

Q1. Give an example of 4 node graph that does not have a topological ordering.

Answer)

A directed graph does not have a topological ordering if it contains a cycle.

An example of a 4-node graph with no topological ordering is:

Nodes: A, B, C, D

Edges: $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow A$, and $C \rightarrow D$

This graph contains a cycle ($A \rightarrow B \rightarrow C \rightarrow A$), so a topological ordering is not possible.

Q2. Run the build max heap [procedure (that internally uses the max- heapify procedure) on the following array and report the number of swap operations done:

89, 19, 50, 27, 32, 65, 2, 5, 7

Answer)

To build a max heap, we call max-heapify from index 4 down to index 1.

Array: 89, 19, 50, 27, 32, 65, 2, 5, 7

Heapify at index 4 \rightarrow swap 27 and 7 \rightarrow 1 swap

Heapify at index 3 \rightarrow swap 50 and 65 \rightarrow 1 swap

Heapify at index 2 \rightarrow swap 19 and 32 \rightarrow 1 swap

Heapify at index 1 \rightarrow no swap

Total swaps = 3.

Q3. A group of people wants to cross a river. There are several boats available each with a different upper limit on the number of people it could accommodate. Each boat costs the same and you wish to hire minimum number of boats. Give a greedy strategy that works and discuss its time complexity.

Answer)

A greedy strategy to use the minimum number of boats is:

1. First sort all boats in **decreasing order of capacity** (largest boat first).
2. Start taking boats from the largest capacity and fill each boat with as many people as it can take.
3. Continue this until all people have been placed in some boat.
4. The number of boats used in this process is the minimum needed.

Using the largest boat first is always better because it can carry more people and reduces the remaining people faster. Choosing a smaller boat first may increase the number of boats needed.

Time complexity:

Sorting the boats takes $O(m \log m)$ time, where m is the number of boats.

Filling people into boats takes $O(m)$ time.

So overall time complexity is $O(m \log m)$.

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