

Lecture #2: Functions, Expressions, Environments

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From Last Time

- From last lecture: *Values* are data we want to manipulate and in particular,
- *Functions* are values that perform computations on values.
- *Expressions* denote computations that produce values.
- Today, we'll look at them in some detail at how functions operate on data values and how expressions denote these operations.
- As usual, although our concrete examples all involve Python, the actual concepts apply almost universally to programming languages.

Functions

- For this lecture, we're going to use this notation to show function *values* (which are created by evaluating function *definitions*):

<div style="border: 1px solid black; padding: 5px; display: inline-block;"><i>abs</i>(<i>number</i>):</div>	<div style="border: 1px solid black; padding: 5px; display: inline-block;"><i>add</i>(<i>left</i>, <i>right</i>)</div>
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(We'll simplify this in a bit to make it easier to write.)

- The green parenthesized lists indicate the number of *parameter values* or *inputs* the functions operate on (this information is also known as a function's *signature*).
- For our purposes, the blue name is simply a helpful comment to suggest what the function does, and the specific (green) parameter names are likewise just helpful hints.
- (Python actually maintains this *intrinsic name* and the parameter names internally, but this is not a universal feature of programming languages.)

Pure Functions

- The fundamental operation on function values is to *call* or *invoke* them, which means giving them one value for each formal parameter and having them produce the result of their computation on these values:

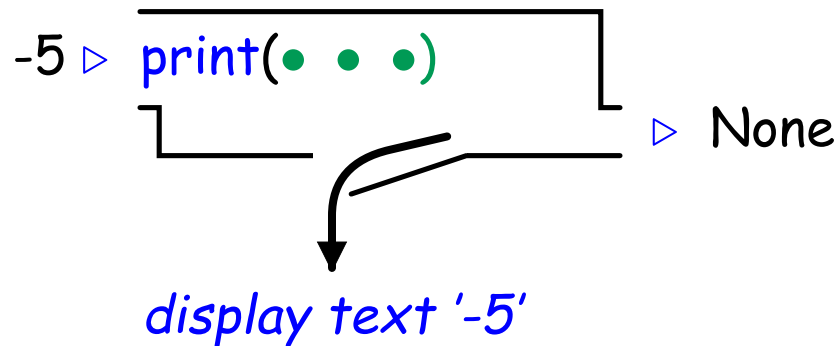
-5 ▷ `abs(number):` ▷ 5

(29, 13) ▷ `add(left, right)` ▷ 42

- These two functions are *pure*: their output depends only on their input parameters' values, and they do nothing in response to a call but compute a value.

Impure Functions

- Functions may do additional things when called besides returning a value.
- We call such things *side effects*.
- Example: the built-in `print` function:



- Displaying text is `print`'s side effect. Its value, in fact, is generally useless (always the null value).

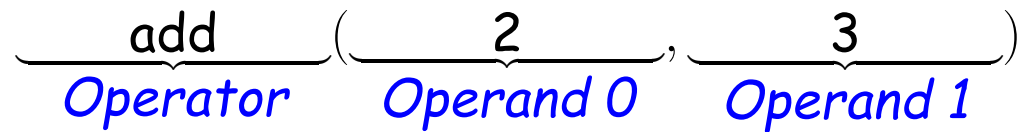
Other Kinds of Impurity

- Most side-effects involve changing the value of some variable.
- Example: the function `random.randint`:

```
>>> random.randint(0, 100)    # Random number in 0--100.  
13  
>>> random.randint(0, 100)  
55                             # Something must have changed!
```

Call Expressions

- A call expression denotes the operation of calling a function.
- Consider `add(2, 3)`:



- The operator and the operands are all themselves expressions (recursion again).
- To evaluate this call expression:
 - Evaluate the operator (let's call the value C);
 - Evaluate the operands in the order they appear (let's call the values P_0 and P_1)
 - Call C (which must be a function) with parameters P_0 and P_1 .
- Together with the definitions for base cases (mostly literal expressions and symbolic names), this describes how to evaluate any call.

