SET THEORY

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SET THEORY

- **SET**

  The collection of well-defined distinct objects is known as a set.

- **For example:**

  1. The collection of children in class VII whose weight exceeds 35 kg represents a set.

  2. The collection of all the intelligent children in class VII.

  It does not represent a set because the word intelligent is vague

- **Notation of a Set:**

-A set is usually denoted by capital letters and elements are denoted by small letters

- **Elements of Set:**

-The different objects that form a set are called the elements of a set.

-The elements of the set are written in any order and are not repeated.

-Elements are denoted by small letters.

- **For example:**
  - The collection of vowels in the English alphabet.
  V = [a, e, i, o, u].

PROPERTIES OF SET

- **The change in order of writing the elements does not make any changes in the set.**

For Example:

Set A = {4, 6, 7, 8, 9} is same as setA = {8, 4, 9, 7, 6}

- **If one or many elements of a set are repeated, the set remains the same.**

For Example:

U = {letters of the word 'COMMITTEE'};

then U = {C, O, M,I, T, E}

Representation of a Set

Statement form method	• A={ Even numbers greater than 6 and less than 14 }
Roster or tabular form method	• A= { 8, 10, 12}
Rule or set builder form method	• A = {x | x is an even number, $6 < x < 14$}

Different Notations in Sets

$\in$	Belongs to
$\notin$	Does not belongs to
: or |	Such that
$\emptyset$	Null set or empty set
n(A)	Cardinal number of the set A
$\cup$	Union of two sets
$\cap$	Intersection of two sets
N	Set of natural numbers = {1, 2, 3, ……}
W	Set of whole numbers = {0, 1, 2, 3, ………}
I or Z	Set of integers = {………, -2, -1, 0, 1, 2, ………}
Z+	Set of all positive integers
Q	Set of all rational numbers
Q+	Set of all positive rational numbers
R	Set of all real numbers
R+	Set of all positive real numbers
C	Set of all complex numbers

Standard Sets of Numbers

N = Natural numbers

= Set of all numbers starting from 1 → Statement form

= Set of all numbers 1, 2, 3, ………..

= {1, 2, 3, …….} → Roster form

= {x :x is a counting number starting from 1} → Set builder form

Therefore, the set of natural numbers is denoted by N i.e., N = {1, 2, 3, ……}

W = Whole numbers

= Set containing zero and all natural numbers $\rightarrow$ Statement form

= {0, 1, 2, 3, …….} $\rightarrow$ Roster form

= {x :x is a zero and all natural numbers} $\rightarrow$ Set builder form

Therefore, the set of whole numbers is denoted by W i.e., W = {0, 1, 2, …….}

Z or I = Integers

= Set containing negative of natural numbers, zero and the natural numbers

→ Statement form

= {………, -3, -2, -1, 0, 1, 2, 3, …….} → Roster form

= {x :x is a containing negative of natural numbers, zero and the natural numbers}

→ Set builder form

Therefore, the set of integers is denoted by I or Z i.e., I = {…., -2, -1, 0, 1, 2, ….}

E = Even natural numbers.

= Set of natural numbers, which are divisible by 2 → Statement form

= {2, 4, 6, 8, ……….} → Roster form

= {x :x is a natural number, which are divisible by 2} → Set builder form

Therefore, the set of even natural numbers is denoted by E i.e., E = {2, 4, 6, 8,…….}

O = Odd natural numbers.

= Set of natural numbers, which are not divisible by 2 → Statement form

= {1, 3, 5, 7, 9, ……….}

→ Roster form

= {x :x is a natural number, which are not divisible by 2} → Set builder form

Therefore, the set of odd natural numbers is denoted by **O** i.e., **O** = {1, 3, 5, 7, 9,…….}

Exercise 1

1) Solve using the three methods of representation of a set:

   The set of integers lying between -2 and 3.

2) Write the set of vowels used in the word 'UNIVERSITY'.

3) For each statement, given below, state whether it is true or false along with the explanations.

(i) $\{9, 9, 9, 9, 9, \ldots\ldots\} = \{9\}$

(ii) $\{p, q, r, s, t\} = \{t, s, r, q, p\}$

1) **Statement form:** {I is a set of integers lying between -2 and 3}

Roster form: I = {-1, 0, 1, 2}

Set builder form: I = $\{x : x \in I, -2 < x < 3\}$

2) Set V = {U, I, E}

3) (i) {9, 9, 9, 9, 9, ……..} = {9}

True, since repetition of elements does not change the set.

(ii) {p, q, r, s, t} = {t, s, r, q, p}

True, since the change in order of writing the elements does not change the set.

