Welding Technology
ME692

Quiz = 10%

→ No formula sheet

→ Calculator is allowed

Quiz 1

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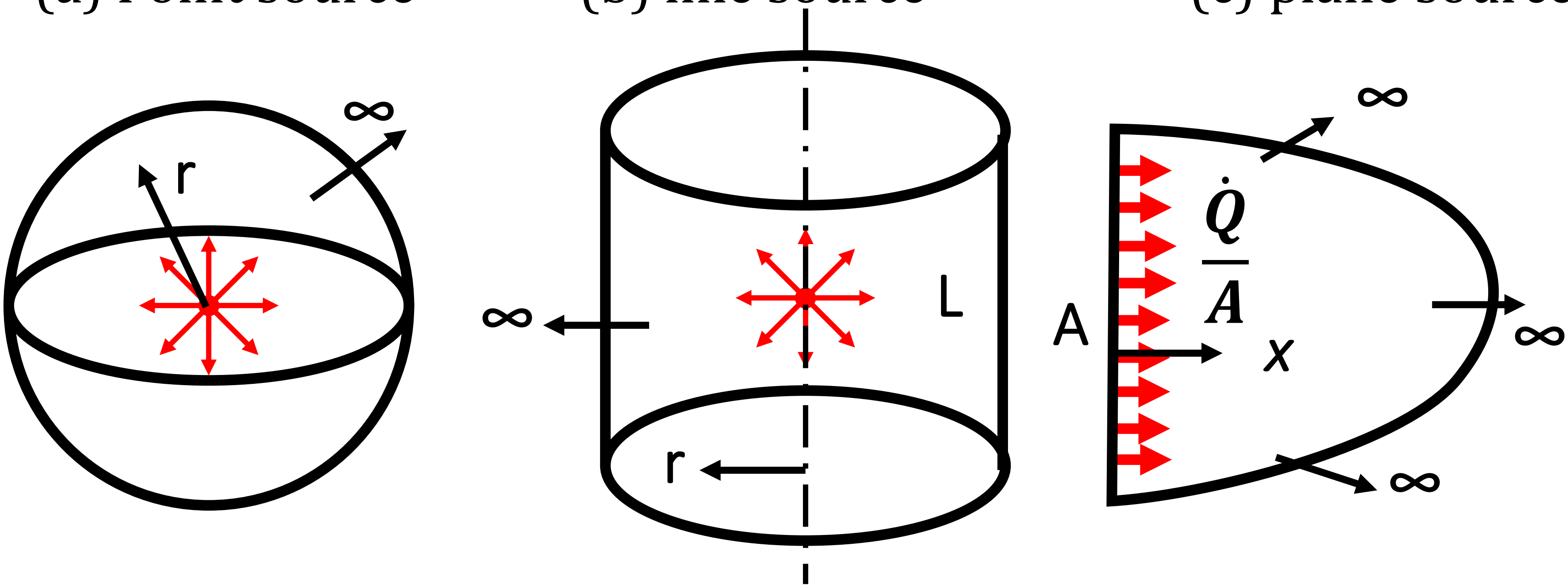
Source/Sink in Welding

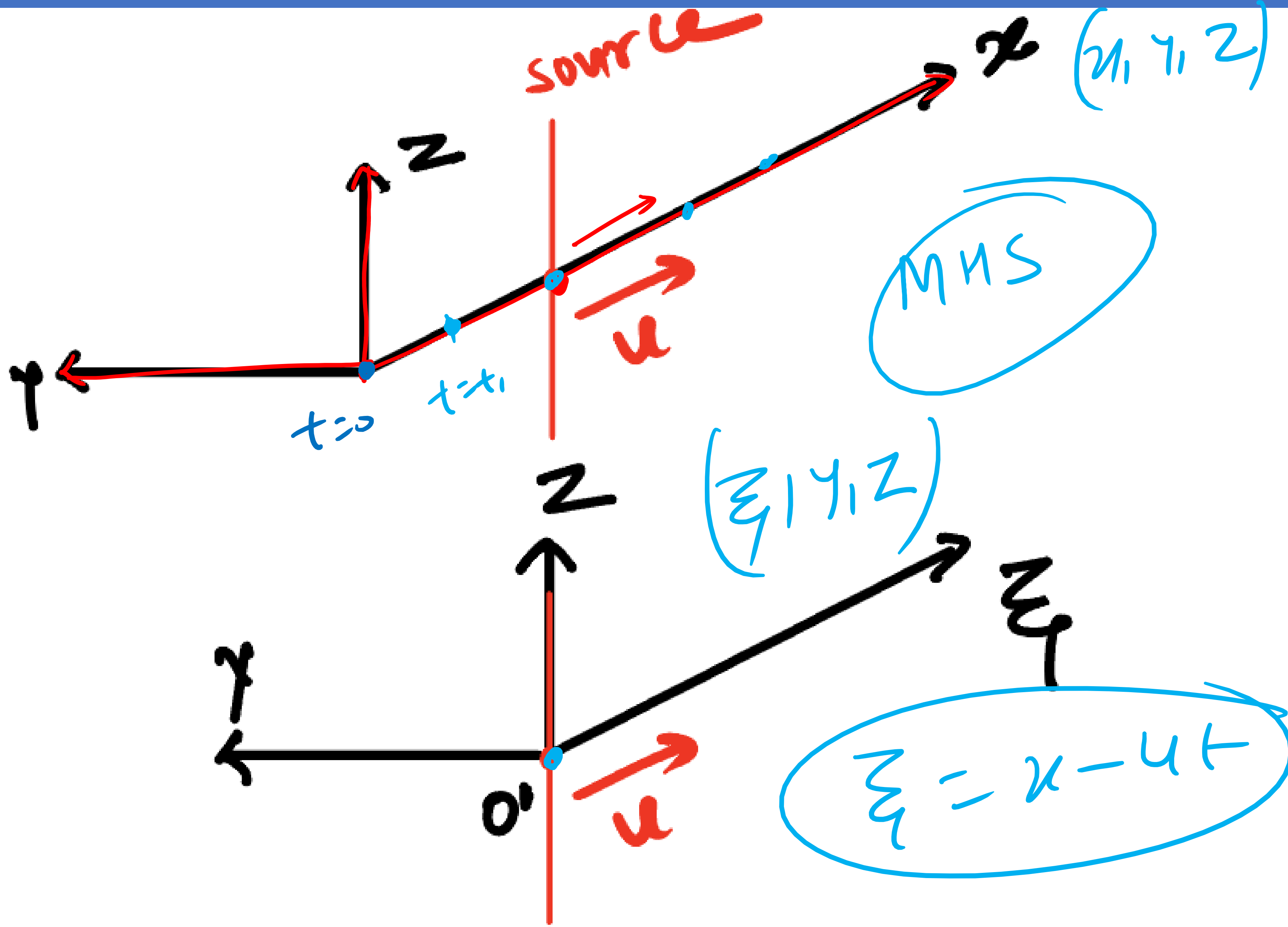
Heat transfer with heat sources or sinks

Sources of constant heat production rate:

(a) Point source (b) line source (c) plane source



Moving Heat Sources



Moving Heat Sources

(I) Assumptions

all physical properties are constant, independent of temp.

(II) uniform velocity of source. $= u$

(III) Rate heat input $=$ constant

(IV) $\alpha = \frac{1}{2\lambda}$ or $2\lambda = \frac{1}{\alpha}$

Thermal diffusivity

Moving Heat Sources

G.E.

$T(x, y, z, t)$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} = 2\lambda \frac{\partial T}{\partial t} \quad (1)$$

$\xi = x - ut$

$T(\xi, y, z, t)$

$$\frac{\partial T(\xi, y, z, t)}{\partial t} = \frac{\partial T}{\partial \xi} * \frac{\partial \xi}{\partial t} + \frac{\partial T}{\partial y}\cancel{\frac{\partial y}{\partial t}} + \frac{\partial T}{\partial z}\cancel{\frac{\partial z}{\partial t}} + \frac{\partial T}{\partial t}\cancel{\frac{\partial t}{\partial t}}$$

$\frac{\partial \xi}{\partial t} \to -u$, $\frac{\partial y}{\partial t} \to 0$, $\frac{\partial z}{\partial t} \to 0$, $\frac{\partial t}{\partial t} \to 1$

$$\frac{\partial \xi}{\partial t} = -u$$

$$= -u\frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial t} \quad (2)$$

Moving Heat Sources

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \xi_1}\cancel{\frac{\partial \xi_1}{\partial x}}^{\,1} + \frac{\partial T}{\partial t}\cancel{\frac{\partial t}{\partial x}}^{\,0} = \frac{\partial T}{\partial \xi_1} \quad \text{— 1A}$$

$\xi_1 = x - ut$

$\frac{\partial \xi_1}{\partial x} = 1$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial^2 T}{\partial \xi_1^2} \quad \text{— 2B}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial y^2} \quad \text{— 2}$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{\partial^2 T}{\partial z^2} \quad \text{— 2C}$$

Moving Heat Sources

$T(\xi, y, z, t)$

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 2\lambda \left\{ -u \frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial t} \right\} \quad \text{--- } (2)$$

Quasi steady state :— $\frac{\partial T}{\partial t} = 0$

$$\boxed{\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -2\lambda u \frac{\partial T}{\partial \xi} \quad \text{--- } (3)}$$

$$T \propto e^{-\lambda u \xi} \, \phi(\xi, y, z)$$

Moving Heat Sources

assuming Heat $T = T_0 + e^{-\lambda u \xi} \phi(\xi, y, z)$

$$\frac{\partial T}{\partial \xi} = e^{-\lambda u \xi} \frac{\partial \phi}{\partial \xi} + \phi e^{-\lambda u \xi} (-\lambda u)$$

$$\frac{\partial^2 T}{\partial \xi^2} = e^{-\lambda u \xi} \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} e^{-\lambda u \xi} (-\lambda u)$$

$$+ (-\lambda u) \left\{ e^{-\lambda u \xi} \frac{\partial \phi}{\partial \xi} + \phi e^{-\lambda u \xi} (-\lambda u) \right\}$$

$$= e^{-\lambda u \xi} \frac{\partial^2 \phi}{\partial \xi^2} - 2 \lambda u \frac{\partial \phi}{\partial \xi} e^{-\lambda u \xi} + (\lambda u)^2 \phi e^{-\lambda u \xi}$$

