EE208 EXPERIMENT 11

NONLINEAR SYSTEM DYNAMICS ON SIMULINK FOR DIFFERENT LYAPUNOV CONTROL DESIGNS

Group no. 9

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Objective

To incorporate different Lyapunov control designs in the crane trolley system introduced in Expt. #8 and #10, and to simulate the system on Simulink.

Tasks

1. Given four energy-based function components, generate various Lyapunov functions and design control inputs for the same.

A. Proportionate to $\textit{square of linear potential energy} := K_{PE}^{l} \cdot (x_{1,ref} - x_1)^2$
B. Proportionate to $\textit{linear kinetic energy} := K_{KE}^{l} \cdot x_2^2$
C. Proportionate to $\textit{square of rotary potential energy} := K_{PE}^{r} \cdot x_3^2$
D. Proportionate to rotary kinetic energy $:= K_{KE}^{r} \cdot x_4^2$

2. Using step, impulse, and ramp inputs for $x_{1,ref}$ (the reference position of the carriage), study the dynamics of the crane from one steady-state to another, using traces of all four state variables as well as the single output variable.
3. Conclude the best Lyapunov control for the nonlinear system to work with.
4. Repeat the studies for different values of load mass that the crane is lifting and discuss its effect.

Overview

- Possible Lyapunov functions are formed from the given components “A, B, C, D”.
- Appropriate control inputs, if any, are found.
- The control is implemented in Simulink and tested against various reference inputs.
- We shall discuss system responses for the stable equilibrium – (0,0,0,0) since the unstable equilibrium (0,0,π,0) cannot occur in the crane-trolley system in real-life scenarios.
- We wish to perform control for the purpose of moving from one stable equilibrium to another, along the horizontal axis, with minimal oscillation of the load mass. This is the criterion we will use to judge various control laws.

Lyapunov Functions and Control Inputs

From the given system equations, we can obtain the state-space form –

$$\dot{\boldsymbol{x}} = \begin{pmatrix} x_2 \\ \dfrac{l\, m_L \sin(x_3)\, {x_4}^2 + u + g m_L \cos(x_3) \sin(x_3)}{m_L \sin^2(x_3) + m_C} \\ x_4 \\ \dfrac{l\, m_L \cos(x_3) \sin(x_3)\, {x_4}^2 + u \cos(x_3) + g(m_C + m_L) \sin(x_3)}{(-l) \times (m_L \sin^2(x_3) + m_C)} \end{pmatrix}$$

$$y = x_1 + l \sin(x_3)$$

Now, in order to simplify our control analysis, we shall perform partial feedback linearization using the following input –

$$u \ = -l m_L \sin(x_3)\, {x_4}^2 + v(m_L \sin^2(x_3) + m_C) - g m_L \cos(x_3) \sin(x_3)$$

Which gives us a simpler effective system –

$$\dot{\mathbf{x}} = \begin{pmatrix} x_2 \\ v \\ x_4 \\ -\dfrac{v\, cos(x_3) + g\, sin(x_3)}{l} \end{pmatrix}$$

We can already see that the load and crane masses have been removed from the system using feedback linearization.

Now, we can try Lyapunov functions of the form $\sum k_i x_i^2$.

$L_1 = k_1 \left(x_1 - x_{ref}\right)^2$

This gives –

$$\dot{L_1} = 2k_1 x_2 \left(x_1 - x_{ref}\right)$$

The input variable v doesn't occur at all in this equation, so control is not possible. We should try another function.