CARDINALITY OF A SET

- The number of distinct elements in a given set A is called the cardinal number of A or Cardinality of set A.
- It is denoted by n(A).

For example:

- A $\{x : x \in N, x < 5\}$

A = {1, 2, 3, 4}

Therefore, n(A) = 4

- B = set of letters in the word ALGEBRA

B = {A, L, G, E, B, R}

Therefore, n(B) = 6

TYPES OF SETS

- <u>EMPTY SET OR NULL SET</u>:
- A set which does not contain any element is called an empty set, or the null set or the void set and it is denoted by $\emptyset$ and is read as phi.
- In roster form, $\emptyset$ is denoted by $\{\}$.
- An empty set is a finite set, since the number of elements in an empty set is finite, i.e., 0.
- Example: $N = \{x : x \in N, 3 < x < 4\} = \emptyset$

* Its cardinality is 1.

- SINGLETON SET:

A set which contains only one element is called a singleton set.

For example:

- A = {x : x is neither prime nor composite}

It is a singleton set containing one element, i.e., 1.

- B = {x : x is a whole number, x < 1}

This set contains only one element 0 and is a singleton set.

* Its cardinality is 1

- FINITE SET:

A set which contains a definite number of elements is called a finite set. Empty set is also called a finite set.

For example:

- The set of all colors in the rainbow.
- $N = \{x : x \in N, x < 7\}$
- $P = \{2, 3, 5, 7, 11, 13, 17, \ldots\ldots 97\}$

- INFINITE SET:

The set whose elements cannot be listed, i.e., set containing never-ending elements is called an infinite set.

For example:

- Set of all points in a plane
- $A = \{x : x \in N, x > 1\}$
- Set of all prime numbers
- $B = \{x : x \in W, x = 2n\}$

Note:

All infinite sets cannot be expressed in roster form.

For example:

The set of real numbers since the elements of this set do not follow any particular pattern.

- EQUIVALENT SETS:
- Two sets A and B are said to be equivalent if their cardinal number is same, i.e.,

  $n(A) = n(B)$.
- The symbol for denoting an equivalent set is '$\leftrightarrow$'.

For example:

$A = \{1, 2, 3\}$ Here $n(A) = 3$

$B = \{p, q, r\}$ Here $n(B) = 3$

Therefore, $A \leftrightarrow B$

- EQUAL SETS:
- Two sets A and B are said to be equal if they contain the same elements.
- Every element of A is an element of B and every element of B is an element of A.

  For example:

  $A = \{p, q, r, s\}$

  $B = \{p, s, r, q\}$

  Therefore, $A = B$
- NOTE:

  Equal sets are always equivalent.
  Equivalent sets may not be equal

- OVERLAPPING SETS:
- Two sets A and B are said to be overlapping if they contain at least one element in common.

  For example;

  - $A = \{a, b, c, d\}$

  $B = \{a, e, i, o, u\}$

  - $X = \{x : x \in N, x < 4\}$

  $Y = \{x : x \in I, -1 < x < 4\}$

  Here, the two sets contain three elements in common, i.e., (1, 2, 3)

- <u>DISJOINT SETS</u>:
- Two sets A and B are said to be disjoint, if they do not have any element in common.

  For example;

  A = {x : x is a prime number}

  B = {x : x is a composite number}.

  Clearly, A and B do not have any element in common and are disjoint sets.

SUBSET

- If A and B are two sets, and every element of set A is also an element of set B, then A is called a subset of B and we write it as $\mathbf{A \subseteq B}$ or $\mathbf{B \supseteq A}$

- The symbol $\subset$ stands for 'is a subset of' or 'is contained in'
  - Every set is a subset of itself, i.e., $A \subset A$, $B \subset B$.
  - Empty set is a subset of every set.
  - Symbol '$\subseteq$' is used to denote 'is a subset of' or 'is contained in'.
  - $A \subseteq B$ means A is a subset of B or A is contained in B.
  - $B \subseteq A$ means B is superset of A or B contains A.

SUBSET

- **For example;**

  Let $A = \{2, 4, 6\}$

  $B = \{6, 4, 8, 2\}$

  Here A is a subset of B

  Since, all the elements of set A are contained in set B.

  But B is not the subset of A

  Since, all the elements of set B are not contained in set A.

- **Notes:**
  - If $A \subset B$ and $B \subset A$, then $A = B$, i.e., they are equal sets.
  - Every set is a subset of itself.
  - **Null set** or $\emptyset$ is a subset of every set.