Moving Heat Sources

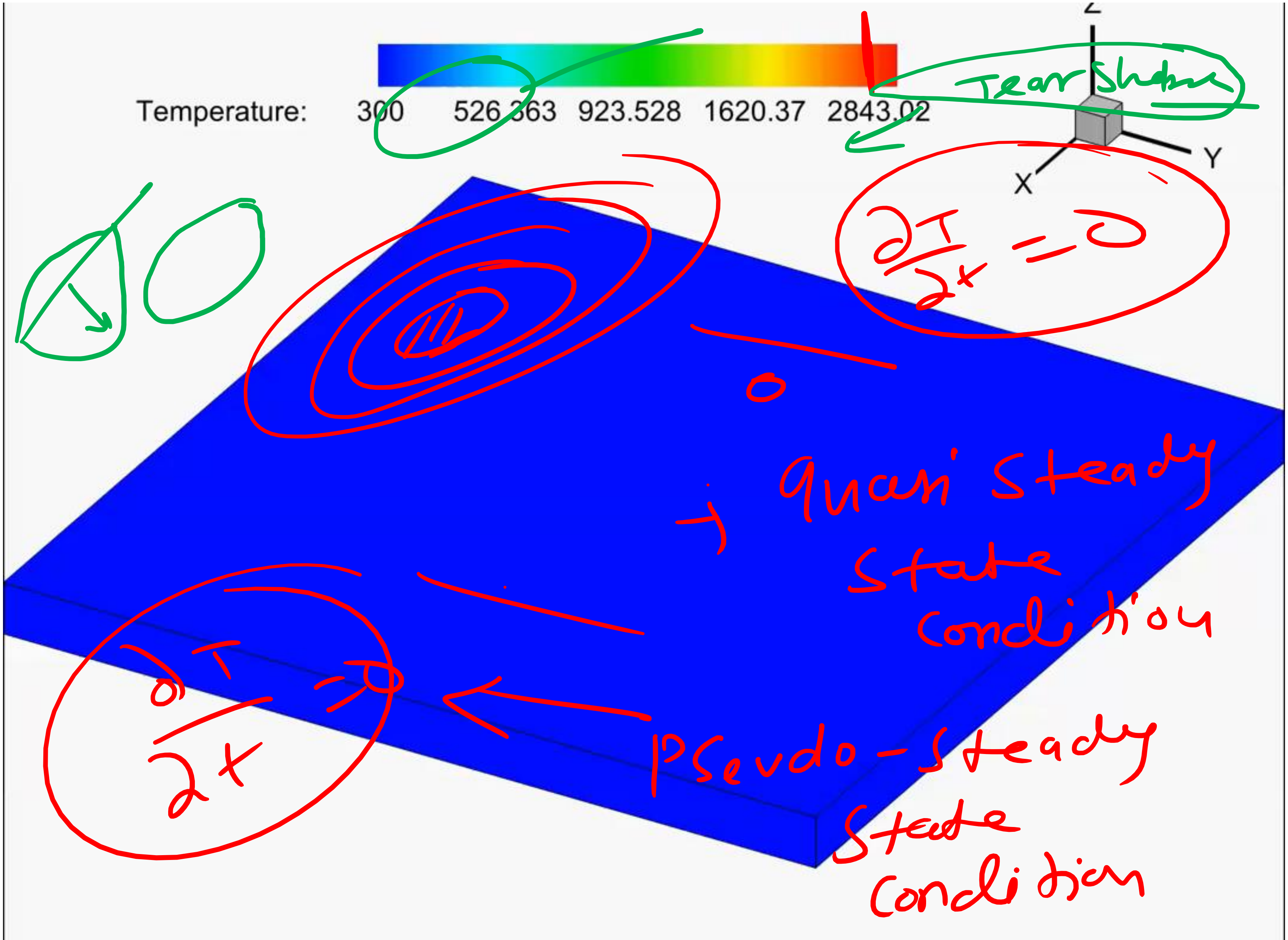
$$\frac{\partial^2 T}{\partial y^2} = e^{-\lambda u \xi} \frac{\partial^2 \phi}{\partial y^2}$$

$$\frac{\partial^2 T}{\partial z^2} = e^{-\lambda u \xi} \frac{\partial^2 \phi}{\partial z^2}$$

$$-2\lambda u \underline{\frac{\partial T}{\partial \xi}} = -2\lambda u \left\{ e^{-\lambda u \xi} \frac{\partial \phi}{\partial \xi} + \phi e^{-\lambda u \xi} \right\}$$

$$= -2\lambda u \, e^{-\lambda u \xi} \frac{\partial \phi}{\partial \xi} + (-2\lambda u) \phi e^{-\lambda u \xi} (-\lambda u)$$

Moving Heat Sources



Moving Heat Sources

$$e^{-\lambda u \xi_1}\frac{\partial^2\phi}{\partial\xi_1^2} \cancel{- 2\lambda u \frac{\partial\phi}{\partial\xi_1} e^{-\lambda u\xi_1}} \cancel{+ (\lambda u)^2 \phi e^{-\lambda u \xi_1}}$$

$$+ e^{-\lambda u\xi_1}\frac{\partial^2\phi}{\partial y^2} + e^{-\lambda u\xi_1}\frac{\partial^2\phi}{\partial z^2} = \cancel{-2\lambda u e^{-\lambda u \xi_1}\frac{\partial\phi}{\partial\xi_1}} \cancel{- 2\lambda u\phi e^{-\lambda u\xi_1}(-\lambda u)}$$

$$\boxed{T = T_0 e^{-\lambda u \xi_1}\phi}$$

$$\frac{\partial^2\phi}{\partial\xi_1^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} = (\lambda u)^2\phi \quad \text{—— } 3$$

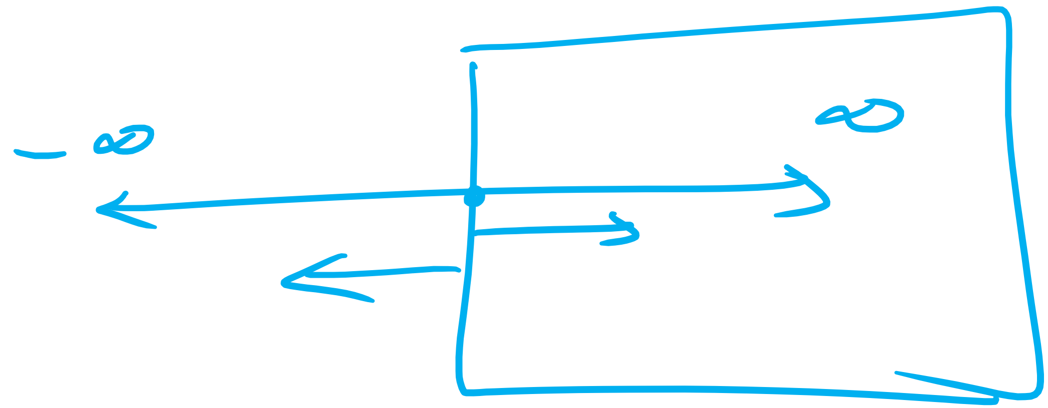
Moving Heat Sources

$$\boxed{\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-(\lambda v)^2\phi=0} \quad (4)$$

$$\boxed{\nabla^2\phi-(\lambda v)^2\phi=0} \quad (5)$$

Moving Heat Sources

Case 1:- for infinite or semi infinite Body with linear flow of heat:-

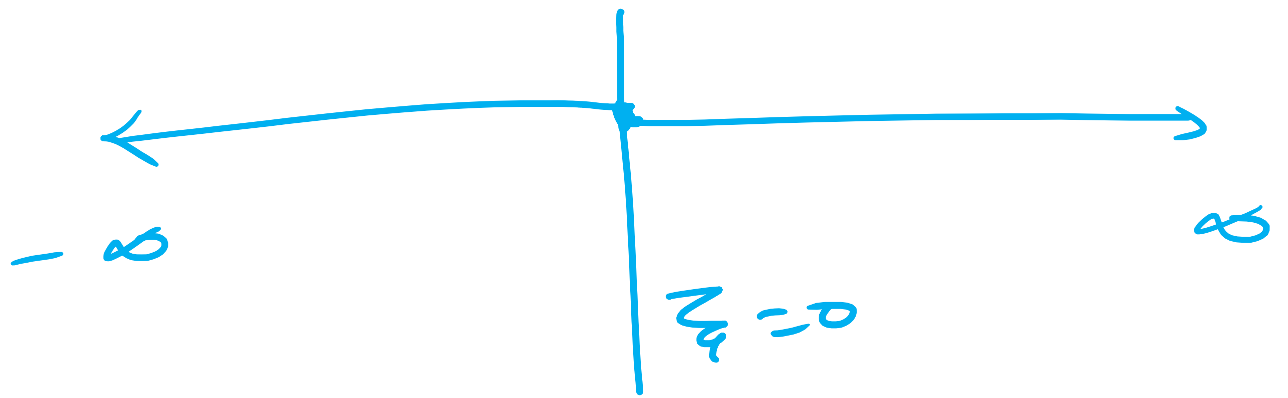


$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0$$

$$\boxed{\frac{\partial^2 \phi}{\partial \xi^2} - (\lambda v)^2 \phi = 0} \quad \text{--- (7)}$$

B.C. $\xi \to -\infty, +\infty$, $\frac{\partial T}{\partial \xi} \to 0$ --- (8)

