Project Report

Time Series Analysis

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Analysis of Indian Rupees to U.S. Dollar Foreign Exchange Rate (Spot)

What is foreign exchange rate?

Foreign exchange, often abbreviated as forex or FX, refers to the global marketplace for trading and exchanging currencies. In this market, participants can buy, sell, exchange, and speculate on the value of different currencies relative to one another. The exchange rate, or forex rate, is the rate at which one currency can be exchanged for another. For example:

The exchange rate for USD/INR represents how many Indian Rupees you can get in exchange for one US Dollar. For instance, if the exchange rate is 75 USD/INR, it means that for every 1 US Dollar, you can exchange it for 75 Indian Rupees.

Exchange rates are determined by a variety of factors, including supply and demand for each currency, interest rates, economic stability, geopolitical events, and market sentiment. Exchange rates can fluctuate frequently, and these fluctuations are influenced by a wide range of factors. Exchange rates are usually quoted in pairs, where one currency is expressed in terms of another.

Foreign exchange markets serve several purposes, including facilitating international trade, investment, and travel. They are essential for businesses and individuals who need to convert one currency into another for various purposes. Additionally, forex trading is also a speculative activity where traders and investors aim to profit from currency price movements. Central banks, financial institutions, corporations, governments, and individual traders all participate in the foreign exchange market, making it one of the largest and most liquid financial markets in the world.

Factors affecting foreign exchange rate

The foreign exchange (forex) market is influenced by a wide range of factors that can cause exchange rates to fluctuate. Here are 5 of the most important factors that affect the forex market:

1. **Interest Rates**: Differentials in interest rates between two countries can have a significant impact on exchange rates. *Higher interest rates in one country can attract foreign capital and increase demand for that country's currency, leading to an appreciation of its currency.*

2. **Economic Indicators**: Economic data, such as GDP growth, employment figures, inflation rates, and trade balances, can influence currency values. *Strong economic performance can lead to a stronger currency, while weak economic indicators can lead to depreciation.*

3. **Political Stability and Economic Performance**: *Countries with stable governments and strong economic fundamentals often have stronger currencies.*

4. **Market Sentiment**: *Investor perception and sentiment play a critical role in currency markets.* News, events, and geopolitical developments can influence market sentiment, leading to rapid and significant price movements.

5. **Central Bank Policies**: *Central banks, such as the Reserve Bank of India in India, have a considerable influence on their respective currencies through monetary policy decisions.* Interest rate changes, quantitative easing, and forward guidance can all affect exchange rates.

These factors interact and overlap, making the forex market complex and often subject to sudden and significant price swings. Traders and investors in the forex market need to consider these factors when making trading decisions.

How well a business cycle manifests itself in forex data?

The foreign exchange (forex) market is influenced by a wide range of factors, and while *business cycles can be one of them, they are just one of many elements that impact currency exchange rates. It is important to note that the relationship between business cycles and forex rate movements can be more complex and indirect than a simple, observable oscillatory pattern.*

1. **Complex Interaction of Factors**: Currency exchange rates are influenced by numerous factors, including interest rates, inflation, trade balances, central bank policies, geopolitical events, and market sentiment. Business cycles are just one of these factors, and their impact can be influenced by various other domestic and global economic conditions.

2. **Differing Timeframes**: Business cycles can have varying durations, and they may not necessarily align with the timeframes typically used in forex market analysis (e.g., daily, weekly, or monthly data). The effects of business cycles on exchange rates can be gradual and may not manifest as short-term oscillations.

3. **Lagged Effects**: The impact of business cycles on exchange rates might not be immediate or direct. Changes in economic conditions that are tied to business cycles, such as interest rate adjustments or fiscal policies, can have a lagged effect on currency exchange rates.

Hence, it is wise to conclude that while business cycles can play a role in exchange rate movements, they are just one piece of the puzzle.

About our dataset

In the analysis done as part of our project work, we have chosen the dataset of Indian Rupees to U.S. Dollar Spot Exchange rate prepared by *the Board of Governors of the Federal Reserve System (US).* The data was in the form of monthly averages of the Forex from 1973 to 2022. For ease in analysis of both trend component and oscillatory component, we converted the monthly averages into yearly averages.

Following table shows the annual averages.

Serial Number	Year	(y_t) Exchange Rate USD between INR
1	1973	7.658466667
2	1974	8.032375
3	1975	8.4098
4	1976	8.971291667
5	1977	8.767241667
6	1978	8.195616667
7	1979	8.15725
8	1980	7.885491667
9	1981	8.679758333
10	1982	9.482733333
11	1983	10.10589167
12	1984	11.3548
13	1985	12.33981667
14	1986	12.5986
15	1987	12.94775833
16	1988	13.90281667
17	1989	16.20713333
18	1990	17.5021
19	1991	22.71985
20	1992	28.13509167
21	1993	31.25691667
22	1994	31.39435833
23	1995	32.43238333
24	1996	35.51641667
25	1997	36.36454167
26	1998	41.32943333
27	1999	43.12118333
28	2000	45.00093333
29	2001	47.227625
30	2002	48.62203333
31	2003	46.594675
32	2004	45.27628333

33	2005	44.011275
34	2006	45.17219167
35	2007	41.19918333
36	2008	43.411425
37	2009	48.319475
38	2010	45.65086667
39	2011	46.61216667
40	2012	53.35910833
41	2013	58.5255
42	2014	61.01738333
43	2015	64.102925
44	2016	67.16683333
45	2017	65.10150833
46	2018	68.401825
47	2019	70.40413333
48	2020	74.11025833
49	2021	73.92673333
50	2022	78.57661667

The data is sourced from the following website:
https://fred.stlouisfed.org/series/DEXINUS

Decomposition and analysis

The decomposition of any time series data is important as it enables us to isolate and better identify the components influencing the time series and assists in better forecasting.

Trend Component Analysis: Exponential Curve Fitting

Serial Number	Year	(y_t) Exchange Rate USD between INR	(x_t) First differences of y_t	(w_t) Second differences of x_t	(z_t) log y_t	First differences of z_t / log y_t	Ratio of the first differences of y_t/Ratio of x_t	Ratio of the first differences of logarithmns of y_t (z_t)	t	t^2	tz_t	T_t	Detrended Data
1	1973	7.658466667	0.374	0.003	2.03581	0.048	1.008	0.96	-49	2401	-99.7548	7.55002	0.108447504
2	1974	8.032375	0.377	0.184	2.08348	0.046	1.488	1.41	-47	2209	-97.9236	7.9569	0.075471061
3	1975	8.4098	0.561	-0.765	2.1294	0.065	-0.364	-0.35	-45	2025	-95.8229	8.38572	0.024083495
4	1976	8.971291667	-0.204	-0.368	2.19403	-0.023	2.804	2.91	-43	1849	-94.3433	8.83764	0.133653078
5	1977	8.767241667	-0.572	0.534	2.17102	-0.067	0.066	0.07	-41	1681	-89.0119	9.31392	-0.546673939
6	1978	8.195616667	-0.038	-0.234	2.1036	-0.005	7.158	6.8	-39	1521	-82.0404	9.81586	-1.620243423
7	1979	8.15725	-0.272	1.066	2.09891	-0.034	-2.919	-2.82	-37	1369	-77.6596	10.3449	-2.187605309
8	1980	7.885491667	0.794	0.009	2.06502	0.096	1.011	0.92	-35	1225	-72.2759	10.9024	-3.016867413
9	1981	8.679758333	0.803	-0.18	2.16099	0.088	0.776	0.73	-33	1089	-71.3128	11.4899	-2.810149446
10	1982	9.482733333	0.623	0.626	2.24947	0.064	2.005	1.83	-31	961	-69.7337	12.1091	-2.626387254
11	1983	10.10589167	1.249	-0.264	2.31312	0.117	0.789	0.71	-29	841	-67.0804	12.7617	-2.655812275
12	1984	11.3548	0.985	-0.726	2.42964	0.083	0.263	0.25	-27	729	-65.6003	13.4495	-2.094656245
13	1985	12.33981667	0.259	0.09	2.51283	0.021	1.347	1.29	-25	625	-62.8208	14.1743	-1.83445615
14	1986	12.5986	0.349	0.606	2.53359	0.027	2.736	2.63	-23	529	-58.2725	14.9382	-2.339551121
15	1987	12.94775833	0.955	1.349	2.56092	0.071	2.413	2.15	-21	441	-53.7794	15.7432	-2.795437935
16	1988	13.90281667	2.304	-1.009	2.63209	0.153	0.562	0.5	-19	361	-50.0097	16.5916	-2.688810151
17	1989	16.20713333	1.295	3.923	2.78545	0.077	4.029	3.39	-17	289	-47.3527	17.4858	-1.278647556
18	1990	17.5021	5.218	0.197	2.86232	0.261	1.038	0.82	-15	225	-42.9348	18.4281	-0.926022612
19	1991	22.71985	5.415	-2.293	3.12324	0.214	0.577	0.49	-13	169	-40.6021	19.4212	3.298601093
20	1992	28.13509167	3.122	-2.985	3.33702	0.105	0.044	0.04	-11	121	-36.7072	20.4679	7.667195014
21	1993	31.25691667	0.137	0.901	3.44224	0.004	7.577	8.25	-9	81	-30.9802	21.571	9.685966447
22	1994	31.39435833	1.038	2.046	3.44663	0.033	2.971	2.76	-7	49	-24.1264	22.7334	8.660908909
23	1995	32.43238333	3.084	-2.236	3.47916	0.091	0.275	0.26	-5	25	-17.3958	23.9586	8.473785429
24	1996	35.51641667	0.848	4.117	3.57	0.024	5.855	5.33	-3	9	-10.71	25.2498	10.26664472
25	1997	36.36454167	4.965	-3.173	3.59359	0.128	0.361	0.33	-1	1	-3.59359	26.6105	9.754011871
26	1998	41.32943333	1.792	0.088	3.72157	0.042	1.049	1.02	1	1	3.72157	28.0446	13.28481188
27	1999	43.12118333	1.88	0.347	3.76401	0.043	1.185	1.12	3	9	11.292	29.556	13.56518432
28	2000	45.00093333	2.227	-0.833	3.80668	0.048	0.626	0.6	5	25	19.0334	31.1488	13.85210577
29	2001	47.227625	1.394	-3.421	3.85498	0.029	-1.454	-1.48	7	49	26.9849	32.8275	14.40012835
30	2002	48.62203333	-2.027	0.709	3.88408	-0.043	0.65	0.67	9	81	34.9567	34.5966	14.02540096
31	2003	46.594675	-1.318	0.053	3.84149	-0.029	0.96	0.97	11	121	42.2563	36.4611	10.13356483
32	2004	45.27628333	-1.265	2.426	3.81278	-0.028	-0.918	-0.93	13	169	49.5662	38.4261	6.850215148
33	2005	44.011275	1.161	-5.134	3.78445	0.026	-3.422	-3.54	15	225	56.7667	40.4969	3.514353502
34	2006	45.17219167	-3.973	6.185	3.81048	-0.092	-0.557	-0.57	17	289	64.7782	42.6794	2.492814665
35	2007	41.19918333	2.212	2.696	3.71842	0.052	2.219	2.06	19	361	70.65	44.9794	-3.780265817
36	2008	43.411425	4.908	-7.577	3.77072	0.107	-0.544	-0.53	21	441	79.1852	47.4035	-3.992051527
37	2009	48.319475	-2.669	3.63	3.87783	-0.057	-0.36	-0.37	23	529	89.1902	49.9581	-1.638664312
38	2010	45.65086667	0.961	5.786	3.82102	0.021	7.021	6.43	25	625	95.5256	52.6505	-6.999611028
39	2011	46.61216667	6.747	-1.581	3.84186	0.135	0.766	0.68	27	729	103.73	55.4879	-8.875744605
40	2012	53.35910833	5.166	-2.674	3.97704	0.092	0.482	0.46	29	841	115.334	58.4783	-5.119151165
41	2013	58.5255	2.492	0.594	4.06946	0.042	1.238	1.17	31	961	126.153	61.6298	-3.104263232
42	2014	61.01738333	3.086	-0.022	4.11116	0.049	0.993	0.96	33	1089	135.668	64.9511	-3.933724115
43	2015	64.102925	3.064	-5.129	4.16049	0.047	-0.674	-0.66	35	1225	145.617	68.4514	-4.34852017
44	2016	67.16683333	-2.065	5.365	4.20718	-0.031	-1.598	-1.58	37	1369	155.666	72.1404	-4.973589363
45	2017	65.10150833	3.3	-1.298	4.17595	0.049	0.607	0.59	39	1521	162.862	76.0282	-10.92669785
46	2018	68.401825	2.002	1.704	4.2254	0.029	1.851	1.76	41	1681	173.241	80.1255	-11.72368465
47	2019	70.40413333	3.706	-3.89	4.25425	0.051	-0.05	-0.04	43	1849	182.933	84.4436	-14.03949121
48	2020	74.11025833	-0.184	4.834	4.30555	-0.002	-25.272	-30.5	45	2025	193.75	88.9945	-14.88419245
49	2021	73.92673333	4.65		4.30307	0.061			47	2209	202.245	93.7905	-19.86379631
50	2022	78.57661667			4.36407				49	2401	213.84	98.8451	-20.26846157
Maximum value			6.747	6.185		0.261	7.577	8.25					
Minimum value			-3.973	-7.577		-0.092	-25.272	-30.5					
Difference			10.72	13.762		0.353	32.849	38.75					
Sum					165.378				0	41650	1093.1		

The trend represents the smooth, regular, and long-term movement of the time series. It can either be upward, downward or both at the same time.

For fitting the suitable curve, computation of first and second order difference was done but it was observed that the differences are not constant. So, the first differences of the logarithmic values of the actual observations are computed and it was observed that the ratio of the first difference changes by a constant percentage.

Therefore, we fit an exponential curve for the given set of data.

