EE208 EXPERIMENT 10

DYNAMIC STUDIES OF A NONLINEAR MECHANICAL SYSTEM ON SIMULINK

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OBJECTIVE

Dynamic studies of a given crane trolley system on Simulink, using a detailed nonlinear state space system simulation in four state variables.

PROJECT STATEMENT

The following differential equations represent a simplified model of an overhead crane:

$$[m_L + m_C]\cdot \ddot{x}_1(t) + m_L l \cdot \left[\ddot{x}_3(t)\cdot \cos x_3(t) - \dot{x}_3^2(t)\cdot \sin x_3(t)\right] = u(t)$$

$$m_L \cdot [\ddot{x}_1(t)\cdot \cos x_3(t) + l\cdot \ddot{x}_3(t)] = -m_L g \cdot \sin x_3(t)$$

The model will involve certain parameters that are constant, some that are constant but adjustable, and some that are constant and arbitrary.

The parameters are –

Parameters	Description	Value	Remarks
m_C	Mass of trolley	10 kg	Constant
m_L	Mass of hook and load	10 kg (hook) + load from 0 kg to several hundred kg	Constant for a particular crane operation
l	Length of rope	1 m or longer	Constant for a particular crane operation
g	Acc. due to gravity	9.8 m/s^2	Constant

Variables for the problem include –

Variables	Description	Type	Units
$\boldsymbol{u}$	Force applied to the trolley	Input	N
$\boldsymbol{y = x_1 + l * sin(x_3)}$	Position of load	Output	m
$\boldsymbol{x_1}$	Position of trolley	State	m
$\boldsymbol{x_2}$	Speed of trolley	State	m/s
$\boldsymbol{x_3}$	Rope angle	State	rad
$\boldsymbol{x_4}$	Angular speed of rope	State	rad/s

For the nonlinear system as given above –

- Create a complete four-state, single output simulation model for the crane using nonlinear simulation blocks in Simulink.
- Check how each parameter can be programmed for individual dynamic studies.
- The simulation evolved should be for the complete nonlinear set of equations, and should not be confused with the linearised studies undertaken earlier.

For observations and discussions –

- Using step, impulse, and ramp inputs for u(t), study the dynamics of the crane from one steady-state to another, using traces of all four state variables as well as the single output variable.
- Accordingly, conclude the best input for the nonlinear system to work with.
- Repeat the studies for different values of load mass that the crane is lifting. Discuss thoroughly if the load mass makes any difference to system dynamics.
- Discussions should refer to the results from Expt. #8, and as and when useful, may be discussed in view of the earlier studies.
- Include the detailed patched block diagram for the overall system from the Simulink main screen, including a brief write-up on the simulation, involving notes and comments.

THE SIMULINK MODEL

Using MATLAB, we can write the given pair of coupled nonlinear ODEs in general state-space form –

$$\dot{\boldsymbol{x}} = \begin{pmatrix} x_2 \\ \dfrac{l\, m_L \sin(x_3)\, {x_4}^2 + u + g m_L \cos(x_3) \sin(x_3)}{m_L \sin^2(x_3) + m_C} \\ x_4 \\ \dfrac{l\, m_L \cos(x_3) \sin(x_3)\, {x_4}^2 + u \cos(x_3) + g(m_C + m_L) \sin(x_3)}{(-l) \times (m_L \sin^2(x_3) + m_C)} \end{pmatrix}$$

$$y = x_1 + l \sin(x_3)$$

Note – The RHS in the first equation will be referred to as the *system* $\boldsymbol{f}$ *matrix*.

This can be implemented in Simulink with integrators and function blocks.

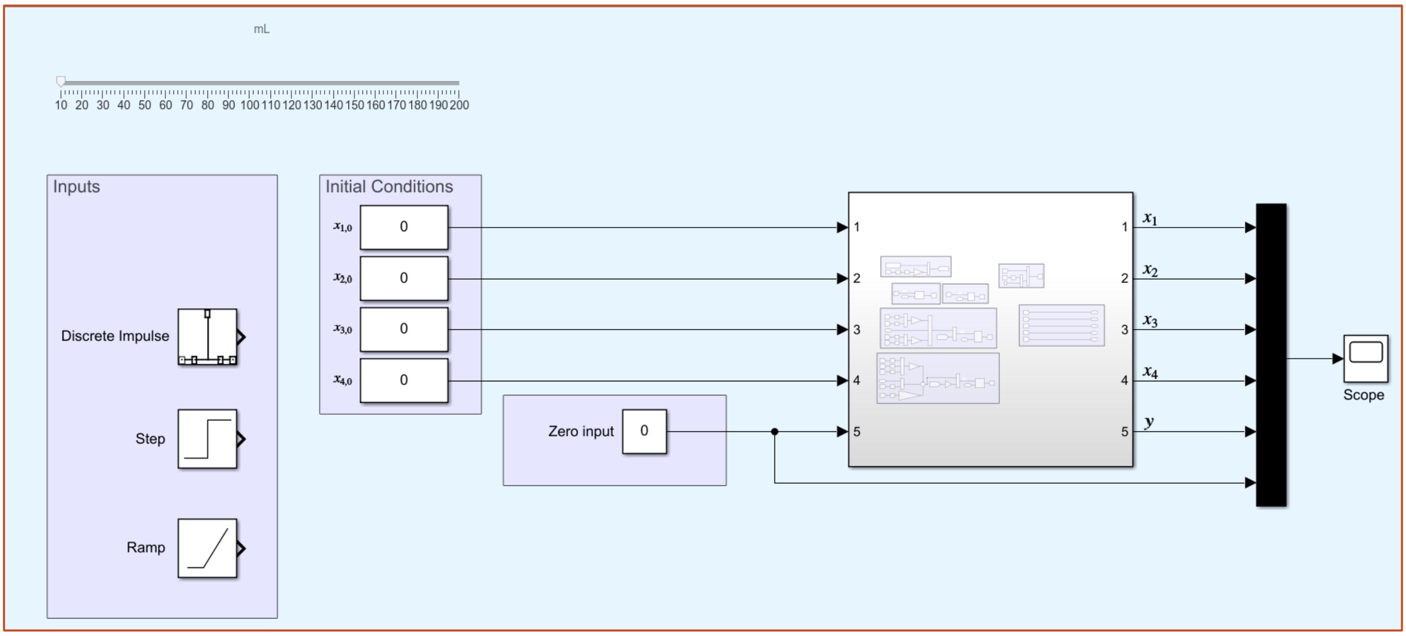
OVERALL MODEL

The complete model is shown below.

- We have put the system of differential equations inside a subsystem for convenience.
- Initial conditions and input signal can be fed to the subsystem as inputs, and states obtained as outputs which can be observed with the Scope block.
- The parameter m_L can be changed using the slider block, this will be required when we wish to examine the effect of loading conditions.
- The nominal parameter values are –
  - $m_C = 10$ kg, constant
  - $g = 9.81\ \mathrm{m/s^2}$, constant
  - $l = 2$ m, constant but changeable if required
  - $m_L = 20$ kg, can be changed using the slider block
- The parameters are stored inside the Model Workspace –

Name	Value	DataType	Dimensions	Complexity
L	1	double (auto)	[1 1]	real
g	9.8	double (auto)	[1 1]	real
mC	10	double (auto)	[1 1]	real
mL	20	double (auto)	[1 1]	real

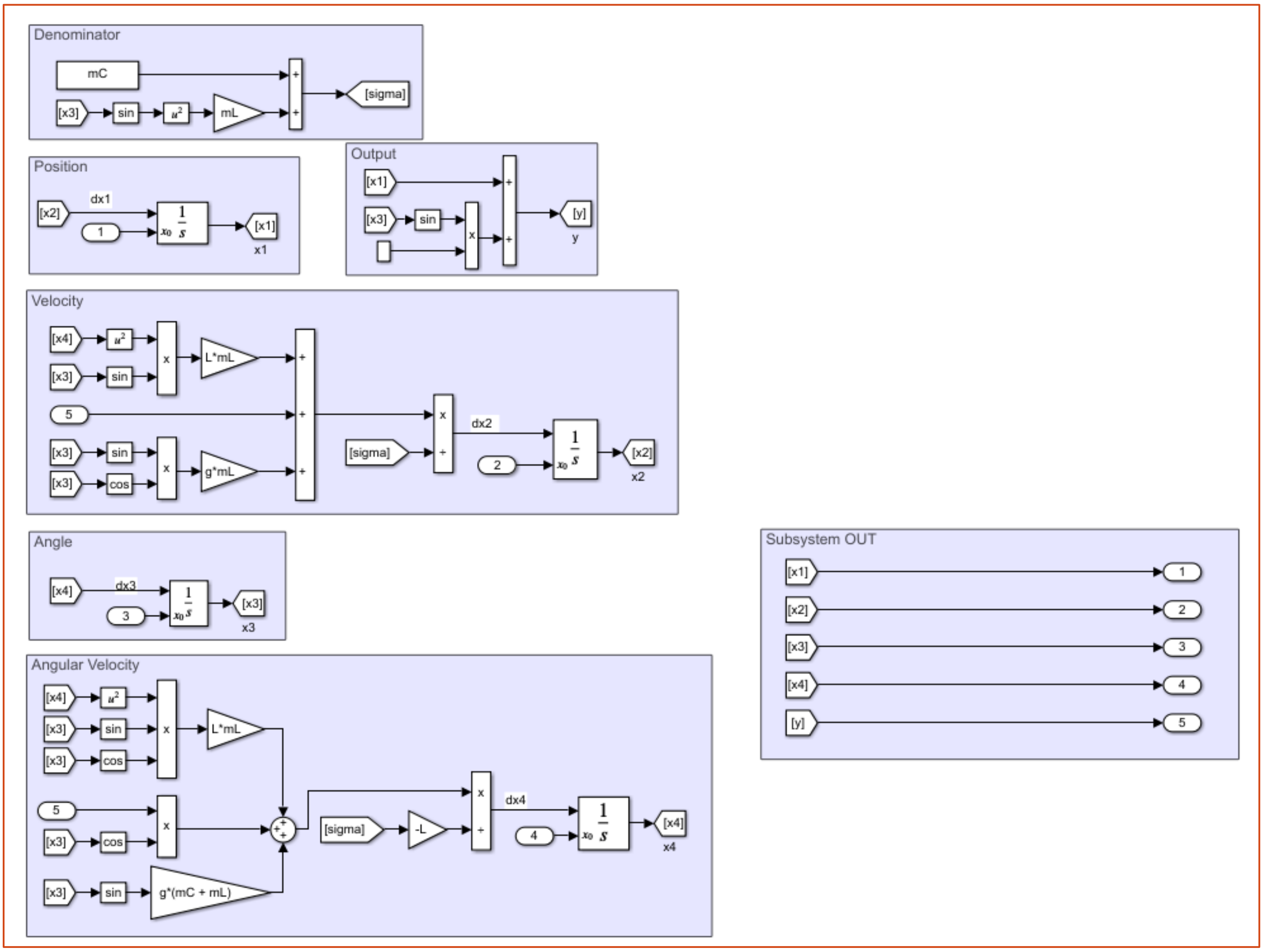
- Different inputs can be given to the system as desired – impulse, step, ramp.
- The maximum step size is 0.0001s, and the solver used is ode23.


mL
10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
Inputs
Discrete Impulse
Step
Ramp
Initial Conditions
$x_{1,0}$
$x_{2,0}$
$x_{3,0}$
$x_{4,0}$
0
0
0
0
Zero input
0
x_1
x_2
x_3
x_4
y
Scope

NONLINEAR SYSTEM MODEL

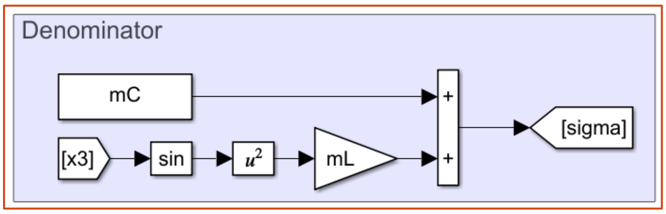
The nonlinear system model involves implementing the equations in Simulink using function blocks and integrators.

The complete subsystem is –



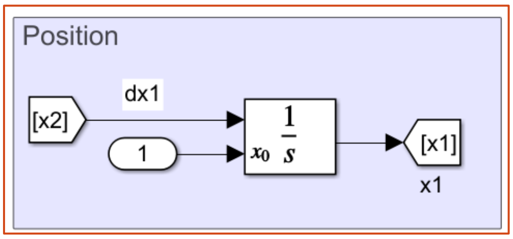
We shall take a look at each segment individually. We have used Goto-From blocks for clarity, due to the large number of terms involved in the equations.

DENOMINATOR



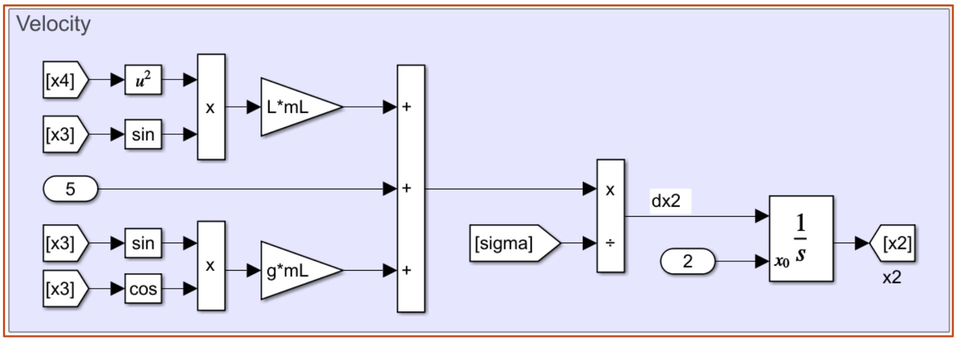
This implements the denominator term common to the 2nd and 4th rows in the $\boldsymbol{f}$ matrix.

x_1 : POSITION



The state x_1 is simply the integral of the x_2, as can be seen from the $\boldsymbol{f}$ matrix. The integrator block also has a functionality to set the initial state, which is fed as an input to the subsystem.

x_2 : VELOCITY



As before, we can see that the initial state (Input-2 of the subsystem to be specific) is given as an input to the integrator. The parameters L, g, etc. are stored in the Model Workspace.