Moving Heat Sources



$\xi = 0$, Temp. continuity should maintain

$$\xi \to 0, \quad \boxed{-k\frac{\partial T}{\partial \xi} = q''}$$

Moving Heat Sources

$$\frac{\partial^2 \phi}{\partial \xi_1^2} - (\lambda u)^2 \phi = 0$$

$$\boxed{(D^2 - a^2) y = 0}$$

$$\boxed{y = c_1 e^{-ax} + c_2 e^{ax}}$$

$a = \lambda u$

$$\boxed{\phi = c_1 e^{-\lambda u \xi_1} + c_2 e^{\lambda u \xi_1}}$$

$$T = T_0 + e^{-\lambda u \xi_1} \phi(\xi_1, y, z)$$

$$\boxed{T = T_0 + c_1 e^{-2\lambda u \xi_1} + c_2}$$

Moving Heat Sources

$$\boxed{T = T_0 + C_1 e^{-2\lambda u \xi_1} + C_2}$$

$-\infty < \xi_1 < \infty$

$\xi_1 < 0$

$\frac{\partial T_2}{\partial \xi_1} \to 0$

$\xi_1 \to \infty$

$\xi_1 > 0$

$\frac{\partial T}{\partial \xi_1} = 0$

$\xi_1 \to \infty$

$C_2 = 0$

$\frac{\partial T}{\partial \xi_1} = C_1 (-2\lambda u) e^{-2\lambda u \xi_1}$

$C_1 = 0$

Moving Heat Sources

$T_0 + C_1 e^{-2.14\xi} + C_2$

$\xi < 0$: $C_1 = 0$

$\xi > 0$: C_2^+

$T = T_0 + C_2^-$

$T = T_0 + C_1^+ e^{-2.14\xi}$

at $\xi = 0$

$T - T_0 = C_2^- \Leftarrow\Rightarrow T - T_0 = C_1^+ e^{-2.14\xi}$

$\xi = 0$

$T - T_0 = C_1^+$

$C_1^+ = C_2^-$

Moving Heat Sources

$\xi > 0$

RHL

$$T = T_0 + C_1^+ e^{-2\lambda\mu\xi} \cancel{+ C_2}$$

$$\frac{\partial T}{\partial \xi} = -C_1^+ (2\lambda\mu) e^{-2\lambda\mu\xi}$$

$$-k \left.\frac{\partial T}{\partial \xi}\right|_{\xi=0} = q''$$

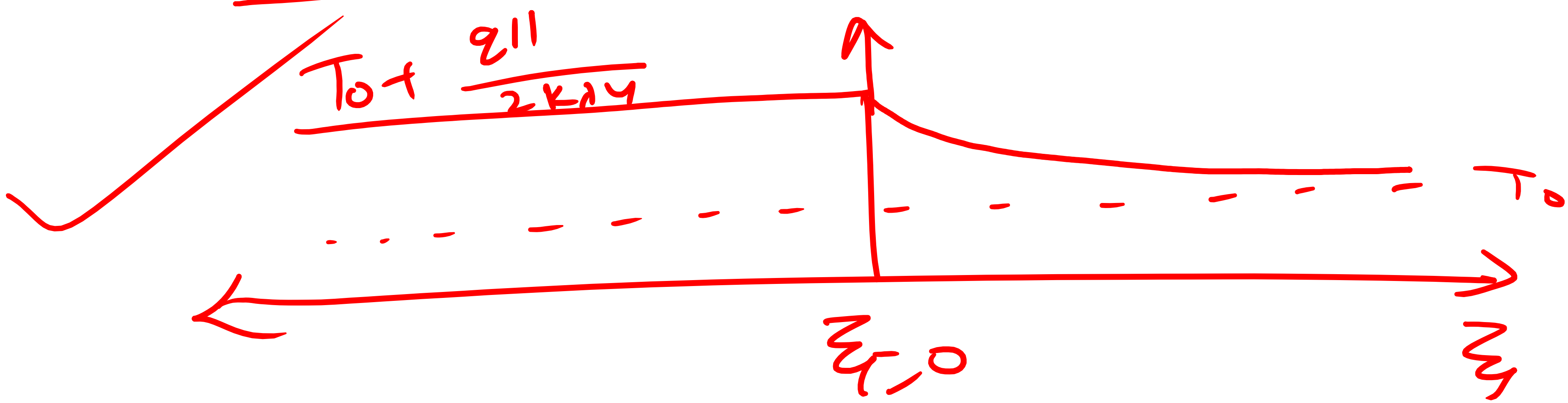
$$\cancel{k}\, C_1 (2\lambda\mu) = q''$$

$$\boxed{C_1^+ = \frac{q''}{2\lambda\mu k}} = C_2^-$$

Moving Heat Sources

$$T = T_0 + \frac{q''}{2\lambda K u}, \quad \xi < 0$$

$$T = T_0 + \frac{q''}{2\lambda K u} e^{-\lambda u \xi}, \quad \xi > 0$$



Moving Heat Sources

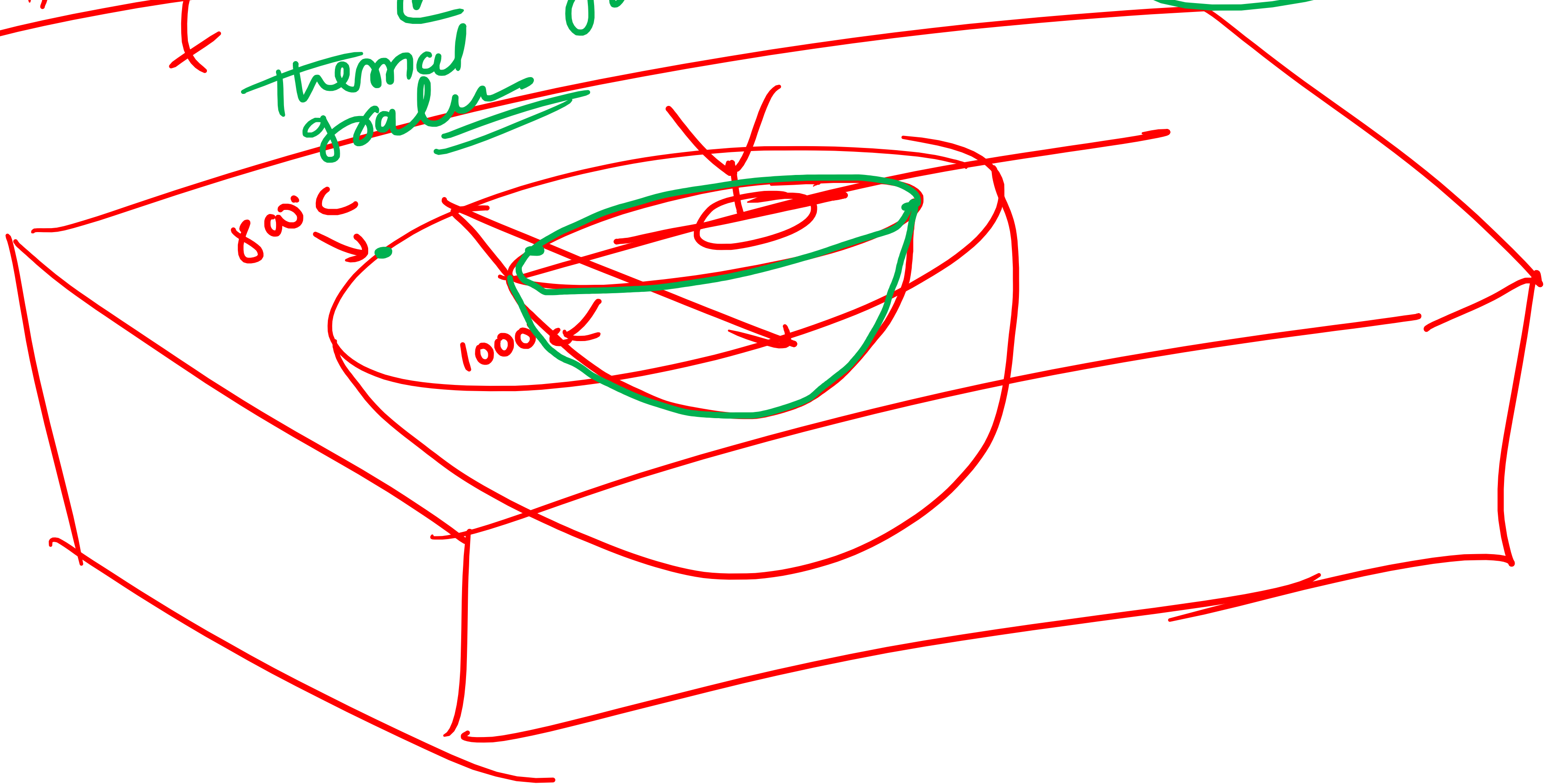
$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial x} \times \frac{\partial x}{\partial t}$$

thermal gradient

growth rate

$T_m = 1000°C$

$$q'' = -k\frac{dT}{dx}$$



Moving Heat Sources

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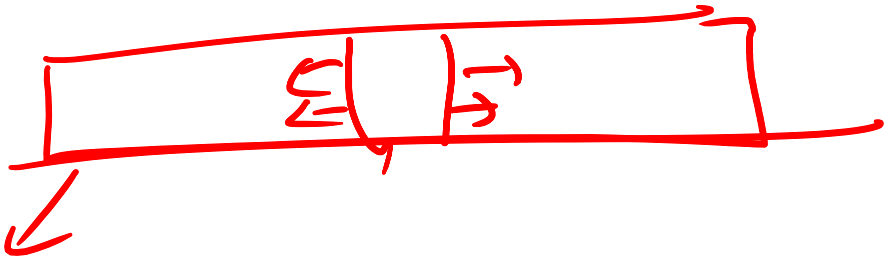
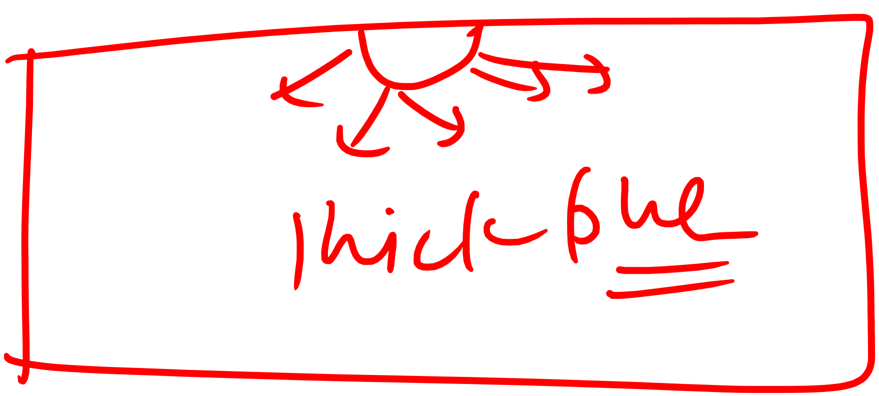
Moving Heat Sources-2D

$$\# \quad \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = (\lambda y)^2 \phi$$

cylindrical co-ord

& thin plate welding





Moving Heat Sources-2D

$$\frac{\partial^2 \phi}{\partial \xi_1^2} + \frac{\partial^2 \phi}{\partial y^2} = (\lambda y)^2 \phi$$

$\cancel{x}$

$\xi_1 = x - ut$

$$\frac{\partial T}{\partial \xi_1} \to 0, \quad \xi_1 \to \pm\infty$$

$$\frac{\partial T}{\partial y} \to 0, \quad y \to \mp\infty$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) = (\lambda y)^2 \phi$$

