Welding Technology
ME692

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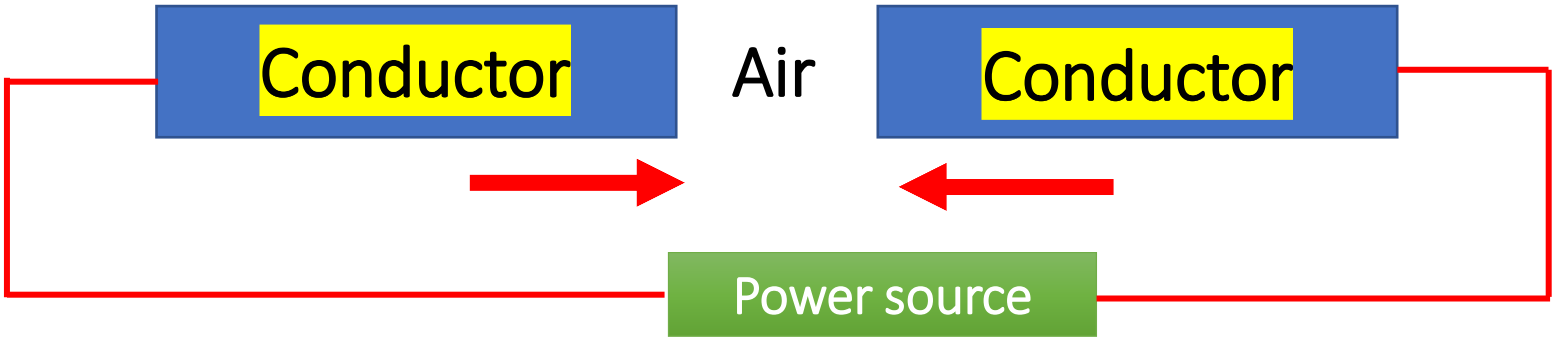
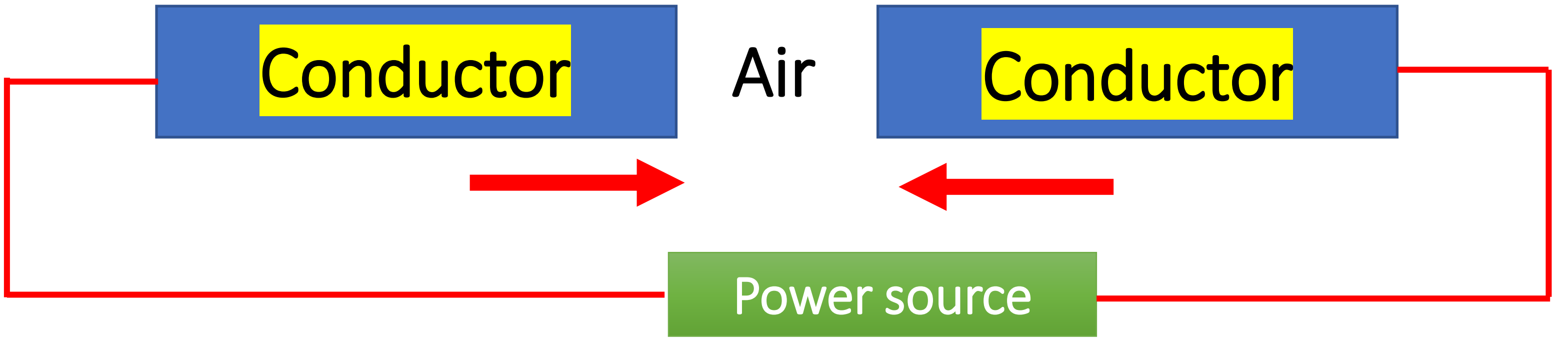
Discharge



- ✓ Decrease in the gap between conductors: no current flow
- ✓ If we apply more energy between two conductors: **Thermionic emission**
- ✓ Further decrease in the gap as well as apply high energy... air ionized
- ✓ Thermionic emission is the liberation of electrons from an electrode by its temperature.

Discharge



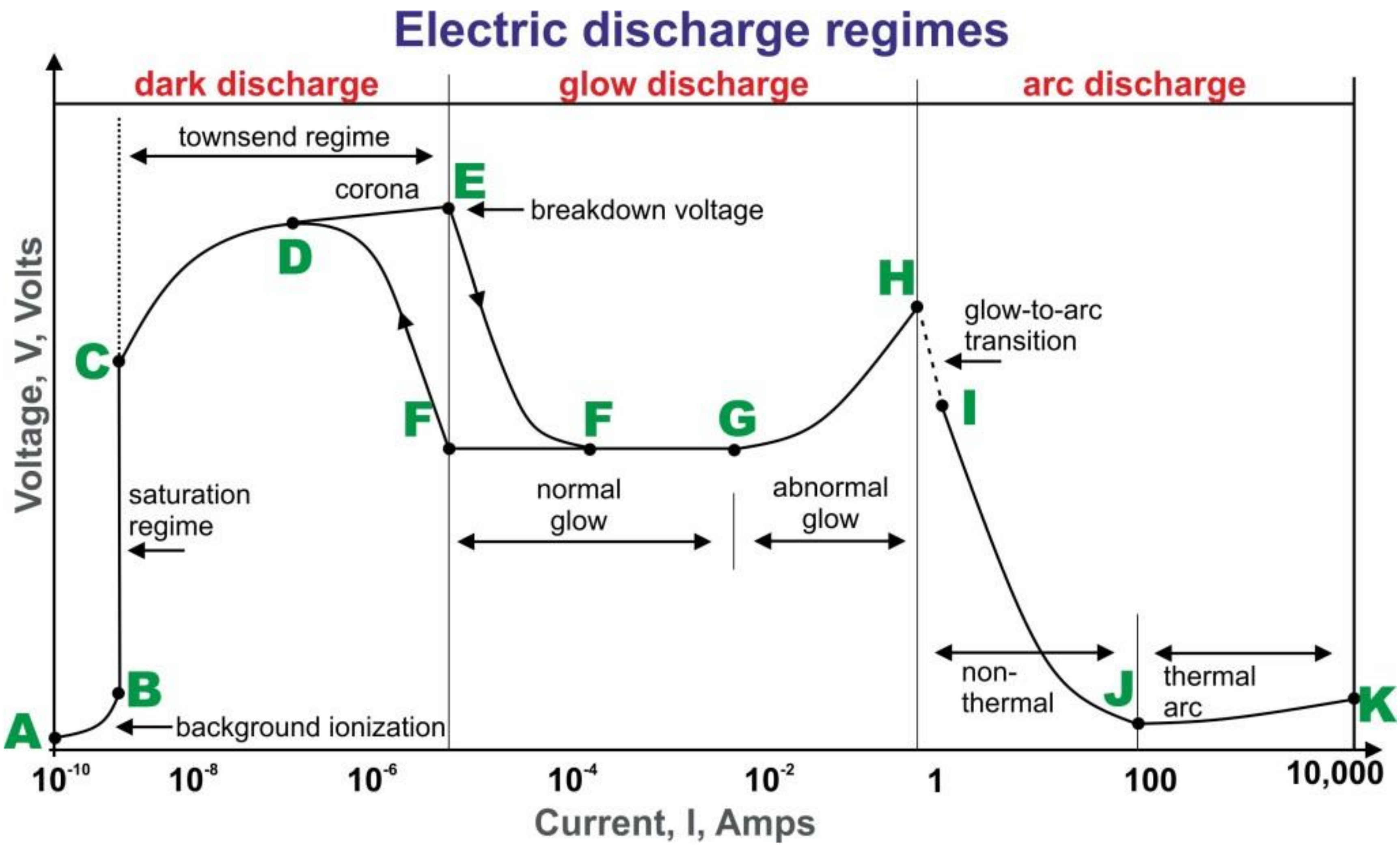
- ✓ More electrons will interact with air, so gas atoms and molecules will become ionized.
- ✓ Ionized: gas atoms or molecules lose electrons during the process, so then they become positive ions. This process creates more energy carriers (electrons, ions). It conducts energy and charges.
- ✓ This process is known as discharge.

Discharge

- ✓ Discharge example: lightning. Thunderbolt
- ✓ When the carrier passes through the gas medium, three types of discharge:
  - ✓ A. Momentary (0.01s): Spark and lightning
  - ✓ B. Townsend discharge, glow discharge, and arc discharge
    - ✓ Townsend discharge: photovoltaic effects (high voltage and low current)
    - ✓ Glow discharge: house lighting (medium V and normal current)
    - ✓ Arc discharge: low voltage and high current

Discharge

Electric discharge: Avalanche of ions & electrons



https://www.plasma-universe.com/electric-glow-discharge/

ARC

- ✓ Arc: A sustained electric discharge in a gas: movement of free electrons and ions of atoms and molecules in between a potential difference.

- ✓ A welding arc is a gaseous electrical conductor that changes electrical energy into heat.
- ✓ It involves electrical discharges formed and sustained by the development of conductive charge carriers in a gaseous medium.