SUBSET

- Let $A = \{1, 2, 3, 4\}$

  $B = \{4, 5, 6, 7\}$

  Here $A \not\subset B$ and also $B \not\subset C$

  [$\not\subset$ denotes 'not a subset of']

- The set N of natural numbers is a subset of the set Z of integers and we write $N \subset Z$.
-

SUPERSET

- Whenever a set A is a subset of set B, we say the B is a superset of A and we write, $B \supseteq A$.

  Symbol $\supseteq$ is used to denote 'is a super set of'

- **For example;**

  $A = \{a, e, i, o, u\}$

  $B = \{a, b, c, \ldots\ldots\ldots\ldots, z\}$

  Here $A \subseteq B$ i.e., A is a subset of B but $B \supseteq A$ i.e., B is a super set of A

PROPER SUBSET

- If A and B are two sets, then A is called the proper subset of B if $A \subseteq B$ but $B \supseteq A$ i.e., $A \neq B$. The symbol '$\subset$' is used to denote proper subset. Symbolically, we write $A \subset B$.

- **For example;**

  $A = \{1, 2, 3, 4\}$

  $B = \{1, 2, 3, 4, 5\}$

  We observe that, all the elements of A are present in B but the element '5' of B is not present in A.

  So, we say that A is a proper subset of B. Symbolically, we write it as $A \subset B$

- **Notes:**

- No set is a proper subset of itself.
- Null set or ∅ is a proper subset of every set.
- Empty set is proper subset of every set.

POWER SET

- The collection of all subsets of set A is called the power set of A. It is denoted by P(A). In P(A), every element is a set.

- **For example;**

If $A = \{p, q\}$ then all the subsets of A will be

$P(A) = \{\emptyset, \{p\}, \{q\}, \{p, q\}\}$

Number of elements of $P(A) = n[P(A)] = 4 = 2^2$

In general,

$n[P(A)] = 2^m$

where m is the number of elements in set A.

UNIVERSAL SET

- A set which contains all the elements of other given sets is called a **universal set**.
- The symbol for denoting a universal set is $\cup$ or ξ.

For example;

1. If A = {1, 2, 3} B = {2, 3, 4} C = {3, 5, 7}

then U = {1, 2, 3, 4, 5, 7}

[Here $A \subseteq U$, $B \subseteq U$, $C \subseteq U$ and $U \supseteq A$, $U \supseteq B$, $U \supseteq C$]

2. If P is a set of all whole numbers and Q is a set of all negative numbers then the universal set is a set of all integers.

3. If A = {a, b, c} B = {d, e} C = {f, g, h, i}

then U = {a, b, c, d, e, f, g, h, i} can be taken as universal set.

OPERATIONS ON SETS

- UNION

Union of the sets A and B is defined to be the set of all those elements which belong to

A or B or both and is denoted by Au B, i.e.,

A v B = {x: x E A or x E B}

Let A= {1, 2, 3}, B = {3, 4, 5, 6},

Au B = {1, 2, 3, 4, 5, 6}.

- INTERSECTION

Intersection of two sets A and B is the set of a ll those elements which belong to both A and Band is denoted by An B,

i.e., A n B = {x: x E A and x E B}

Let A= {a, b, c, d}, B ={a, b, l, m},

An B ={a, b}.

- **DIFFERENCE**

The difference of two sets A and B is a set of all those elements which belong to A but do not belong to B and is denoted

by A - B or A/B,

i.e., A - B = {x: x E A and x e/ B}

e.g., Let A= {a, b, c, d}, B = {d, l, m, n}

then A- B ={a, b, c}.

Note. The set A- B or A/B is also known as relative complement of B w.r.t. A.

- **COMPLEMENT**

The complement of a set A is a set of all those elements of the universal set which do not belong to A and is denoted by N,

i.e., N = U- A= {x: x E U and x e A} = {x: x e A}

e.g., Let U be the set of all natural numbers.

Let Then A= {1, 2, 3}, A^c ={all natural numbers except 1, 2, 3}.

Note. The set N is also known as absolute complement of the set A

SYMMETRIC DIFFERENCE

The symmetric difference of two sets A and B is the set containing all the elements that are in A or in B but not in both and is denoted by A ΘB.

i.e., $A \oplus B = (A \cup B) \setminus (A \cap B)$ or

$A \oplus B = (A \setminus B) \cup (B \setminus A)$

Eg. A={1,2,3,4,5}   B={4,5,6,7,8}

(A u B)= { 1,2,3,4,5,6,7,8}

(A n B)= {4,5}

$A \oplus B$ = (A u B) - (A n B) = { 1,2,3,6,7,8}

General Identities of Set Theory

(i) Idempotent Laws

a) AuA =A

b) An A = A

(ii) Associative Laws

(a) (Au B) u C = A u (B u C)