$$L_2 = k_1\left(x_1 - x_{ref}\right)^2 + k_2(x_2)^2$$

This gives –

$$\dot{L_2} = 2k_1x_2\left(x_1 - x_{ref}\right) + 2k_2vx_2$$

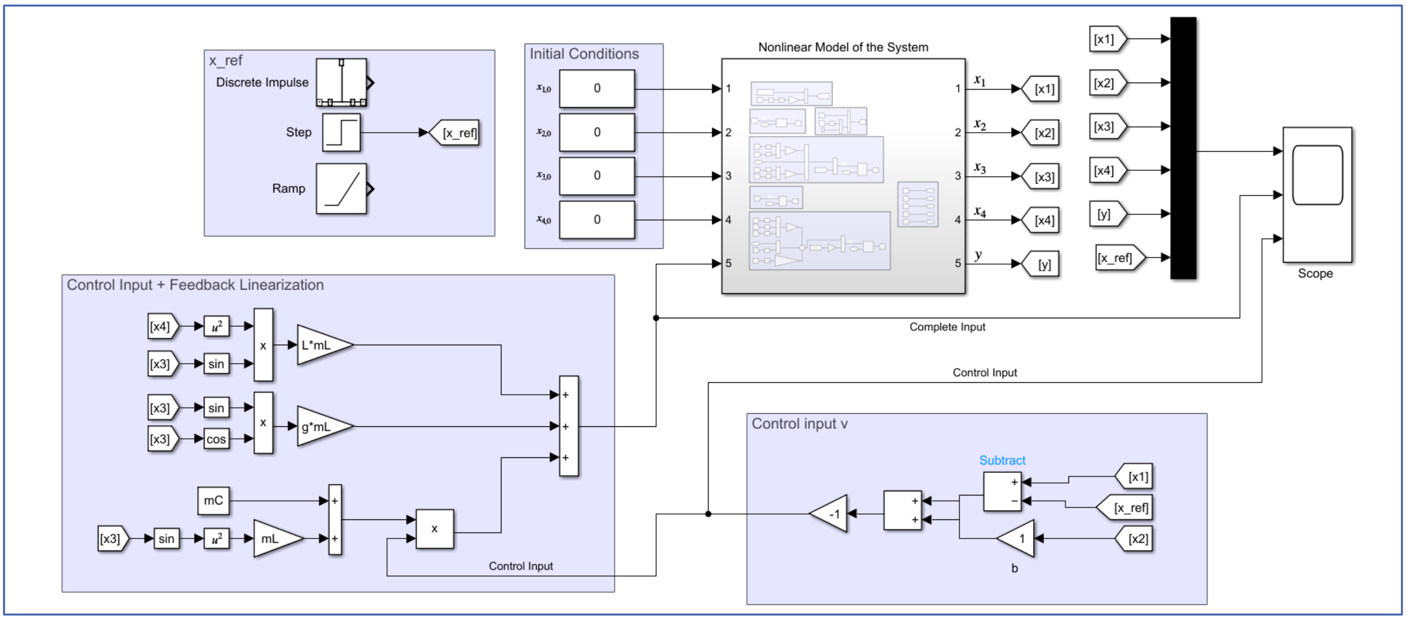
Now, for $\dot{L_2} \leq 0$, we can choose a suitable v such that $\dot{L_2} = -bx_2^2,\ b \neq 0$, which is always ≤ 0. Equality is achieved at $x_2 = 0$.

This v is $-\left(\frac{b\,x_2 + 2\,k_1\,x_1 - 2\,k_1\,x_{ref}}{2\,k_2}\right)$.

Setting $k_1 = k_2 = 0.5$ to simplify the expression we obtain $-\left(b\,x_2 + x_1 - x_{ref}\right)$.

Here b is a tuneable parameter.

