Monetary Policy Analysis with Heterogeneous Unemployment Dynamics

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Abstract

This paper examines the welfare effects of monetary policy rules targeting unemployment rates across skill levels within a DSGE framework. I develop a New Keynesian model with heterogeneous agents and asymmetric search and matching frictions, calibrated to reflect the empirical responses of low- and high-skill unemployment rates to identified monetary policy shocks. I find that the optimal monetary policy rule should respond to both low- and high-skill unemployment, alongside inflation. This need arises because, unlike standard New Keynesian models, the inclusion of skill-specific matching frictions introduces a congestion externality that elevates unemployment levels inefficiently. While a robust response to inflation remains welfare improving, responding to output consistently reduces welfare outcomes.

Keywords: Monetary Policy, Unemployment, Heterogeneity, Search and Matching

JEL Codes: E24, E32, E52, J64

1 Introduction

In recent years, central banks, including the Federal Reserve, have emphasized inflation targeting or price stability as core policy objectives, often assigning relatively small weights to output stabilization and even less attention to indicators such as unemployment. This approach relies on the premise that stabilizing prices reduces the trade-off between output and inflation volatility, allowing inflation targeting with minimal effects on output. However, this approach typically assumes models with nominal rigidities and Walrasian labor markets.

In contrast, the Federal Open Market Committee (FOMC) has recently highlighted the importance of a broader policy focus, especially on strengthening labor markets for low- and moderate-income communities, recognizing the limitations imposed by the zero lower bound. Under these conditions, traditional monetary policy tools may struggle to stimulate labor markets without potentially increasing inflation (Powell, 2021). This paper examines the role of responding to additional real economic variables in a framework with sticky prices, non-Walrasian labor markets, and real wage rigidities, exploring the potential welfare gains from targeting demographic-specific unemployment rates.

The empirical analysis of my paper reveals that low-skilled unemployment rates are significantly more responsive to monetary policy shocks than high-skilled unemployment, highlighting distinct sensitivities across skill groups. By focusing on skill levels as a proxy for income groups, this analysis captures the heterogeneous responses of unemployment rates to monetary policy shocks—suggesting that skill-specific variables can effectively represent broader income disparities in labor market dynamics Blair et al. (2021). These findings align with the Federal Open Market Committee's recent emphasis on improving labor market outcomes, especially for low- and moderate-income communities, indicating that a broader approach to policy design could yield substantial welfare benefits. Including skill-specific unemployment rates in optimal monetary policy rules enables policymakers to better navigate the distinct welfare trade-offs that arise in labor markets where skill groups differ in their responsiveness to policy changes.

The empirical impulse responses reveal that unemployment rates for low-skilled workers are significantly more sensitive to a 0.25 percentage point reduction in the federal funds rate compared to high-skilled workers. The peak response of low-skilled unemployment is more than double that of high-skilled unemployment, and the latter shows a relatively prolonged recovery period. This differential sensitivity underlines the need for a framework that incorporates nominal and real rigidities, as well as asymmetric responses across skill levels. To that end, I employ a New Keynesian DSGE model with sticky prices à la Calvo (1983), monopolistic competition, and labor market frictions following Mortensen and Pissarides (1999). This setup allows for the examination of frictional unemployment and the persistent trade-off between volatile inflation and inefficient unemployment. Such trade-offs, absent in standard New Keynesian models, are essential for understanding whether optimal monetary policy should deviate from strict inflation targeting in favor of more robust labor market interventions. To ensure a more accurate welfare analysis across demographic groups, I incorporate real wage rigidity, which has been shown to reconcile discrepancies between matching friction models and observed labor market dynamics (Hall (2005), and Shimer (2005)). To capture these dynamics accurately, I estimate eight skill-specific parameters—including real wage rigidity—by minimizing the distance between the model-generated and empirical impulse response functions (as in Christiano et al. (2005)), allowing the model to reflect observed variations in unemployment responses.

In my framework, the economy encounters three primary inefficiencies that persist both in the short and long term. First, monopolistic competition in the goods market drives output below its efficient level, suggesting that minor deviations from strict inflation targeting may improve welfare outcomes. Second, price stickiness à la Calvo (1983) introduces staggered price rigidity that distort output and make inflation stabilization essential to mitigate these costs. Lastly, labor market frictions generate a congestion externality, where high unemployment or excessive vacancies reduce the probability of successful job matches, leading to tighter labor market. In this setup, the extent of vacancy creation versus job searching depends on worker bargaining power; an imbalance in this bargaining share can result in either excessive vacancy creation or an excessive number of job seekers, as discussed in Hosios (1990). When the labor market inefficiency results in a suboptimal level of employment, the monetary authority is incentivized to address unemployment fluctuations alongside inflation stabilization.

Previous studies have explored welfare implications of monetary policy under labor market frictions using various methodological approaches. Faia (2008), for example, combines a constrained Ramsey framework with a numerical evaluation of policy rules to derive optimal responses in a representative-agent model without capital. Blanchard and Galí (2006) take a linear quadratic approach with Hosios conditions (Hosios (1990)), focusing on wage rigidity and removing search externalities. In contrast, the present analysis evaluates welfare outcomes across a range of monetary policy rules in a more complex setting with heterogeneous agents. This approach enables an examination of welfare impacts under realistic frictions, including persistent skill-specific unemployment, which cannot be fully captured by traditional Ramsey optimization.

The recent optimal monetary policy literature analyses the role of distortions through various channels, with each approach offering distinct insights. Debortoli and Galí (2017) study optimal policy within a TANK framework, incorporating hand-to-mouth agents to highlight how redistributive mechanisms influence welfare outcomes. While their model provides insights into how monetary policy affects consumption inequality, it lacks the examination of how labor market interventions can improve overall welfare. Acharya et al. (2023) analyze optimal policy within a HANK framework, suggesting that output stabilization can mitigate consumption inequality by reducing income risk. Their findings emphasize the importance of countercyclical income risk, which they argue justifies a stronger focus on output stabilization. Similarly, Dávila and Schaab (2023) examine optimal policy under discretion and commitment, where redistribution plays a central role. They find that interest rate cuts can aid high marginal utility debtors, illustrating how redistribution goals can be integrated into monetary policy design. Both studies highlight alternative perspectives on heterogeneous agents, yet neither addresses the labor market-driven heterogeneity noted in recent Federal Open Market Committee (FOMC) statements, which emphasize a broader policy focus on supporting labor market outcomes for lower- and middle-income groups. This paper instead focuses on labor market-driven heterogeneity by analyzing skill-specific unemployment rates, showing how targeted labor market interventions can improve welfare within an optimal monetary policy framework.

In contrast to the standard New Keynesian framework, where stabilizing inflation alone achieves efficiency, my model demonstrates that strictly targeting only inflation is no longer optimal.[1] The inclusion of asymmetric search and matching frictions, specific to skill groups, introduces a congestion externality that results in inefficiently low employment levels across both high- and low-skill sectors, persisting in the steady state and during economic fluctuations. In this context, optimal policy requires addressing skill-specific unemployment alongside inflation, as these search externalities generate a trade-off between minimizing price adjustment costs and achieving a more efficient allocation of employment across skill groups. To my best knowledge, this is the first study within heterogeneous-agent frameworks to incorporate asymmetric search and matching frictions for optimal policy analysis. This approach offers valuable insights into the potential welfare gains from targeting skill-specific unemployment rates, which could inform more tailored and effective monetary policy strategies.

The remainder of the paper is organized as follows: Section 2 presents the empirical evidence, analyzing the heterogeneous impacts of monetary policy shocks on unemployment rates for low- and high-skilled workers. Section 3 details the theoretical framework, intro-

[1]This finding is consistent with the conclusions of Faia (2008), who demonstrates that the presence of search frictions in Representative Agent New Keynesian (RANK) setup requires a policy response that includes unemployment, in addition to inflation stabilization, to achieve welfare optimization.

ducing a New Keynesian DSGE model with asymmetric search and matching frictions and calibrating it to reflect the U.S. economy. Section 4 explores the model's dynamic properties through impulse response analysis, highlighting how labor market responses vary across skill groups following monetary policy interventions. Section 5 conducts a welfare analysis, assessing the optimality of policy rules targeting skill-specific unemployment, and Section 6 concludes.

2 Monetary Policy and Heterogeneous Unemployment: Empirical Evidence from Local Projections

This section presents the empirical impulse response of the unemployment rate to a monetary policy shock, using the local projections (LP) method developed by Jordà (2005). The LP approach estimates impulse responses by running a series of predictive regressions, where the variable of interest is regressed on the identified shock across chosen prediction horizons. The impulse responses are then captured by the sequence of regression coefficients associated with the shock.

To find the effect of the change in nominal interest rate (federal funds rate) on high- and low-skilled unemployment rates, first, I estimate the following equation

$$u_{s,t+j} = \alpha_j + \beta_s(j)\, shock_t + \gamma_{s,j} u_{s,t+j-1} + \varepsilon_{s,t+j}$$

to capture the responses of two groups' unemployment rates on an identified monetary policy shock, where j is tjhe length of horizon, $\varepsilon_{s,t+j}$ is a prediction error term with variance $\mathbb{V}(\varepsilon_{s,t+j}) = \sigma_j^2$, $u_{s,t+j}$ is quarterly unemployment rate for $s = \{l, h\}$, $shock_t$ is the identified monetary policy shock (GK) [2] and $u_{s,t+j-1}$ represents one period lag of $u_{s,t+j}$.[3] The dynamic multiplier $\beta_s(j)$ captures the responses of unemployment rates on an identified monetary policy shock.

Furthermore, I conducted joint hypothesis testing on the impulse responses to a monetary policy shock separately for high- and low-skilled unemployment rates across 16 forecast horizons. In my analysis for the high-skilled unemployment rates, I found a chi-square statistic of 70.40 with a p-value of less than 0.0001. This indicates a statistically significant effect of the monetary policy shock on high-skilled unemployment rate at least at one of the horizons examined. For low-skilled unemployment rates, the analysis produced a chi-square statistic of 52.55 with a p-value of less than 0.0001, providing strong evidence that

[2] GK Shock stands for Gertler and Karadi (2015) identified monthly (monetary policy) shocks, current futures.

[3] For more details about the data set see Appendix A

the impulse responses are not jointly equal to zero. These findings show that both high- and low-skilled labor market segments exhibit statistically significant responses to monetary policy shocks, as the impulse responses for both skill groups are statistically different from zero.

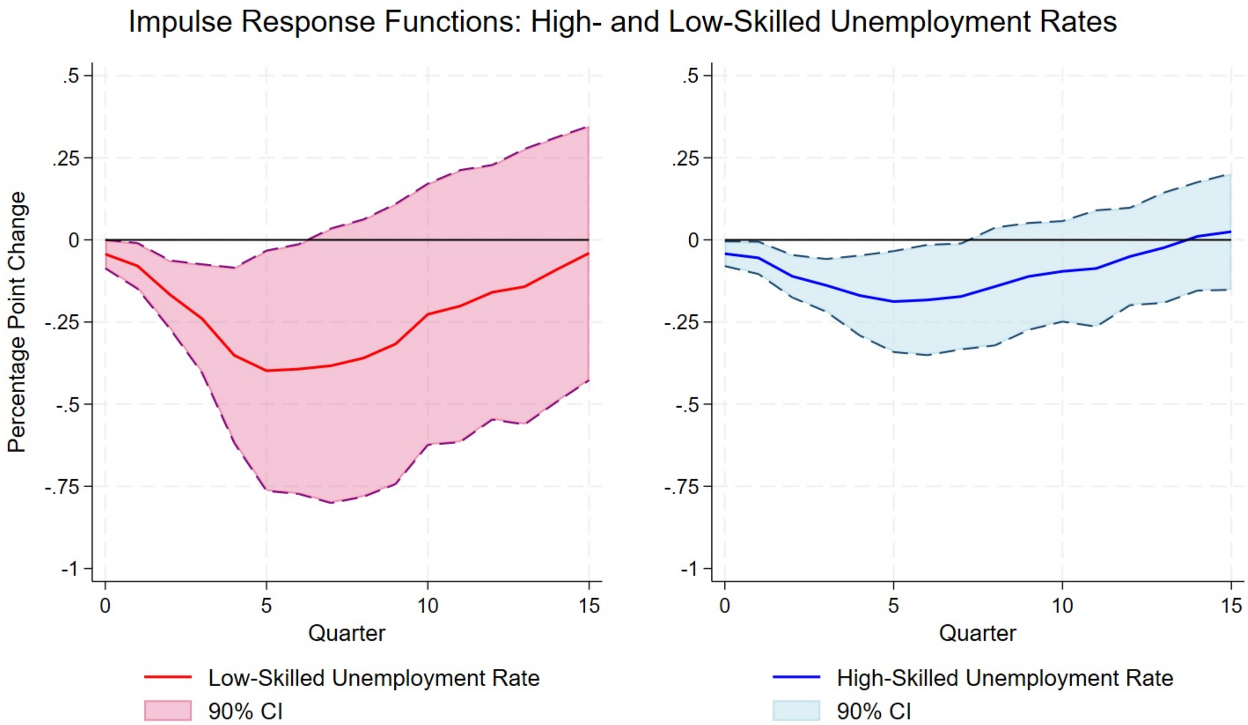

Figure 1. Impulse responses of low- and high-skilled unemployment rates to a 0.25 p.p. decrease in FED funds rate (using GK shocks)

To further analyze the differential impact of monetary policy shocks across skill groups, I regressed the difference between low- and high-skilled unemployment rates on the same identified monetary policy shock and performed the same joint test. This analysis produced a chi-square statistic of 77.67 with a p-value of less than 0.0001, indicating that the difference in unemployment rates is statistically different from zero at least in one period, within the 90% confidence interval.

Figure 2 illustrates the impulse response function for the difference between low- and high-skilled unemployment rates following the monetary policy shock, supporting the results of the joint test that the difference between low- and high-skilled unemployment rates in response to the same identified monetary policy shock is statistically different from zero. These findings indicate that monetary policy shocks impact unemployment rates differently across skill groups in the United States, highlighting the varying sensitivity of high- and low-skilled labor markets to such policy changes.

To interpret dynamic multipliers of unemployment rates in terms of the responses to 0.25 percentage point decrease in fed funds rate, I use local projection estimation and run a sequence of regressions of a federal funds rate on the same identified monetary policy shocks

$$r_{t+j} = \alpha_{r,j} + \beta_r(j)\, shock_t + \gamma_{r,j} r_{t+j-1} + \varepsilon_{r,t+j}$$

to get the dynamic responses of fed funds rate on the shock. After estimation, I normalize responses of high- and how-skilled unemployment rates ($\beta_s(j)\ \ s \in \{l, h\}$) by the factor $0.25/\beta_r(j=0)$ to interpret the responses of different demographic groups to a 0.25 percentage point decrease in the fed funds rate on impact.

