Welding Technology
ME692

Dr. Virkeshwar Kumar

Department of Mechanical Engineering
IIT Kanpur
Email: virkeshwar@iitk.ac.in

Numerical: moving thermal analysis

Q

Consider stringer bead deposition (GMAW) on a thick plate of low alloy steel under the following conditions:

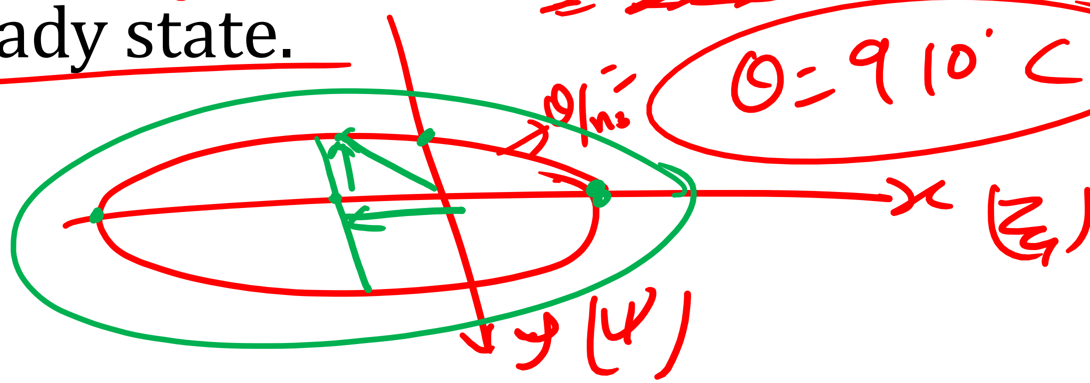
$$\frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_4}$$

I=300A, Voltage=28V, V=4mm/s, efficiency: 0.8,
T_0=20°C, T_c=T_m= 1520°C, α=5 mm^2/s, ΔH=7.5J/mm^3

$\theta = 1$ $\quad$ $\theta = \theta_m = 1520°$

Sketch the contours of the fusion boundary and the Ac3-isotherm (910°C) in the ξ-ψ(x-y) plane at a pseudo-steady state.

$\theta = 910°C$



Numerical: moving thermal analysis

Case I — when $T = T_m = 1520°C$

$$\theta = \theta_m = \frac{T - T_0}{T_c - T_0} = \frac{1520 - 20}{1520 - 20} = 1$$

$$n_3 = \frac{q_0 V}{4\pi\alpha^2 (H_c - H_0)} = \frac{0.8 \times 300 \times 28 \times \cancel{4}}{\cancel{4} \times \pi \times 5^2 \times 7.5}$$

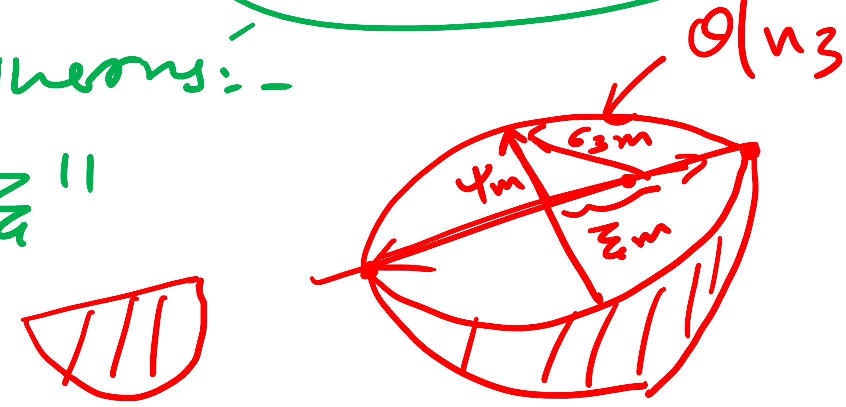
$$= 11.89$$

$q_0 = \eta I V$

$$\frac{\theta}{n_3} = 0.084 \quad | \quad \frac{n_3}{\theta} = 11.89$$

Part (a) Length of isotherms:-

ξ_1', ξ_1''



Numerical: moving thermal analysis

$$\xi_4' = \frac{1}{2}\ln\left(\frac{n_3}{\theta \xi_4'}\right)$$

$$\xi_4' = \frac{1}{2}\ln\left(\frac{11.89}{\xi_4'}\right)$$

$$\xi_4' = 1.162$$

$$\xi_4' = \frac{V \cancel{x'}}{2\alpha}$$

$$x' = \xi_4' \times \frac{\cancel{2\times 5}}{\cancel{4}} \; 2.5$$

$$= 1.162 \times 2.5$$

$$= 2.905 \text{ mm}$$

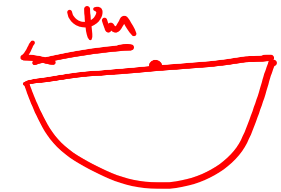
$$\xi_4'' = -\frac{n_3}{\theta} = -11.89$$

$$x'' = -11.89 \times 2.5$$

$$= -29.725 \text{ mm}$$

Numerical: moving thermal analysis

Max^m width of isotherm



$$\psi_m = \zeta_m = \frac{\sigma_{3m}}{\sigma_{3m}+1}\sqrt{1+2\sigma_{3m}}$$

θ_m

$$\frac{\theta_p}{n_3} = \frac{1}{\sigma_{3m}}\, e^{-\frac{\sigma_{3m}}{\sigma_{3m}+1}}$$

$$0.084 = \frac{1}{\sigma_{3m}}\, e^{-\frac{\sigma_{3m}}{\sigma_{3m}+1}}$$

$$\sigma_{3m} = 5.15$$

$R = 5.15 \times 2.5 =$

$$\psi_m = \zeta_m = \frac{5.15}{1+5.15}\sqrt{1+2\times 5.15}$$

$$= 2.814 \rightarrow Y = 7.037\ \text{mm}$$

Numerical: moving thermal analysis

$$\xi_m = \sqrt{\sigma_{3m}^2 - \psi_m^2}$$

$$= \sqrt{5.15^2 - 2.914^2}$$

$$= \underline{4.31}$$

$$x_m = \underline{10.\cancel{78}} \text{ mm}$$

Numerical: moving thermal analysis

For the intersection with $\psi(y)$- axis

in That case

$$\xi_1 = 0, \quad \zeta = 0$$

$$\sigma_3 = \sqrt{\cancel{\xi_1}^2 + \psi^2 + \cancel{\zeta_1}^2}$$

$$\sigma_3 = \psi$$

$$\frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_1}$$

(2)