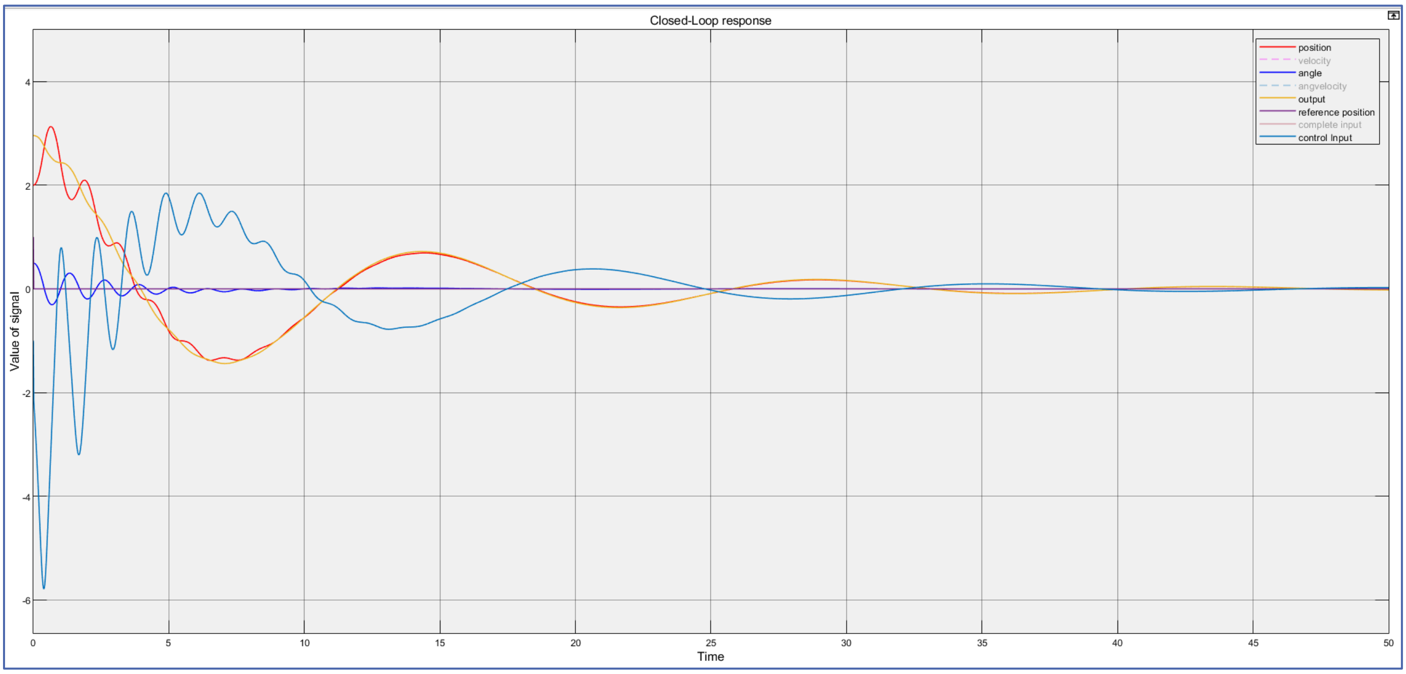
This can be implemented in Simulink –



Here we have re-used the model from Expt #10, and added the feedback loop comprising of the Lyapunov-based control law and the Partial Feedback Linearization scheme.