$r = \sqrt{\xi_1^2 + y^2}$

Moving Heat Sources-2D

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) - \left(\frac{u}{2\alpha}\right)^2\phi = 0$$

$$\boxed{0 < r < \infty}$$

$\frac{\partial T}{\partial r} \to 0$, $r \to \infty$

at $r \to 0$, $\boxed{-2\pi r k \frac{\partial T}{\partial r} = q'}$

$\frac{\text{watt}}{\text{length}}$

m

Bessel fn: —

2nd kind of zero order

Moving Heat Sources-2D

$$T \propto K_0(\lambda u r) \propto -\ln(\lambda u r)$$

K_0 → 2nd kind zero order

$$T = C\,K_0(\lambda u r)$$

$$-2\pi r k \frac{\partial T}{\partial r} = q'$$

$$C = \frac{-q''}{2\pi K}$$

Moving Heat Sources-2D

$$\boxed{T = T_0 + \frac{q'}{2\pi k} e^{-\lambda u \xi} K_0(\lambda u r)}$$

Moving Heat Sources-2D

Moving Heat Sources-3D

$$\frac{\partial^2\phi}{\partial\xi^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = (\lambda v)^2\phi$$

$$\frac{\partial T}{\partial \xi} \to 0 \quad , \quad \xi \to \pm\infty$$

$$\frac{\partial T}{\partial y} = 0 \quad , \quad y \to \pm\infty$$

$$\frac{\partial T}{\partial z} = 0 \quad , \quad z \to \pm\infty$$

Moving Heat Sources-3D

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial \phi}{\partial R}\right) = (\lambda u)^2 \phi$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$-4\pi R^2 k \frac{\partial T}{\partial R} = q, \quad R \to 0$$

$$R\phi = c_1 e^{-\lambda u R}$$

$$\phi = \frac{c_1}{R} e^{-\lambda u R}$$

Moving Heat Sources-3D

$$\boxed{T = T_0 + \frac{q}{2\pi k} e^{-\lambda u \xi} \frac{e^{-\lambda u R}}{R}}$$

$2\lambda = \frac{1}{\alpha}$

$\alpha = \frac{1}{2\lambda}$

$\lambda = \frac{1}{2\alpha}$

$$\boxed{T = T_0 + \frac{q}{2\pi k} e^{-\frac{u\xi}{2\alpha}} \frac{e^{-\frac{uR}{2\alpha}}}{R}}$$

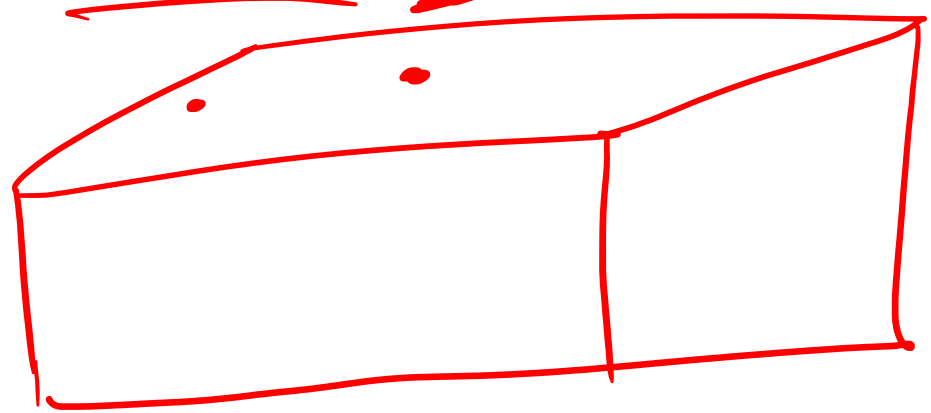
Moving Heat Sources-3D

Moving Heat Sources-3D

Numerical: Thick plate

Determine the temperature of a point (-40, 20, 0) mm w.r.t the arc as an origin for depositing a single weld bead by MIG using 200A and 20V at a welding speed of 100mm/min on a wide steel plate assuming that semi-infinite and thicker plate.

efficiency: 0.7, T_0=20°C, k=64.167J/m.s °C,
α=1.74e-5 m^2/s



Moving Heat Sources: Thick plate

$$T = T_0 + \frac{q}{2\pi k} e^{-\frac{u\xi}{2\alpha}} \frac{e^{-\frac{uR}{2\alpha}}}{R}$$

$T(-\overset{mm}{40}, \overset{mm}{20}, \overset{mm}{0})$

$P(\xi, y, z)$

$$\frac{q}{2\pi k} e^{-\lambda \xi u} \frac{e^{-uR\lambda}}{R} =$$

$$R = \sqrt{\xi^2 + y^2 + z^2}$$

$$= \sqrt{40^2 + 20^2 + 0}$$

$$= 0.04472 \text{ m}$$

$q_0 = \eta I V$

$= 0.7 \times 200 \times 20$

$= 2800 \text{ J/s} = 2800 \text{ watt}$

$V = \frac{100 \text{mm}}{\text{min}} = 0.00167 \text{ m/s},$

$$\lambda = \frac{1}{2\alpha} = 2800 \text{ s/m}^2$$

Moving Heat Sources: Thick plate

$$-\lambda 4\xi = -28800 \times 0.00167 \times \frac{-40}{1000}$$

$$= 1.92$$

$$e^{-\lambda 4\xi} = 6.82$$

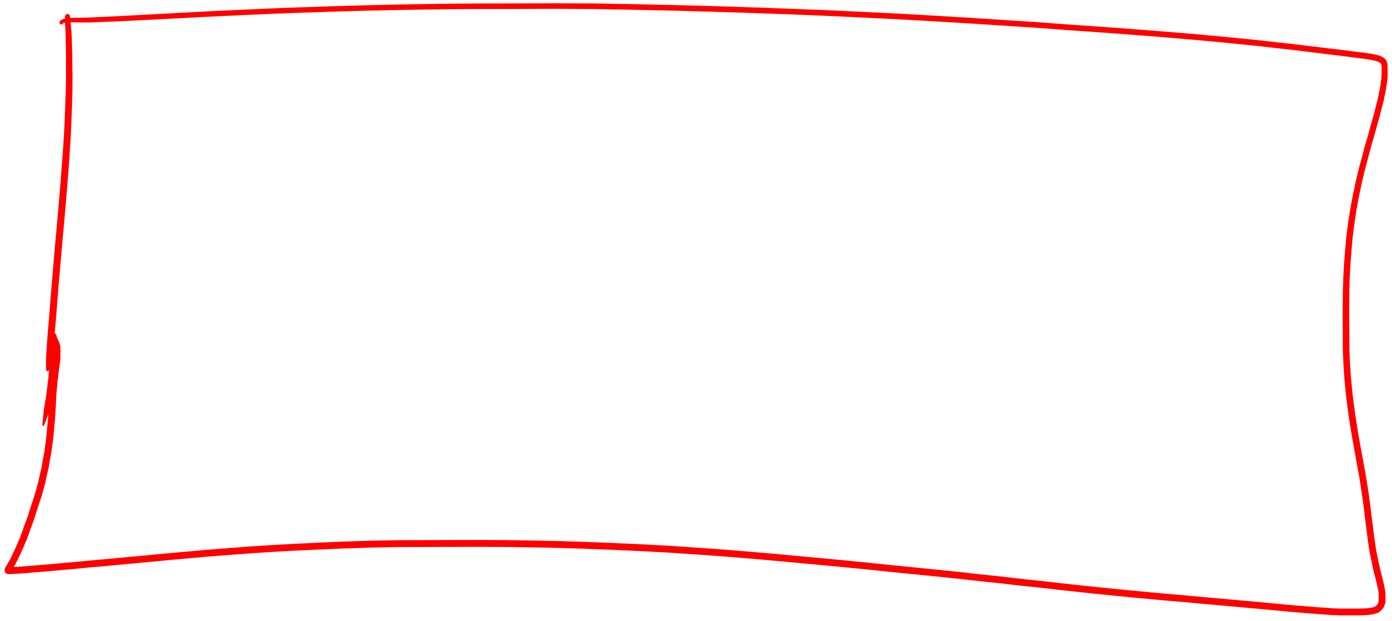
$$\frac{e^{-\lambda 4R}}{R} = \frac{e^{-28800 \times 0.00167 \times 0.0472}}{0.0472} = 12.77$$

$$T = T_0 \pm \frac{2800}{2 \times 3.14 \times 64.16} \times 12.77 \times 6.82$$

$$T - T_0 = \cancel{6.4\ldots} \; 605.82$$

$$\boxed{T - T_0 = 605.82\,^\circ C}$$

Moving Heat Sources: Thick plate



Moving Heat Sources: Thick plate

Moving Heat Sources: Thick plate

$$T - T_0 = \frac{q_0}{2\pi K} e^{-\frac{V x'}{2\alpha}} \frac{e^{-\frac{VR}{2\alpha}}}{R} \quad \text{--- (1)}$$

$$x' = x_0 - Vt$$

$$T - T_0 = \frac{q_0}{2\pi K} \frac{e^{-\frac{V}{2\alpha}\{x' + R\}}}{R} \quad \text{--- (1.1)}$$

Dimension less Radius factor

$$\sigma_3 = \frac{VR}{2\alpha} \quad \text{--- (1.2)}$$

Dimension less time factor

$$\tau = \frac{V^2 t}{2\alpha} \quad \text{--- (1.3)}$$

Dimension less temp $\theta = \frac{T - T_0}{T_c - T_0} =$

Moving Heat Sources: Thick plate

Dimension les x- coordinate

$$\xi = \frac{v x'}{2\alpha} \quad \text{--- } (1.4) \qquad x' = x_0 - vt$$

Dimension les y- coor.

$$\psi = \frac{vy}{2\alpha} \quad \text{--- } (1.5)$$

Dimensionles z- coor.

zeta, $$\zeta = \frac{vz}{2\alpha} \quad \text{--- } (1.6)$$

Dimension les Process Parameter.

$$n_3 = \frac{q_0 V}{4\pi\alpha^2 \rho c (T_c - T_0)} = \frac{q_0 V}{4\pi\alpha^2 (H_c - H_0)} \quad \text{--- } (1.7)$$

Moving Heat Sources: Thick plate

$$T - T_0 = \frac{q_0}{2\pi K} \frac{1}{R} e^{-\frac{V}{2\alpha}\{x' + R\}}$$

$$\sigma_3 = \frac{VR}{2\alpha}$$

$$\xi_1 = \frac{Vx'}{2\alpha}$$

θ

$$\frac{T - T_0}{T_c - T_0} = \frac{q_0}{2\pi K} \frac{1}{R} \frac{1}{T_c - T_0} e^{-\sigma_3 - \xi_1}$$

$$\theta = \frac{q_0}{2\pi K \cancel{R} (T_c - T_0) \frac{VR}{2\alpha}} \times \frac{2\alpha}{V\cancel{R}} e^{-\sigma_3 - \xi_1}$$

$$= \frac{q_0 \times V}{\sigma_3 \; 4\pi K \alpha \frac{(T_c - T_0)\rho c}{\rho c}} e^{-\sigma_3 - \xi_1}$$

$H_c - H_0$

$$= \frac{q_0 V}{\sigma_3 \; 4\pi K \alpha \frac{(H_c - H_0)}{\rho c}} e^{-\sigma_3 - \xi_1}$$