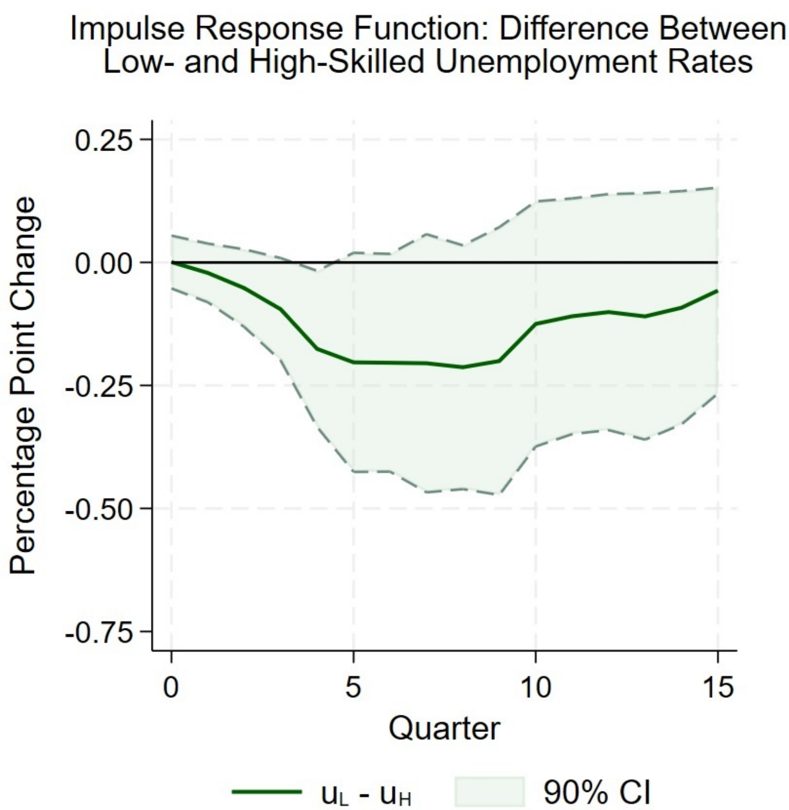

Figure 2. Impulse responses of low- and high-skilled unemployment rates to a 0.25 p.p. decrease in FED funds rate (using GK shocks)

Based on the local projections (LP) estimation results, a 0.25 percentage point reduction in the federal funds rate, reveals distinct sensitivities and recovery patterns within high- and low-skilled unemployment rates. The dynamic multipliers obtained from the LP methodology, indicates a more pronounced immediate response in the low-skilled unemployment rate ($u_{l,t}$), which undergoes a sharper decrease following the policy shock. This phenomenon suggests that low-skilled unemployment is more susceptible to changes in monetary policy, a finding that aligns with expectations given the typically higher cyclicality of low-skilled unmployment.

Conversely, as depicted in the Figure 1, high-skilled unemployment ($u_{h,t}$) exhibits a more moderate initial response, with the effect of the monetary policy shock attenuating more rapidly compared to its low-skilled counterpart. This attenuation of effects in the high-skilled labor market is demonstrated by the impulse response of high-skilled unemployment rate approaching zero faster compared to low-skilled unemployment rate. The peak responses observed in the data further support these findings, with the low-skilled unemployment rate experiencing a more substantial deviation from baseline levels before the effect of the shock begins to weaken. This analysis shows the variation in responses across different segments of the labor market to monetary policy changes, with low-skilled unemployment showing greater sensitivity to shocks. In contrast, high-skilled unemployment appears more stable and recovers more quickly after an unanticipated shock. These results emphasize the relative resilience of high-skilled workers compared to their low-skilled counterparts.

These empirical insights contribute to our understanding of labor market behavior, highlighting the differential impact of monetary policy on various skill groups. The faster recovery seen in high-skilled unemployment rates after a shock suggests a relative robustness and inherent steadiness of high skilled individuals, in contrast to the increased sensitivity and extended path to recovery observed in low-skilled unemployment rates.

3 The Model Economy

3.1 High- and Low-Skilled Households

There are two types of households. A constant fraction $1 - \omega$ of households consists only by high-skilled labor force. High-skilled households have unconstrained access to financial markets and maximize the following expected utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t \left[\ln\left(c_{h,t} - \chi_h c_{h,t-1}\right)\right] \tag{1}$$

where $c_{h,t}$ is an aggregate consumption of high-skilled household, χ_h is habit formation parameter with $0 < \chi_h < 1$, and β_h is the discount factor $0 < \beta_h < 1$. For clarity, throughout the text, I will use the subscript h to denote variables specific to high-skilled households.

Households supply labor hours, l_h, inelastically and it is normalized to 1. The household supplies $n_{h,t}$ units of labor and k_t units of capital at the real wage rate, $w_{h,t}$, and the capital remuneration rate r_t^k, respectively, to each basic good producing firm during period t. The wage contract between the worker and the firm is determined through a Nash bargaining process.It is assumed that workers can insure themselves against earning uncertainty and unemployment, meaning that wage earnings are net of insurance costs. Each agent also invests in non-state contingent nominal bonds, b_t, which pay a gross nominal interest rate $(1 + r_t^n)$ one period later. Additionally, households receive nominal profits $\Xi_{I,t}$ and $\Xi_{B,t}$ from the intermediate and basic goods producing firms, respectively. Household pays nominal lump-sum tax, $\tau_{h,t}$, in each period to finance fixed unemployment benefit, g_h, of unemployed individuals, $u_{h,t}$, in the household. The household uses its income for consumption, $c_{h,t}$, investment, i_t, and lump-sum tax $\tau_{h,t}$, and carries b_t bonds into period $t+1$. Let p_t be the price level (the price index) associated to the final output y_t, then the high-skilled household's real budget constraint will be given by:

$$\begin{aligned} c_{h,t} + \frac{i_t}{1-\omega} + \frac{b_t}{p_t}\frac{1}{1-\omega} + \tau_{h,t} = \frac{b_{t-1}}{p_t}\frac{\left(1+r_{t-1}^n\right)}{1-\omega} + w_{h,t}\frac{n_{h,t}e_{h,t}}{1-\omega} + r_t^k\frac{k_t}{1-\omega} \\ + \left(\frac{\Xi_{I,t}}{p_t} + \frac{\Xi_{B,t}}{p_t}\right)\frac{1}{1-\omega} + g_h u_{h,t} \end{aligned} \tag{2}$$

for all $t = 0, 1, 2, ...$ By investing i_t units of output during period t, the household increases the capital stock k_{t+1} available during period $t+1$ according to

$$k_{t+1} = (1-\delta_k)\, k_t + \phi\,(i_t, i_{t-1}) \tag{3}$$

where the depreciation rate satisfies $0 < \delta_k < 1$ and the function $\phi\,(\cdot)$ summarizes the technology which transforms current and past investment into installed capital for use in the following period. Investment adjustment cost is given by

$$\phi\,(i_t, i_{t-1}) = \left(1 - e\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{4}$$

where, $e\,(1) = e'\,(1) = 0$ and $\tau_i \equiv e''\,(1) > 0$.[4]

Therefore, households choose $\{c_{h,t}, b_t, i_t, k_{t+1}\}_{t=0}^{\infty}$ taking $\left\{p_t, w_t, r_t^k, r_t^n\right\}_{t=0}^{\infty}$ as given to maximize Equation (1) subject to Equation (2) and Equation (3). Letting $\pi_{t+1} = \frac{p_{t+1}}{p_t}$ denote the gross inflation rate, $\lambda_{h,t}$ the non-negative Lagrange multiplier on the high-skilled budget constraint, and $\varkappa_t$ the non-negative multiplier on the law of capital accumulation,

[4] If we assume that $e\left(\frac{i_t}{i_{t-1}}\right) = \frac{\iota}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$, then $e'\left(\frac{i_t}{i_{t-1}}\right) = \iota\left(\frac{i_t}{i_{t-1}} - 1\right)$ and $e''\left(\frac{i_t}{i_{t-1}}\right) = \iota$.

the first order conditions associated to this problem are

$$\lambda_{h,t} = (c_{h,t} - \chi_h c_{h,t-1})^{-1} - \beta_h \chi_h \left(\mathbb{E}_t \left[c_{h,t+1} - \chi_h c_{h,t}\right]\right)^{-1} \tag{5}$$

$$\psi_t = \beta_h \mathbb{E}_t \left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}} \left[r_{t+1}^k + \psi_{t+1}(1-\delta_k)\right]\right] \tag{6}$$

$$\lambda_{h,t} = \beta_h (1 + r_t^n) \mathbb{E}_t \left[\frac{\lambda_{h,t+1}}{\pi_{t+1}}\right] \tag{7}$$

and

$$\begin{aligned} 1 = \psi_t &\left(\left(1 - \frac{\iota}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) - \iota\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}}\right) \\ &+ \beta_h \mathbb{E}_t \left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}} \psi_{t+1} \iota \left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right]. \end{aligned} \tag{8}$$

where $\pi_{t+1} = \frac{p_{t+1}}{p_t}$ is a gross inflation rate and $\psi_t = (1-\omega)\frac{\varkappa_t}{\lambda_{h,t}}$ is the present discounted value of the rental rate on capital. According to Equation (5), marginal utility of consumption equals to the Lagrange multiplier. Equation (6) is the Euler equation for capital, linking intertemporal marginal utility of consumption to the real remuneration rate of capital. Equation (7) describes high-skilled household's optimal consumption decision and lastly, Equation (8) shows the optimal investment decision.

The remaining share, ω, of households are assumed to be hand to mouth. They consume their current labor income, possibly (but not necessarily) because they do not have access to financial markets and they maximize the following expected utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_l^t \left[\ln(c_{l,t} - \chi_l c_{l,t-1})\right] \tag{9}$$

where $c_{l,t}$ is an aggregate consumption of low-skilled household, χ_l is habit formation parameter with $0 < \chi_l < 1$, and β_l is the discount factor $0 < \beta_l < 1$. For clarity, throughout the text, I will use the subscript l to denote variables specific to high-skilled households.

Furthermore, the household pays lump-sum tax $\tau_{l,t}$ each period in order to finance fixed unemployment benefit, g_l, (throughout the text, I will use the subscript l to denote variables specific to low-skilled households) of unemployed individuals in the household. Since, p_t is assumed to be the price level (the price index) associated to the final output y_t, then the low-skilled household's real budget constraint will be given by:

$$c_{l,t} + \tau_{l,t} = w_t \frac{n_t e_{l,t}}{\omega} + g_l u_{l,t} \tag{10}$$

for all $t = 0, 1, 2, \ldots$. Hence, the low-skilled households choose $\{c_{l,t}\}_{t=0}^{\infty}$ to maximize the utility (Equation 9) subject to the budget constraint (Equation 10) for all $t = 0, 1, 2, \ldots$. Letting $\lambda_{l,t}$ be the non-negative Lagrange multiplier on the budget constraint, the first order condition associated to the low-skilled hosehold's problem is

$$\lambda_{l,t} = \left(c_{l,t} - \chi_l c_{l,t-1}\right)^{-1} - \beta_l \chi_l \left(\mathbb{E}_t\left[c_{l,t+1} - \chi_l c_{l,t}\right]\right)^{-1} \tag{11}$$

stating that marginal utility of consumption equals to the (low-skill household) Lagrange multiplier.

The two types of agents (high- and low-skilled households) do not face any form of idiosyncratic uncertainty. Furthermore, I assume that they take the wage as given (defined by the Nash Bargaining process) and are happy to supply as much labor as demanded by firms.

3.2 The Production Sector

There is a final good producing firm that aggregates a continuum of intermediate outputs, where intermediate good producers live on the interval $i \in [0, 1]$ and intermediate good producers itself buy basic good producers' output. Labor-firm relations are modeled within the standard Mortensen and Pissarides (1999) framework. Firms engage in the recruitment of workers from the pool of unemployed individuals, where the process of searching for suitable candidates involves a fixed vacancy posting cost. The determination of wages is structured through a decentralized bargaining process based on Nash bargaining principles, in which the basic goods producers and the workers negotiate the wage rate. This process ensures that wages are determined in a manner that accounts for the individual bargaining power of each party.

3.2.1 Labor Market Search and Matching

The process of searching for a worker incurs a fixed cost κ_s, where $s \in \{h, l\}$, and the likelihood of successfully finding a worker is determined by a constant returns to scale matching function. This technology converts the pool of unemployed workers $u_{s,t}$, and vacancies $\nu_{s,t}$ into matches, $m\left(u_{s,t}, \nu_{s,t}\right)$:

$$m\left(u_{s,t}, \nu_{s,t}\right) = m_s u_{s,t}^{\eta_s} \nu_{s,t}^{1-\eta_s}. \tag{12}$$

If we define the labor market tightness as $\theta_{s,t} \equiv \frac{\nu_{s,t}}{u_{s,t}}$, then under this definition the firms find unemployed workers at rate $q_{s,t}\left(\theta_{s,t}\right) \equiv \frac{m(u_{s,t}, \nu_{s,t})}{\nu_{s,t}} = m_s \theta_{s,t}^{-\eta_s}$, while the unemployed workers meet vacancies at rate $\theta_{s,t} q\left(\theta_{s,t}\right) = m_s \theta_{s,t}^{1-\eta_s}$.

Matches are terminated at a rate $\delta_s(e_{s,t})$, where $e_{s,t}$ represents the utilization of labor input. The rationale is that, while labor input may be predetermined for the period, firms have the flexibility to adjust the intensity of labor usage based on the prevailing economic conditions. As a result, the separation rate now depends on labor input utilization, which is normalized to one in the steady state. Specifically:

$$\delta_s(e_{s,t}) = \delta_{s,0} + \varphi_{s,1}(e_{s,t}-1) + \frac{\varphi_{s,2}}{2}(e_{s,t}-1)^2 .$$

The labor force is normalized to one for both types of workers. The number of employed individuals at time t is determined by the number of those employed at time $t-1$, along with the flow of new matches formed during period $t-1$

$$n_{s,t} = (1-\delta_s(e_{s,t}))\, n_{s,t-1} + \nu_{s,t-1} q_{s,t-1}(\theta_{s,t-1}), \tag{13}$$

and after determining $n_{s,t}$, unemployment is defined as the difference between the total labor force and the number of employed workers

$$u_{s,t} = 1 - n_{s,t}. \tag{14}$$

3.2.2 Intermediate Good Producers

During each period $t = 0, 1, 2, ...$, the representative finished-goods-producing firm uses $y_t(i)$ units of each intermediate good producer $i \in [0,1]$, purchased at nominal price $p_t(i)$, to produce y_t units of the finished product at a constant returns to scale technology

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\mu-1}{\mu}} di \right)^{\frac{\mu}{\mu-1}} \tag{15}$$

where $\mu > 1$ is the elasticity of substitution among different goods. The final good producer's profit maximization problem yields the following demand schedule for intermediate varieties

$$y_t(i) = \left(\frac{p_t(i)}{p_t} \right)^{-\mu} y_t. \tag{16}$$

Equation (16) is a function of the price of intermediate good i, $p_t(i)$, aggregate price index, p_t, and aggregate output, y_t. Profit maximization requires the final goods output price to equal its marginal cost

$$p_t = \left(\int_0^1 p_t(i)^{1-\mu} di \right)^{\frac{1}{1-\mu}} . \tag{17}$$

Intermediate good producers buy basic goods producers' output, $y_{B,t}$, at $p_{B,t}$, and repackage it such that $y_t(i) = y_{B,t}$.[5] As a result, their nominal profit for a period t is:

$$\Xi_{I,t}(i) = p_t y_t(i) - p_{B,t} y_{B,t}$$

or in real terms

$$\xi_{I,t}(i) = \left(\frac{p_t(i)}{p_t}\right)^{1-\mu} y_t - \frac{p_{B,t}}{p_t}\left(\frac{p_t(i)}{p_t}\right)^{-\mu} y_t \tag{18}$$

where $\xi_{I,t}(i) = \frac{\Xi_{I,t}(i)}{p_t}$ and $y_{B,t} = y_t(i)$.