Now we shall look at various responses –

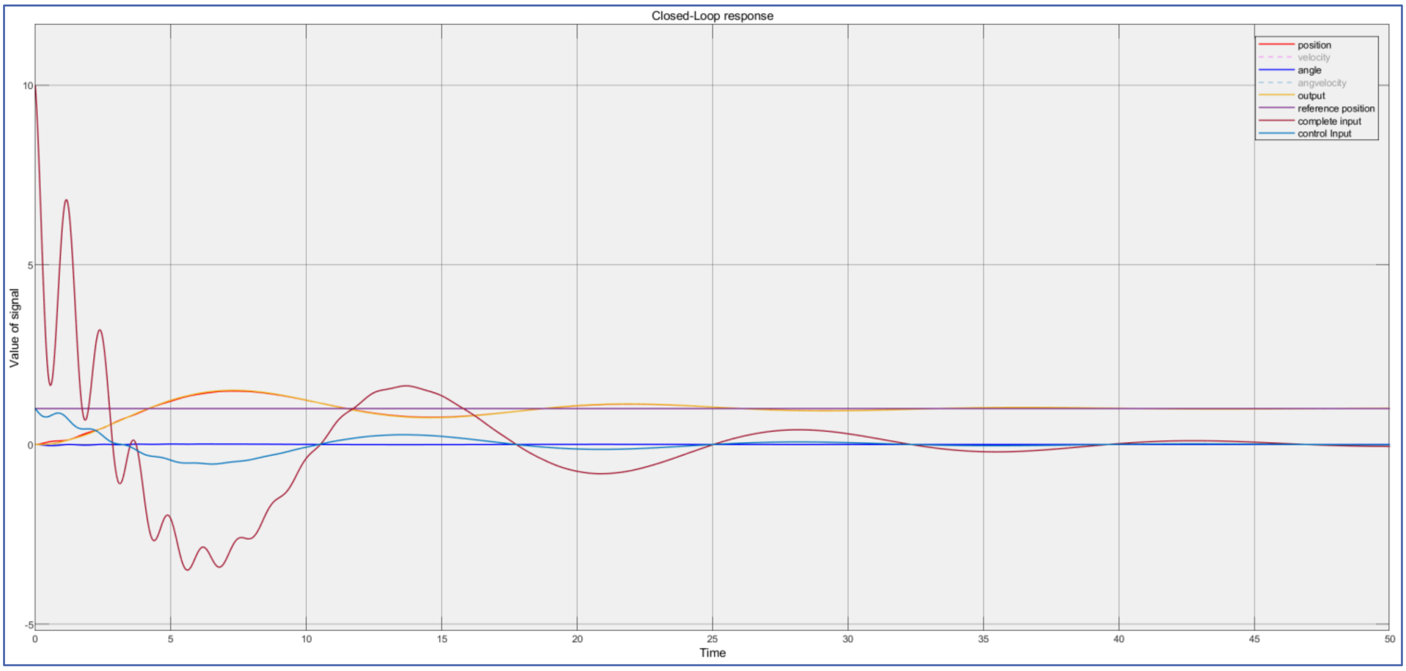
Impulse Response



This is the Closed-Loop impulse response with initial conditions $x_{1,0} = 2$ and $x_{3,0} = 0.5$ (rest are 0).

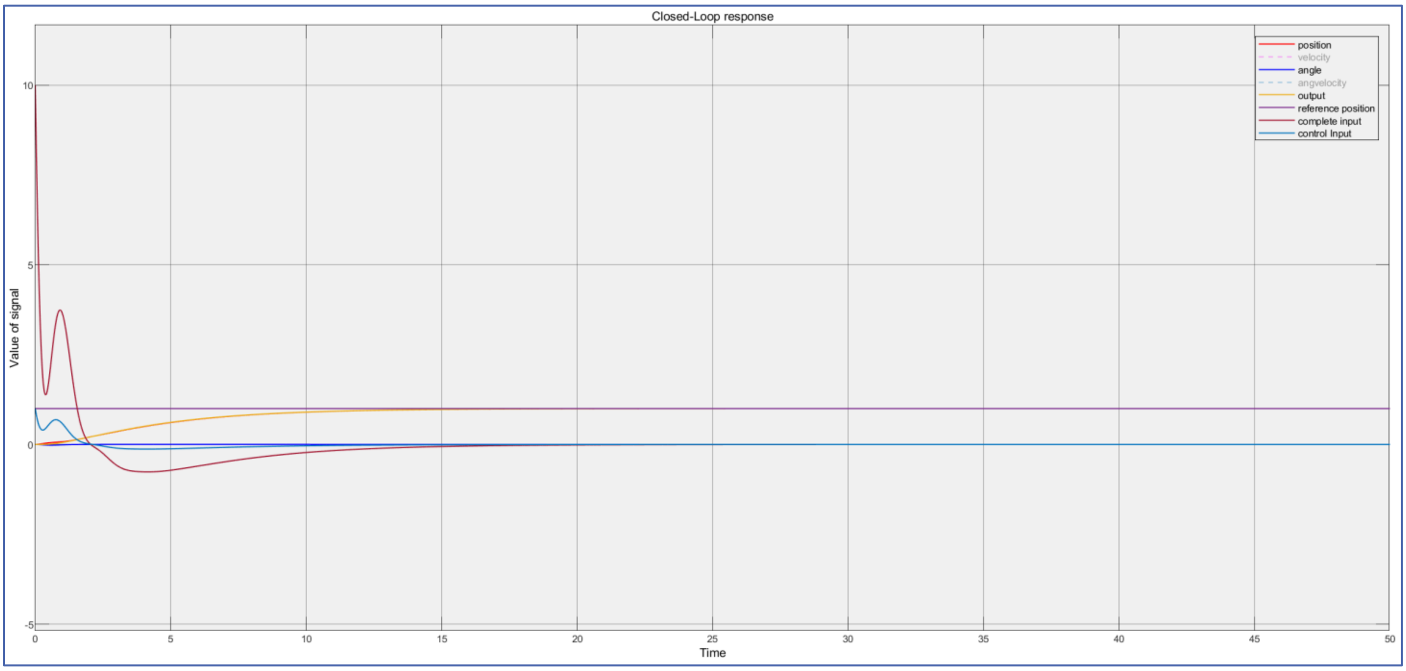
Near the stable equilibrium the dynamics are similar to the zero-input response, and we can see the effect of the controller in driving the system towards equilibrium.

Step Response



This is the step response for the system at stable equilibrium. The control law works well but the response is very slow. The cart position takes ~40 seconds to settle.

