Since we're interested in maintaining steady axial flow, we set acceleration to 0 and solve the balance of forces equation, noting that $F_{net} = \sum F = 0$

Note: The following equations hold when we consider a uniformly cylindrical region of the pipe where the fluid is flowing in the direction of the circular area of cross-section. We consider an arbitrary Δx , representing a differential length along the axis of the pipe. We have downward pressure $P + \Delta P$ acting on area $A = \pi R^2$, which is counteracted by pressure P in the upward direction acting on the same area A , and by the shear stress τ_w . Since we are considering a cylinder, the wall is the total curved surface area $= 2\pi R \Delta x$. We ignore perpendicular forces since we're concerned with balance of forces only in the direction of flow. Remembering that $F = P * A$ and from balance of forces, we get

$$(-(P + \Delta P) + P)\pi R^2 - \tau_w 2\pi R \Delta x = 0$$

Substituting our value for τ_w , and rearranging this, we get $-\frac{\Delta P}{\Delta x} = CQ_0$, where $C = \frac{2\mu^*(q+2)}{\pi R^4}$. As with our shear stress, the pressure gradient is proportional to q (measure of turbulence), linearly proportional to Q_0 (total flow rate) and μ^* (effective viscosity). Once again, the most important factor is radius R , but this time there is a quartic inversely proportional relationship. As a final observation, the negative sign on the left shows us that the slurry flows from region of high pressure to low pressure

2.4 Velocity Profile Characteristics

The velocity profile q is expressed as a function of the slurry's Reynolds number

$$q = q(R_{ec}) + c_1 R_{ec} + c_2$$

where c_1 c_2 are constants We can rearrange this using earlier equations to get

$$q^2 - (\gamma^* + c_2)q - 2\gamma^* = 0$$

where $\gamma^* = \frac{2c_1 Q \rho^*}{\pi R \mu^*}$. It is clear that q is essential to all our governing equations. Since we have 2 possible values for q , as given by $q(R_{ec}) = \frac{1}{2}((\gamma^* + c_2) \pm \sqrt{(\gamma^* + c_2)^2 + 8\gamma^*})$, we must pick one. A negative value of q makes no physical sense, since for $q \leq 0$, $v(0) = \inf$ or undefined. Hence we must choose the positive root. Focusing on γ^* , which is simply a combination of other variables and constants, we note that our value of q is proportional to flow rate Q and effective density ρ^* , while it is inversely proportional to R and μ^* . Unlike previous terms, the exact mathematical relation between the variables and q isn't clear, but the general trends have been established

2.5 Joule Heating

Joule heating is defined as "the process of pumping an electrical current through a mass to directly heat it." Here, we make another simplifying assumption: as per the project spec "The fluid is heated uniformly (imagine that the flow is moving through an infinitely long pipe and that the heating current is uniform

everywhere and started simultaneously).” As a result of this assumption, we treat the current solely as a function of time, rather than one of time and position. A more accurate function would account for factors like the relative radial coordinate of a point in the binder, and also the dimensions of the binder itself. Thus, position and dimensions are some of the factors that get left out. On the topic of heating, because we assume heat sources other than Joule heating are negligible, we are able to easily get an expression for $J(t)$ from the First Law of Thermodynamics, and we also assume that the only nontrivial mode of thermal loss to the environment is in the form of convection, giving us a relatively straightforward expression for $S(t)$ as well. For the context of this project, we have a fixed temperature increase rate $\dot{\theta}$ of 50 K/s. From the First Law of Thermodynamics, we have $\rho^*C^*\dot{\theta} = H - S$, where the terms are effective density, effective specific heat capacity, rate of temperature change, heating power and all of the heat losses, respectively. Since we assume we have only Joule heating, $H = a\frac{J^2}{\sigma^*}$, where a is the absorption coefficient and σ^* is effective conductivity. By discretising the First Law of Thermodynamics and solving for the temperature, we get

$$\theta(t + \Delta(t)) = \theta(t) + \frac{\Delta t}{\rho^*C^*}(H - S) = \theta(t) + \frac{\Delta t}{\rho^*C^*}\left(a\frac{J^2}{\sigma^*} - S(t)\right)$$

Solving for $J(t)$, we get

$$J(t) = \sqrt{\frac{\sigma^*}{a}(\rho^*C^*\dot{\theta} + S(t))}$$

The only term that is currently unknown/remains to be calculated is $S(t)$. As established earlier, this accounts for all the thermal losses, which we assume is equivalent to all the convective losses. Hence, $S(t) = hA(\theta(t) - \theta_a)$, where h is the convection coefficient, A is area over which convection occurs, $\theta(t)$ is temperature at time t and θ_a is ambient temperature. To keep our equations consistent, we express this in a per unit basis, by dividing by volume. For some length Δx of the pipe, area $A = \pi R^2$, while volume is $\pi R^2 \Delta x$. As such, dividing by volume yields

$$S(t) = \frac{h(\theta(t) - \theta_a)}{R}$$

Note that the heat equation scales appropriately.