(b) (An B) n C = An (B n C)

(iii) Commutative Laws

(a) Au B = BuA

(b) An B = B n A

(iv) Distributive Laws

(a) A u (B n C) = (Au B) n (A u C)

(b) An (B u C)= (An B) u (An C)

* Refer book for proofs of these identities

(v) De Morgan's Laws

(a) (Au B)^c = A^c n B^c

(b) (A n B)^c = A^c u B^c

(vi) Identity Laws

(a) Au Φ =A

(b) An U = A

(c) Au U = U

(d) A n Φ = Φ

(vii) Complement Law

a) Au A^c =U

b) An A^c = Φ

c) U^c = Φ

d) Φ^c = U

(viii) Involution Law

a) (A^c)^c =A

CARTESIAN PRODUCT OF TWO SETS

- The Cartesian product of two sets P and Q in that order is the set of all ordered pairs whose first member belongs to the set P and second member belongs to set Q and is denoted by P x Q

i.e., P x Q = {(x, y): x € P, y € Q}.

Ax B x C ={(a, b, c): a € A, b € B, c € C}.

- Let P = {a, b, c) and Q = {k, l, m, n}.

PXQ= {(a, k), (a, l), (a, m), (a, n),(b, k), (b, l),
(b, m), (b, n),(c, k), (c, l), (c, m), (c, n) }

- Let A= {1, 2, 4, 5}, B = {a, b, c, f}, C = {a, 5}

Sol: Au C = {1, 2, 4, 5, a}

(Au C) x B = {1, 2, 4, 5, a} x {a, b, c, f}

= {(1, a), (1, b), (1, c), (1, f), (2, a), (2, b),(2, c), (2, f), (4, a), (4, b), (4, c), (4, f), (5, a), (5, b),
(5, c), (5, f),(a, a), (a, b), (a, c), (a, f)}

EXERCISE

1) Determine the power set P(A) of the set A = {1, 2, 3}.
2) If A = {1, 2, 5, 6}, B = {2. 5, 7},

   C = {1, 3, 5, 7, 9}. U = {1, 2, 3, 4, 5, 6, 7, 8, 9},

   Find

   a) AΠ B
   b) B u C
   c) A^c
   d) A- B
   e) A – C
   f) A $\oplus$ B
   g) A $\oplus$ C
   h) (A v C) – B
   i) (A v B)^c
   j) (B $\oplus$ C) - A

SOLUTIONS

1) P(A) = {{1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}, Φ}.

2) (a) An B = {2, 5}

(b) B u C = {1, 2, 3, 5, 7, 9}

(c) A^c = {3, 4, 7, 8, 9}

(d) A- B = {1, 6}

(e) A- C = {2, 6}

(f) A ⊕ B = {1, 6, 7}

(g) A ⊕ C = {2, 3, 6, 7, 9}

(h) (Au C)- B = {1, 3, 6, 9}

(i) (Au B)^c = {3, 4, 8, 9}

(j) (B ⊕ C) - A= {3, 9}

FINITE SETS & COUNTING

- Sets can be finite or infinite.
- A set S is said to be finite if S is empty or if S contains exactly m elements where m is a positive integer; otherwise S is infinite.
- Ex: The set A of the letters of the English alphabet and the set D of the days of the week are finite sets.

  Specifically, A has 26 elements and D has 7 elements. So both are finite sets.

Counting Elements in Finite Sets:

- The notation $n(S)$ or $|S|$ will denote the number of elements in a set S.

  Thus $n(A) = 26$, where A is the letters in the English alphabet,

  $n(D) = 7$, where D is the days of the week.

  $n(\emptyset) = 0$ since the empty set has no elements.

- **Suppose A and B are finite disjoint sets**.

  Then $A \cup B$ is finite and

  $n(A \cup B) = n(A) + n(B)$

  $n(A \setminus B) = n(A) - n(A \cap B)$

For example: Let an art class A has 25 students and 10 of them are taking a biology class B. Then the number of students in class A which are not in class B is: $n(A \setminus B) = n(A) - n(A \cap B) = 25 - 10 = 15$

Inclusion–Exclusion Principle

- Suppose A and B are finite sets and they are not disjoint.

  Then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

- Also suppose A, B, C are finite sets.

  Then $A \cup B \cup C$ is finite and

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

- <u>Exercise Que</u>:

  Suppose a list A contains the 30 students in a mathematics class, and a list B contains the 35 students in an English class, and suppose there are 20 names on both lists. Find the number of students:
  (a) only on list A, (b) only on list B,
  (c) on list A or B (or both), (d) on exactly one list.