$$\frac{\theta}{n_3} = \frac{1}{\psi} e^{-\psi}$$

$$\boxed{0.084 = \frac{1}{\psi} e^{-\psi}}$$

Numerical: moving thermal analysis

$\psi = 1.856$

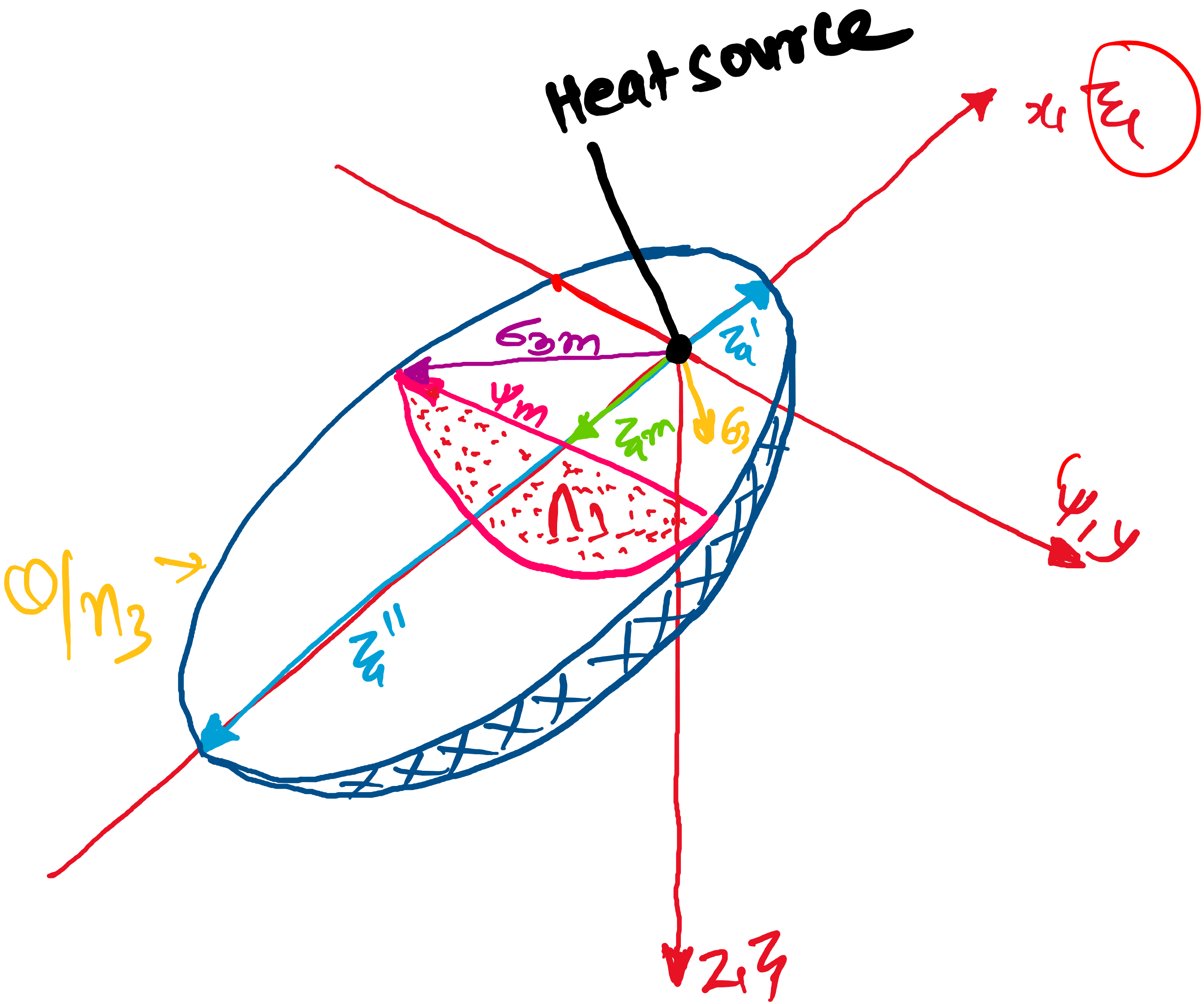
$Y = 1.856 \times 2.5$

$= 4.64$ mm

Numerical: moving thermal analysis

Numerical: moving thermal analysis

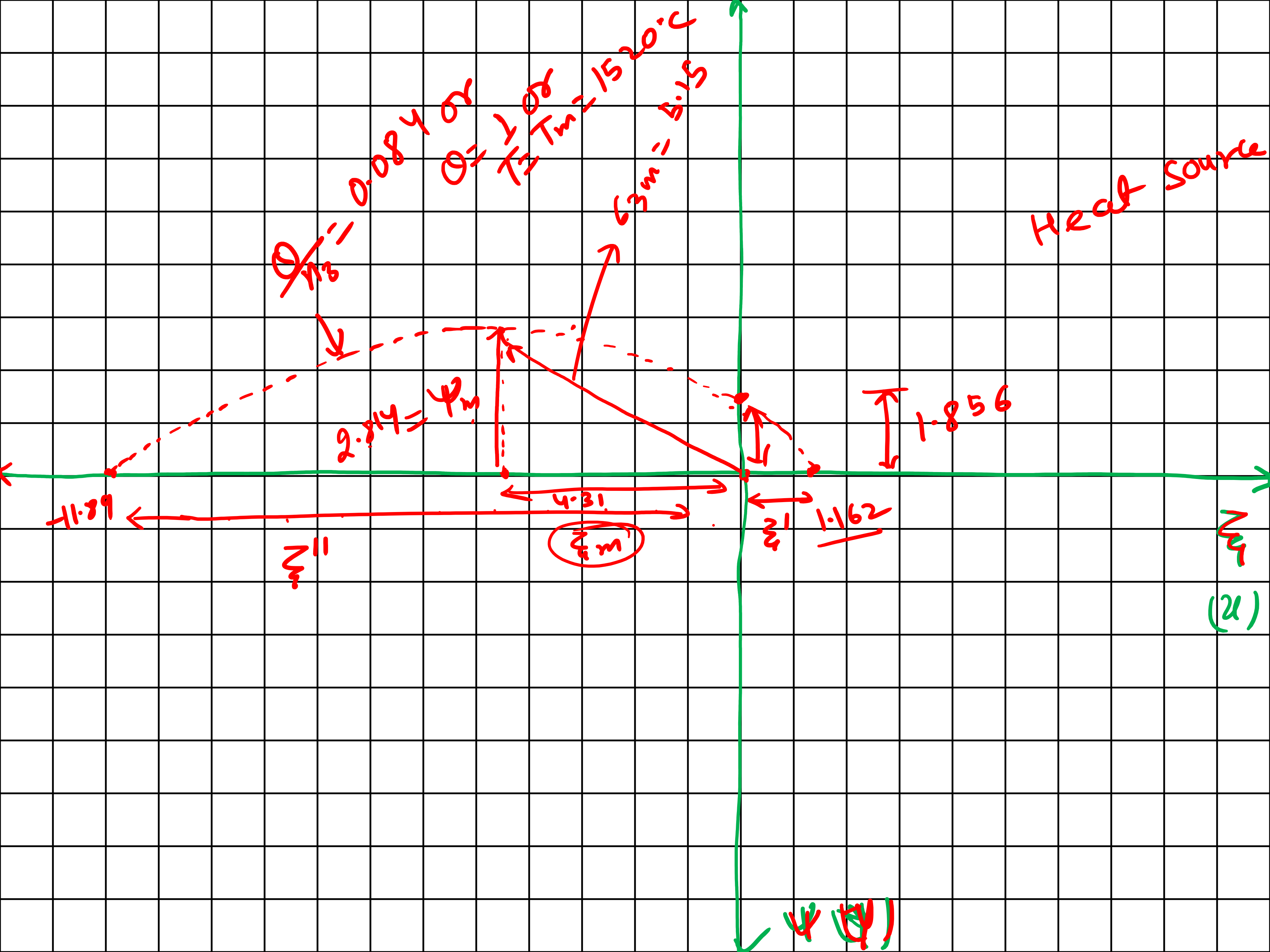
Numerical: moving thermal analysis



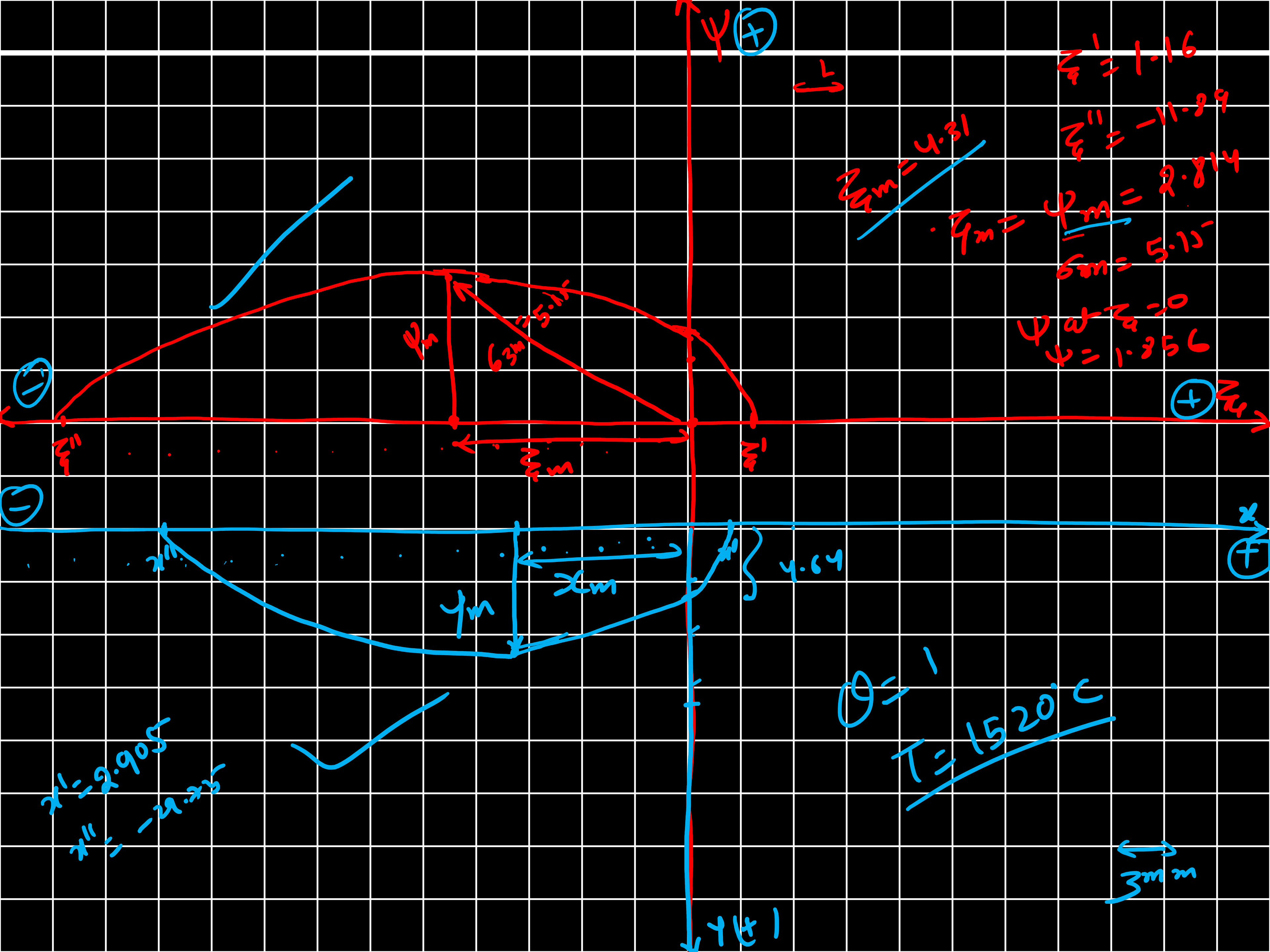
Heat source
x_1
ξ_1
σ_{3m}
ψ_m
z_a'
z_{am}
σ_3
Λ_1
$\psi_1 y$
Ω/η_3
z_a''
$z_1 \zeta$

Numerical: moving thermal analysis

Numerical: moving thermal analysis



Θ1/3 = 0.084 or
Θ = 1 or
T = Tm = 1520°C
ξ3m = 5.15
Heat source
2.814 = ψm
1.856
4.31
-11.89
ξ''
ξm'
ξ'
1.162
ξ
(x)
ψ (y)



ψ
+
−
ξ
ξ''
ξ'
ξm
ψm
6m = 5.15
ξ' = 1.16
ξ'' = -11.89
ξm = 4.31
ηm = ψm = 2.814
6m = 5.15
ψ at ξ = 0
ψ = 1.856
x
x''
x'
xm
ym
4.64
y(+)
θ = 1
T = 1520°C
x' = 2.905
x'' = -29.235
3mm

Numerical: moving thermal analysis

Part B

$\theta = \frac{T - T_0}{T_c - T_0}$

$= \frac{910 - 20}{1500} = 0.59$

$T = 910^\circ C$

$n_3 = 11.89$

$\frac{\theta}{n_3} = 0.05$

$\boxed{\frac{n_3}{\theta} = 20}$

$\xi' \Rightarrow \quad \xi' = \frac{1}{2} \ln\left(\frac{20}{\xi'}\right)$

$\xi' = 1.348$ or $x' = 3.37$

$\xi'' = -\frac{n_3}{\theta} = -20$ or $x'' = -50$

Numerical: moving thermal analysis

$\sigma_{3m} \Rightarrow \quad \frac{\theta_p}{n_3} = \frac{1}{\sigma_{3m}} e^{-\frac{\sigma_{3m}}{1+\sigma_{3m}}} = 0.05$

$\sigma_{3m} = 8.1 \quad \Rightarrow \quad R_m = 20.28 \text{ mm}$

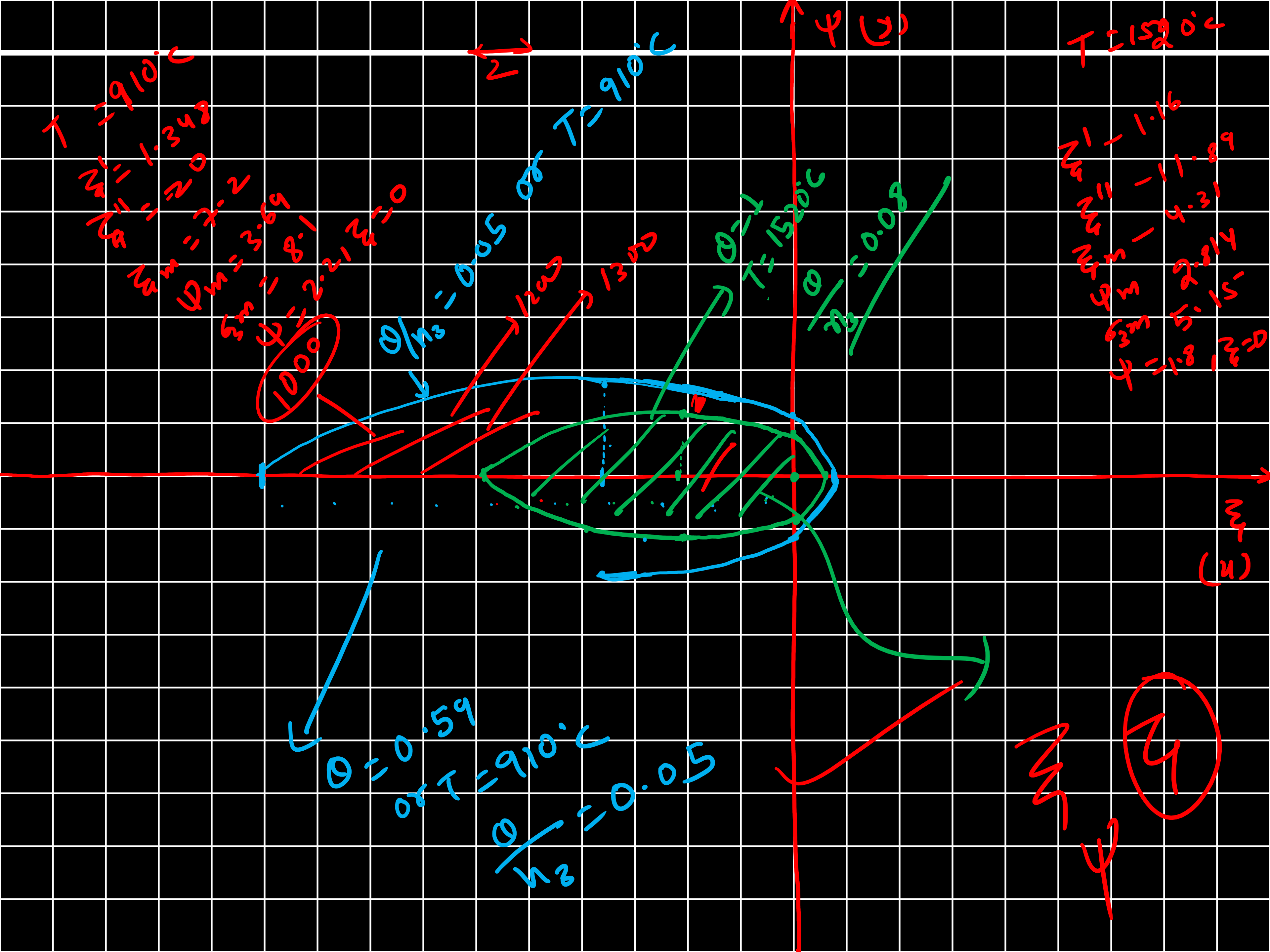
$\psi_m = Z_m = 3.69 \quad , \quad Y_m = 9.23 \text{ mm}$

$\xi_m = 7.21 \quad , \quad x_m = 18.02 \text{ mm}$

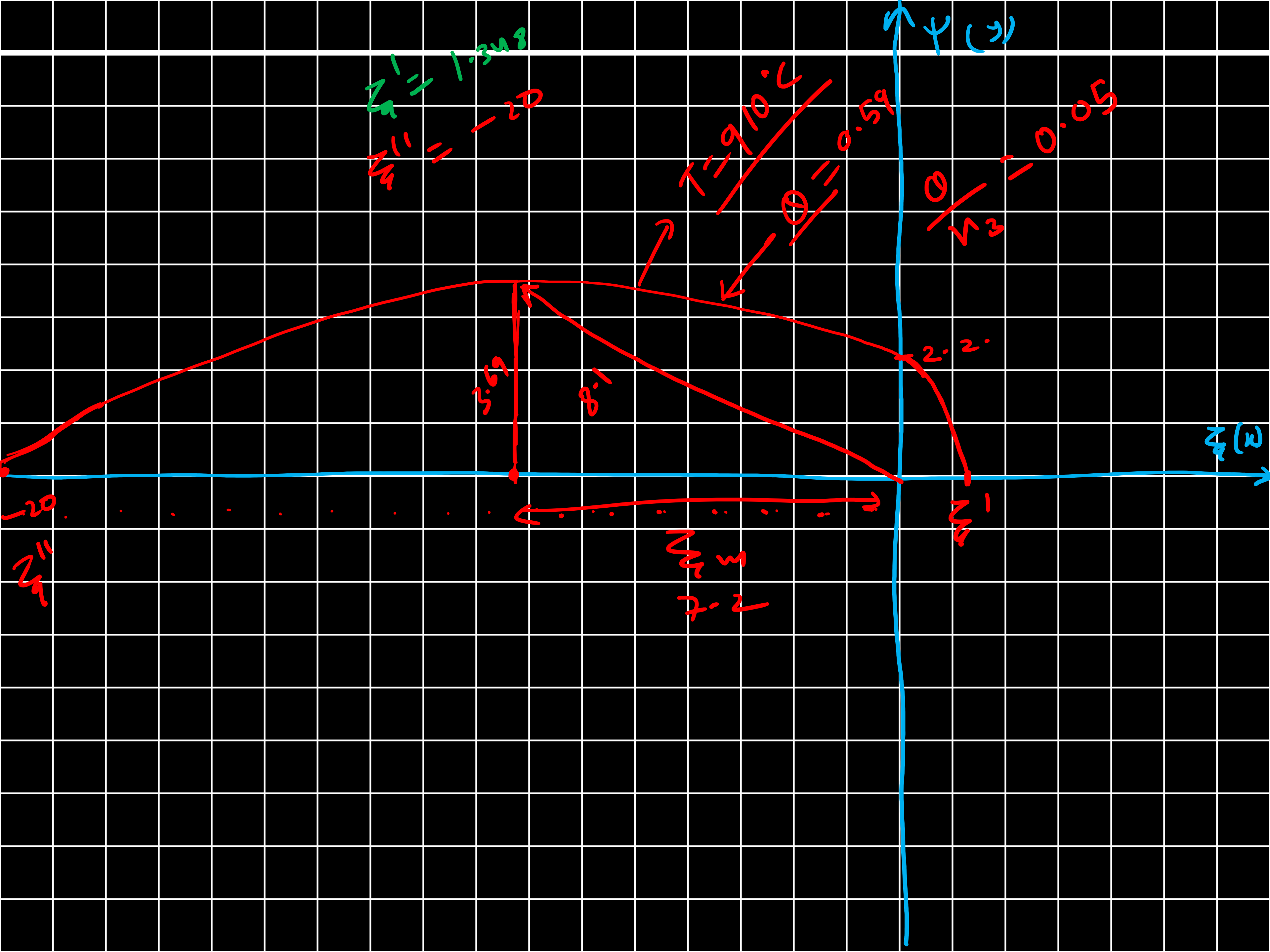
intersection point $\xi = 0 \quad \psi = 0,$

$\cdot \; \psi = \sigma_3 = 2.2$

5.5 mm



T = 910°C
T = 1520°C
ψ (y)
ξ (x)
Θ/n3 = 0.05 or T = 910°C
1000
1200
1300
Θ = 1 T = 1520°C
Θ/n3 = 0.08
Θ = 0.59 or T = 910°C
Θ/n3 = 0.05



ζ'q = 1.348
ζ''q = -20
T = 910°C
Θ = 0.59
Θ/√3 = 0.05
Ψ(y)
ζq(x)
2.2
3.69
8.1
-20
ζ''q
ζ'q
ζqm
7.2

Numerical: moving thermal analysis

Consider GTA welding on a thick plate of low alloy steel under the following conditions:

I=150A, Voltage=23V, V=3mm/s, efficiency: 0.5, T_0=20°C, T_c=T_m= 1520°C, α=5 mm^2/s, ΔH=7.5J/mm^3

Calculate the weld pool volume, the weld bead cross-section, the width of the fully transformed HAZ, the cooling time from 800 to 500°C, and the cooling rate at the onset of the austenite to ferrite transformation (e.g., at 650°C).