Moving Heat Sources: Thick plate

$$\theta = \frac{q_0 V}{\sigma_3 \times 4\pi K \alpha \frac{(H_c - H_0)}{\rho C}} e^{-\sigma_3 - \xi_1}$$

$$\frac{K}{\rho C_p} = \alpha$$

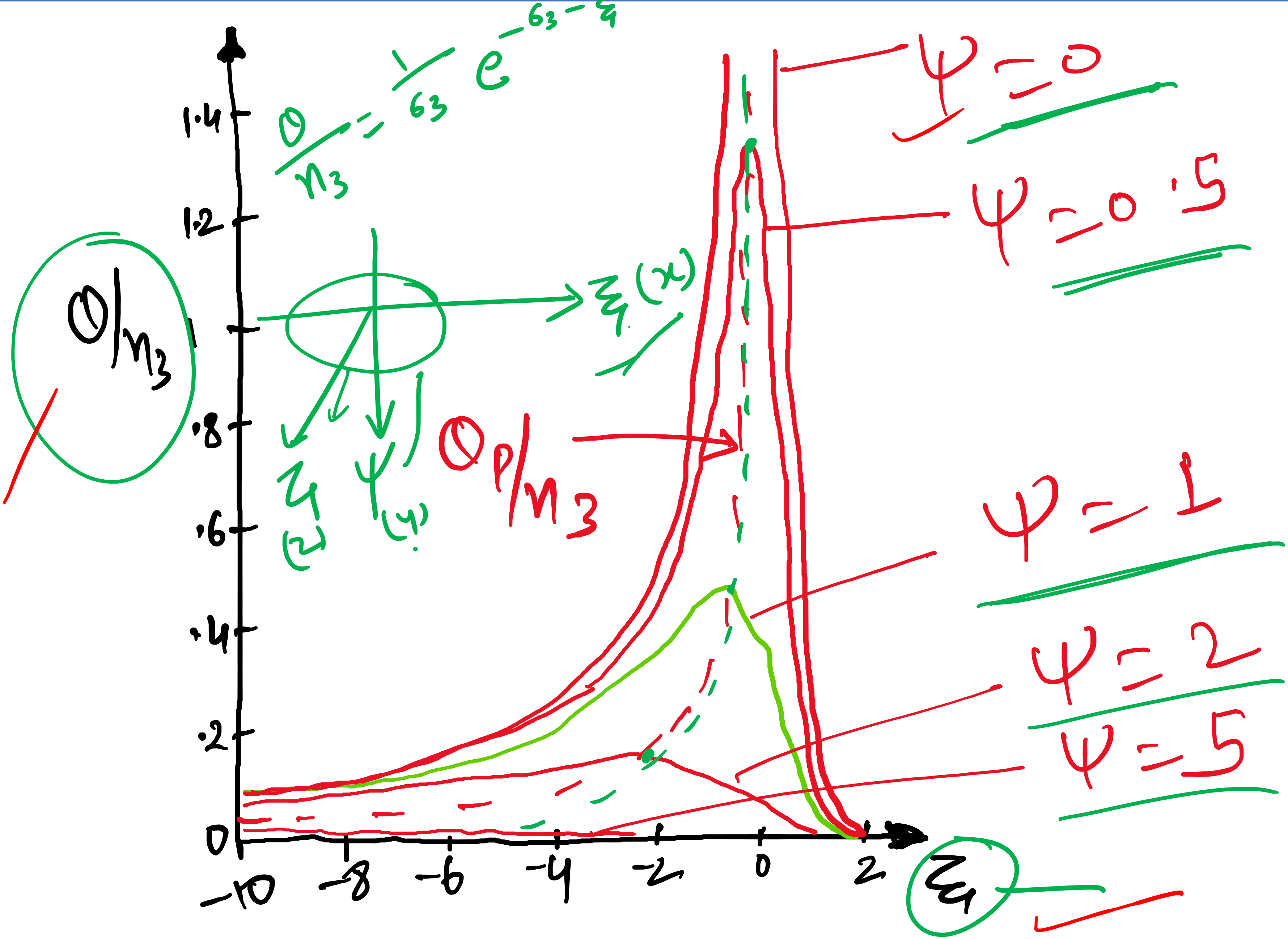
$$\theta = \frac{q_0 V}{\sigma_3 \, 4\pi \alpha^2 (H_c - H_0)} e^{-\sigma_3 - \xi_1}$$

(i.e.) $\Rightarrow$

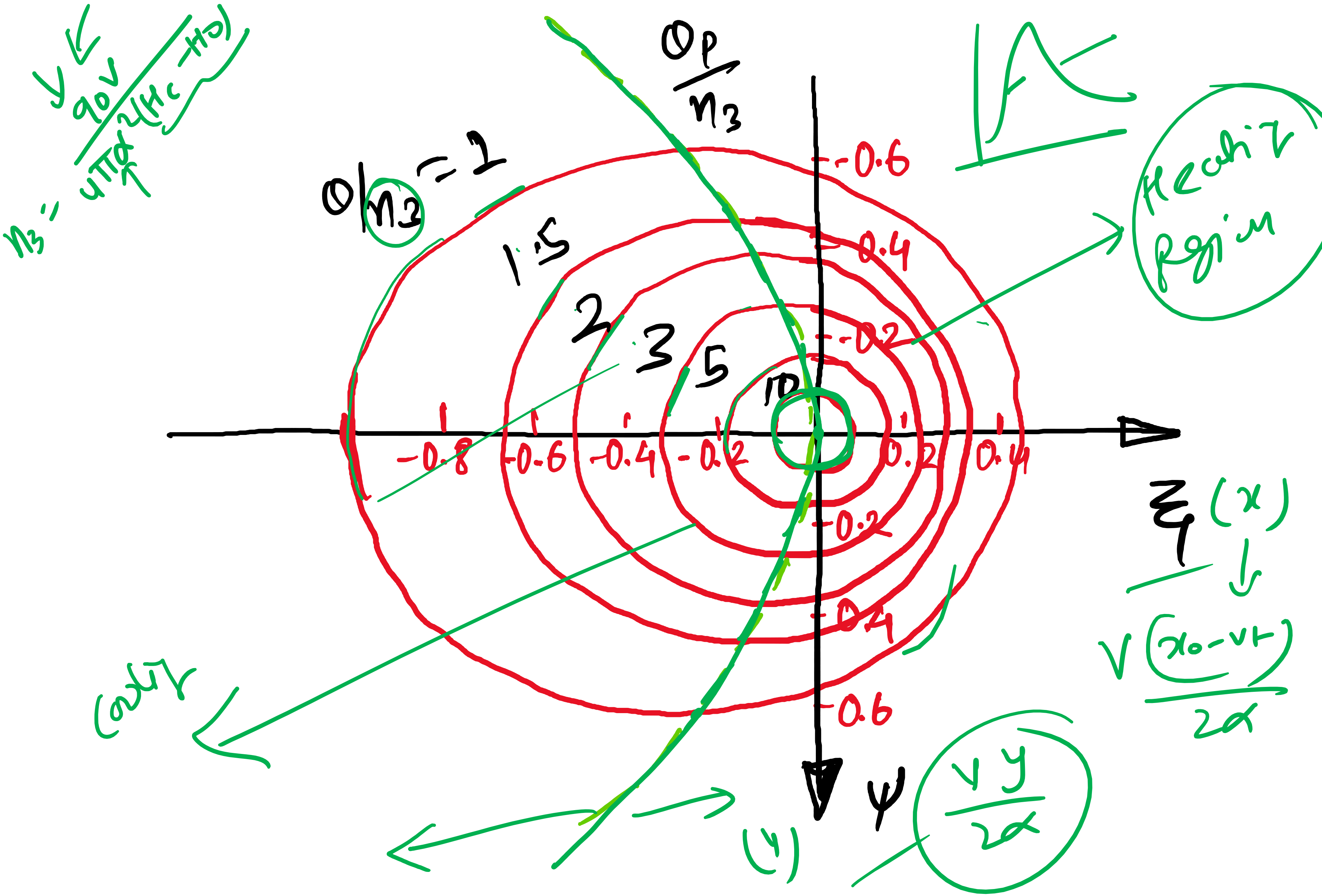
$$\theta = \frac{n_3}{\sigma_3} e^{-\sigma_3 - \xi_1}$$

$$\boxed{\frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_1}} \quad (2)$$

