A* Search Algorithm: Intelligent Pathfinding in AI

Search algorithms play a crucial role in Artificial Intelligence, especially in problems involving **pathfinding**, **planning**, and **navigation.** Among them, the **A*** (A-star) search algorithm is widely used because it efficiently finds the **shortest path** while intelligently guiding the search.

This blog covers:

- Introduction to A*
- How A* differs from Greedy Best-First and other algorithms
- How to implement A*
- Properties: optimality, completeness, time & space
- Role of heuristics in optimality
- A solved example using A*

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1. Introduction to A* Search

A* is a **best-first search algorithm** that finds the shortest path from a start node to a goal node using both:

- The **actual cost** from the start
- A **heuristic estimate** to the goal

It evaluates nodes using:

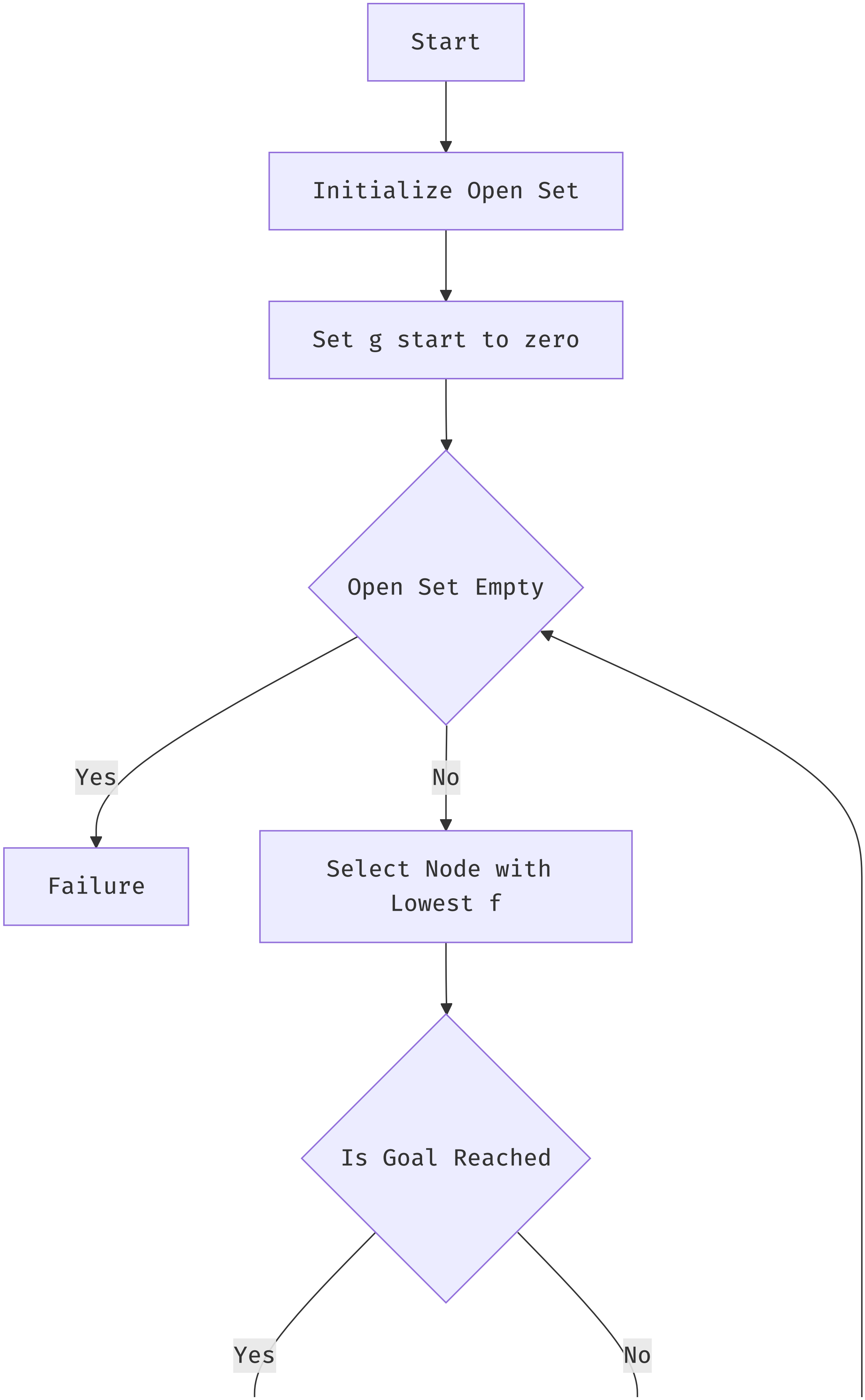
$$f(n) = g(n) + h(n)$$