Numerical: moving thermal analysis

$$n_3 = \frac{q_0 V}{4\pi\alpha^2(H_c - H_0)} = \frac{0.5 \times 150 \times 23 \times 3}{4 \times \pi \times 5^2 \times 7.5} = \underline{2.2}$$

$$\theta_m = 1 \rightarrow T = \underline{1520^\circ C}$$

weld pool

$$\frac{\theta_m}{n_3} = \frac{1}{2.2} = \underline{0.455}$$

dimension less vol.

$$\xi' = \frac{1}{2}\ln\left(\frac{n_3}{\theta\,\xi'}\right) = \frac{1}{2}\ln\left(\frac{1}{0.455\,\xi'}\right)$$

$$\xi' = 0.627$$

$$\xi'' = -\frac{n_3}{\theta} = -2.2$$

$$\Gamma = \frac{\pi}{12}\left\{3\left(\frac{n_3}{\theta}\right)^2 - 3(\xi')^2 - 4(\xi')^3\right]$$

$$\Gamma = 3.22$$

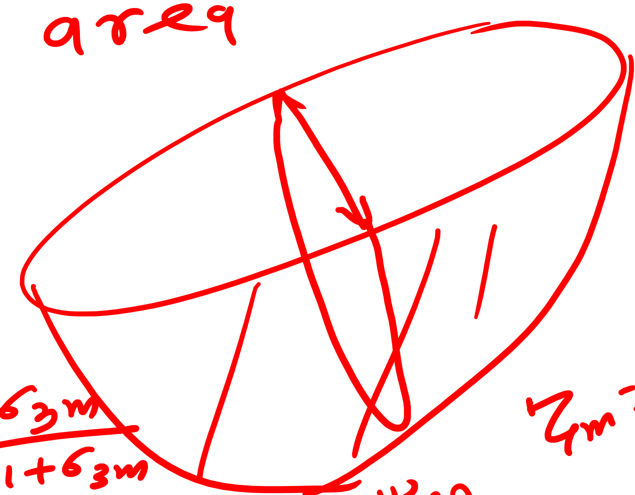
Numerical: moving thermal analysis

$$V = \frac{8\alpha^3}{\nu^3}\,\Gamma = \frac{8\times 5^3}{3^3}\times 3.22$$

$$= 120\ \text{mm}^3$$

Dimensionless cross-section area

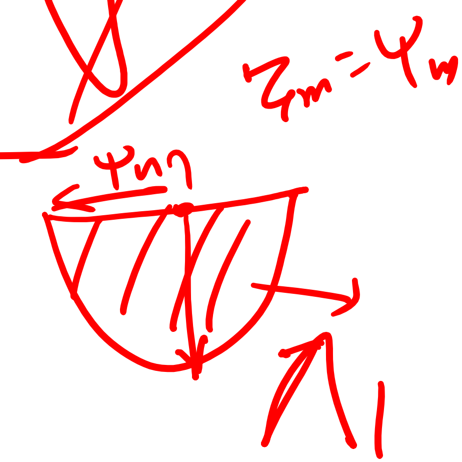
$$\Lambda_1 = \frac{\pi}{2}\,\psi_m^2$$



$\psi_m \Rightarrow \sigma_{3m}$?

$$0.455 = \frac{Q}{n_3} = \frac{1}{\sigma_{3m}}\,e^{-\frac{\sigma_{3m}}{1+\sigma_{3m}}}$$

$$\boxed{\sigma_{3m} = 8.94}$$



Numerical: moving thermal analysis

$$\psi_m = \zeta_m = \frac{\sigma_{3m}}{\sigma_{3m}+1}\sqrt{1+2\sigma_{3m}}$$

$$= \frac{8.94}{8.94+1}\sqrt{1+2\times 8.94} = \underline{3.9}$$

Dimension less area $\Rightarrow$

$$\Lambda_1 = \frac{\pi}{2}\times \psi_m^2 = 23.977$$

$$A_1 = \frac{4\alpha^2}{v^2}\times \Lambda_1 = \frac{4\times 25}{9}\times 23.977$$

$$= \underline{266.4\ mm^2}$$

Numerical: moving thermal analysis

$T_p = 910^\circ C$

$\frac{n_3}{\theta_p} = 2.196 \left(\frac{1520-20}{910-20}\right) = 3.7$

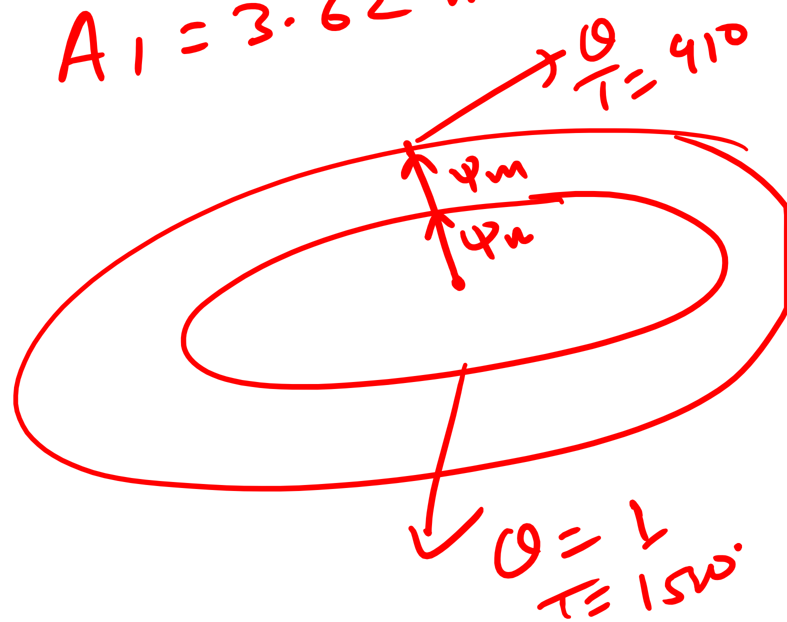
$\sigma_{3m} = 1.91$

$\psi_m = 1.44$

$\Lambda = 3.26$, $A_1 = 3.62\ mm^2$

$\Delta\psi_m = \psi_{m\ at\ 1520^\circ C} - \psi_{m\ at\ 910^\circ C}$

$= 3.9 - 1.44$



Numerical: moving thermal analysis

$\boxed{\Delta t_m = 2.46}$ → Dimensionless HA2

$\therefore \quad \Delta y = 2.46 \times \frac{2\alpha}{V}$

$= 2.46 \times \frac{2 \times 5}{3}$

$= \underline{6.15 \text{ mm}}$

$\frac{Vy}{2\alpha}$

Numerical: moving thermal analysis

Cooling time b/w 800°C — 500°C

$$\Delta\tau = \frac{n_3}{\theta_2} - \frac{n_3}{\theta_1}$$

$$= n_3 \left\{ \frac{1}{\theta_2} - \frac{1}{\theta_1} \right\}$$

$$= 2.2 \left\{ \frac{1}{0.32} - \frac{1}{0.52} \right\}$$

$$= 2.4644$$

$\theta_2 \Rightarrow 500°C$

$\theta_1 \Rightarrow 800°C$

$\frac{800-20}{1500} = 0.52$

$\frac{500-20}{1500} = 0.32$

$$\tau = \frac{V^2 t}{2\alpha}, \quad \Delta\tau = \frac{V^2 \Delta t}{2\alpha}$$

Numerical: moving thermal analysis

$$\Delta\tau = \frac{V^2 \Delta t}{2\alpha} = 2.644$$

$$\Delta t_{gw-sec} = 2.644 \times \frac{2 \times \alpha}{V^2}$$

$$= 2.644 \times \frac{2 \times 5}{3^2}$$

$$= 2.93 \text{ sec}$$

Numerical: moving thermal analysis

Cooling rate at 650°C

$$\boxed{C.R. = \frac{2\pi K}{(q/v)}(T_c - T_o)^2}$$

$q_0 = 0.5 \times 150 \times 23$

$$= \frac{2 \times \pi \times 0.025}{\frac{0.5 \times 150 \times 23}{3}} (650 - 20)^2$$

$$= \underline{108°C/sec}$$

Numerical: moving thermal analysis

$$C \cdot R_0 = \frac{2\pi K}{\left(\frac{q_0}{V}\right)} (T_c - T_0)^2$$

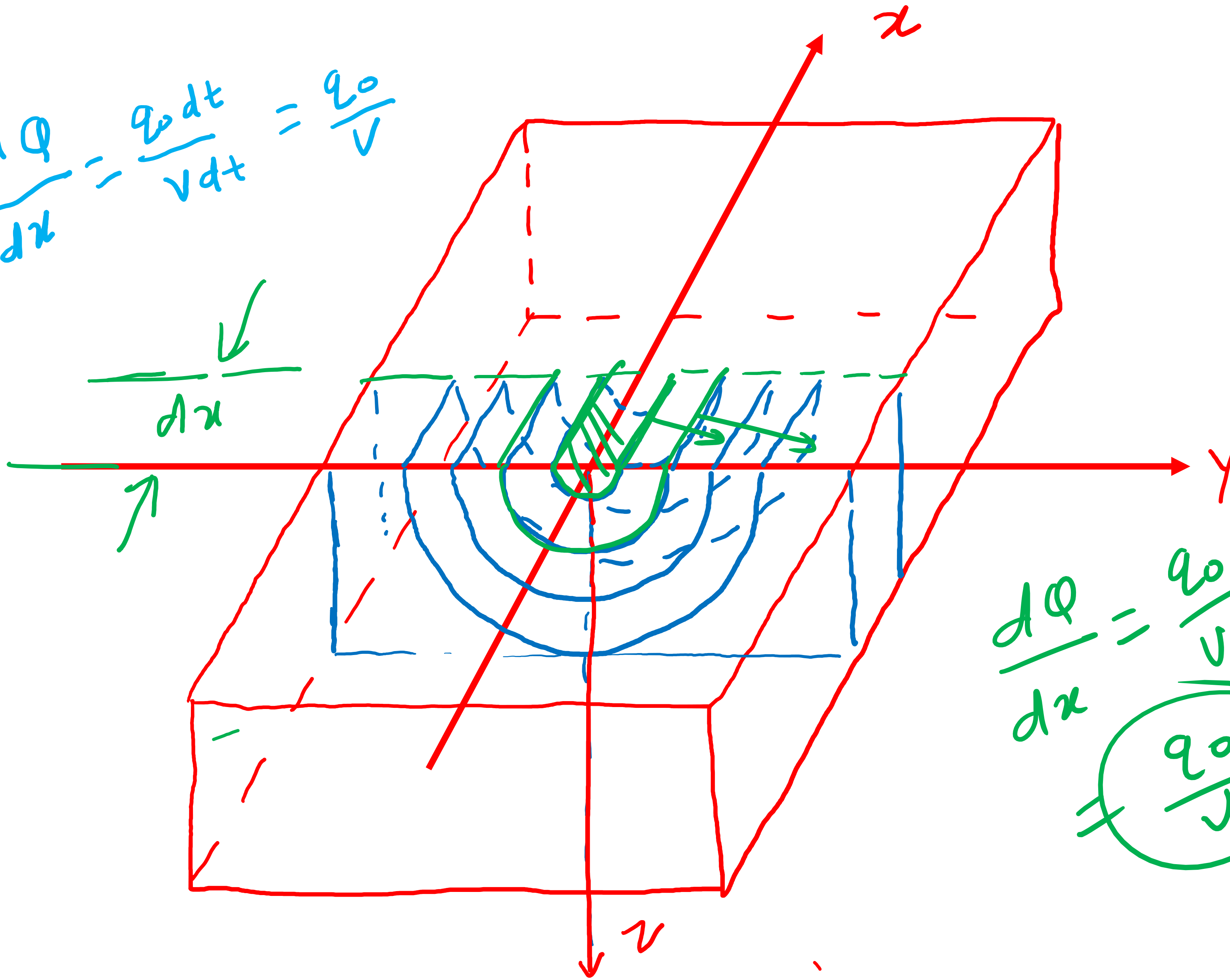
for thick plate

$$H_{net} = \frac{\eta I \times V}{speed}$$

$$C \cdot R = \frac{2\pi K}{H_{net}} (T_c - T_0)^2$$

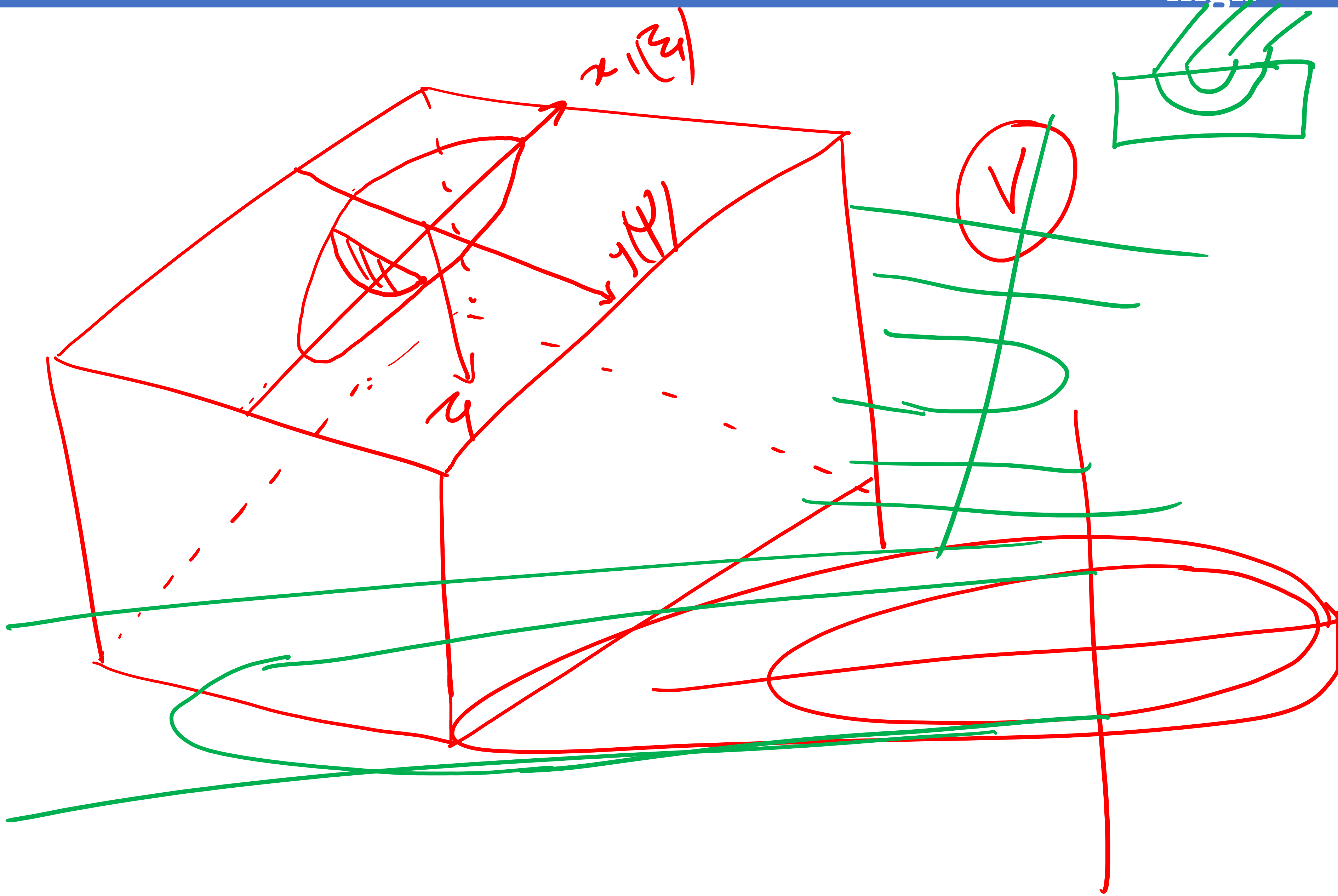
Moving Heat Sources: thick plate: V_{high}

$$\frac{dQ}{dx} = \frac{q_0\,dt}{V\,dt} = \frac{q_0}{V}$$



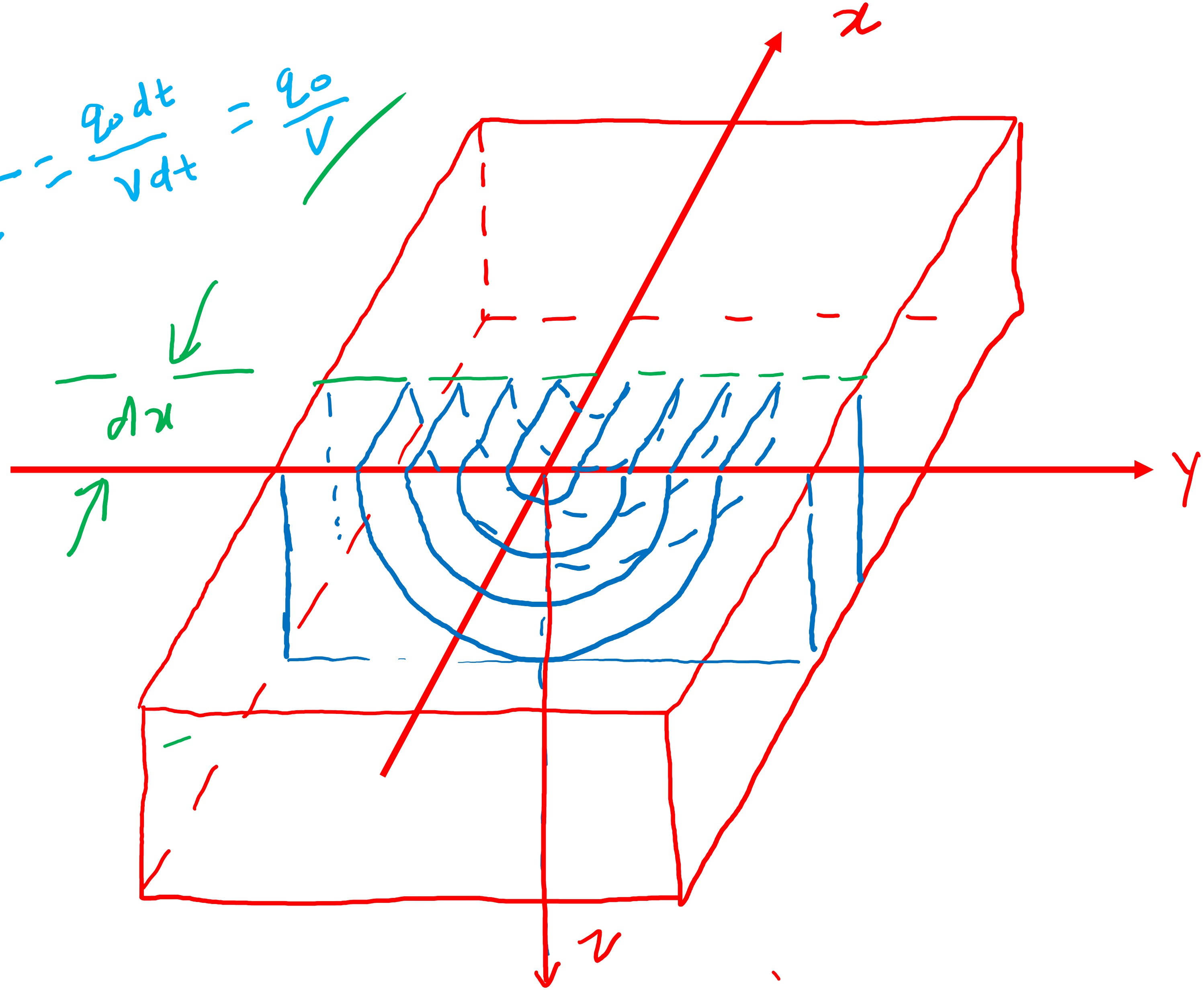
$$\frac{dQ}{dx} = \frac{q_0\,dt}{V\,dt} = \frac{q_0}{V}$$