Example: From Expression to Value

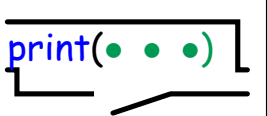
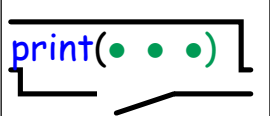
Let's evaluate the expression `mul(add(2, mul(0x4, 0x6)), add(0x3, 005))`.
In the following sequence, values are shown in boxes.
Everything outside a box is an expression.


- $\underbrace{\text{mul}(\underbrace{\text{add}(2, \underbrace{\text{mul}(0x4, 0x6)})}, \underbrace{\text{add}(0x3, 005)})}$
- $\boxed{\text{mul}(\text{left}, \text{right})} \quad (\text{add}(2, \text{mul}(0x4, 0x6)), \text{add}(0x3, 005))$
- $\boxed{\text{mul}(\text{left}, \text{right})} \quad (\boxed{\text{add}(\text{left}, \text{right})} \quad (\boxed{2} , \boxed{\text{mul}(\text{left}, \text{right})} \quad (\boxed{4} , \boxed{6})), \text{add}(0x3, 005))$
- $\boxed{\text{mul}(\text{left}, \text{right})} \quad (\boxed{\text{add}(\text{left}, \text{right})} \quad (\boxed{2} , \boxed{24}), \text{add}(0x3, 005))$
- $\boxed{\text{mul}(\text{left}, \text{right})} \quad (\boxed{26} , \text{add}(0x3, 005))$
- $\boxed{\text{mul}(\text{left}, \text{right})} \quad (\boxed{26} , \boxed{\text{add}(\text{left}, \text{right})} \quad (\boxed{3} , \boxed{5}))$
- $\boxed{\text{mul}(\text{left}, \text{right})} \quad (\boxed{26} , \boxed{8})$
- $\boxed{208} .$



Example: Print

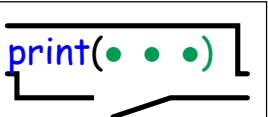
What about an expression with side effects?

1. `print(print(1), print(2))`

2.  ( (`1`), `print(2)`)

3.  (`None` , `print(2)`)
and print '1'.

4.  (`None` ,  (`2`))

5.  (`None` , `None`)
and print '2'.

6. `None`
and print 'None None'.

Names

- Evaluating expressions that are literals is easy: the literal's text gives all the information needed.
- But how did I evaluate names like `add`, `mul`, or `print`?
- Deduction: there must be another source of information.
- We'll first try a simple approach: *substitution* of values for names.
- This won't cover all the cases, however, and so we'll introduce the concept of an *environment*.

Substitution

- Let's try to explain the effect of

$x = 3$

$y = x * 3$

$z = y ** x$

by treating each assignment (=) as a *definition*.

- Thus, we get

$x = 3$

$x = 3$

$x = 3$

$x = 3$

$y = x * 2$

$y = 3 * 2$

$y = 6$

$y = 6$

$z = y ** x$

$z = y ** 3$

$z = 6 ** 3$

$z = 216$

- That is, we *replace names by their definitions (values)*.

Substitution and Functions

- Now consider a simple function definition:

```
def compute(x, y):  
    return (x * y) ** x  
print(compute(3, 2))
```

- A **def** statement is sort of like an assignment, but specialized to functional values.
- The **def** statement above defines **compute** to be "the function of **x** and **y** that returns $(xy)^x$."
- Here, I'll use a common notation for that (due to Church):

$$\lambda x, y : (xy)^x.$$

- So after substitution for **compute**, we have

```
print( (  $\lambda x, y : (xy)^z$  ) (3, 2) )
```

- Now what?

Substitution and Formal Parameters

- A function call such as

$$(\lambda x, y : (xy)^z) (3, 2)$$

from last slide is like a *simultaneous assignment* to or substitution for x and y .

- So we replace the whole expression with

$$(3 \cdot 2)^3$$

and (eventually), just 216.

Getting Fancy

- What about this?

```
def incr(n):  
    def f(x):  
        return n + x  
    return f
```

```
print(incr(5)(6))
```

- The `incr` function returns a function. The argument to `print` then calls this function on 6.
- What happens?