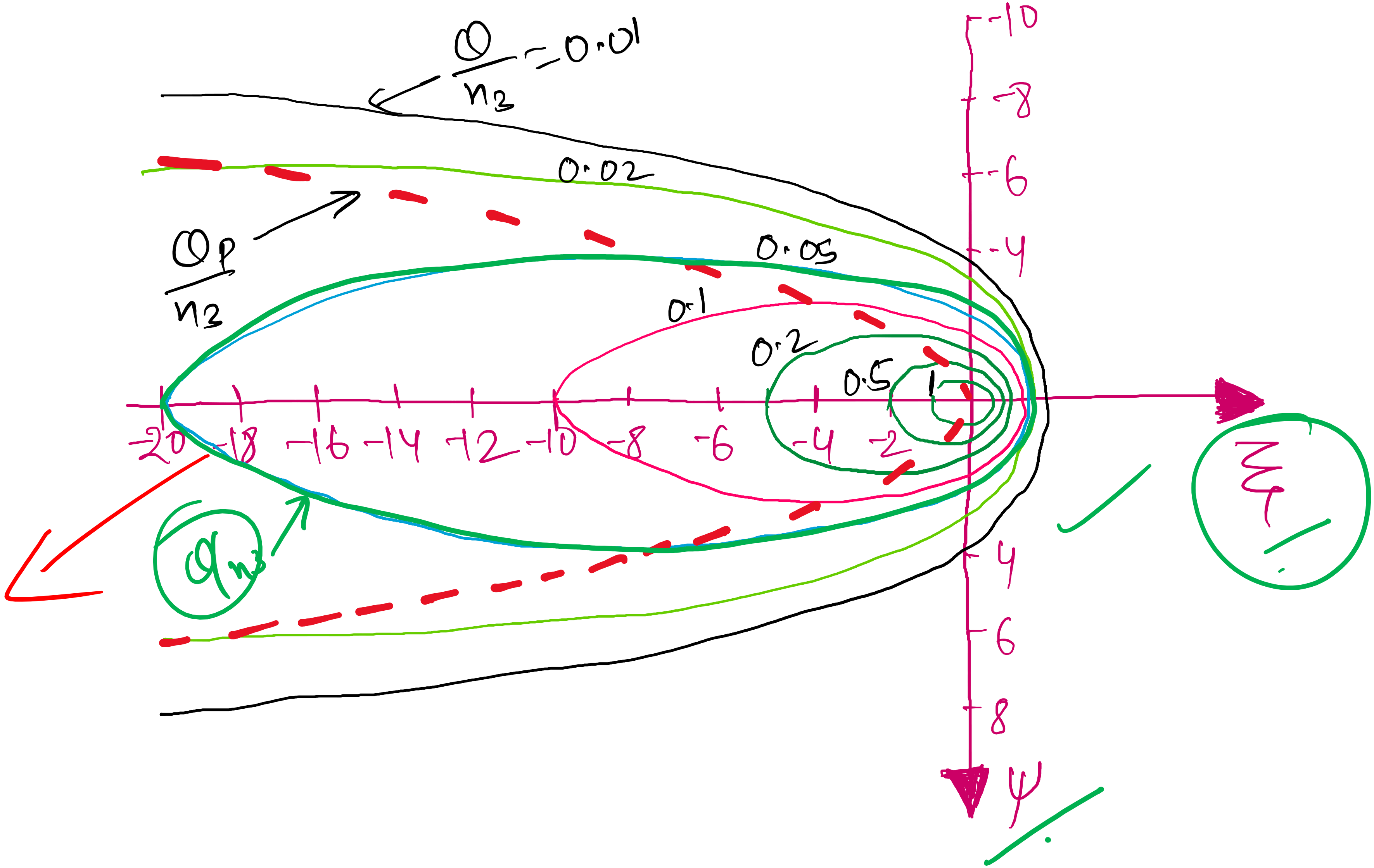
Moving Heat Sources



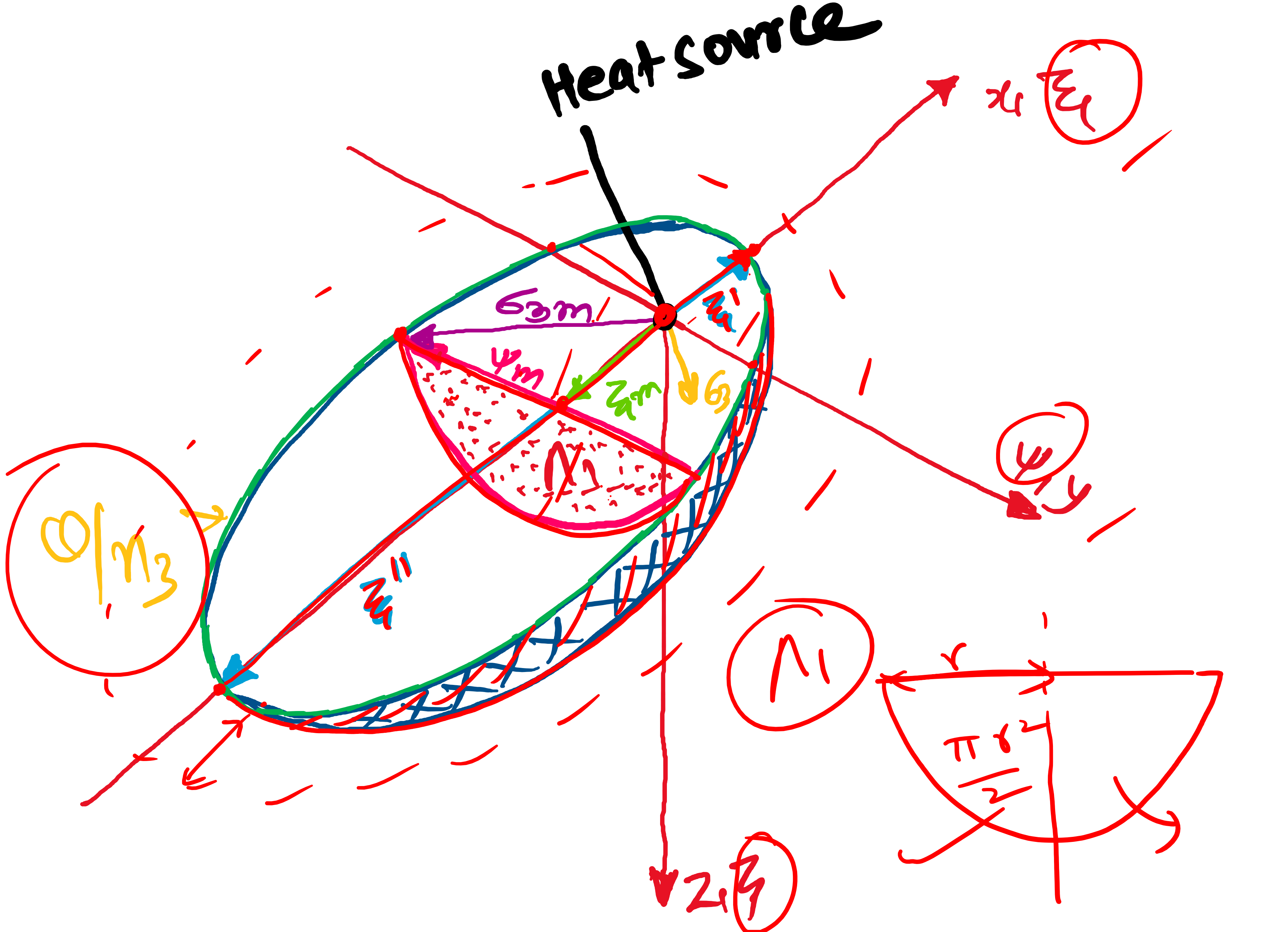
Moving Heat Sources



Moving Heat Sources



Moving Heat Sources

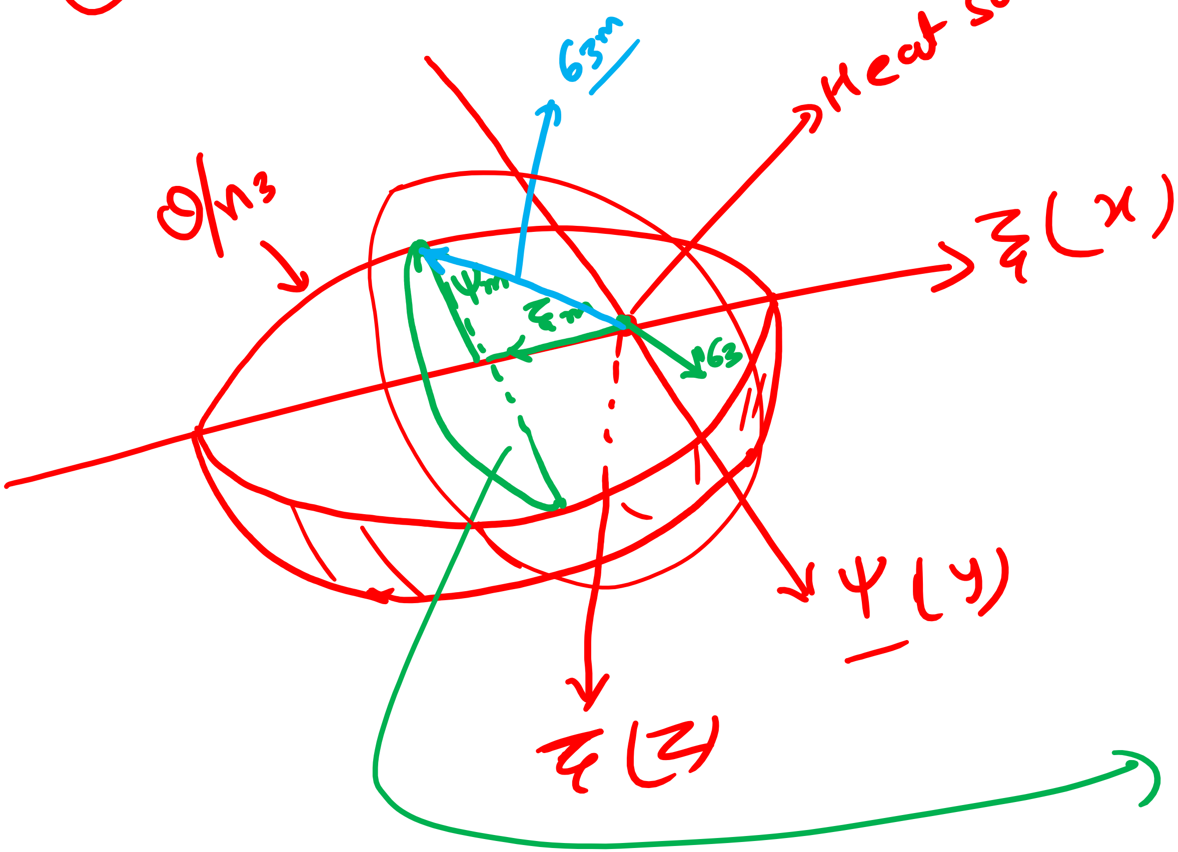


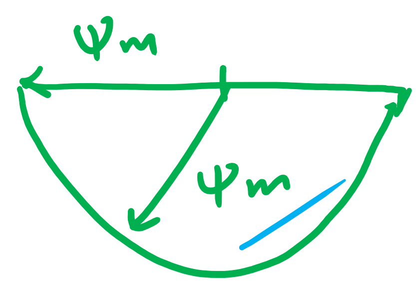
Moving Heat Sources: Thick plate

$$\frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_1} \quad \text{— } (2)$$

$$\sigma_3 = \frac{vR}{2\alpha}$$

(1) isothermal zone width :-





Moving Heat Sources: Thick plate

$$\frac{\partial\left(\frac{\theta}{\eta_3}\right)}{\partial \sigma_3} = 0 \quad \text{——} \quad 3$$

$$\sigma_3 = \frac{vR}{2\alpha} = \sqrt{\xi_4^2 + \psi^2 + \zeta_4^2}$$

$$\frac{\partial \sigma_3}{\partial \xi_4} = \frac{1}{\cancel{2}} \frac{1}{\sqrt{\xi_4^2+\psi^2+\zeta_4^2}} \times \cancel{2}\,\xi_4$$

$$= \frac{\xi_4}{\sigma_3}$$

$$\frac{\partial \sigma_3}{\partial \xi_4} = \frac{\xi_4}{\sigma_3}$$

Moving Heat Sources: Thick plate

$$\frac{\partial\left(\frac{\theta}{n_3}\right)}{\partial \sigma_3} = 0 \qquad \frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_4}$$

$$\Rightarrow \cancel{\frac{1}{\sigma_3}} \cancel{e^{-\sigma_3 - \xi_4}} \left\{ -1 - \frac{\partial \xi_4}{\partial \sigma_3} \right\} + \cancel{e^{-\sigma_3 - \xi_4}} \left(-\frac{1}{\cancel{\sigma_3^2}} \right) = 0$$

$$\Rightarrow -1 - \frac{\partial \xi_4}{\partial \sigma_3} - \frac{1}{\sigma_3} = 0 \qquad \frac{\partial \sigma_3}{\partial \xi_4} = \frac{\xi_4}{\sigma_3}$$

$$\Rightarrow -1 - \frac{\sigma_3}{\xi_4} - \frac{1}{\sigma_3} = 0$$

$$-1 - \frac{1}{\sigma_3} = \frac{\sigma_3}{\xi_4} \Rightarrow \frac{-(1+\sigma_3)}{\sigma_3} = \frac{\sigma_3}{\xi_4}$$

$$\xi_4 = -\frac{\sigma_3^2}{1+\sigma_3}$$

Moving Heat Sources: Thick plate

$$\boxed{\xi_{1m} = -\frac{\sigma_3 m^2}{1+\sigma_3 m}}$$

$- 4$

$$2 \rightarrow \quad \frac{\theta}{\eta_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_1}$$

$$-\sigma_3 m - \left(-\frac{\sigma_3 m^2}{1+\sigma_3 m}\right)$$

$$\frac{\theta_p}{\eta_3} = \frac{1}{\sigma_3 m} \times e^{-\sigma_3 m + \frac{\sigma_3 m^2}{1+\sigma_3 m}}$$

$$= \frac{1}{\sigma_3 m} e^{\frac{-\sigma_3 m \cancel{- \sigma_3 m^2} \cancel{+ \sigma_3 m}}{1+\sigma_3 m}}$$

$$= \frac{1}{\sigma_3 m} e^{-\frac{\sigma_3 m}{1+\sigma_3 m}}$$

$$\boxed{\frac{\theta_p}{\eta_3} = \frac{1}{\sigma_3 m} e^{-\frac{\sigma_3 m}{1+\sigma_3 m}}}$$