Moving Heat Sources: thick plate: V $_{high}$



Moving Heat Sources: thick plate: V_{high}

$$\frac{dQ}{dx} = \frac{q_0\,dt}{V\,dt} = \frac{q_0}{V}$$



Moving Heat Sources: thick plate: V_{high}

Instantenery heatsource : line source.

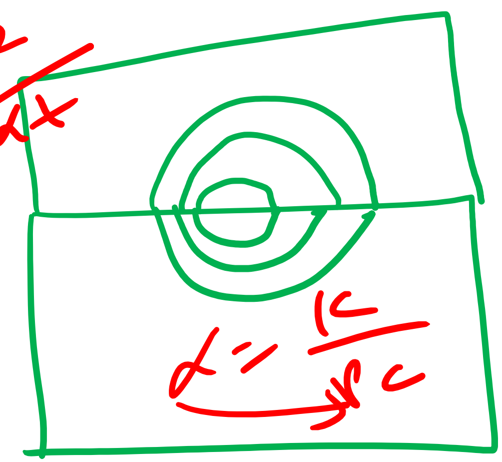
$$T = T_0 + \frac{Q/d}{\rho c\,(4\pi\alpha t)}\, e^{\left(-\frac{r^2}{4\alpha t}\right)}$$

$$\frac{dQ}{dx} = \frac{q_0}{v}$$

$$= \frac{2 q_0}{v}$$

$$= T_0 + \frac{\cancel{2}\, q_0 / v}{\rho c\, 2\cancel{4}\pi\alpha t}\, e^{-\frac{r^2}{4\alpha t}}$$

$$= T_0 + \frac{q_0}{2\pi k t v}\, e^{-\frac{r^2}{4\alpha t}}$$



$$\alpha = \frac{k}{\rho c}$$

$$\rho c \alpha = k$$

Moving Heat Sources: thick plate: V $_{high}$

$$T = T_0 + \frac{q_0}{2\pi K t\, v} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

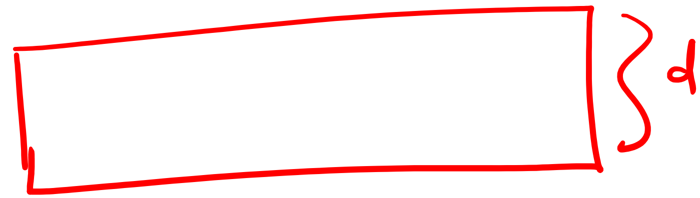
If speed of welding some in moving with high velocity.

$V \to \infty$

Moving Heat Sources: thin plate

✓ $$T = T_0 + \frac{q'}{2\pi K} e^{-\frac{V\xi}{2\alpha}} K_0\left(\frac{Vr}{2\alpha}\right)$$

$$x' = x_0 - Vt$$



$r =$ Hit

$$T = T_0 + \frac{q'}{2\pi K} e^{-\frac{Vx'}{2\alpha}} K_0\left(\frac{Vr}{2\alpha}\right)$$

rate of heat per unit length watt/m

$$q' = \frac{q_0}{d} = \frac{\eta I \times V}{d}$$

Moving Heat Sources: thin plate

$$T = T_0 + \frac{q_0}{d\,2\pi K}\, e\left(-\frac{v x'}{2\alpha}\right) K_0\left(\frac{v r}{2\alpha}\right)$$

$\rightarrow K_0$ = Bessel fn. of 2nd kind & zero order

$q' = q_0/d$

Dimension less Temp.

$$\theta = \frac{T - T_0}{T_c - T_0}$$

$$\delta = \frac{v d}{2\alpha}$$ $\rightarrow$ Dimension less plate thickness

$$\xi = \frac{v x'}{2\alpha}$$, Dimension less x-dirn coordinate.

$$\psi = \frac{v y}{2\alpha}$$, Dimension less y-dirn co-ordinate.

Moving Heat Sources: thin plate

$\sigma_3 = \frac{V r}{2\alpha}$ for thick plate

$r = \sqrt{\xi^2 + \psi^2 + \cancel{z^2}}$

$\sigma_4 = \frac{V r}{2\alpha}$ Dimension les radius

$$\boxed{r = \sqrt{\xi^2 + \psi^2}}$$

$$\eta_3 = \frac{q_0 V}{4\pi\alpha^2 \rho c (T_c - T_0)} = \frac{q_0 V}{4\pi\alpha^2 (H_c - H_0)}$$

ΔH

Moving Heat Sources: thin plate

(1) $\rightarrow$ $$T = T_0 + \frac{q^0}{2\pi k d} \exp\left(\frac{-V x'}{2\alpha}\right) K_0\left(\frac{V r}{2\alpha}\right)$$

$$\frac{T - T_0}{T_c - T_0} = \frac{q_0}{2\pi K d \times \frac{V}{2\alpha} \times \frac{2\alpha}{V}} \times \frac{1}{(T_c - T_0)} \exp(-\xi_1) K_0(\sigma_1)$$

$$\theta = \frac{q_0 V}{4\pi K \delta \alpha (T_c - T_0)} \times \frac{1}{\frac{\rho c}{\rho c}} \exp(-\xi_1) K_0(\sigma_1)$$

$\rightarrow \Delta H = H_c - H_0$

$$\theta = \frac{q_0 V}{4\pi \alpha^2 \delta \Delta H} \exp(-\xi_1) K_0(\sigma_1)$$

Moving Heat Sources: thin plate

$$\theta = \frac{n_3}{\delta} \exp(-\xi_4)\, K_0(\sigma_4)$$

$$\delta = \frac{vd}{2\alpha}$$

thin

$$\boxed{\frac{\theta \delta}{n_3} = \exp(-\xi_4)\, K_0(\sigma_4)}$$

$$\boxed{\frac{\theta}{n_3} = \frac{1}{\sigma_3} e^{-\sigma_3 - \xi_4}}$$

thick plate

Moving Heat Sources: thin plate

$$\frac{\theta\delta}{n_3} = \exp(-\xi_4)\, K_0(\sigma_4) \quad \text{—— (2)}$$

Isothermal zone width ?

$\psi_m = ?$

$$\frac{\partial\left(\frac{\theta\delta}{n_3}\right)}{\partial \xi_4} = \exp(-\xi_4)(-1)\, K_0(\sigma_4) + \exp(-\xi_4)\left\{-K_1(\sigma_4) \times \frac{\partial \sigma_4}{\partial \xi_4}\right\} = 0$$

$$\boxed{\frac{d K_0(z)}{dz} = -K_1(z)}$$

Property of Bessel fn.

$$\sigma_4 = \sqrt{\xi_4^2 + \psi^2}$$

$$\frac{\partial \sigma_4}{\partial \xi_4} = \frac{1}{\cancel{2}} \frac{\cancel{2}\xi_4}{\sqrt{\xi_4^2+\psi^2}} = \xi_4/\sigma_4$$

Moving Heat Sources: thin plate

$$-K_0(\sigma_4) - K_1(\sigma_4) \times \frac{\xi_4}{\sigma_4} = 0$$

$$\boxed{\xi_m = -\frac{K_0(\sigma_{4m})}{K_1(\sigma_{4m})}\sigma_{4m}}$$

K_0 = 2nd kind zero order B.S.

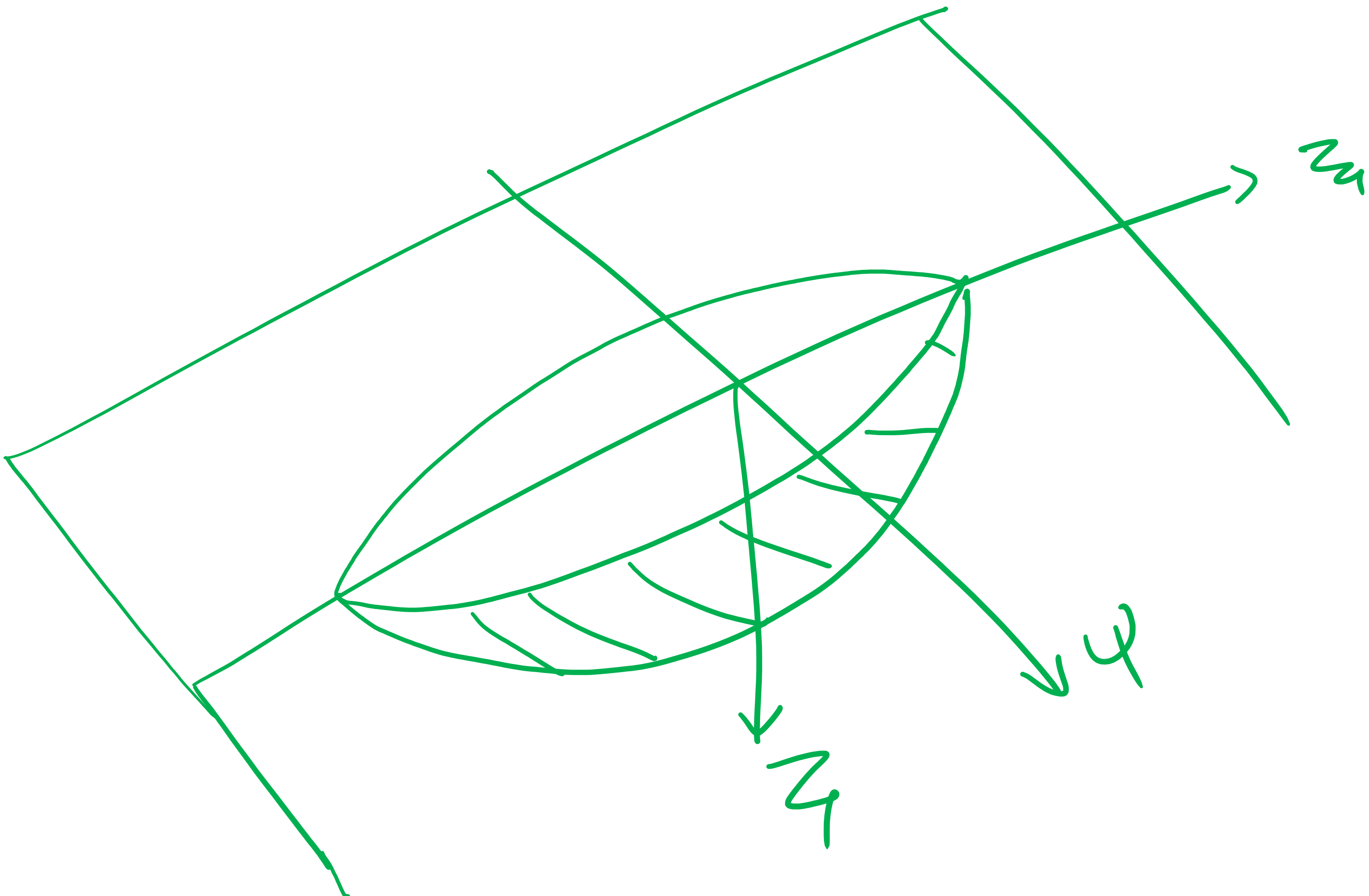
K_1 = 2nd kind 1st order B.f.

$$\boxed{\frac{\theta_p \delta}{n_3} = \exp\left(\frac{K_0(\sigma_{4m})}{K_1(\sigma_{4m})}\sigma_{4m}\right) K_0(\sigma_{4m})}$$

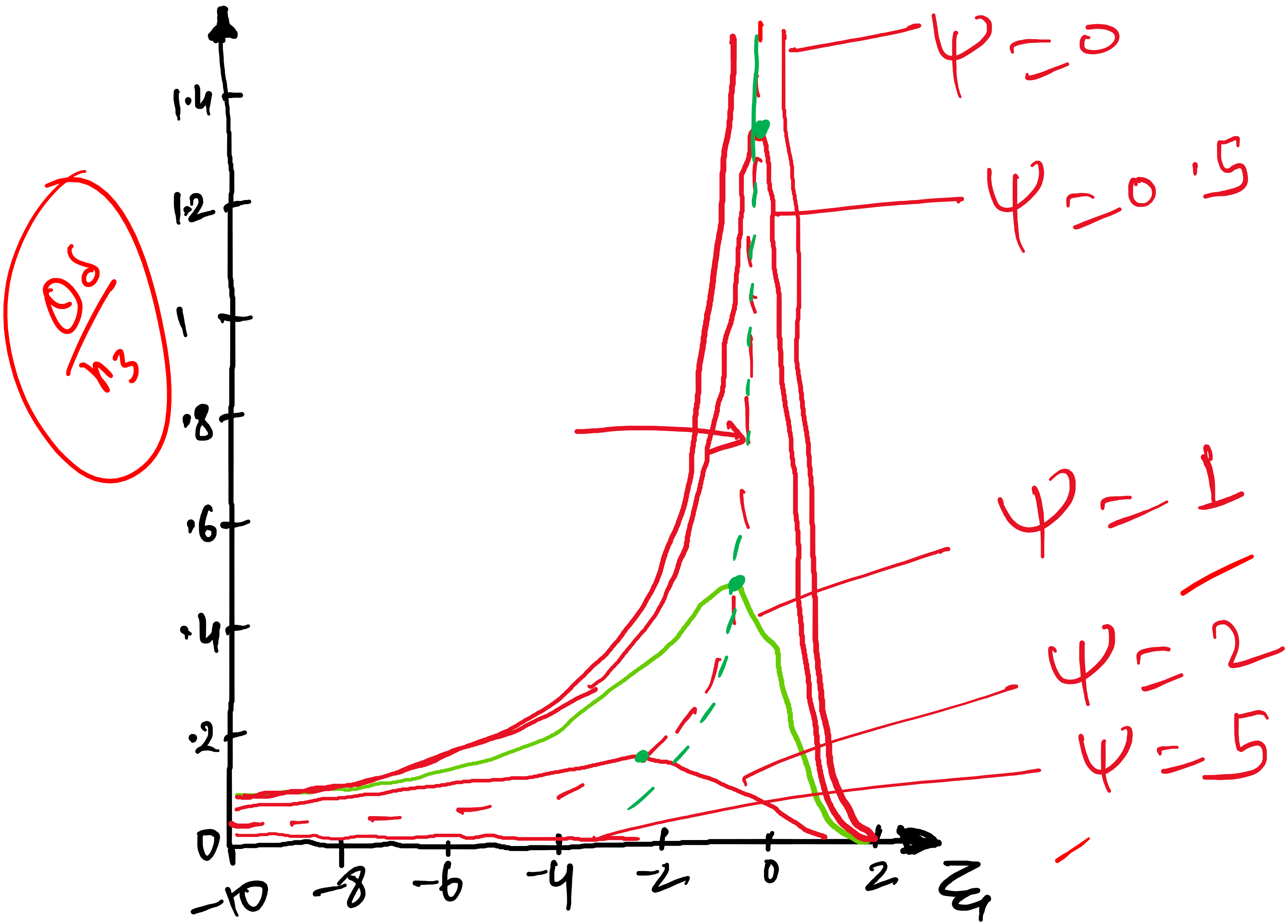
$$\psi_m = \sqrt{\sigma_{4m}^2 - \xi_m^2}$$

$$\psi_m = \sigma_{4m}\sqrt{1 - \frac{[K_0(\sigma_{4m})]^2}{[K_1(\sigma_{4m})]^2}}$$