Answer

- First, deal with `incr`:

```
def incr(n):  
    def f(x):  
        return n + x  
    return f
```

```
print(incr(5)(6))
```

```
print(( $\lambda$  n: return  $\lambda$  x: n + x)(5)(6))
```

- The 5 now gets substituted for `n`:

```
print(( $\lambda$  x: 5 + x)(6))
```

- And 6 for `x`:

```
print(5 + 6)
```

- Finally giving

```
print(11)
```


Trouble

- Alas, this relatively simple (if tedious) approach doesn't work.
- Example:

```
x = 4
x = 8
print(x)
```

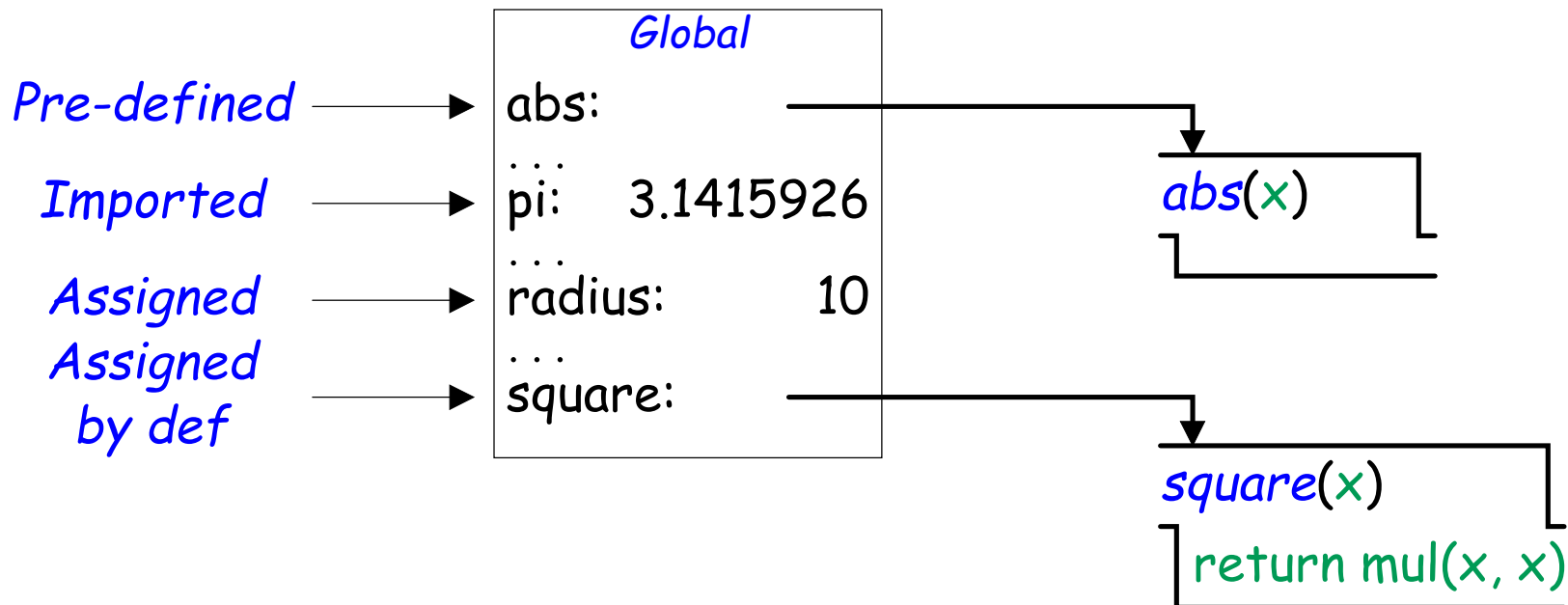
- If we just substitute for the first `x` as before:

```
x = 4
x = 8      # or even worse: 4 = 8
print(4)
```

- ...we get a wrong result (4 instead of 8).
- After one substitution, `x` isn't around any more to substitute for.
- We need something stronger.