We shall now see the effect of changing *b*. We set *b* = 5. The step response is now –

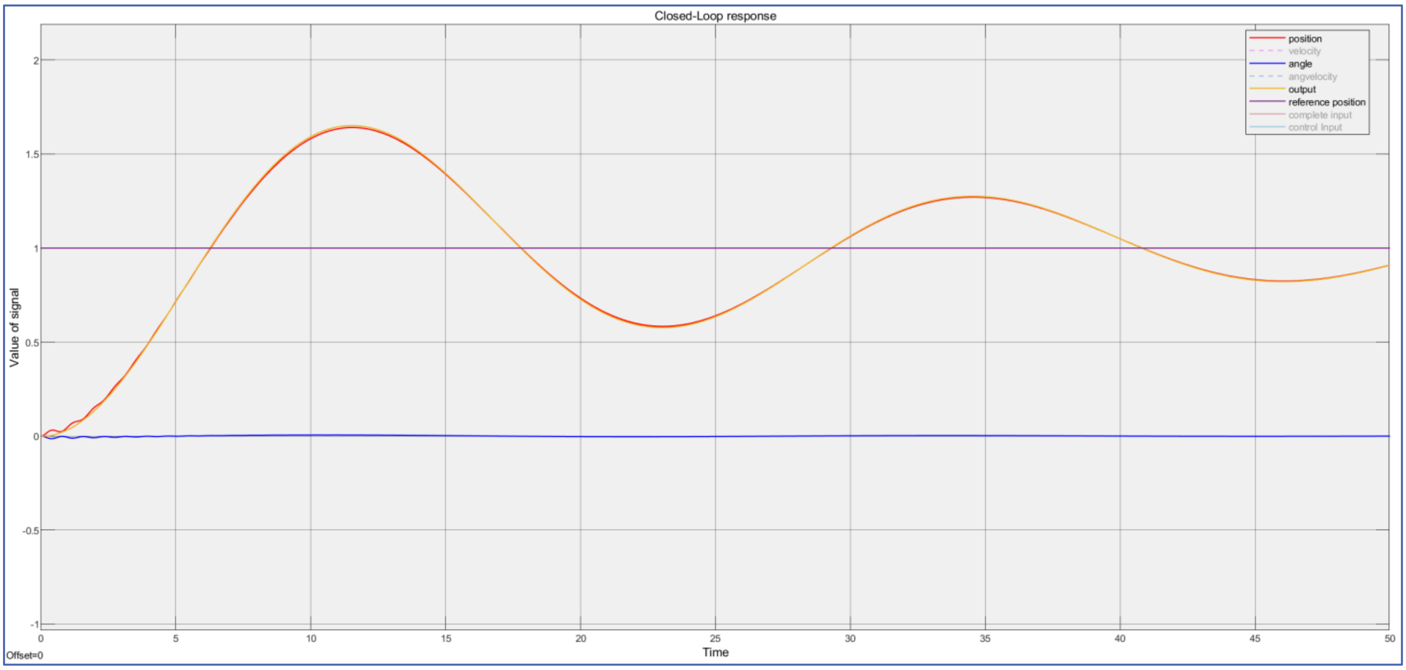


The response is now lag-type, with the position having a 2% settling time of ~16 seconds, a great improvement from the previous value.

Through more precise tuning the critical value of *b* can be found if needed.