ARC Welding

- ✓ Path of electron and ion is decided by polarity.
- ✓ The electron moves from the cathode to the anode.
- ✓ Positive ions: anode to cathode.
- ✓ Negative ions: cathode to anode
- ✓ IN DCEN: Electrons move to WP, and ions move to the electrode.
- ✓ Energy transfer by the motion of energy carriers (electrons and ions).

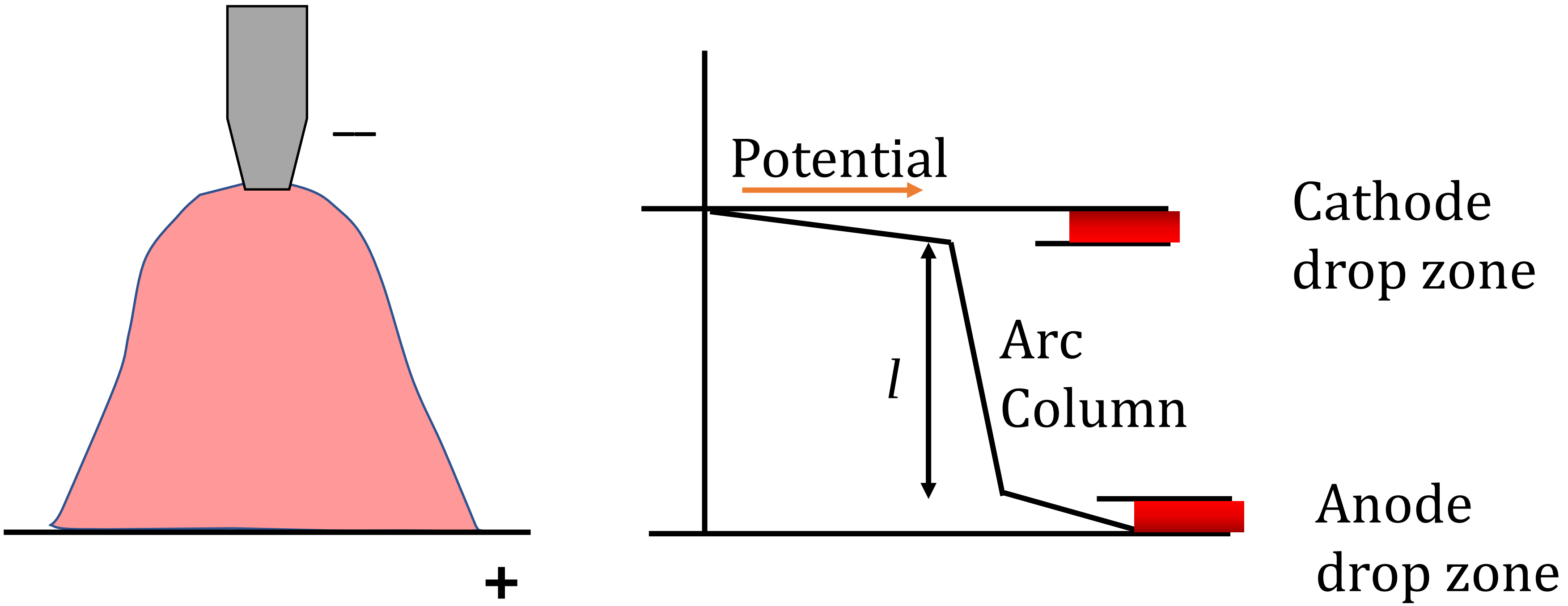
ARC Welding

- ✓ Electrons are much smaller than +ve ions but have higher kinetic energy (higher velocities)
- ✓ Electron has less mass and number is very high
- ✓ Even ions are heavy; during the collisions between ions and electrons carry more heat.

ARC Welding

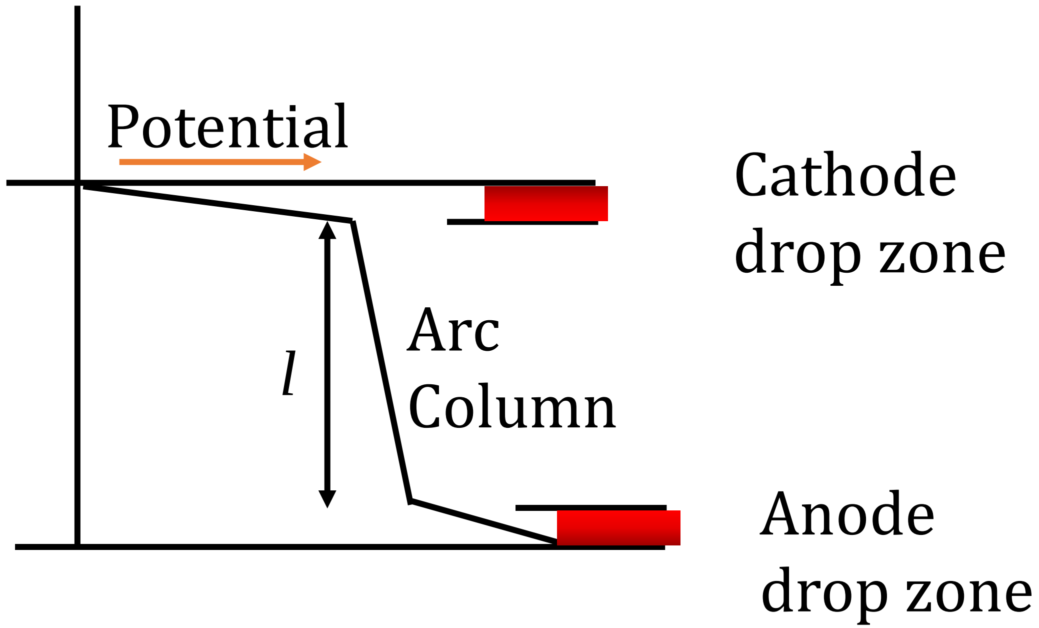
- ✓ Arc consists of thermally emitted electrons and positive ions from the electrode and workpiece.
- ✓ Electrons and ions are accelerated by the potential field (voltage) between the source and the work. Heat is produced when electrons and ions collide with the oppositely charged element.

Electric Arc: Arc Electrical Features



Cathode drop zone: the gaseous region which has a positive space charge, so a voltage drop is necessary as the electrons are to be pulled across this region.

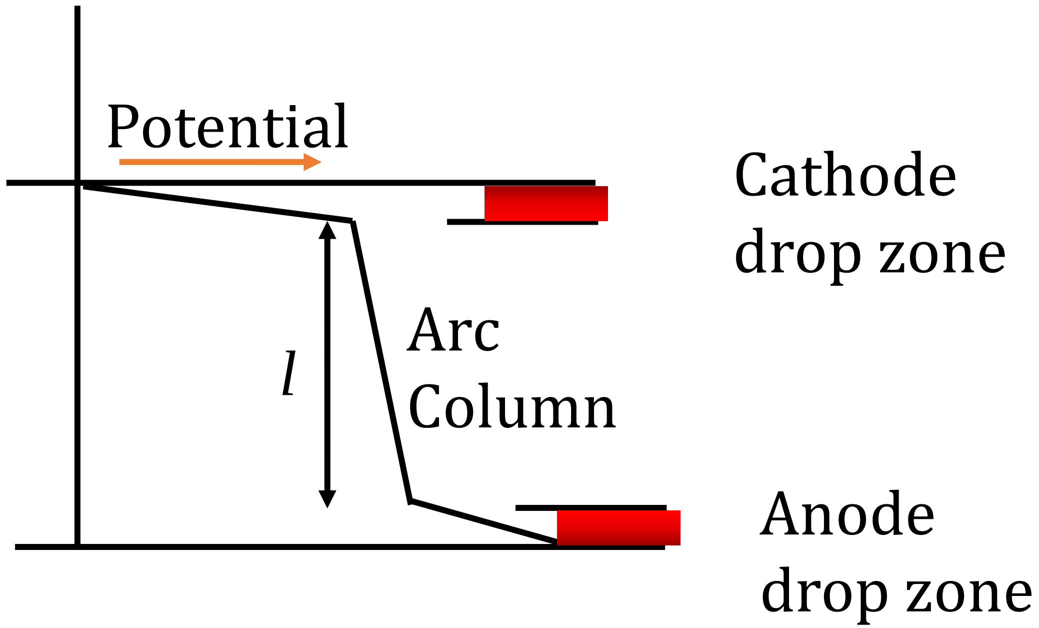
Electric Arc: Arc Electrical Features



Arc Column: This is the visible portion of the arc consisting of plasma where the voltage drop is not sharp.

✓ Arc consists of thermally emitted electrons and positive ions from the electrode and workpiece.

