DISCRETE MATHEMATICS (IT-14501) (2 MARKS)

Q: Let $A = \{1,2,3,4\}$. Define a relation on A which is reflexive and transitive but not symmetric.

Q- For any set $A = \{a, b, c\}$, write all Proper and improper subsets of set A.

Q- Give an example of a func which is onto but not one-one.

Q- What do you mean by Partial Ordered Set

Q- Verify the proposition $p \vee \neg(p \wedge q)$ is a tautology

Q- State Pigeonhole Principle

Q- Find the chromatic number of a bipartite graph $K_{3,5}$.

Q- Define Tree and minimum spanning tree.

Q- Find the numbers of eleven - letters words that can be formed using MATHEMATICS

Q- Define monoid with example.

Q- Prove that $(A \cup B)^c = A^c \cap B^c$

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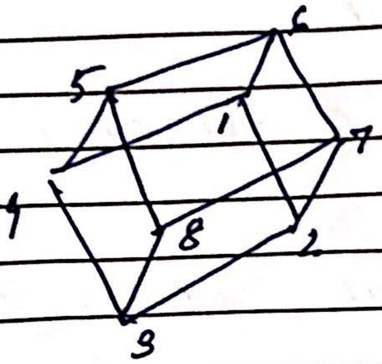
Q- Use Venn Diagram to show
i) $A^c \cap (B-C)$ ii) ~~A^c~~ $A^c \cup (B \cap C)$

Q- construct the truth table of
$(p \Rightarrow q) \rightarrow (p \wedge q)$

Q- Define Total Order Relation with example.

Q Find chromatic number of graph



Q- What is inclusion and exclusion principal

Q- What is bijective fun^c, give an example

Q- How many nodes are required to construct a graph with exactly 6 edges in which each node is of degree 2?

Q- Find the chromatic number of $K_{4,3}$.

Q- Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be defined by $f(x) = 2x+1$ and $g(x) = x^2 - 2$. Find $g \circ f$.

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Q- State significance of hash value?

Q How many diff teams of 7 players can be choosen from 10 players.

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Q- State and prove Euler Theorem

Q- Define cyclic groups with example.

Q- Find cardinality of set Z of integers

Q- ~~Let~~ Define integral domain

Q- What is difference between semigroup and sub-semigroup.

Q- What are the conditions to make two groups isomorphic.

Q- What is chromatic number of $K_{2,3}$

Q- Prove that $A \to A \wedge (A \vee B)$ is tautology

Q- What do you mean by chromatic number? Find chromatic number for bipartite Graph $(K_{5,9})$.

Q- Define partial order relation

Q- A class consists of 40 G and 60 B. In how many ways can be a president, vice-pres, treasurer and secretary be chosen if treasurer must be girl, the secretary must be a boy and a student may not hold more than one office?

Q- Let R be a relation $R = \{(1,1)\ (2,2)\ (2,3)\}$
on set $A = \{1, 2, 3\}$
Find reflexive and symmetric closure
of R.

Q- Define homomorphic and isomorphic
graph.

Q - Define hashing function

Q - Define spanning sub graph

Q - Define inverse relation

Q- Define generating function

Q - Define automorphism

Q- Define integral domain

DISCRETE MATHEMATICS (IT-14501) (5 MARKS)

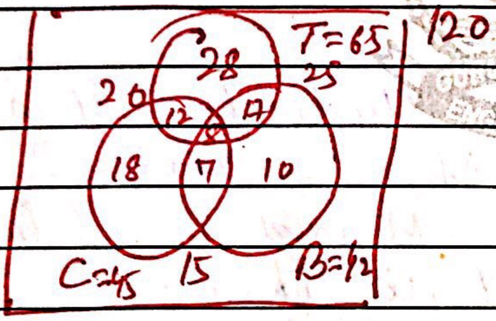
Q- Prove that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

Q- In a survey of 120 people, 65 like T, 20 like both T and C, 45 like C, 25 like both T and B, 42 like B, 15 like both C and B and 8 like all

a) Find no. of people who like atleast one of the three

b) Find no. of people who like only one.