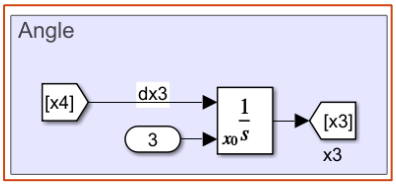
The benefit of the Goto-From blocks can be seen here in making the block diagram cleaner visually.

The block diagram can be understood better as follows –

- Each of the inputs to the Sum block is a *force* on the Crane's body.
    - The first and third terms are contributed by the motion of the mass on the rope (basically a pendulum), such as centripetal force and weight of the mass.
    - The second term is simply the input u, the horizontal force being applied to the crane.

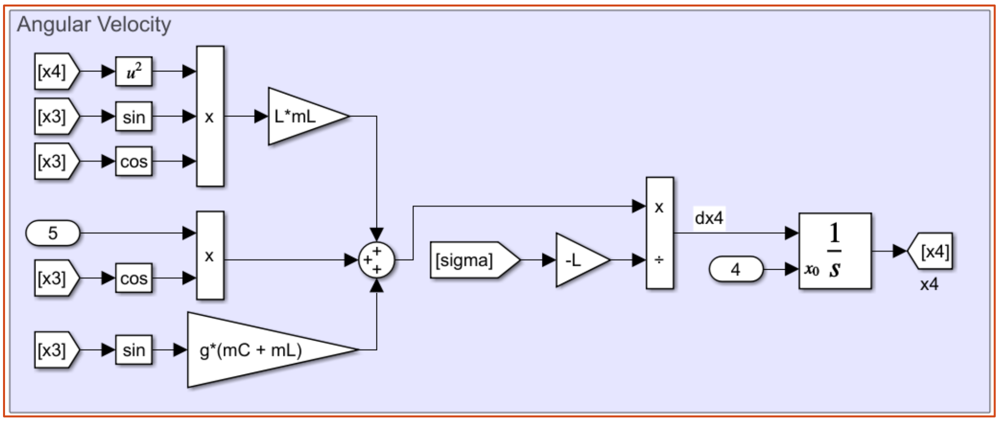
- The denominator can be understood as a sort of equivalent or effective mass.
- Division gives us the acceleration of the crane, which is integrated to give the velocity.

x_3 : ANGLE



This is simply the integral of angular velocity. The initial condition here is quite significant in terms of the equilibria.

x_4 : ANGULAR VELOCITY



Rather than force, it is torque we focus on this time. Our explanation is only qualitative, since the expressions have already been simplified to some extent.

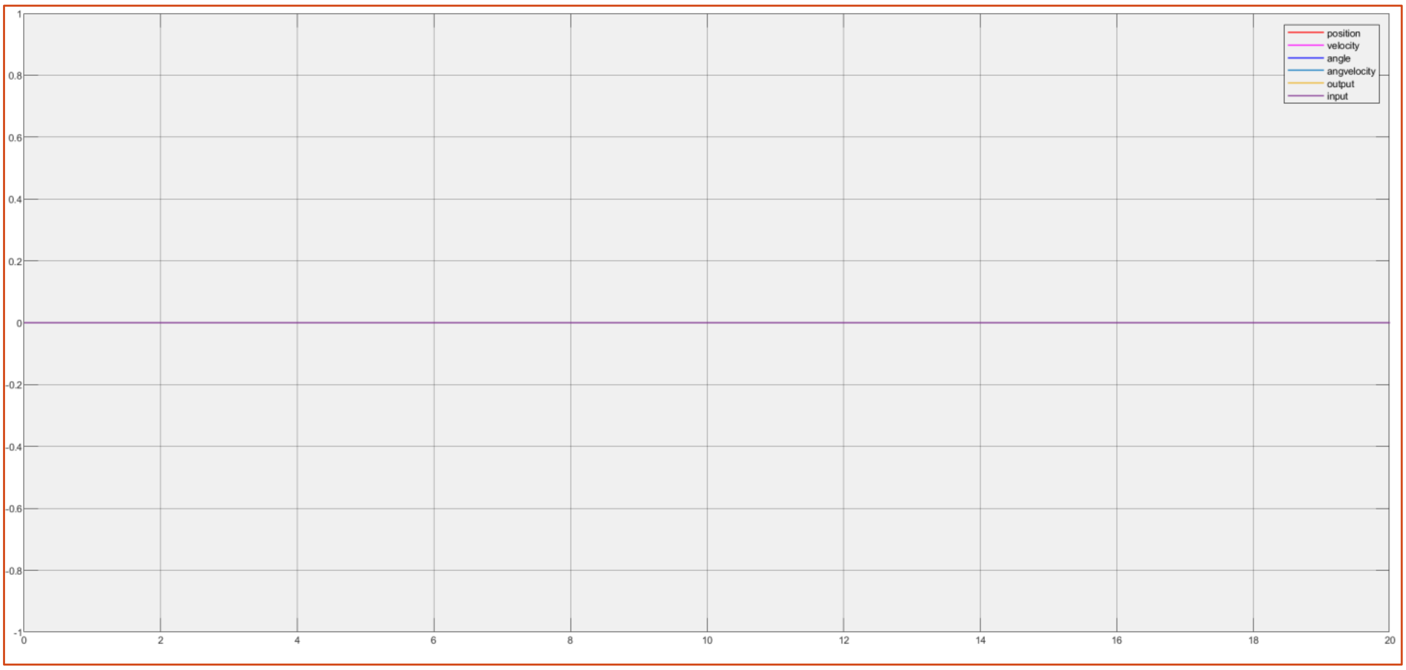
- The first term is the torque corresponding to centripetal force.
- The second term is torque due to the applied force.
- The third term is torque due to gravity.
- The denominator is equivalent to a moment of inertia (divided by l).
- The entire equation is already divided by l, so the description doesn't match exactly with the expressions.

ZERO INPUT RESPONSES TO INITIAL CONDITIONS

We shall first look at the natural responses of the system at different initial conditions. There is a lot of variation in the natural responses depending on the initial state.

CASE 1 - [0, 0, 0, 0]

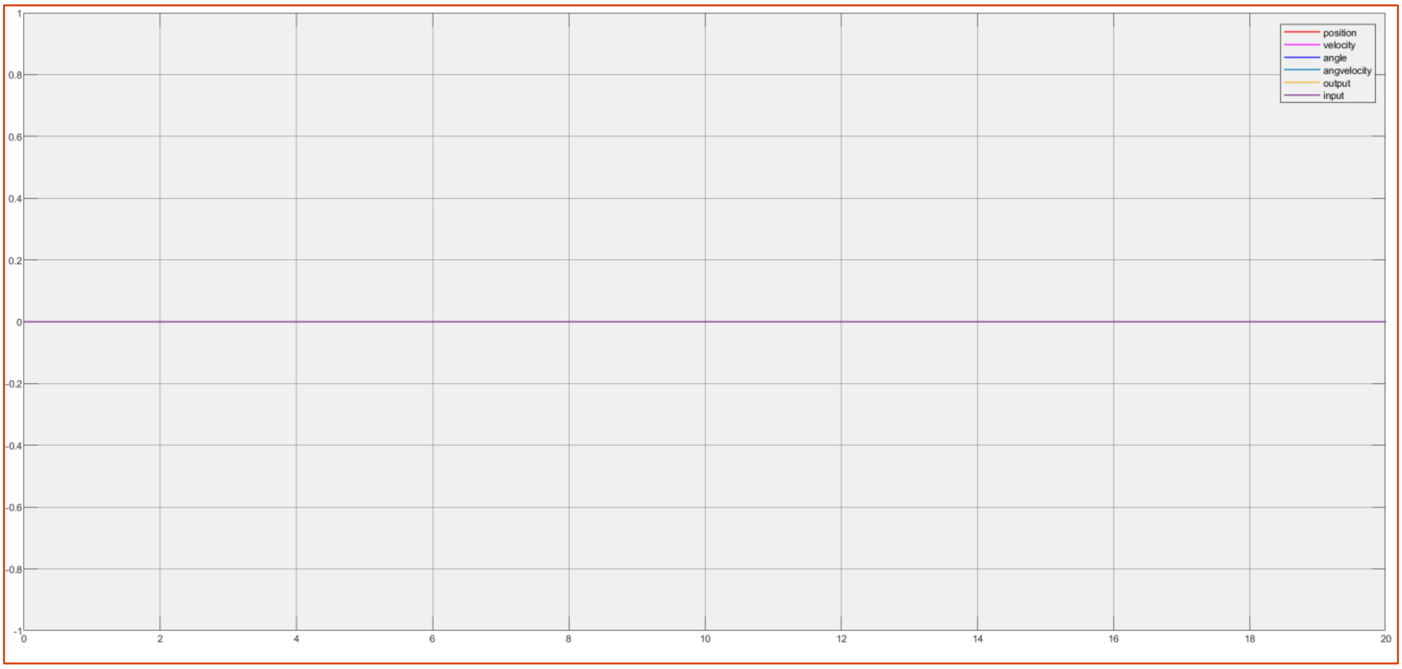
This is one of the equilibria of the system.



As expected, at equilibrium the system is at rest.

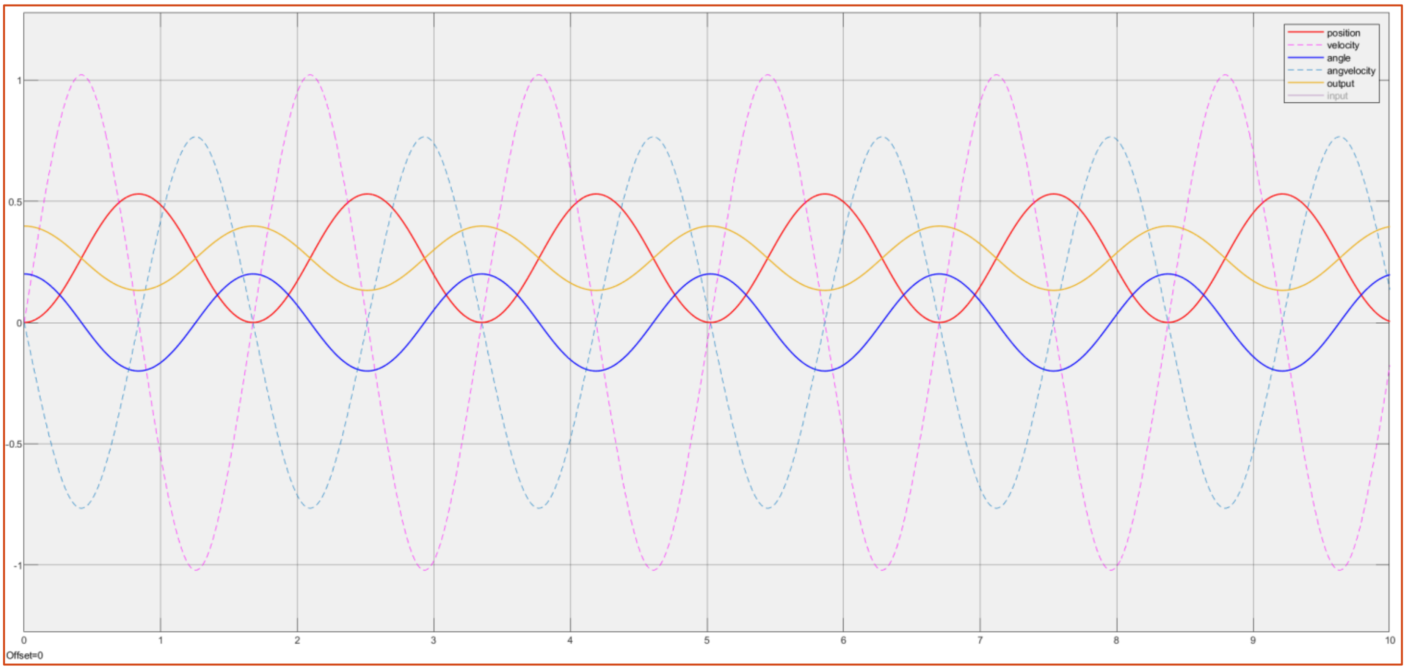
CASE 2 – [0, 0, π, 0]

This is the second type of equilibrium this system possesses.



CASE 3 – [0, 0, 0.2, 0]

Let us observe the response near Eqb 1.