The general equation of exponential curve is given by:

$$y_t = ab^t$$

$\log y_t$=loga + tlogb

z_t=A+Bt

On solving the normal equations obtained, we get:

$\hat{a}$= antilog(A)

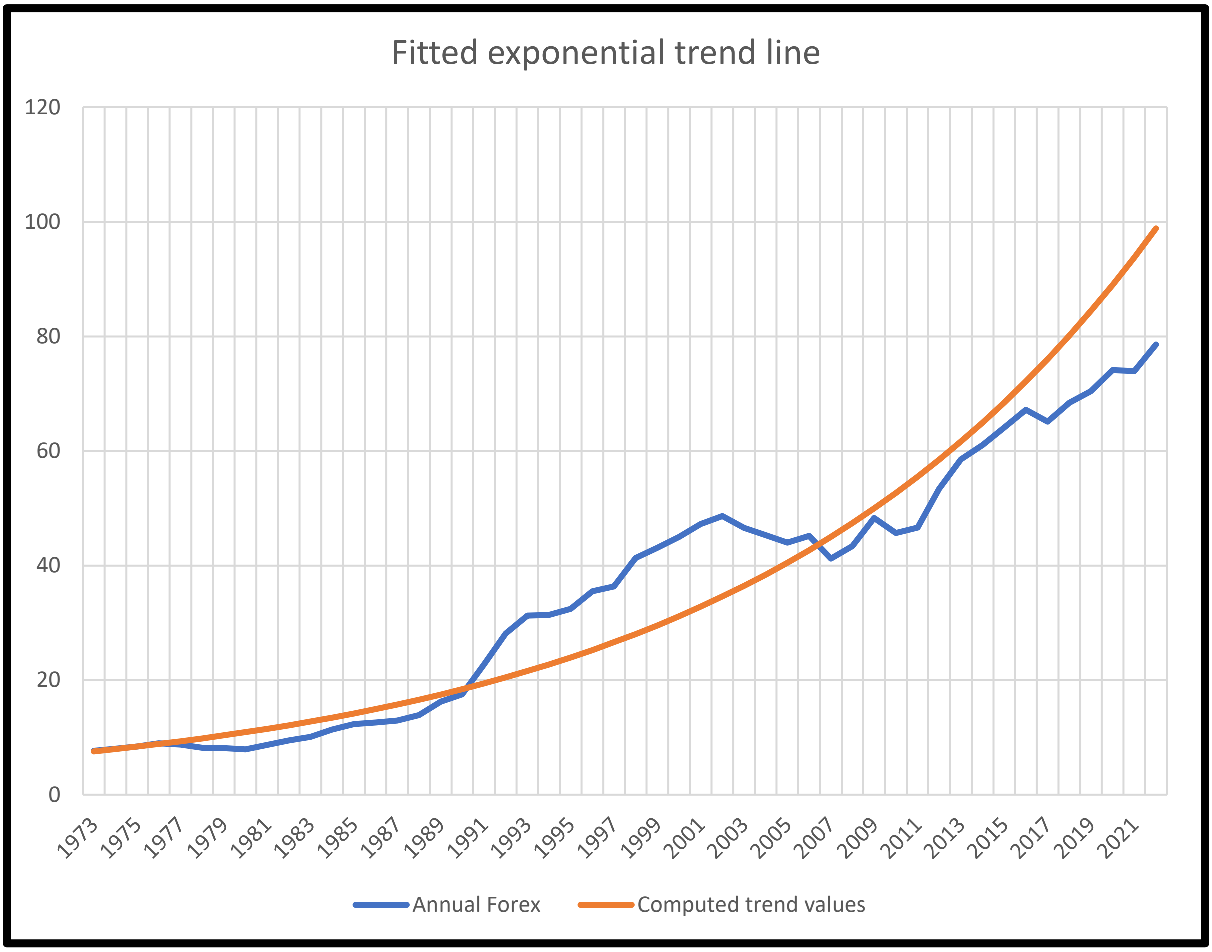
where A = $\sum z_t$ /n

$\hat{b}$= antilog(B)

where B=$\sum t * z_t$ /t

Then, the trend component T_t was computed using the parameters obtained above and is plotted.

Post the computation of the trend component, we computed the detrended values. Detrending refers to removing the trend estimates from a given time series*;* $y_t - T_t$ *gives us the detrended time series*. The graph of the same too is plotted as shown in the next page.


Fitted exponential trend line
120
100
80
60
40
20
0
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1977
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1995
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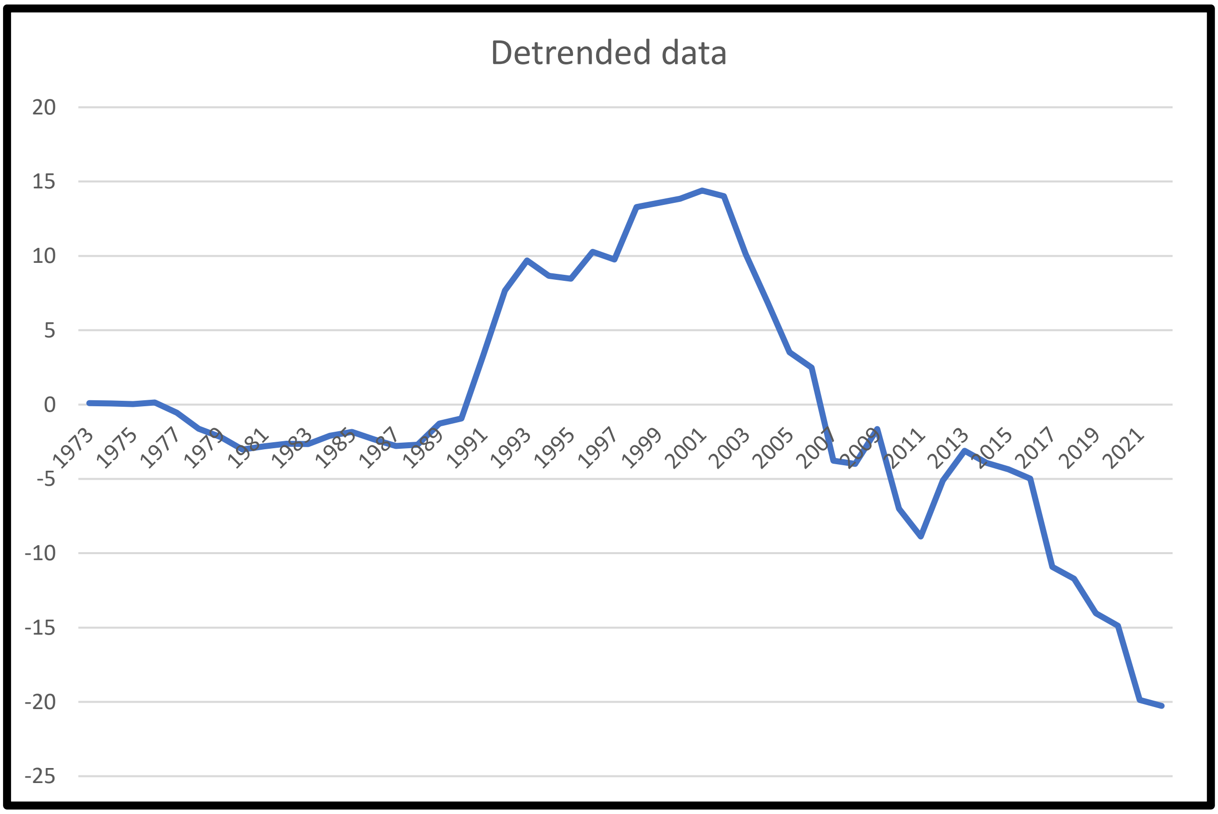
2003
2005
2007
2009
2011
2013
2015
2017
2019
2021
Annual Forex
Computed trend values


Detrended data
20
15
10
5
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-5
-10
-15
-20
-25
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1981
1983
1985
1987
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2021

Trend Component Analysis: Polynomial Fitting

Another way to fit a trend line is to determine the degree of the systematic part of the time series and then plot the polynomial of the same degree to see if is able to capture the trend of the time series. To find out the degree of the polynomial i.e., the systematic part to be fitted, the variate difference method is employed.

t	Year	(y_t) Exchange Rate USD between INR	(a_t) First difference, y_t-y_{t-1}	(b_t) Second difference, a_t-a_{t-1}	(c_t) Third order difference, b_t-b_{t-1}	(d_t) Fourth order difference, c_t-c_{t-1}	(e_t) Fifth order difference, d_t-d_{t-1}	(f_t) Sixth order difference, e_t-e_{t-1}
1	1973	7.658467						
2	1974	8.032375	0.3739083					
3	1975	8.4098	0.377425	0.0035167				
4	1976	8.971292	0.5614917	0.1840667	0.18055			
5	1977	8.767242	-0.20405	-0.7655417	-0.949608	-1.130158		
6	1978	8.195617	-0.571625	-0.367575	0.3979667	1.347575	2.4777333	
7	1979	8.15725	-0.038367	0.5332583	0.9008333	0.5028667	-0.844708	-3.3224417
8	1980	7.885492	-0.271758	-0.2333917	-0.76665	-1.667483	-2.17035	-1.3256417
9	1981	8.679758	0.7942667	1.066025	1.2994167	2.0660667	3.73355	5.9039
10	1982	9.482733	0.802975	0.0087083	-1.057317	-2.356733	-4.4228	-8.15635
11	1983	10.10589	0.6231583	-0.1798167	-0.188525	0.8687917	3.225525	7.648325
12	1984	11.3548	1.2489083	0.62575	0.8055667	0.9940917	0.1253	-3.100225
13	1985	12.33982	0.9850167	-0.2638917	-0.889642	-1.695208	-2.6893	-2.8146
14	1986	12.5986	0.2587833	-0.7262333	-0.462342	0.4273	2.1225083	4.8118083
15	1987	12.94776	0.3491583	0.090375	0.8166083	1.27895	0.85165	-1.2708583
16	1988	13.90282	0.9550583	0.6059	0.515525	-0.301083	-1.580033	-2.4316833
17	1989	16.20713	2.3043167	1.3492583	0.7433583	0.2278333	0.5289167	2.10895
18	1990	17.5021	1.2949667	-1.00935	-2.358608	-3.101967	-3.3298	-3.8587167
19	1991	22.71985	5.21775	3.9227833	4.9321333	7.2907417	10.392708	13.722508
20	1992	28.13509	5.4152417	0.1974917	-3.725292	-8.657425	-15.94817	-26.340875

21	1993	31.25692	3.121825	-2.2934167	-2.490908	1.2343833	9.8918083	25.839975
22	1994	31.39436	0.1374417	-2.9843833	-0.690967	1.7999417	0.5655583	-9.32625
23	1995	32.43238	1.038025	0.9005833	3.8849667	4.5759333	2.7759917	2.2104333
24	1996	35.51642	3.0840333	2.0460083	1.145425	-2.739542	-7.315475	-10.091467
25	1997	36.36454	0.848125	-2.2359083	-4.281917	-5.427342	-2.6878	4.627675
26	1998	41.32943	4.9648917	4.1167667	6.352675	10.634592	16.061933	18.749733
27	1999	43.12118	1.79175	-3.1731417	-7.289908	-13.64258	-24.27718	-40.339108
28	2000	45.00093	1.87975	0.088	3.2611417	10.55105	24.193633	48.470808
29	2001	47.22763	2.2266917	0.3469417	0.2589417	-3.0022	-13.55325	-37.746883
30	2002	48.62203	1.3944083	-0.8322833	-1.179225	-1.438167	1.5640333	15.117283
31	2003	46.59468	-2.027358	-3.4217667	-2.589483	-1.410258	0.0279083	-1.536125
32	2004	45.27628	-1.318392	0.7089667	4.1307333	6.7202167	8.130475	8.1025667
33	2005	44.01128	-1.265008	0.0533833	-0.655583	-4.786317	-11.50653	-19.637008
34	2006	45.17219	1.1609167	2.425925	2.3725417	3.028125	7.8144417	19.320975
35	2007	41.19918	-3.973008	-5.133925	-7.55985	-9.932392	-12.96052	-20.774958
36	2008	43.41143	2.2122417	6.18525	11.319175	18.879025	28.811417	41.771933
37	2009	48.31948	4.90805	2.6958083	-3.489442	-14.80862	-33.68764	-62.499058
38	2010	45.65087	-2.668608	-7.5766583	-10.27247	-6.783025	8.0255917	41.713233
39	2011	46.61217	0.9613	3.6299083	11.206567	21.479033	28.262058	20.236467
40	2012	53.35911	6.7469417	5.7856417	2.1557333	-9.050833	-30.52987	-58.791925
41	2013	58.5255	5.1663917	-1.58055	-7.366192	-9.521925	-0.471092	30.058775
42	2014	61.01738	2.4918833	-2.6745083	-1.093958	6.2722333	15.794158	16.26525
43	2015	64.10293	3.0855417	0.5936583	3.2681667	4.362125	-1.910108	-17.704267
44	2016	67.16683	3.0639083	-0.0216333	-0.615292	-3.883458	-8.245583	-6.335475
45	2017	65.10151	-2.065325	-5.1292333	-5.1076	-4.492308	-0.60885	7.6367333
46	2018	68.40183	3.3003167	5.3656417	10.494875	15.602475	20.094783	20.703633
47	2019	70.40413	2.0023083	-1.2980083	-6.66365	-17.15853	-32.761	-52.855783
48	2020	74.11026	3.706125	1.7038167	3.001825	9.665475	26.824	59.585
49	2021	73.92673	-0.183525	-3.88965	-5.593467	-8.595292	-18.26077	-45.084767
50	2022	78.57662	4.6498833	4.8334083	8.7230583	14.316525	22.911817	41.172583

	N	50	50	50	50	50	50	50
	n_k	50	49	48	47	46	45	44
	k	0	1	2	3	4	**5**	6
(μ'_2)*(k^{th} order difference)	(Summation of square of k^{th} order differences)/n_k	1745.423	6.995	7.836	21.826	66.996	217.645	740.520
	$^{2k}C_k$	1	2	6	20	70	252	924
	σ^2 (hat)	1745.423	3.497	1.306	1.091	0.957	0.864	0.801
n=25	$H_{(k,n)}$	6.932	13.884	18.608	22.104	24.711	26.645	28.056
	R_k	6.918	8.700	3.058	2.719	2.412	**1.920**	

It is observed that we fail to reject the null hypothesis established when k=5, as 1.92<1.96, so we get the degree of the polynomial as k-1=4.

Therefore, degree of the systematic part = 4

The 4th degree polynomial: $y_t = a + bt + ct^2 + dt^3 + et^4$ is fitted to the given data and the values of a, b, c, d and e are estimated to compute the trend values, T_t.