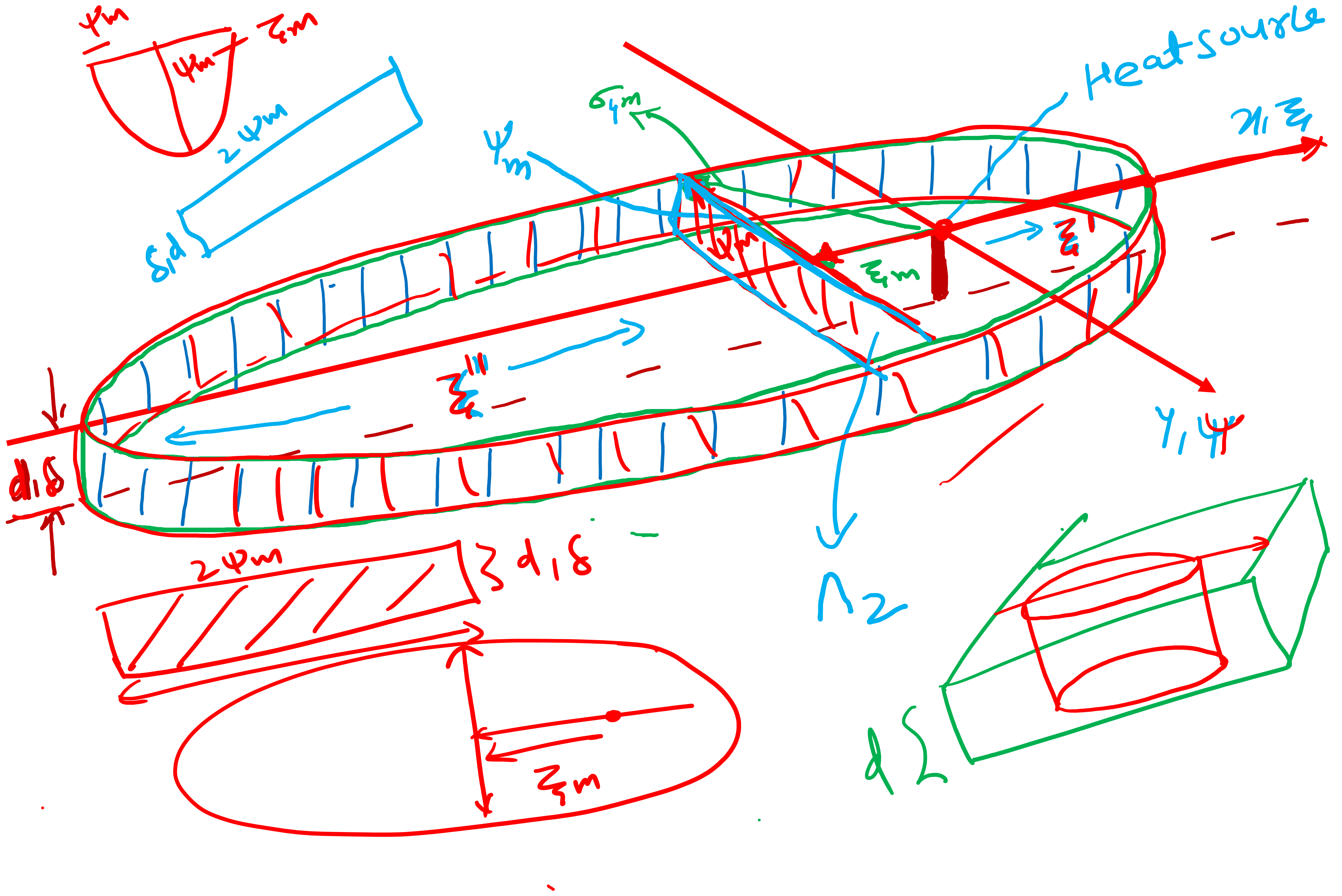
Moving Heat Sources: thin plate



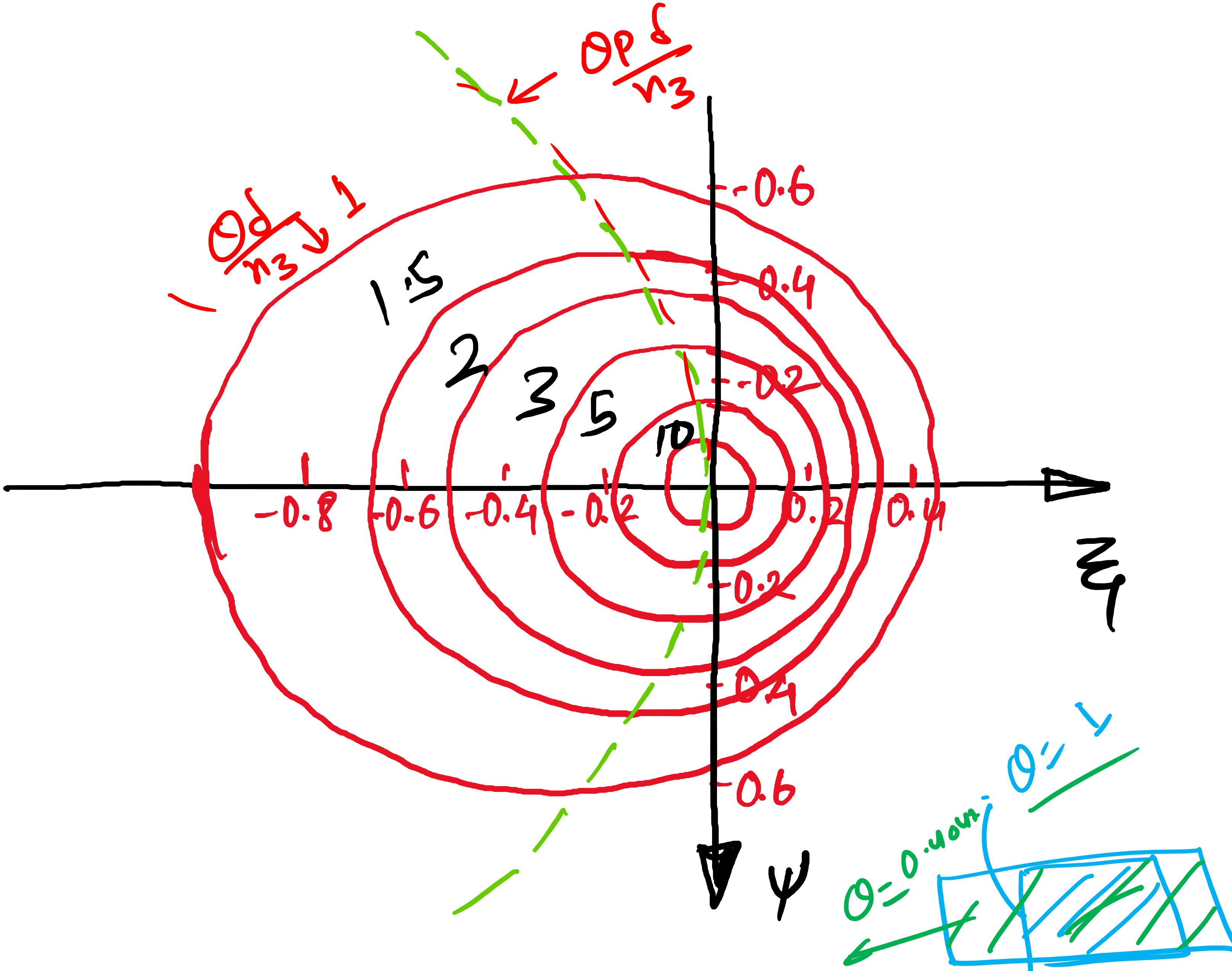
Moving Heat Sources: thin plate



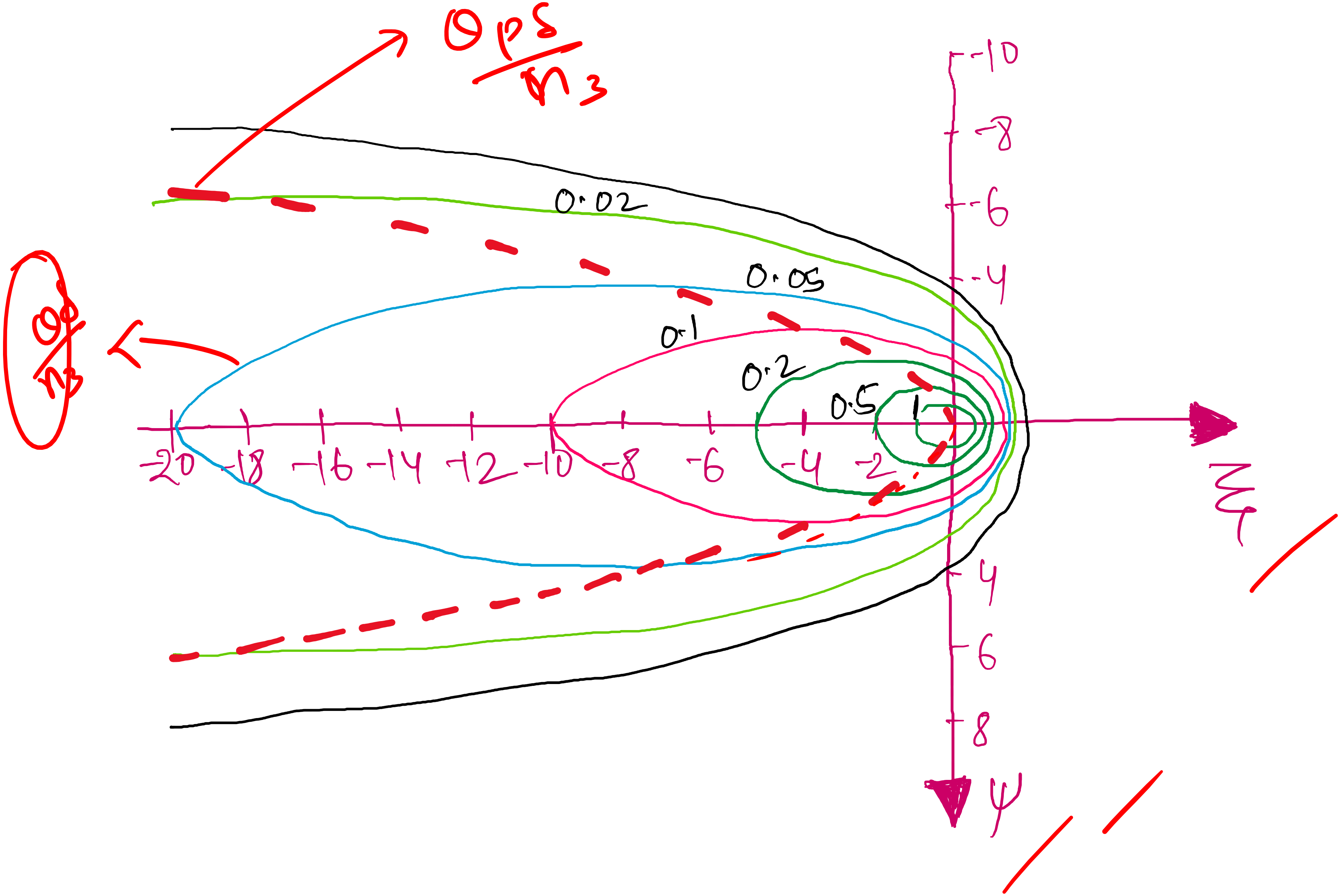
Moving Heat Sources: thin plate



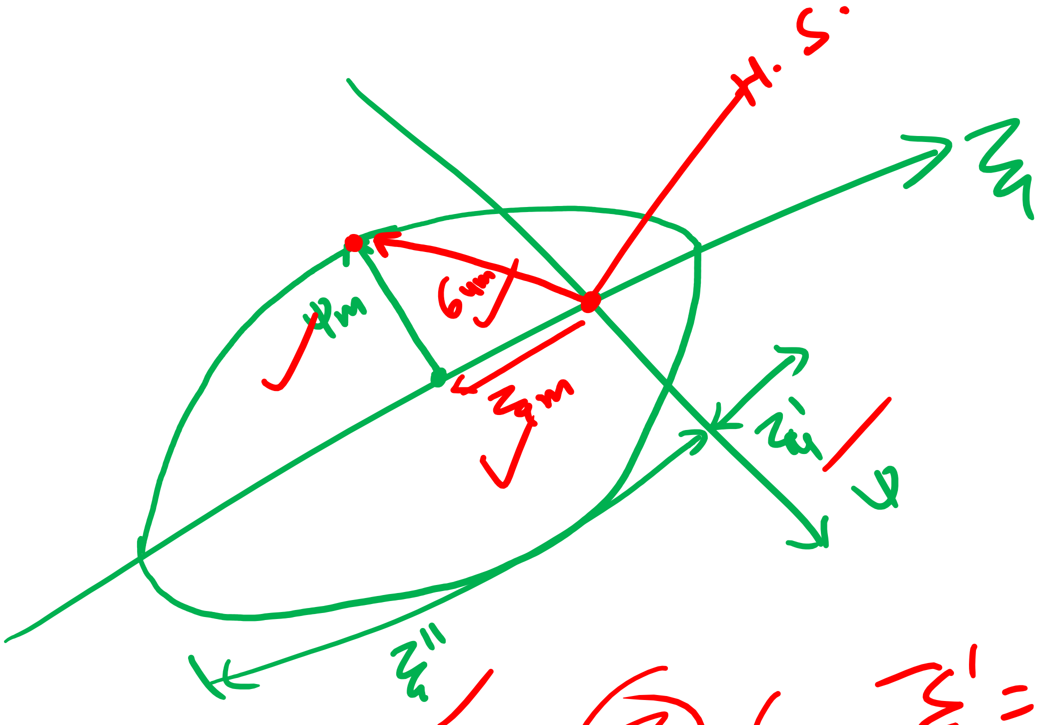
Moving Heat Sources: thin plate



Moving Heat Sources: thin plate



Moving Heat Sources: thin plate



Length of isotherm

ξ_1', ξ_1''