Intermediate good producers face a constant hazard, $1-\gamma$, of being able to adjust their price. They discount future real profit flows by the stochastic discount factor of high-skilled household (because high-skilled households own the firms). An intermediate producer with the opportunity to adjust its price therefore solves:

$$\max_{p_t(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} \left[\left(\frac{p_{t+j}(i)}{p_{t+j}}\right)^{1-\mu} y_{t+j} - \frac{p_{B,t+j}}{p_{t+j}}\left(\frac{p_{t+j}(i)}{p_{t+j}}\right)^{-\mu} y_{t+j}\right]$$

and the first-order condition that characterizes the problem of an intermediate goods producing firm adjusting its price is given by

$$p_t^* = \frac{\mu}{\mu-1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} \frac{p_{B,t+j}}{p_{t+j}} (p_{t+j})^{\mu} y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} (p_{t+j})^{\mu-1} y_{t+j}} \tag{19}$$

which represents the optimal reset price for the firm.

The equation for the optimal reset price can also be expressed in terms of inflation, π_t^*, driven by the price adjusting firms in period t[6]

$$\pi_t^* = \frac{\mu}{\mu-1} \frac{\mathcal{P}_{B,t}}{\mathcal{P}_t} \tag{20}$$

where, $\pi_t^* = \frac{p_t^*}{p_t}$,

$$\mathcal{P}_{B,t} = \frac{p_{B,t}}{p_t} y_t + \gamma \mathbb{E}_t \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} (\pi_{t+1})^{\mu} \mathcal{P}_{B,t+1}$$

and

$$\mathcal{P}_t = y_t + \gamma \mathbb{E}_t m_{t,t+1} (\pi_{t+1})^{\mu-1} \mathcal{P}_{t+1}.$$

[5] Because basic good producers are perfectly competitive, they choose the same allocations and take the price of their output as given - we can have $y_{B,t}$ without index i.

[6] More details in the Appendix B

3.2.3 Basic Good Producers

Wages are determined by the bargaining process, which will be addressed in the subsequent section. In this section, we outline the dynamic optimization problem faced by a basic goods producer, who seeks to choose the optimal number of employees, $n_{s,t}$, and vacancies $\nu_{s,t}$, in order to maximize the discounted value of future profits. The wage schedule, determined through bargaining, is taken as given in this optimization.

During each period $t = 0, 1, 2, ...,$ a representative (perfectly competitive) basic good producing firm hires $n_{h,t}$ and $n_{l,t}$ units of labor from the representative high- and low-skilled household, in order to produce $y_{B,t}$ units of basic good according to the following production technology

$$y_{B,t} = a_t k_t^{\alpha} \left((a_l n_{l,t} e_{l,t})^{\rho} + (a_h n_{h,t} e_{h,t})^{\rho} \right)^{\frac{1-\alpha}{\rho}} \tag{21}$$

where $0 < \alpha < 1$ defines the capital input share of production, ρ is the elasticity of substitution between labor inputs, $n_{s,t} e_{s,t}$ is labor services, and a_t is the neutral technology shock, which follows the autoregressive process

$$\ln a_t = \rho_a \ln a_{t-1} + \epsilon_{a,t}$$

with $0 < \rho_a < 1$ and zero-mean serially uncorrelated innovation $\varepsilon_{a,t}$ that is normally distributed with standard deviation σ_a.

The representative intermediate goods producing firm maximizes its total market value given by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \left(\beta^t \frac{\lambda_{h,t}}{p_t} \right) \Xi_{B,t}$$

, where $\beta^t \dfrac{\lambda_{h,t}}{p_t}$ measures the marginal utility value to the representative high-skilled household of an additional "currency" in profits received during period t and

$$\Xi_{B,t} = p_{B,t} y_{B,t} - p_t r_t^k k_t - p_t \sum_{s \in \{h,l\}} w_{s,t} n_{s,t} e_{s,t} - p_t \sum_{s \in \{h,l\}} \kappa_s \nu_{s,t} \tag{22}$$

for all $t = 0, 1, 2, ...$ Therefore, the firm chooses $\{n_{s,t}, \nu_{s,t}, e_{s,t}, k_t, \}$, $s \in \{h, l\}$, to maximize its profits $\Xi_{B,t}$ subject to the law of employment accumulation

$$n_{s,t} = (1 - \delta_s (e_{s,t})) n_{s,t-1} + \nu_{s,t-1} q_{s,t-1} (\theta_{s,t-1})$$

where

$$\delta_s (e_{s,t}) = \delta_{s,0} + \varphi_{s,1} (e_{s,t} - 1) + \frac{\varphi_{s,2}}{2} (e_{s,t} - 1)^2 .$$

Let's define $\phi_{s,t}$, the Lagrange multiplier on the aw of employment accumulation constraint as the marginal value of one additional worker. Therefore, The first-order conditions for the problem outlined above are as follows:

- $e_{l,t}$ and $e_{h,t}$

$$\begin{aligned}
&\phi_{l,t}\left(\varphi_{l,1}+\varphi_{l,2}\left(e_{l,t}-1\right)\right)n_{l,t-1}= \\
&\frac{p_{B,t}}{p_t}\left(1-\alpha\right)a_l n_{l,t} y_{B,t}\frac{\left(a_l n_{l,t} e_{l,t}\right)^{\rho-1}}{\left(a_l n_{l,t} e_{l,t}\right)^{\rho}+\left(a_h n_{h,t} e_{h,t}\right)^{\rho}}-w_{l,t} n_{l,t}
\end{aligned} \tag{23}$$

$$\begin{aligned}
&\phi_{h,t}\left(\varphi_{h,1}+\varphi_{h,2}\left(e_{h,t}-1\right)\right)n_{h,t-1}= \\
&\frac{p_{B,t}}{p_t}\left(1-\alpha\right)a_h n_{h,t} y_{B,t}\frac{\left(a_h n_{h,t} e_{h,t}\right)^{\rho-1}}{\left(a_l n_{l,t} e_{l,t}\right)^{\rho}+\left(a_h n_{h,t} e_{h,t}\right)^{\rho}}-w_{h,t} n_{h,t}
\end{aligned} \tag{24}$$

- $n_{l,t}$ and $n_{h,t}$

$$\begin{aligned}
\phi_{l,t}=&\frac{p_{B,t}}{p_t}\left(1-\alpha\right)a_l e_{l,t} y_{B,t}\frac{\left(a_l n_{l,t} e_{l,t}\right)^{\rho-1}}{\left(a_l n_{l,t} e_{l,t}\right)^{\rho}+\left(a_h n_{h,t} e_{h,t}\right)^{\rho}}-w_{l,t} e_{l,t} \\
&\beta_h\mathbb{E}_t\left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}}\left(1-\delta_l\left(e_{l,t+1}\right)\right)\phi_{l,t+1}\right]
\end{aligned} \tag{25}$$

$$\begin{aligned}
\phi_{h,t}=&\frac{p_{B,t}}{p_t}\left(1-\alpha\right)a_h e_{h,t} y_{B,t}\frac{\left(a_h n_{h,t} e_{h,t}\right)^{\rho-1}}{\left(a_l n_{l,t} e_{l,t}\right)^{\rho}+\left(a_h n_{h,t} e_{h,t}\right)^{\rho}}-w_{h,t} e_{h,t} \\
&\beta_h\mathbb{E}_t\left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}}\left(1-\delta_h\left(e_{h,t+1}\right)\right)\phi_{h,t+1}\right]
\end{aligned} \tag{26}$$

- $\nu_{l,t}$ and $\nu_{h,t}$

$$\frac{\kappa_l}{q_{l,t}\left(\theta_{l,t}\right)}=\beta_h\mathbb{E}_t\left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}}\phi_{l,t+1}\right] \tag{27}$$

$$\frac{\kappa_h}{q_{h,t}\left(\theta_{h,t}\right)}=\beta_h\mathbb{E}_t\left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}}\phi_{h,t+1}\right] \tag{28}$$

- k_t

$$r_t^k=\frac{p_{B,t}}{p_t}\alpha\frac{y_{B,t}}{k_t}. \tag{29}$$

Combining equations (25) and (27) (and equations (26) and (28) respectively) and rearranging we obtain the basic good producing firm's real marginal cost (the same as price of basic good) $\frac{p_{B,t}}{p_t}$

$$\frac{p_{B,t}}{p_t} = \frac{\phi_{l,t} - \frac{\kappa_l}{q_{l,t}(\theta_{l,t})}\left(1-\delta_l\left(e_{l,t}\right)\right)}{mpl_{l,t}} + \frac{w_{l,t}e_{l,t}}{mpl_{l,t}}$$

and

$$\frac{p_{B,t}}{p_t} = \frac{\phi_{h,t} - \frac{\kappa_h}{q_{h,t}(\theta_{h,t})}\left(1-\delta_h\left(e_{s,t}\right)\right)}{mpl_{h,t}} + \frac{w_{h,t}e_{l,t}}{mpl_{h,t}}$$

which depends on marginal productivity of worker $\left(w_{s,t}e_{s,t}\right)/mpl_{s,t}$ plus the future value of current employee, $\left(\phi_{s,t} - \frac{\kappa_s}{q_{s,t}(\theta_{s,t})}\left(1-\delta_h\left(e_{s,t}\right)\right)\right)\Big/ mpl_{s,t}$.[7] Since, posting a vacancy incurs a cost, a successful match today holds value as it lowers future search costs. Additionally, the future value of a current employee is affected by the dynamics of unemployment. An increase in the number of unemployed workers raises the likelihood of filling future vacancies, which in turn reduces the future value of the current employee.

3.2.4 Nash Bargaining and Wage Setting

The wage schedule is determined through the solution of a Nash bargaining process. To get the wage schedule, we must first derive the marginal values of a match for both firms and workers, as these values will be incorporated into the sharing rule of the bargaining process. Let $\mathcal{V}^J_{s,t}$ denote the marginal discounted value of a match (with worker type $s \in \{h, l\}$) for a firm. Using equations (25) and (26) (generalizing it for both types) and noticing that $\mathcal{V}^J_{s,t} = \phi_{s,t}$, then we have

$$\mathcal{V}^J_{s,t} = \frac{p_{B,t}}{p_t} mpl_{s,t} - w_{s,t}e_{s,t} + \beta_h \mathbb{E}_t\left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}}\left(1-\delta_s\left(e_{s,t}\right)\right)\mathcal{V}^J_{s,t+1}\right] \tag{30}$$

where $mpl_{s,t} = (1-\alpha)\, a_s e_{s,t} y_{B,t} \frac{(a_s n_{s,t} e_{s,t})^{\rho-1}}{\sum_{s\in\{h,l\}}(a_s n_{s,t} e_{s,t})^{\rho}}$. The Equation (30) states that the marginal value of a match depends on real revenues minus the real wage plus the discounted continuation value. With probability $\left(1-\delta_s\left(e_{s,t}\right)\right)$, the job remains filled and generates its expected value, while with probability $\delta_s\left(e_{s,t}\right)$, the job is terminated and holds no value. Using equations (25) and (27) (and equations (26) and (28) respectively for high-skilled workers) we arrive at the following result:

$$\phi_{s,t} = \frac{p_{B,t}}{p_t} mpl_{s,t} - w_{s,t}e_{s,t} + \left(1-\delta_s\left(e_{s,t}\right)\right)\frac{\kappa_s}{q_{s,t}\left(\theta_{s,t}\right)}, \tag{31}$$

[7]The marginal product of labor input is denoted as $mpl_{s,t} = (1-\alpha)\, a_s e_{s,t} y_{B,t} \frac{(a_s n_{s,t} e_{s,t})^{\rho-1}}{\sum_{s\in\{h,l\}}(a_s n_{s,t} e_{s,t})^{\rho}}$

and if we substitute $\phi_{s,t}$ in Equation (25) (and Equation (26) respectively) and recall that $\mathcal{V}^J_{s,t} = \phi_{s,t}$, we will have

$$\frac{\kappa_s}{q_{s,t}\left(\theta_{s,t}\right)} = \beta_h \mathbb{E}_t \left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}} \mathcal{V}^J_{s,t+1}\right] \tag{32}$$

since, free entry ensures that cost of posting/creating a vacancy is zero in every state of the economy. Moreover, Equation (32) is an arbitrage condition for posting vacancies for each type of worker. It implies that in equilibrium the cost of posting a vacancy must be equal to the discounted expected return from posting the vacancy.

The values of being employed, $\mathcal{V}^E_{s,t}$, and unemployed, $\mathcal{V}^U_{s,t}$, are specified for each worker type as follows:

$$\mathcal{V}^E_{s,t} = w_{s,t} e_{s,t} + \beta_h \mathbb{E}_t \left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}} \left(\left(1 - \delta_s\left(e_{s,t}\right)\right) \mathcal{V}^E_{s,t+1} + \rho \mathcal{V}^U_{s,t+1}\right)\right] q_{s,t}\left(\theta_{s,t}\right) \tag{33}$$

$$\mathcal{V}^U_{s,t} = g_s + \beta_h \mathbb{E}_t \left[\frac{\lambda_{h,t+1}}{\lambda_{h,t}} \left(\theta_{s,t} q_{s,t}\left(\theta_{s,t}\right) \mathcal{V}^E_{s,t+1} + \left(1 - \theta_{s,t} q_{s,t}\left(\theta_{s,t}\right)\right) \mathcal{V}^U_{s,t}\right)\right] \tag{34}$$

where g_s denotes real unemployment benefits for unemployed workers, $s \in \{h, l\}$.

Firms and workers participate in a Nash bargaining process to negotiate wages. The optimal sharing rule within the framework of standard Nash bargaining is given by

$$\left(\mathcal{V}^E_{s,t} - \mathcal{V}^U_{s,t}\right) = \frac{\zeta_s}{1 - \zeta_s} \mathcal{V}^J_{s,t} \tag{35}$$

where ζ_s is the the bargaining power of each type of worker.