3 Results and Discussion

3.1 Part 1

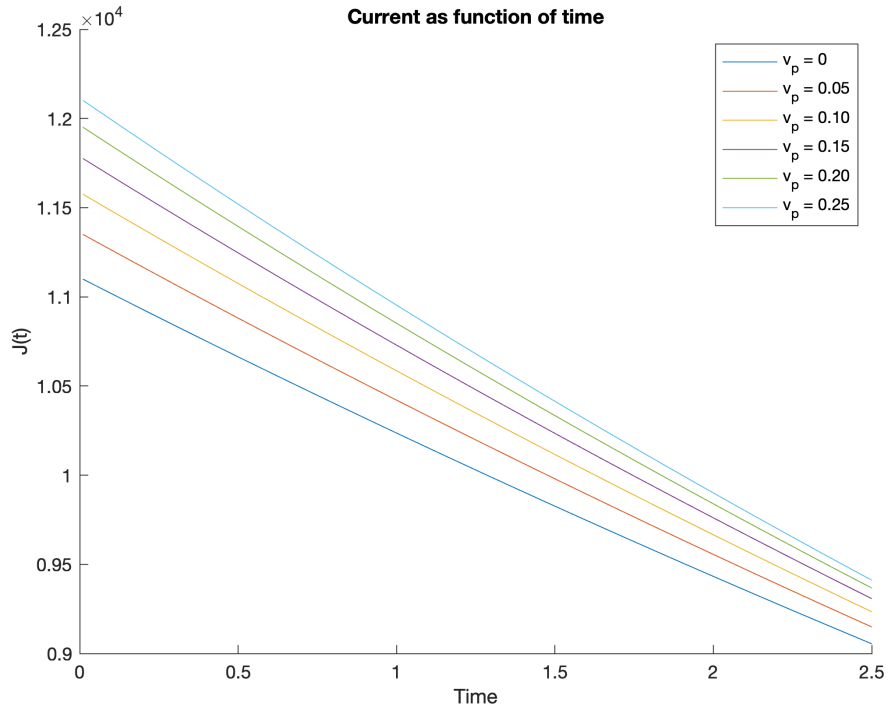


Figure 2: $J(t)$ vs. t

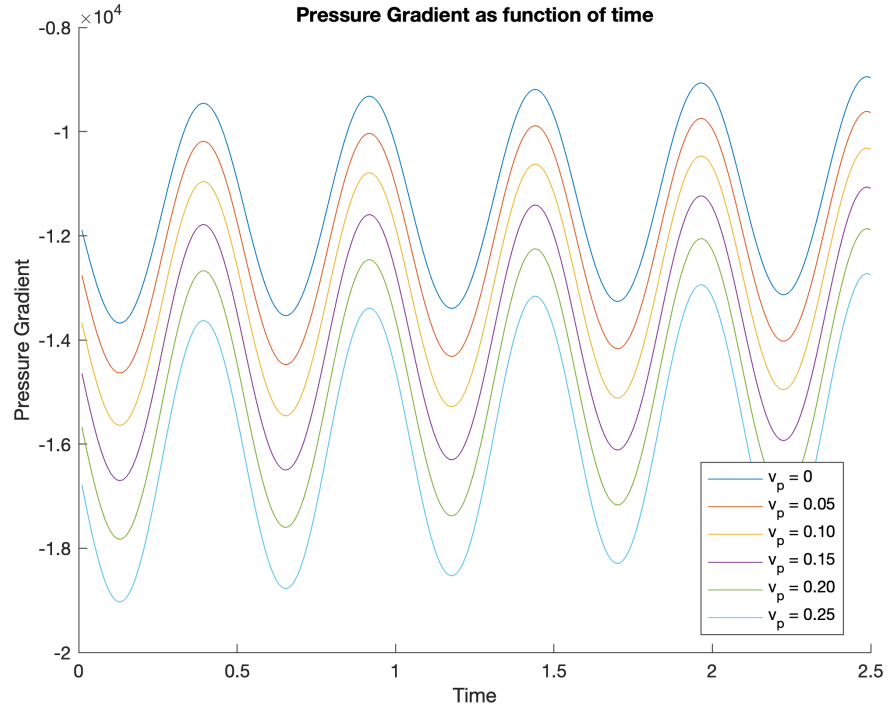


Figure 3: $\frac{\Delta P}{\Delta x}$ vs. t

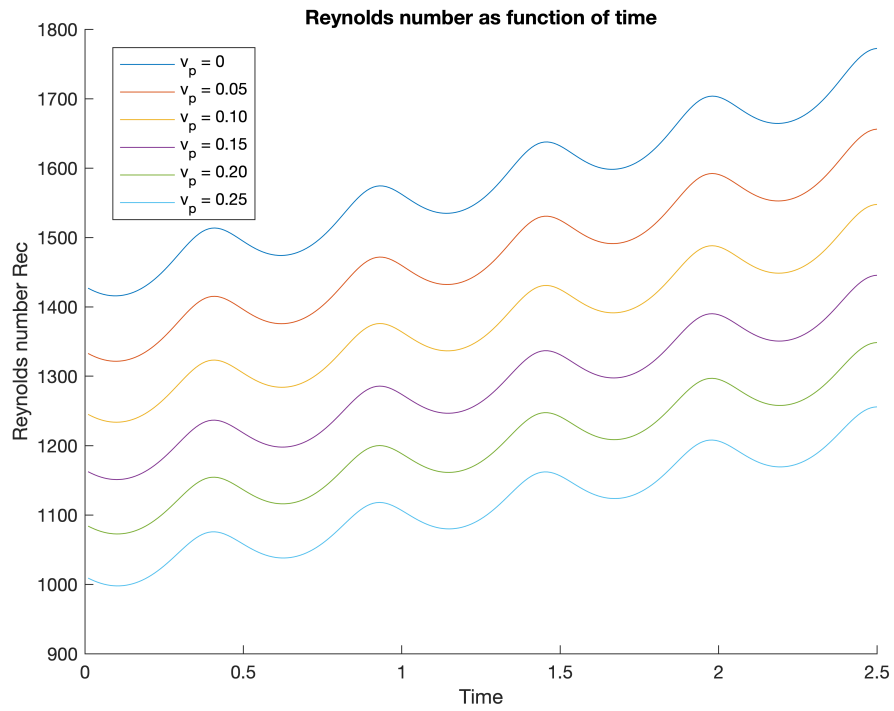


Figure 4: Rec vs. t

3.2 Part 2

We solve for steady state radius R_{SS} . We know that $\frac{dR}{dt} = \nu * \max(\frac{\mu^* Q_0(q+2)}{\pi \tau^{crit} R^3} - 1, 0)$. For steady state, derivative must be 0, hence we solve for the case when the first term is 0, yielding