$G_u = \pm \xi_1$

$$G_u = \begin{cases} \xi_1 & \xi_1' = \ln\left(\dfrac{n_3 K_0(\xi_1')}{\theta\delta}\right), \quad \xi_1 > 0 \\[2ex] -\xi_1 & \xi_1'' = \ln\left(\dfrac{n_3 K_0(-\xi_1'')}{\theta\delta}\right), \quad \xi_1'' < 0 \end{cases}$$

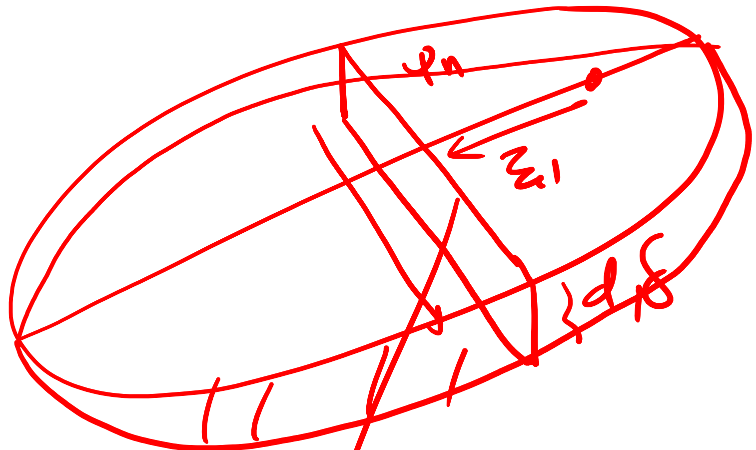
Moving Heat Sources: thin plate

$$\Lambda_2 = 2\psi_n \times \delta$$

$$\Lambda_2 = 2\sigma u m \sqrt{1 - \frac{[K_0(\sigma u m)]^2}{[K_1(\sigma u m)]^2}}$$

$$\boxed{A_2 = \frac{4\alpha^2}{v^2}\Lambda_2}$$

Dimensional area



Λ_2

Dimensionless area

Moving Heat Sources: thin plate

Cooling time :- $\sigma_u = -\xi_1 = \tau$

$$= \frac{v^2 t}{2\alpha}$$

Dimension cooling time

discussed in thick plate class

$$\frac{\theta\delta}{n_3} \sim \sqrt{\frac{\pi}{2t}}$$

$$\boxed{\Delta\tau = \frac{\pi}{2}\left(\frac{n_3}{\delta}\right)^2 \left[\frac{1}{\theta_2^2} - \frac{1}{\theta_1^2}\right]}$$

θ_2 θ_1

$$\Delta\tau = \frac{v^2 \Delta t}{2\alpha}$$

$$\boxed{\Delta t = \frac{2\alpha\, \Delta\tau}{v^2}}$$

Moving Heat Sources: thin plate

Cooling rate: –

$$-\frac{\partial\left(\frac{\theta\delta}{n_3}\right)}{\partial\tau} = \frac{1}{\pi}\left(\frac{\theta\delta}{n_3}\right)^3$$

$$\text{Cooling rate} = \frac{q_0 v^2}{4\pi^2\alpha^2\rho c d}\left(\frac{\theta\delta}{n_3}\right)^2$$

$$\boxed{C \cdot R = \frac{2\pi K \rho c}{\left(\frac{q_0}{v d}\right)^2}(T-T_0)^3}$$

°C/s / K/s

Moving Heat Sources: thin plate

$$C.R. = \frac{2\pi K \rho C}{\left(\frac{q_0}{Vd}\right)^2}(T-T_0)^3$$

$$= \frac{2\pi K \rho C}{\left(\frac{H_{net}}{\text{thickness}}\right)^2}(T-T_0)^3$$

$$= 2\pi K \rho C \left(\frac{\text{thickness}}{H_{net}}\right)^2 (T-T_0)^3$$

$$\frac{q_0}{V} = H_{net} = \frac{\eta I V}{V_{\text{speed of weld}}}$$

Moving Heat Sources: thin plate

Numerical: thin plate

Consider GTA butt welding of a 2mm thick sheet of cold-rolled aluminum (Al-Mg alloy) under the following conditions:

I=110A, Voltage=15V, V=4mm/s, efficiency: 0.6, T_0=20°C, T_c=T_m= 650°C, k=0.149W/mm.°C, ρC=0.0027J/mm^3.°C, α=55 mm^2/s, ΔH=1.7J/mm^3

Sketch the contours of the fusion boundary in the ξ-ψ(x-y) plane at a pseudo-steady state.
Calculate also the cross-sectional area of the fully recrystallized HAZ and the cooling rate at 275°C for points located within this region.

Numerical: thin plate

Sol^n

(a) $\theta = \frac{T - T_0}{T_c - T_0} = \frac{650 - 20}{650 - 20} = 1$

$\delta = \frac{V d}{2\alpha} = \frac{4 \times 2}{2 \times 55} = 0.07273$

$\frac{\theta \delta}{\eta_3} = \frac{\theta \times \frac{\cancel{V} d}{\cancel{2\alpha}}}{\frac{q_0 \cancel{V}}{2 \cancel{4} \pi \cancel{\alpha^2} \Delta H}} = \frac{\theta d \, 2\pi \Delta H}{q_0}$

cur.

vol.

$q_0 = \eta I \times V$

$= \frac{1 \times 2 \times 2 \times \pi \times 1.7}{0.6 \times 110 \times 15}$

$= 1.186$

$\boxed{\frac{\eta_3}{\theta \delta} = 0.84}$

Numerical: thin plate

$$\xi_q' = \ln\left(\frac{n_3 K_0(\xi_q')}{\Theta \delta}\right)$$

$$\boxed{\xi_q' = \ln(0.84\, K_0(\xi_q'))}$$

$\xi_q' = 0.25$

$\xi_q' = \frac{v x'}{2\alpha}$

$x' = 0.25 \times \frac{2\alpha}{v} \rightarrow 27.5$

$= 0.25 \times \frac{\cancel{2} \times 55}{1/2}$

$= 6.875$ mm

$$\xi_q'' = \ln\left(\frac{n_3}{\Theta\delta} K_0(-\xi_q'')\right)$$

$= -0.9$

$x'' = -0.9 \times 27.5$

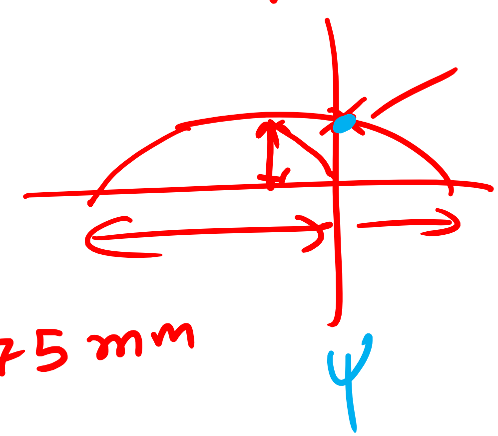
$= -24.75$ mm

$\xi_q',\ \xi_q''$

$\left\{\begin{array}{l}\psi_m,\ \sigma_{qm} \\ \xi_q m\end{array}\right.$

ψ, at $\xi_q = 0$

intersection point



Numerical: thin plate

$\sigma_{4m} = ?$

$\psi_m = \sqrt{\sigma_{4m}^2 - \xi_{4m}^2}$

$\dfrac{\theta_p \delta}{n_3} = \exp\left(\sigma_{4m} \dfrac{K_0(\sigma_{4m})}{K(\sigma_{4m})}\right) K_0(\sigma_{4m})$

$\xi_m = -\dfrac{K_0(\sigma_{4m})}{K_1(\sigma_{4m})} \sigma_{4m}$

at $\theta_p = 1$

$1.186 =$

$\sigma_{4m} = 0.5$

$\sigma_{4m} \rightarrow \theta = 0.4047$

$r_{4m} = 0.5 \times 27.5 = 13.75$ mm

Numerical: thin plate

$$\xi_{qm} = \frac{-K_0(\sigma_{4m})}{K_1(\sigma_{4m})} \sigma_{4m}$$

$$= \frac{-K_0(0.5)}{K_1(0.5)} \times 0.5 = -0.29$$

$$x_m = -0.29 \times 27.5$$

$$= -7.98 \text{ mm}$$

$$\psi_m = \sqrt{\sigma_{4m}^2 - \xi_{qm}^2} = \sqrt{0.5^2 - (-0.29)^2}$$

$$= 0.41$$

$$y_m = 0.41 \times 27.5$$

$$= 11.275 \text{ mm}$$

Numerical: thin plate

Intersection point on $\psi(y)$ and when $\xi = 0$

$$\sigma_4 = \sqrt{\xi_4^2 + \psi^2}$$

$$\sigma_4 = \psi$$

$$\frac{\theta \delta}{n_3} = \exp(0)\, K_0(\psi)$$

$$1$$

$$1.186 = K_0(\psi)$$

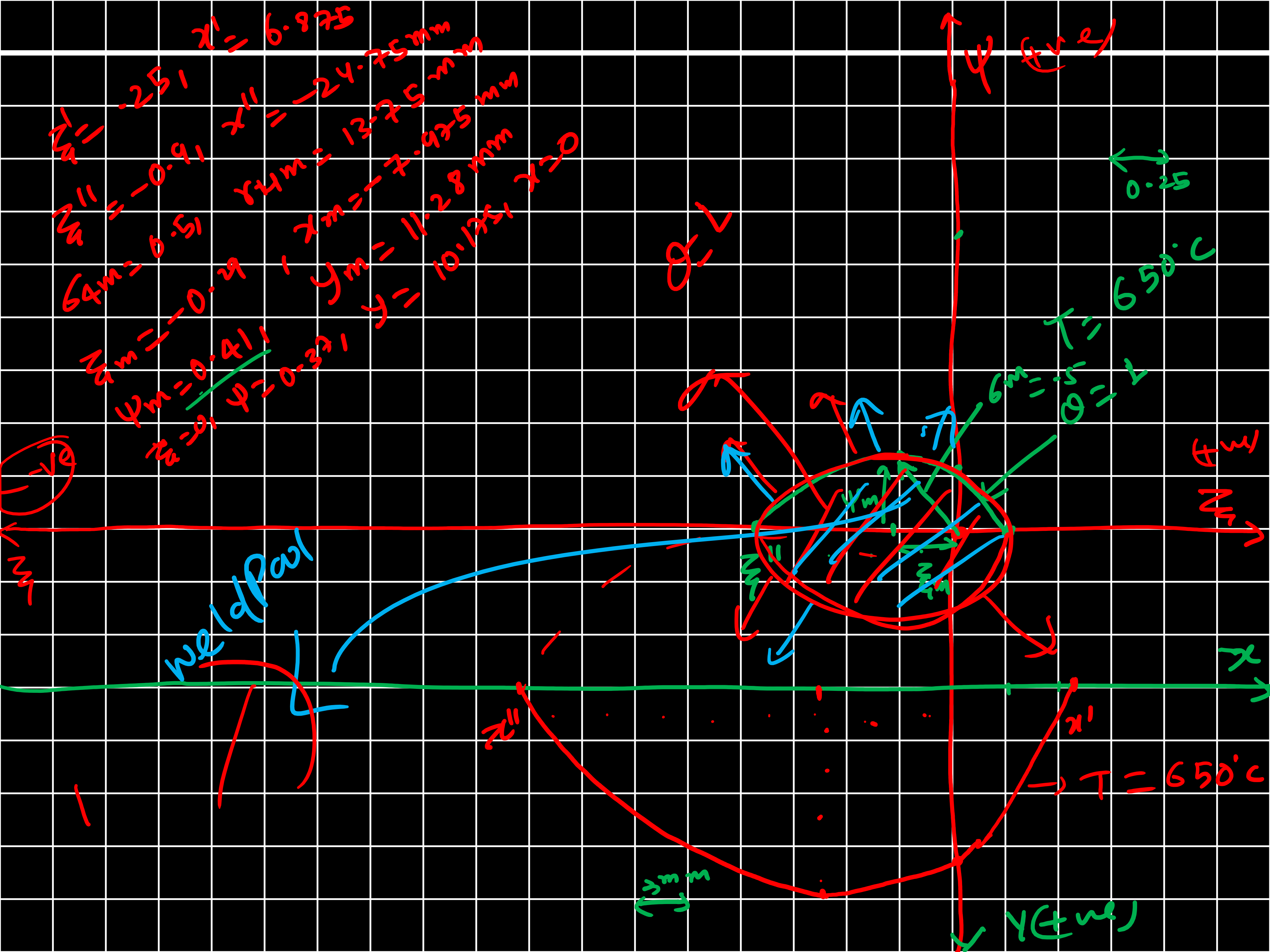
$$\psi = 0.37$$

$$y = 0.37 \times 27.5$$

$$= 10.175 \text{ mm}$$

② $$\frac{\theta \delta}{n_3} = \exp(-\xi)\, K_0(\sigma_4)$$

Numerical: thin plate

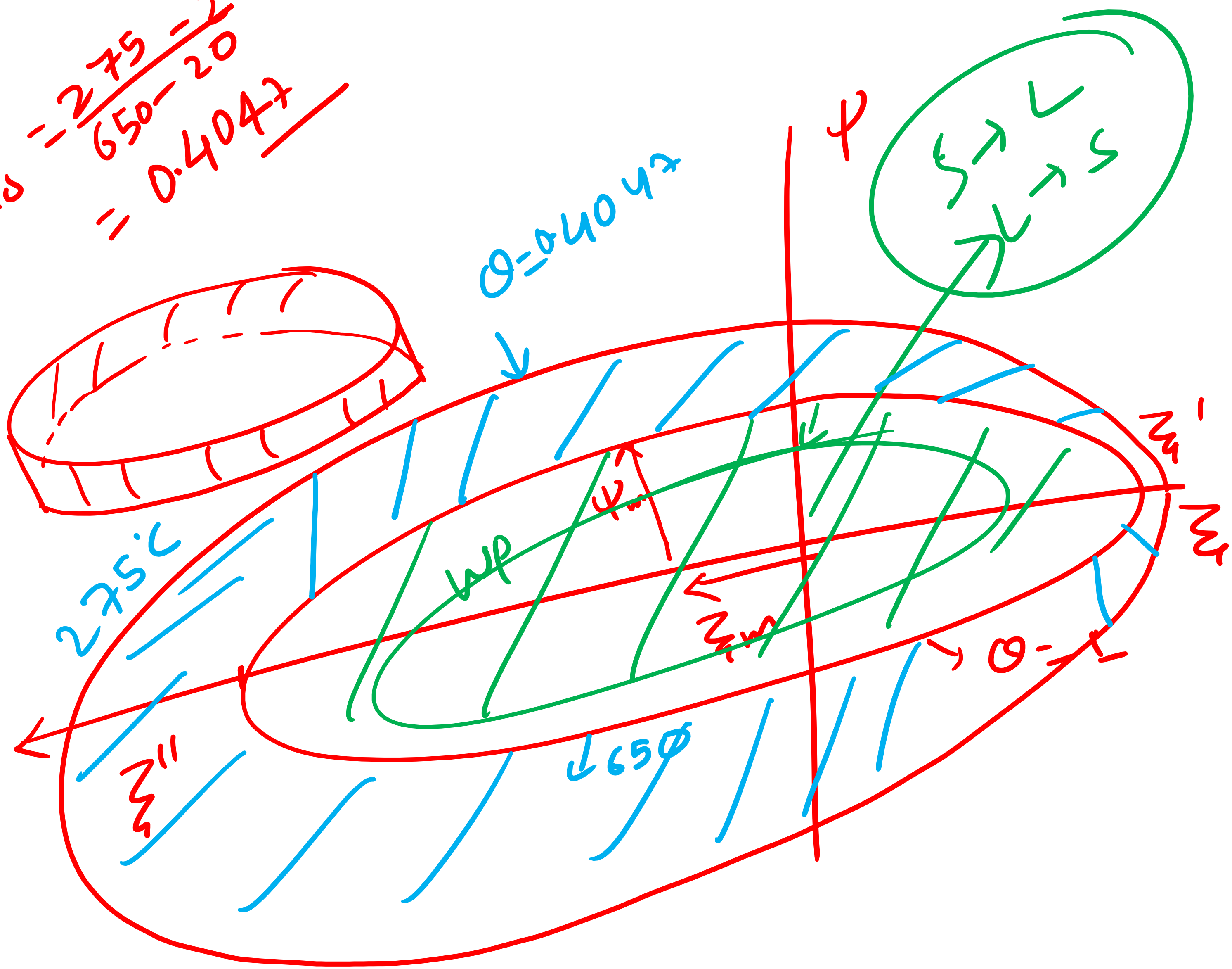


x' = 6.875
ξa' = -.25
x'' = -24.75 mm
ξa'' = -0.91
xψm = 13.75 mm
σm = 0.51
xm = -7.975 mm
ξm = -0.29
ym = 11.28 mm
y = 10.125, x = 0
ξm = -0.29
ψm = 0.41
ψ = 0.37
ξa = .91
θ = 1
ψ (+ve)
0.25
T = 650°C
σm = .5
θ = 1
(+ve)
ξa
(-ve)
ξa
weldpool
ψm
ξ''
ξm
x
x''
x'
T = 650°C
3mm
Y(+ve)