Hints



a) $T \cup C \cup B = 100$

b) $T + C + B = 28 + 18 + 10$
$= 56.$

Q- In how many ways, 6 M books and 5 E books can be arranged on a bookshelf

Also find no. of ways

a) E books should be kept together always

b) All E books kept together either at start of shelf or at end of shelf.

Hint - a) $7! \times 5!$

b) $2! \times 6! \times 5!$

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Q- Using laws of algebra of propositions show that $\neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$

Q- What do you mean by Partial Order Relation?
Give an example of relation which is

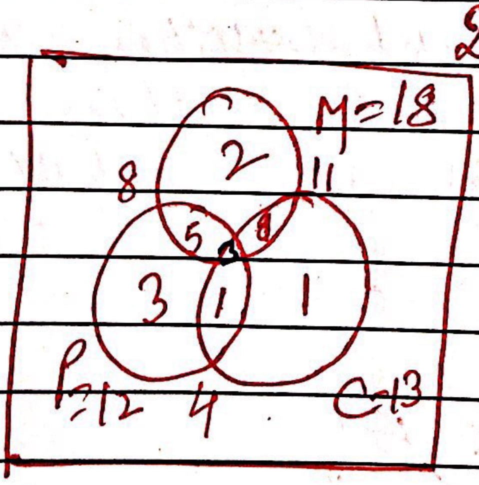
a) Symmetric and Transitive but not Reflexive

b) Irreflexive and Transitive

Q- In a survey of 25 students, it was found that 18 took M, 12 took P, 13 took C, 11 took M and C, 8 took M and P, 4 took P and C, 3 had all three.

a) Find the students that had taken only one of the subjects.

b) Also find the no. of students that had taken none of subjects.



a) $M + P + C = 2 + 3 + 1 = 6$

b) $25 - (M \cup P \cup C)$

$25 - \{18 + 12 + 13 - 8 - 11 - 4 + 3\}$

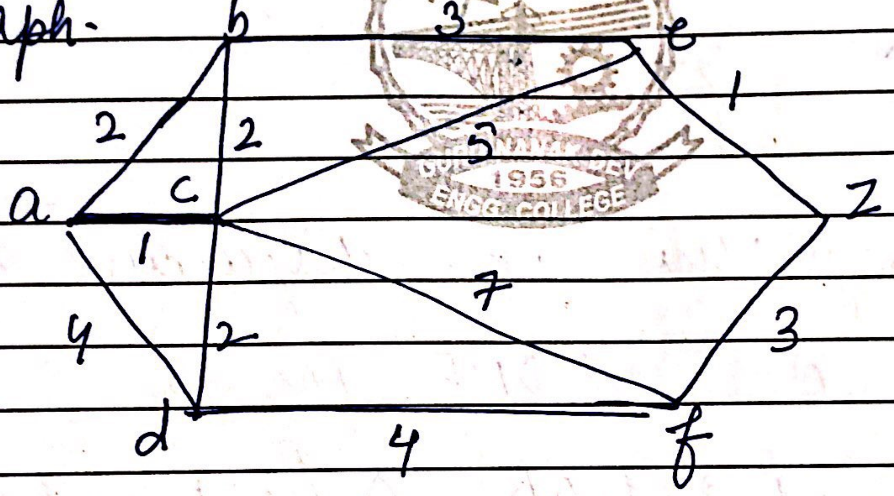
$25 - \{23\} = 2$ Ans

Q. - If p, q, r are 3 statements, then test the validity of the argument $(S_1, S_2; S)$ where $S_1 = p \to q$ $S_2 = p \to r$

$$S : p \to (q \wedge r)$$

Q - Show that every subgroup of a cyclic group is also cyclic

Q - Use Dijkstra Algo to find the shortest path from node a to node z in following graph.