$$R_{SS} = (\frac{\mu^* Q_0(q+2)}{\pi \tau^{crit}})^{1/3}$$

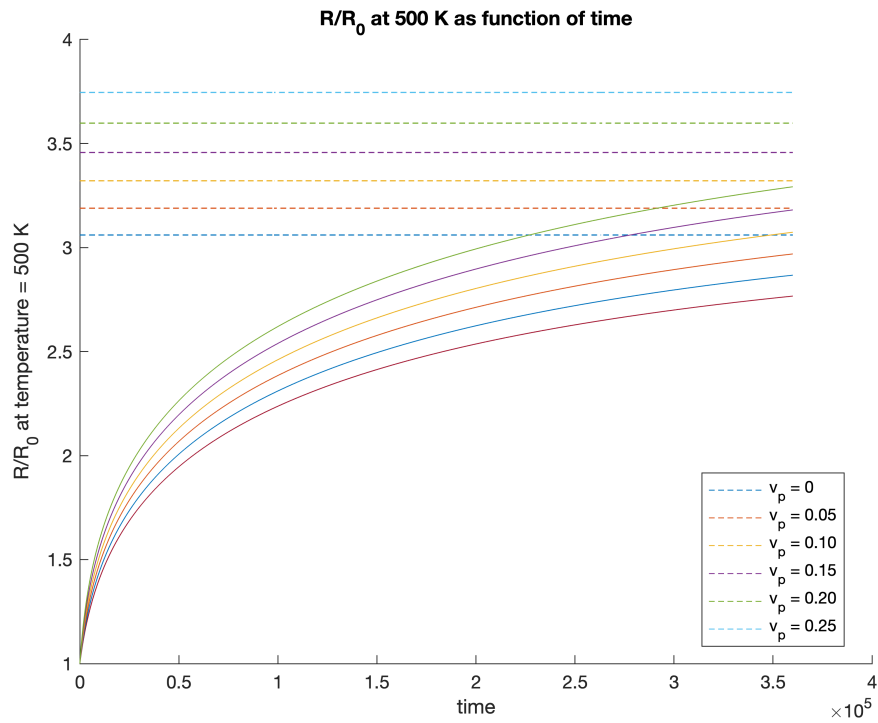


Figure 5: $\frac{R}{R_0}$ vs. t at 500K

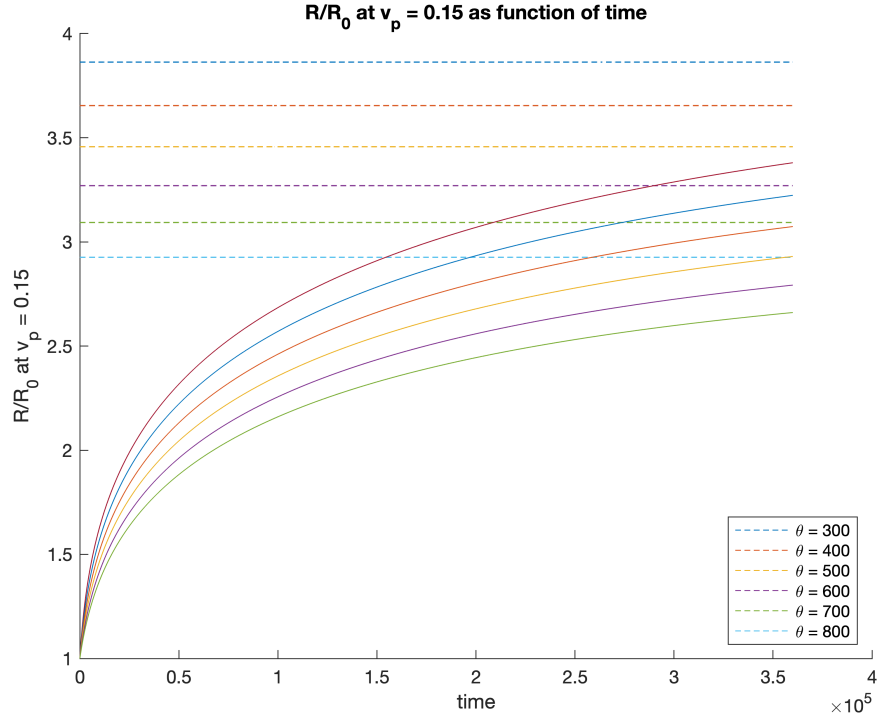


Figure 6: $\frac{R}{R_0}$ vs. t at $v_p = 0.15$

We note that for a given volume fraction, there seems to be a greater variation in normalised radius for various temperatures than there is in the case when normalised radii are