Moving Heat Sources: Thick plate

$$\boxed{\frac{\theta_p}{n_3} = \frac{1}{\sigma_{3m}} e^{-\frac{\sigma_{3m}}{1+\sigma_{3m}}}} \quad \text{—} \quad (5)$$

$$\xi_m = \frac{-\sigma_{3m}^2}{1+\sigma_{3m}} \quad (4)$$

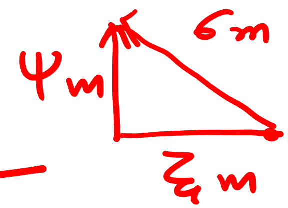
$\psi_m = ?$

$$\sigma_m^2 = \psi_m^2 + \xi_m^2$$

$$\psi_m^2 = \sigma_m^2 - \xi_m^2$$

$$= \sigma_{3m}^2 - \left(\frac{-\sigma_{3m}^2}{1+\sigma_{3m}}\right)^2$$

$$= \sigma_{3m}^2 - \frac{\sigma_{3m}^4}{(1+\sigma_{3m})^2}$$



Moving Heat Sources: Thick plate

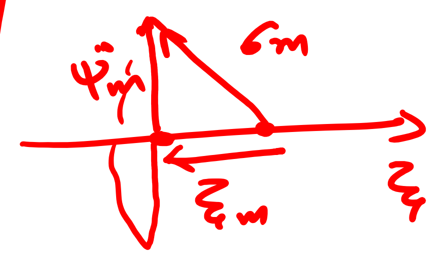
$$\psi_m^2 = \sigma_{3m}^2 - \frac{\sigma_{3m}^4}{(1+\sigma_{3m})^2}$$

$$= \frac{\sigma_{3m}^2(1+\cancel{\sigma_{3m}^2}+2\sigma_{3m}) - \cancel{\sigma_{3m}^4}}{(1+\sigma_{3m})^2}$$

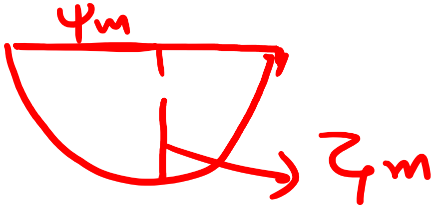
$$= \frac{\sigma_{3m}^2(1+2\sigma_{3m})}{(1+\sigma_{3m})^2}$$

(6)

$$\boxed{\psi_m = \frac{\sigma_{3m}\sqrt{1+2\sigma_{3m}}}{1+\sigma_{3m}}}$$

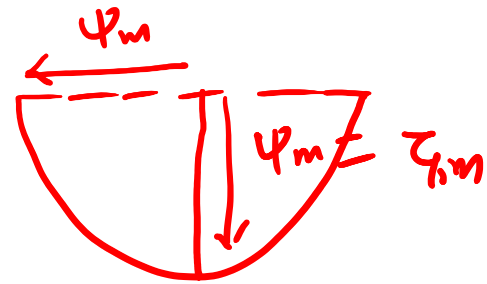


$$\boxed{\psi_m = \zeta_m = \frac{\sigma_{3m}\sqrt{1+2\sigma_{3m}}}{1+\sigma_{3m}}}$$



Moving Heat Sources: Thick plate

⇒ Cross-Section Area Λ_1 - Semi-Circular shape



$$\Lambda_1 = \frac{\pi}{2} \psi_m^2$$

$$\Lambda_1 = \frac{\pi}{2} \frac{\sigma_{3m}^2 (1+2\sigma_{3m})}{(1+\sigma_{3m})^2}$$

$$\Lambda_1 = A \frac{v^2}{4\alpha^2}$$

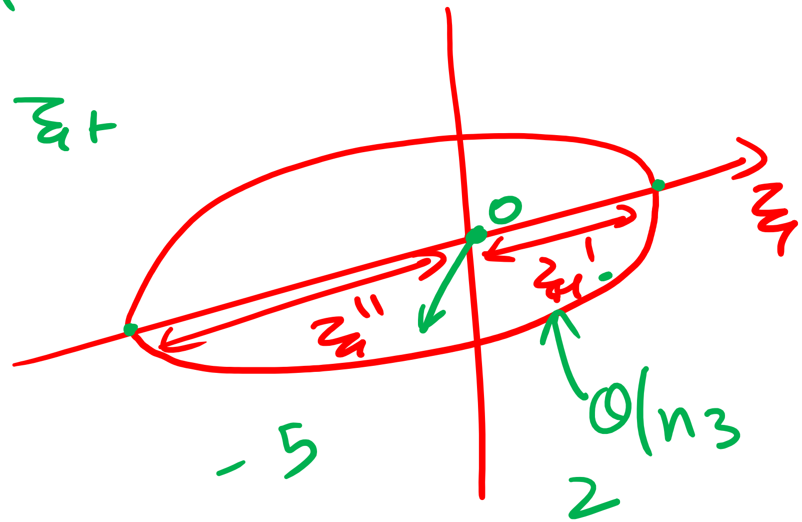
$$\Rightarrow \boxed{A = \Lambda_1 \frac{4\alpha^2}{v^2} = \frac{4\alpha^2}{v^2} \frac{\pi}{2} \frac{\sigma_{3m}^2 (1+2\sigma_{3m})}{(1+\sigma_{3m})^2}}$$

Moving Heat Sources: Thick plate

\# Length of the isotherm

total length of an isotherm $\xi_1 t$

$$= \xi_1' - \xi_1''$$



distance from the heat source to the front point

distance from the heat source to the rear point

ξ_1', ξ_1''

$$\sigma_3 = \pm \xi_1 \quad (2)$$

Moving Heat Sources: Thick plate

for $\sigma_3 = \xi_1 = \xi'$

$$\frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_1}$$

$$\boxed{\frac{\theta}{n_3} = \frac{1}{\xi'} e^{-2\xi'}}$$

$$\ln\left(\frac{\theta}{n_3}\right) = \ln\frac{1}{\xi_1} + \left(-2\xi'\right)$$

$$-2\xi' = \ln\frac{1}{\xi_1} - \ln\left(\frac{\theta}{n_3}\right)$$

$$\boxed{\xi' = \frac{1}{2}\ln\frac{n_3}{\theta\xi'}} \quad \text{— } 7$$

Moving Heat Sources: Thick plate

for $\xi_1''\uparrow$ $\quad \sigma_3 = -\xi_1$ in eq^n (2)

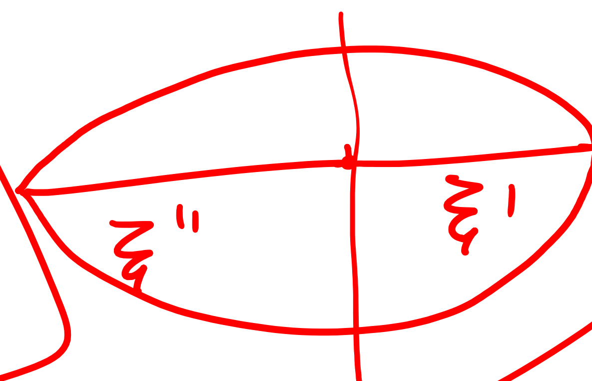
$$\frac{\Theta}{n_3} = \frac{1}{-\xi_1''}\, e^{+\cancel{\xi_1''} - \cancel{\xi_1''}}$$

$$\boxed{\xi_1'' = -\frac{n_3}{\Theta}} \quad (8)$$

$$\xi_1^{t} = \frac{v\,x'}{2\alpha}$$

$$x' = \xi_1' \frac{2\alpha}{v}$$

$$\boxed{\xi_1^{t} = \xi_1' - \xi_1'' = \frac{1}{2}\ln\left(\frac{n_3}{\Theta\,\xi_1'}\right) + \frac{n_3}{\Theta}}$$