Environments

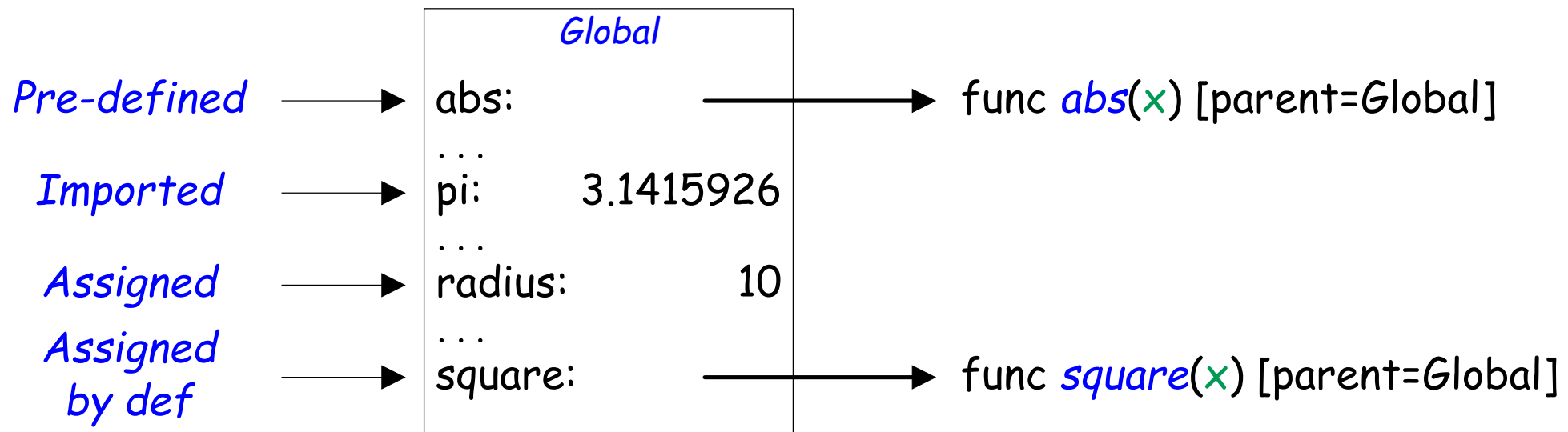
- An **environment** is a mapping from names to values.
- We say that a name is **bound to** a value in this environment.
- In its simplest form, it consists of a single *global environment frame*:



```
from math import pi
radius = 10
def square(x): return x**2
```

Slight Change of Notation

- You'll be using the Python Tutor from time to time, which uses a somewhat different notation for function values. Might as well get used to it (we'll explain the "parent=" stuff in a later lecture):



```
from math import pi
radius = 10
def square(x): return x**2
```