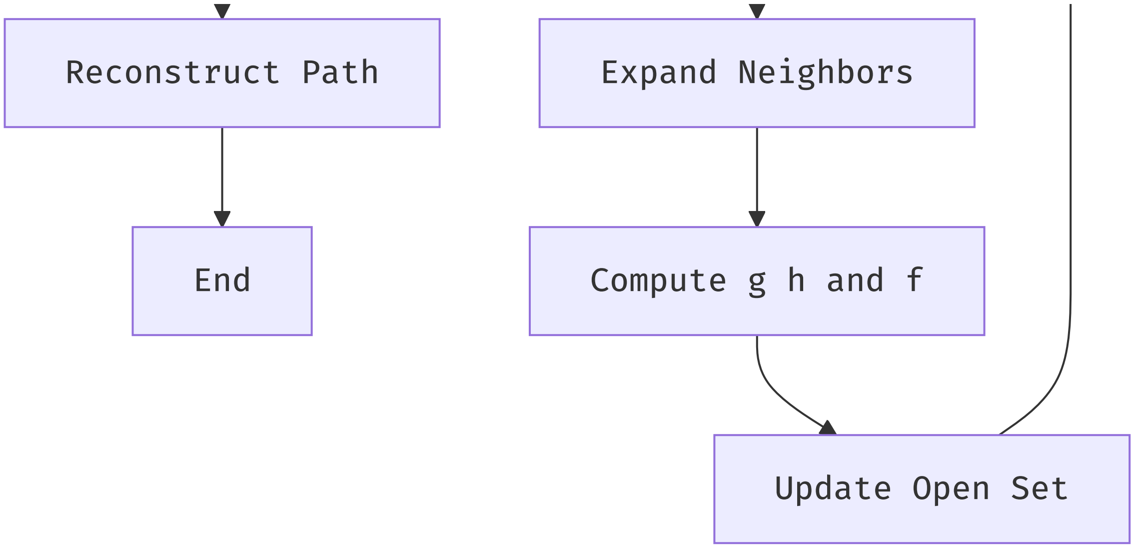
Where:

- **g(n)** = cost from start to node *n*
- **h(n)** = estimated cost from *n* to the goal
- **f(n)** = total estimated cost

By balancing real cost and estimated future cost, A* explores fewer nodes than uninformed algorithms like BFS or Dijkstra.

A* Search Flowchart


Start
Initialize Open Set
Set g start to zero
Open Set Empty
Yes
No
Failure
Select Node with Lowest f
Is Goal Reached
Yes
No



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2. How A* is Different from Greedy Best-First and Other Algorithms

Algorithm	Uses g(n)?	Uses h(n)?	Optimal?	Fast?
BFS	❌	❌	✅	❌
Dijkstra	✅	❌	✅	❌
Greedy Best-First	❌	✅	❌	✅
A*	✅	✅	✅	✅

Key Differences

- **Greedy Best-First Search** only uses `h(n)`
  → Fast, but not guaranteed to find the shortest path
- **Dijkstra's Algorithm** only uses `g(n)`
  → Always optimal, but explores many nodes
- **A*** uses both
  → Efficient *and* optimal (with the right heuristic)