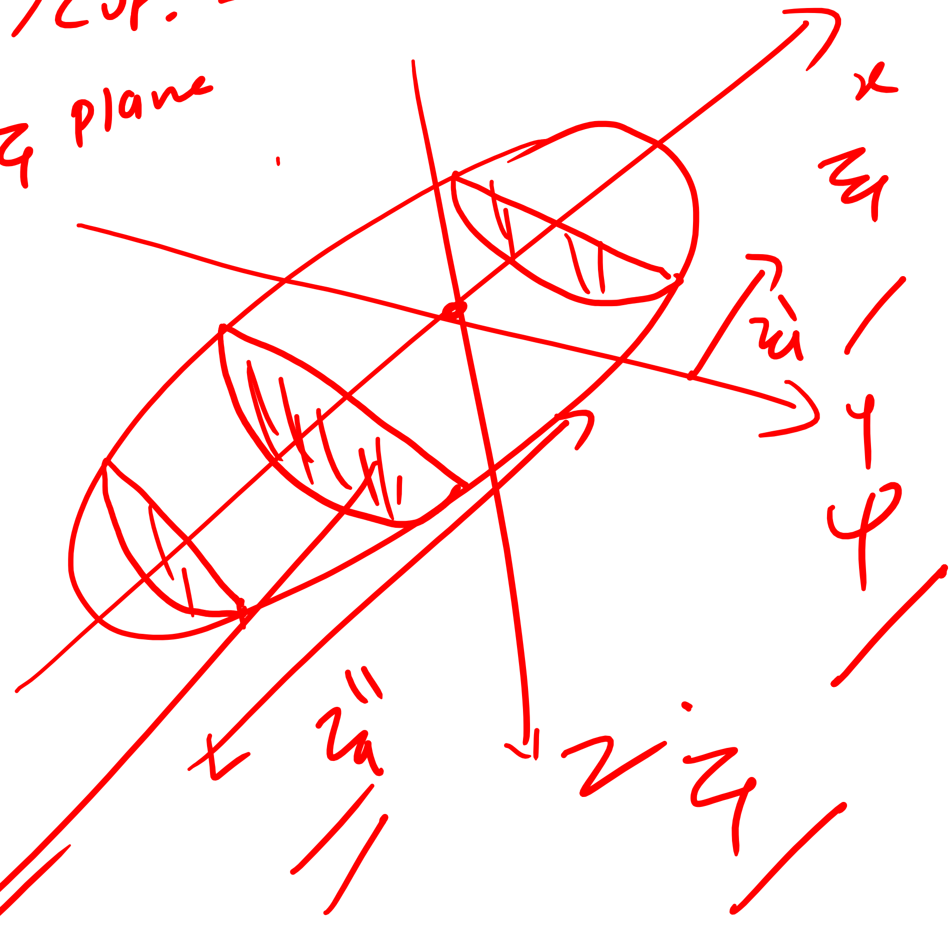


$x' =$

Moving Heat Sources: Thick plate

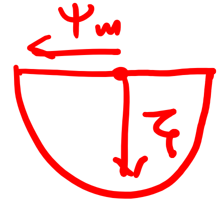
\# Vol. of isothermal zone/cup:-

= Integration of semi circular isotherm in $\psi-\xi$ plane from ξ'' to ξ'



$$\Gamma = \int_{\xi''}^{\xi'} \frac{\pi}{2}\psi^2 \, d\xi$$

$$= \frac{\pi}{12}\left[3\left(\frac{n_3}{\theta}\right)^2 - 3(\xi')^2 - 4(\xi')^3\right]$$



$\psi = \xi$

Moving Heat Sources: Thick plate

$$\text{Volume} = \frac{8\alpha^3}{V^3}\,\Gamma\text{-}$$

speed

Dimension less vol. of isothermal cup

$$\text{Volume} = \frac{8\alpha^3}{V^3}\,\frac{\pi}{12}\left[3\left(\frac{n_3}{\theta}\right)^2 - 3\left(\xi_1'\right)^2 - 4\left(\xi_1'\right)^3\right]$$

speed

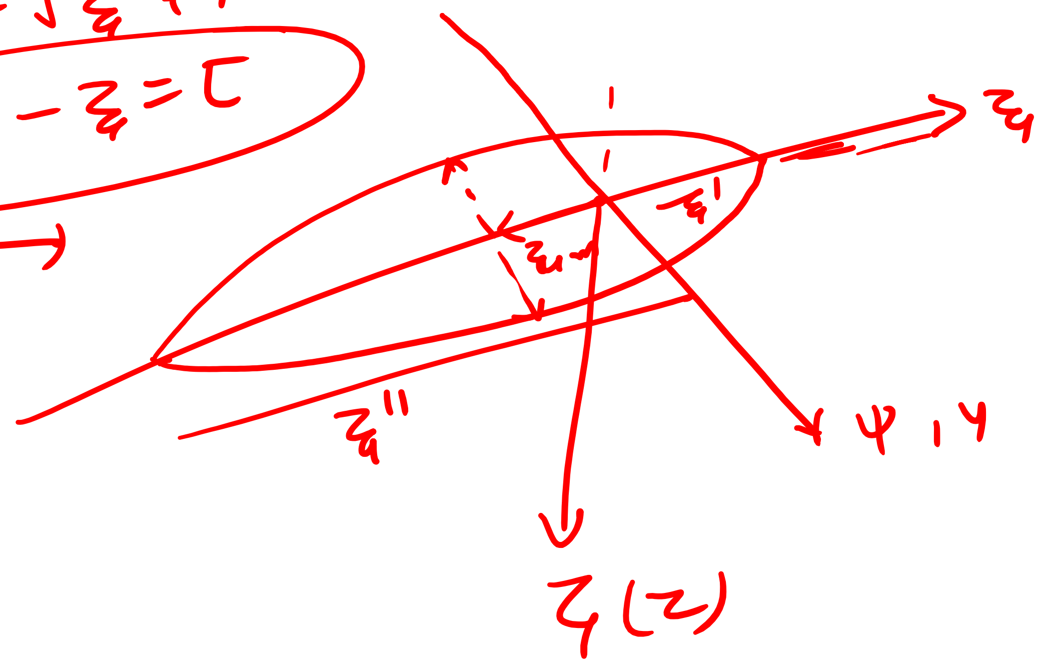
Γ

Moving Heat Sources: Thick plate

Cooling condition close to the centre weld line

$\psi = \zeta = 0 \quad | \quad \sigma_3 = \sqrt{\zeta^2 + \psi^2 + \zeta^2}$

$\sigma_3 = -\zeta = \tau$



(2) $\frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \zeta}$

$\frac{\theta}{n_3} = \frac{1}{\tau} \times 1$

$\tau = \frac{n_3}{\theta}$

$\tau = \frac{v^2 t}{2\alpha}$

Moving Heat Sources: Thick plate

$$\Delta \tau = n_3 \left(\frac{1}{\theta_2} - \frac{1}{\theta_1} \right) = \frac{v^2 \Delta t}{2\alpha}$$

Cooling time b/w two temp. like 800-500°C

$$\boxed{\Delta t = \frac{2\alpha}{v^2} n_3 \left(\frac{1}{\theta_2} - \frac{1}{\theta_1} \right)}$$

Dimensionless

$$C.R. \Rightarrow -\frac{d(\theta/n_3)}{d\tau} = -\left(-\frac{1}{\tau^2}\right) = \left(\frac{\theta}{n_3}\right)^2$$

$$C.R. = \frac{q_0 v^2}{8\pi \alpha^3 \rho c} \left(\frac{\theta}{n_3}\right)^2$$

$$\boxed{C.R = \frac{2\pi K}{q_0/v} (T - T_0)^2}$$

Moving Heat Sources: Thick plate

Moving Heat Sources: Thick plate

Numerical: moving thermal analysis

Consider stringer bead deposition (GMAW) on a thick plate of low alloy steel under the following conditions:

I=300A, Voltage=28V, V=4mm/s, efficiency: 0.8, T_0=20°C, T_c=T_m= 1520°C, , α=5 mm^2/s, ΔH=7.5J/mm^3

Sketch the contours of the fusion boundary and the Ac3-isotherm (910°C) in the ξ-ψ(x-y) plane at a pseudo-steady state.

Numerical: moving thermal analysis

Case I — when $T = T_m = 1520^\circ C$

$$\theta = \theta_m = \frac{T - T_0}{T_c - T_0} = \frac{1520 - 20}{1520 - 20} = 1$$

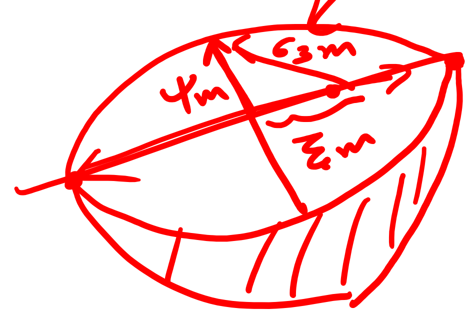
$$n_3 = \frac{q_0 v}{4\pi\alpha^2 (H_c - H_0)} = \frac{0.8 \times 300 \times 28 \times \cancel{4}}{\cancel{4} \times \pi \times 5^2 \times 7.5}$$

$$= 11.89$$

$q_0 = \eta I V$

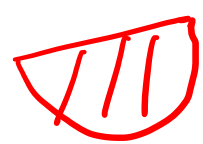
$$\frac{\theta}{n_3} = 0.084 \quad \Big| \quad \frac{n_3}{\theta} = 11.89$$

θ / n_3



Part ⓐ length of isotherms:-

ξ', ξ''



Numerical: moving thermal analysis

$$\xi_4' = \frac{1}{2}\ln\left(\frac{n_3}{\theta \xi_4'}\right)$$

$$\xi_4' = \frac{1}{2}\ln\left(\frac{11.89}{\xi_4'}\right)$$

$$\xi_4' = 1.162$$

$$\xi_4'' = -\frac{n_3}{\theta} = -11.89$$

$$X'' = -11.89 \times 2.5$$

$$= -29.725 \text{ mm}$$

$$\xi_4' = \frac{V X'}{2\alpha}$$

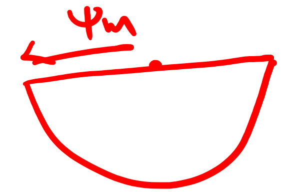
$$X' = \xi_4' \times \frac{\cancel{2 \times 5}}{\cancel{4}}\ 2.5$$

$$= 1.162 \times 2.5$$

$$= 2.905 \text{ mm}$$

Numerical: moving thermal analysis

Max^{m} width of isotherm



$$\psi_m = \zeta_m = \frac{\sigma_{3m}}{\sigma_{3m}+1}\sqrt{1+2\sigma_{3m}}$$

θ_m

$$\frac{\theta_p}{n_3} = \frac{1}{\sigma_{3m}}\, e^{-\frac{\sigma_{3m}}{\sigma_{3m}+1}}$$

$$0.084 = \frac{1}{\sigma_{3m}}\, e^{-\frac{\sigma_{3m}}{\sigma_{3m}+1}}$$

$$\sigma_{3m} = 5.15 \qquad R = 5.15 \times 2.5$$

$$\psi_m = \zeta_m = \frac{5.15}{1+5.15}\sqrt{1+2\times 5.15}$$

$$= 2.814 \rightarrow Y = 7.037 \text{ mm}$$

Numerical: moving thermal analysis

$$\xi_m = \sqrt{\sigma_{3m}^2 - \psi_m^2}$$

$$= \sqrt{5.15^2 - 2.914^2}$$

$$= \underline{4.31}$$

$$x_m = \underline{10.78} \text{ mm}$$

Numerical: moving thermal analysis

For the intersection with $\psi(y)$- axis

in that case

$$\xi_1 = 0, \ \zeta = 0$$

$$\sigma_3 = \sqrt{\cancel{\xi_1^2} + \psi^2 + \cancel{\zeta^2}}$$

$$\sigma_3 = \psi$$

$$\frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_1}$$

0

(2)

$$\frac{\theta}{n_3} = \frac{1}{\psi} e^{-\psi}$$

$$\boxed{0.084 = \frac{1}{\psi} e^{-\psi}}$$

Numerical: moving thermal analysis

$\psi = 1.856$

$Y = 1.856 \times 2.5$

$= 4.64$ mm