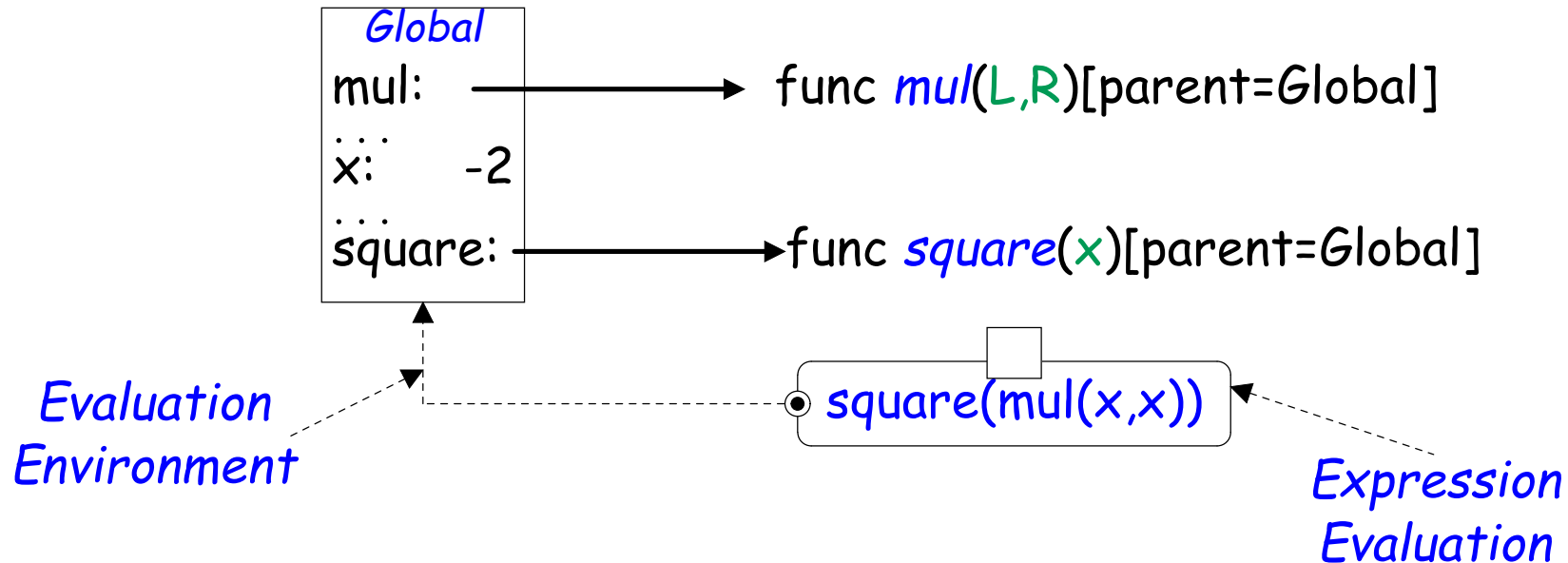
Environments and Evaluation

- Every expression is evaluated in an environment, which supplies the meanings of any names in it.
- Evaluating an expression typically involves first evaluating its subexpressions (the operators and operands of calls, the operands of conventional expressions such as $x^*(y+z)$, ...).
- These subexpressions are evaluated in the same environment as the expression that contains them.
- Once their subexpressions (operator + operands) are evaluated, calls to user-defined functions must evaluate the expressions and statements from the definition of those functions.

Evaluating User-Defined Function Calls

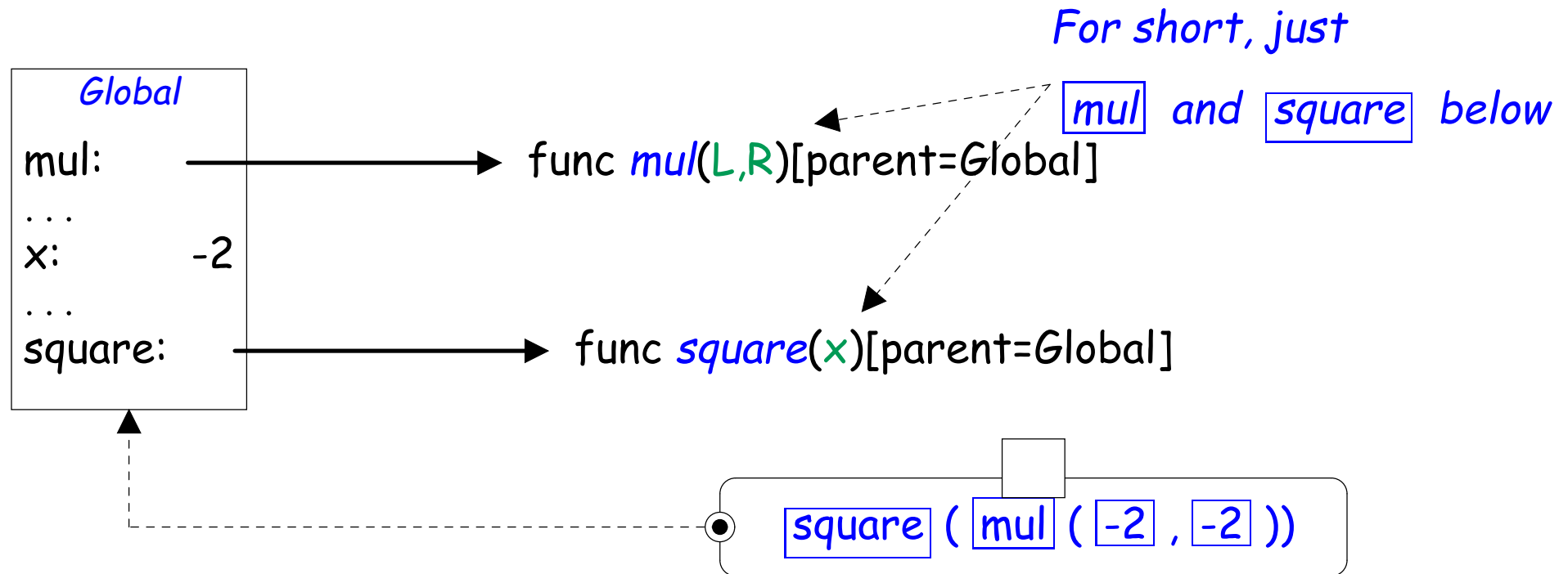
- Consider the expression `square(mul(x, x))` after executing

```
from operator import mul
def square(x):
    return mul(x,x)
x = -2
```