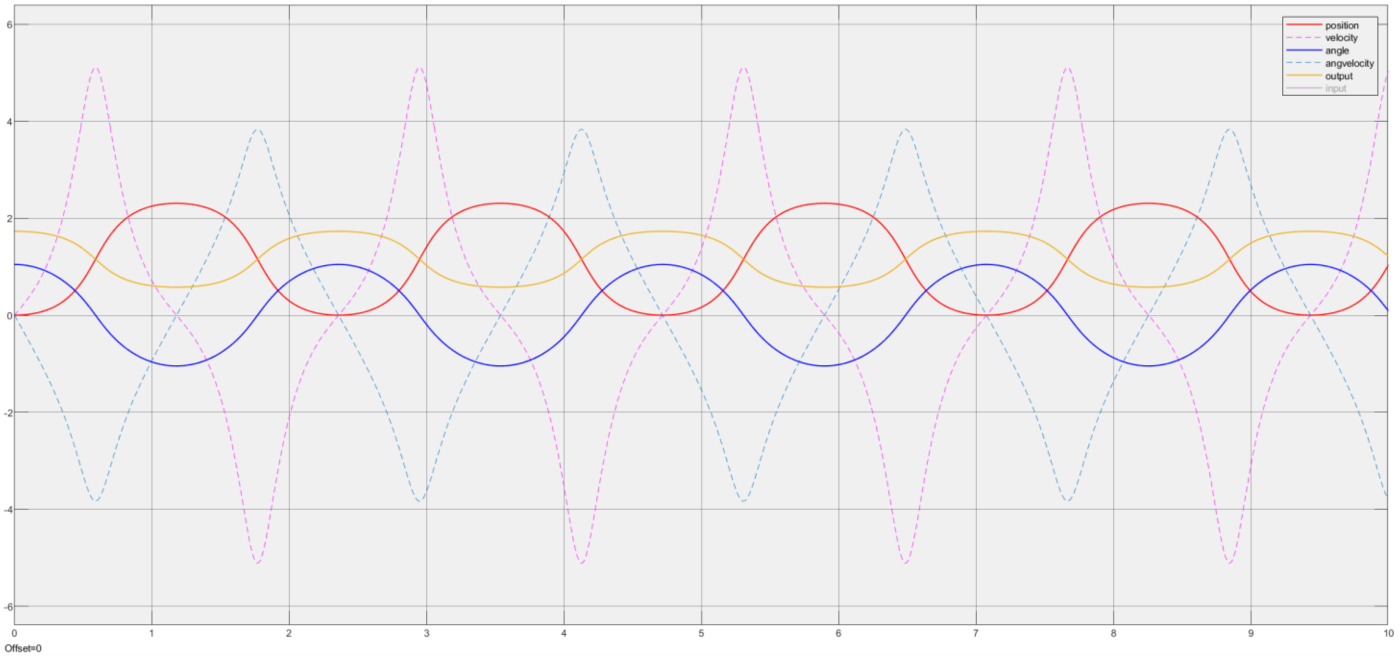
The response is oscillatory, which is expected due to the absence of damping in the model.

It is also almost sinusoidal, again expected since the initial conditions are in close range of the equilibrium and the linearization is valid.

The physical understanding of this is that the crane oscillates about its position with a time period of ~1.674 s. The load mass is oscillating at the same frequency with a phase shift of 90°. As the load mass oscillates towards one side, it pushes the crane to the opposite side, and so on.

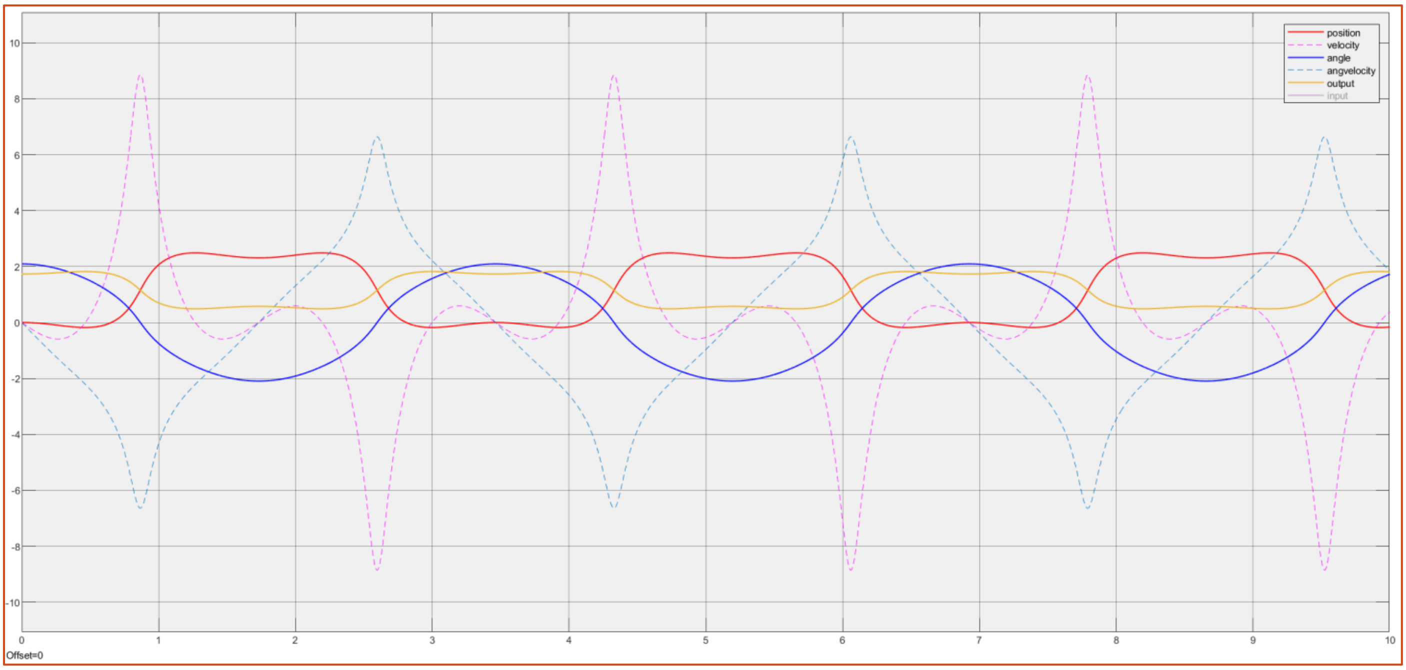
CASE 4 – [0, 0, $\pi/3$, 0]

Here we have increased the initial angle of the load mass. It can clearly be seen that higher order effects have come into play and while the dynamics are still periodic, they are not perfectly sinusoidal.



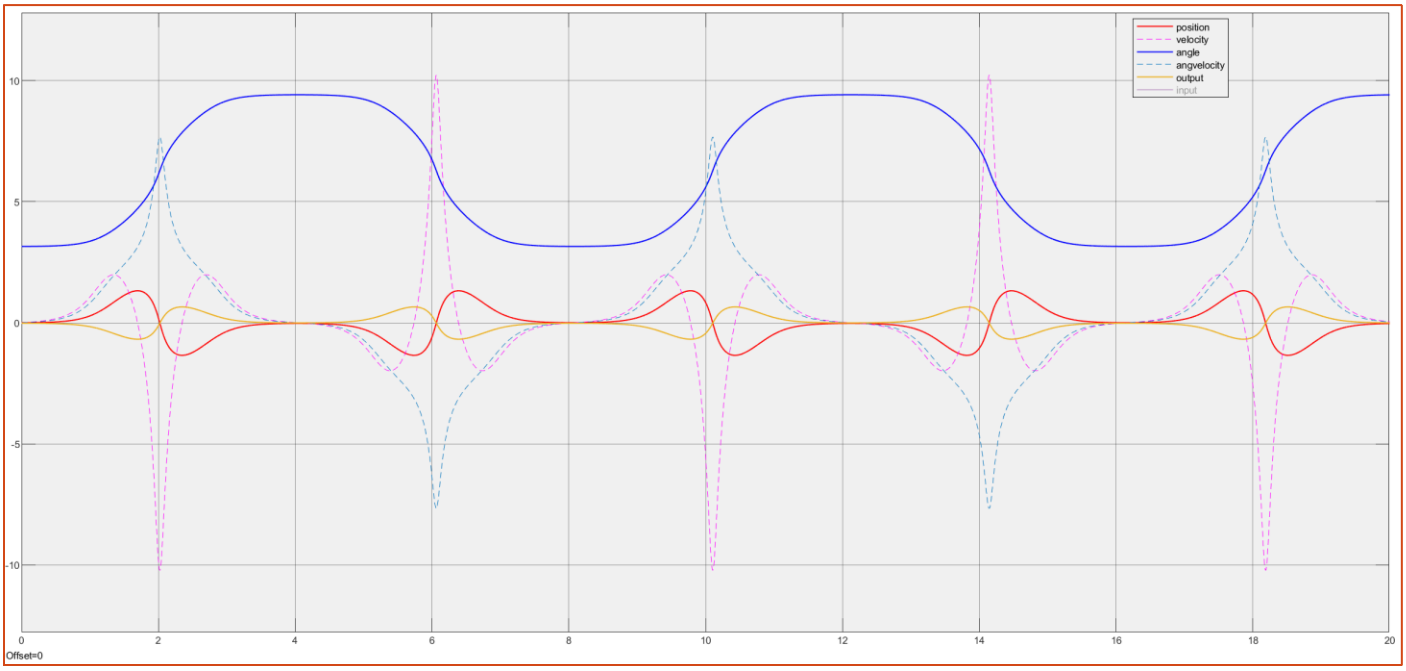
CASE 5 – [0, 0, $2\pi/3$, 0]

A second type of oscillation can be clearly seen, especially in the red curve. The linear approximation obviously fails at very large angles.



CASE 6 - [0, 0, $\pi + 0.01$, 0]

Let us see what happens as we slightly perturb Eqb 2.



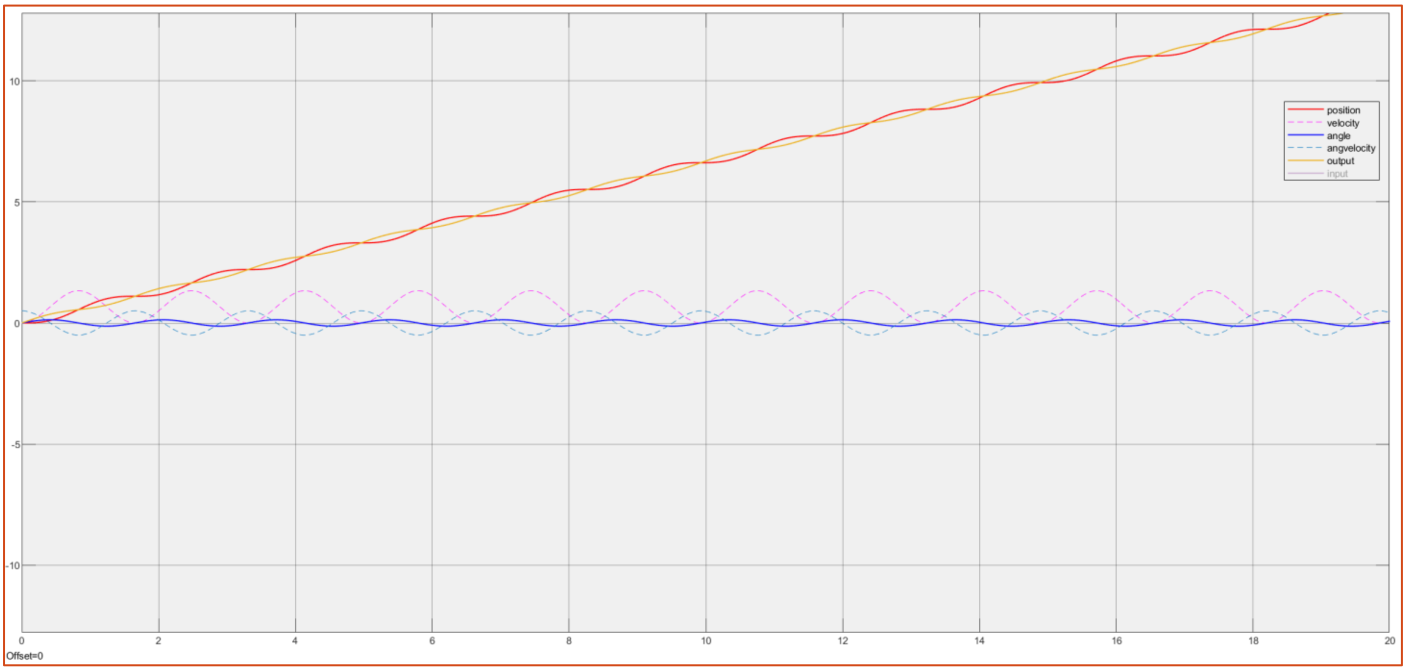
We obtain a different type of dynamics, in which the system oscillates with a period of ~8 seconds, with a small period of stasis before rapidly speeding up and reaching the same angle but from the opposite direction. The equilibrium is unstable.

Due to the absence of friction, this motion will continue forever. If friction was present, the system would eventually move towards Eqb 1 as energy would keep getting dissipated.