Numerical: thin plate

$T = 275^\circ C$

$$\theta = \frac{T - T_0}{T_c - T_0} = \frac{275 - 20}{650 - 20} = 0.4047$$



Numerical: thin plate

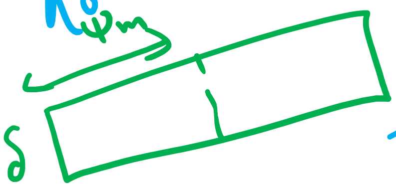
at 275°C , $\theta = 0.4047$

$$\frac{\theta \delta}{n_3} = \frac{\theta \times d \times 2\pi\alpha(\Delta H)}{q_0}$$

$$= \frac{0.4047 \times 2 \times 2 \times \pi \times 55 \times 1.7}{0.6 \times 110 \times 15}$$

$$= 0.4047 \times 1.186 = 0.48$$

$A_0 = 0.6 \times 110 \times 15$



for $\theta = 0.4047$, $\psi_m = ?$

$\theta = 0.4047$

$\psi_m = 1.5$

$A_2 = 2\psi_m \times \delta$

Numerical: thin plate

$\theta = 1$

$\psi_m = 0.41$

$\theta = 0.4047$

$\psi_m = 1.5$

$\Lambda_2 = 2\psi_m \delta$

$\Delta\Lambda_2 = 2\Delta\psi_m \delta$

$= 2(1.5 - 0.41) \times \delta \rightarrow 0.07273$

$= 2 \times 1.09 \times 0.07273$

$= \underline{0.0158}$

$\Lambda_2 =$

$A_2 = \frac{4\alpha^2}{V^2}\Lambda_2$

$\Delta A_2 = \frac{4\alpha^2}{V^2} \times \Delta\Lambda_2$

Numerical: thin plate

$$= \frac{4 \times 55^2}{4^2} \times 0.0158$$

$$= 11.99\ mm^2$$

Cooling rate at $T \Rightarrow 275°C$

$$C.R. = \frac{2\pi k \rho C}{\left(\frac{90}{Vd}\right)^2} \left(T - T_0\right)^3$$

$T = 275°C$

Numerical: thin plate

$$= \frac{2 \times \pi \times 0.149 \times 0.0027}{\left(\frac{0.6 \times 110 \times 15}{4 \times 2}\right)^2} \times (275-20)^3$$

$q_0 = 0.6 \times 110 \times 15$

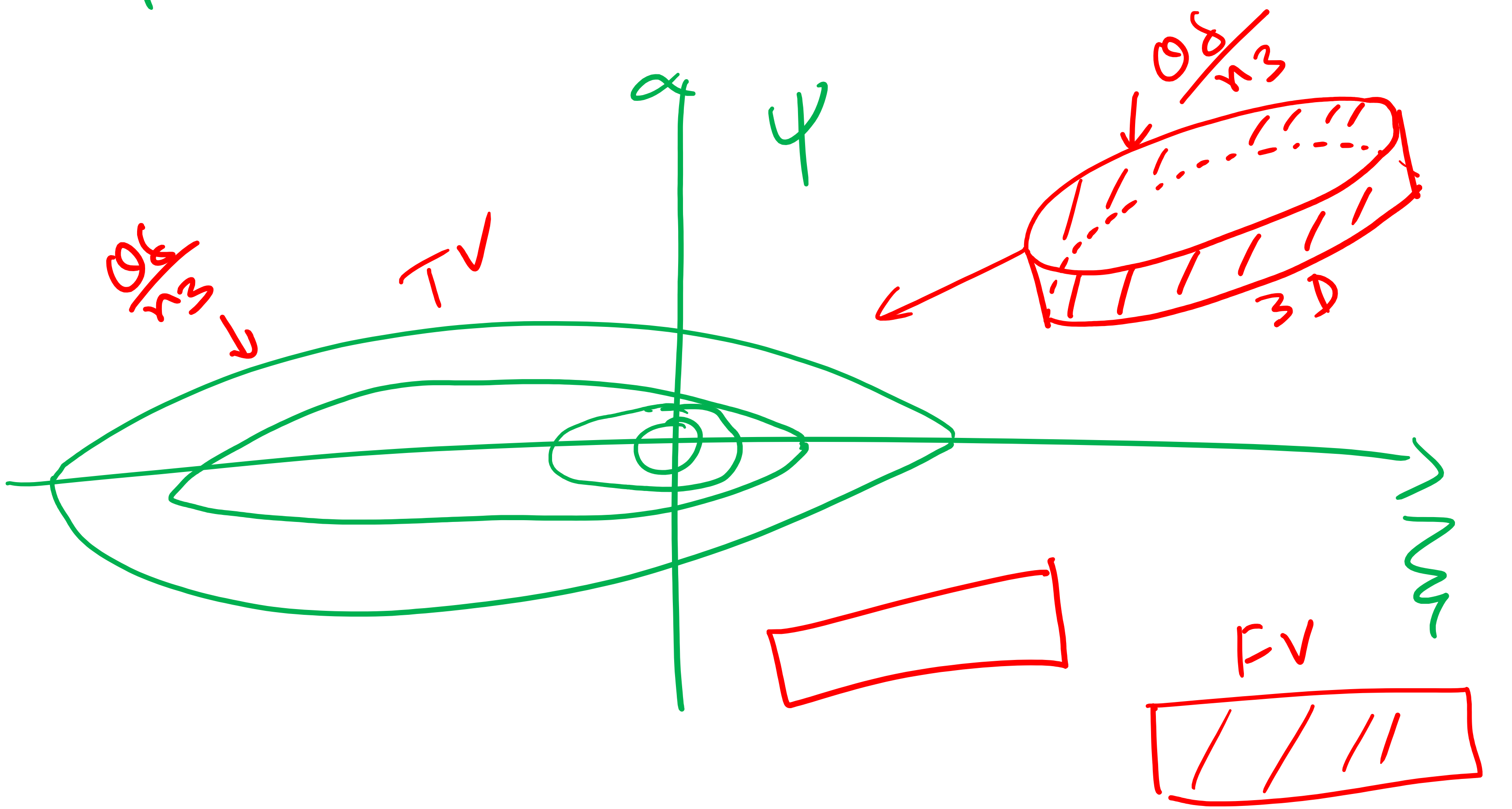
$$= \underline{\underline{2.7\,°C/sec}}$$

Numerical: thin plate

Numerical: thin plate

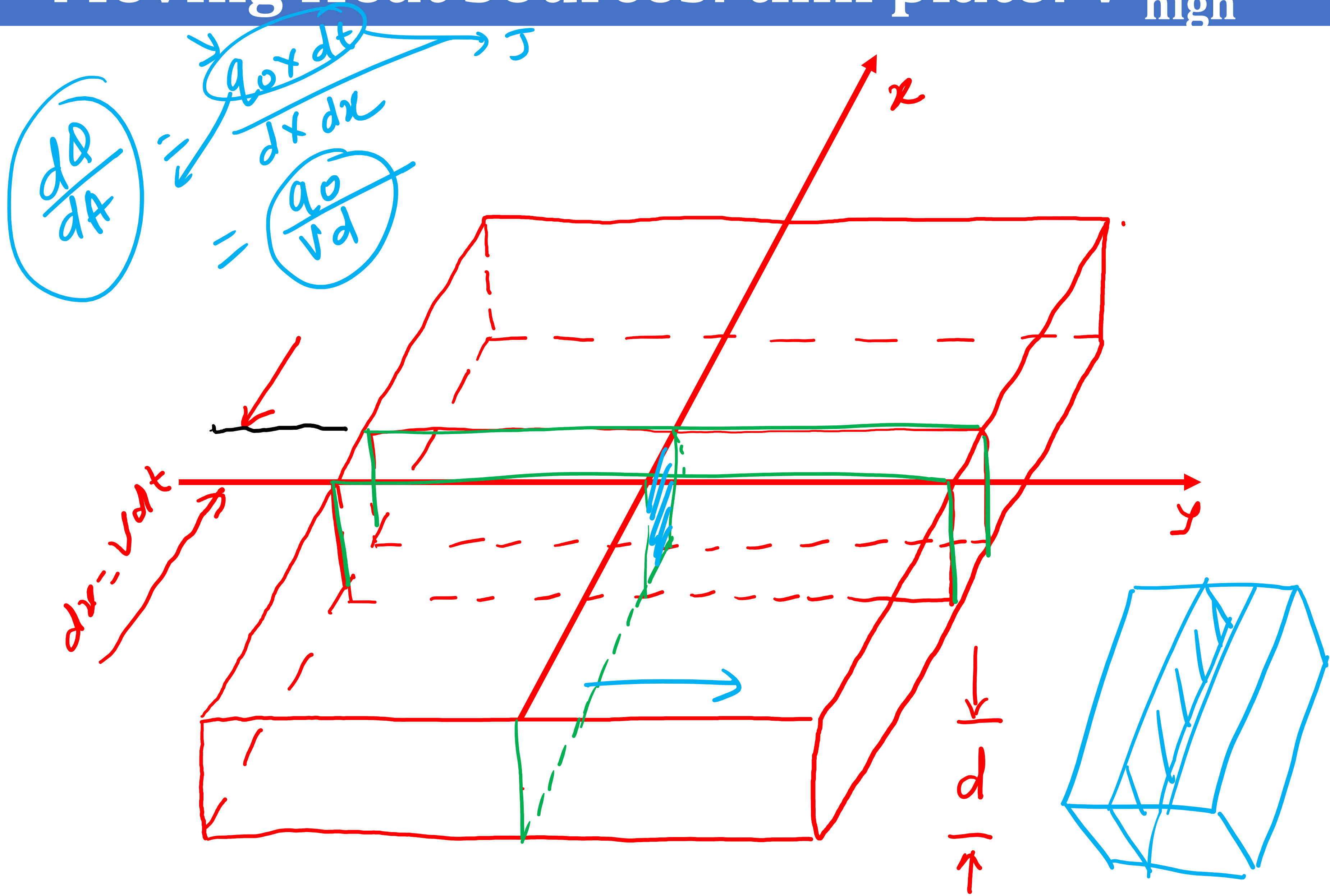
Moving Heat Sources: thin plate: $\vec{V}_{high}$

$$T = T_0 + \frac{q_0}{2\pi k d} \exp\left(-\frac{Vx}{2\alpha}\right) K_0\left(\frac{Vr}{2\alpha}\right)$$



Moving Heat Sources: thin plate: V_{high}

Moving Heat Sources: thin plate: V_{high}



Moving Heat Sources: thin plate: V $_{high}$

Temp. distribution in plane heat source condition

Instantaneus heat condition

$$\frac{Q}{A} = \frac{q_0}{Vd}$$

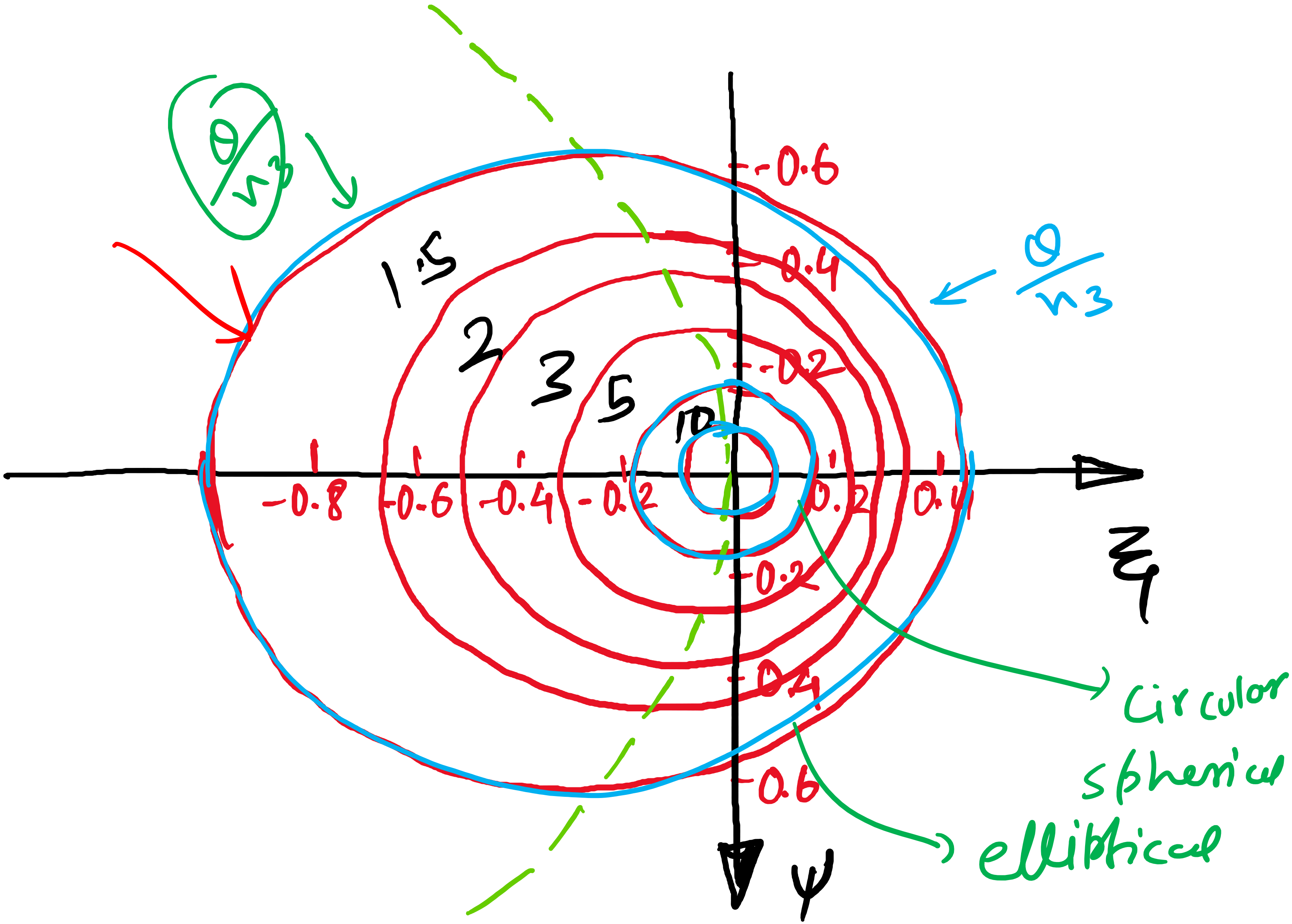
$$T = T_0 + \frac{Q/A}{\rho c\,(4\pi\alpha t)^{1/2}} \exp\left(\frac{-y^2}{4\alpha t}\right)$$

$$T - T_0 = T_0 + \frac{\frac{q_0}{Vd}}{\rho c\,(4\pi\alpha t)^{1/2}} \exp\left(\frac{-y^2}{4\alpha t}\right)$$