By substituting the previously defined value functions for both the basic goods-producing firm and the workers, we can derive the following wage schedule:

$$w_{s,t} e_{s,t} = \zeta_s \left(\frac{p_{B,t}}{p_t} mpl_{s,t} + \theta_{s,t} \kappa_s\right) + \left(1 - \zeta_s\right) g_s \tag{36}$$

stating that the agreed-upon wage represents a convex combination of the worker's outside option, namely the unemployment benefit, and the firm's real value, which includes its real revenue and the vacancy cost per unemployed worker. When the worker's bargaining power approaches zero, the wage converges toward the level of the unemployment benefit. Conversely, as the worker's bargaining power increases, the firm is required to offer a wage that is significantly higher relative to the unemployment benefit.

It is important to highlight that Shimer (2005) and Hall (2005) observed that in a Mortensen-Pissarides style matching model, wages exhibit excessive volatility due to limited adjustment along the employment margin. They also noted that introducing real wage rigidity helps address some of the puzzling aspects of the standard matching model. Therefore, following Hall (2005), I assume that the individual real wage is a weighted average of the wage derived from the Nash bargaining process and the one determined in the steady state solution.

$$w_{s,t}e_{s,t} = \psi_s \left(\zeta_s \left(\frac{p_{B,t}}{p_t} mpl_{s,t} + \theta_{s,t}\kappa_s \right) + (1-\zeta_s)\, g_h \right) + (1-\psi_s)\, w_{s,ss}e_{s,ss} \tag{37}$$

where ψ_s is the real wage rigidity for each type of worker, $s \in \{h, l\}$.

3.2.5 The Central Bank

During each period $t = 0, 1, 2, ...$, the central bank implements monetary policy through an interest rate reaction function of the following form

$$\ln\left(\frac{1+r_t^n}{1+r^n}\right) = \phi_r \ln\left(\frac{1+r_{t-1}^n}{1+r^n}\right) + (1-\phi_r)\left(\phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) + \phi_u \ln\left(\frac{u_t}{u}\right) + \phi_{u,l} \ln\left(\frac{u_{l,t}}{u_l}\right) + \phi_{u,h} \ln\left(\frac{u_{h,t}}{u_h}\right)\right) + \varepsilon_{r,t} \tag{38}$$

where $1 + r^n$, y, π, u, u_l and u_h are the steady-state values of the nominal interest rate, output, gross inflation rate, aggregate unemployment, low-skilled unemployment and high-skilled unemployment, respectively. The last term in the modified Taylor rule is serially uncorrelated innovation $\varepsilon_{r,t}$ that is normally distributed with standard deviation σ_r.

The monetary authority seeks to maximize the welfare (as defined in Section 5) of agents, $s \in \{h, l\}$, subject to the constraints defined by the economic relationships and the class of monetary policy rules outlined in Equation (38). I conduct a numerical search for the set of parameters $\{\phi_\pi, \phi_y, \phi_u, \phi_{u,l}, \phi_{u,h}\}$ that maximizes households' (aggregate) welfare and evaluate the welfare rankings of rules that impose different constraints on the Equation (38).

3.2.6 Government

The government receives lump-sum taxes $\tau_{h,t}$ and $\tau_{l,t}$ from both households in order to balance fixed unemployment benefits (g_h and g_l) payed to unemployed household members $\sum_{s\in\{h,l\}} g_s u_{s,t}$. Hence, in each period $t = 0, 1, 2, ...$ government balances its budget according to the following equation

$$(1-\omega)\,\tau_{h,t} + \omega\tau_{l,t} = (1-\omega)\, g_h u_{h,t} + \omega g_l u_{l,t} \tag{39}$$

where, for simplicity, I assume $\tau_{h,t} = g_h u_{h,t}$ and $\tau_{l,t} = g_l u_{l,t}$.

3.2.7 Aggregate Conditions

Since the economy consists of two types of households, with a constant proportion $1-\omega$ representing high-skilled households and the remaining share ω representing low-skilled households, the aggregate resource constraint is a weighted sum of the budget constraints for both household types. After substituting in real profits of intermediate and basic goods firms' profits, $\Xi_{I,t}/p_t$ and $\Xi_{B,t}/p_t$, imposing a zero net supply of nominal bonds, and applying the government's balanced budget condition, the following result is obtained:

$$y_t = c_t + i_t + \sum_{s\in\{h,l\}} \kappa_s \nu_{s,t} \tag{40}$$

where $c_t = (1-\omega)\, c_{h,t} + \omega c_{l,t}$ is the aggregate consumption and $\sum_{s\in\{h,l\}} \kappa_s \nu_{s,t}$ is the total cost of posting vacancies for both types of workers.

Using properties of Calvo pricing, the aggregate price level in terms of inflation evolves according to

$$1 = (1-\gamma)\,(\pi_t^*)^{1-\mu} + \gamma \pi_t^{\mu-1}. \tag{41}$$

Furthermore, Integrating relative demand (intermediate good producers) over all i (and recalling that $y_t\,(i) = y_{B,t}$) will provide the aggregate production function

$$y_{B,t} = y_t \int_0^1 \left(\frac{p_t\,(i)}{p_t}\right)^{-\mu} di$$

where $\int_0^1 \left(\frac{p_t(i)}{p_t}\right)^{-\mu} di$ is a price dispersion between intermediate and final good's price indices. If we denote $\int_0^1 \left(\frac{p_t(i)}{p_t}\right)^{-\mu} di$ as d_t^p then the price dispersion can be written as

$$d_t^p = (1-\gamma)\,(\pi_t^*)^{-\mu} + \gamma \pi_t^{\mu} d_{t-1}^p \tag{42}$$

and, the aggregate production function will be

$$a_t k_t^{\alpha} \left((a_l n_{l,t} e_{l,t})^{\rho} + (a_h n_{h,t} e_{h,t})^{\rho}\right)^{\frac{1-\alpha}{\rho}} = y_t d_t^p. \tag{43}$$

3.2.8 Calibration

The model is calibrated on quarterly frequencies using U.S. data. The values for all parameters are described below and presented in Table 1. In this model, I assume a discount factor,

β_h, of 0.995 for high-skilled households and $\beta_l = 0.9$ for low-skilled, hand-to-mouth, consistent with the literature on heterogeneous agents. The lower discount factor for low-skilled households reflects their immediate consumption behavior and limited saving capacity, a characteristic often associated with hand-to-mouth consumers, as discussed in Kaplan et al. (2018) and Carroll (1997). High-skilled households, with a higher discount factor, are assumed to have a longer-term consumption horizon, aligning with more patient consumer behavior. Furthermore, I assume the same habit formation parameter for both high-skilled and low-skilled households, a common practice in macroeconomic models with heterogeneous agents to maintain consistency in consumption smoothing. For instance, Iacoviello (2005) applies a uniform consumption smoothing parameter across different household types, specifically borrowers and savers, in his model of the business cycle. Similarly, Krueger et al. (2016) adopt consistent behavioral parameters, including habit formation, across wealth and income groups in their study of household heterogeneity. This approach allows for simplification without undermining the essential differences in household behavior with respect to consumption and saving. Additionally, the capital depreciation rate is calibrated to 0.025, following Smets and Wouters (2007), who justify this value based on average depreciation observed in macroeconomic datasets.

For the calibration of household shares, I set the share of low-skilled households, denoted by ω, to 0.2, based on Bilbiie et al. (2023). In their model, this parameter represents the proportion of hand-to-mouth households—those who do not have access to financial markets—corresponding to the low-skilled households in my framework. In the welfare analysis section, I also consider an alternative share of $\omega = 0.4$, consistent with Aguiar et al. (2024), to analyze how changes in the proportion of hand-to-mouth households impact welfare outcomes. This variation allows for an exploration of the policy implications when the share of hand-to-mouth households shifts in the economy.

In calibrating the parameters of the production sector, the capital share, α, is set to 0.4 in line with studies such as King and Rebelo (1999) and Ireland (2004). The choice of setting the Calvo probability to 0.75 is supported by the work of Eichenbaum and Fisher (2003), who empirically test the Calvo model of sticky prices. They find that firms adjust prices approximately once every four quarters, implying a price adjustment probability of 0.75, a value consistent with the degree of price rigidity typically observed in DSGE models. Furthermore, I set the elasticity of substitution between high- and low-skilled labor inputs to 0.75, which is consistent with the empirical findings of Ciccone and Peri (2005), who estimate a long-run elasticity between 0.6 and 0.9 for more and less educated workers in the U.S. And lastly, in line with Basu and Fernald (1997), I set the value-added markup of prices over marginal cost to 0.2. This corresponds to a price elasticity of demand, μ, equal to 6.

Table 1. Calibrated Parameter Values

Parameters	Description	Values
ω	Share of low-skilled, hand-to-mouth households in the economy	0.2
$\beta_l; \beta_h$	Discount factors for low- and high-skilled households	$0.9; 0.995$
$\chi_l; \chi_h$	Habit formation parameters for low- and high-skilled households	0.9
$\delta_{l,0}; \delta_{h,0}$	Steady state job destruction rates for low- and high-skilled households	$0.15; 0.08$
δ_k	Capital depreciation rate	0.025
α	Capital input share	0.4
γ	Share of price adjusting intermediate goods producing firms	0.75
μ	Price elasticity of demand	6
ρ	Elasticity of substitution between labor inputs	0.75
ϕ_r	Monetary policy smoothing parameter	0.85
ϕ_y	Interest rate reaction to output	$0.5/4$
ϕ_π	Interest rate reaction to inflation	1.5
ρ_a	Autoregressive coefficient, technological progress	0.95
$a_l; a_h$	Productivity terms of low and high-skilled workers	$1; 3$
$\zeta_l; \zeta_h$	Surplus share (bargaining powers of low- and high-skilled workers)	$0.3299; 0.6872$
$\psi_l; \psi_h$	Real wage rigidity for low- and high-skilled wage schedules	$0.3156; 0.7312$
$\varphi_{l,2}; \varphi_{h,2}$	Utilization cost adjustment parameter for low- and high-skilled	$0.3080; 5.2194$
$\kappa_l; \kappa_h$	Cost of posting low- and high-skill vacancy	$0.9928; 1.8057$

The matching technology in the labor market, for both types of workers is a homogenous of degree one function and is characterized by the parameter η_l and η_h for low- and high-killed workers respectively. In line with Dolado et al. (2021) I assume symmetric matching elasticities and set the value to 0.5 for both types. The steady state job separation probability for high-skilled workers, $\delta_{h,0}$, is set to 0.08, and the steady state job separation probability for low-skilled workers, $\delta_{l,0}$, is set to 0.15, which is compatible with those used in the literature which range from 0.07 (Merz (1995)) to 0.15 (Andolfatto (1996)). Given that $\delta_{h,0} < \delta_{l,0}$, the low-skill labor market can be described as fluid, while the high-skill labor market is more sclerotic, consistent with the classification by Blanchard and Galí (2010). Additionally, I calibrate the unemployment benefit for both worker types to achieve a steady-state ratio of $g_s/w_s = 0.5$ consistent with the average value observed in industrialized countries, as reported by Nickell and Nunziata (2001).

The remaining skill-specific parameters, ζ_s (bargaining power), ψ_s (real wage rigidity), $\varphi_{s,2}$ (utilization cost adjustment parameter), and κ_s (cost of posting vacancy), for $s \in \{h, l\}$, are estimated by minimizing a measure of the distance between the model and empirical impulse response functions following Christiano et al. (2005). To calibrate the eight skill-specific parameters, I first smoothed the empirical impulse responses of high- and low-skilled unemployment rates to an identified monetary policy shock. I then minimized the distance between these smoothed empirical impulse responses and their corresponding model-generated impulse responses. The empirical responses were smoothed using Gaussian smoothing, which applies a weighted moving average with Gaussian weights to reduce the variability and smooth out sharp variations. This step is particularly valuable, as empirical responses often exhibit sharper variations compared to model-generated responses.

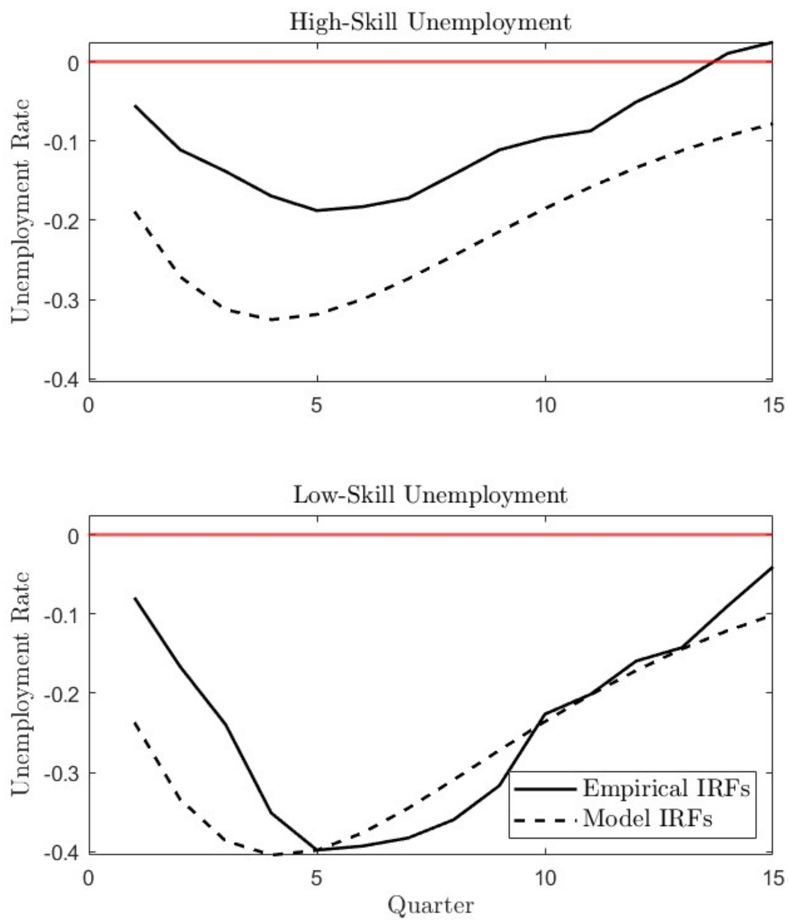

Figure 3. Empirical versus model impulse responses of high- and low-skilled unemployment rates after minimizing a measure of the distance between the model and empirical impulse response functions.

Figure 3 illustrates the limited information estimation results, where model-derived impulse responses are plotted alongside empirical impulse responses.[8] Notably, the calibrated parameters effectively align the model's low-skill unemployment response with the empirical counterpart more closely than for high-skill unemployment. Nonetheless, significant differences remain, particularly in the sensitivity of low- and high-skill unemployment to monetary policy shocks. Further details on the smoothing process, along with the smoothed empirical and model IRFs, are provided in Appendix C.