Q - In a town of 20,000 families, it was found that 40% families buy Tribune, 20% buy HT and 10% buy Times. 5% buy Tribune and HT, 3% HT and Ti
4% buy Times and Tribune.
If 2% families buy all three.
Find number of families who buy

1) Tribune only
ii) HT only
iii) None of three.

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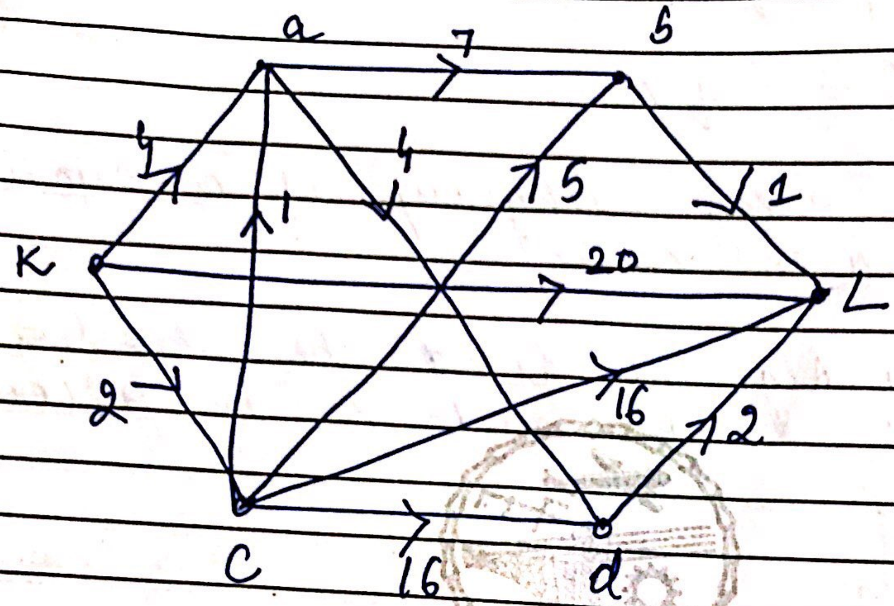
Q: Find the shortest path using Dijkstra's Algo.



Q: Define conjuctive and disjunctive normal forms. Also find CNF and DNF for S.

$$S \Leftrightarrow (P \to (Q \wedge R)) \wedge (\sim P \to (\sim Q \wedge \sim R))$$

Q: If $f, g, h : R \to R$ are defined by

$f(x) = x+2$ $\quad g(x) = \frac{1}{x^2+1}$ and $h(x) = 3$

Find

i) $g(h(f(x)))$

ii) $f^{-1} \circ g \circ f(x)$

iii) $f(g(f(x)))$

iv) $g \circ f^{-1} \circ f(x)$

v) $f(g(h(x)))$

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DISCRETE MATHS (IT-14501) (5 MARKS).

Q: Let repetition of digits in number is allowed. How many 4 digit numbers can be formed using digits 1, 2, 3, 5, 7, 8. How many of them are

a) even → $6 \times 6 \times 6 \times 2 = 432$

b) Multiple of 5 → $6 \times 6 \times 6 \times 1 = 216$

c) Less than 4000 → $3 \times 6 \times 6 \times 6 = 648$

Q: How many integers between 1 and 300 (inclusive) are

a) Divisible by atleast one of 3, 5, 7

b) Div by 3 and 5, not by 7

c) Div by 5 but neither by 3 or 7.

Q: Find the range of following function

$$F(x) = \frac{1}{(2x-3)} \quad (x+1)$$

Q: Show that equivalence R on semigroup (S, +) of positive integers, defined by aRb if and only if $a \equiv b \pmod 2$ is congruence relation.

