Problem Set 1A

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Cal Poly CSC 445, Fall 2024

1 Instructions for Submission

The **regular deadline** for this problem set is **Thursday, October 3, at 11:59pm.** The **late deadline** for this problem set is **Friday, October 4, at 11:59pm.** Please upload a PDF or image of your work to Canvas. Legible handwriting or typing is fine. Please submit **just your answers, and not the problem set document.**

2 How to solve/use this problem set

The required questions are designed to help you develop and sharpen the problem-solving tools from class. The practice questions are intended to help you gain more practice with the building blocks from lecture.

Some questions are graded on effort. These require a credible, reasonable attempt for credit. Any such attempt will receive full credit. Other questions are graded on correctness. **Questions that are graded on correctness are indicated as such in bold.**

3 Required Problems

1. **(Graded on correctness) (25 points)** Given a bit string (a string of 1's and 0's) w, the *parity* of w is defined to be 1 if w has an odd number of 1's, and 0 if w has an even number of 1's. Give a recursive definition for the parity of a bit string w, with ε (the empty string) as the base case. (See the lecture notes on string length for inspiration.)

    - ε has parity 0, since it has an even number of 1's (0 of them): $parity(\varepsilon) = 0$.
    - If $w \in L$ and has parity p, then $w0$ has parity p, since number of 1's has not changed, so $parity(w0) = parity(w) = p$.
    - If $w \in L$ and has parity p, then $w1$ has parity $1 - p$, since the number of 1's has changed by 1, so $parity(w1) = 1 - parity(w) = 1 - p$.

2. Give a recursive definition for each of the following languages:

   (a) **(Graded on effort) (10 points)** The set of all strings of parentheses over the alphabet $\Sigma = \{(,)\}$ that are balanced.

   - The empty string ε is balanced and so $\varepsilon \in L$.
   - If $x \in L$ and $y \in L$, where x and y are balanced strings, then $(x)y \in L$.

   (b) **(Graded on effort) (15 points)** The set of all bit strings over the alphabet $\Sigma = \{0, 1\}$ **not** containing the string 11.

   - $\varepsilon \in L$, 0 $\in L$, 1 $\in L$
   - If $w \in L$, then $0w \in L$ and $10w \in L$

3. Give a mathematical set-builder definition for each of the following languages. Set-builder notation means, for example, an answer such as

$$L = \{a^{3k} \mid k \geq 0\}$$

or

$$L = \{(aaa)^k \mid k \geq 0\}.$$

Do not use a description in words.

   (a) **(Graded on effort) (10 points)** The set of all strings over the alphabet $\Sigma = \{a\}$ whose length is equal to a multiple of 3 plus a multiple of 5.

$$L = \{a^{3k+5l} \mid k, l \geq 0\}$$

   (b) **(Graded on correctness) (15 points)** The set of all strings over the alphabet $\Sigma = \{a, b\}$ with exactly twice as many b's as a's, and with no occurrences of the substring ba.

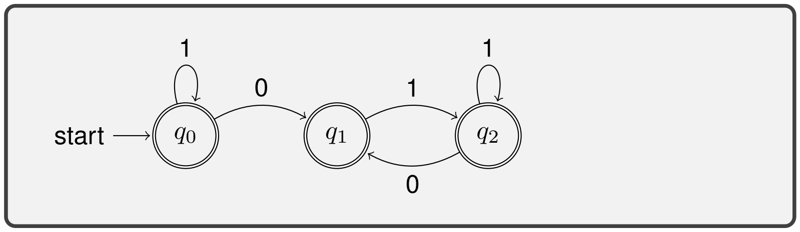
$$L = \{a^k b^{2k} \mid k \geq 0\}$$

4. Prove the following languages are regular by providing a DFA that recognizes each:

   (a) **(Graded on effort) (10 points)**

$$L_1 = \{w \in \Sigma^* \mid w \text{ does not contain the substring } 00\}$$

where $\Sigma = \{0, 1\}$.



(b) **(Graded on correctness)(15 points)**

$L_2 = \{w \in \Sigma^* \mid w$ contains at least two 0's and does not contain the substring $00\}$

where $\Sigma = \{0, 1\}$.
Note that $L_2 \subseteq L_1$ i.e. L_2 is a subset of L_1.