Lastly, the aggregate productivity shock is modeled as an $AR(1)$ process, and following the RBC literature, its standard deviation is calibrated to 0.008 with a persistence of 0.95. This calibration is consistent with the values used by King and Rebelo (1999) in real business cycle models. When considering the Taylor rule smoothing parameter, I follow Smets and Wouters (2007) and Christiano et al. (2005), setting it equal to 0.85. This value reflects the interest rate inertia typically observed in monetary policy, capturing the gradual adjustment of interest rates over time, aligning with empirical evidence on central bank behavior.

4 Dynamic Properties of The Model

Before addressing the welfare implications of various monetary policy regimes, it is essential to explore the model's dynamic properties under different monetary policy rules. This examination allows for an evaluation of the model's ability to replicate key stylized facts about the labor market. In this section, I will focus on the impulse response functions of several economic variables in the context of productivity shocks. This analysis provides a clearer understanding of how different policy rules shape the model's dynamic responses, especially regarding labor market behavior, thereby offering insights into the model's effectiveness in capturing observed economic patterns. I consider three types of monetary policy rules in the model:

1. a rule with a strong response to inflation, where $\phi_\pi = 4$, $\phi_y = 0$, $\phi_{u,l} = 0$ and $\phi_{u,l} = 0$;

2. a standard Taylor rule, specified by $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, $\phi_{u,l} = 0$ and $\phi_{u,l} = 0$;

3. a rule that incorporates a response to both low-skilled and high-skilled unemployment, with parameters $\phi_\pi = 1.5$, $\phi_y = 0$, $\phi_{u,l} = 1.2/4$ and $\phi_{u,h} = 0.6/4$;

For this last rule, the coefficient on inflation is set to 1.5, consistent with the Taylor Rule, allowing for a direct comparison under a similar inflation response while targeting unemployment rather than output. The coefficients on unemployment for both skill types

[8] Impulse responses of other selected endogenous variables on monetary policy shock with estimated skill-specific parameters can be found in Appendix D

are set significantly higher than the typical output response in the standard Taylor Rule. This is because, in the model, low- and high-skill unemployment rates exhibit greater volatility than output, prompting a more aggressive policy response to unemployment fluctuations.[9] Additionally, the difference in the coefficients for low-skilled and high-skilled unemployment is set at a ratio of two to one, reflecting empirical evidence from a local projections approach, which indicates that low-skilled unemployment rate is approximately twice as sensitive to an expansionary monetary policy shock compared to high-skilled unemployment rate. This specification aligns the model's response with observed labor market behavior, allowing for a more accurate reflection of how different household types respond to monetary policy.

Figure 4 presents the impulse responses of selected skill-specific labor market variables, as well as aggregate variables such as inflation and output, to a positive productivity shock. Following the shock, output rises, resulting in a decrease in inflation. As firms expand production, they increase vacancies for both low- and high-skilled workers, leading to a tightening of their respective labor markets. This increase in vacancies drives up real wages for both skill types, while unemployment falls, with low- and high-skilled unemployment rates moving in opposite directions to vacancies, thus tracing skill-specific Beveridge curves.

The impulse responses indicate that both low-skilled and high-skilled labor market tightnesses are pro-cyclical, showing a positive response to the productivity shock across all policy rules. Labor market tightness for each skill type increases with economic expansion. Furthermore, although not shown in the figure, the aggregate labor market tightness—a convex combination of the skill-specific tightnesses weighted by the respective shares of each household type—also displays pro-cyclicality. This result aligns with Faia (2008), which suggest that aggregate labor market tightness moves with the business cycle, and is consistent with empirical evidence on aggregate labor market dynamics. Additionally, the model indicates that unemployment exhibits a high degree of persistence following a productivity shock, further reflecting empirical observations.

In response to a productivity shock, the impulse response functions reveal key distinctions in the effects of each monetary policy rule on both nominal and real variables. The strict inflation-targeting rule demonstrates a strong stabilizing effect on inflation, which declines sharply and remains stable throughout the time horizon. However, strict inflation stabilization tends to amplify fluctuations in labor market variables. Labor market tightness and vacancies for both low- and high-skilled workers initially spike, indicating a strong adjustment reaction, followed by a more prolonged and persistent response. Strict targeting of

[9]This choice also reflects considerations related to model determinacy issues inherent in models with search-and-matching frictions in the labor market. The implications of these determinacy concerns for the selection of coefficient values will be further discussed in the following section.

inflation, however, comes at the expense of labor market stability, causing more substantial fluctuations in unemployment across both skill types.

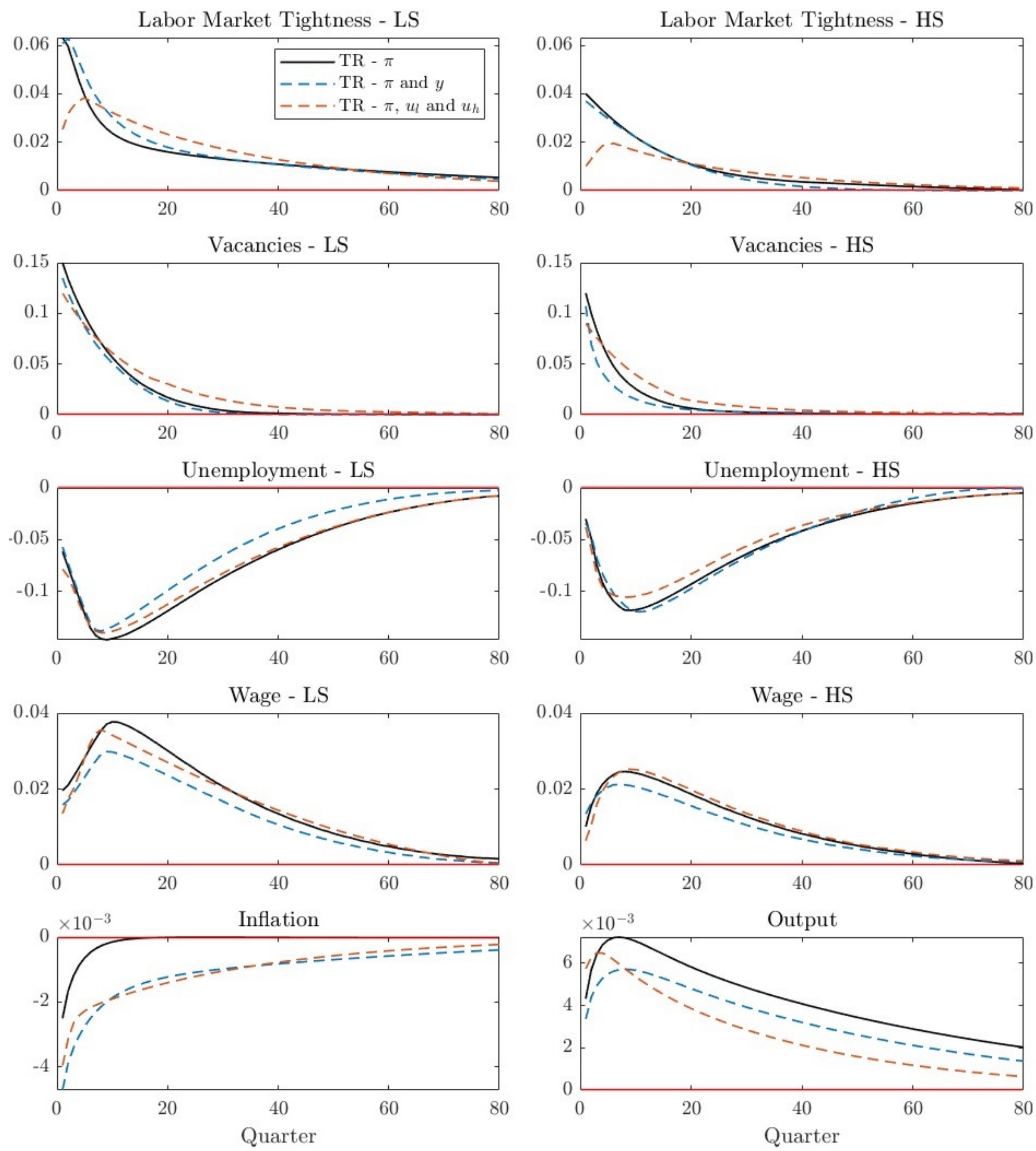

Figure 4. Impulse responses of selected variables to productivity shocks ($\sigma_a = 0.008$) under each of the following three rules: 1. strong response to inflation: $\phi_\pi = 4$, $\phi_y = 0$, $\phi_{u,l} = 0$ and $\phi_{u,l} = 0$; 2. standard Taylor rule: $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, $\phi_{u,l} = 0$; $\phi_\pi = 1.5$, 3. response to heterogeneous unemployment $\phi_\pi = 1.5$, $\phi_y = 0$, $\phi_{u,l} = 2.4/4$ and $\phi_{u,h} = 1.2/4$.

In contrast, the standard Taylor Rule achieves a more balanced effect, providing smoother adjustments in both inflation and labor market variables. Inflation stabilization is achieved to a lesser degree than under strict inflation targeting, but the volatility in labor market variables is notably reduced. For both low- and high-skilled labor market tightness and vacancies, the responses under the Taylor Rule are more moderate and gradual. This rule yields a more stable response in unemployment, as the adjustment path is less severe and shorter-lived than under strict inflation targeting. The simple Taylor Rule thus supports a more stable real economy by balancing its responses between inflation and output.

By further refining the policy approach, the rule responding to skill-specific unemployment rates offers an even more effective stabilization of both nominal and real variables. While inflation declines steadily, it does not exhibit the extreme initial decline seen under the strict inflation-targeting rule. The responses of labor market variables under this rule are smoother and less volatile, as the policy response is directly aligned with stabilizing labor market conditions by targeting unemployment. Unemployment rates for both low- and high-skilled workers show quicker adjustments back to equilibrium compared to the other rules, indicating that this approach enhances stability for the real economy. By addressing skill-specific unemployment fluctuations, this rule reduces volatility across both inflation and labor market variables, providing a balanced response that supports stability in both nominal and real terms.

Overall, the results indicate that while strict inflation targeting stabilizes nominal variables effectively, it does so at the expense of labor market stability. The standard Taylor Rule achieves a more balanced approach, reducing volatility across both inflation and labor market indicators. The rule that responds to skill-specific unemployment rates further refines this balance, directly addressing labor market dynamics and promoting stability in both nominal and real variables. This finding aligns with Faia (2008), who demonstrated a similar outcome within a representative household framework.

5 Welfare Analysis

In this framework, the optimal monetary policy problem is formulated under the assumption that the monetary authority seeks to maximize low- and high-skilled households' aggregate welfare, taking into account the competitive equilibrium conditions and the monetary policy rules specified in the Equation (38). The goal is to identify parameters for interest rate rules that are straightforward, based only on observable variables, and can ensure a unique rational expectations equilibrium. In this model, I search for parameter sets that maximize the expected lifetime utility of high-skilled and low-skilled households separately, recognizing that their welfare may respond differently to policy changes. To capture the overall impact, I also examine parameter combinations that optimize aggregate welfare, represented by a weighted sum of the lifetime utilities of both household types. However, since the conditions that maximize each household type's welfare are not necessarily aligned, the parameters that yield the highest aggregate welfare may not coincide with those that maximize the welfare of each household type separately.

Some key considerations are necessary when computing welfare in this context. Standard first-order approximation methods are insufficient for accurately comparing the welfare implications of various monetary policy arrangements. This is due to the presence of a distorted steady state, where stochastic volatility influences both the mean and variance of variables that are essential for welfare analysis. In a first-order approximation, the expected value of a variable aligns with its non-stochastic steady-state, inherently neglecting the impact of volatility on mean values. Therefore, to accurately rank policy arrangements, one must rely on a higher-order approximation of the policy functions, which can effectively account for these volatilities Kim and Kim (2003).

In this analysis, welfare is computed using the unconditional ergodic mean, representing the theoretical long-run average welfare level across various economic states. This approach captures the steady-state welfare implications, providing an assessment of the economy's average welfare over time, independent of initial conditions. Although conditional expected discounted utility can account for transitional effects as the economy moves from deterministic to stochastic steady states under different policy rules, the focus here is on evaluating the long-term welfare outcomes of each policy rule rather than on transitional dynamics. Thus, the unconditional mean offers a stable reference point for comparing policies in terms of their average welfare impact over the long run, making it a reliable benchmark for assessing policy impacts within the steady-state distribution, where the economy is expected to operate over time. Therefore, the welfare functions for both low-skilled and high-skilled households are represented by Bellman equations, capturing the present discounted value of the corresponding utilities over time. The welfare for the low-skilled household, $\mathcal{W}_{l,t}$, is defined as follows:

$$\mathcal{W}_{l,t} = \ln\left(c_{l,t} - \chi_l c_{l,t-1}\right) + \beta_l \mathbb{E}_t\left[\mathcal{W}_{l,t+1}\right]. \tag{44}$$

This expression reflects the current period utility, $\ln\left(c_{l,t} - \chi_l c_{l,t-1}\right)$, which accounts for consumption adjustments based on past consumption, plus the discounted expected future welfare, $\beta_l \mathbb{E}_t\left[\mathcal{W}_{l,t+1}\right]$. Similarly, the welfare for the high-skilled household, $\mathcal{W}_{h,t}$, is given by:

$$\mathcal{W}_{h,t} = \ln\left(c_{h,t} - \chi_h c_{h,t-1}\right) + \beta_h \mathbb{E}_t\left[\mathcal{W}_{h,t+1}\right] \tag{45}$$

where β_h is the high-skilled household's discount factor, reflecting its specific time preference.

The aggregate welfare, $\mathcal{W}_t$, is the weighted sum of the welfare values for both household types:

$$\mathcal{W}_t = \omega \mathcal{W}_{l,t} + (1 - \omega) \mathcal{W}_{h,t}. \tag{46}$$

Here, ω represents the share of low-skilled households in the economy, with the remaining portion, $(1 - \omega)$, attributed to high-skilled households. These equations provide a recursive framework that includes both current utility and expected future welfare, reflecting the welfare effects of consumption choices and policy impacts over time.

5.1 Simple Rules Versus Optimal Policy Rule

Evaluating the relative effectiveness of simple versus optimal policy rules is crucial for assessing welfare implications in the face of economic uncertainty. Analyzing these policy frameworks provides insights into the welfare effects of alternative monetary policy specifications under different macroeconomic conditions. In this analysis, I simulate the model economy under aggregate uncertainty driven by productivity shocks. Although incorporating additional shocks—such as government expenditure or cost-push shock—would add to the robustness and depth of the analysis, isolating productivity shocks provides valuable insights into the welfare effects of real economic fluctuations alone.

To explore the welfare implications under this framework, I begin by evaluating welfare across various (ad hoc) specifications of the following monetary policy rules:

(i) Simple Taylor rule: $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, and $\phi_r = \phi_u = \phi_{u,l} = \phi_{u,h} = 0$.