CASE 7 – [0, 0, 0, 0.5]

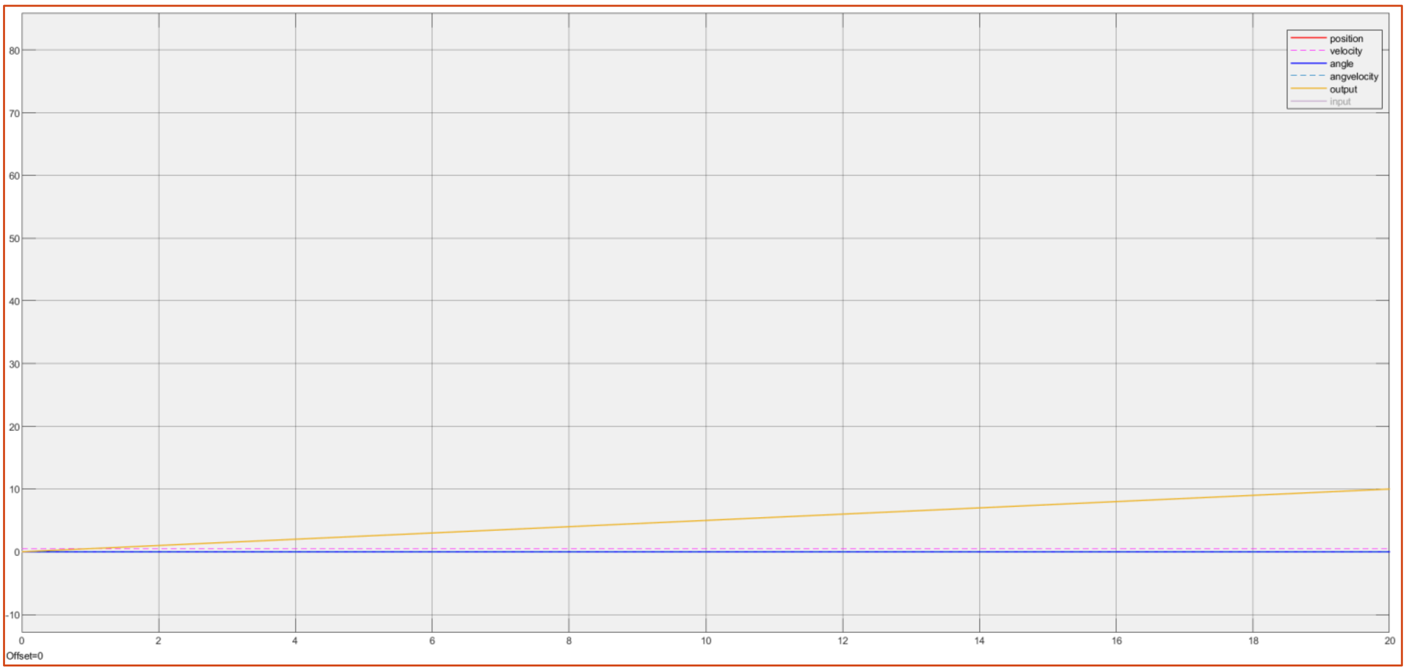
We can clearly see the effect of non-zero initial angular velocity – the crane keeps moving forward, with the load mass slowly oscillating. Since there is no friction, energy is conserved.

The crane's velocity is never negative, so the crane always moves forward, though it slows down periodically. The physical explanation is that the phase difference between x_2 and x_4 is such that in this case, the force applied by the load mass doesn't oppose the cart's motion, hence it keeps moving in one direction with some oscillation, as opposed to earlier cases where the load mass' motion pushed on the crane out-of-phase, thus the crane position kept oscillating.



CASE 8 – [0, 0.5, 0, 0]

The only state affected by x_2 is x_1, so this response is simple to explain – the crane moves forward with a constant velocity, the load mass is always at angle 0.



Both Case 7 and Case 8 have a continuously increasing position.

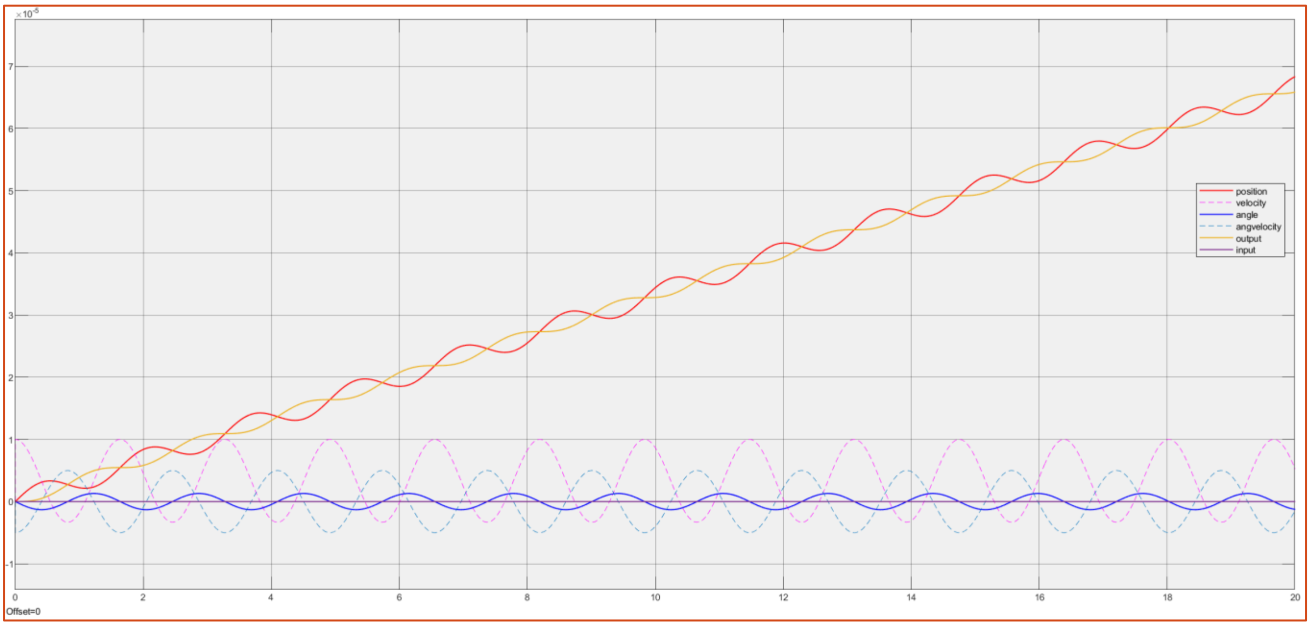
IMPULSE RESPONSE

We shall study the impulse response by providing input with the Discrete Impulse block.

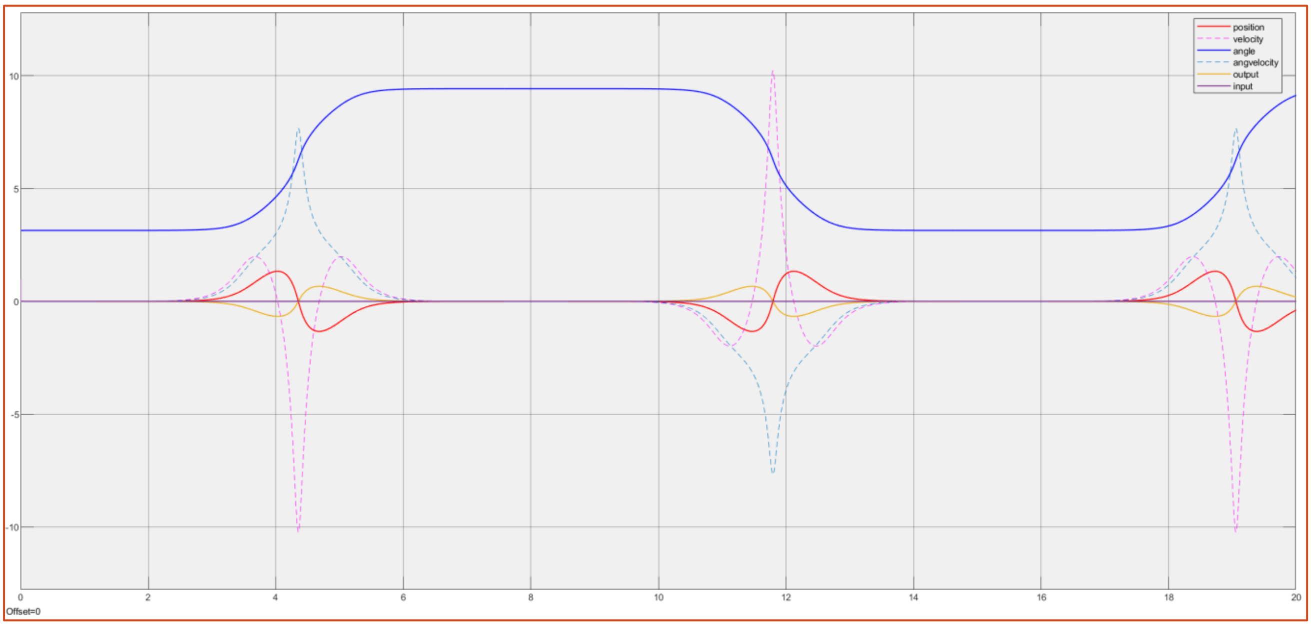
CASE 1 - [0, 0, 0, 0]

We are not showing the input here because it is very large compared to the system response.

This response can be thought of as a combination of Cases 7 and 8 from the previous section, with non-zero initial x_2 and x_4.



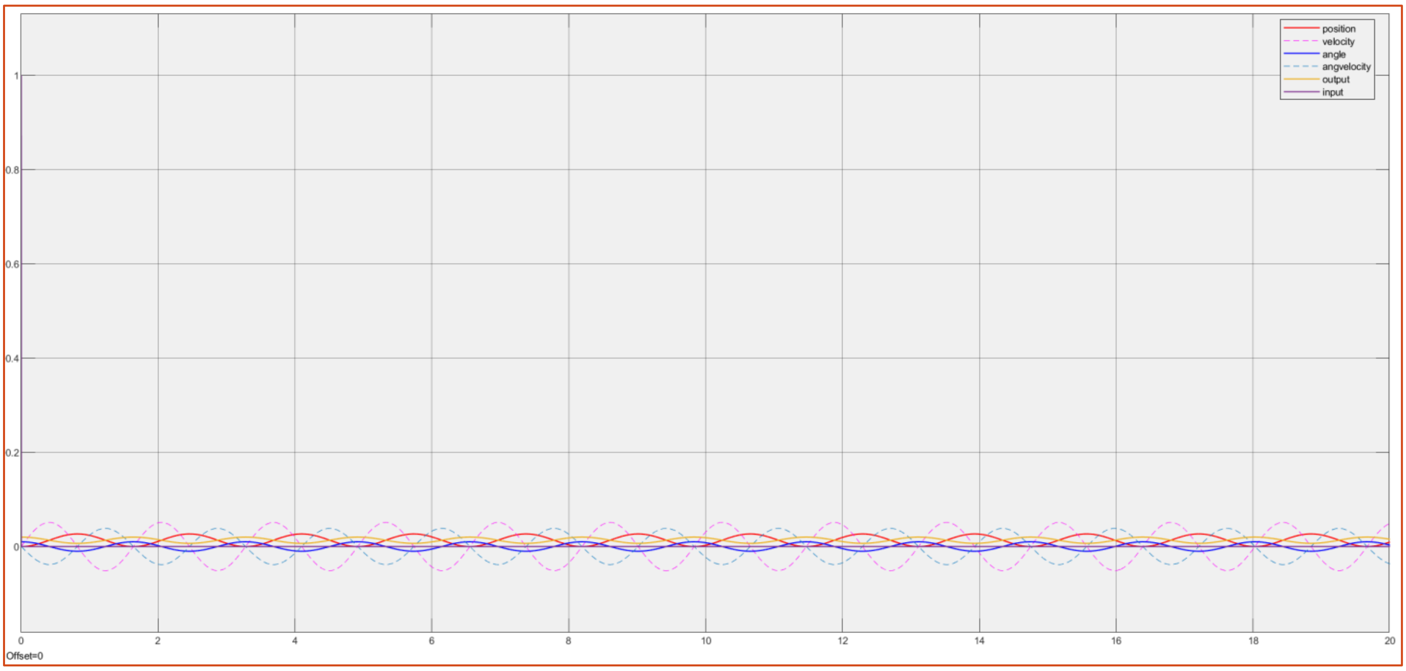
CASE 2 – [0, 0, π, 0]



This response is qualitatively similar to Case 6 from the previous section, and there is not much to say about it other than the reduced frequency of oscillations. The impulse response is very similar for perturbations around this equilibrium, so will skip showing those.

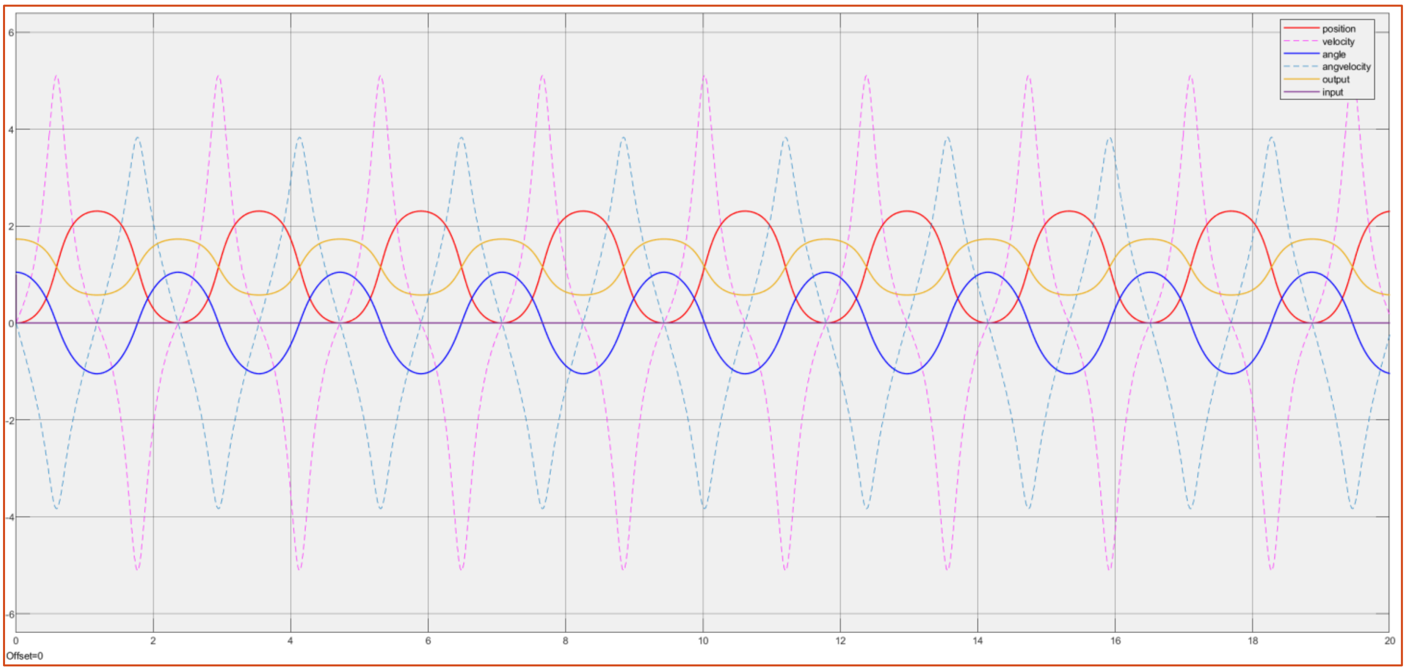
CASE 3 - [0, 0, 0.01, 0]

This response is nearly identical to that of case 3 in the previous section. The load mass' non-zero initial angle has an effect on the phase differences between the states, so the crane position isn't unbounded in this case.