Serial Number	Year	(y_t) Exchange Rate USD between INR	t	t^2	t^3	t^4	t^5	t^6	t^7
1	1973	7.658466667	-49	2401	-117649	5764801	-282475249	1.384E+10	-6.78223E+11
2	1974	8.032375	-47	2209	-103823	4879681	-229345007	1.078E+10	-5.06623E+11
3	1975	8.4098	-45	2025	-91125	4100625	-184528125	8.304E+09	-3.73669E+11
4	1976	8.971291667	-43	1849	-79507	3418801	-147008443	6.321E+09	-2.71819E+11
5	1977	8.767241667	-41	1681	-68921	2825761	-115856201	4.75E+09	-1.94754E+11
6	1978	8.195616667	-39	1521	-59319	2313441	-90224199	3.519E+09	-1.37231E+11
7	1979	8.15725	-37	1369	-50653	1874161	-69343957	2.566E+09	-94931877133
8	1980	7.885491667	-35	1225	-42875	1500625	-52521875	1.838E+09	-64339296875
9	1981	8.679758333	-33	1089	-35937	1185921	-39135393	1.291E+09	-42618442977
10	1982	9.482733333	-31	961	-29791	923521	-28629151	887503681	-27512614111
11	1983	10.10589167	-29	841	-24389	707281	-20511149	594823321	-17249876309
12	1984	11.3548	-27	729	-19683	531441	-14348907	387420489	-10460353203
13	1985	12.33981667	-25	625	-15625	390625	-9765625	244140625	-6103515625

14	1986	12.5986	-23	529	-12167	279841	-6436343	148035889	-3404825447
15	1987	12.94775833	-21	441	-9261	194481	-4084101	85766121	-1801088541
16	1988	13.90281667	-19	361	-6859	130321	-2476099	47045881	-893871739
17	1989	16.20713333	-17	289	-4913	83521	-1419857	24137569	-410338673
18	1990	17.5021	-15	225	-3375	50625	-759375	11390625	-170859375
19	1991	22.71985	-13	169	-2197	28561	-371293	4826809	-62748517
20	1992	28.13509167	-11	121	-1331	14641	-161051	1771561	-19487171
21	1993	31.25691667	-9	81	-729	6561	-59049	531441	-4782969
22	1994	31.39435833	-7	49	-343	2401	-16807	117649	-823543
23	1995	32.43238333	-5	25	-125	625	-3125	15625	-78125
24	1996	35.51641667	-3	9	-27	81	-243	729	-2187
25	1997	36.36454167	-1	1	-1	1	-1	1	-1
26	1998	41.32943333	1	1	1	1	1	1	1
27	1999	43.12118333	3	9	27	81	243	729	2187
28	2000	45.00093333	5	25	125	625	3125	15625	78125
29	2001	47.227625	7	49	343	2401	16807	117649	823543
30	2002	48.62203333	9	81	729	6561	59049	531441	4782969
31	2003	46.594675	11	121	1331	14641	161051	1771561	19487171
32	2004	45.27628333	13	169	2197	28561	371293	4826809	62748517
33	2005	44.011275	15	225	3375	50625	759375	11390625	170859375
34	2006	45.17219167	17	289	4913	83521	1419857	24137569	410338673
35	2007	41.19918333	19	361	6859	130321	2476099	47045881	893871739
36	2008	43.411425	21	441	9261	194481	4084101	85766121	1801088541
37	2009	48.319475	23	529	12167	279841	6436343	148035889	3404825447
38	2010	45.65086667	25	625	15625	390625	9765625	244140625	6103515625
39	2011	46.61216667	27	729	19683	531441	14348907	387420489	10460353203
40	2012	53.35910833	29	841	24389	707281	20511149	594823321	17249876309
41	2013	58.5255	31	961	29791	923521	28629151	887503681	27512614111
42	2014	61.01738333	33	1089	35937	1185921	39135393	1.291E+09	42618442977
43	2015	64.102925	35	1225	42875	1500625	52521875	1.838E+09	64339296875
44	2016	67.16683333	37	1369	50653	1874161	69343957	2.566E+09	94931877133

45	2017	65.10150833	39	1521	59319	2313441	90224199	3.519E+09	1.37231E+11
46	2018	68.401825	41	1681	68921	2825761	115856201	4.75E+09	1.94754E+11
47	2019	70.40413333	43	1849	79507	3418801	147008443	6.321E+09	2.71819E+11
48	2020	74.11025833	45	2025	91125	4100625	184528125	8.304E+09	3.73669E+11
49	2021	73.92673333	47	2209	103823	4879681	229345007	1.078E+10	5.06623E+11
50	2022	78.57661667	49	2401	117649	5764801	282475249	1.384E+10	6.78223E+11
Sum		1775.260075	0	41650	0	62416690	0	1.113E+11	0

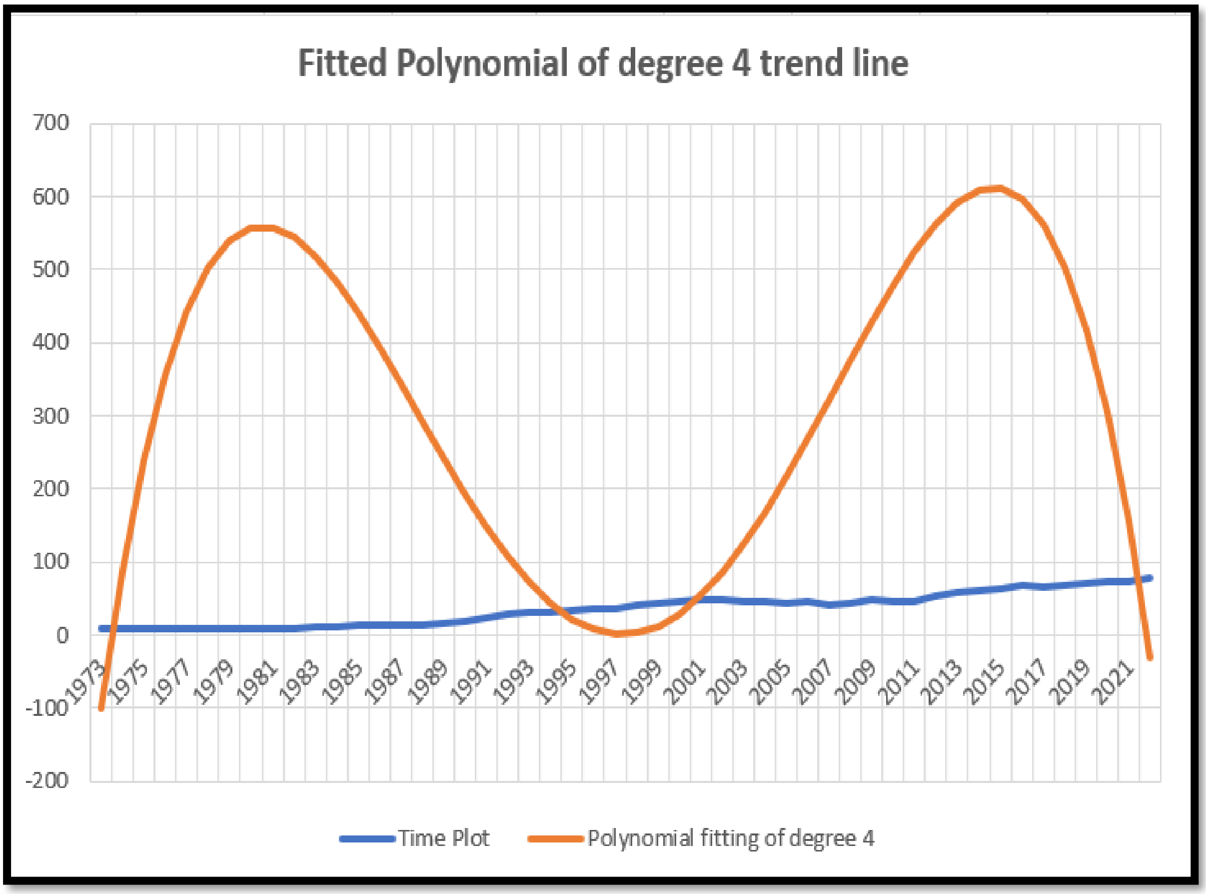
Serial Number	Year	t^8	ty_t	t^2y_t	t^3y_t	t^4y_t	T_t	Detrended data
1	1973	3.32329E+13	-375.265	18387.98	-901011	44149536.3	-100.15	107.80
2	1974	2.38113E+13	-377.522	17743.52	-833945	39195427.7	87.81	-79.78
3	1975	1.68151E+13	-378.441	17029.85	-766343	34485436.1	238.41	-230.00
4	1976	1.16882E+13	-385.766	16587.92	-713280	30671060.9	355.41	-346.44
5	1977	7.98493E+12	-359.457	14737.73	-604247	24774129.6	442.45	-433.69
6	1978	5.35201E+12	-319.629	12465.53	-486156	18960075.6	502.97	-494.78
7	1979	3.51248E+12	-301.818	11167.28	-413189	15287999.8	540.26	-532.10
8	1980	2.25188E+12	-275.992	9659.727	-338090	11833165.9	557.43	-549.55
9	1981	1.40641E+12	-286.432	9452.257	-311924	10293507.7	557.45	-548.77
10	1982	8.52891E+11	-293.965	9112.907	-282500	8757503.37	543.11	-533.63
11	1983	5.00246E+11	-293.071	8499.055	-246473	7147705.16	517.04	-506.94
12	1984	2.8243E+11	-306.58	8277.649	-223497	6034406.27	481.70	-470.35
13	1985	1.52588E+11	-308.495	7712.385	-192810	4820240.89	439.40	-427.06
14	1986	78310985281	-289.768	6664.659	-153287	3525604.82	392.26	-379.66
15	1987	37822859361	-271.903	5709.961	-119909	2518092.99	342.26	-329.31
16	1988	16983563041	-264.154	5018.917	-95359.4	1811828.97	291.21	-277.30
17	1989	6975757441	-275.521	4683.862	-79625.6	1353635.98	240.74	-224.53
18	1990	2562890625	-262.532	3937.973	-59069.6	886043.813	192.34	-174.84
19	1991	815730721	-295.358	3839.655	-49915.5	648901.636	147.32	-124.60
20	1992	214358881	-309.486	3404.346	-37447.8	411925.877	106.83	-78.69
21	1993	43046721	-281.312	2531.81	-22786.3	205076.63	71.85	-40.59
22	1994	5764801	-219.761	1538.324	-10768.3	75377.8544	43.20	-11.81
23	1995	390625	-162.162	810.8096	-4054.05	20270.2396	21.55	10.89
24	1996	6561	-106.549	319.6478	-958.943	2876.82975	7.37	28.15
25	1997	1	-36.3645	36.36454	-36.3645	36.3645417	1.00	35.36
26	1998	1	41.32943	41.32943	41.32943	41.3294333	2.60	38.73
27	1999	6561	129.3636	388.0907	1164.272	3492.81585	12.17	30.95
28	2000	390625	225.0047	1125.023	5625.117	28125.5833	29.54	15.46
29	2001	5764801	330.5934	2314.154	16199.08	113393.528	54.39	-7.16
30	2002	43046721	437.5983	3938.385	35445.46	319009.161	86.20	-37.58
31	2003	214358881	512.5414	5637.956	62017.51	682192.637	124.34	-77.75

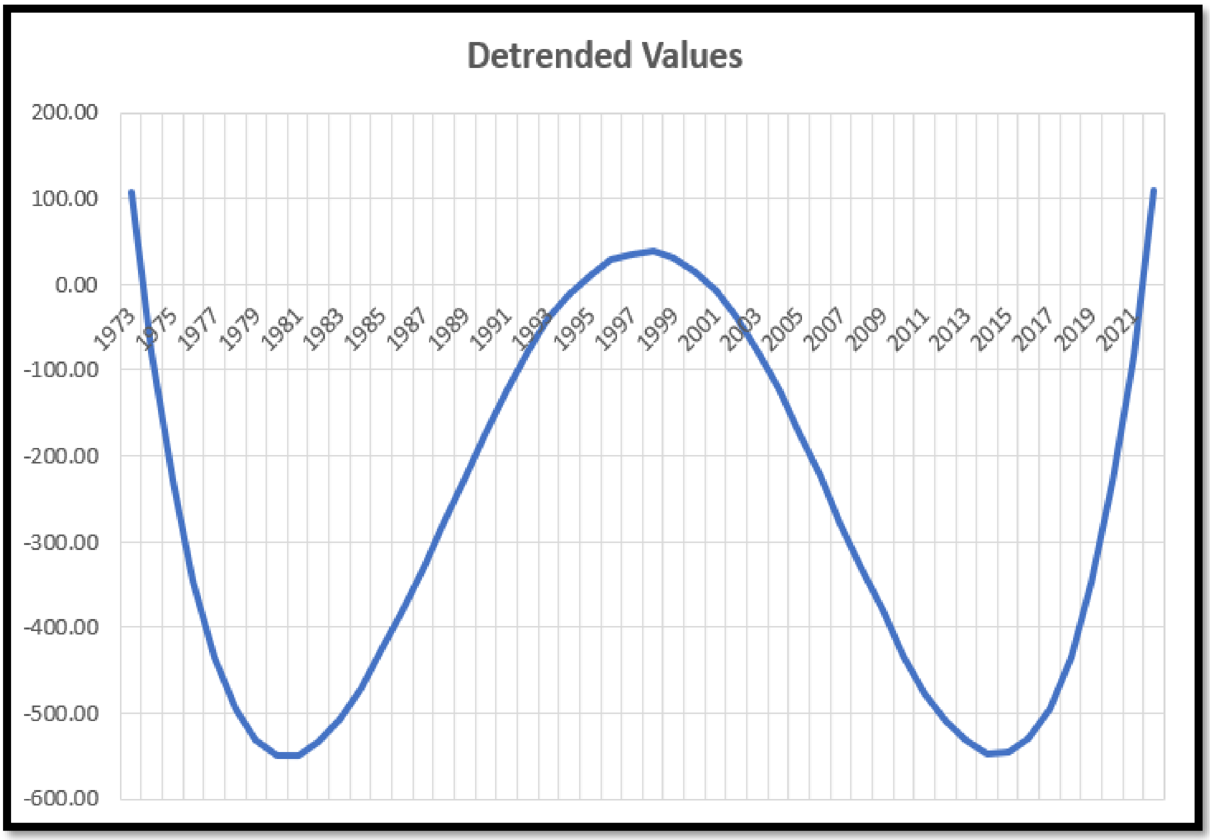
32	2004	815730721	588.5917	7651.692	99471.99	1293135.93	167.97	-122.69
33	2005	2562890625	660.1691	9902.537	148538.1	2228070.8	216.10	-172.09
34	2006	6975757441	767.9273	13054.76	221931	3772826.62	267.59	-222.42
35	2007	16983563041	782.7845	14872.91	282585.2	5369118.77	321.11	-279.91
36	2008	37822859361	911.6399	19144.44	402033.2	8442697.35	375.19	-331.78
37	2009	78310985281	1111.348	25561	587903.1	13521770.2	428.17	-379.85
38	2010	1.52588E+11	1141.272	28531.79	713294.8	17832369.8	478.25	-432.60
39	2011	2.8243E+11	1258.529	33980.27	917467.3	24771616.5	523.45	-476.84
40	2012	5.00246E+11	1547.414	44875.01	1301375	37739883.5	561.64	-508.28
41	2013	8.52891E+11	1814.291	56243.01	1743533	54049528.3	590.51	-531.98
42	2014	1.40641E+12	2013.574	66447.93	2192782	72361796.3	607.59	-546.57
43	2015	2.25188E+12	2243.602	78526.08	2748413	96194451.8	610.25	-546.14
44	2016	3.51248E+12	2485.173	91951.39	3402202	125881460	595.69	-528.52
45	2017	5.35201E+12	2538.959	99019.39	3861756	150608499	560.95	-495.85
46	2018	7.98493E+12	2804.475	114983.5	4714322	193287209	502.91	-434.51
47	2019	1.16882E+13	3027.378	130177.2	5597621	240697721	418.28	-347.88
48	2020	1.68151E+13	3334.962	150073.3	6753297	303898378	303.60	-229.49
49	2021	2.38113E+13	3474.556	163304.2	7675295	360738876	155.25	-81.33
50	2022	3.32329E+13	3850.254	188662.5	9244460	452978558	-30.54	109.12
Sum		2.15974E+14	30996.03	1549738	45782091	2434684090		

A(hat)=	-263.3829432
B(hat)=	0.800624035
C(hat)=	1.000428162
D(hat)=	-0.000037650
E(hat)	-0.000428146

The trend component T_t was computed using the estimates obtained above. However, we see that the 4th degree polynomial is not able to capture the trend well, which can be corroborated to the fact that the fourth order differences of the data values were not constant.