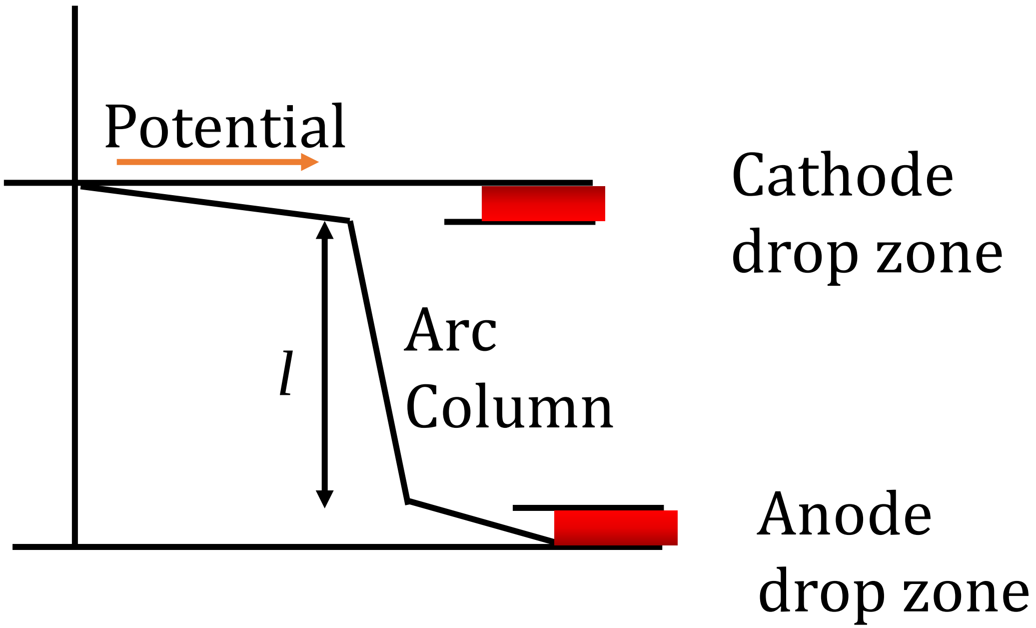
Electric Arc: Arc Electrical Features



Arc Column: This is the visible portion of the arc consisting of plasma where the voltage drop is not sharp.

✓A plasma is the ionized state of a gas, comprised of a balance of negative electrons and positive ions to maintain charge neutrality.

Electric Arc: Arc Electrical Features



✓Anode zone: This is the area in the anode surface where electrons are absorbed. This area is larger than the cathode spot and where a sharp drop in the voltage takes place.

ARC Welding

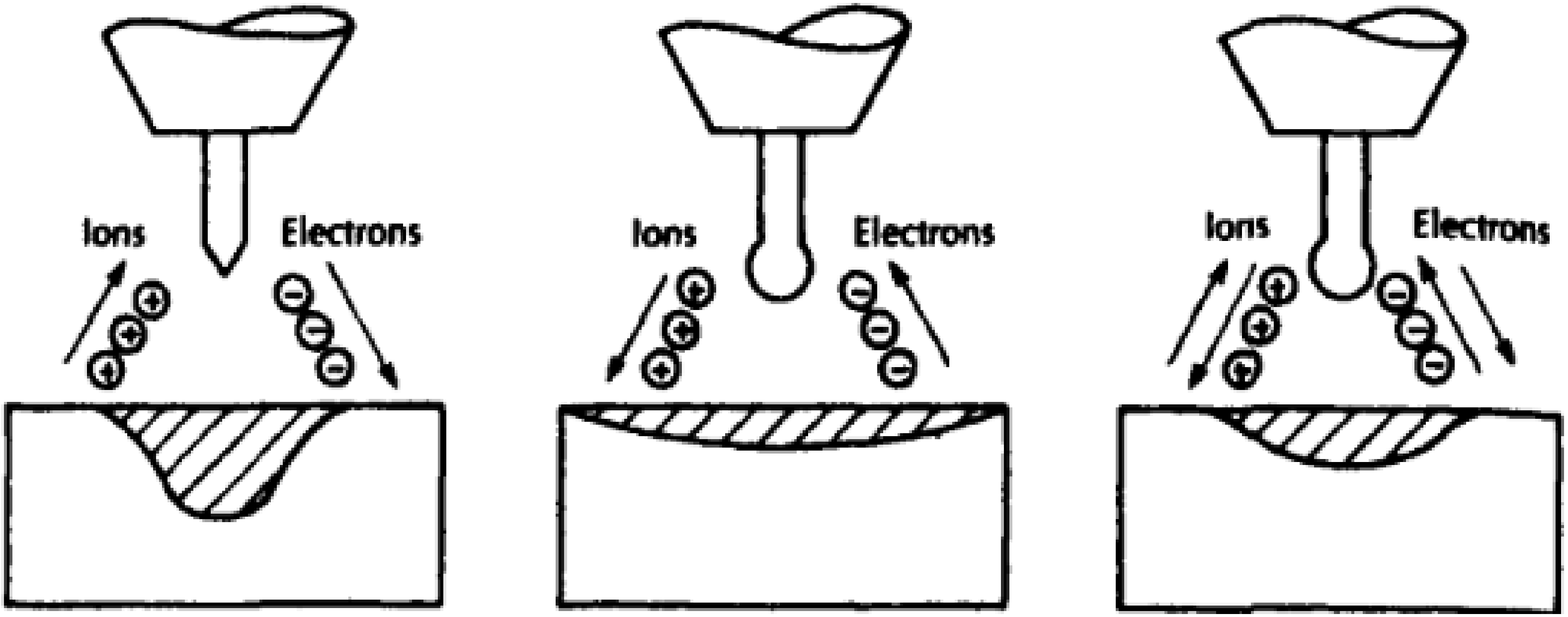
- ✓ IN DCEN: Wp has maximum temperature compared to electrode
- ✓ IN DCEP: Electrode has maximum temperature for consumable electrodes.

ARC Welding

ARC Welding

- ✓ Ions have heavy wt, so Bombardments of ions clean the surfaces.
- ✓ Al has oxide formation, so it needs both cleanings as well as heating, so AC,

Electric Arc: Current Mode



DCSP (DCEN)

- No Surface cleaning
- Deep weld
- 70% heat at work
- 30% heat at electrode
- Filler metal deposition rate is low

DCRP (DCEP)

- Strong cleaning action
- Shallow weld
- 30% heat at work
- 70% heat at electrode
- Filler deposition rate is high

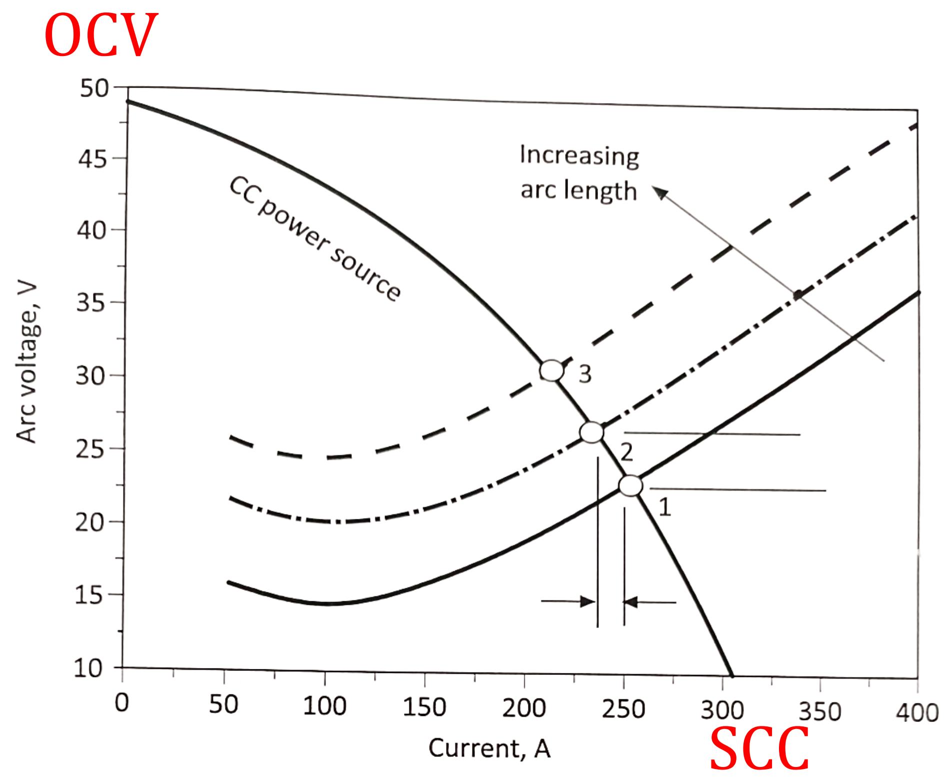
AC

- Cleaning every half cycle
- Sallow weld
- 50% heat at work
- 50% heat at electrode
- Good Electrode current capacity

ARC Welding

Constant-Current Power Sources

- ✓It is well suited for manual operation. It is widely used in SMAW.
- ✓Arc voltage depends on the arc gap or arc length.
- ✓CC shows a small variation in current due to a steep decrease in the V-I relationship.