CASE 4 – [0, 0, $\pi/3$, 0]

The natural response drowns out the effect of the impulse response, giving us a response similar to what we obtained earlier. – [0, 0, $\pi/3$, 0]

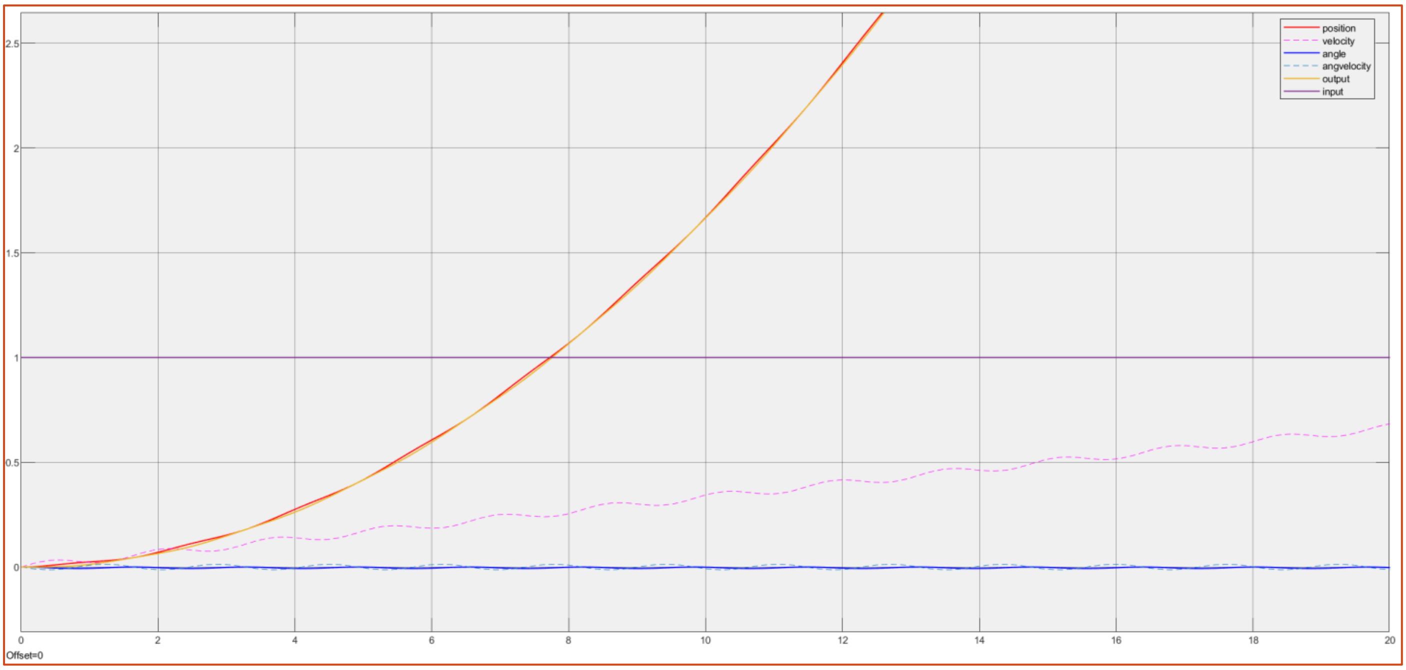


STEP RESPONSES

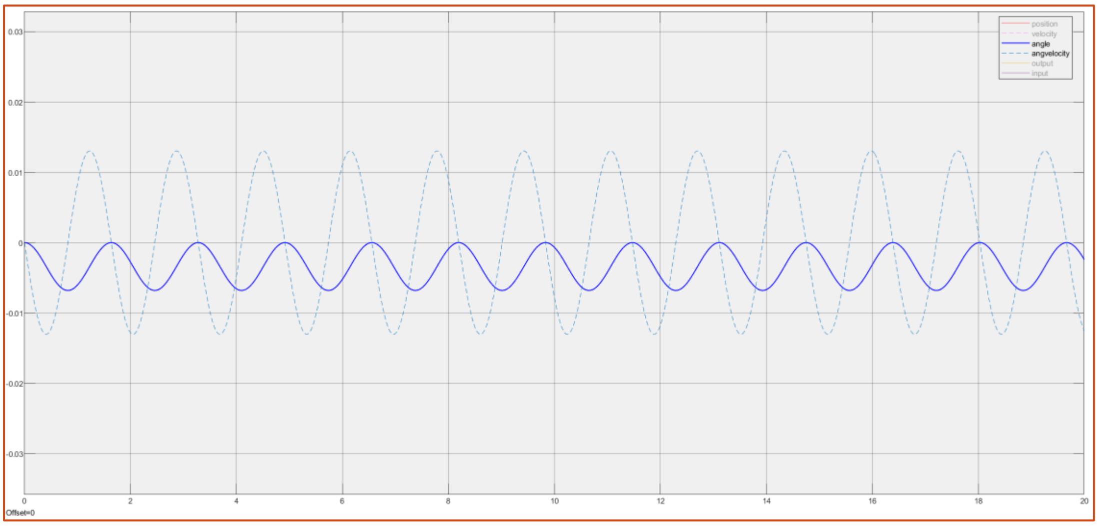
Having observed the impulse responses, we shall now observe the step responses for different initial states as before.

CASE 1 - [0, 0, 0, 0]

We see that the velocity and crane position are continuously increasing. In physical terms the crane is being continuously pushed forward. The load mass oscillations are of a very small amplitude.



OSCILLATIONS OF ROPE ANGLE

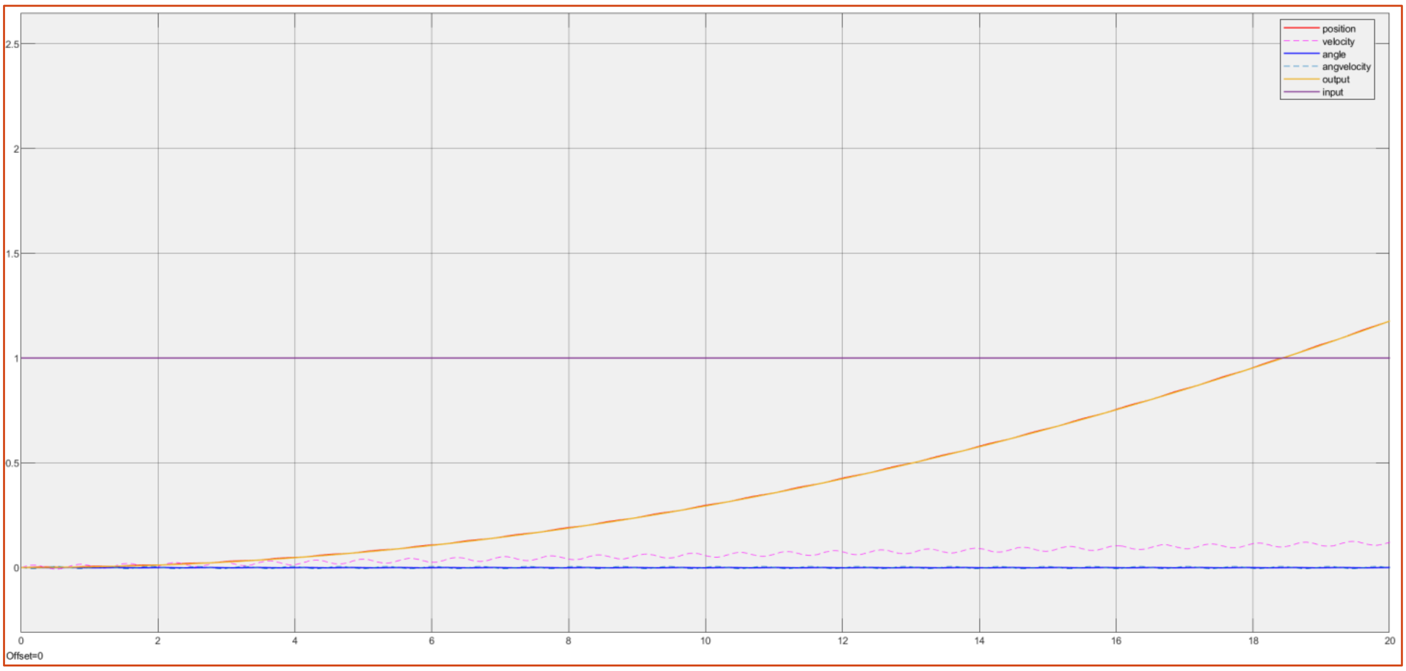


In this case, a constant force is being applied on the system, so the rope angle continuously oscillates around some average value, similar to a pendulum in a moving train.

LOAD MASS INCREASED TO 160 KG

Upon increasing the load mass, we can see that the response has become much slower. Also, looking at the pink velocity curve, we observe that the frequency of oscillations has increased.

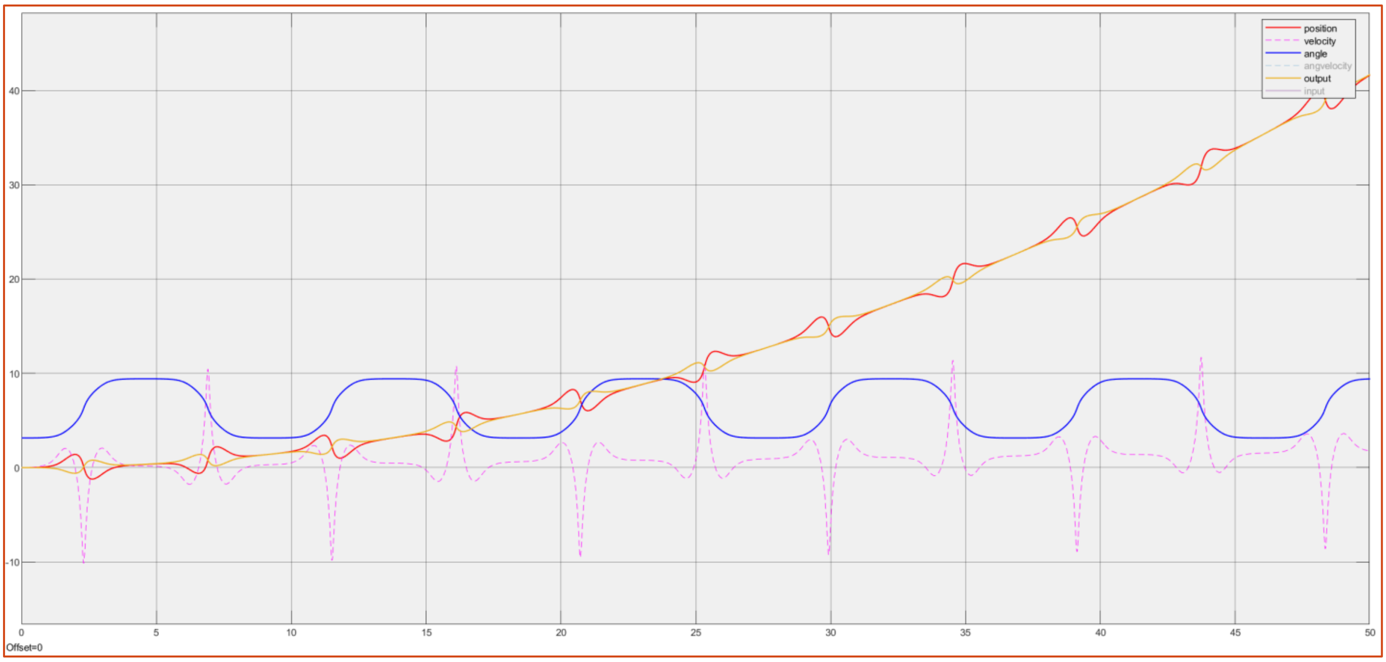
This is consistent with our results from Expt #8 : increasing load mass increases the magnitude of the imaginary eigenvalues (of the linearized system). Larger imaginary eigenvalues correspond to faster oscillations.



CASE 2 – [0, 0, π, 0]

We have hidden the input and angular velocity traces, since they don't offer extra insight into the dynamics.

- The effect of the step input is to move the crane forward, and the oscillations due to the swinging of the load mass can be clearly seen.
- The pink velocity curve is also moving upwards, again because of the step response.
- There does not seem to be any appreciable effect on the rope angle, which oscillates similar to the past cases.