- Solution:

- (a) List A has 30 names and 20 are on list B; hence $30 - 20 = 10$ names are only on list A.
- (b) Similarly, $35 - 20 = 15$ are only on list B.
- (c) We seek $n(A \cup B)$. By inclusion–exclusion, $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 30 + 35 - 20 = 45$.

  In other words, we combine the two lists and then cross out the 20 names which appear twice.
- (d) By (a) and (b), $10 + 15 = 25$ names are only on one list; that is, $n(A \oplus B) = 25$.

CLASSES OF SETS, POWER SETS, PARTITIONS

- Suppose S = {1, 2, 3, 4}.

(a) Let A be the class of subsets of S which contain exactly three elements of S.

Then A = [{1, 2, 3},{1, 2, 4},{1, 3, 4},{2, 3, 4}]

That is, the elements of A are the sets {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, and {2, 3, 4}.

- (b) Let B be the class of subsets of S, each which contains 2 and two other elements of S.

Then B = [{1, 2, 3},{1, 2, 4},{2, 3, 4}]

The elements of B are the sets {1, 2, 3}, {1, 2, 4}, and {2, 3, 4}.

Thus B is a subclass of A, since every element of B is also an element of A

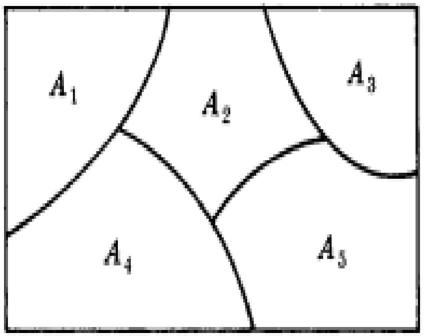
- POWER SET: The class of all subsets of S is called the power set of S.

  n(P (S)) = 2^n(S)

- Suppose S = {1, 2, 3}.

  P(S) has 2^3 = 8 elements.

  Then P (S) = [∅,{1},{2},{3},{1, 2},{1, 3},{2, 3}, S]



- PARTITIONS:
- Let S be a nonempty set.

  A partition of S is a subdivision of S into non-overlapping, nonempty subsets.

  A partition of S is a collection {Ai} of nonempty subsets of S such that:

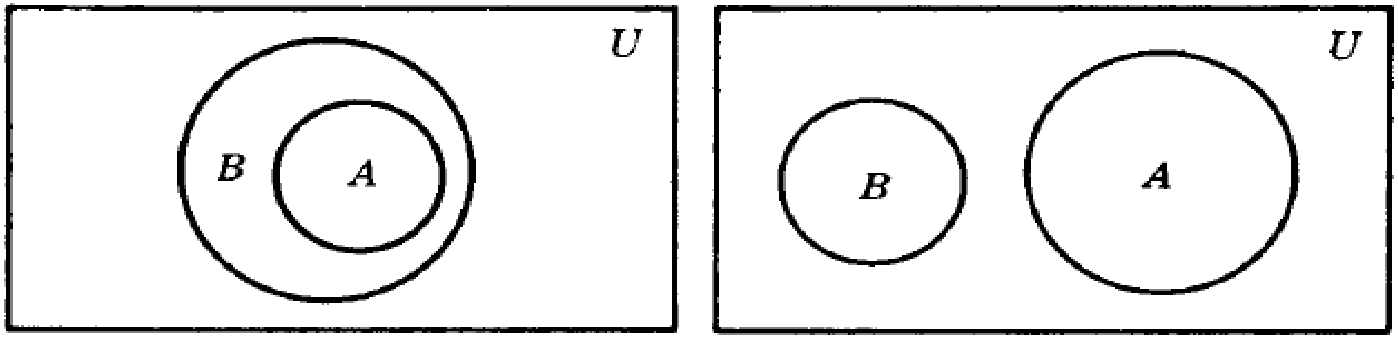
  (i) Each a in S belongs to one of the Ai.