INCREASED LOAD MASS

Upon increasing load mass to 60 kg, we obtain the following plot –

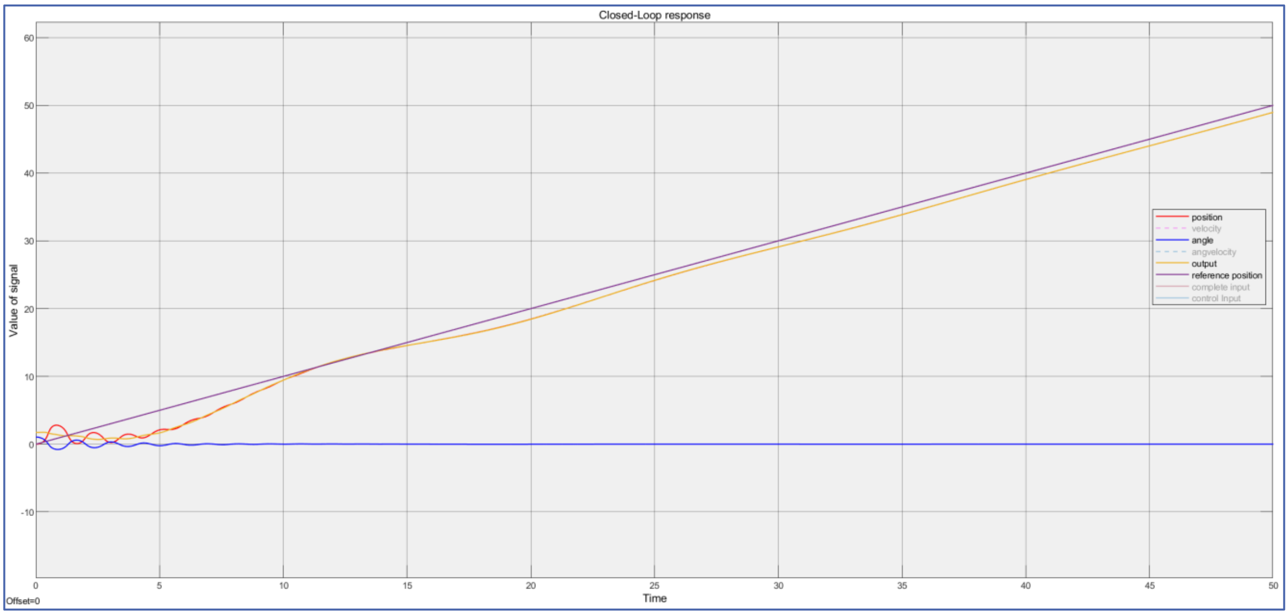


Our observations are the same as in Expt #10 – increased load mass increases frequency of oscillations and increases settling times.

Ramp response

This is the ramp response for initial load mass angle = $\pi/3$, b=1.

We can see that the angle is reduced to 0, as expected. For the cart position however, we see that there is a small offset error that the control cannot remove, but otherwise the tracking works well.



$L_3 = k_3(x_3)^2 + k_4(x_4)^2$

With this function, we obtain v as –

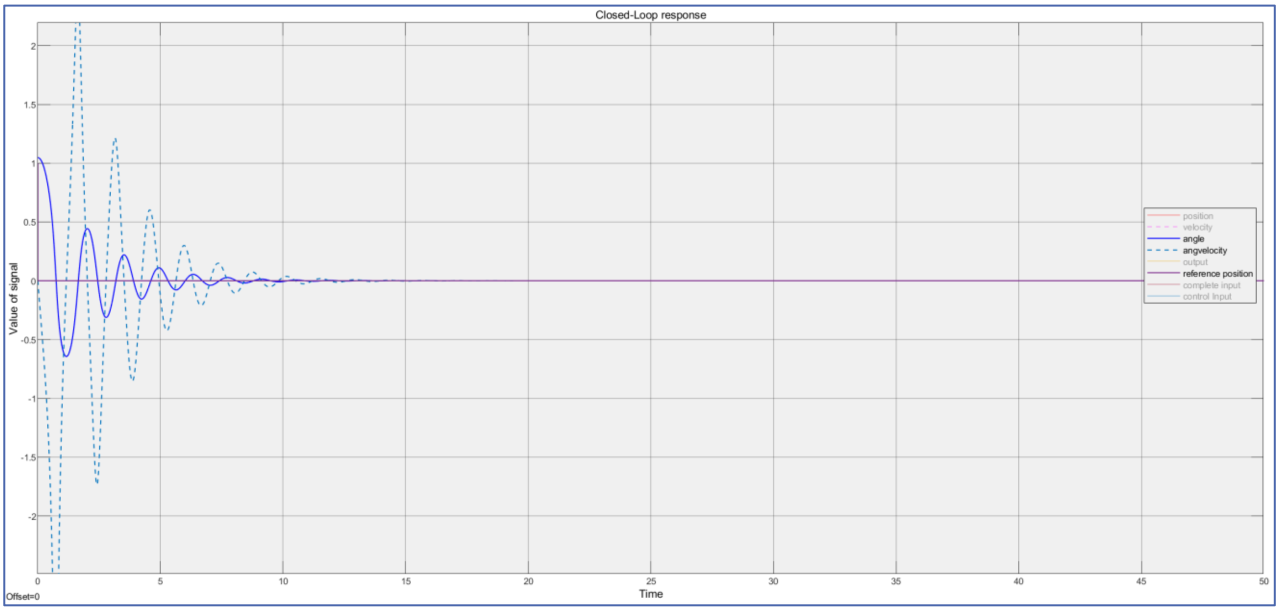
$$\boxed{\frac{b\,l\,x_4 - 2\,g\,k_4\sin(x_3) + 2\,k_3\,l\,x_3}{2\,k_4\cos(x_3)}}$$

Setting k_3 and k_4 to 0.5 as before and implementing this law in Simulink, we obtain the following responses.