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3. How to Write Code for A* Search

```
import heapq

def heuristic(a, b):
    # Manhattan distance
```

```python
    return abs(a[0] - b[0]) + abs(a[1] - b[1])

def a_star(grid, start, goal):
    rows, cols = len(grid), len(grid[0])

    open_set = []
    heapq.heappush(open_set, (0, start))

    came_from = {}
    g_score = {start: 0}

    while open_set:
        _, current = heapq.heappop(open_set)

        if current == goal:
            # Reconstruct path
            path = []
            while current in came_from:
                path.append(current)
                current = came_from[current]
            path.append(start)
            return path[::-1]

        x, y = current
        neighbors = [(x+1,y), (x-1,y), (x,y+1), (x,y-1)]

        for nx, ny in neighbors:
            if 0 <= nx < rows and 0 <= ny < cols and grid[nx][ny] == 0:
                neighbor = (nx, ny)
                tentative_g = g_score[current] + 1

                if neighbor not in g_score or tentative_g < g_score[neighbor]:
                    g_score[neighbor] = tentative_g
                    f = tentative_g + heuristic(neighbor, goal)
                    heapq.heappush(open_set, (f, neighbor))
                    came_from[neighbor] = current

    return None
```

4. Properties of A*

Optimal

A* finds the **shortest path** if the heuristic used is **admissible** (it never overestimates the true cost).

Complete

A* is **complete**, meaning it will always find a solution if:

- A path exists
- All step costs are positive

Time Complexity

In the worst case:

$$O(b^d)$$

Where:

- **b** = branching factor
- **d** = depth of the optimal solution

Space Complexity

$$O(b^d)$$

A* stores all generated nodes in memory, which can be expensive for large problems.

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5. Heuristics and Optimality

What is a Heuristic?

A heuristic is a function that estimates the cost to reach the goal:

$$h(n) \approx \text{distance to goal}$$

It helps guide the search efficiently.

Admissible Heuristic

A heuristic is **admissible** if:

$$h(n) \leq h^*(n)$$

Where:

- (h^*(n)) = true cost to the goal

This guarantees **optimality**.

Consistent Heuristic

A heuristic is **consistent** if:

$$h(n) \leq c(n, n') + h(n')$$

This ensures:

- No re-expansion of nodes
- Better performance

Types of Heuristics

Heuristic	Use Case
Manhattan Distance	Grid movement (4-direction)
Euclidean Distance	Continuous space
Diagonal Distance	8-direction grids
Zero Heuristic	Turns A* into Dijkstra

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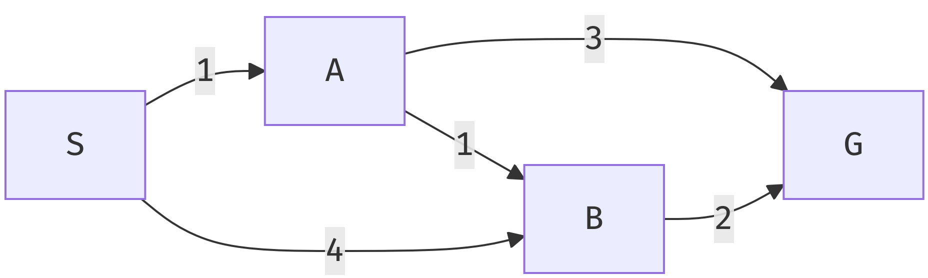
6. Example Problem Using A*

Problem

Find the shortest path from **S** to **G**.



Heuristic Values

Node	h(n)
S	5
A	3
B	2

Node	h(n)
G	0

Step 1: Expanding the Start Node

From **S**, we can go to:

- **A** with cost = 1
- **B** with cost = 4

Calculate the evaluation function:

$$f(A) = g(A) + h(A) = 1 + 3 = 4$$

$$f(B) = g(B) + h(B) = 4 + 2 = 6$$

Since **A** has the smaller (f(n)), A* chooses **A** first.

Step 2: Expanding Node A

From **A**, we can go to:

- **G** with additional cost = 3

Total cost to reach **G**:

$$g(G) = 1 + 3 = 4$$

$$f(G) = 4 + 0 = 4$$

Final Path

The shortest path found by A* is:

S → A → G

Total path cost = **4**

This is the **optimal solution.**

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7. Conclusion

A* is one of the most powerful search algorithms because:

- It is **optimal**
- It is **complete**
- It uses **heuristics** to guide the search
- It is efficient in practice

With a good heuristic, A* provides both **speed** and **accuracy**, making it ideal for pathfinding and AI planning problems.