(ii) Simple Taylor rule with smoothing: $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, $\phi_r = 0.85$, and $\phi_u = \phi_{u,l} = \phi_{u,h} = 0$.

(iii) Strict inflation targeting: $\phi_\pi = 3$, and $\phi_r = \phi_y = \phi_u = \phi_{u,l} = \phi_{u,h} = 0$.

(iv) Response to inflation and aggregate unemployment: $\phi_\pi = 3$, $\phi_u = 6/4$.

(v) Response to strong inflation and skill-specific unemployment: $\phi_\pi = 2.5$, $\phi_{u,l} = 2.4/4$, $\phi_{u,h} = 1.2/4$, and $\phi_r = \phi_y = \phi_u = 0$.

(vi) Response to inflation and skill-specific unemployment: $\phi_\pi = 1.5$, $\phi_{u,l} = 2.4/4$, $\phi_{u,h} = 1.2/4$, and $\phi_r = \phi_y = \phi_u = 0$.

Following this, I conduct a grid search over the parameters $\{\phi_\pi, \phi_y, \phi_u, \phi_{u,l}, \phi_{u,h}\}$ to identify the rule that delivers the highest level of welfare, which I define as the optimal policy rule. This approach allows for a comparison of welfare outcomes between the optimal rule and simpler policy rules. The search is conducted over the following parameter ranges: $[0, 4]$ for ϕ_π, $[0, 2]$ for ϕ_y, ϕ_u, $\phi_{u,l}$, and $\phi_{u,h}$.[10] Additionally, I compare policy rules with interest rate smoothing, $\phi_r = 0.85$, to those without smoothing, $\phi_r = 0$, and a combination of policy parameters is considered acceptable if it leads to a single, well-defined rational expectations equilibrium.

The inclusion of unemployment as an independent variable in policy formulation reflects the fact that central banks, such as the Federal Reserve with its dual mandate, often face the trade-off between stabilizing inflation and promoting maximum employment. In this context, it is relevant to question whether the emphasis on price stability, which has been strongly promoted in recent years, truly represents the optimal policy approach. Furthermore, the idea of incorporating skill-specific unemployment into the policy rule aligns with recent policy shifts, such as the Federal Open Market Committee's (FOMC) decision to prioritize labor market improvements for low- and moderate-income communities.Powell (2021) By acknowledging the differential impacts on these groups, this policy approach aims to address labor market disparities more effectively, highlighting a broader focus on incorporating employment outcomes into monetary policy analysis.

The findings are summarized in Table 2. The table shows the percentage difference in welfare relative to the optimal policy rule across various alternative simple rules. The results indicate that the optimal policy rule involves responding to both inflation and skill-specific unemployment rates, with the coefficients $\phi_\pi = 2.5$, $\phi_{u,l} = 2.4/4$, $\phi_{u,h} = 1.2/4$, and $\phi_r = \phi_y = \phi_u = 0$. This outcome arises due to the presence of matching frictions in the labor market, which introduce a congestion externality: an excessive number of (low- and high-skill) job seekers or vacancies reduces the probability of successful matches, driving unemployment above its efficient level. As a result, the policymaker faces a trade-off between stabilizing unemployment and controlling inflation, prompting a need to respond to both. Focusing solely on inflation fails to address the distortions caused by unemployment fluctuations, which prevents the policymaker from reaching a constrained-efficient allocation. By responding to inefficient unemployment, the policy can move closer to the optimal allocation. However, it is important to note that the optimal rule also includes a strong response to inflation.

Notably, a strong response to inflation remains a central component of the optimal policy rule, consistent with Faia's (2008) findings in her representative-agent framework, which incorporated search and matching frictions. In her model, the combination of robust inflation stabilization and a response to unemployment yielded the most favorable outcomes, with a coefficient of 0.6/4 applied to unemployment. In my analysis, however, I employ a

[10] One should also note that ϕ_y, ϕ_u, $\phi_{u,l}$, and $\phi_{u,h}$ are divided by four, considering the standard quarterly period assumption and the fact that inflation is expressed in annual terms in Taylor-type rules.

significantly larger coefficient of 6/4 for unemployment. This adjustment is necessary to address the model's determinacy issues introduced by search and matching frictions. It is also well-documented that matching frictions can lead to indeterminacy across certain parameter ranges, as shown by Krause and Lubik (2004) and Hashimzade and Ortigueira (2005). [11]

In the context of my analysis, however, skill-specific responses to unemployment yield even greater welfare gains than those obtained from targeting aggregate unemployment alone. Specifically, the welfare differences relative to the optimal policy rule, as shown in the Table 2, illustrate that the rule responding to both types of skill-specific unemployment results in the highest welfare, outperforming a rule that targets only aggregate unemployment.

Table 2. Welfare comparison across alternative monetary policy specifications

% Difference in welfare relative to the optimal policy rule			
		$\omega = 0.2$	
Monetary policy rules	$\mathcal{W}_t$	$\mathcal{W}_{l,t}$	$\mathcal{W}_{h,t}$
Simple Taylor rule	-7.48	-5.62	-7.51
Simple Taylor rule with smoothing	-6.19	-5.172	-6.21
Strict inflation targeting	-4.23	-4.45	-4.23
Response to inflation and aggregate unemployment	-2.54	-35.70	-2.09
Response to strong inflation and skill-specific unemployment	0	0	0
Response to inflation and skill-specific unemployment	-1.73	-1.03	-1.73
		$\omega = 0.4$	
Monetary policy rules	$\mathcal{W}_t$	$\mathcal{W}_{l,t}$	$\mathcal{W}_{h,t}$
Simple Taylor rule	-5.79	-20.25	-5.68
Simple Taylor rule with smoothing	-4.805	-18.64	-4.69
Strict inflation targeting	-3.29	-16.03	-3.20
Response to inflation and aggregate unemployment	-2.00	-14.54	-1.91
Response to strong inflation and skill-specific unemployment	0	0	0
Response to inflation and skill-specific unemployment	-1.33	-3.73	-1.31

Including output as a target in Taylor rule, alongside inflation, leads to a reduction in welfare. This result is consistent with Schmitt-Grohé and Uribe (2007) findings, who demonstrated similar effects within a model featuring capital accumulation and labor markets free of frictions. In the current context, this outcome can be explained by the policymaker's objective to stabilize variables that reflect underlying inefficiencies. Given that the main distortions in this model arise within the labor market—affecting both high- and low-skill

[11] A figure illustrating the regions of determinacy and indeterminacy can be found in the Appendix E.

sectors—prioritizing unemployment over output allows the policymaker to address these distortions more directly. Additionally, the findings indicate that interest rate smoothing consistently improves welfare. This outcome aligns with Schmitt-Grohé and Uribe (2007) findings and can be attributed to the fact that smoothing the interest rate extends the stabilization effects of the monetary policy targets over time.

5.2 Response to Skill-Specific Unemployment

To further explore the relationship between policy coefficients and welfare outcomes, Figure 5 displays the unconditional welfare surface by varying the coefficients for low-skill ($\phi_{u,l}$) and high-skill ($\phi_{u,h}$) unemployment in the monetary policy rule (Equation (38)), while holding the inflation coefficient at its welfare-maximizing value. These results reflect scenarios in which the output coefficient is set to zero. As previously suggested, policy rules that include a positive response to output consistently yield lower welfare relative to cases where the output response remains at zero.

The welfare surface in Figure 5 illustrates the effects of varying the responses to low-skill ($\phi_{u,l}$) and high-skill ($\phi_{u,h}$) unemployment in the monetary policy rule, with the inflation coefficient held fixed at its welfare-maximizing level. The welfare surface shown in the figure indicates that welfare is maximized at $\phi_{u,l} = 2.4/4$ and $\phi_{u,h} = 1.2/4$ with $\phi_\pi = 2.5$. Increasing the response parameter for high-skill unemployment reduces overall welfare. This happens because focusing heavily on stabilizing high-skill unemployment increases the variability and costs associated with low-skill unemployment, outweighing the benefits of reducing high-skill unemployment fluctuations—especially with the inflation response set at its welfare-maximizing level. Similarly, increasing the response to low-skill unemployment also reduces welfare, as placing too much weight on low-skill unemployment fluctuations raises the cost of high-skill unemployment variability. In both cases, an excessive response to one type of unemployment leads to undesirable trade-offs.[12]

The optimal policy rule reflects the distinct sensitivities of low-skill and high-skill unemployment to monetary policy interventions. With the model calibrated to show that low-skill unemployment is roughly twice as responsive to monetary policy shocks as high-skill unemployment, the rule assigns a proportionally greater weight to low-skill unemployment. This differential response justifies a configuration that emphasizes stabilizing low-skill unemployment, thereby maximizing welfare through targeted, skill-specific responses. Furthermore, the determinacy constraint plays a crucial role in shaping the welfare surface by limiting $\phi_{u,l}$ and $\phi_{u,h}$ to parameter combinations that ensure a unique, stable equilibrium. Outside this

[12] The results presented in Figure 5 remain valid even when a positive interest rate smoothing parameter is included.

determinacy-preserving range, the model is prone to indeterminacy, where the conditions for a unique equilibrium are not met. Matching frictions, particularly those involving congestion externalities—where an excessive number of job seekers or vacancies lowers the probability of successful matches—add complexity to the labor market. These frictions require careful calibration of the policy parameters to avoid scenarios with multiple or undefined equilibria and to ensure that the model achieves a stable, unique rational expectations equilibrium.[13]

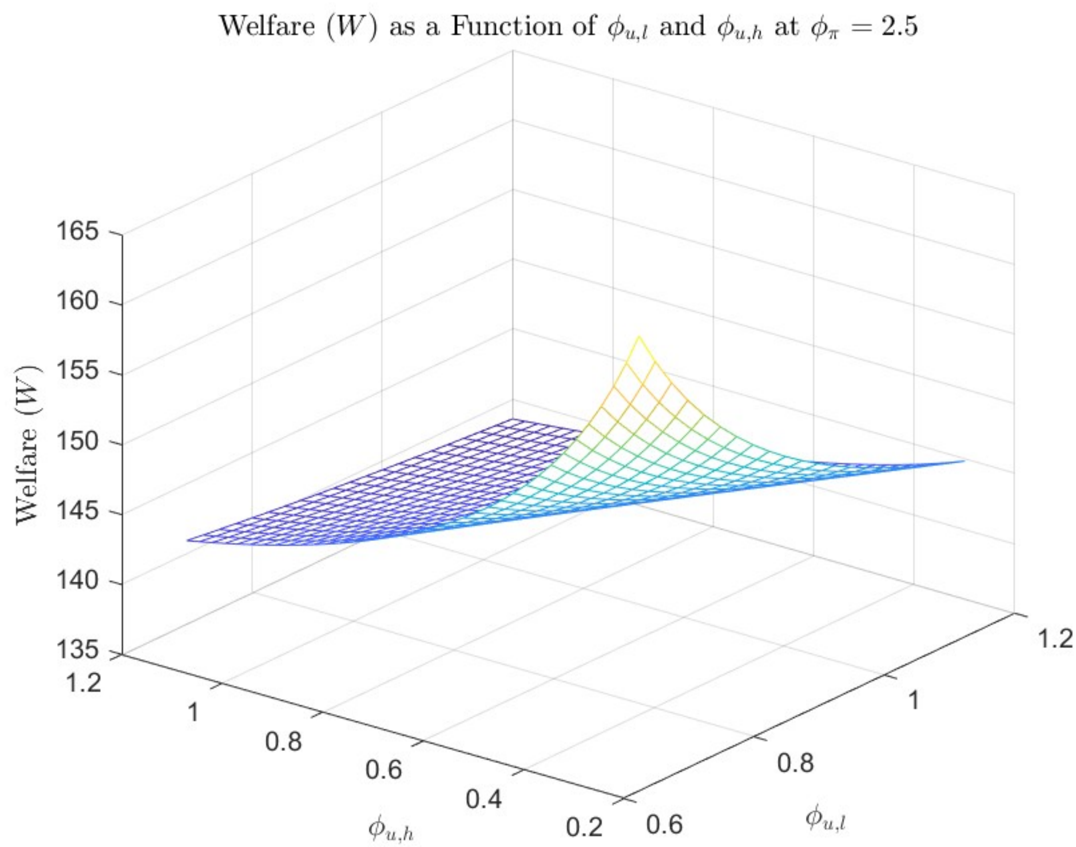

Figure 5. Effect on welfare of varying the response to skill-specific unemployment rates in the Taylor rule, with the inflation coefficient fixed at its welfare-maximizing value (and $\phi_r = 0$).

This finding contrasts with the optimal policy prescriptions commonly derived in standard New Keynesian models, where price rigidity and monopolistic competition are the primary frictions. In these models, policymakers face no trade-off between output and inflation stabilization; by closing the gap between flexible and sticky price allocations, they can achieve the first-best outcome. However, my model incorporates labor market frictions

[13]A figure illustrating the regions of determinacy and indeterminacy can be found in the Appendix E.

that introduce skill-specific unemployment dynamics, resulting in inefficiently high unemployment for low- and high-skill workers alike. This feature drives the monetary authority to move away from strict price stability and instead respond more directly to fluctuations in skill-specific unemployment. Building on Faia (2008), my results suggest that the optimal policy rule must balance inflation stabilization with targeted responses to low- and high-skill unemployment. In doing so, it addresses the inefficiencies introduced by labor market frictions while mitigating the cost of inflation variability, achieving a more effective balance between employment and price stability.

This emphasis on skill-specific responses stands in contrast to the approach taken by Blanchard and Galí (2006), who assume Hosios conditions (Hosios (1990)) that neutralize search externalities and focus primarily on wage rigidity. By retaining search frictions and exploring policy impacts in a heterogeneous-agent setting, the results here provide insight into welfare trade-offs without requiring an optimization constraint. This highlights the welfare effects of skill-specific unemployment responses, which reflect the labor market distortions that emerge under structural frictions. The analysis thus offers a more detailed perspective on policy effectiveness in complex economic environments where search externalities and skill-specific dynamics play a significant role.

6 Conclusion

This paper develops a heterogeneous-agent model with labor market frictions, including skill-specific unemployment dynamics, monopolistic competition, and sticky prices. The model evaluates welfare under various monetary policy rules, exploring how skill-targeted responses to unemployment, alongside inflation stabilization, can improve welfare outcomes in an economy characterized by real wage rigidity and labor market matching frictions.

This analysis concludes that the presence of those frictions implies an optimal policy rule should include targeted responses to both low- and high-skill unemployment, alongside inflation. These frictions introduce a congestion externality, where excessive numbers of job seekers or vacancies reduce the probability of successful matches, making unemployment inefficiently high across skill groups. Consequently, the monetary authority must navigate a trade-off between stabilizing inflation and reducing unemployment fluctuations. This trade-off highlights the importance of incorporating skill-specific responses within the policy rule. Such an approach enables more precise stabilization in a labor market where low- and high-skill groups exhibit different sensitivities to monetary policy shocks.