  (ii) The sets of {Ai} are mutually disjoint; that is,

  if Aj is not = Ak then Aj ∩ Ak = ∅

VENN DIAGRAMS

- Venn diagram is a pictorial representation of sets in which sets are represented by enclosed areas in the plane.
- The universal set U is represented by the interior of a rectangle, and the other sets are represented by disks lying within the rectangle



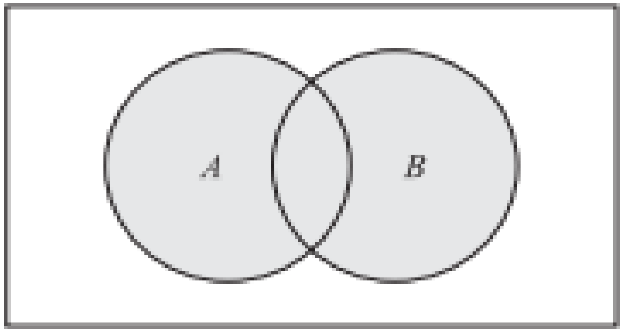
(a) $A \subseteq B$ (b) A and B are disjoint

VENN DIAGRAMS

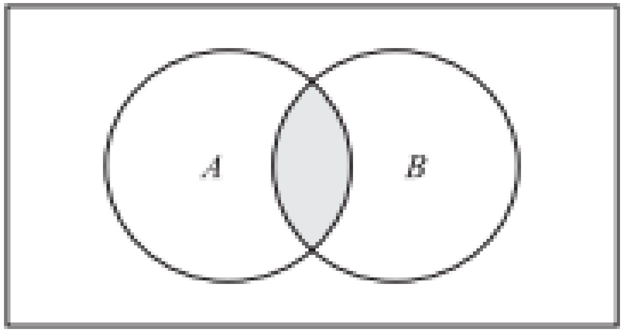
- **Union:** The *union of two sets A and B, denoted by $A \cup B$, is the set of all elements which belong to A or to B;*

  that is, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- **Intersection:** The *intersection of two sets A and B, denoted by $A \cap B$, is the set of elements which belong to both A and B;*

  that is, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



(a) $A \cup B$ is shaded

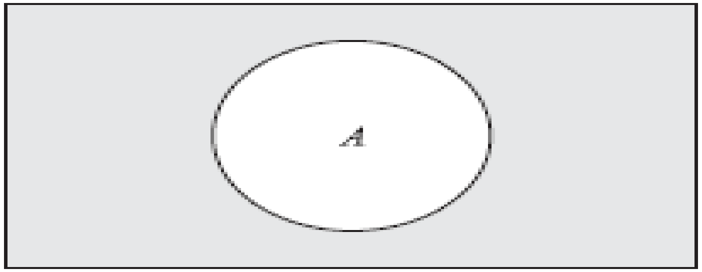


(b) $A \cap B$ is shaded

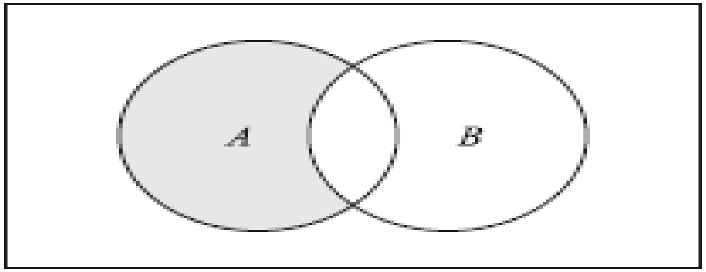
VENN DIAGRAMS

- **Complements :** The *absolute complement or, simply, complement of a set A, denoted by A^c, is the set of elements which belong to U but which* do not belong to *A. That is, A^c= {x | x ∈ U, x / ∈ A}*
- **Differences;** The *relative complement of a set B with respect to a set A or, simply, the difference of A and B, denoted by A \ B, is the set of elements which belong to A but which do not belong to B; that is*

$(A - B)$ or $(A \sim B)$ or $(A \setminus B) = \{x \mid x \in A, x \notin B\}$



(a) A^C is shaded



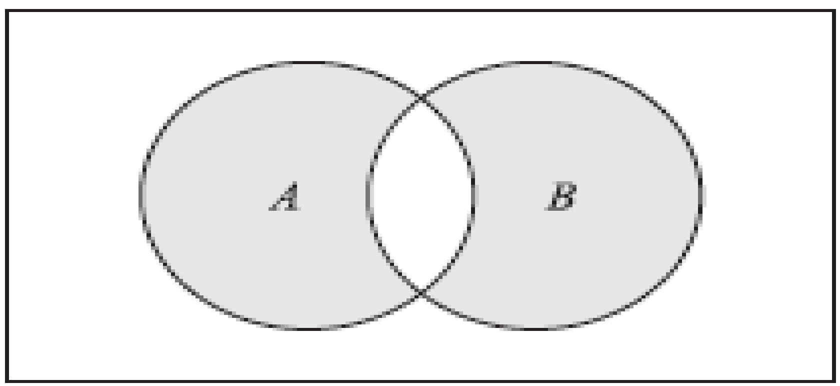
(b) $A \setminus B$ is shaded

VENN DIAGRAMS

- **Symmetric Differences**

The *symmetric difference of sets A and B, denoted by* $A \oplus B$*, consists of those elements which belong to* A or B *but not to both. That is,*

$$A \oplus B = (A \cup B) \setminus (A \cap B) \text{ or } A \oplus B = (A \setminus B) \cup (B \setminus A)$$