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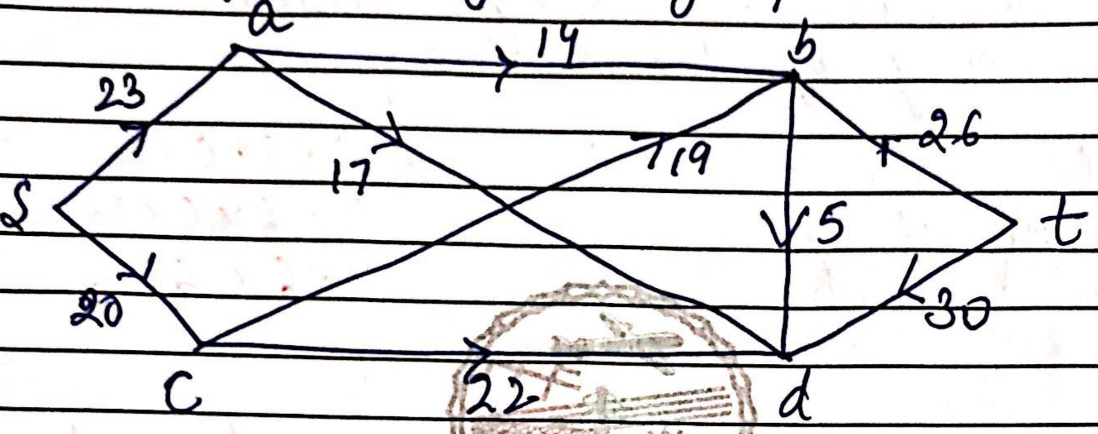
Q: Using BFS algo or Dijkstra's Algo, Find shortest path from S to t in following weighted graph.



Q: Each student in Liberal Arts at some college has a mathematics requirement A and a science requirement B.
A poll of 140 students shows that :

60 completed A , 45 completed B
20 completed A and B.
Use a venn diagram to find no. of student who have completed :

a) Atleast one of A and B
b) Exactly one of A and B
c) neither A nor B.

Q: Show without using truth table that
"It is not sunny this afternoon and it is colder than yesterday."
"We will go swimming only if it is sunny."
"If we donot go swimming, then we will take a canoe trip" and
"If we take a canoe trip, then we will be home by sunset"
lead to conclusion "we will be home by sunset".

Q: State and Prove De-Morgan's Law in Boolean algebra without using Truth Tab

Q: Prove that $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$

Q: If H & K be two subgroups of finite group G, then show that

$[G:H] = [G:K][K:H]$

Q: Solve $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$

where $a_0 = 3$ $a_1 = 7$.

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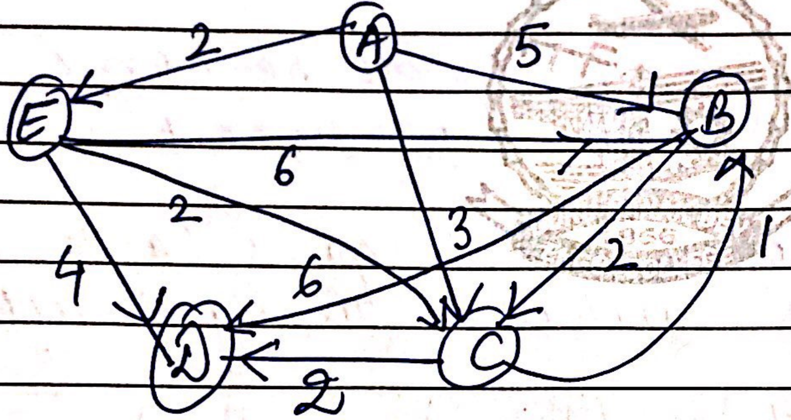
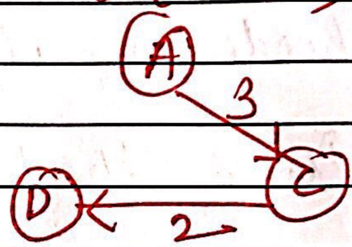
Q: A connected graph G is an Euler graph iff it can be decomposed into circuits. Prove it.