Evaluating User-Defined Function Calls (II)

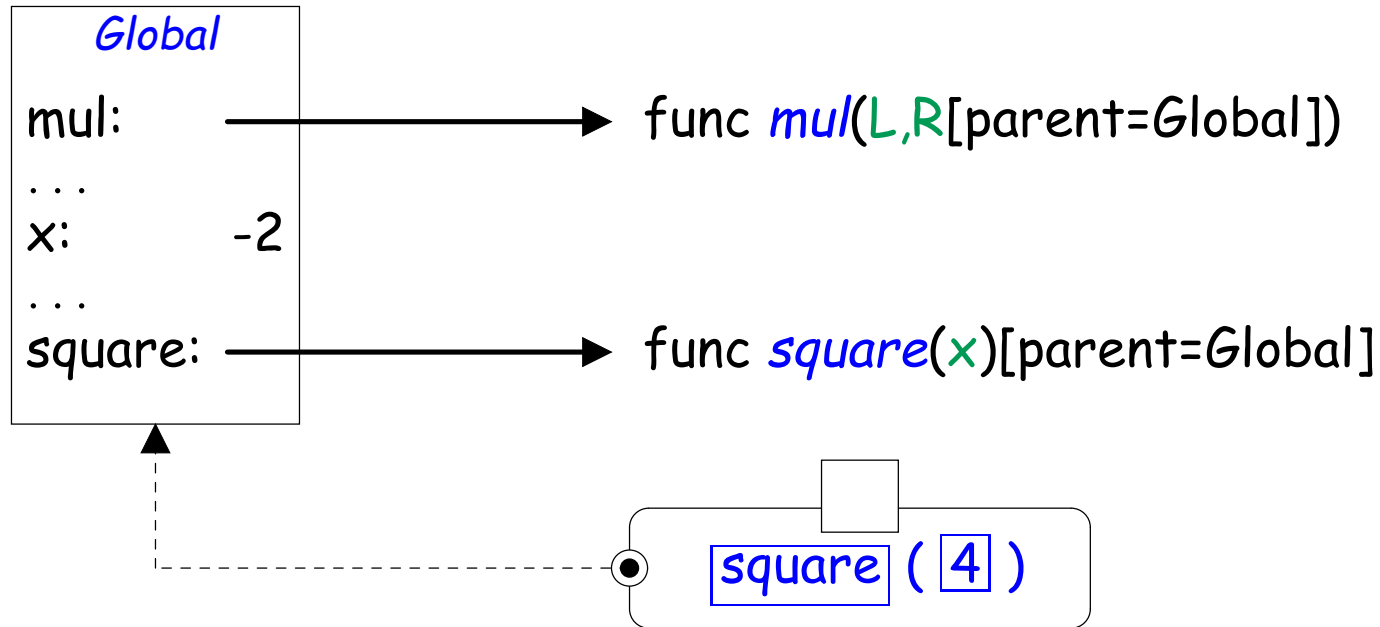
- First evaluate the subexpressions of `square(mul(x, x))` in the global environment:



- Evaluating subexpressions `x`, `mul`, and `square` take values from the expression's environment.