Future research could extend this framework by incorporating heterogeneous agent New Keynesian (HANK) elements, allowing for a more detailed analysis of distributional impacts across different skill and income groups. A HANK model would facilitate a deeper exploration of how monetary policy affects consumption and labor market outcomes, not only at an aggregate level but also across varying segments of the population. This approach could provide valuable insights into the broader welfare implications of skill-specific policy responses, especially in an environment where wealth and income distribution play a crucial role in the transmission of monetary policy.

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Appendix

A Data

The data used in this analysis consists of unemployment rates for individuals with at least a bachelor's degree and those with a high school diploma or less[14], as well as the identified monetary policy shocks from Gertler and Karadi (GK) depicted in Figure 7. The dataset is quarterly, spanning from 1992-Q1 to 2012-Q2, and includes three recession periods.

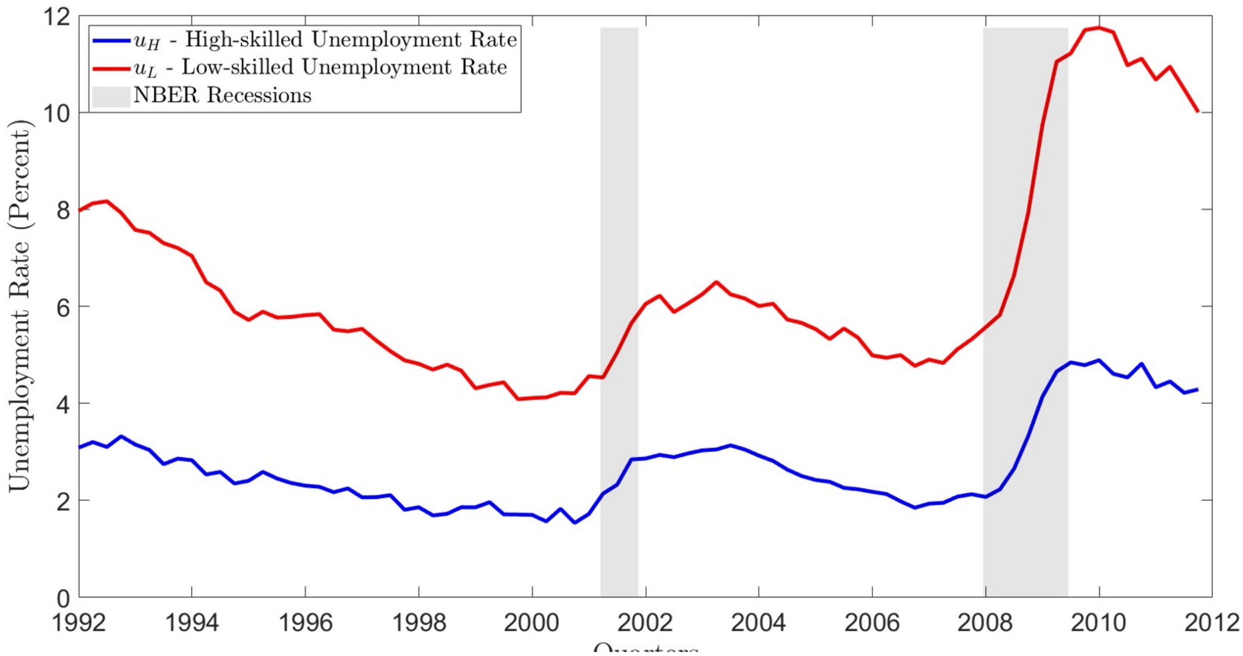

Figure 6. Low- and High-Skilled Unemployment Rates with NBER Recessions (Grey Bars)

Unemployment rates are calculated using seasonally adjusted data from the U.S. Bureau of Labor Statistics. The low-skilled unemployment rate, representing individuals with a high school diploma or less, is computed as:

$$u_{l,t} = \frac{U_{HS,t} + U_{NHS,t}}{L_{HS,t} + L_{NHS,t}}$$

where $U_{HS,t}$ and $L_{HS,t}$ are the unemployment level and labor force for high school graduates aged 25 and older, and $U_{NHS,t}$ and $L_{NHS,t}$ represent the same for individuals without a high school diploma.

[14] Source: U.S. Bureau of Labor Statistics

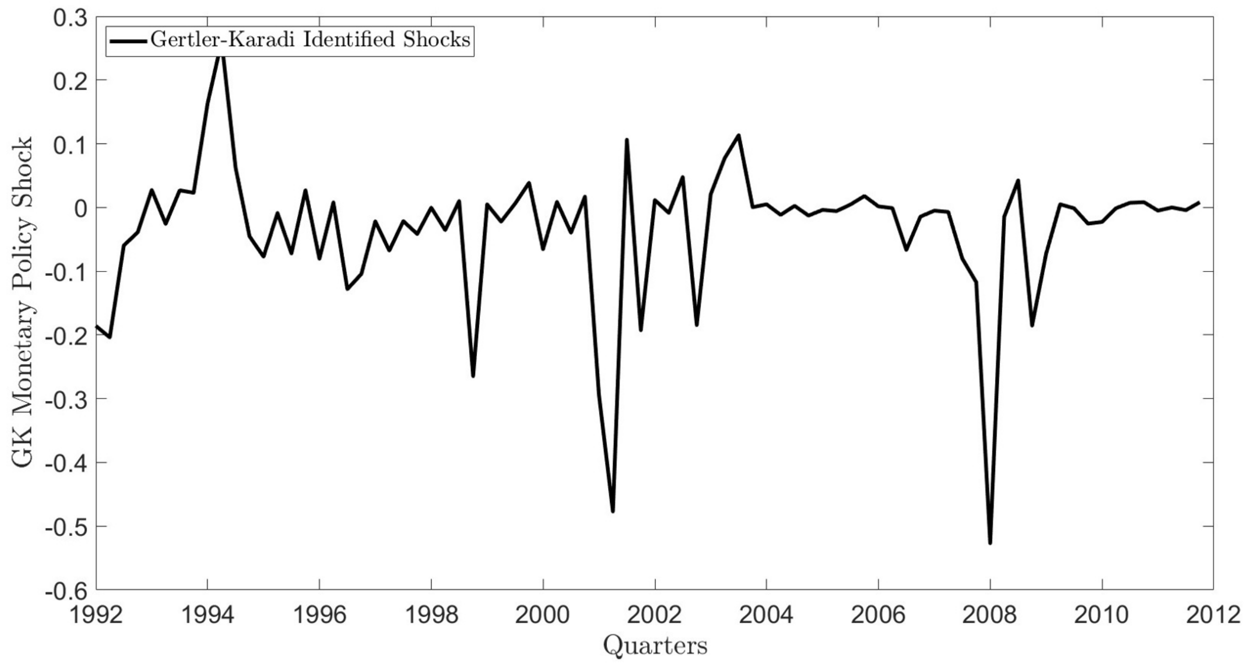

Figure 7. Gertler and Karadi Identified Monetary Policy Shocks

Similarly, the high-skilled unemployment rate for individuals with at least a bachelor's degree is calculated as:

$$u_{h,t} = \frac{U_{BA,t}}{L_{BA,t}}$$

where $U_{BA,t}$ and $L_{BA,t}$ denote the unemployment level and labor force for those aged 25 and older with at least a bachelor's degree.

The time-series of unemployment rates, as depicted in Figure 6, highlight significant differences between the two unemployment series, with low-skilled unemployment exhibiting greater variability ($\mathbb{V}(u_{l,t}) = 4.64$ compared to high-skilled unemployment $\mathbb{V}(u_{h,t}) = 0.84$). Low-skilled unemployment rates also show a more pronounced sensitivity to economic cycles, with sharper increases during recessions and steeper declines during recovery periods, relative to the more stable high-skilled unemployment rates.

B Optimal Reset Price

We can rewrite the numerator of the Equation (19) as follows

$$\mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} \frac{p_{B,t+j}}{p_{t+j}} (p_{t+j})^{\mu} y_{t+j} = \gamma^0 \beta_h^0 \frac{\lambda_{h,t}}{\lambda_{h,t}} \frac{p_{B,t}}{p_t} (p_t)^{\mu} y_t + \mathbb{E}_t \sum_{j=1}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} \frac{p_{B,t+j}}{p_{t+j}} (p_{t+j})^{\mu} y_{t+j}$$

from where

$$\gamma \mathbb{E}_t \sum_{j=1}^{\infty} \gamma^{j-1} \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} \frac{p_{B,t+j}}{p_{t+j}} (p_{t+j})^{\mu} y_{t+j} = \gamma \mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \left(\beta_h^{j+1} \frac{\lambda_{h,t+j+1}}{\lambda_{h,t}} \Big/ \beta^j \frac{\lambda_{h,t+j+1}}{\lambda_{h,t+1}} \right) \times \beta^j \frac{\lambda_{h,t+j+1}}{\lambda_{h,t+1}} \frac{p_{B,t+j+1}}{p_{t+j+1}} (p_{t+j+1})^{\mu} y_{t+j+1}$$

and we arrive at

$$\mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} \frac{p_{B,t+j}}{p_{t+j}} (p_{t+j})^{\mu} y_{t+j} = \frac{p_{B,t}}{p_t} (p_t)^{\mu} y_t + \mathbb{E}_t \beta_h \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \sum_{j=0}^{\infty} \gamma^j \beta^j \frac{\lambda_{h,t+j+1}}{\lambda_{h,t+1}} \frac{p_{B,t+j+1}}{p_{t+j+1}} (p_{t+j+1})^{\mu} y_{t+j+1}.$$

If we denote $\mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} \frac{p_{B,t+j}}{p_{t+j}} (p_{t+j})^{\mu} y_{t+j}$ as

$$\mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+j}}{\lambda_{h,t}} \frac{p_{B,t+j}}{p_{t+j}} (p_{t+j})^{\mu} y_{t+j} \equiv X_{B,t}$$

then

$$\sum_{j=0}^{\infty} \gamma^j \beta^j \frac{\lambda_{t+j+1}}{\lambda_{t+1}} \frac{p_{B,t+j+1}}{p_{t+j+1}} (p_{t+j+1})^{\mu} y_{t+j+1} \equiv X_{B,t+1}.$$

Therefore, we can write

$$X_{B,t} = \frac{p_{B,t}}{p_t} (p_t)^{\mu} y_t + \gamma \mathbb{E}_t \beta_h \frac{\lambda_{h,t+1}}{\lambda_{h,t}} X_{B,t+1}$$

and if we define $\mathcal{P}_{B,t} \equiv X_{1,t} \big/ (p_t)^{\mu}$, then

$$\mathcal{P}_{B,t} = \frac{p_{B,t}}{p_t} y_t + \gamma \beta_h \mathbb{E}_t \frac{\lambda_{h,t+1}}{\lambda_{h,t}} (\pi_{t+1})^{\mu} \mathcal{P}_{B,t+1}.$$

If we use the same logic for the denominator of the Equation (19),

$$X_t \equiv \mathbb{E}_t \sum_{j=0}^{\infty} \gamma^j \beta_h^j \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \left(p_{t+j}\right)^{\mu-1} y_{t+j},$$

and take into account that $X_t = \left(p_t\right)^{\mu-1} y_t + \gamma\beta_h \mathbb{E}_t \frac{\lambda_{h,t+1}}{\lambda_{h,t}} X_{t+1}$, and $\mathcal{P}_t = X_t \big/ \left(p_t\right)^{\mu-1}$, then we will have

$$\mathcal{P}_t = y_t + \gamma\beta_h \mathbb{E}_t \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \left(\pi_{t+1}\right)^{\mu-1} \mathcal{P}_{t+1}.$$

Substituting $X_{B,t}$ and X_t in the Equation (19) will yield

$$p_t^* = \frac{\mu}{\mu-1} \frac{X_{B,t}}{X_t}$$
$$p_t^* = \frac{\mu}{\mu-1} \frac{\mathcal{P}_{B,t}}{\mathcal{P}_t} p_t$$

and if we define $\frac{p_t^*}{p_t}$ by π_t^*, then the above optimal price setting equation will become

$$\pi_t^* = \frac{\mu}{\mu-1} \frac{\mathcal{P}_{B,t}}{\mathcal{P}_t}$$

the optimal reset price (Equation (20)) in terms of inflation.

C Smoothed Empirical Impulse Responses

Gaussian smoothing is a widely used technique that applies a Gaussian-weighted moving average, where observations closer to the center of the window are assigned greater weight, while those farther away contribute less. This approach helps reduce sharp variations in the data, smoothing the transitions while preserving the key characteristics of the impulse response. By applying this method, fluctuations in the empirical impulse responses are minimized, allowing for a clearer comparison with the typically smoother model-generated IRFs. The smoothing process ensures that the essential dynamics of the data are maintained without distorting the overall pattern.

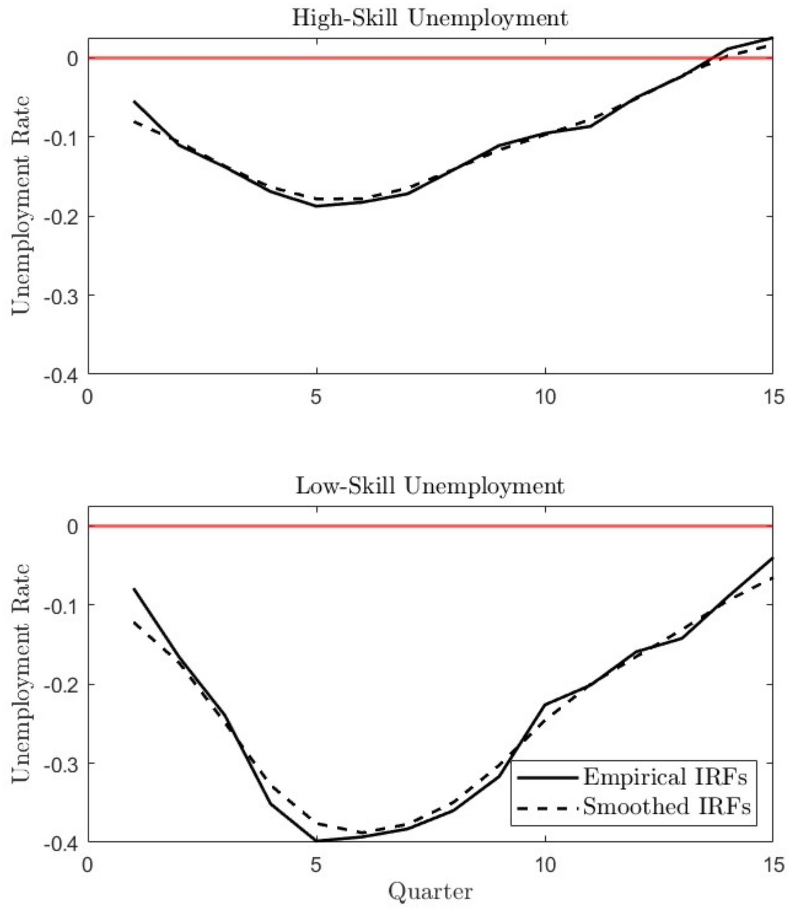

Figure 8. Empirical versus smoothed impulse responses of high- and low-skilled unemployment rates.