Q: Write Prim's algo to find minimal spanning tree.

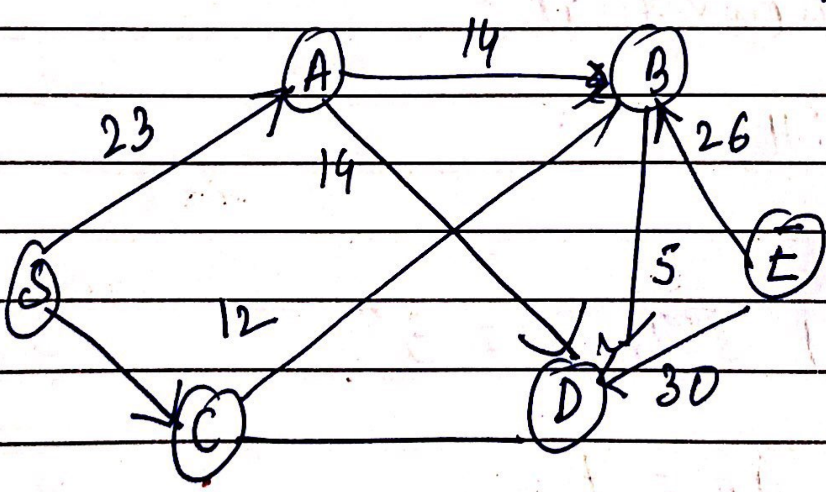
Q: Find shortest path using Dijkstra's algo. Also draw the resultant path.



Hint
Shortest Path
5. (A C D)



Q:) Find Shortest Path using Dykstra's Algo.



Shortest Path
S → E

S C A D B.

DISCRETE MATHS (IT- 14501) (10 MARKS).

Ques: Solve the Recurrence relation using Generation function

Q1) $S_n - 6 S_{n-1} + 8 S_{n-2} = 0$; $n \geq 2$; $S_0 = 10$, $S_1 = 25$

Ans. $S_n = \frac{5}{2} (4^n + 3 \cdot 2^n)$

Q2) $a_K = a_{K-1} + 2 a_{K-2} + 2^K$; $a_0 = 4$ $a_1 = 12$

Ans $\rightarrow$ $G(t) = \frac{38}{9(1-2t)} + \frac{2}{3(1-2t)^2} - \frac{8}{9(1+t)}$

Q3) $a_n - 7 a_{n-1} + 10 a_{n-2} = 6 + 8n$ $n \geq 2$

$a_0 = 1$ and $a_1 = 2$

Q4) $S(K) - 7 S(K-2) + 6 S(K-3) = 0$

$S(0) = 8$ $S(1) = 6$ $S(2) = 22$

Q5) $a_n - 6 a_{n-1} + 8 a_{n-2} = 3^n$

$a_0 = 3$, $a_1 = 7$.

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Q6- $a_{r+2} - 5a_{r+1} + 6a_r = 2$; $a_0 = 1$, $a_1 = 2$

Q7) $a_k = a_{k-1} + 2a_{k-2} + 2^k$

where $a_0 = 4$ and $a_1 = 12$

Q8) $a_n - 4a_{n-1} + 4a_{n-2} = 0$ $n \geq 2$

where $a_0 = 1$ and $a_1 = 2$

Q9) $S(k) - 3S(k-1) - 2 = 0$ $k \geq 1$

where $S(0) = 1$

Ans $S(k) = (-1)^k + 2 \cdot (3)^k$ $k \geq 0$

Q10) $S(k) - 6S(k-1) + 5S(k-2) = 0$

where $S(0) = 1$, $S(1) = 2$

Ans $G(t) = \dfrac{1-4z}{1-6z+5z^2}$

11) If $S_n = 2S_{n-1} + 3S_{n-2}$ $n \geq 2$

where $S(0) = 3$ $S(1) = 1$

Ans $S_n = 3^n + 2(-1)^n$

Q12) If $S(n) + 3\,S(n-1) - 4\,S(n-2) = 0$ $\quad n \geq 2$

where $S(0) = 3$, $S(1) = -2$

Ans – $S(n) = 2 + 5(-4)^n$

Q) State and prove Euler's theorem for Planar Graph

Q) Let G be the set of all 2x2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c, d are real numbers, such that

ab – bc are not equal to zero. Show that G is a non Abelian Group for multiplication of matrices defined as