Post the computation of the trend component, we computed the detrended values. $y_t - T_t$ *gives us the detrended time series.* The graph of the same too is plotted as shown in the next page:


Fitted Polynomial of degree 4 trend line
700
600
500
400
300
200
100
0
-100
-200
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
2017
2019
2021
Time Plot
Polynomial fitting of degree 4


Detrended Values
200.00
100.00
0.00
-100.00
-200.00
-300.00
-400.00
-500.00
-600.00
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
2017
2019
2021

Trend Component Analysis: Spencer's 15 points method

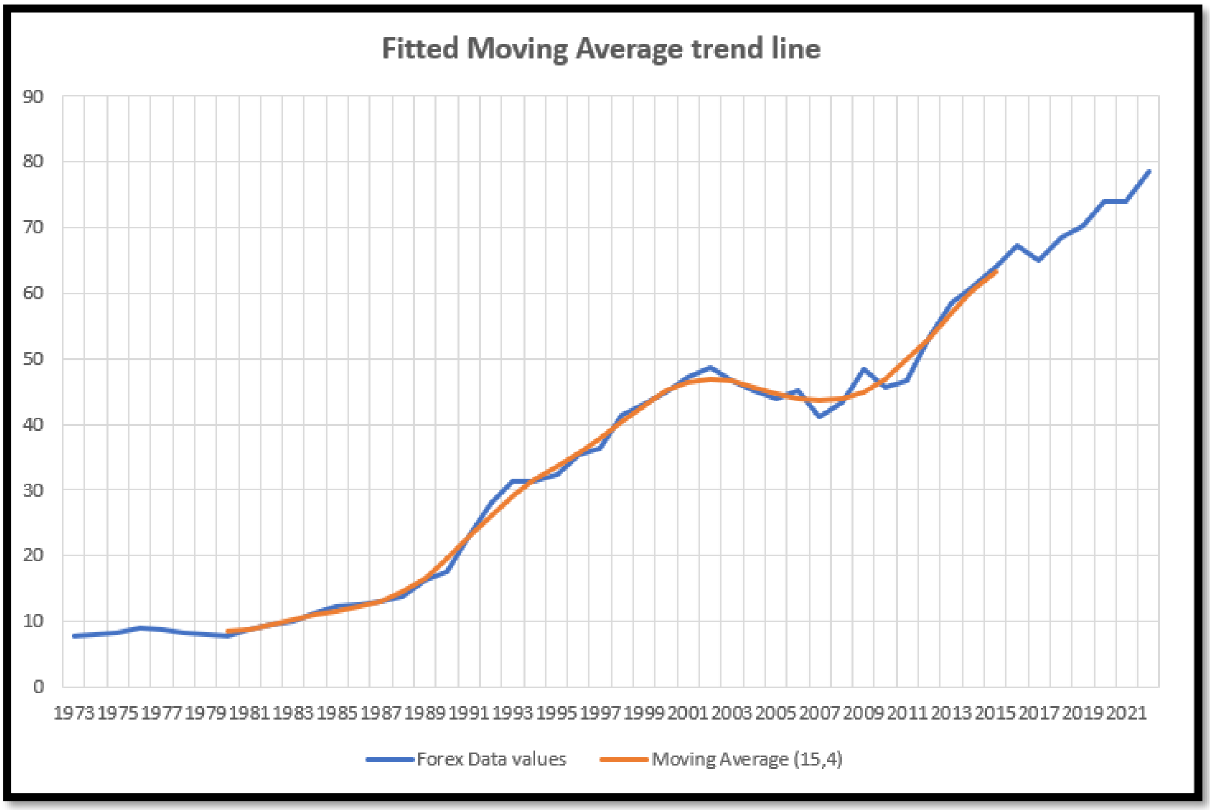
Yet another method for fitting the trend is to fit the trend using Spencer's 15 points method. The weights are as given below:

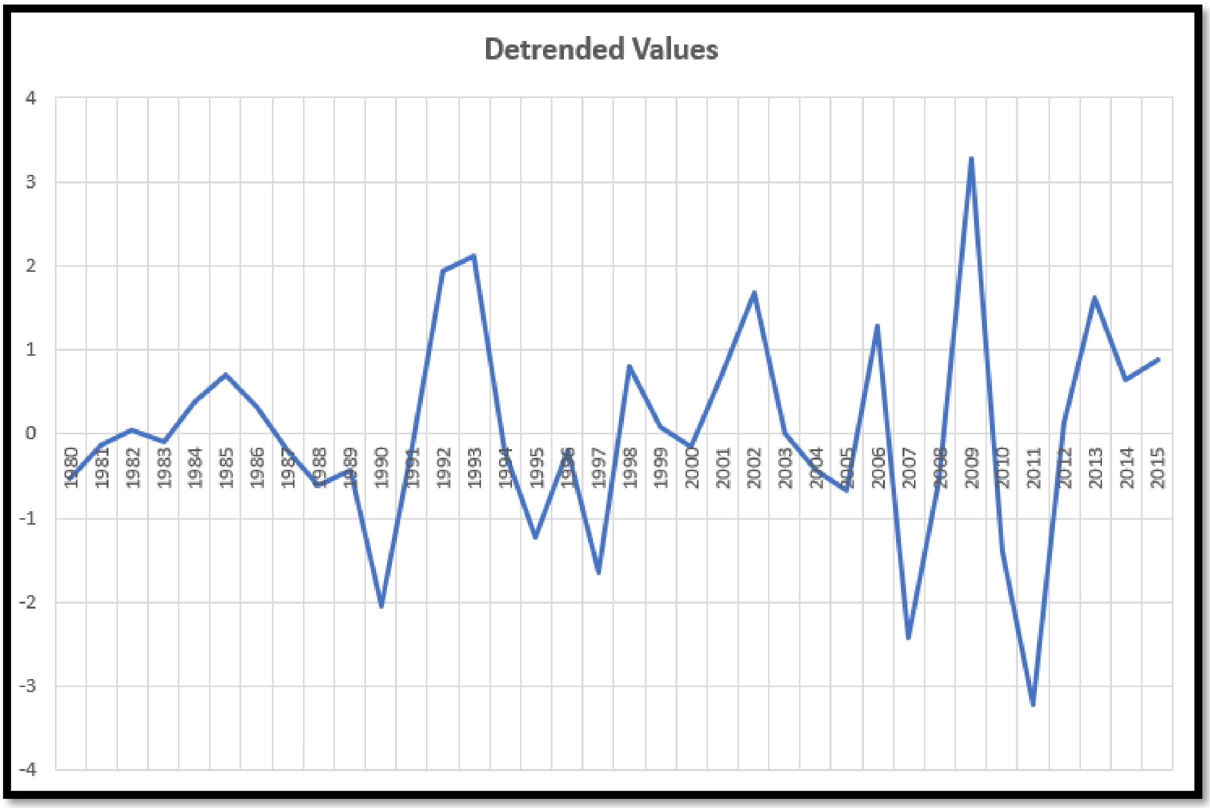
Spencer's 15 pt weights
(-3/320)
(-6/320)
(-5/320)
(3/320)
(21/320)
(46/320)
(67/320)
(74/320)
(67/320)
(46/320)
(21/320)
(3/320)
(-5/320)
(-6/320)
(-3/320)

Serial Number	Year	(y_t) Exchange Rate USD between INR	MA(m,p)	Detrended data
1	1973	7.658466667		
2	1974	8.032375		
3	1975	8.4098		
4	1976	8.971291667		
5	1977	8.767241667		
6	1978	8.195616667		
7	1979	8.15725		
8	1980	7.885491667	8.414911	-0.5294193
9	1981	8.679758333	8.806741	-0.12698307
10	1982	9.482733333	9.433015	0.049718229
11	1983	10.10589167	10.20179	-0.0959006
12	1984	11.3548	10.96541	0.389386641
13	1985	12.33981667	11.63455	0.705264557
14	1986	12.5986	12.28306	0.315542891
15	1987	12.94775833	13.14316	-0.19540458
16	1988	13.90281667	14.51915	-0.61633344
17	1989	16.20713333	16.64498	-0.43784198
18	1990	17.5021	19.53995	-2.03785161

19	1991	22.71985	22.87136	-0.1515069
20	1992	28.13509167	26.19016	1.944928906
21	1993	31.25691667	29.13818	2.11873513
22	1994	31.39435833	31.57816	-0.18380289
23	1995	32.43238333	33.6537	-1.22132052
24	1996	35.51641667	35.71676	-0.20034521
25	1997	36.36454167	38.02143	-1.65689247
26	1998	41.32943333	40.53347	0.79596612
27	1999	43.12118333	43.02952	0.091663906
28	2000	45.00093333	45.14607	-0.14513682
29	2001	47.227625	46.51472	0.712905599
30	2002	48.62203333	46.94538	1.67665276
31	2003	46.594675	46.58584	0.008834219
32	2004	45.27628333	45.71801	-0.44172542
33	2005	44.011275	44.68443	-0.67315893
34	2006	45.17219167	43.88498	1.287210208
35	2007	41.19918333	43.61771	-2.41852807
36	2008	43.411425	43.96758	-0.55615521
37	2009	48.319475	45.04964	3.269837865
38	2010	45.65086667	47.03905	-1.38818797
39	2011	46.61216667	49.8417	-3.22952914
40	2012	53.35910833	53.23734	0.121769036
41	2013	58.5255	56.90596	1.619539661
42	2014	61.01738333	60.37376	0.643622344
43	2015	64.102925	63.21432	0.888603594
44	2016	67.16683333		
45	2017	65.10150833		
46	2018	68.401825		
47	2019	70.40413333		
48	2020	74.11025833		
49	2021	73.92673333		
50	2022	78.57661667		

Post the computation of the trend component, we computed the detrended values. The graph of the same is plotted as shown in the next page:


Fitted Moving Average trend line
0
10
20
30
40
50
60
70
80
90
1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013 2015 2017 2019 2021
Forex Data values
Moving Average (15,4)


Detrended Values
4
3
2
1
0
-1
-2
-3
-4
1980 1981 1982 1983 1984 1985 1986 1988 1989 1990 1991 1992 1993 1994 1995 1997 1998 1999 2000 2001 2002 2003 2005 2006 2007 2009 2010 2011 2012 2013 2014 2015

Trend Component Analysis: Model of best fit

Three different methods were employed in order to capture the trend and represent the secular trend component of the time series. The three different methods were, exponential fitting (obtained by computing the first differences of the logarithms of the data values which came out to be constant), 4^{th} degree polynomial fitting (obtained by employing the Variate Difference Method which gave us the degree of the systematic part polynomial) and lastly Spencer's 15-point formula which fitted a moving average of extent/ period 15.

Via means of visual inspection, we notice how the trend fitted using the Moving average method best represents the data and captures the secular trend component of the time series. Hence, the trend so obtained using Spencer's 15-point formula is decided to be the best fit.

Oscillatory Component Analysis

The oscillatory component represents that component of the time series which is periodic in nature having the period of oscillations of more than a year, it causes wavelike movement in the time series. It is well known that economic and business series are influenced by changes of prosperity, recession, depression and recovery, these stages manifest themselves as a wave like movement in the time series.

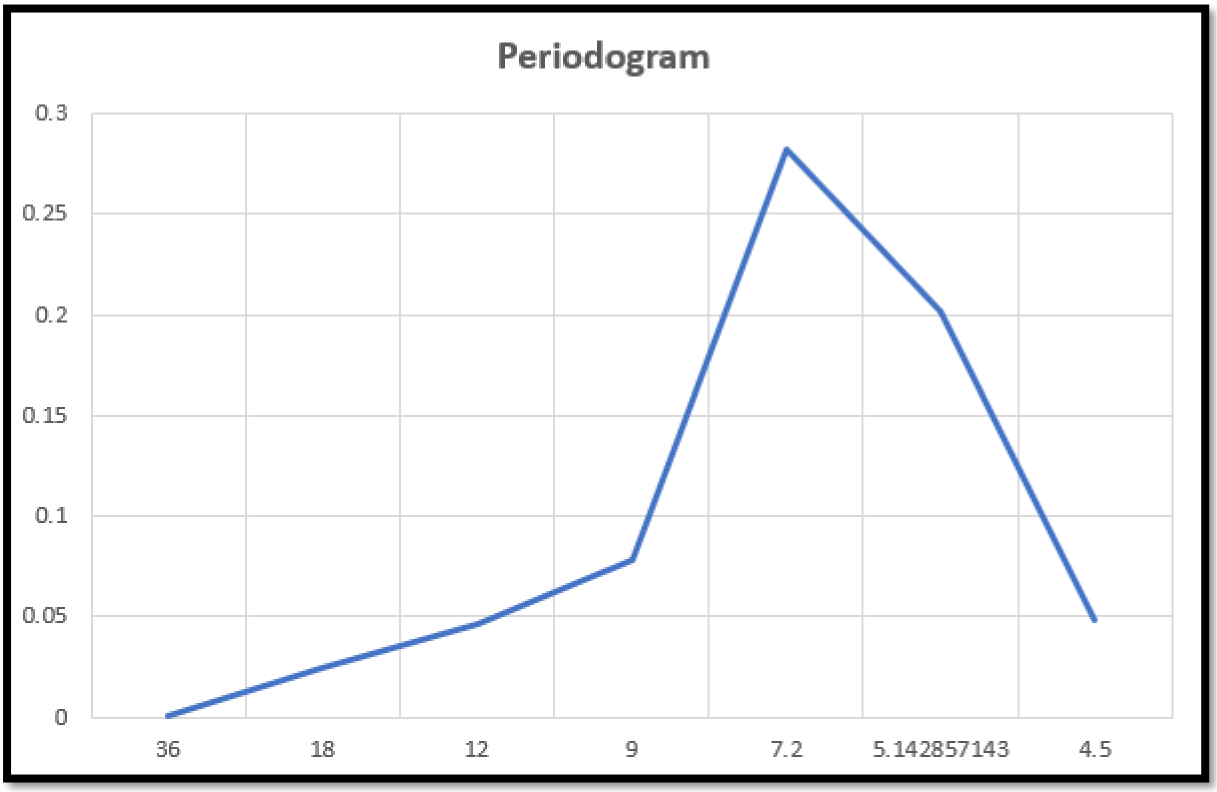
Using the harmonic analysis, the harmonic terms can be fitted to estimate the oscillatory component.