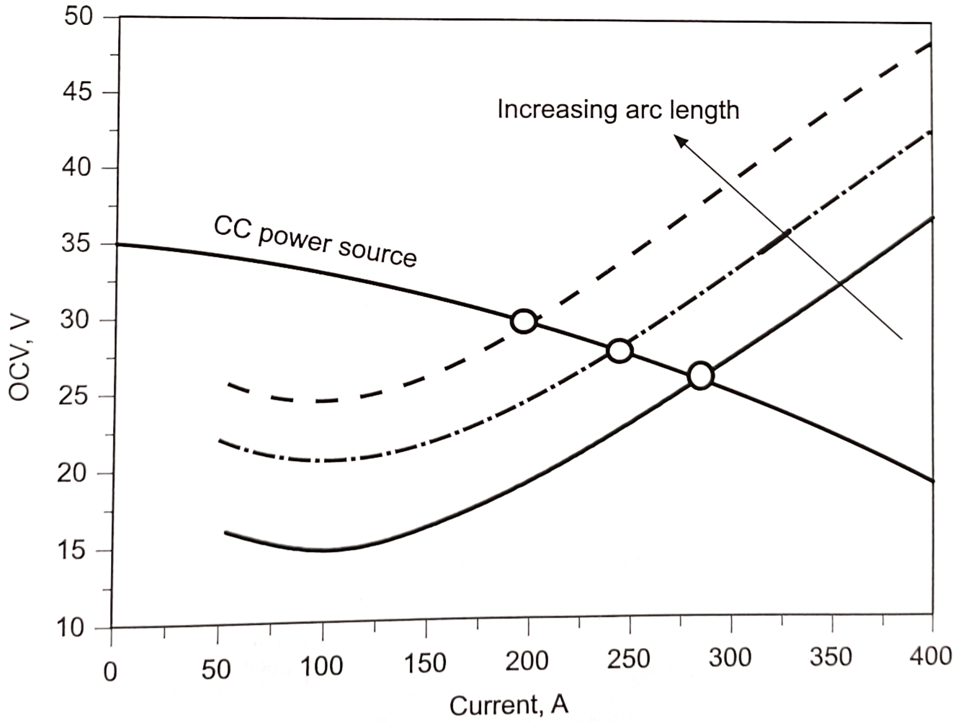
dI/dV should minimum (ignore sign)
V α arc length, V α –I

- ✓CC sources are sometimes called "drooping sources" or "droopers" because of the substantial downward slope or droop of the V-A curves.

ARC Welding

Constant-Voltage Power Sources

- ✓It is well suited for semi-automatic operation. It maintains constant arc voltage irrespective of current.
- ✓Used for consumable process.
- ✓In this cases arc gap is maintained by controllers so deposition rate can be controlled by changes in current.



Numerical Problems

1. For the manual arc welding process: what is most suitable among the following V-I relationships?

a. $V=80-0.1I$

b. $V=80-0.8I$

c. $V=80-0.5I$

d. $V=60-0.3I$

Numerical Problems

2. Determine the maximum power for a $4V+2I=240$ DC power source.

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Numerical Problems

3. The voltage-length characteristic of a DC arc is given by $V=20+40l$ volts, where l is the length of the arc in cm. The power source characteristic is approximated by a straight line with an open circuit voltage =80V and a short circuit current =1000A. Determine the optimum arc length and the corresponding arc power.

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Numerical Problems

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Numerical Problems

Duty Cycle

- It is specified for a welding machine that is apart from the Ampere-Volt rating.
- Duty cycle is % of the 10min period that equipment operates at rated power. This means there is no overheating

- Example: if welding equipment is the specified rating of 60% duty cycle, current of 300A, and voltage of 30V.
- It means the machine will work safely for 6 min at 300A and 30V.

- Machine duty cycle is based on output current rating, not on power.

Duty Cycle

- $I^2T = I_r^2T_r$

- Where T is % rated duty cycle, I is rated current
- T_r is required duty cycle, I_r is maximum required current output at desired duty cycle

Numerical problem:

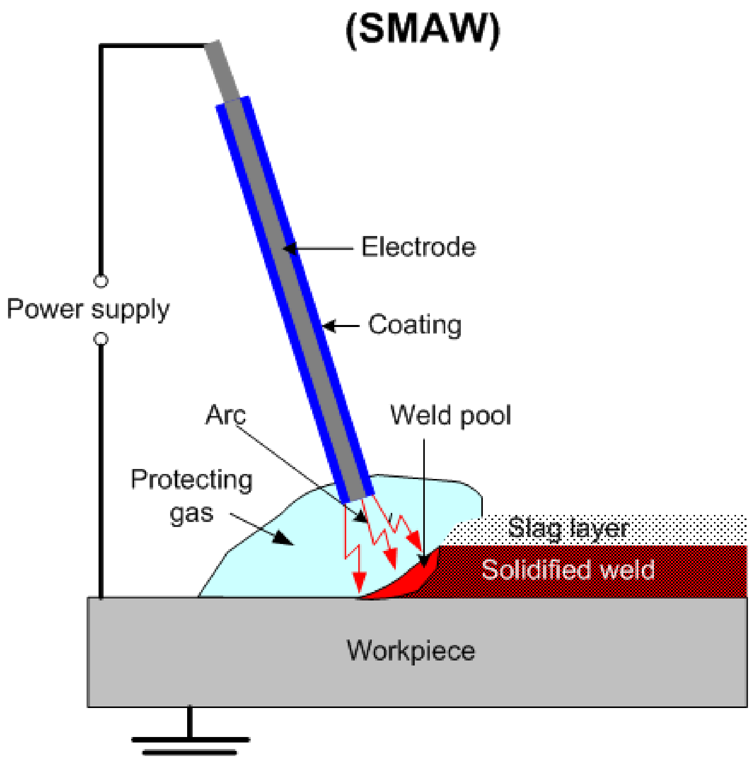
1. What will be the duty cycle with a 200A power supply rated at 60% duty cycle operated at the current 250A output?

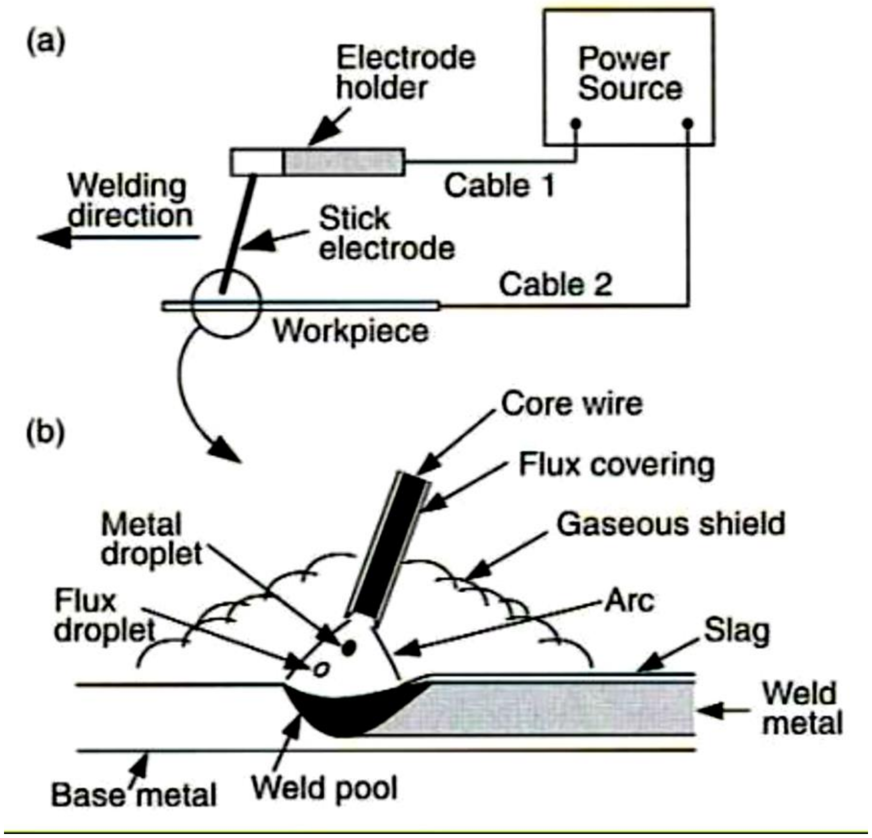
Numerical Problems: Duty Cycle

1. What will be the duty cycle with a 200A power supply rated at 60% duty cycle operated at the current 250A output?

Shielded-Metal Arc Welding (SMAW)