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{bmatrix}$$

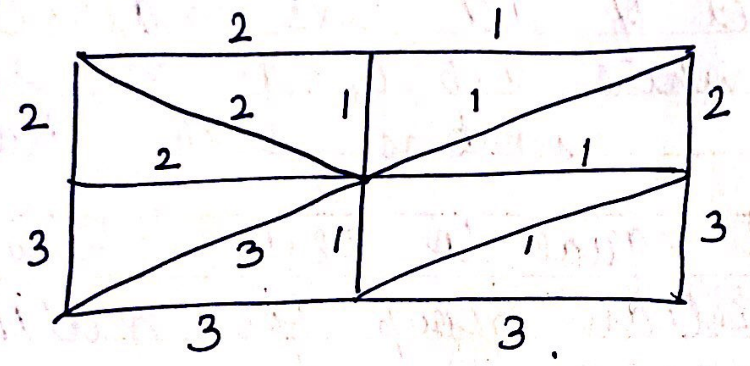
Q) Prove that the set $G = \{1, 2, 3, 4\}$ is finite Abelian Group of order 4 wrt multiplication modulo 5.

Q) Explain and give an example of a graph which has:

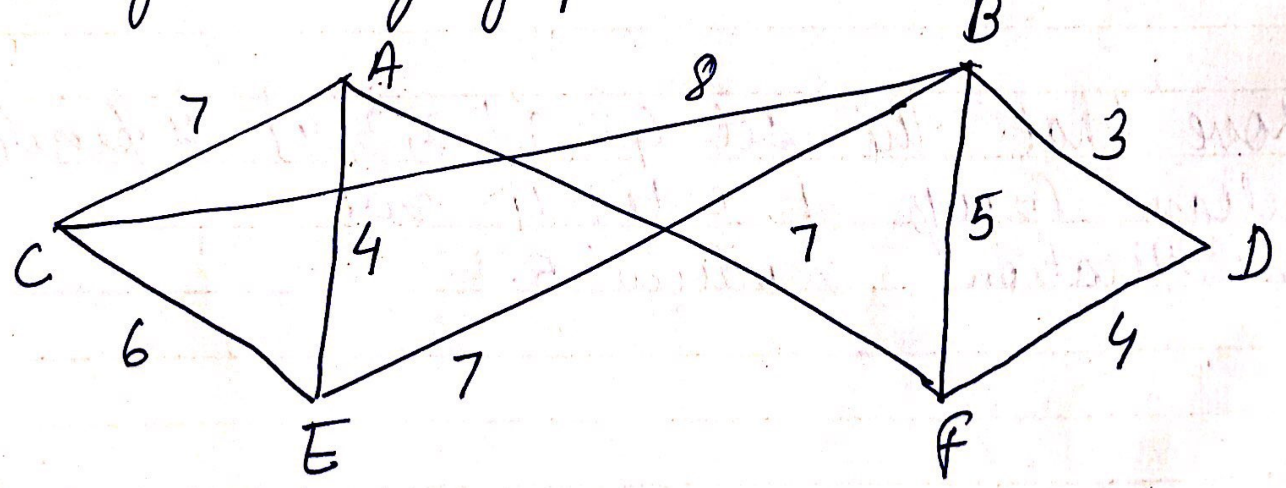
i) Euler circuit but not Hamiltonian circuit.

ii) Neither Hamiltonian circuit nor Euler circuit.

Q) Explain Kruskal and Prim's algo.
Find min spanning tree of following graph using both algos.



Q) Find minimum Spanning Tree of following graph.



