We know that:

$$\bar{X}_{12} = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2}{N_1 + N_2}$$

$$8 = \frac{50(10) + 50(\bar{X}_2)}{100}$$

$$800 = 500 + 50\bar{X}_2$$

$$300 = 50\bar{X}_2$$

$$\therefore \quad \bar{X}_2 = \frac{300}{50} = 6$$

$$d_1 = \bar{X}_1 - \bar{X}_{12} = 10 - 8 = 2 \Rightarrow d_1^2 = 4$$

$$d_2 = \bar{X}_2 - \bar{X}_{12} = 6 - 8 = -2 \Rightarrow d_2^2 = 4$$

$$\sigma_{12} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

Substituting the values, we get

$$\sqrt{10.5} = \sqrt{\frac{50 \times 4 + 50\sigma_2^2 + 50 \times 4 + 50 \times 4}{100}}$$

Squaring both sides,

$$10.5 = \frac{200 + 50\sigma_2^2 + 200 + 200}{100}$$

$$10.5 \times 100 = 600 + 50\sigma_2^2$$

$$1050 = 600 + 50\sigma_2^2$$

$$\therefore \quad 50\sigma_2^2 = 450$$

$$\therefore \quad \sigma_2^2 = \frac{450}{50} = 9$$

$$\Rightarrow \quad \sigma_2 = 3$$

Thus, $\bar{X}_2 = 6$, $\sigma_2 = 3$

Example 26. Find the missing information from the following:

	Group I	Group II	Group III	Combined
Number	50	—	90	200
Standard Deviation	6	7	—	7.746
Mean	113	—	115	116

Solution: We are given:

$$N = N_1 + N_2 + N_3 = 200 \quad N_1 = 50, \ N_3 = 90$$

$$\therefore \quad N_2 = N - (N_1 + N_3) = 200 - 140 = 60$$

Now, $$\overline{X}_{123} = \frac{N_1\overline{X}_1 + N_2\overline{X}_2 + N_3\overline{X}_3}{N_1 + N_2 + N_3}$$

We are given: $\overline{X}_1 = 113$, $\overline{X}_3 = 115$, $\overline{X}_{123} = 116$

Substituting the values, we get

$$116 = \frac{(50)(113) + (60)(\overline{X}_2) + (90)(115)}{200}$$

$$116 \times 200 = 50 \times 113 + 60\overline{X}_2 + 90 \times 115$$

$$23200 = 5650 + 60\overline{X}_2 + 10350$$

$$\therefore \quad 60\overline{X}_2 = 23200 - 5650 - 10350 = 7200$$

$$\therefore \quad \overline{X}_2 = \frac{7200}{60} = 120$$

$$d_1 = \overline{X}_1 - \overline{X}_{123} = 113 - 116 = -3 \Rightarrow d_1^2 = 9$$

$$d_2 = \overline{X}_2 - \overline{X}_{123} = 120 - 116 = 4 \Rightarrow d_2^2 = 16$$

$$d_3 = \overline{X}_3 - \overline{X}_{123} = 115 - 116 = -1 \Rightarrow d_3^2 = 1$$

$$\sigma_{123} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_2\sigma_3^2 + N_1 d_1^2 + N_2 d_2^2 + N_3 d_3^2}{N_1 + N_2 + N_3}}$$

We are given: $\sigma_{123} = 7.745$, $\sigma_1 = 6$, $\sigma_2 = 7$

Substituting the values, we get

$$7.746 = \sqrt{\frac{50(36) + 60(49) + 90\sigma_3^2 + 50(9) + 60(16) + 90(1)}{50 + 60 + 90}}$$

$$7.746 = \sqrt{\frac{1800 + 2940 + 90\sigma_3^2 + 450 + 960 + 90}{200}}$$

$$7.746 = \sqrt{\frac{6{,}240 + 90\sigma_3^2}{200}}$$

Squaring both sides,

$$(7.746)^2 = \frac{6{,}240 + 90\sigma_3^2}{200}$$

$$12000 = 6240 + 90\sigma_3^2$$

$$\Rightarrow \quad 90\sigma_3^2 = 12000 - 6240 = 5760$$

$$\Rightarrow \quad \sigma_3^2 = \frac{5760}{90} = 64$$

$$\Rightarrow \quad \sigma_3 = \sqrt{64} = 8$$

Thus, $N_2 = 60$, $\overline{X}_2 = 120$, $\sigma_3 = 8$