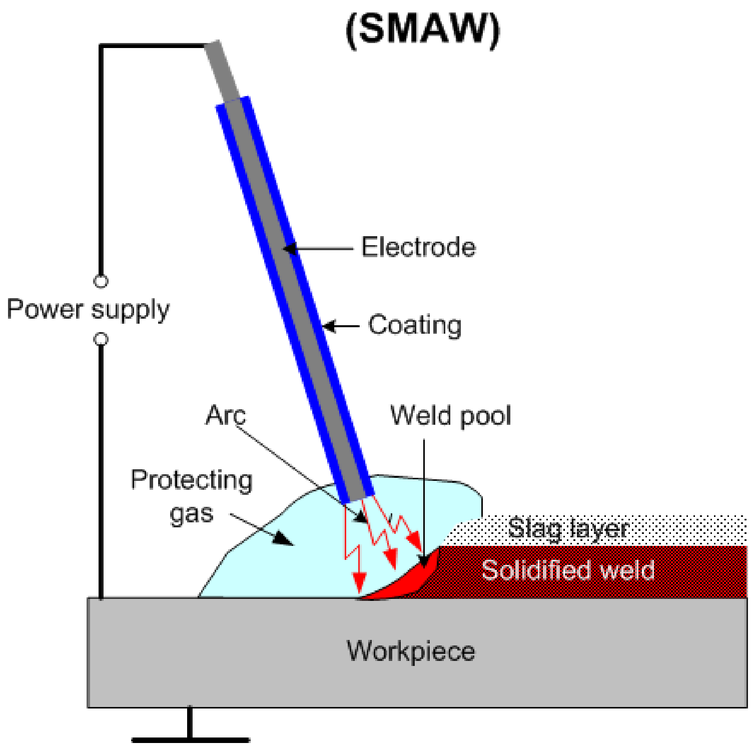
- ✓Manual metal arc welding
- ✓Arc by consumable flux coated metal electrode
- ✓Shielding by flux which produces slag covers weld pool

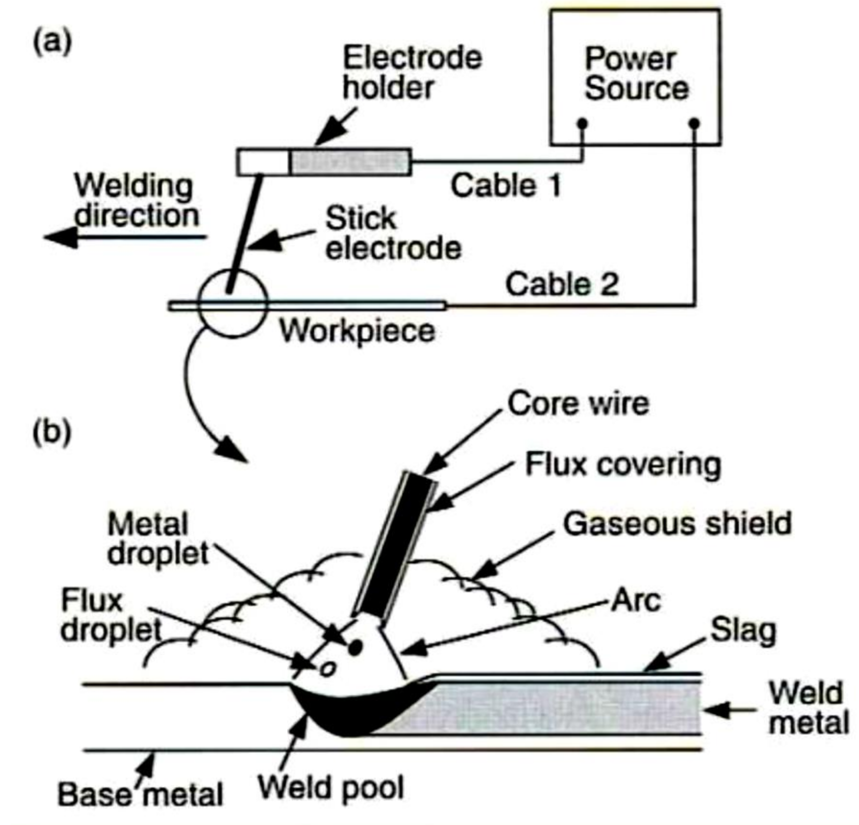




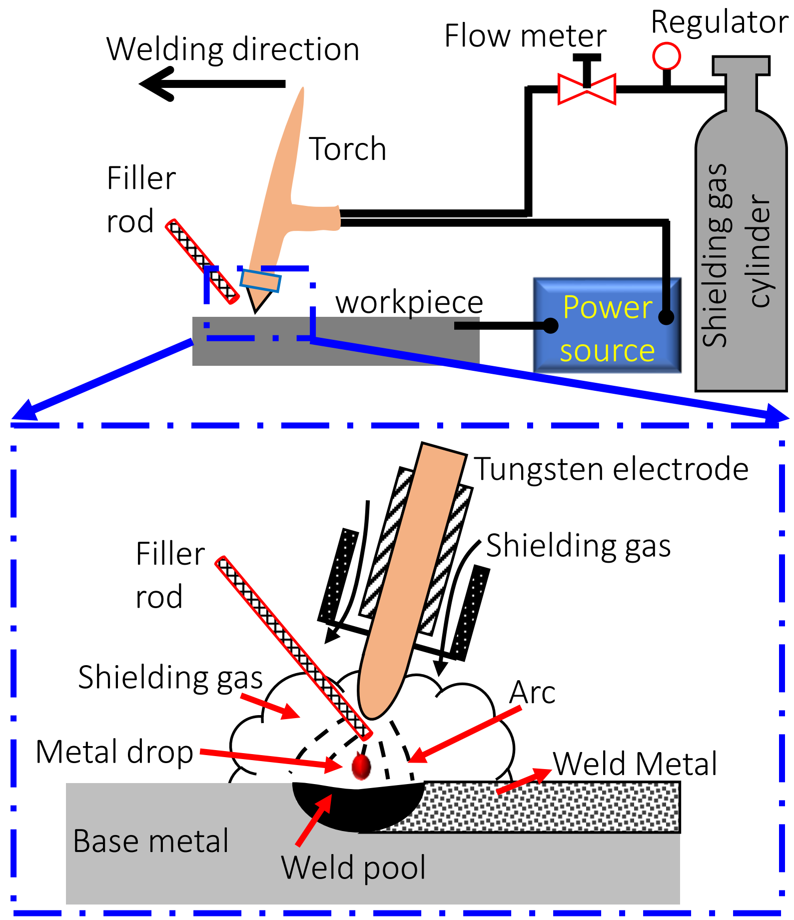
Shielded-Metal Arc Welding (SMAW)

✓Molten metal in weld pool solidifies in weld metal while lighter molten flux floats on top surface and solidifies as a slag layer.

