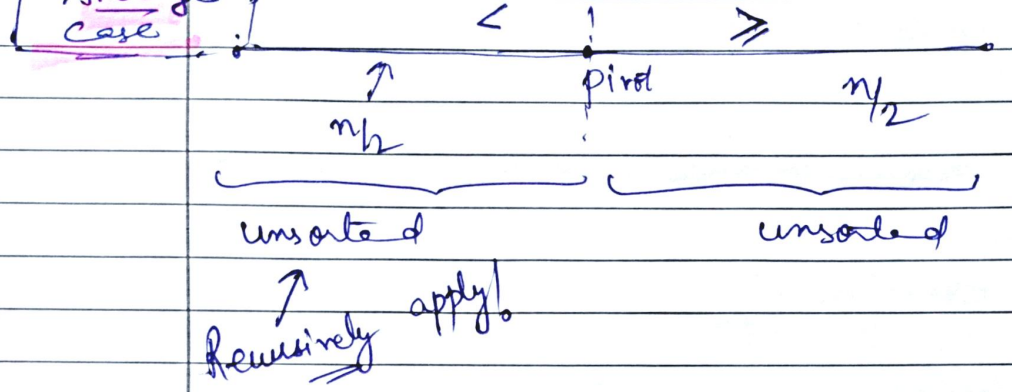
Quick sort (D&C)

Average case



Analysis of Average case of Quicksort :-

$$T(n) = T(n/2) + T(n/2) + bn$$

partitioning $\propto n$ $= bn$

$$T(n) = 2T\left(\frac{n}{2}\right) + bn$$

$$= 2\left[2T\left(\frac{n}{4}\right) + \frac{bn}{2}\right] + bn$$

$$= 4T\left(\frac{n}{4}\right) + bn + bn$$

$$= 4T\left(\frac{n}{4}\right) + 2bn$$

$n = 2^k$

$\Rightarrow n = 4$

$2^2 = 4$

$2^k = 4 \Rightarrow \boxed{k = 2}$

$\Rightarrow \log_2 n = \log_2 2^k$

$k * \log_2 2 = \log_2 n$

$\boxed{k = \log_2 n}$

$T(n) = 4\,T\left(\frac{n}{4}\right) + 2bn$

$= 2^2\,T(1) + k\,bn$

$= 2^k * a + k\,bn$

$= an + \log n\, bn$

$= an + b\, n \log n$

$\rightarrow O(n \log n)$

$T(1) = a$ (any constant)

$\boxed{k = \log n}$

Analysis of:-

Worst case of Quicksort

$\boxed{T(n) = T(1) + T(n-1) + bn}$

1 (pivot) — (n-1) elements

$T(n) = T(1) + T(n-1) + bn$

$\Rightarrow$ constant $\Rightarrow a$

$T(n) \Rightarrow T(n-1) + bn$

$= [T(n-2) + b(n-1)] + bn$

$T(n) = [T(n-3) + b(n-2)] + b(n-1) + bn$

$= T(n-3) + b(n-2) + b(n-1) + bn$

$= T(n-3) + bn + b(n-1) + b(n-2)$

$\Rightarrow$ for $n=4$, $T(n-3) = T(4-3) = T(1) \Rightarrow a$ (any constant)

$bn + b(n-1) + b(n-2)$ is like:-

$b[n + (n-1) + (n-2)]$

$= b[4 + 3 + 2]$

$= b[2+3+4]$

$= b[(1+2+3+4) - 1]$

$= b\left[\frac{n(n+1)}{2} - 1\right]$

$= \frac{bn(n+1)}{2} - b$ ignoring constant

$= b\left[\frac{n^2+n}{2}\right] = \frac{bn^2}{2} + \frac{bn}{2}$

It is becomes:-

$T(n) = T(1) + b[n + (n-1) + (n-2)]$ (constant)

$= b\left[\frac{n(n+1)}{2} - 1\right] = \frac{bn(n+1)}{2} - b$ (constant)

$= \frac{bn^2+bn}{2} = \frac{bn^2}{2} + \frac{bn}{2}$

$\Rightarrow O(n^2)$