Problem Set 1B: Regular Languages

Daniel Frishberg

Cal Poly CSC 445, Fall 2024

1 Instructions for Submission

The **regular deadline** for this problem set is **Monday, April 29, at 11:59pm.** The **late deadline** for this problem set is **Tuesday, April 30, at 11:59pm.** Please upload a PDF or image of your work to Canvas. Legible handwriting or typing is fine. Please submit **just your answers, and not the problem set document**.

2 How to solve/use this problem set

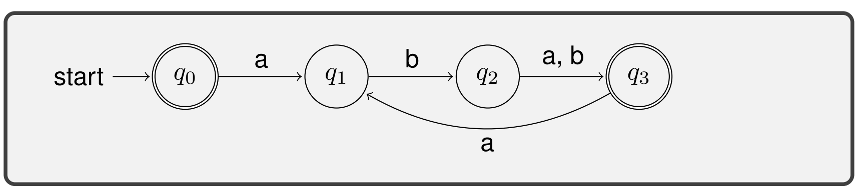
Some questions are graded on effort. These require a credible, reasonable attempt for credit. Any such attempt will receive full credit. Other questions are graded on correctness. **Questions that are graded on correctness are indicated as such in bold.**

3 Required Problems

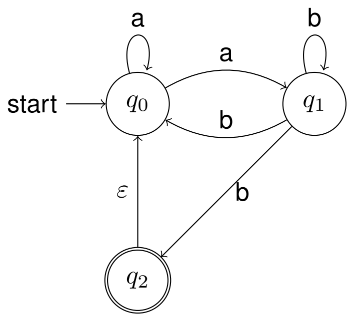
1. **(Graded on correctness) (20 points)** Construct an NFA that recognizes the language defined by the regular expression

$$(ab(a \cup b))^*$$

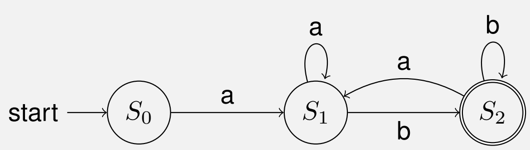
Use the regex-to-NFA conversion procedure from the "Equivalences" lecture in class. You just need to draw the final NFA obtained via this procedure, with all resulting states and transitions.



2. **(Graded on correctness) (20 points)** Consider the following NFA.



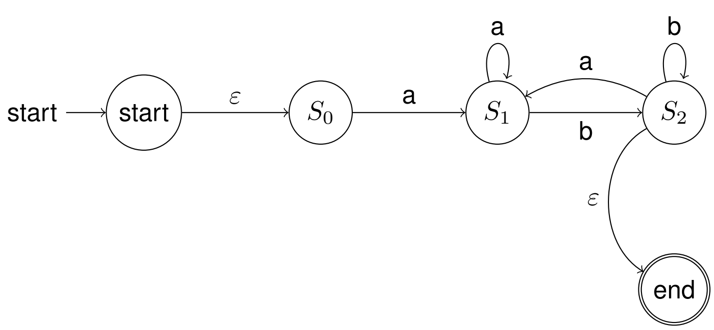
Let L be the language this NFA recognizes. Construct a DFA that recognizes L, **using the procedure shown in the "Equivalences" lecture.** You can just draw the final DFA, but **do label the states in the DFA as subsets of the states in the NFA. You do not have to draw unreachable or dead states.**



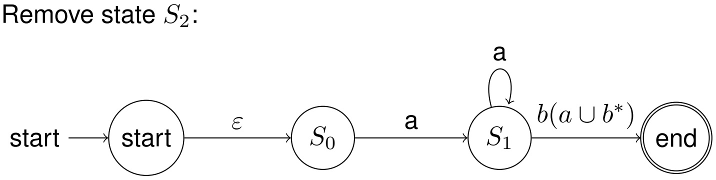
Where $S_0 = \{q_0\}$, $S_1 = \{q_0, q_1\}$, and $S_2 = \{q_0, q_1, q_2\}$.

3. **(Graded on effort) (20 points)** Convert the DFA you obtained in your answer to Question 2 to a regular expression using the procedure shown in the lecture "Equivalences". **Show each intermediate machine in the procedure.**

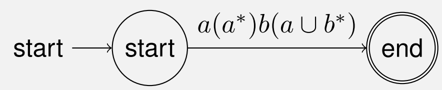
Add start and end states:



Remove state S_2:



Remove states S_0 and S_1:



Final regular expression: $a(a^*)b(a \cup b^*)$.

4. **(Graded on correctness) (20 points)** Let $L_1 = \{a^k b^l \mid k \geq l \geq 0\}$. Prove that the langauge L is not regular using the Pumping Lemma.

Suppose L_1 is regular. Let p be the pumping length given by the Pumping Lemma. Consider the string $w = a^p b^p$. Since $w \in L_1$ and $|w| \geq p$, the Pumping Lemma guarantees that w can be split into three parts, $w = xyz$, such that:

(a) $|xy| \leq p$

(b) $|y| > 0$

(c) For all $i \geq 0$, $xy^i z \in L_1$

Since $|xy| \leq p$, y consists of only a's. Let $w' = xy^0 z = xz$. Then w' has more b's than a's, so $w' \notin L_1$. This contradicts the Pumping Lemma, so L_1 is not regular.

5. **(Graded on correctness) (20 points)** Let $L = \{w \mid$ the number of a's in w is $\geq$ the number of b's $\}$. Use the fact that L_1 (from part (4)) is not regular, along with closure properties, to prove that L is not regular.

Suppose L is regular.
Let $L_1 = \{a^k b^l \mid k \geq l \geq 0\}$, from part (4).
Let $L_2 = a^* b^*$ and is regular.
$L \cap L_2 = L_1$ is regular by closure properties.
By closure properties, L_1 is regular.
However, we know that L_1 is not regular (contradiction).
Therefore, L is not regular.