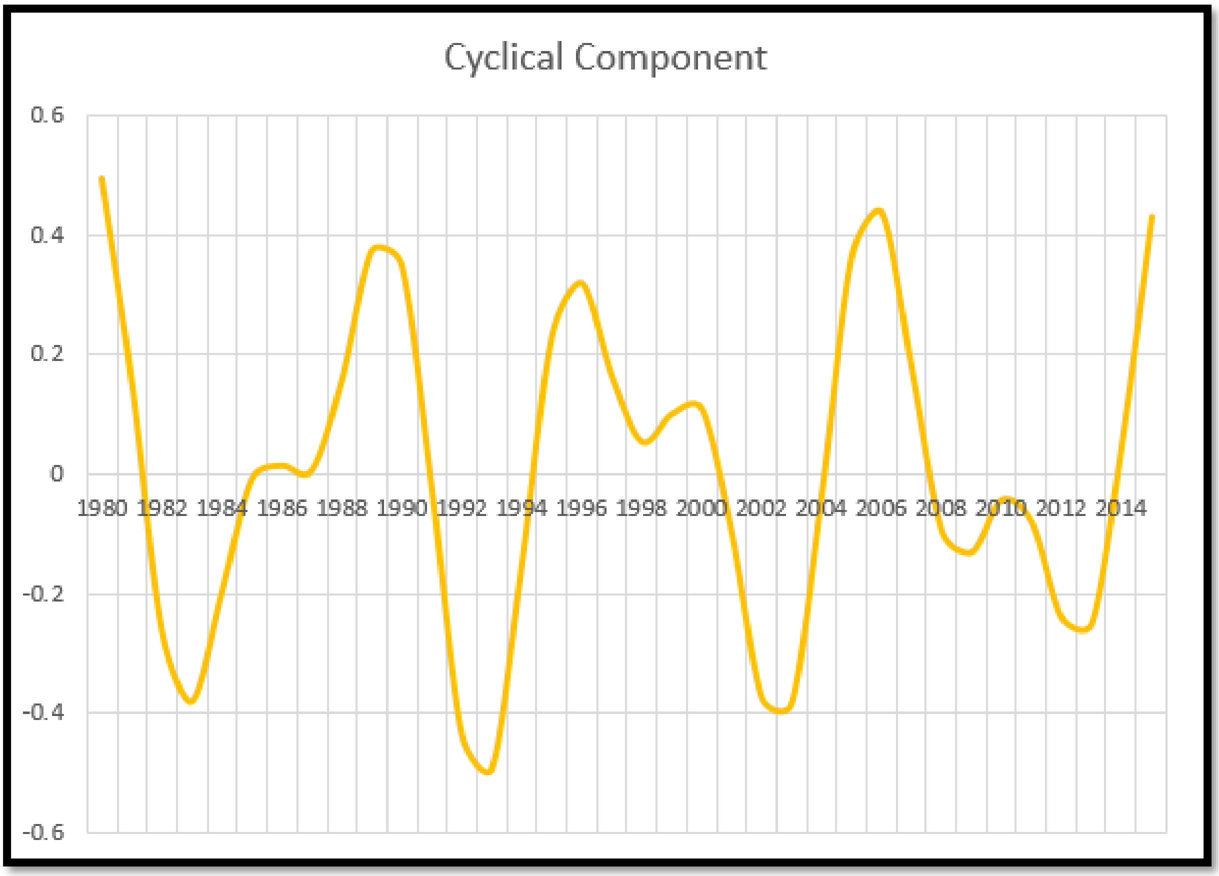
Because the trend values were best represented by Spencer's 15 points method, we will carry out the harmonic analysis for the Detrended values that were computed using the same method.

Trial Periods, μ	A	B	S^2_{μ}
36	-0.00855	0.026867	0.000795
18	0.141878	0.166884	0.04798
12	0.289682	0.069216	0.088707
9	-0.3697	-0.11782	0.150556
7.2	0.656992	-0.33572	0.544345
5.143	-0.2027	-0.59042	0.389687
4.5	0.077087	-0.29542	0.093218

a_o(hat)=	0.009282	λ_1	7.2
a_1(hat)=	-0.00855	λ_2	5.142857
a_2(hat)=	0.026867	λ_3	9
a_3(hat)=	0.141878		
a_4(hat)=	0.166884		
a_5(hat)=	0.289682		
a_6(hat)=	0.069216		
a_7(hat)=	-0.3697		
a_8(hat)=	-0.11782		

Serial Number	Month	Detrended	C_t	C_t'	C_t''
1	1980	-0.5294193	0.024365	0.22971	0.49611
2	1981	-0.1269831	0.037227	0.035813	0.154281
3	1982	0.0497182	0.030124	-0.17619	-0.26109
4	1983	-0.0959006	0.008132	-0.13158	-0.38012
5	1984	0.3893866	-0.01304	0.097704	-0.19818
6	1985	0.7052646	-0.01826	0.197202	-0.00758
7	1986	0.3155429	-0.00381	0.032835	0.014973
8	1987	-0.1954046	0.019999	-0.1704	0.007017
9	1988	-0.6163334	0.036149	-0.13074	0.158947
10	1989	-0.437842	0.033105	0.109349	0.37575
11	1990	-2.0378516	0.013042	0.23208	0.350547
12	1991	-0.1515069	-0.00971	0.063879	-0.02102
13	1992	1.9449289	-0.01889	-0.18759	-0.43613
14	1993	2.1187351	-0.00795	-0.19693	-0.49282
15	1994	-0.1838029	0.015307	0.054735	-0.15005
16	1995	-1.2213205	0.034255	0.250211	0.232349
17	1996	-0.2003452	0.035363	0.143657	0.321075
18	1997	-1.6568925	0.017837	-0.12404	0.165641
19	1998	0.7959661	-0.0058	-0.21115	0.055255
20	1999	0.0916639	-0.01866	-0.01725	0.101219
21	2000	-0.1451368	-0.01156	0.194752	0.109854
22	2001	0.7129056	0.010432	0.150144	-0.09839
23	2002	1.6766528	0.031603	-0.07914	-0.37503
24	2003	0.0088342	0.036827	-0.17864	-0.38342
25	2004	-0.4417254	0.022373	-0.01427	-0.03213
26	2005	-0.6731589	-0.00143	0.188965	0.366383
27	2006	1.2872102	-0.01758	0.149299	0.438981
28	2007	-2.4185281	-0.01454	-0.09078	0.175616
29	2008	-0.5561552	0.005523	-0.21352	-0.09505
30	2009	3.2698379	0.028272	-0.04531	-0.13021
31	2010	-1.388188	0.037455	0.206156	-0.04238
32	2011	-3.2295291	0.02651	0.215496	-0.08039
33	2012	0.121769	0.003257	-0.03617	-0.24095
34	2013	1.6195397	-0.01569	-0.23165	-0.24951
35	2014	0.6436223	-0.0168	-0.12509	0.052325
36	2015	0.8886036	0.000727	0.142605	0.432287


Periodogram
0.3
0.25
0.2
0.15
0.1
0.05
0
36
18
12
9
7.2
5.142857143
4.5


Cyclical Component
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1980 1982 1984 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008 2010 2012 2014

We computed the intensity functions for 7 trial periods, and plotted them on a periodogram. The Periodogram analysis gave a clear picture with the intensity function peaking at μ=7.2 and subsequently having high values of intensity functions existed at μ=9 and μ=5.14. With these 3 values of trial periods as the wavelengths, the cyclical component was calculated, using the formula:

$$C_t = \bar{y} + \frac{2}{N}\left(\sum y_j \cos\frac{2\pi j}{\lambda}\right) \cos\frac{2\pi t}{\lambda} + \frac{2}{N}\left(\sum y_j \sin\frac{2\pi j}{\lambda}\right) \sin\frac{2\pi t}{\lambda}$$

Where

$$A= \frac{2}{N}\left(\sum y_j \cos\frac{2\pi j}{\lambda}\right)$$

$$B= \frac{2}{N}\left(\sum y_j \sin\frac{2\pi j}{\lambda}\right)$$

$\mu^2 = A^2+B^2$

μ^2 being the intensity function

The graph so obtained represented the cyclical movement of the forex data.

Conclusion

We decomposed the time series into 2 components and thoroughly studied what each of them entailed in our data. By virtue of the nature of the data that we have diagnosed, we avoided to isolate and carry out computation of seasonal component, as it would be negligible. Forex, is a pure economic variable which is not affected by the change of seasons or months but rather by change of phases of the business cycle. The random component over the entire time frame, nullifies its own positive or negative effect as the random shocks tend to happen in both the direction.

From the analysis of the trend component using various methods, we chose the spencer's 15 points method which best fitted the given data and w*e infer that over the years the Indian currency has depreciated as the graph is an increasing curve. The curve tells how over time more INRs were required to get one single USD.*
The rationale behind the same can be the trade deficit of India which is a huge $26.72 billion (as of September 2022). *Due to the piled-up trade deficit, Indian economy is on the lookout to export more so that it can reduce the deficit, moreover, a depreciated currency helps in increasing exports as one unit of the foreign currency is worth more in that case thus allowing more purchase of Indian goods.* The other reason can be to increase FDI and FPI in the economy, *when more INRs can be bought using one single unit of the foreign currency the FDIs and FPIs increase as it is beneficial for investors. This can be viewed simply as a consequence of the* ***law of supply***.

Talking about the cyclical component we see *how the currency was made to depreciate in 90s*, this is owing to the *balance of payment crisis that the Indian economy was facing during the late*

80s. The economy was liberalized and the currency was intentionally weakened in order to increase the exports and do away the crisis looming on the balance of payments.

Though, the cyclical component is present, it should not be confused with the business cycle containing the phases of prosperity, recession, depression, and recovery. This is because, though the currency was depreciated heavily during 90s, that period is marked by high FDI and FPI, along with increase in other economic indicators. Hence, it would be wise to say that the Forex cyclical component manifests itself in response to the business cycle however, it is not the mirror reflection of the same.

Forecasting and its different types

Forecasting in time series refers to the process of predicting future values based on past observations and patterns in a time-ordered sequence of data points. Forecasting is a crucial aspect of various fields, including finance, economics, weather prediction, stock market analysis, sales, and more.
Key components and concepts related to time series forecasting include:

1.**Past Data:** Time series forecasting relies on historical data to identify patterns and trends that can be used to make predictions about future values.

2.**Temporal Structure (Decomposition):** Time series data often exhibits temporal structures, such as trends, seasonality, and cyclic patterns. Understanding and capturing these patterns are essential for accurate forecasting.

3.**Forecast Horizon:** The forecast horizon is the time period for which predictions are made. It could be short-term, medium-term, or long-term, depending on the specific application.

4.**Models and Methods:** Various statistical models are used for time series forecasting. Common methods include autoregressive integrated moving average (ARIMA) models, exponential smoothing methods, Brown’s Distributed Regression Procedure, etc.

5.**Evaluation Metrics (Diagnostic Checks/Model adequacy Checks):** The accuracy of a time series forecast is evaluated using metrics such as mean absolute error (MAE), mean squared error (MSE), or root mean squared error (RMSE). These metrics quantify the difference between the predicted values and the actual values.

6.**Stationarity:** Stationarity is often a prerequisite for time series forecasting models. If a time series is non-stationary (meaning its statistical properties change over time), pre-processing steps such as differencing may be applied to make it stationary.

The few different types of forecasting are as follows:

1.**ARIMA (AutoRegressive Integrated Moving Average):** A classical statistical method that combines autoregressive and moving average components.

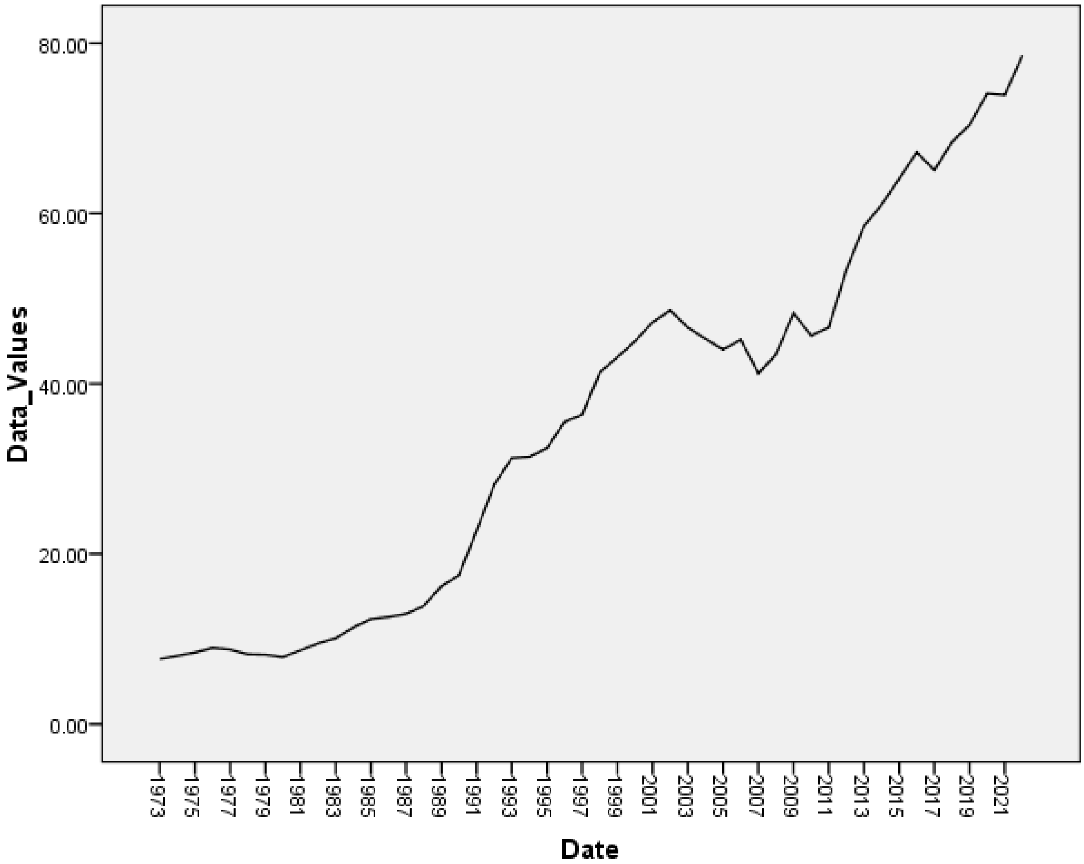
2.**Exponential Smoothing Models:** Includes methods like Simple Exponential Smoothing (SES), Double Exponential Smoothing (Holt's method), and Triple Exponential Smoothing (Holt-Winters method).

3.**Brown's Distributed Regression Procedure:** The Brown's distributed regression procedure is a statistical method for analysing time series data. It involves breaking down a time series into different components and modelling each component separately to capture various patterns and trends. The procedure is designed to handle distributed lag effects in regression models.

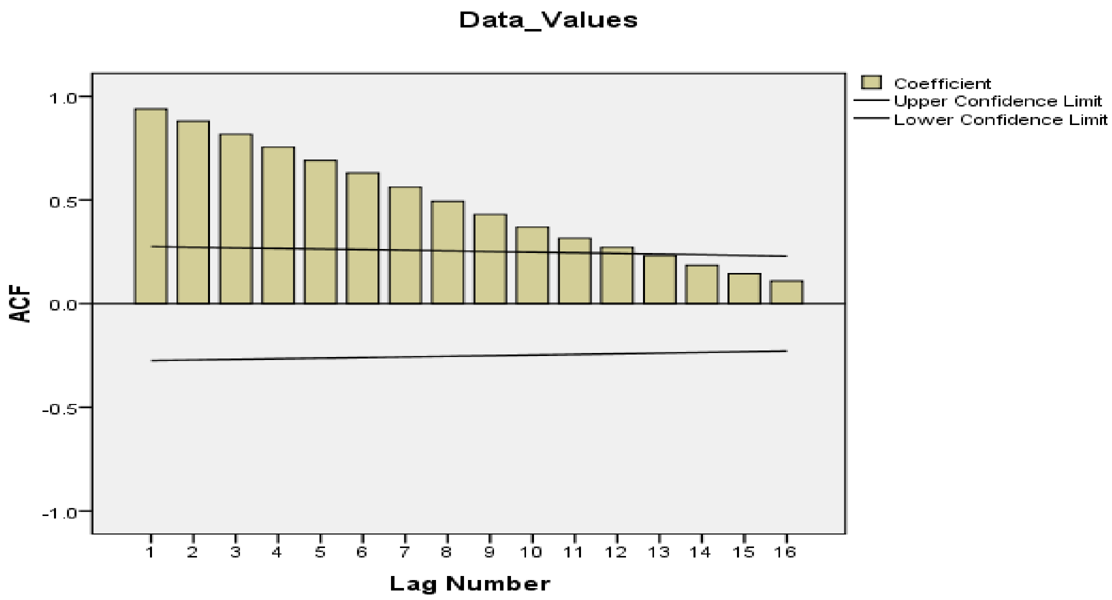
Time series forecasting is a dynamic field with ongoing research and development, particularly in the integration of advanced machine learning techniques for improved accuracy in predicting future values.