Venn diagrams

- (i) There are *$m = 2$^n such fundamental products*.
- (ii) Any two such fundamental products are disjoint.
- (iii) The universal set U is the union of all fundamental products.
- Venn diagram of three sets *A, B, C. The following lists the $m = 2$^$3 = 8$*
- Fundamental products of the sets *A, B, C:*

$P1 = A \cap B \cap C$, $P2 = A \cap B \cap C$^c,

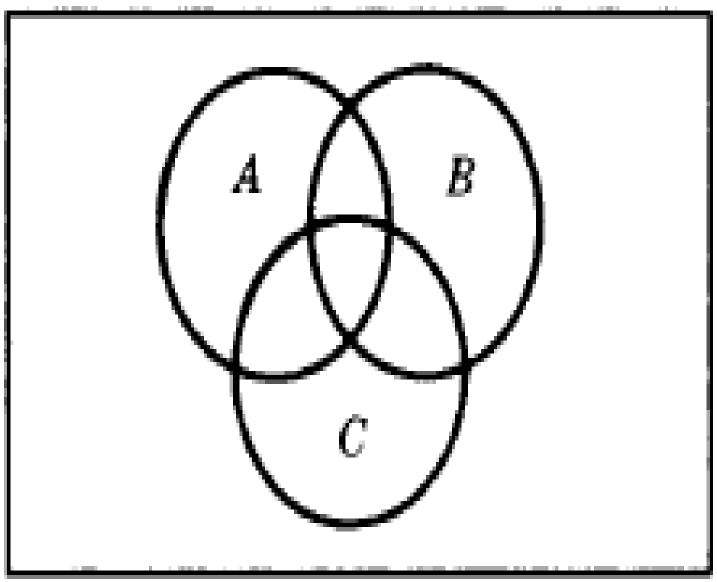
$P3 = A \cap B$^$c \cap C$, $P4 = A \cap B$^$c \cap C$^c,

$P5 = A$^$c \cap B \cap C$, $P6 = A$^$c \cap B \cap C$^c,

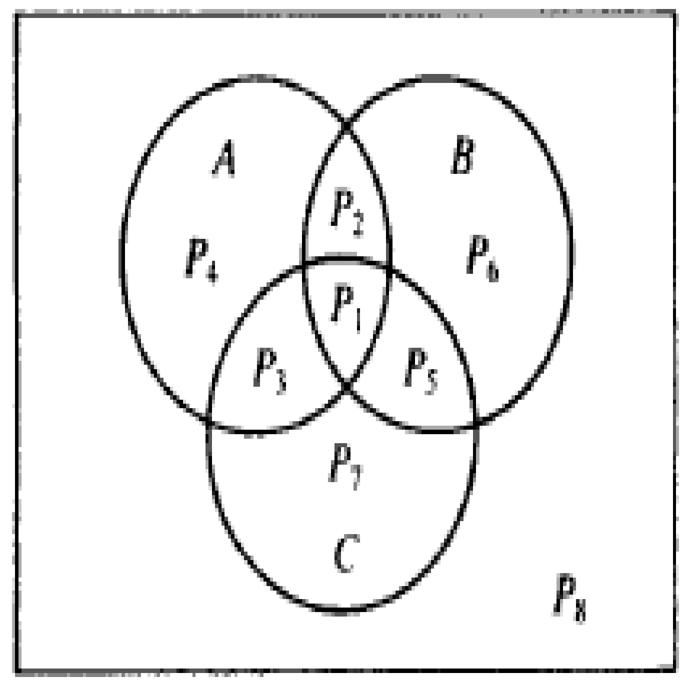
$P7 = A$^$c \cap B$^$c \cap C$, $P8 = A$^$c \cap B$^$c \cap C$^c

Fundamental products of the sets *A, B, C*:

- $P1 = A \cap B \cap C$, $P2 = A \cap B \cap C^c$,
- $P3 = A \cap B^c \cap C$, $P4 = A \cap B^c \cap C^c$,
- $P5 = A^c \cap B \cap C$, $P6 = A^c \cap B \cap C^c$,
- $P7 = A^c \cap B^c \cap C$, $P8 = A^c \cap B^c \cap C^c$



(a)



(b)

EXERCISE:

Q1) Which of these sets are equal:

$\{x, y, z\}, \{z, y, z, x\}, \{y, x, y, z\}, \{y, z, x, y\}$?