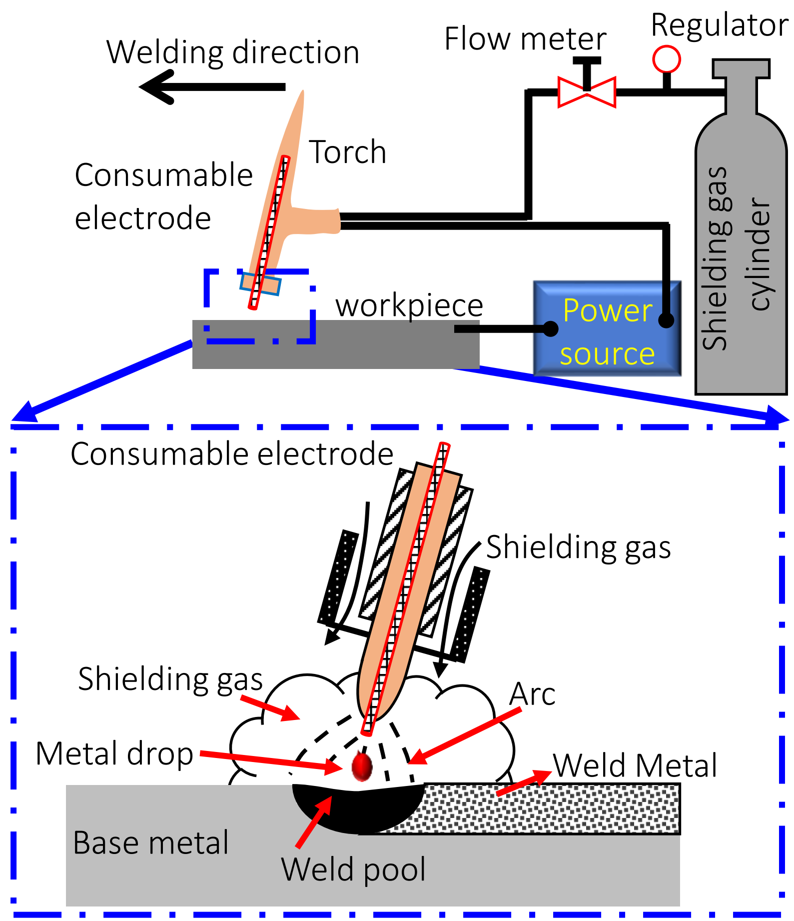




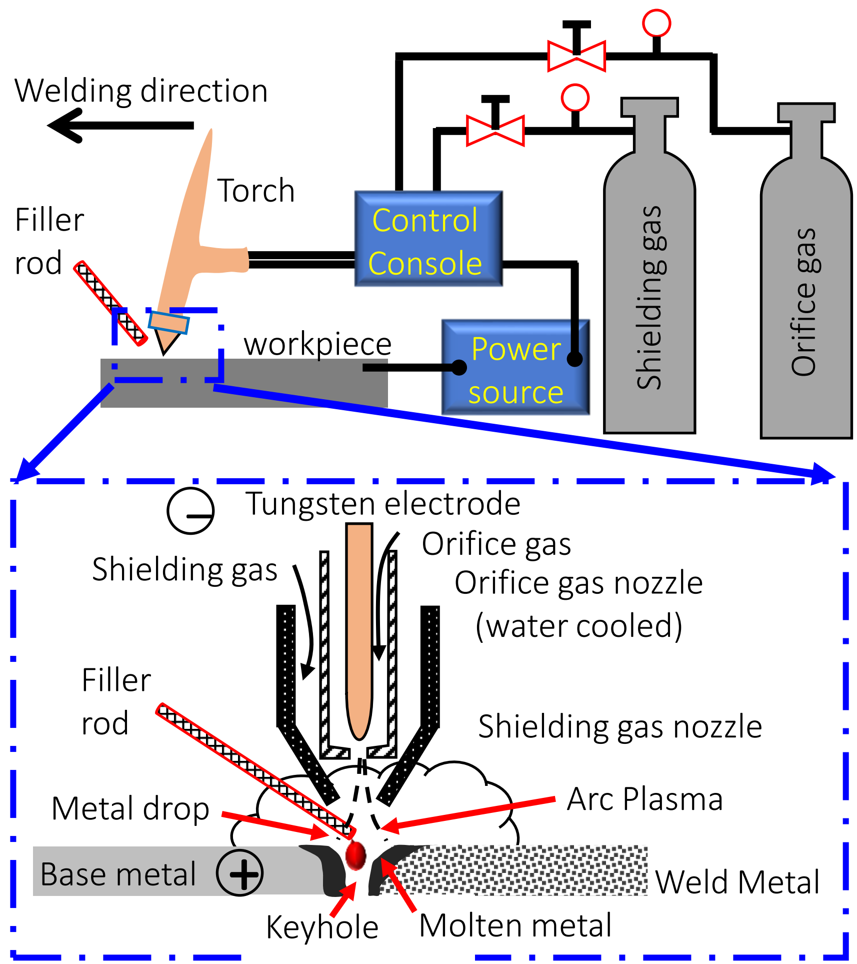
Gas Tungsten Arc Welding (GTAW) or Tungsten Inert Gas (TIG)



Gas Metal Arc Welding (GMAW)/MIGW



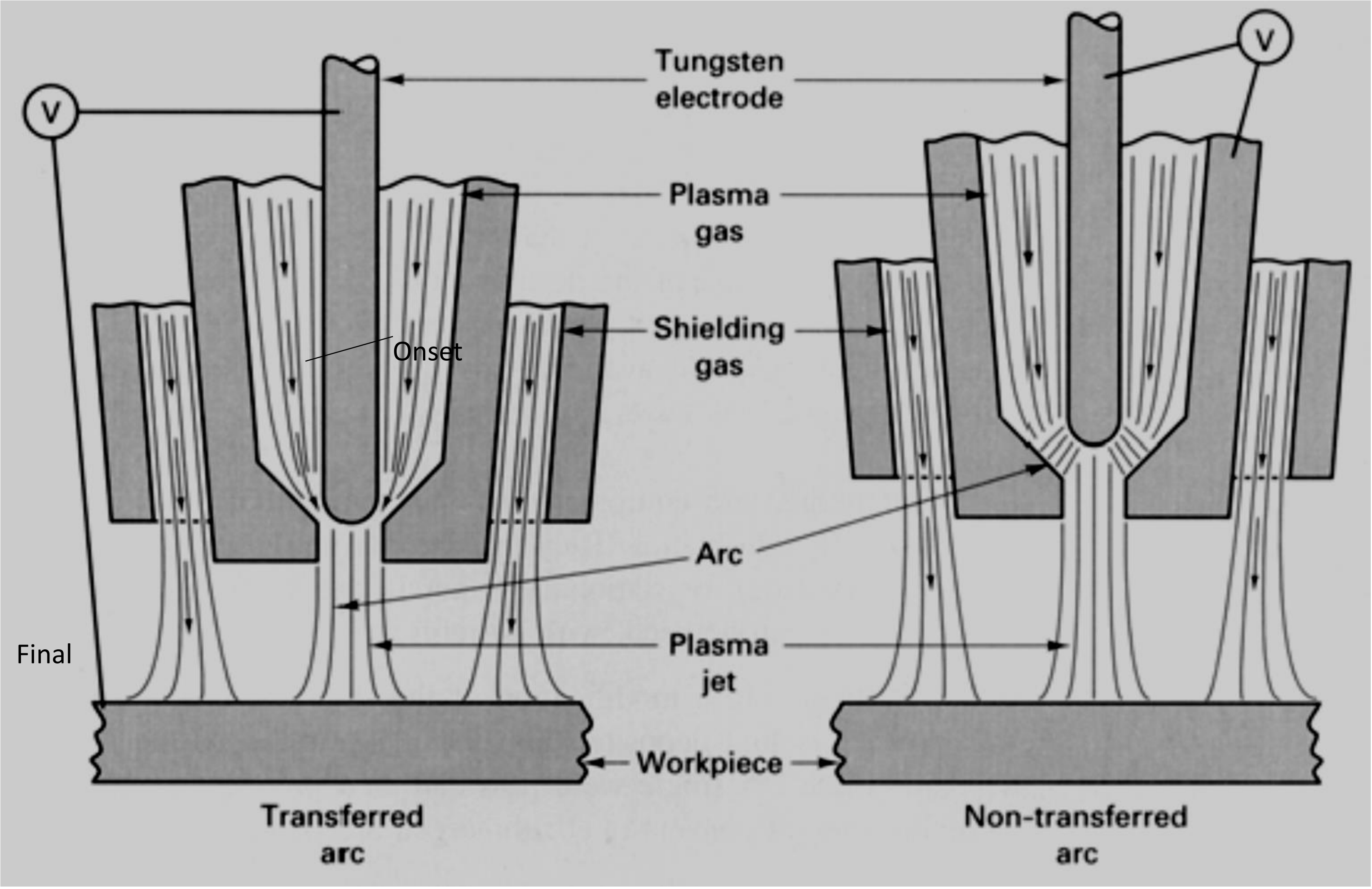
Plasma Arc Welding (PAW)



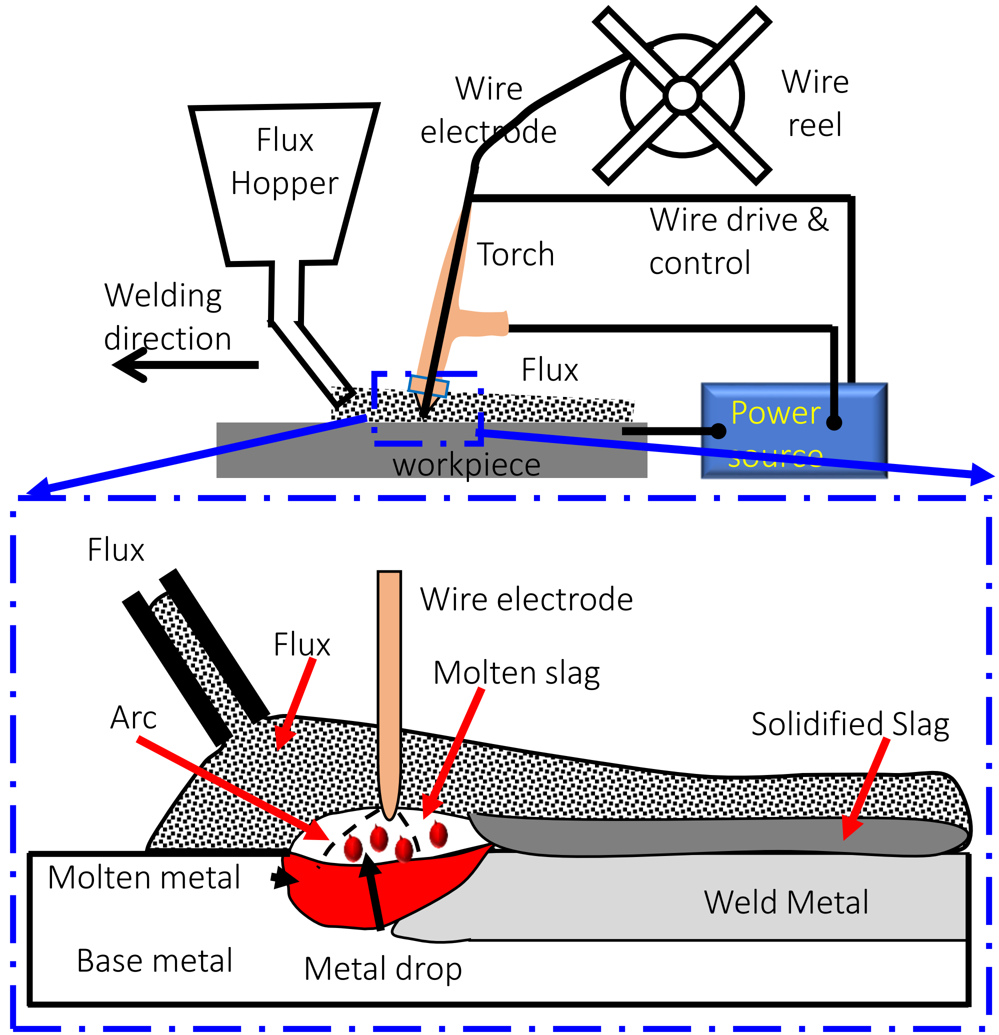
Plasma Arc Welding (PAW)