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DISCRETE MATHS (IT-14501) (10 MARKS)

Q) Let $*$ be relation defined on set $Q' = Q - \{1\}$ of rational nos. such as

$$a * b = a + b - ab$$

Show that $(Q', *)$ is a monoid.
Also find a^{-1}.

Q) If two operations $*$ and σ on set Z of integers are defined as

$$a * b = a + b - 1, \quad a \sigma b = a + b - ab$$

for all $a, b \in Z$.

Show that $(Z, *, \sigma)$ is a commutative ring. Is it a ring with unity?

Q) Consider the set Z together with binary composition $\oplus$ and $\odot$ defined by

$$a \oplus b = a + b - 1$$
$$a \odot b = a + b - ab$$

Show that $(Z, \oplus, \odot)$ is a ring

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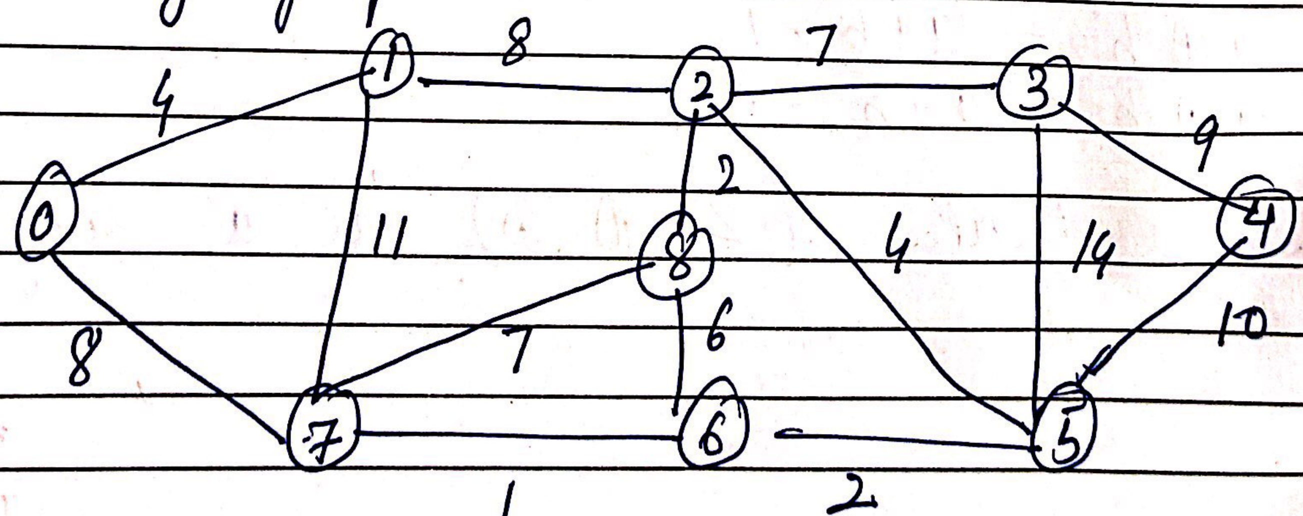
Q: Consider group G (1, 2, 3, 4, 5) under multiplication modulo 6.

a) Find multiplication table of G
b) Prove that G is a group
c) Find 2^{-1}, 3^{-1}, 1^{-1}
d) Find the orders and subgroups generated by 2 and 3.
e) Is G cyclic ? Justify your answer.

Hint-

$\times_6$	1	2	3	4	5
1	1	2	3	4	5
2	2	4	0	2	4
3	3	0	3	0	3
4	4	2	0	4	2
5	5	1	2	2	1

) State and use Dijkstra's Algo to find shortest path from node 0 to 4 in the following graph.



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Q a) Prove that if $(Z_4, +_4, \times_4)$ and $(Z_3, +_3, \times_3)$ are rings then $Z_4 \times Z_3$ is a ring

b) Prove that every subgroup of a cyclic group G is cyclic.