Date
Page

GREEDY ALGORITHM

TERMS:-

Constraints → fulfil → feasible sol. → objective achieve → OPTIMAL SOLUTION

(maximize or minimizes → objective func.)

↓ max profit ↓ min cost

GREEDY TECHNIQUE

a ← input

for all values

x = Select (a)

← objective? max? min?

if x is feasible?

← constraints?

x is (Optimality), feasible

Solution = Include it in the solution

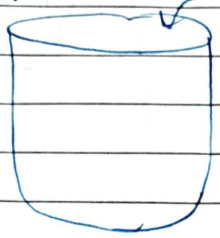
↑ x

Union

Greedy Algorithms

__/__/__

Knapsack Problem



x_i → fraction of object i

profit earned $= p_i x_i$

weight occupied $= w_i x_i$

Knapsack (or bag) of capacity $= m$

(i) e.g. if $x_i = 2$

$p_i x_i = 2 * p_i$

$w_i x_i = 2 * w_i$

(ii) if $x_i = 1/2$

$p_i x_i = p_i/2$

$w_i x_i = w_i/2$

⇒ Constraint to be fulfilled :-

$$\sum_{1 \le i \le n} w_i x_i \le m$$

Objective function :- To maximize profit

$$\text{Maximize} \sum_{1 \le i \le n} p_i x_i$$

where $0 \le x_i \le 1$

$1 \le i \le n$

$n \rightarrow$ no. of elements given

Consider the following values:-

$m = 25$

	p	w
element 1	100	50
element 2	90	25

Options

Case I →

p	w	x_i
100	50	1/2

p_i	w_i
50	25

Total profit = 90

Case II →

p	w	x_i
90	25	1

p_i	w_i
90	25

Total profit = 90

So, the best way is to compute profit per weight & consider the elements on the basis of decreasing order of P/w.

Element	p	w	P/w
element 1	100	50	100/50 = 2
element 2	90	25	90/25 = 3.6

In decreasing order of P/w :-

Element	p	w	P/w
element 2	90	25	3.6
element 1	100	50	2

Element 2 is considered first :-

p = 90, w = 25 ↓ profit = 90, $x_i = 1$ (for $m = 25$)

__/__/____

Algorithm GreedyKnapsack (m, n)

(m: capacity; n: no. of elements)

```
{
for i := 1 to n do x[i] := 0.0   // initialize x
                                 (No fraction is selected)
U := m
for( i=1 to n do
{ if (w[i] > U) then break;      (more than the capacity of knapsack)
  x[i] = 1.0;   U = U - w[i];    (wt of element added)
}
(select the complete element;  U: Net weight/capacity of knapsack left;  U - w[i]: current weight of knapsack)
if (i ≤ n) then x[i] = U/w[i];
}
```

The time complexity of Knapsack Problem solved by greedy technique is $O(n)$

Numerical :-

Ques:- Solve the following Knapsack problem using Greedy Algorithm:-

No. of elements = 3 = n
Capacity of Knapsack = 20 = m

n = 3
m = 20

profits ⇒ (25, 24, 15) = (P_A, P_B, P_C)
weights ⇒ (18, 15, 10) = (W_A, W_B, W_C)

Solution:-

element	P	W	P/W
E = A	25	18	1.4
E = B	24	15	1.6
E = C	15	10	1.5

} Computing profit per weight

Ordering the elements in decreasing order of P/W :-

Elements to be considered	P	W	P/W
i=1 (element B)	24	15	1.6
i=2 (element C)	15	10	1.5
i=3 (element A)	25	18	1.4

for $m = 20$

	p	w	p/w	
$i=1$	24	15	1.6	element B
$i=2$	15	10	1.5	element C
$i=3$	25	18	1.4	element A

$U = m$

$U = 20$

for $i = 1$ to n

$x_1 = 0, \quad x_2 = 0, \quad x_3 = 0$ (initialize)

for $i = 1$ to n :-

$i = 1$ if $w[i] > U$?

$w[1] > U$?

$15 > 20$ No

$x[i] = 1.0$

$x[1] = 1$ (select) element B (is taken as whole)

$U = U - w[i]$

$= 20 - 15$

$= 5$

profit

$p[1] = 24$

for $i = 2$

if $w[2] > U$

$10 > 5$ Yes

break

if $(i \leq n)$

$2 \leq 3$ Yes then → $x[i] = U / w[i]$

$x[2] = 5/10 = \frac{1}{2}$

select $x[2] = 1/2$ i.e. element C is taken 1/2

$x[2] = 1/2$ ⇒ element 3 is taken as $1/2$

⇒ profit $= \frac{15}{2} = 7.5$

for $i=1$ Element B ⇒ $x_i = 1$ ⇒ $p_i = p_1 = 24$ | $w = 15$

$i=2$ Element C ⇒ $x_i = 1/2$ ⇒ $p_i = p_2 = 15/2 = 7.5$ | $w = 1/2 = 5$

(Element A not selected)

Net profit $= 24 + 7.5$

$= 31.5$

$\sum p_i x_i$

Net weight $= 15 + 5$

$= 20$

$\sum w_i x_i = 20$

Answer (element A, element B, element C)

fraction selected ⇒

x (0 , 1 , 1/2)

p (0 , 24 , 7.5) Net profit $= 31.5$

w (0 , 15 , 5) Net weight $= 15 + 5 = 20 \leq m$