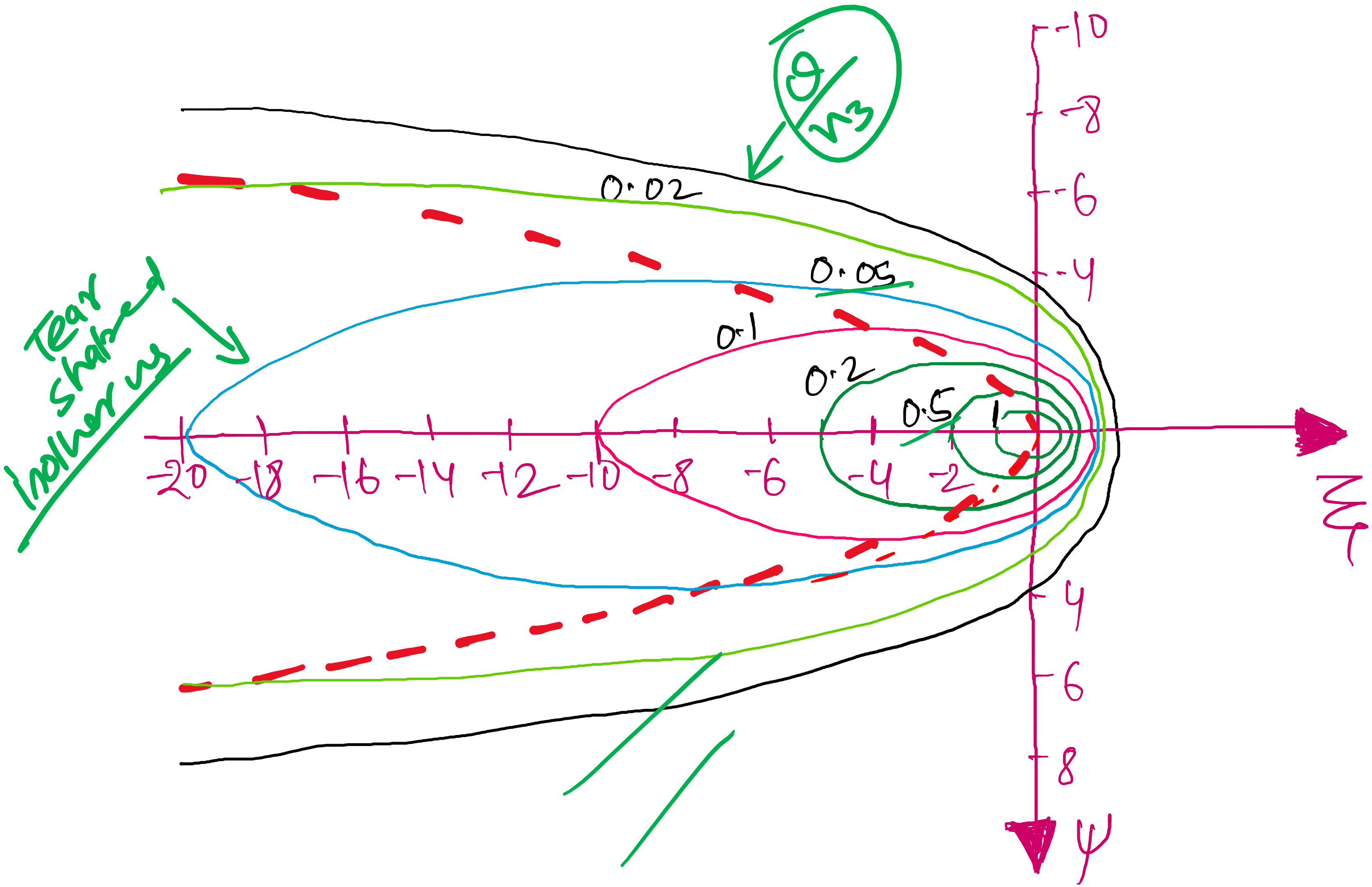
$$\boxed{T = T_0 + \frac{q_0}{Vd\,\rho c\,(4\pi\alpha t)^{1/2}} \exp\left(\frac{-y^2}{4\alpha t}\right)}$$

Moving Heat Sources: thin plate: V_{high}

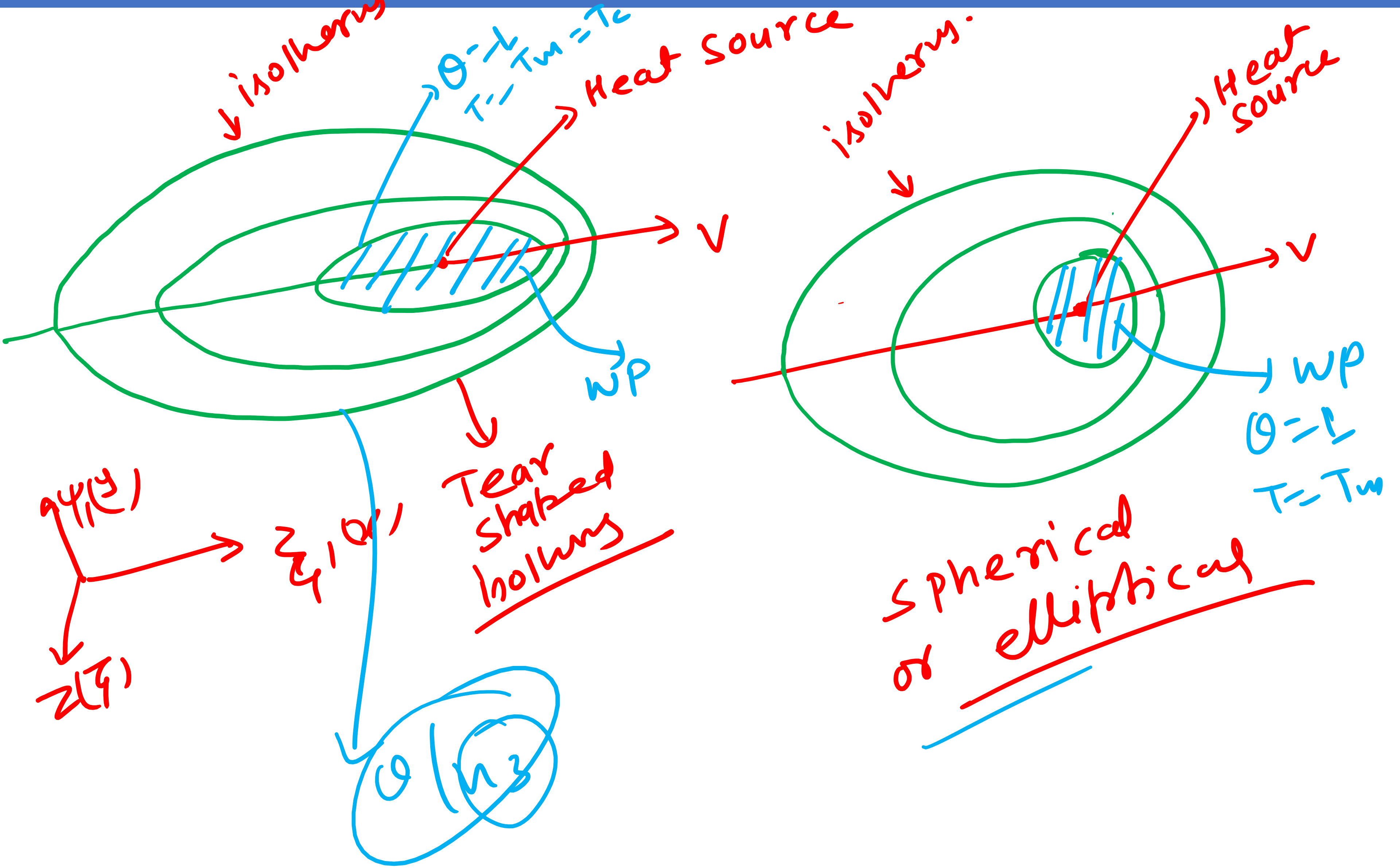
Type of isotherms



Type of isotherms



Type of isotherms



Type of isotherms

$\eta_3 = \dfrac{\nu\, q_0 v}{4\pi \alpha^2 \Delta H}$

$\dfrac{\theta}{\eta_3} =$ given θ → high

low

low

Tear shaped isotherms

high

→ high speed of welding
High heat input
→ low thermal diffusivity
like austenitic steinless steel

spherical or elliptical isotherms

low arc input,
low speed
or
thermal diffusivity should high
like Al

$\dfrac{\theta}{\eta_3} =$ low

given $\theta = 1$ $\dfrac{\theta}{\eta_3} =$ low

Adam's Solution: Peak Temperature

- Predicting metallurgical transformations at that specific location
- For a single pass, the distribution of peak temperatures T_P in the base material adjacent to the weld is given by

$$\frac{1}{T_P - T_0} = \frac{(2\pi e)^{0.5}\rho C t}{Hnet} Y + \frac{1}{T_m - T_0},$$

$$Hnet = \frac{\eta I V}{U} (J/mm)$$

$$\frac{1}{T_P - T_0} = \frac{4.13\rho C t}{Hnet} Y + \frac{1}{T_m - T_0}$$

t = Thickness of the base material (mm)

Y is the position of point from the fusion zone boundary

Numerical: Peak Temperature

1. A single full penetration weld pass is made on low alloy steel using the following parameters: I=200A, Voltage=20V, welding speed= 5mm/s, the thickness of plate= 5 mm, arc efficiency: 0.90, T_0=25°C, Tc=Tm= 1510°C, ρC_p=0.0044 J/mm^3.°C. Use Adam's approach to solve the problem.

a. Calculate the peak temperatures at 1.5 and 3 mm distances from the weld fusion boundary.

b. Estimate the width of heat affected zone if $T_{recrystalization}$=730°C.

c. Estimate the heat-affected zone's width if the sample plate was tempered at 430°C as well as preheated at 200$^{\circ}C$.

d. Estimate the width of heat affected zone if the sample plate was tempered at 430°C without preheating.

e. Estimate the width of heat affected zone if the sample plate was tempered at 430°C without preheating, but the heat input rate is increased by 50%.

Numerical: Peak Temperature

Numerical: Peak Temperature

Cooling rate

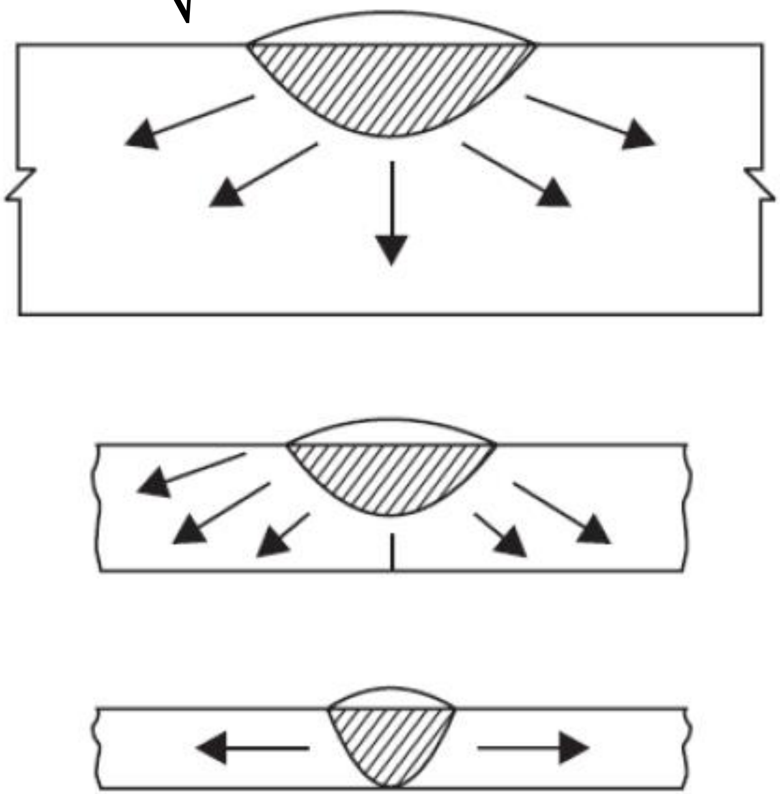
- For thick plates: $R = \frac{2\pi k(T_C - T_0)^{\mathbf{2}}}{Hnet}$
- For thin plates: $R = 2\pi k \boldsymbol{\rho C}(T_C - T_0)^{\mathbf{3}} \left(\frac{\boldsymbol{h}}{\boldsymbol{Hnet}}\right)^{\mathbf{2}}$

- R=*Cooling rate at the weld centreline* (K/s)
- C=*Heat capacity*
- Tc=*Temperature at which the cooling rate is calculated* (K)
- k=*Thermal Conductivity* (J/mm.s.K)
- h= *Thickness of the base metal* (mm)
- To = *Initial Temperature*

Cooling rate

$$\tau = h\sqrt{\frac{\boldsymbol{\rho C}((T_C - T_0)}{Hnet}}, \tau < 0.6: thin - plate\ approximation$$

$$\tau = h\sqrt{\frac{\boldsymbol{\rho C}((T_C - T_0)}{Hnet}}, \tau > 0.9: thick - plate\ approximation$$



Three dimensional heat flow $\tau > 0.9$

Intermediate condition $0.6 < \tau < 0.9$

Two dimensional heat flow $\tau < 0.6$

Cooling rate

$$\tau = h\sqrt{\frac{\boldsymbol{\rho C}((T_C - T_0)}{Hnet}},$$

$$\tau < 0.75: thin - plate\ approximation\ is\ more\ appropriate$$

$$\tau = h\sqrt{\frac{\boldsymbol{\rho C}((T_C - T_0)}{Hnet}},$$

$$\tau > 0.75: thick - plate\ approximation\ is\ more\ appropriate$$

Numerical: Cooling rate

1. A steel plate of thickness 5mm is arc welded with an arc voltage of 25V and arc current of 280A. Find out the critical cooling rate of steel if it is found that welding speed above 10mm/s leads to cracking in HAZ.

Given: ambient temperature of 25°C, Tc=Tm= 530°C, arc efficiency: 0.85, ρC_p=0.0044 J/mm^3.K, and k = 0.028 J/mm.sec.K,

Numerical: Cooling rate

1. Find the best welding speed to be used for welding 6 mm steel plates with an ambient temperature of 30°C with the welding transformer set at 25V and current passing is 300A. The arc efficiency is 0.9, and the possible weld speed ranges from 5-10 mm/s. The limiting cooling rate for satisfactory performance is 6°C/s at a temperature of 550°C.

Given: ρC_p=0.0044 J/mm^3.K, and k = 0.028 J/mm-sec-°C.