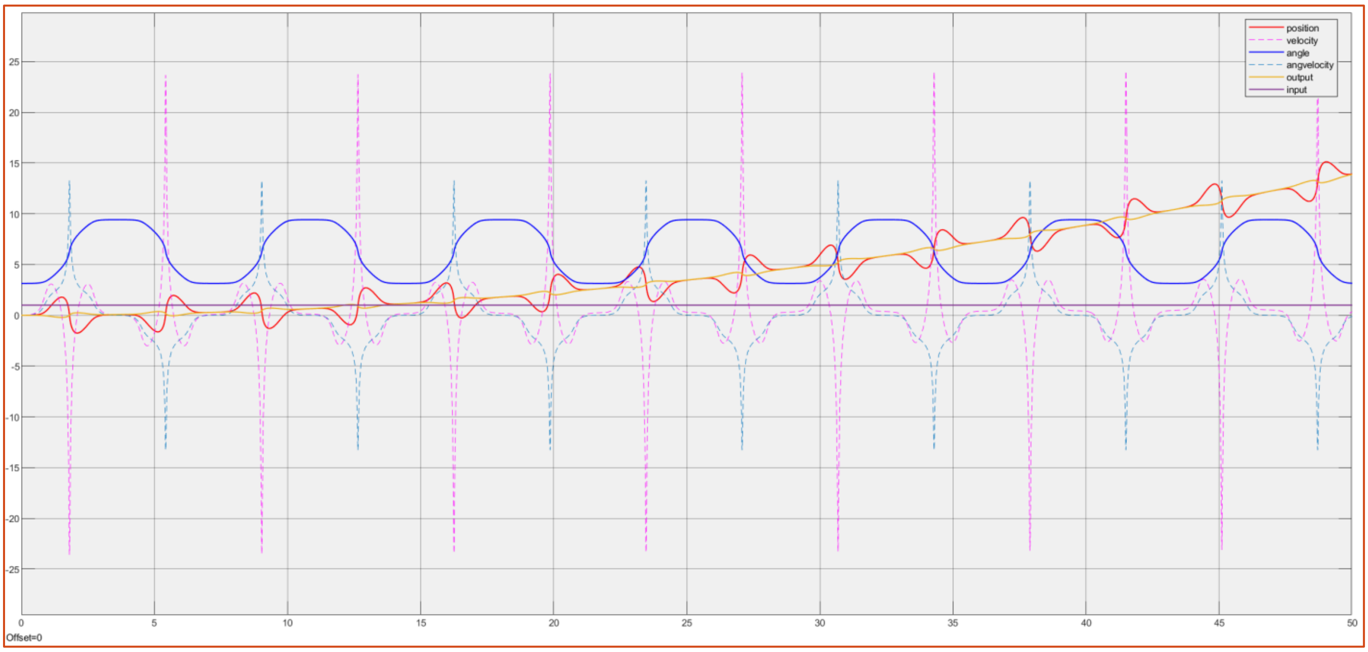


LOAD MASS INCREASED TO 80 KG

Oscillation frequency is increased, and the transition period seems to have shortened.

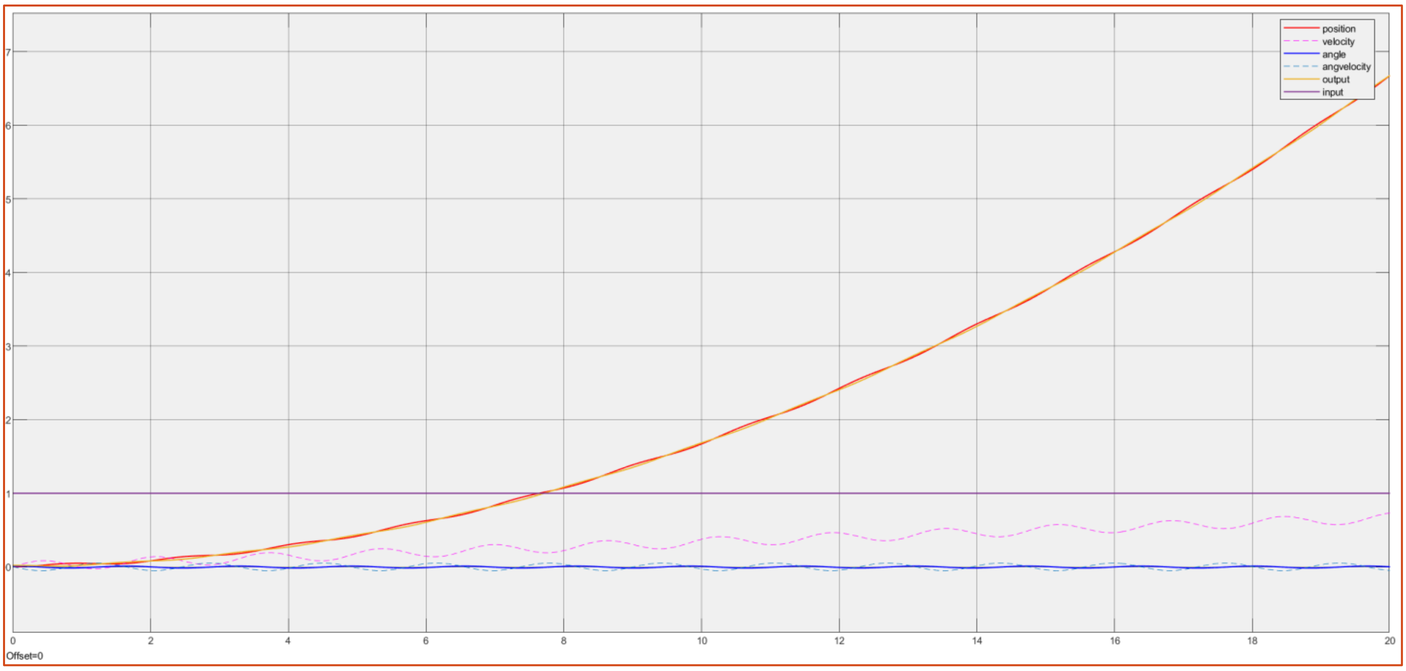
This is directly explained by our observations in Expt #8 – increasing load mass increases the magnitude of the real eigenvalues of the linearized system about this equilibrium. This corresponds to faster dynamics.

(PTO)



CASE 3 - [0, 0, 0.01, 0]

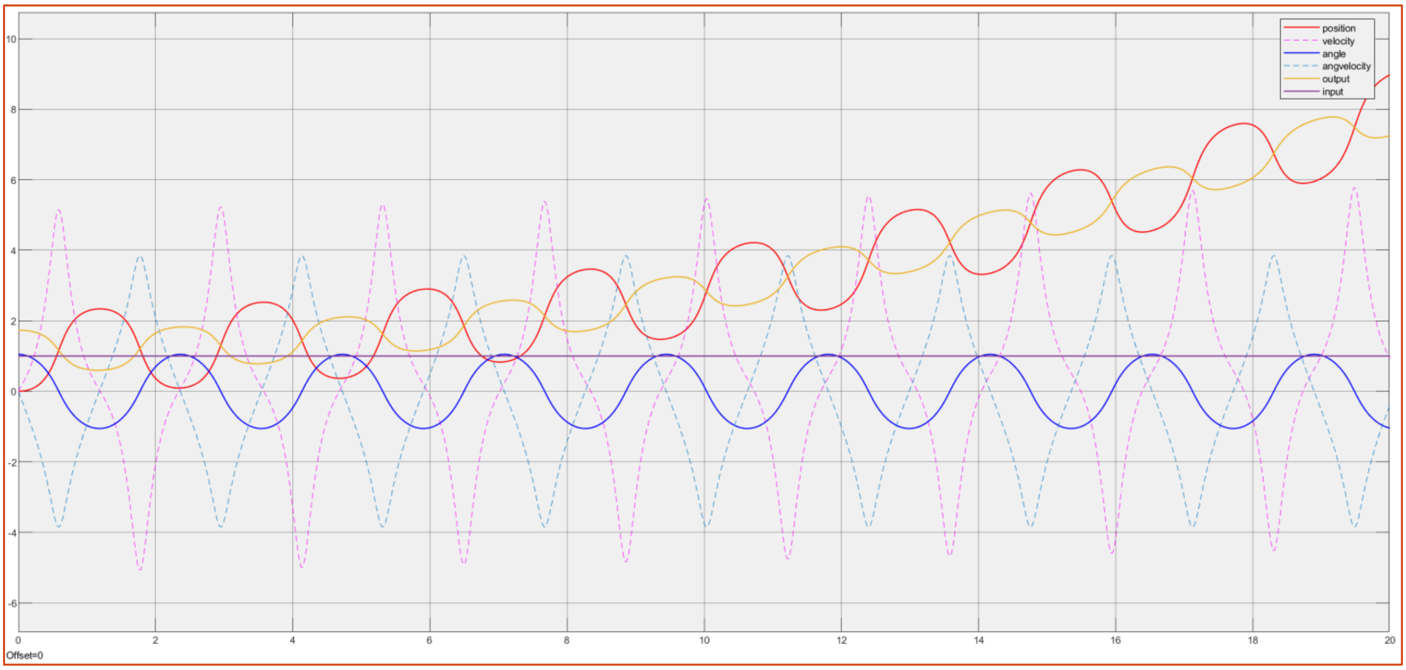
This is very similar to Case 1; the linear approximation holds good here. We will see oscillations of large amplitude if we use a larger initial angle.



CASE 4 – [0, 0, $\pi/3$, 0]

Quite similar to Case 4 in the previous section, with the same differences as in the other cases – continuously increasing crane position and velocity.

Increasing mass has the same effect as in previous cases, so we are not showing it here again.



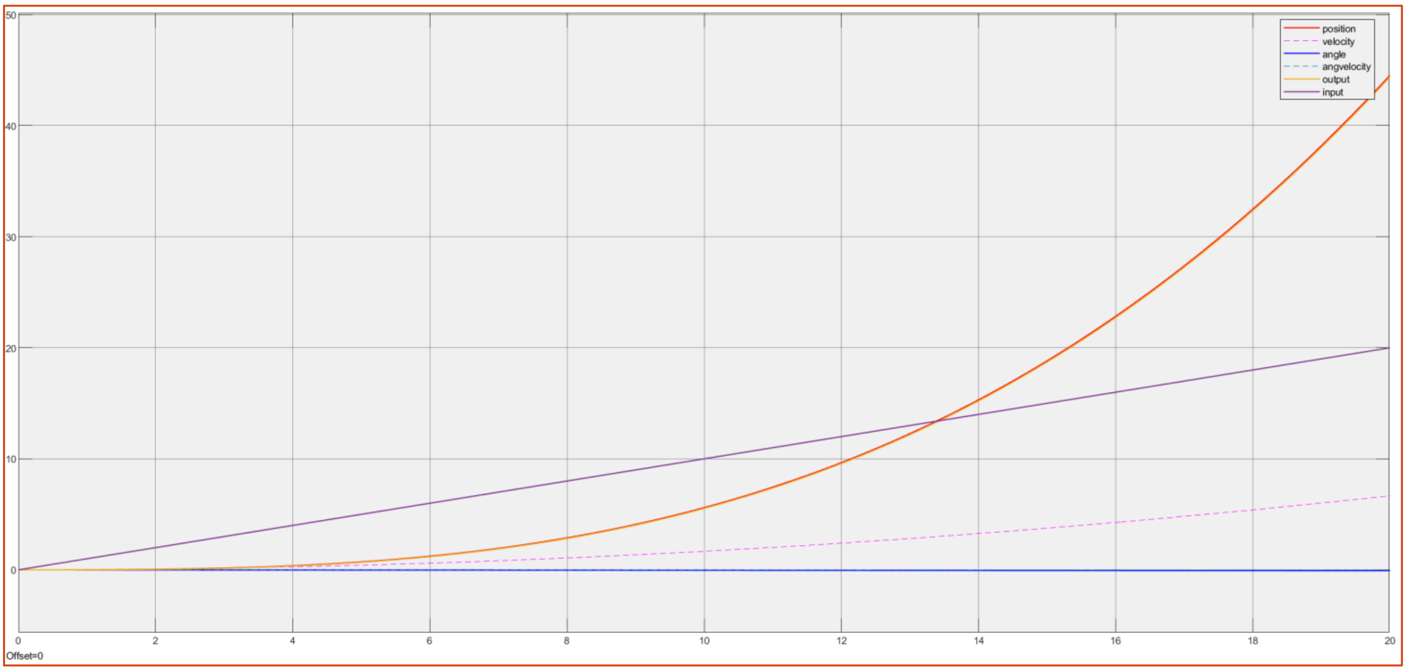
RAMP RESPONSES

Finally, we shall observe the ramp responses for various initial conditions.