(a) Transferred Arc
- **used for welding/cutting**

(b) Non-transferred Arc
- **used for thermal spraying**



Submerged Arc Welding (SAW)



Conduction mode

- ✓When energy is transferred to a workpiece and immediately causes heating by a combination of conduction of heat in the flame, arc, or, to a much lesser extent, beam, and conversion of kinetic energy.
- ✓If the rate at which energy is being deposited **exceeds** the rate at which heat is being conducted away, the temperature will rise to eventually cause melting and produce a fusion weld.
- ✓This mode of energy deposition and weld production is called the **melt-in mode or the conduction mode**.

Conduction mode

- ✓In the melt-in or conduction mode, the temperature is maximum at the point of deposition, on or near the surface.
- ✓From that point outward into the mass of the workpiece, the temperature drops off. Depending on the total amount and rate of energy deposition, as well as the thermal conductivity and mass of the workpiece, heat energy is partitioned between melting to produce a fusion zone and just heating to produce a heat-affected zone.

Keyhole mode

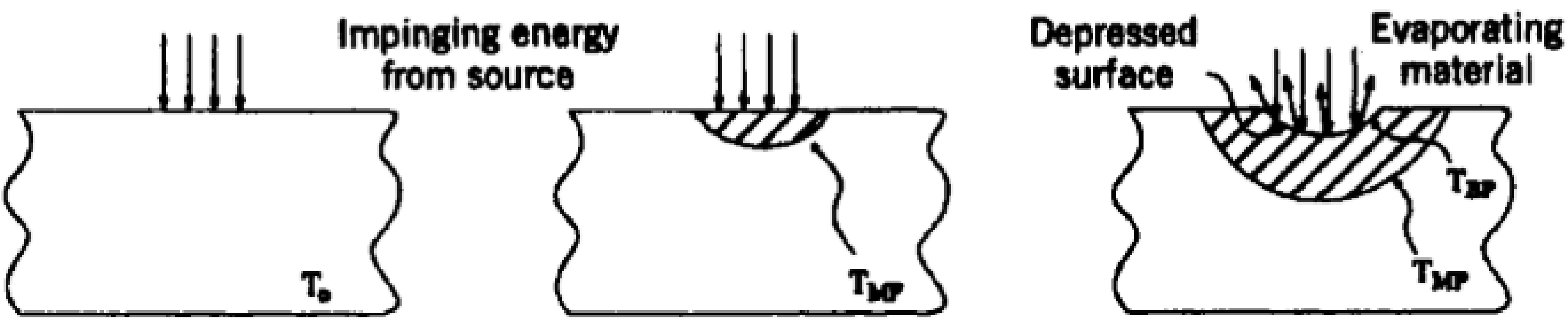
- ✓If the density of the energy coming from a source is high enough, the rate at which it is deposited **greatly exceeds** the rate at which it is lost by being conducted into the workpiece(s).
- ✓The material at the point of deposition rises in temperature not just to the melting point, but well above that.
- ✓The temperature can rise to the boiling point, converting liquid to vapor, superheating the vapor.
- ✓When this occurs, the energy source is said to be operating in the **keyhole mode**.

Keyhole mode

Initial deposition of impinging energy on the surface

Surface heating and inward propagating of heat to cause melting anywhere T> Tm;

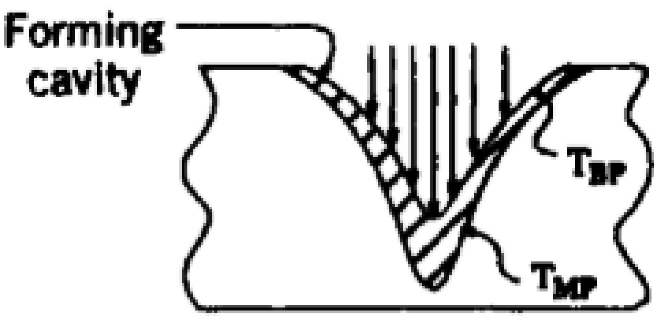
Continued inward propagation of heat to raise a larger volume of material to above Tm, and elevation of temperature near the surface to above T_{BP}, causing vaporization and a downward force on the liquid;

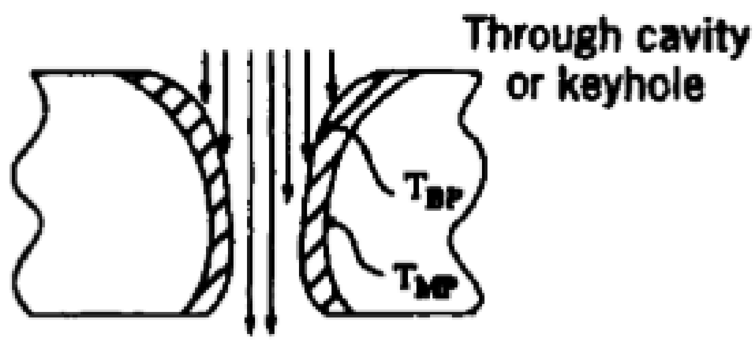


Keyhole mode

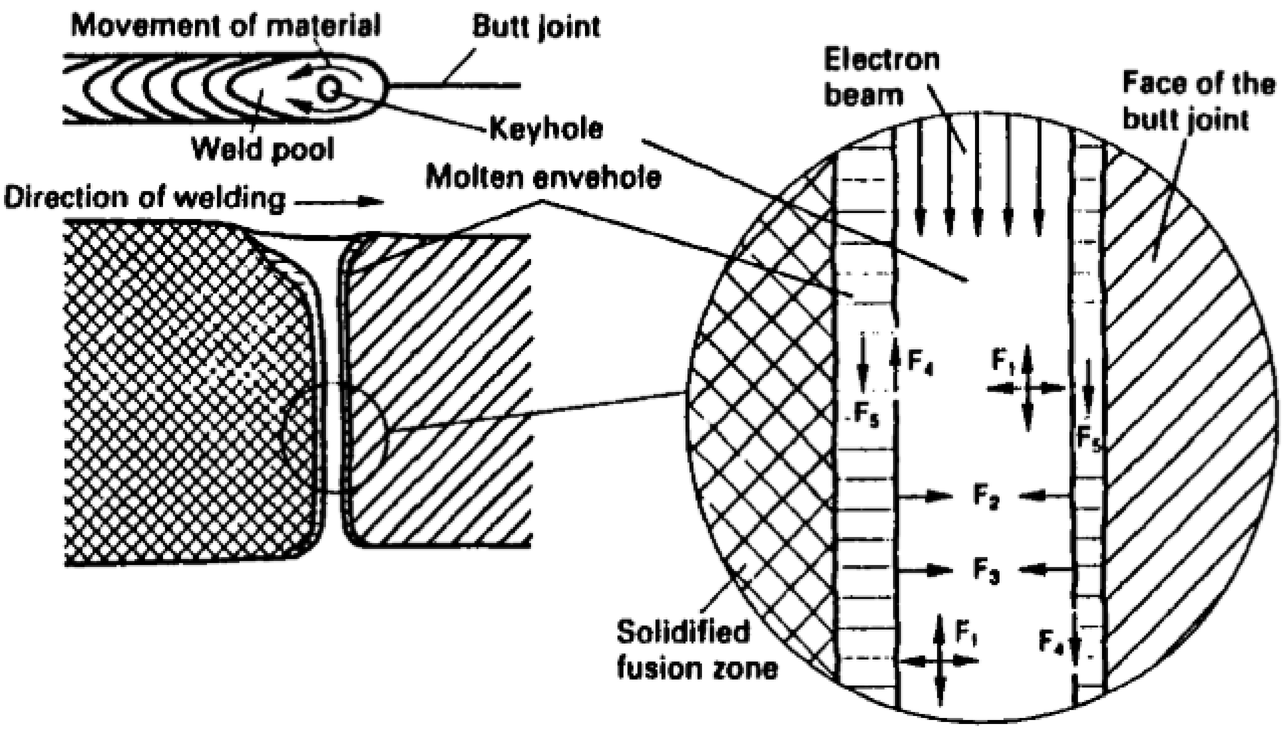
Continued deposition of heat, increased vaporization and greater depression of liquid, and increased growth of melted volume;