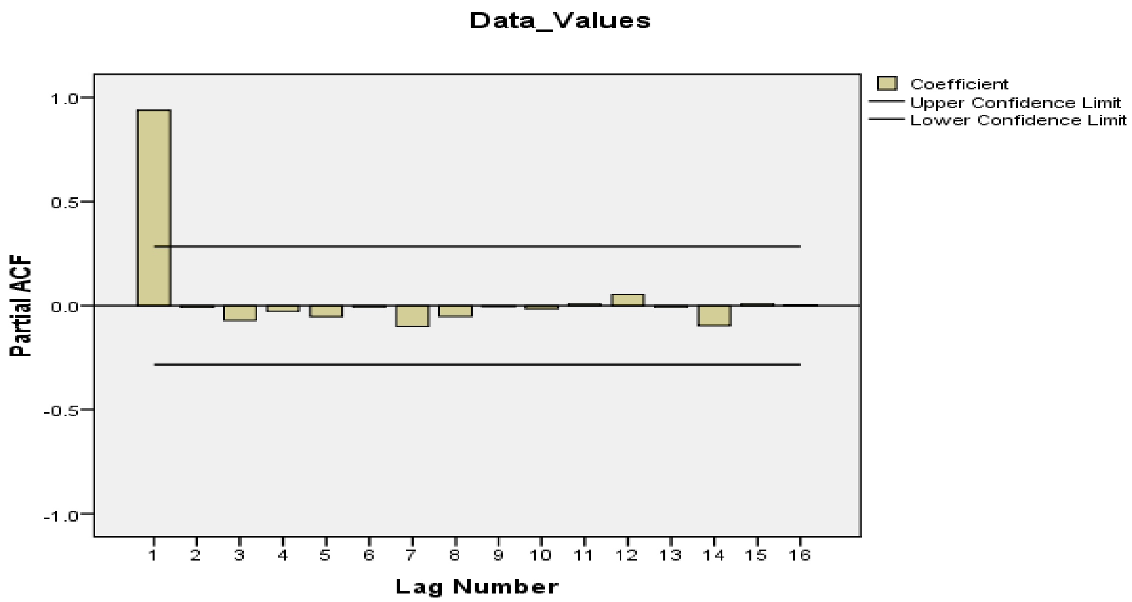
Methodology

We used a highly structured methodology for our model determination. *SPSS was put to use for developing the forecasting models and assist us in determining which of all the models was the best*.
The time plot of the actual data, i.e., the forex rate is as shown below:



It is easily observable by method of mere visual inspection that there is a major trend component which in turn implies that there is presence of non-stationarity as the mean grows over time i.e., from 1973 to 2022.
The plot of autocorrelation function (ACF) against the lag or correlogram and the plot of Partial autocorrelation function (PACF) against the lag are given as follows:

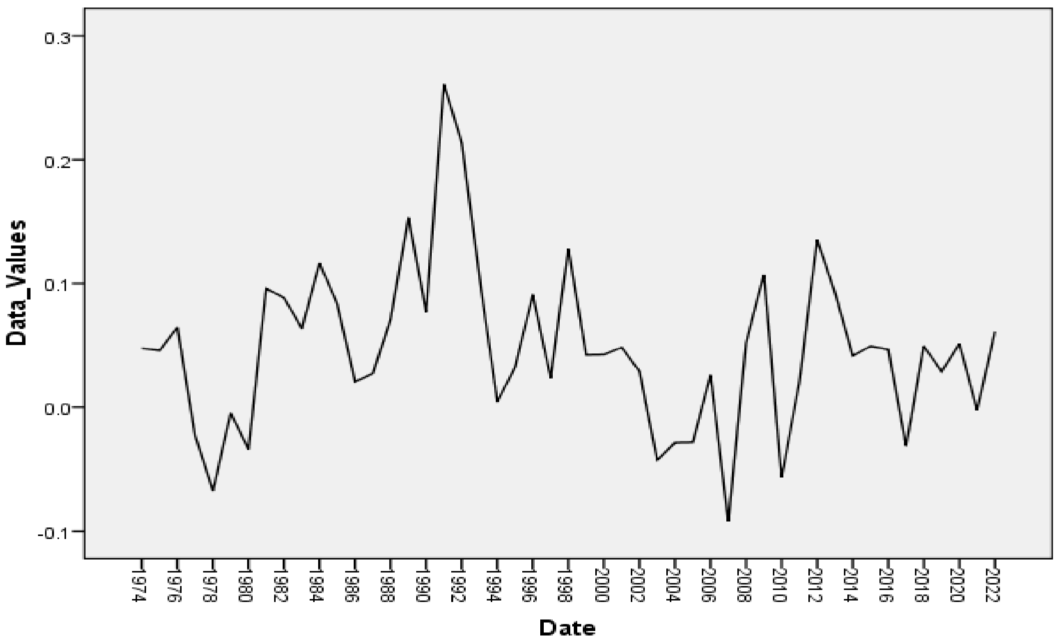




The correlogram or plot of ACF against the lag period shows a gradual decline with almost all the values being significant, this conveys that the process is not stationary. In order to make the process stationary, differencing of order 1 was done, which yielded in the following time plot:

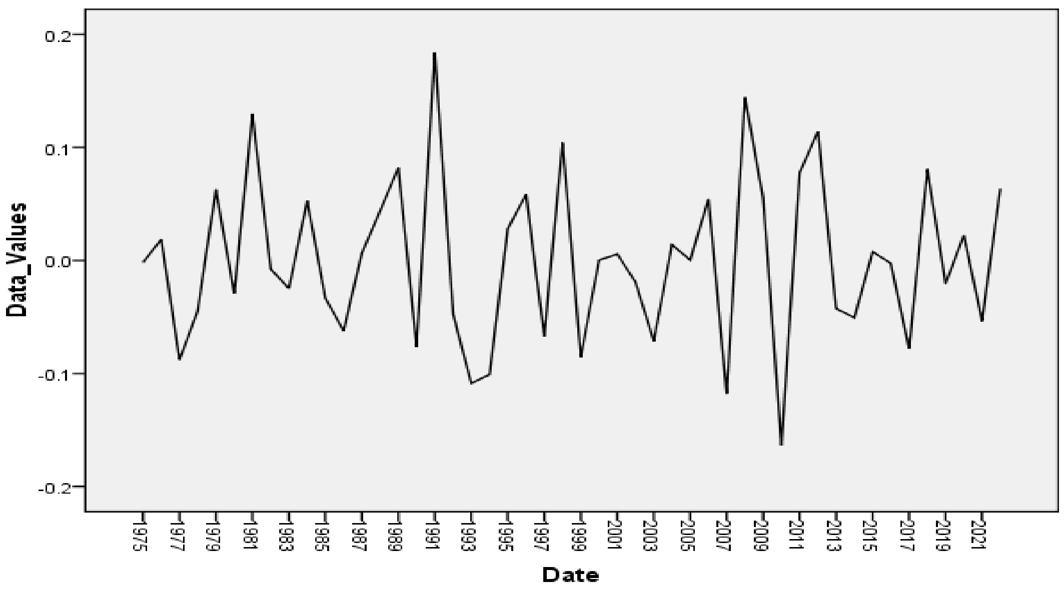
Transforms: difference(1)

It is observable that the width/range keeps on increasing as time progresses. As we move from left to right i.e., from 1973 to 2022 the range in which the data values are distributed increases. *This again is an indication of non-stationarity, to eliminate the same, log transformations are required*. Hence, we plot the time plot with differencing 1 and log transformation:



Transforms: natural log, difference(1)

It is visible by means of visual inspection that we approach stationarity as the values lie almost in a band about a central value. In order to enhance stationarity, differencing of order 2 is done along with log transformation, the following plot is obtained.



Transforms: natural log, difference(2)

Since the graph so obtained reflects near stationarity, we plot the autocorrelation functions and Partial autocorrelation functions against the lag period.

The ACF plot conveys that there is at max 1 spike consecutively which is significant and the PACF plot conveys that there are at max 2 spikes which are consecutively significant.

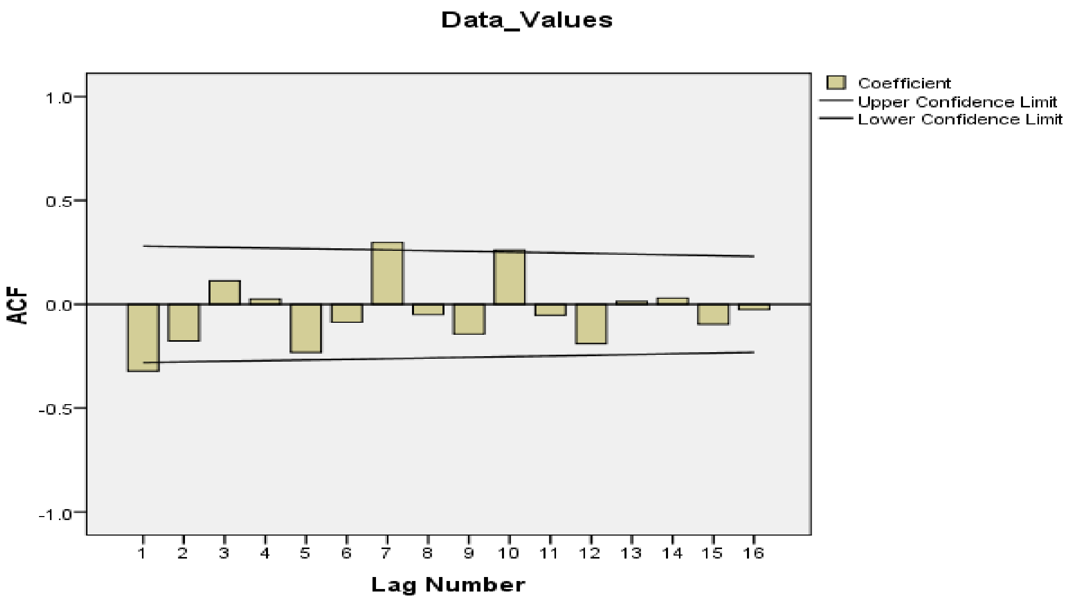
We already know that the differencing done is of order 2 along with log transformations.

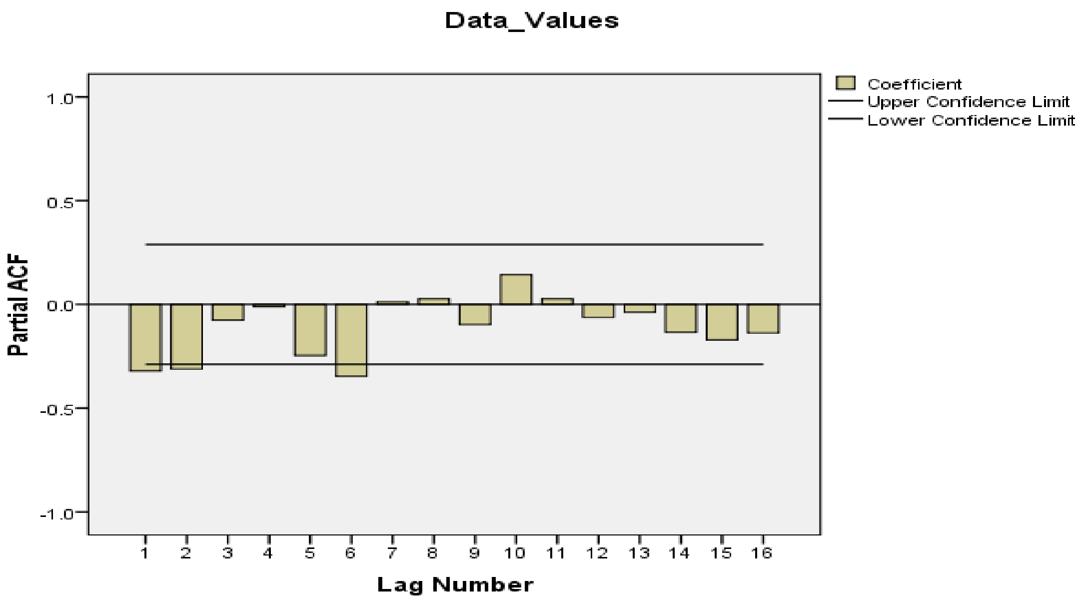
Hence, the parameters of ARIMA so obtained become:

p= 2; d= 2; q= 0,1

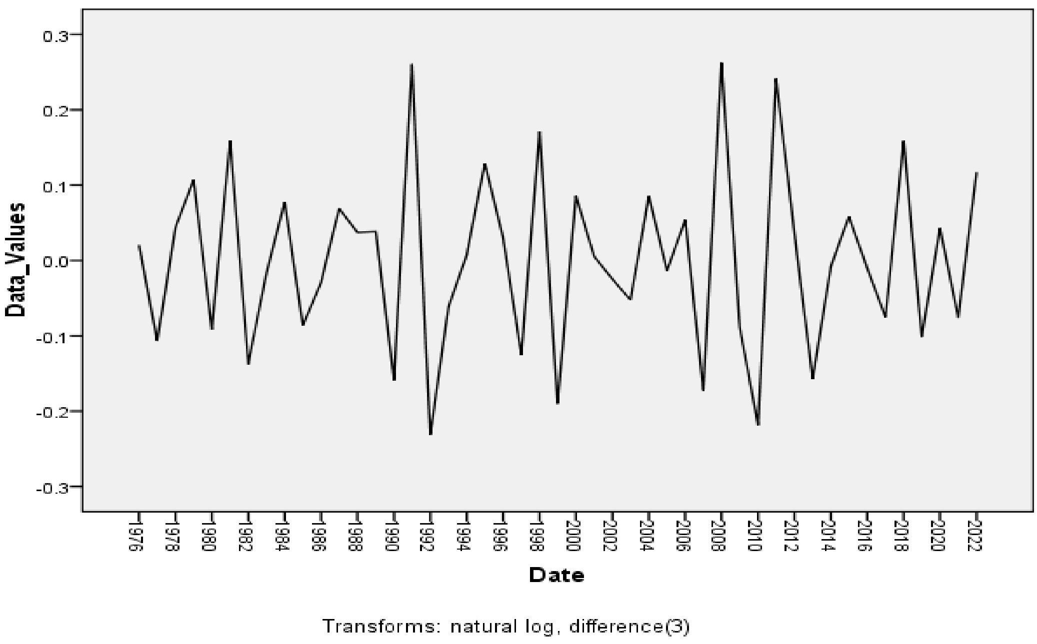
Hence, 2 candidate models become:

ARIMA (2,2,0)
ARIMA (2,2,1)





To, test out more models, we employ differencing of order 3 along with log transformations. The plot so obtained indicates how stationarity is being approached.