plotted for various volume fractions for a given temperature

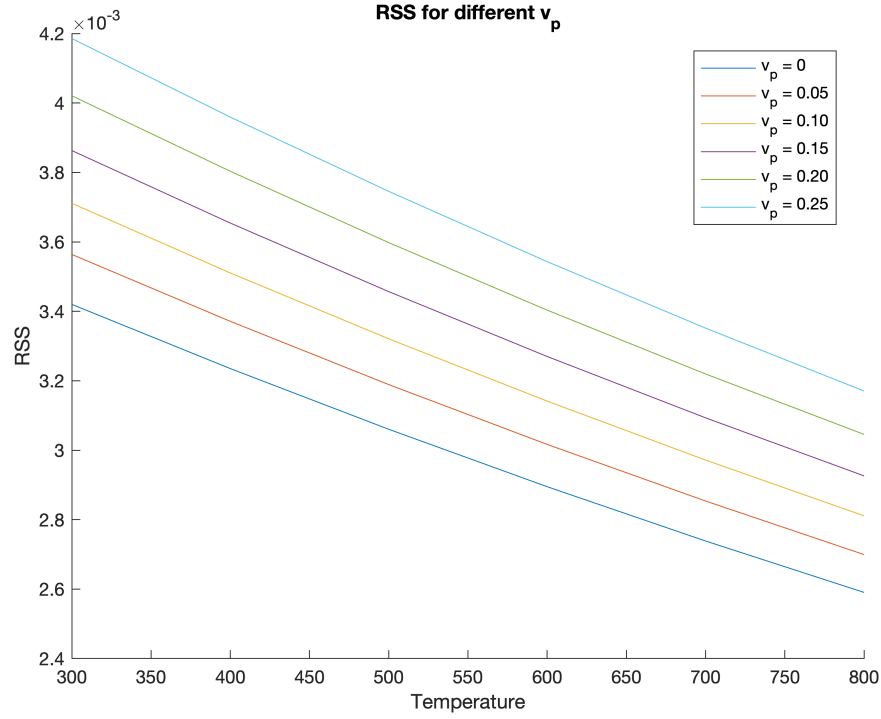


Figure 7: $R_S S$ vs. $Temperature$

I wasn't able to include a table of percentage differences between $R(T)$ and R_{SS} , but I would like to clarify that lower percentages would mean that combination of particle volume and temperature is decaying faster than the rest, approaching steady state more rapidly