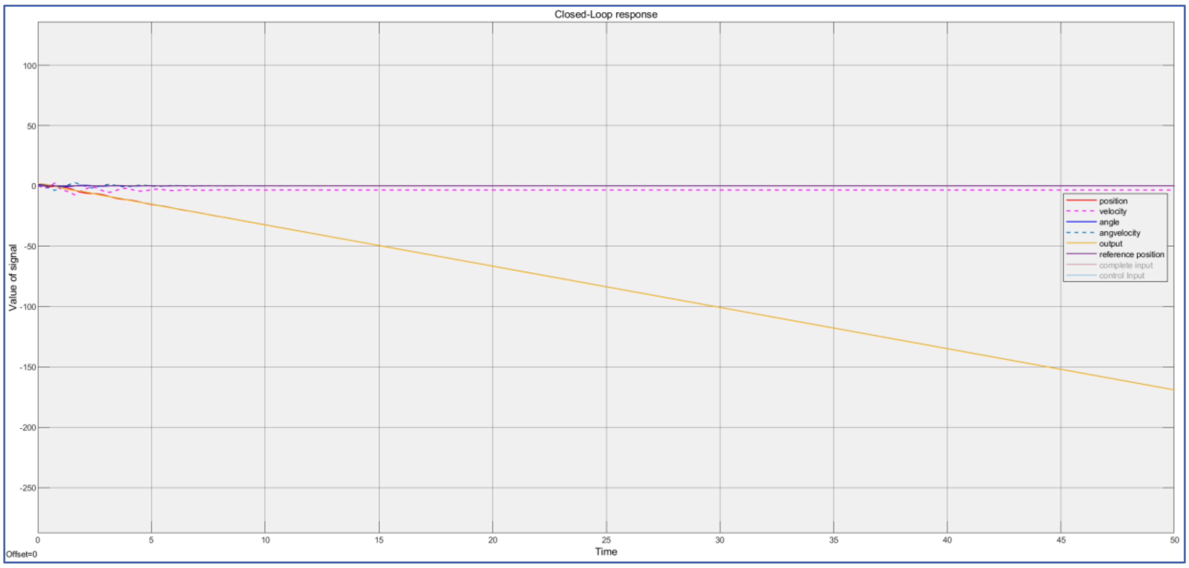
Impulse Response

The initial angle is $\pi/3$.

The angle is reduced to 0 in ~11 seconds, as expected.



However, since we did not take x_1 and x_2 into account in our Lyapunov function, we obtain unstable dynamics in in those states –



State x_2 is affected by x_3 and x_4. The Control law focusses on moving x_3 and x_4 to 0, leaving the other two states unoptimized.

This Lyapunov function is clearly not very useful, and we will not discuss it further.

$$L_4 = k_1\left(x_1 - x_{ref}\right)^2 + k_4(x_4)^2$$

With this function, we obtain v as –

$$\frac{l\left((x_1 - x_{\mathrm{ref}})\, x_2 + b\, x_4{}^2 - \frac{g\, x_4 \sin(x_3)}{l}\right)}{x_4 \cos(x_3)}$$

This type of expression is undesirable, the x_4 term in the denominator causes numerical instability and is difficult to analyse.

$$L_5 = k_1\left(x_1 - x_{ref}\right)^2 + k_2(x_2)^2 + k_3(x_3)^2 + k_4(x_4)^2$$

With this Lyapunov function, we were unable to find a suitable v that did not cause numerical errors and was more useful than the second Lyapunov function derived.

We recommend the addition of cross-terms such as x_1x_2, x_3x_4, etc.

Conclusions

In this experiment we used Lyapunov's 2nd Method to design control laws for the Crane-Trolley system.

We recommend the control law $\left(b\,x_2 + x_1 - x_{ref}\right)$ with the feedback linearization discussed before to control the system.

We also recommend the usage of more complex terms in the Lyapunov functions to possibly obtain better results.

Simulink File

https://drive.google.com/file/d/1XPZM9ivFmmBCdhVKHGMOmoGsqRB2AFUn/view?usp=sharing