I chose a window size of 5 because the empirical IRFs exhibit a peak response at horizon 5. By using a window of this size, the smoothing process aligns with the natural behavior of the data, ensuring that the peak response is preserved while reducing variability in other parts of the IRFs. This helps retain the key features of the impulse responses, particularly around the relevant horizon, while making the empirical IRFs smoother and more comparable to the model-generated counterparts.

D Impulse Responses Of Selected Variables to Monetary Policy Shock

These impulse response functions (IRFs) are derived from IRF-matching calibration estimation, which refines model parameters to align with observed empirical responses. This method facilitates accurate estimation of parameters specific to each skill level, thereby capturing the distinct behavioral dynamics of low- and high-skill groups.

Following the expansionary monetary policy shock, high-skill consumption peaks higher than low-skill consumption, suggesting a stronger response among high-skilled individuals, likely due to greater disposable incomes or wealth. In contrast, low-skill unemployment declines more sharply in the short run, indicating higher sensitivity among low-skilled workers to shifts in labor demand. Over time, consumption decreases, and unemployment rises across both skill levels, reflecting the temporary nature of the expansionary impact.

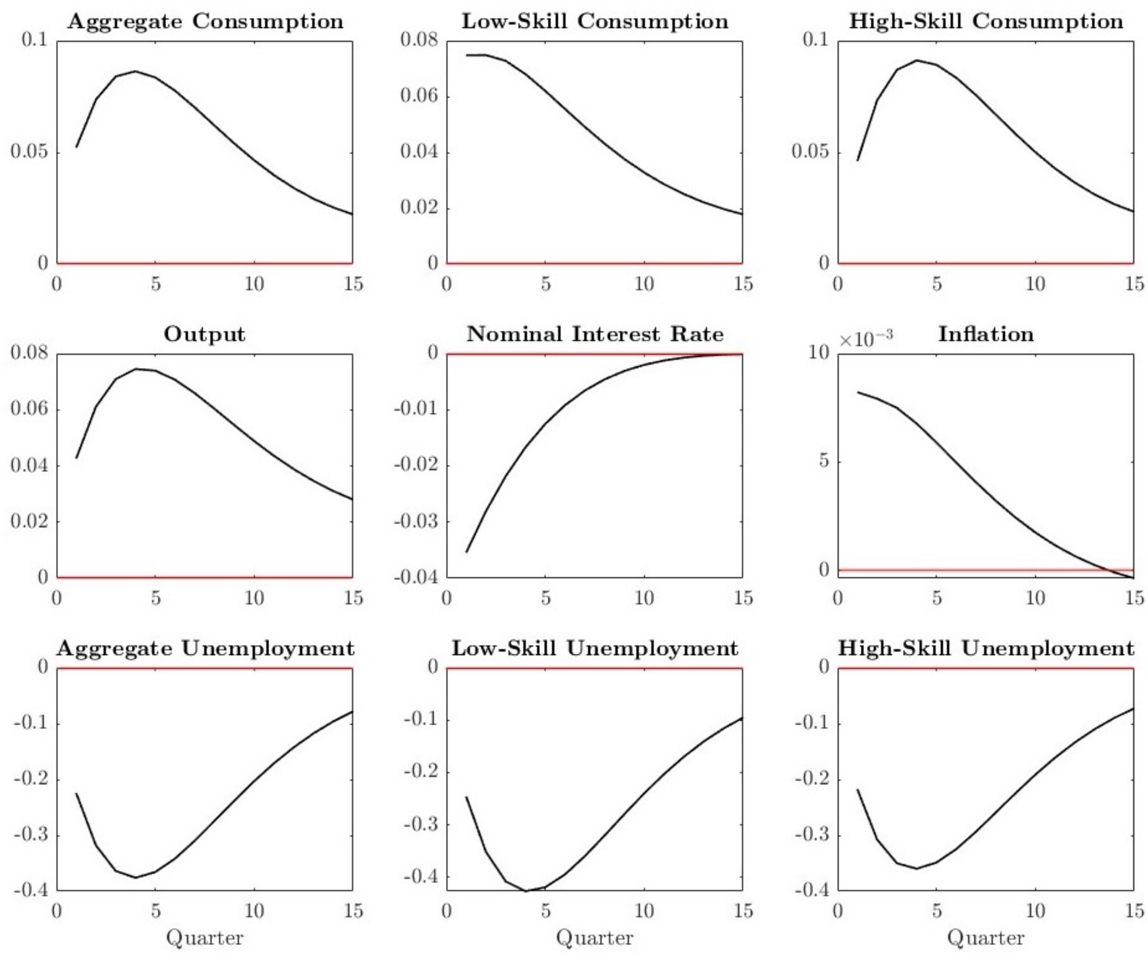

Figure 9. Impulse responses to a monetary policy shock (0.25 p.p.): y-axis for inflation and nominal interest rate in annual percentage points and for all other variables in percent.

Conversely, low-skill unemployment exhibits a more pronounced short-run decline, suggesting that low-skilled workers experience more sensitive response to labor demand fluctuations. This sharper response may reflect the relatively elastic nature of low-skilled labor demand, as firms are likely to adjust hiring practices for lower-cost, more flexible positions in response to improved economic conditions.

Aggregate consumption and unemployment follow similar patterns. Aggregate consumption rises initially, driven by increased demand across both skill groups, before eventually declining, reflecting the temporary effects of the expansionary policy.

E Determinacy And Indeterminacy Regions

E.1 Response to Aggregate Unemployment and Inflation

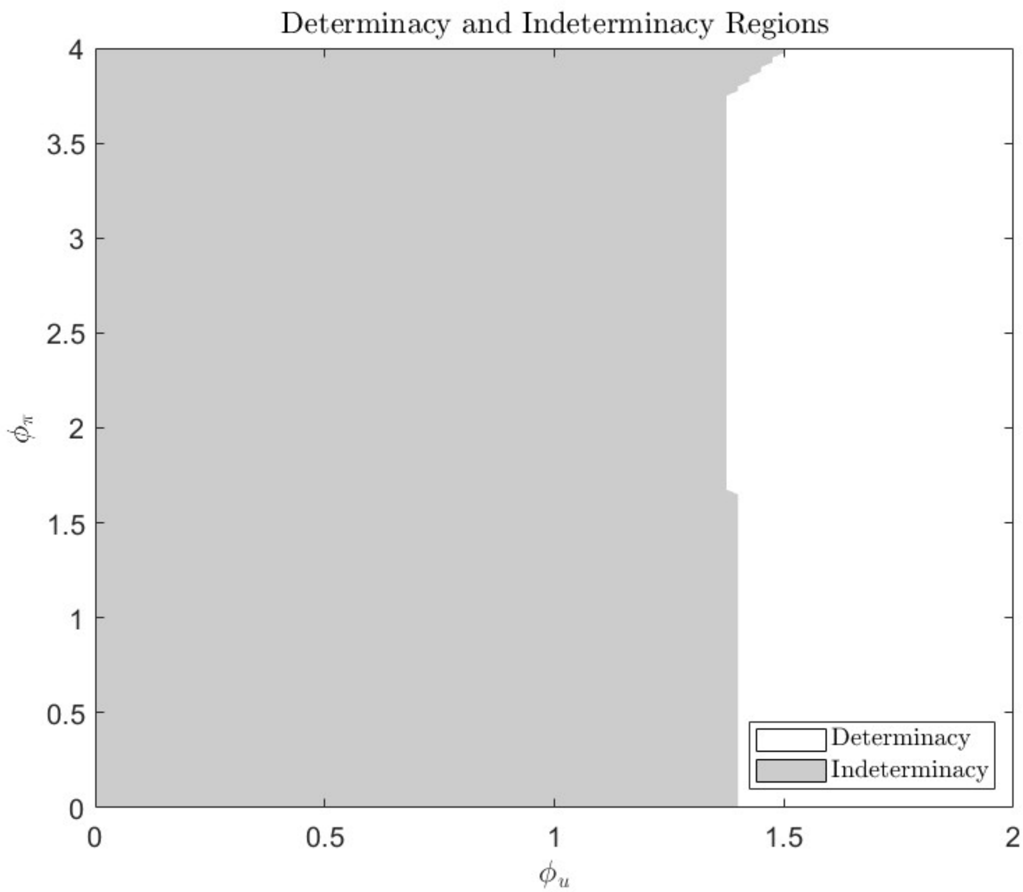

Figure 10. Determinacy versus indeterminacy regions for varying coefficients of aggregate unemployment and inflation in the policy rule.

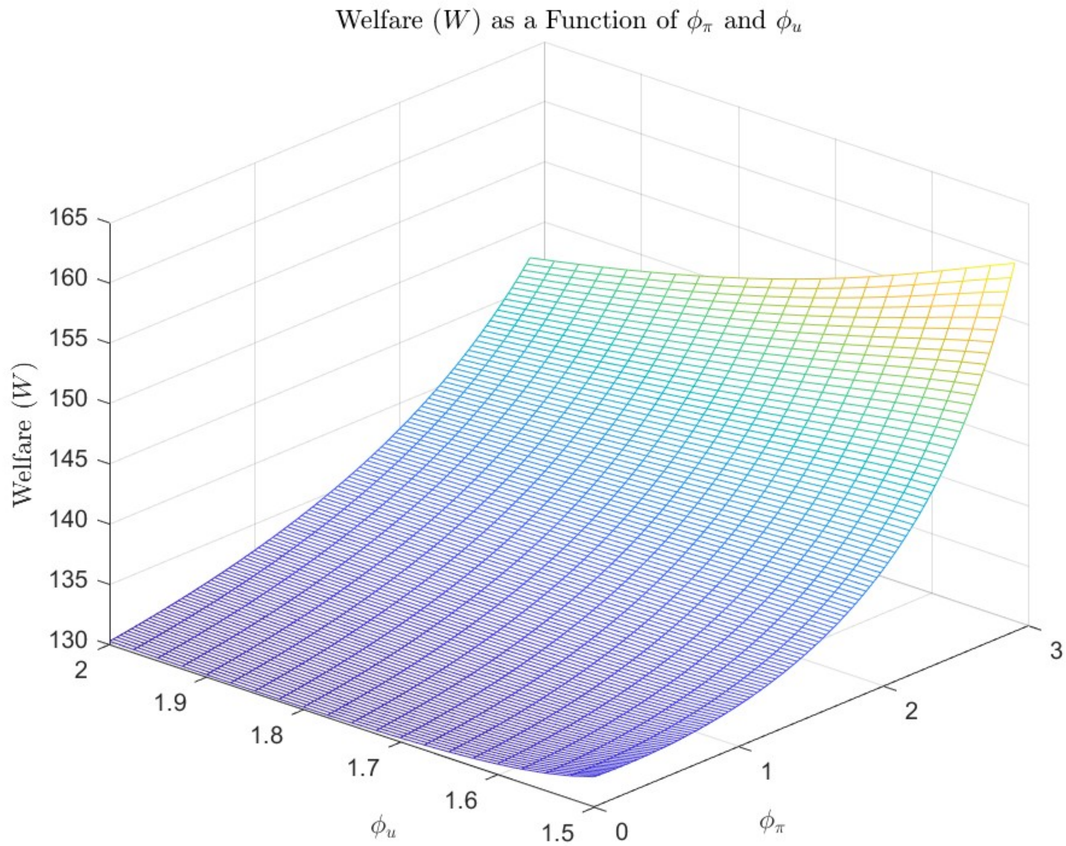

Figure 11. Welfare effects of adjusting the Taylor rule's response to aggregate unemployment and inflation ($\phi_r = 0$).

E.2 Response to Low- and High-Skill Unemployment with Inflation at Its Welfare-Maximizing Value

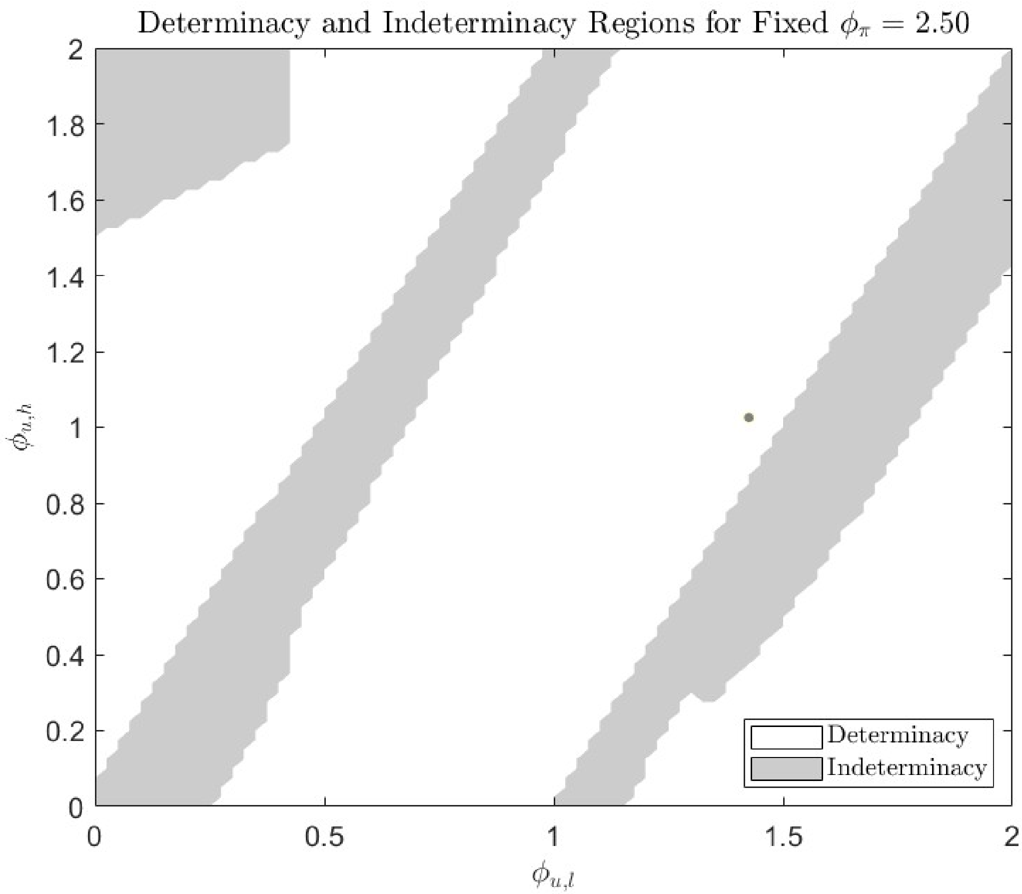

Figure 12. Determinacy versus indeterminacy regions for varying coefficients low- and high-skilled unemployment with he inflation coefficient fixed at its welfare-maximizing value.