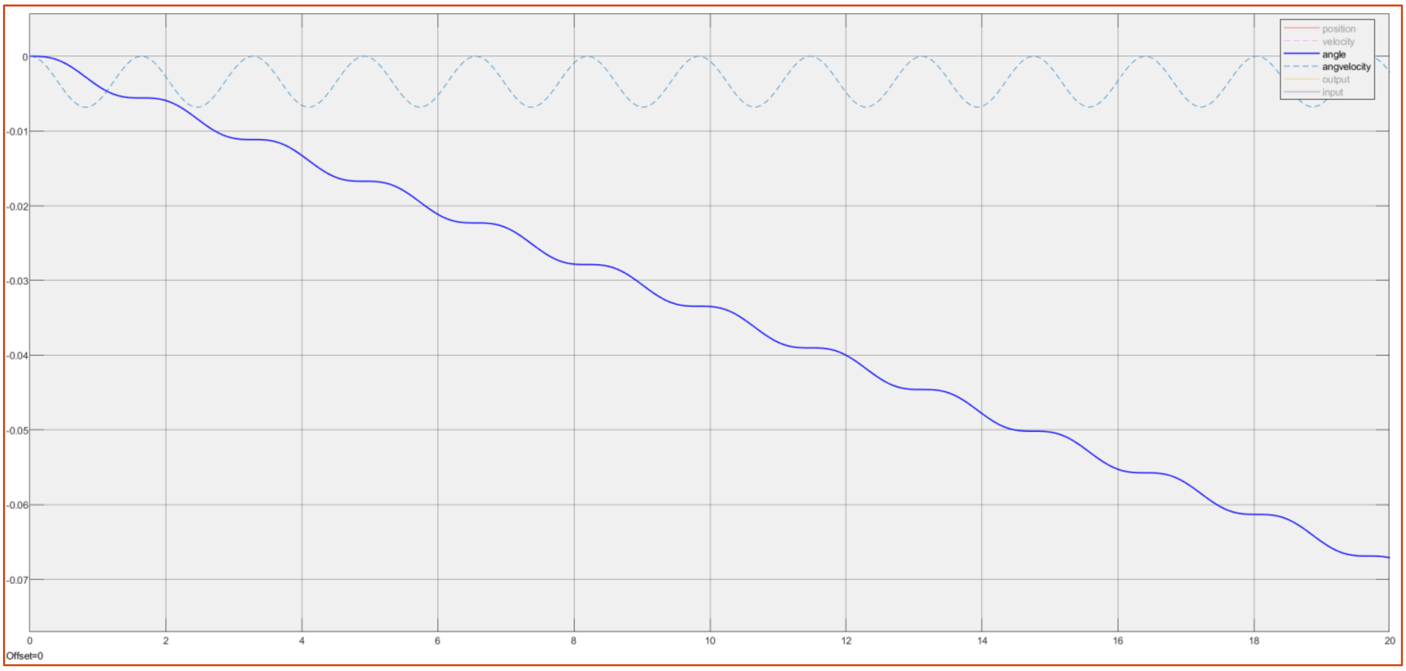
We shall use a ramp function of slope 1.

CASE 1 – [0, 0, 0, 0]

It is clear that this response is very similar to the step response – crane position and velocity increase continuously.



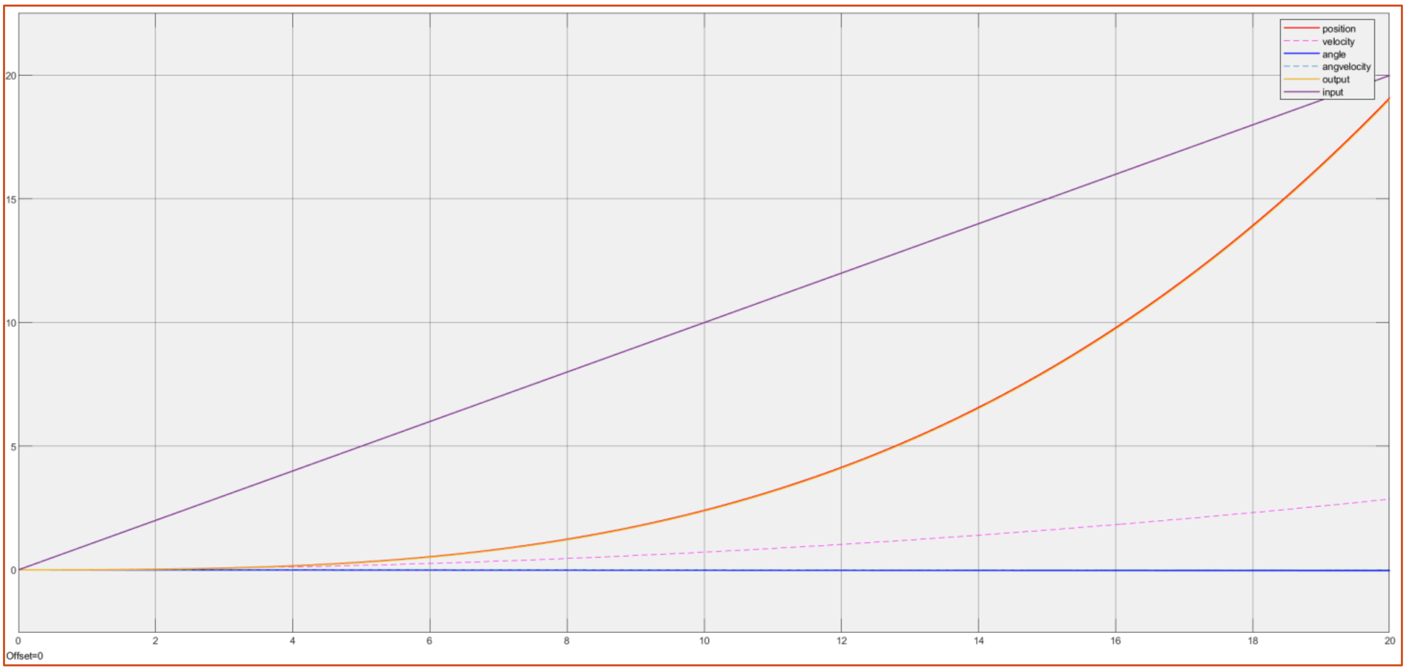
Focusing only on the rope angle and angular velocity –

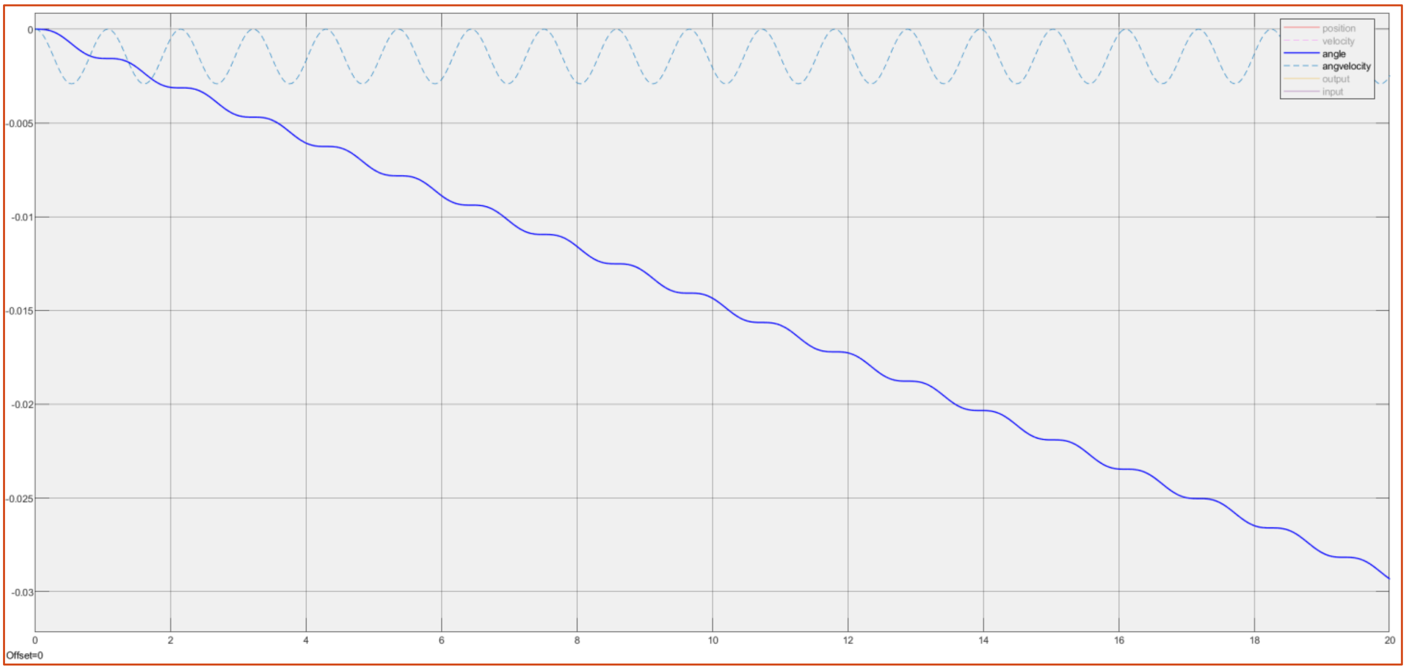


We see that angular velocity oscillates as in previous cases, but rope angle is continuously increasing. Physically, this can be understood by continuing the pendulum in a train example from earlier, but now increasing the acceleration of the train. The “rest” angle of the pendulum keeps increasing, which we can see in the graph above as well.

INCREASING LOAD MASS TO 60 KG

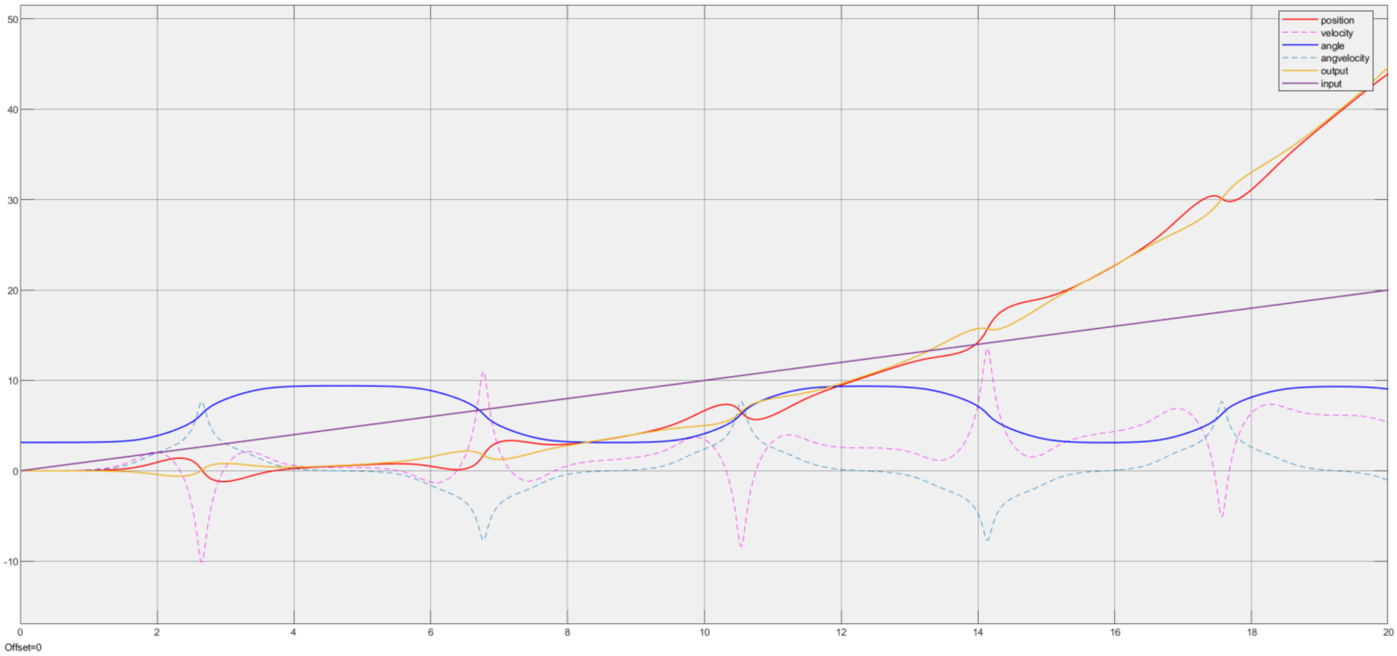
Again, we observe the same effect. Comparing with the plot on the previous page, we can see the frequency of oscillation increase and the growth of crane position decrease.





CASE 2 – $[0, 0, \pi, 0]$

There is not much to say about this case, it is very similar to the step response in the previous section, except now the oscillation of the rope angle is also affected similar to Case 1 of this section.



