

# Problem Set 1A

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## 1 Instructions for Submission

The **regular deadline** for this problem set is **Thursday, October 3, at 11:59pm**. The **late deadline** for this problem set is **Friday, October 4, at 11:59pm**. Please upload a PDF or image of your work to Canvas. Legible handwriting or typing is fine. Please submit **just your answers, and not the problem set document**.

## 2 How to solve/use this problem set

The required questions are designed to help you develop and sharpen the problem-solving tools from class. The practice questions are intended to help you gain more practice with the building blocks from lecture.

Some questions are graded on effort. These require a credible, reasonable attempt for credit. Any such attempt will receive full credit. Other questions are graded on correctness. **Questions that are graded on correctness are indicated as such in bold.**

## 3 Required Problems

1. **(Graded on correctness) (25 points)** Given a bit string (a string of 1's and 0's)  $w$ , the *parity* of  $w$  is defined to be 1 if  $w$  has an odd number of 1's, and 0 if  $w$  has an even number of 1's. Give a recursive definition for the parity of a bit string  $w$ , with  $\varepsilon$  (the empty string) as the base case. (See the lecture notes on string length for inspiration.)
  - $\varepsilon$  has parity 0, since it has an even number of 1's (0 of them):  $\text{parity}(\varepsilon) = 0$ .
  - If  $w \in L$  and has parity  $p$ , then  $w0$  has parity  $p$ , since number of 1's has not changed, so  $\text{parity}(w0) = \text{parity}(w) = p$ .
  - If  $w \in L$  and has parity  $p$ , then  $w1$  has parity  $1 - p$ , since the number of 1's has changed by 1, so  $\text{parity}(w1) = 1 - \text{parity}(w) = 1 - p$ .

2. Give a recursive definition for each of the following languages:

- (a) **(Graded on effort) (10 points)** The set of all strings of parentheses over the alphabet  $\Sigma = \{ (, ) \}$  that are balanced.

- The empty string  $\varepsilon$  is balanced and so  $\varepsilon \in L$ .
- If  $x \in L$  and  $y \in L$ , where  $x$  and  $y$  are balanced strings, then  $(x)y \in L$ .

- (b) **(Graded on effort) (15 points)** The set of all bit strings over the alphabet  $\Sigma = \{0, 1\}$  **not** containing the string 11.

- $\varepsilon \in L, 0 \in L, 1 \in L$
- If  $w \in L$ , then  $0w \in L$  and  $10w \in L$

3. Give a mathematical set-builder definition for each of the following languages. Set-builder notation means, for example, an answer such as

$$L = \{a^{3k} \mid k \geq 0\}$$

or

$$L = \{(aaa)^k \mid k \geq 0\}.$$

**Do not use a description in words.**

- (a) **(Graded on effort) (10 points)** The set of all strings over the alphabet  $\Sigma = \{a\}$  whose length is equal to a multiple of 3 plus a multiple of 5.

$$L = \{a^{3k+5l} \mid k, l \geq 0\}$$

- (b) **(Graded on correctness) (15 points)** The set of all strings over the alphabet  $\Sigma = \{a, b\}$  with exactly twice as many  $b$ 's as  $a$ 's, and with no occurrences of the substring  $ba$ .

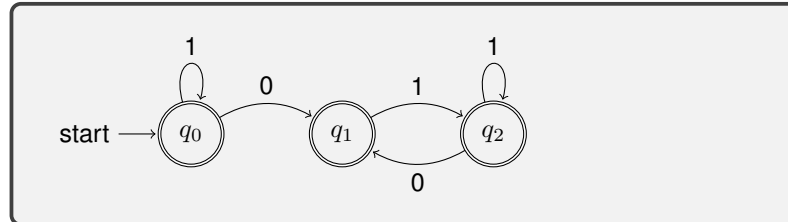
$$L = \{a^k b^{2k} \mid k \geq 0\}$$

4. Prove the following languages are regular by providing a DFA that recognizes each:

- (a) **(Graded on effort) (10 points)**

$$L_1 = \{w \in \Sigma^* \mid w \text{ does not contain the substring } 00\}$$

where  $\Sigma = \{0, 1\}$ .



(b) (Graded on correctness)(15 points)

$L_2 = \{w \in \Sigma^* \mid w \text{ contains at least two 0's and does not contain the substring } 00\}$

where  $\Sigma = \{0, 1\}$ .

Note that  $L_2 \subseteq L_1$  i.e.  $L_2$  is a subset of  $L_1$ .