Q2) List the elements of each set where $N = \{1, 2, 3, \ldots\}$

(a) $A = \{x \in N \mid 3 < x < 9\}$

(b) $B = \{x \in N \mid x \text{ is even}, x < 11\}$

(c) $C = \{x \in N \mid 4 + x = 3\}$

Q3) Let **U = {1,2, …, 9} be the universal set, and let**

$A = \{1, 2, 3, 4, 5\}$, $C = \{5, 6, 7, 8, 9\}$, $E = \{2, 4, 6, 8\}$,

$B = \{4, 5, 6, 7\}$, $D = \{1, 3, 5, 7, 9\}$, $F = \{1, 5, 9\}$.

Find: (a) $A \cup B$ *and* $A \cap B$; (b) $A \cup C$ *and* $A \cap C$;

(c) $D \cup F$ *and* $D \cap F$. (d) $A^\wedge C$, $B^\wedge C$, $D^\wedge C$, $E^\wedge C$;

(e) $A \setminus B$, $B \setminus A$, $D \setminus E$; (f) $A \oplus B$, $C \oplus D$, $E \oplus F$.

EXERCISE:

Q4) In a survey of 120 people, it was found that:

65 read *Newsweek magazine,* *20 read both Newsweek and Time,*

45 read *Time,* *25 read both Newsweek and Fortune,*

42 read *Fortune,* *15 read both Time and Fortune,*

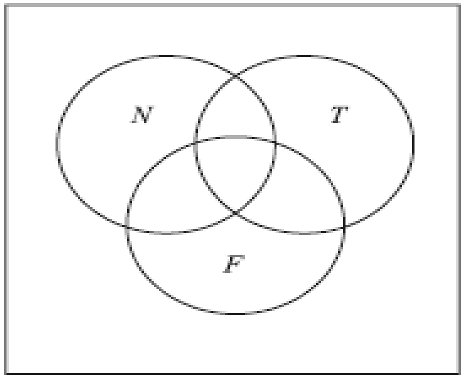
8 read all three magazines.

(*a*) *Find the number of people who read at least one of the three magazines.*

(*b*) *Fill in the correct number of people in each of the eight regions of the Venn diagram in Fig where*

N, T , and F denote the set of people who read Newsweek, Time, and Fortune, respectively.

(*c*) *Find the number of people who read exactly one magazine.*



SOLUTIONS

1)They are all equal. Order and repetition do not change a set.

2) (a)A = {4, 5, 6, 7, 8} ;

(b)B = {2, 4, 6, 8, 10};

(c) No positive integer satisfies 4 + x = 3; hence C = ∅, the empty set.

3) (a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ $A \cap B = \{4, 5\}$

(b) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} = U$ $A \cap C = \{5\}$

(c) $D \cup F = \{1, 3, 5, 7, 9\} = D$ $D \cap F = (1, 5, 9) = F$

(d) $A^\wedge C = \{6, 7, 8, 9\}$; $B^\wedge C = \{1, 2, 3, 8, 9\}$;
$D^\wedge C = \{2, 4, 6, 8\} = E$; $E^\wedge C = \{1, 3, 5, 7, 9\} = D$.

(e) $A \backslash B = \{1, 2, 3\}$; $B \backslash A = \{6, 7\}$;
$D \backslash E = \{1, 3, 5, 7, 9\} = D$; $F \backslash D = \emptyset$.

(f) $A \oplus B = \{1, 2, 3, 6, 7\}$; $C \oplus D = \{1, 3, 6, 8\}$;
$E \oplus F = \{2, 4, 6, 8, 1, 5, 9\} = E \cup F$.

SOLUTIONS

4) (*a) We want to find n(N* $\cup$ *T* $\cup$ *F). (Inclusion–Exclusion Principle),*

$n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap T) - n(N \cap F) - n(T \cap F) + n(N \cap T \cap F)$

$= 65 + 45 + 42 - 20 - 25 - 15 + 8 = 100$

(*b) The required Venn diagram is obtained as follows:*

8 read all three magazines,

20 − 8 = 12 read *Newsweek and Time but not all three magazines,*

25 − 8 = 17 read *Newsweek and Fortune but not all three magazines,*

15 − 8 = 7 read *Time and Fortune but not all three magazines,*

65 − 12 − 8 − 17 = 28 read only *Newsweek,*

45 − 12 − 8 − 7 = 18 read only *Time,*

42 − 17 − 8 − 7 = 10 read only *Fortune,*

120 − 100 = 20 read no magazine at all.

(*c) 28+ 18 + 10 = 56 read exactly one of the magazines.*