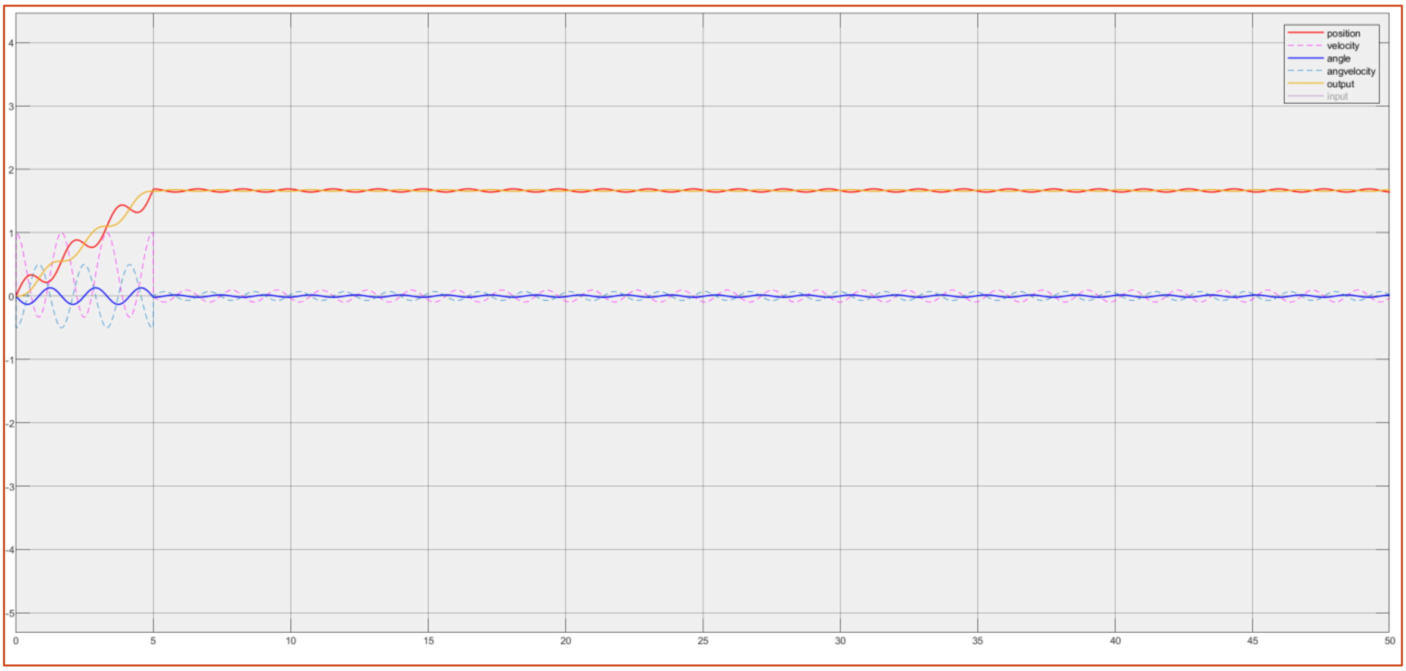
The other two cases are very similar, and the main differences have been discussed already, so we shall not show those responses.

COMPARING RESPONSES

IMPULSE INPUT

The impulse input may be useful for moving the crane horizontally. In the figure below, we gave an impulse at t=0, and a negative impulse at t=5s. This allows us to move the crane to a different horizontal position. In a real system, the oscillations will die out due to friction.

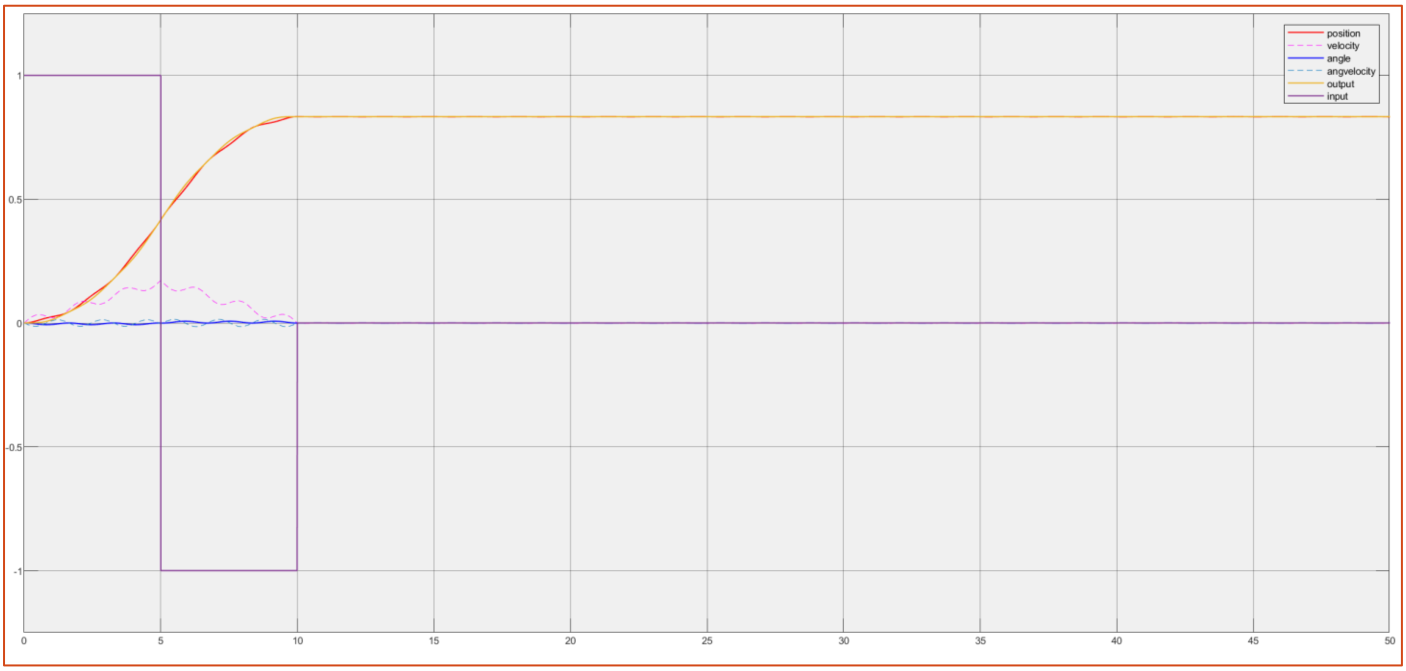
We had to multiply the impulse input by a gain of 10^4 units. So, it is not a very practical method.



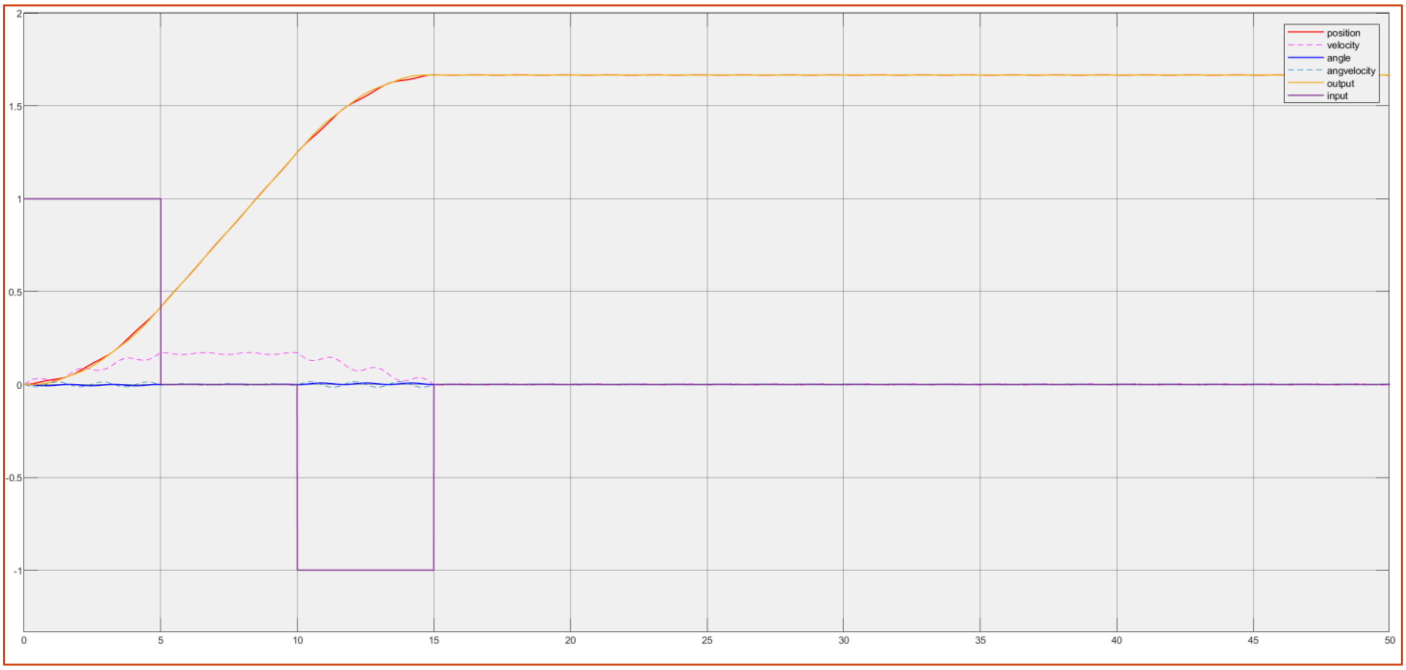
STEP INPUT

Using a combination of step responses, we can also achieve a similar response, but with much smaller amplitude of oscillations. By precisely setting the switching times, we can choose the final position.

The step input is much more useful than the impulse input, and is far easier to implement.

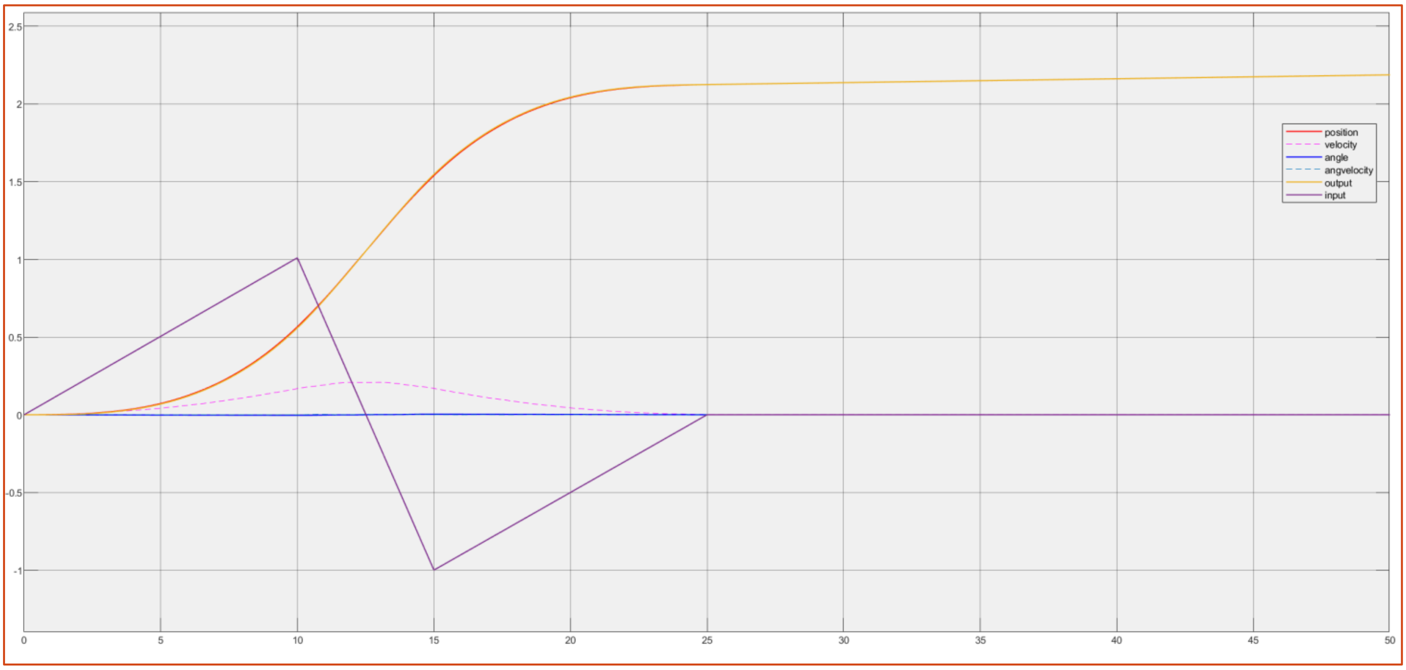


We can take various combinations of step inputs to move the crane to the desired position, as long as the total area is 0.



RAMP INPUTS

A similar control is possible using ramp inputs. While the output appears to keep increasing in the plot, in practical situations it will settle down due to friction.



RECOMMENDATIONS

- Comparing all three inputs together, the impulse input is the hardest to implement and performs the worst, with large amplitudes of oscillation and a very barebones control.
- The step and ramp inputs perform similarly, but it is much easier to implement the simple toggling input as in the step input case.
- The step input is also much more flexible as shown in the second image of that section.
- For the ramp input, the combination of signals required to achieve the desired control is far more complex, making it impossible to use if the only tool available is a simple ramp source, while a simple step source can perform as desired.

Thus, the best input to use for the application of moving the crane horizontally should be the step input.

We have not discussed the Eqb 2 with respect to the inputs since a real-life overhead crane is very unlikely to lift the load over itself.

*Furthermore, although we have discussed the cases for rope angles near π, since the project statement talks of a **rope** and not a **rod**, such a situation is not even possible. We have only discussed those cases for the sake of completeness.*

CONCLUSIONS

In this experiment we simulated a nonlinear crane-pulley system on Simulink, observed its responses to various inputs for various initial conditions, and examined the viability of different inputs for a simple open-loop control task.

Our findings are as follows –

- Using only Open-Loop input control (out of the three given input sources), the only task we can properly achieve is to move the crane from one horizontal position to another when it is at rest. It is difficult to perform any complex angle-based control without state or output feedback.
- The most useful input source from those given in the problem statement is the **Step Input**, for reasons given in the previous section.
- The Step Input source can be used to move the crane horizontally from one position to another with little oscillation of the load mass.

We also studied the effect of load mass on the system dynamics, and found that the major effect is to increase frequency of oscillations and also slow down the horizontal movement of the crane system. The qualitative nature of the system dynamics remains the same. This is a consequence of the characteristics of the System Jacobian's eigenvalues around the equilibria, as discussed in Expt# 8.

Finally, we recommend closed loop methods such as State-Feedback or Output-Feedback control for advanced control tasks such as holding the load mass at some given angle or following a desired trajectory. State-space control can be done around the equilibria of interest, using pole placement algorithms or optimizations such as LQR control.

SIMULINK FILE

https://drive.google.com/file/d/1XdwWpdK9fXi70sSy8qNmRIhkgRPXZGa5/view?usp=sharing