Since the graph so obtained reflects near stationarity, we plot the autocorrelation functions and Partial autocorrelation functions against the lag period.

The ACF plot conveys that there is at max 1 spike consecutively which is significant and the PACF plot conveys that there are at max 3 spikes which are consecutively significant.

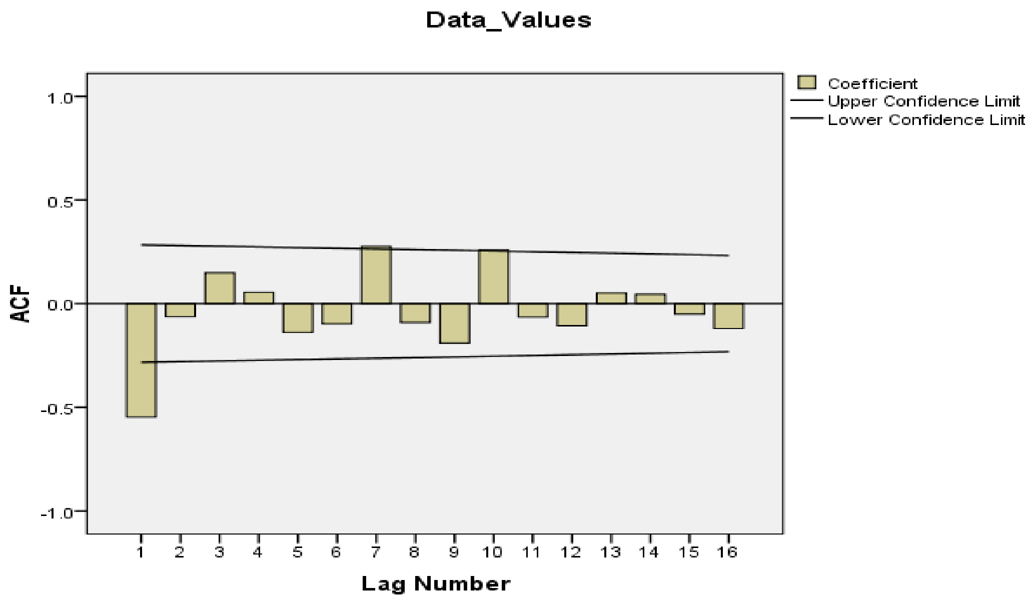
We already know that the differencing done is of order 3 along with log transformations.

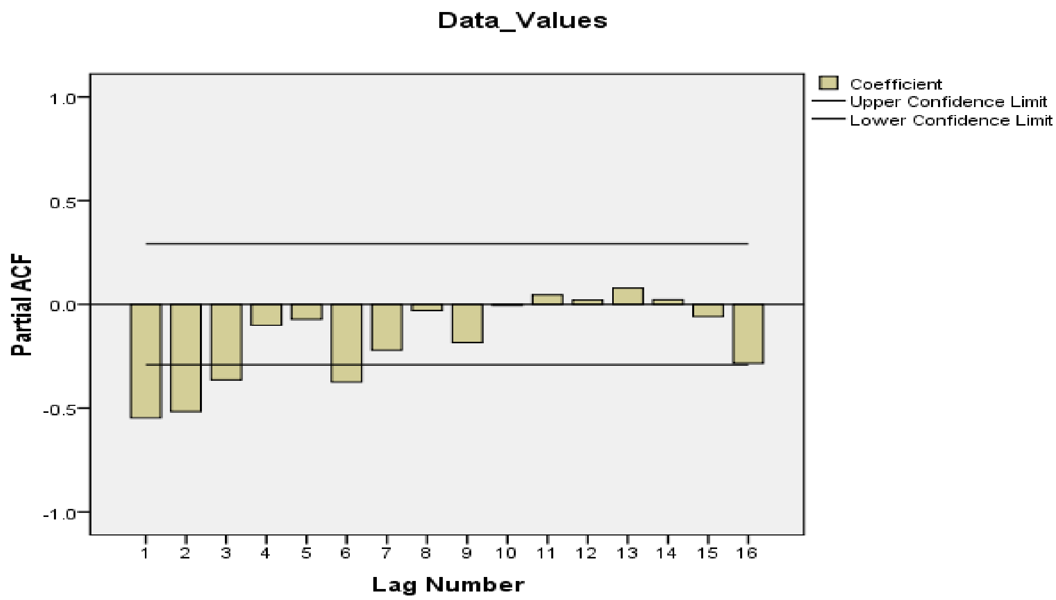
Hence, the parameters of ARIMA so obtained become:

p= 3; d= 3; q= 0,1

Hence, 2 more candidate models become:

ARIMA (3,3,0)
ARIMA (3,3,1)





Apart from the said models we also employ 2 expert models as suggested by SPSS.

In the first suggestion, SPSS employed multiple methods like ARIMA and Exponential smoothening to arrive on a good fit. The model so obtained was Brown's distributed Regression Procedure model, which is generally used for time series with a major trend component or time series which are not stationary.

In the second suggestion, SPSS employed only ARIMA to arrive on a good fit and it fitted an ARIMA (1,1,0) model.

Hence, 2 more candidate models become

Brown's distributed Regression Procedure
ARIMA (1,1,0)

Candidate Models and their Model diagnostic checks

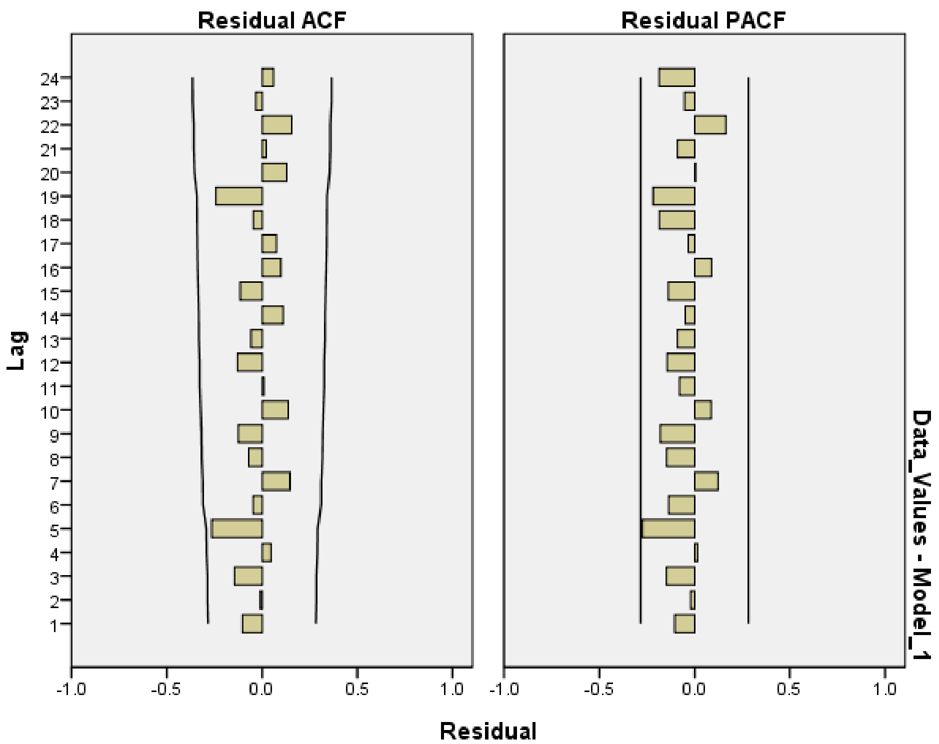
Following are the candidate models:

1) ARIMA (2,2,0)
2) ARIMA (2,2,1)
3) ARIMA (3,3,0)
4) ARIMA (3,3,1)
5) ARIMA (1,1,0)
6) Brown's distributed regression procedure

Following are the values of the Model diagnostic checks of each of the model as calculated by SPSS:

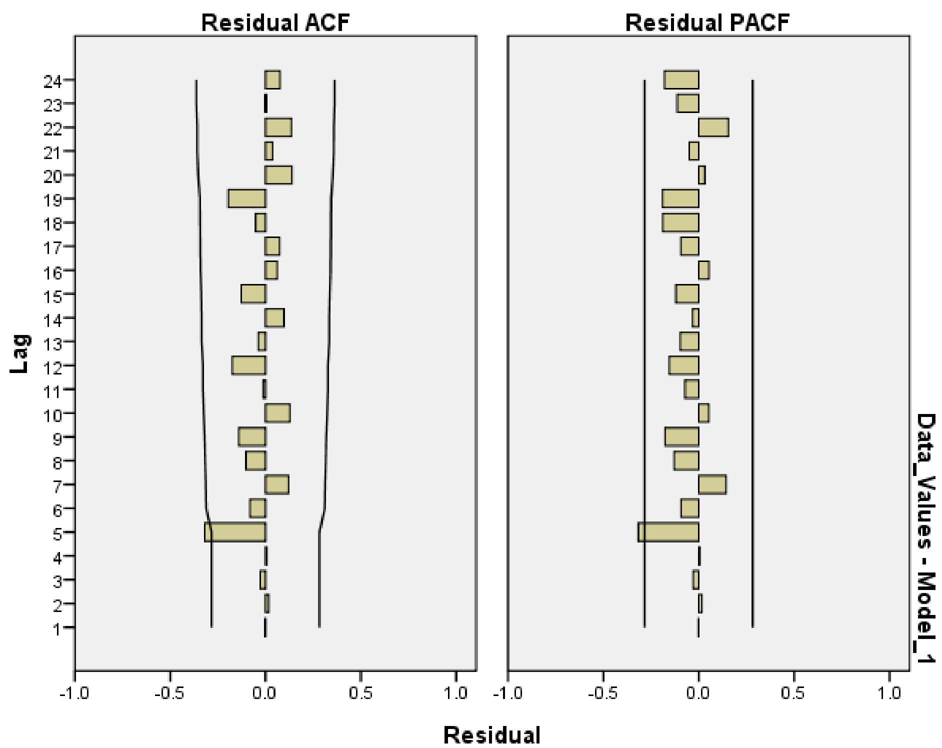
ARIMA (2,2,0)

Fit Statistic	Mean	SE	Minimum	Maximum	Percentile						
					5	10	25	50	75	90	95
Stationary R-squared	.328	.	.328	.328	.328	.328	.328	.328	.328	.328	.328
R-squared	.989	.	.989	.989	.989	.989	.989	.989	.989	.989	.989
RMSE	2.370	.	2.370	2.370	2.370	2.370	2.370	2.370	2.370	2.370	2.370
MAPE	4.961	.	4.961	4.961	4.961	4.961	4.961	4.961	4.961	4.961	4.961
MaxAPE	16.802	.	16.802	16.802	16.802	16.802	16.802	16.802	16.802	16.802	16.802
MAE	1.746	.	1.746	1.746	1.746	1.746	1.746	1.746	1.746	1.746	1.746
MaxAE	4.996	.	4.996	4.996	4.996	4.996	4.996	4.996	4.996	4.996	4.996
Normalized BIC	1.967	.	1.967	1.967	1.967	1.967	1.967	1.967	1.967	1.967	1.967



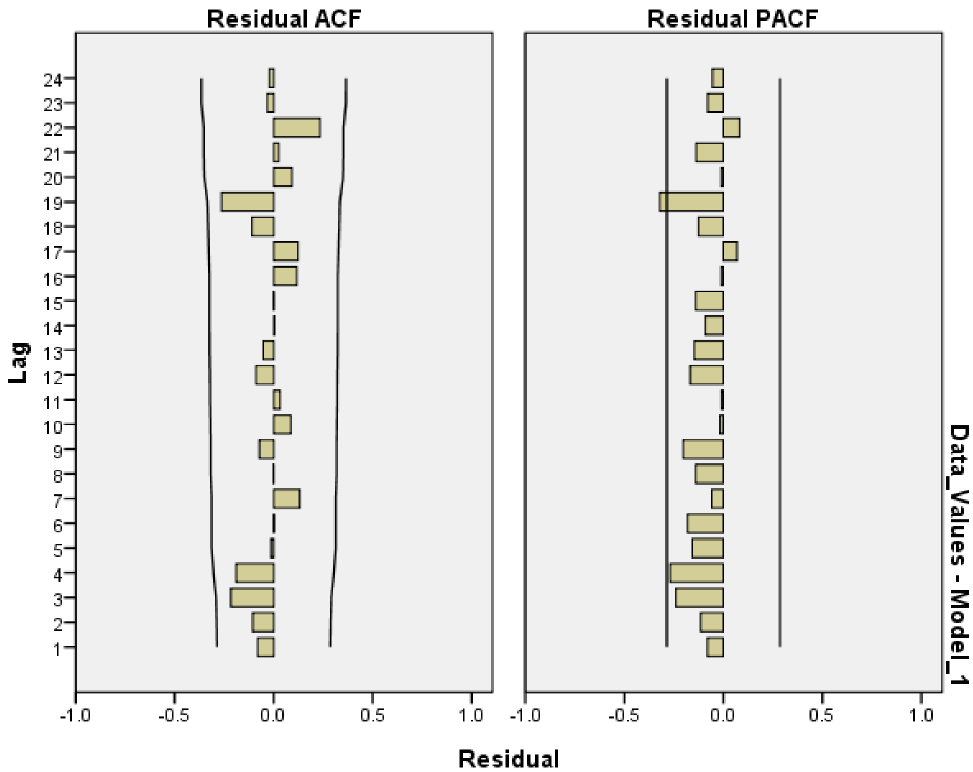
ARIMA (2,2,1)

Fit Statistic	Mean	SE	Minimum	Maximum	Percentile						
					5	10	25	50	75	90	95
Stationary R-squared	.352	.	.352	.352	.352	.352	.352	.352	.352	.352	.352
R-squared	.989	.	.989	.989	.989	.989	.989	.989	.989	.989	.989
RMSE	2.352	.	2.352	2.352	2.352	2.352	2.352	2.352	2.352	2.352	2.352
MAPE	4.937	.	4.937	4.937	4.937	4.937	4.937	4.937	4.937	4.937	4.937
MaxAPE	17.734	.	17.734	17.734	17.734	17.734	17.734	17.734	17.734	17.734	17.734
MAE	1.677	.	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.677
MaxAE	5.512	.	5.512	5.512	5.512	5.512	5.512	5.512	5.512	5.512	5.512
Normalized BIC	2.033	.	2.033	2.033	2.033	2.033	2.033	2.033	2.033	2.033	2.033