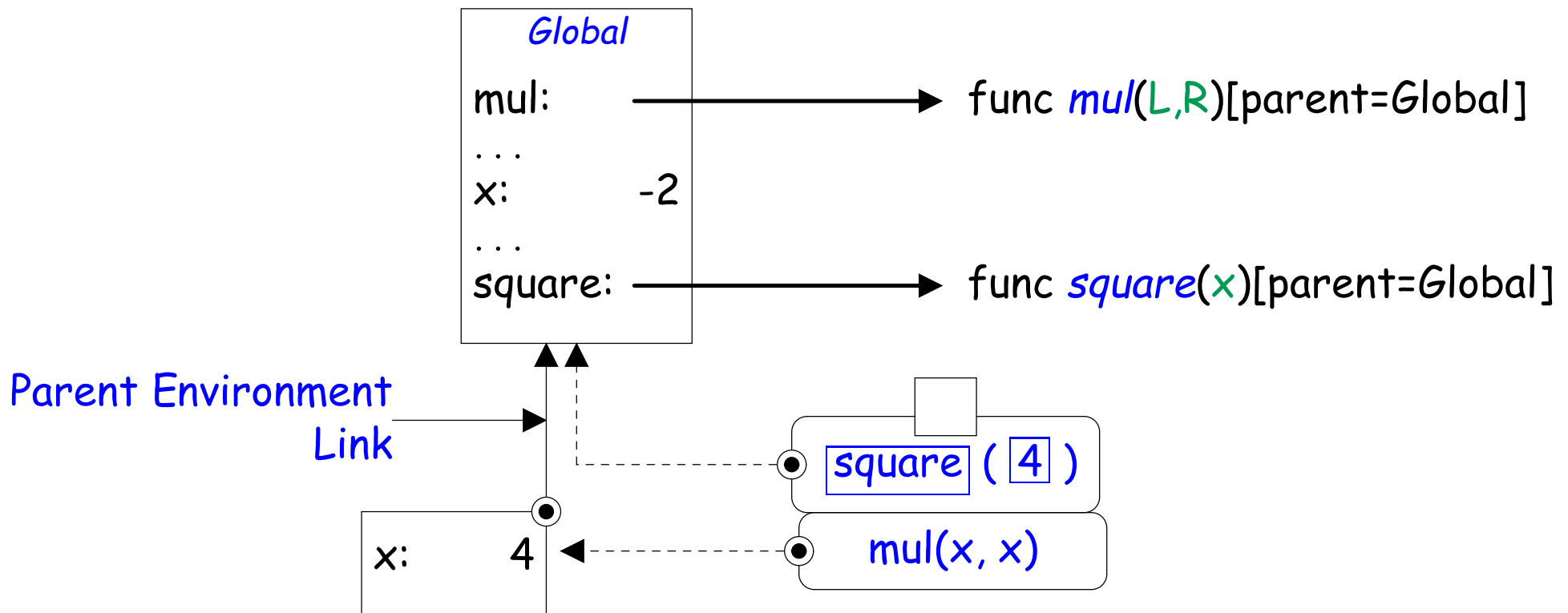
Evaluating User-Defined Functions Calls (III)

- Then perform the primitive multiply function:



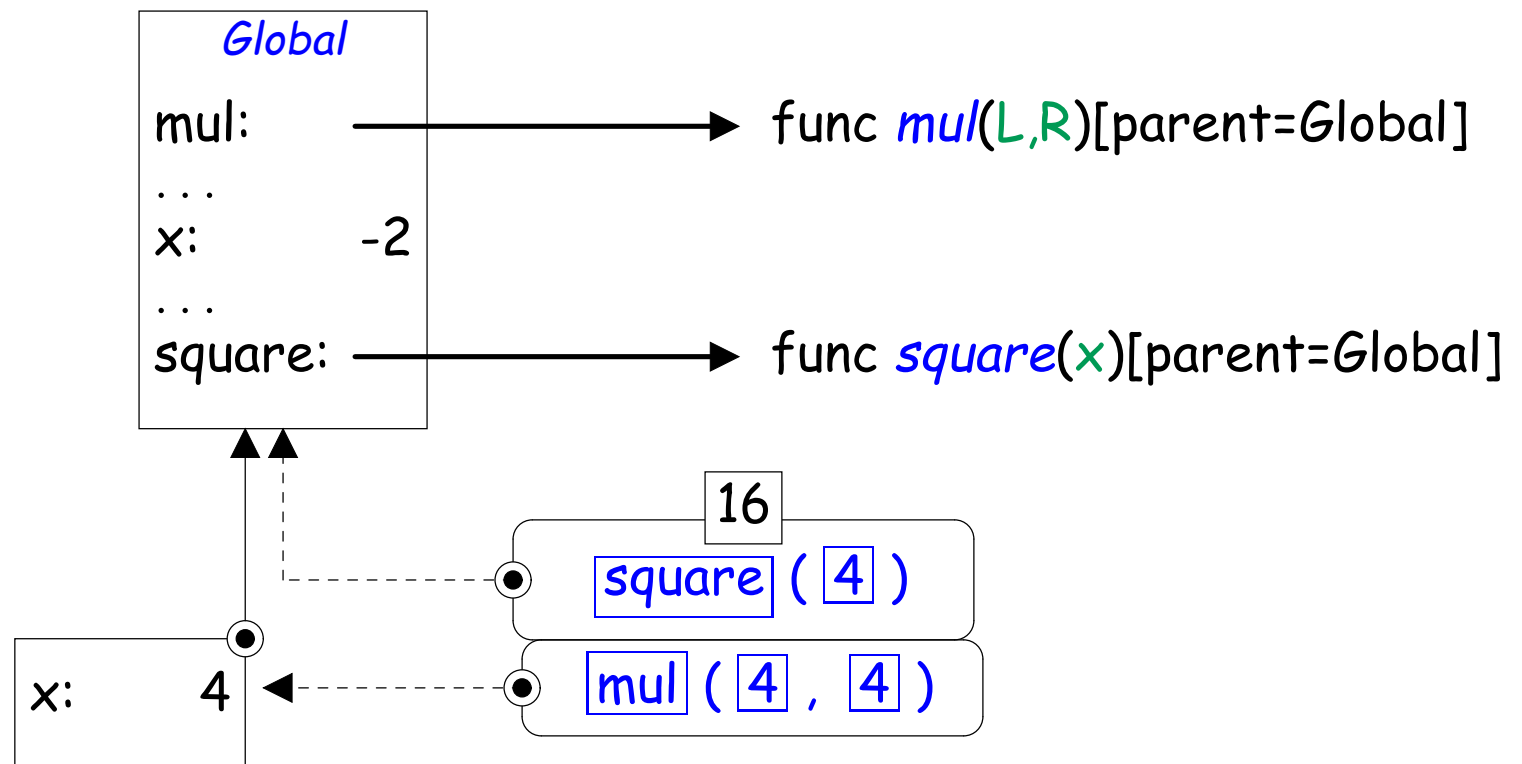
Evaluating User-Defined Functions Calls (IV)

- To explain parameter to user-defined `square` function, extend environment with a *local environment frame*, attached to the frame in which `square` was defined (the global one in this case), and giving `x` the operand value.
- Now replace original call with evaluating body of `square` in the new local environment.



Evaluating User-Defined Functions Calls (V)

- When we evaluate `mul(x, x)` in this new environment, we get the same value as before for `mul`, but the local value for `x`.
- When evaluating an identifier in a chain of environments, follow the parent environment links to the first frame containing its definition.



So How Does This Help?

- The original problem that led to this whole environment diagram thing was how to deal with:

```
x = 4  
x = 8  
print(x)
```

- Now it's easy. Each time we assign to *x*, we create a new binding for it in the current evaluation frame (replacing the old one, if any).
- We get the new (last assigned) value when we look up *x* in the modified environment.