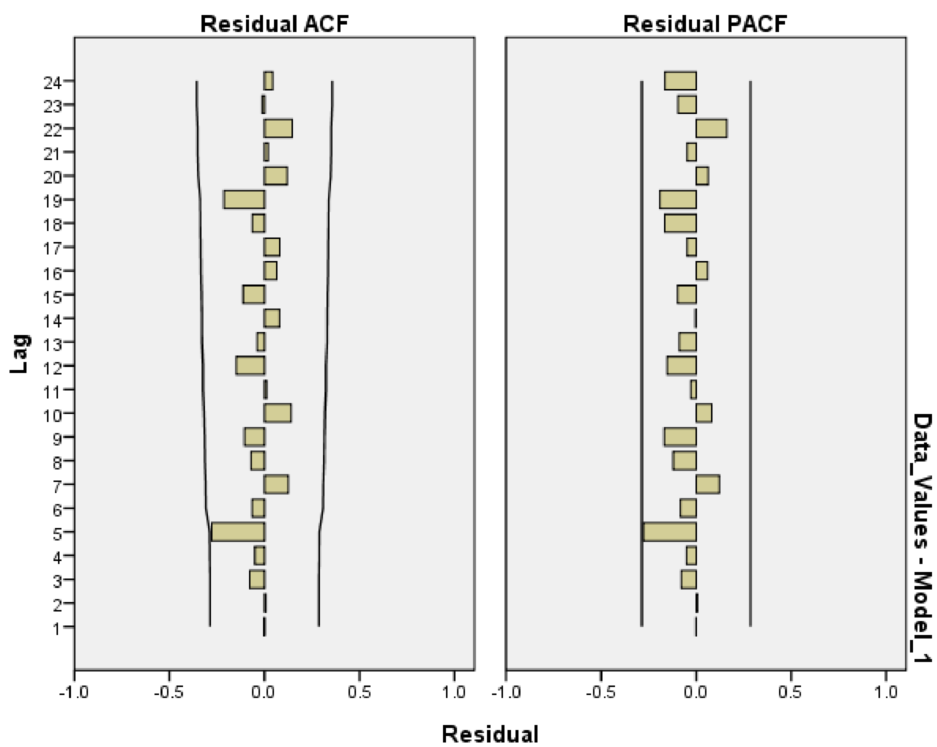
ARIMA (3,3,0)

Fit Statistic	Mean	SE	Minimum	Maximum	Percentile						
					5	10	25	50	75	90	95
Stationary R-squared	.690	.	.690	.690	.690	.690	.690	.690	.690	.690	.690
R-squared	.985	.	.985	.985	.985	.985	.985	.985	.985	.985	.985
RMSE	2.721	.	2.721	2.721	2.721	2.721	2.721	2.721	2.721	2.721	2.721
MAPE	5.719	.	5.719	5.719	5.719	5.719	5.719	5.719	5.719	5.719	5.719
MaxAPE	17.348	.	17.348	17.348	17.348	17.348	17.348	17.348	17.348	17.348	17.348
MAE	1.974	.	1.974	1.974	1.974	1.974	1.974	1.974	1.974	1.974	1.974
MaxAE	6.898	.	6.898	6.898	6.898	6.898	6.898	6.898	6.898	6.898	6.898
Normalized BIC	2.330	.	2.330	2.330	2.330	2.330	2.330	2.330	2.330	2.330	2.330



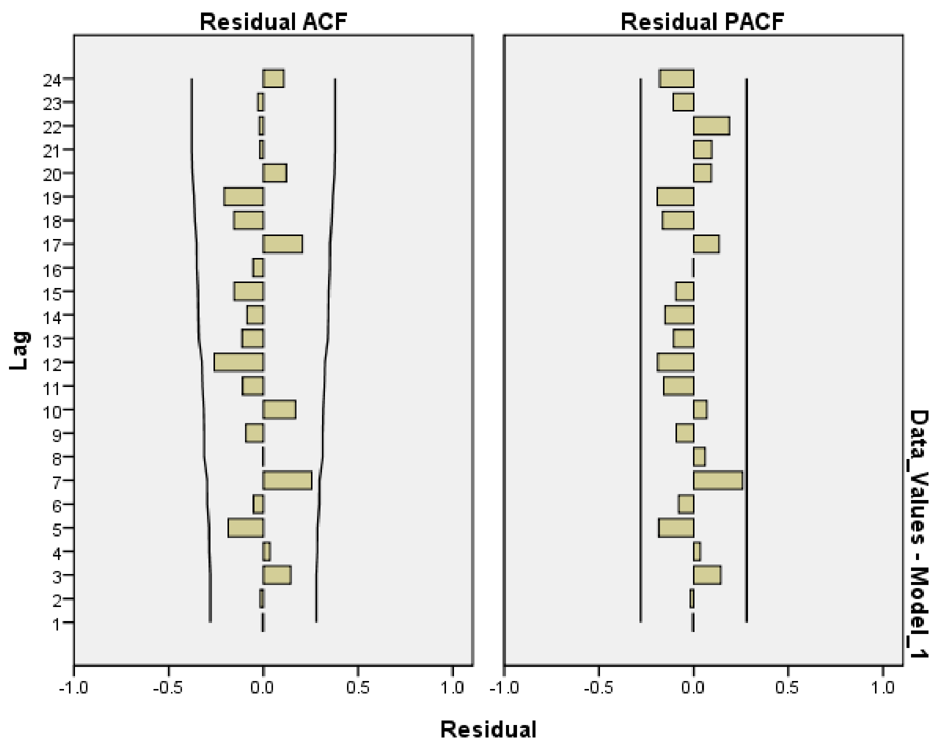
ARIMA (3,3,1)

Fit Statistic	Mean	SE	Minimum	Maximum	Percentile 5	10	25	50	75	90	95
Stationary R-squared	.755	.	.755	.755	.755	.755	.755	.755	.755	.755	.755
R-squared	.988	.	.988	.988	.988	.988	.988	.988	.988	.988	.988
RMSE	2.446	.	2.446	2.446	2.446	2.446	2.446	2.446	2.446	2.446	2.446
MAPE	5.327	.	5.327	5.327	5.327	5.327	5.327	5.327	5.327	5.327	5.327
MaxAPE	17.126	.	17.126	17.126	17.126	17.126	17.126	17.126	17.126	17.126	17.126
MAE	1.798	.	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798	1.798
MaxAE	5.606	.	5.606	5.606	5.606	5.606	5.606	5.606	5.606	5.606	5.606
Normalized BIC	2.199	.	2.199	2.199	2.199	2.199	2.199	2.199	2.199	2.199	2.199



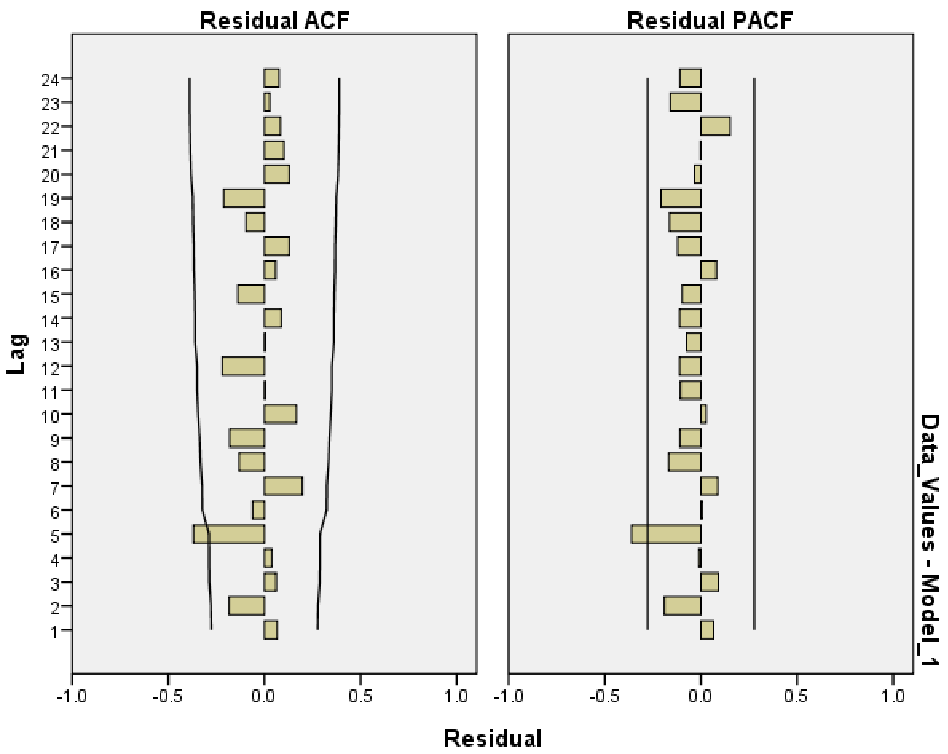
ARIMA (1,1,0)

Fit Statistic	Mean	SE	Minimum	Maximum	Percentile 5	10	25	50	75	90	95
Stationary R-squared	.158	.	.158	.158	.158	.158	.158	.158	.158	.158	.158
R-squared	.989	.	.989	.989	.989	.989	.989	.989	.989	.989	.989
RMSE	2.314	.	2.314	2.314	2.314	2.314	2.314	2.314	2.314	2.314	2.314
MAPE	4.395	.	4.395	4.395	4.395	4.395	4.395	4.395	4.395	4.395	4.395
MaxAPE	18.121	.	18.121	18.121	18.121	18.121	18.121	18.121	18.121	18.121	18.121
MAE	1.531	.	1.531	1.531	1.531	1.531	1.531	1.531	1.531	1.531	1.531
MaxAE	6.317	.	6.317	6.317	6.317	6.317	6.317	6.317	6.317	6.317	6.317
Normalized BIC	1.837	.	1.837	1.837	1.837	1.837	1.837	1.837	1.837	1.837	1.837



Brown's distributed regression procedure

Fit Statistic	Mean	SE	Minimum	Maximum	Percentile						
					5	10	25	50	75	90	95
Stationary R-squared	.312	.	.312	.312	.312	.312	.312	.312	.312	.312	.312
R-squared	.989	.	.989	.989	.989	.989	.989	.989	.989	.989	.989
RMSE	2.298	.	2.298	2.298	2.298	2.298	2.298	2.298	2.298	2.298	2.298
MAPE	5.014	.	5.014	5.014	5.014	5.014	5.014	5.014	5.014	5.014	5.014
MaxAPE	17.095	.	17.095	17.095	17.095	17.095	17.095	17.095	17.095	17.095	17.095
MAE	1.626	.	1.626	1.626	1.626	1.626	1.626	1.626	1.626	1.626	1.626
MaxAE	6.284	.	6.284	6.284	6.284	6.284	6.284	6.284	6.284	6.284	6.284
Normalized BIC	1.742	.	1.742	1.742	1.742	1.742	1.742	1.742	1.742	1.742	1.742



In all the models, the plots of residual PACF and residual ACF indicate that the estimates of the error term are random and the error term are uncorrelated. Hence, all of the models can be accepted as candidate models.

However, the model with the best fit would exhibit the following properties:

1) Lowest BIC
2) Highest R^2
3) Lowest RMSE
4) Lowest MAE

In accordance to the criteria discussed above, we see that the model which can be the best fit is

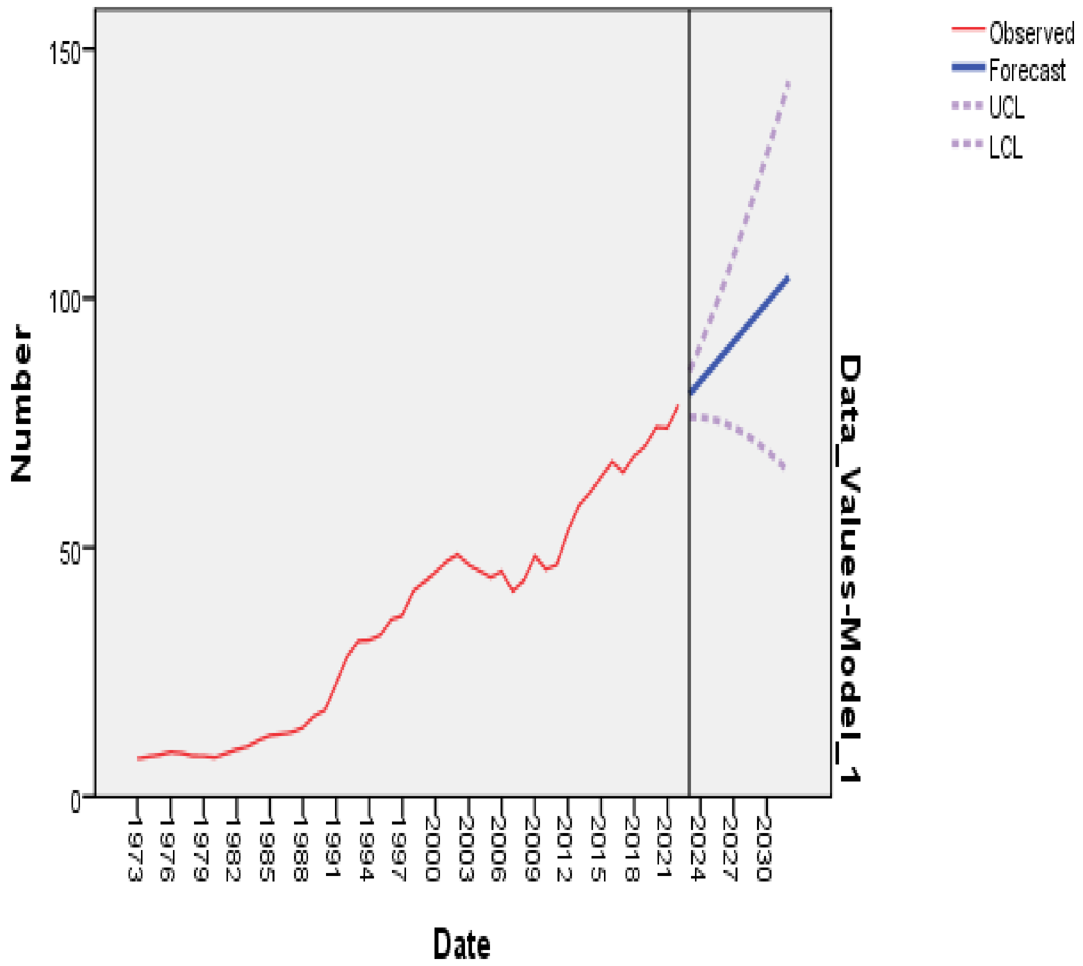
Brown's Distributed Regression Procedure

Forecasted Data values using the best fit model

The Brown's distributed Regression model is used for obtaining the 10-year forecasts i.e., from 2023 to 2032.

Following are the forecasts:

2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
80.78	83.39	86.01	88.62	91.23	93.85	96.46	99.08	101.69	104.30
85.40	90.61	96.22	102.15	108.39	114.91	121.69	128.72	135.97	143.44
76.16	76.17	75.80	75.09	74.08	72.78	71.23	69.44	67.41	65.17



Conclusion:

Using the methodology discussed in depth before, we obtained different candidate models which were then used to arrive at the best model via method of diagnostic checks. Once, the best model was determined the forecasting was done using the same to arrive at forecasted values of the forthcoming 10 years.

The interpretation of our model and forecasted data tells us that keeping in mind the trend of depreciation of INR against USD over the past 65+ years which concern our analysis, the INR is expected to keep depreciating against USD with the expected forex rate of INR 104.3 for USD 1.