Eventual penetration of vapor cavity through thickness to produce a keyhole of vapor surrounded by molten material





Keyhole mode



Keyhole mode EBW

F_1 = Vapour pressure

F_2 = Force resulting from surface tension

F_3 = Hydrostatic pressure

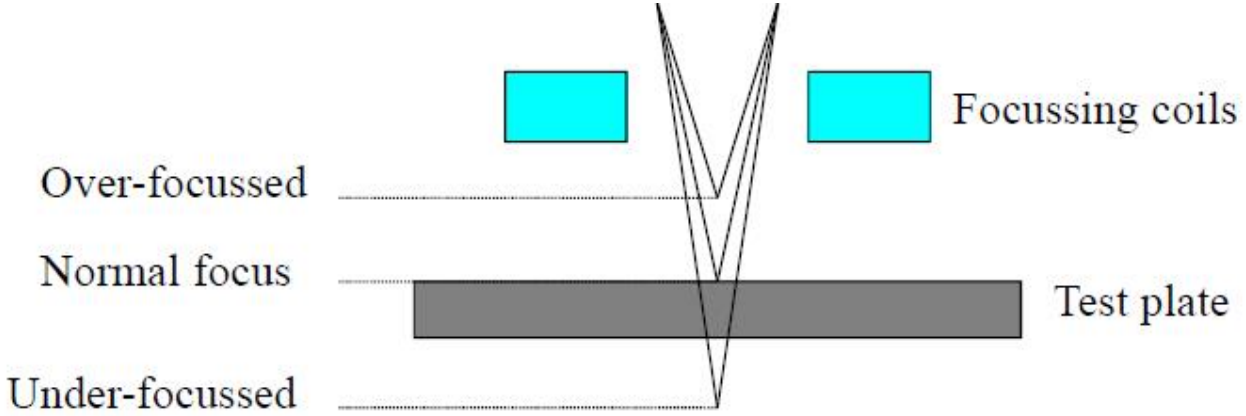
F_4= Frictional force from the escaping metal vapour

F_5 = Weight of the molten liquid.

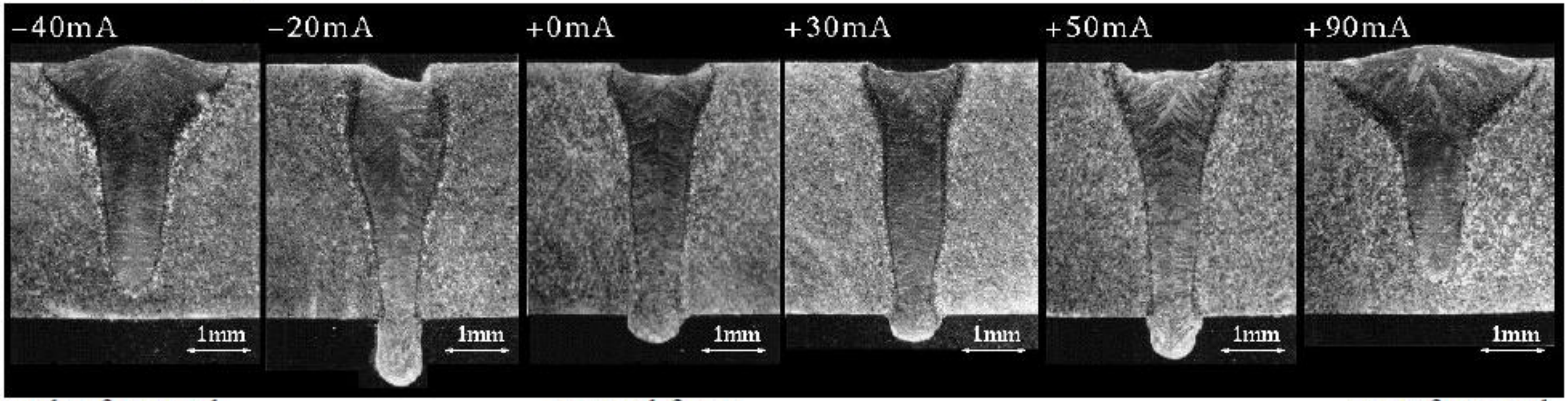
Keyhole mode

- ✓ Formed vapour (expand and less dense) moves upward away from the surface, and produces a reaction force that presses the melt downward and sideways.
- ✓ The result is a depression that permits additional photons (from a laser beam), electrons (from an electron beam), or electrons and ions (from a plasma arc) to impinge upon fresh material, which is then heated in the same way.
- ✓ The depression becomes larger and transforms to a keyhole, the entire central core of which consists of vapor surrounded on all sides by an envelope of liquid.
- ✓ For sufficient energy input, this keyhole will penetrate entirely through the thickness of the workpiece, even if this is several centimetres or inches.
- ✓ In this way, the faces of the two joint elements can be melted, the molten material flows back into the molten weld pool, metallurgical continuity is obtained, and solidification can occur to produce a weld.
- ✓ Key-hole mode (as opposed to the melt-in mode) begins to occur at energy densities above around 10^9 W/m^2

Effect of Beam Defocus (EBW)



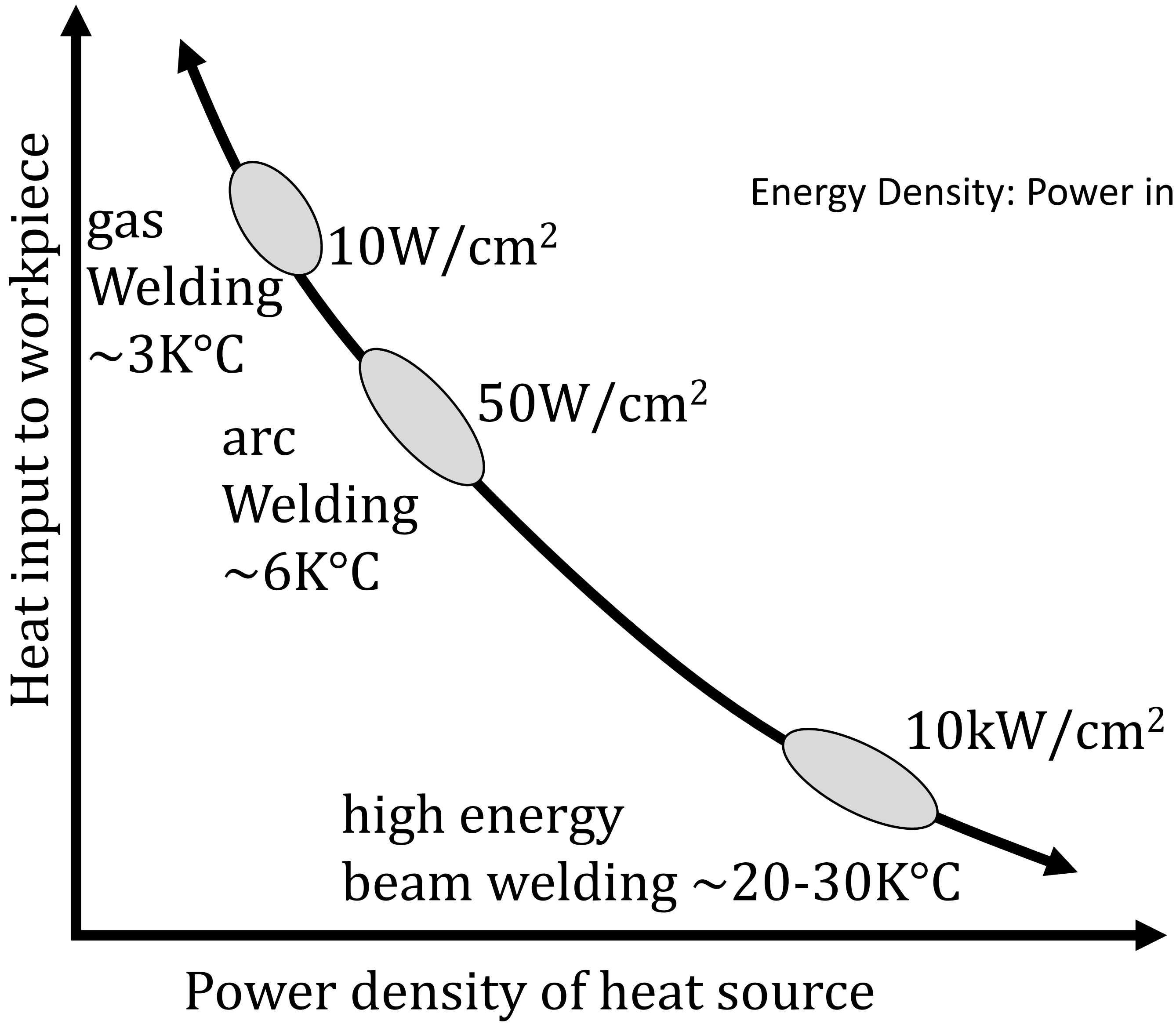
Metallographic sections



under-focussed normal focus over-focussed

Fusion welding: Energy Density

Energy Density: Power input/Effective area

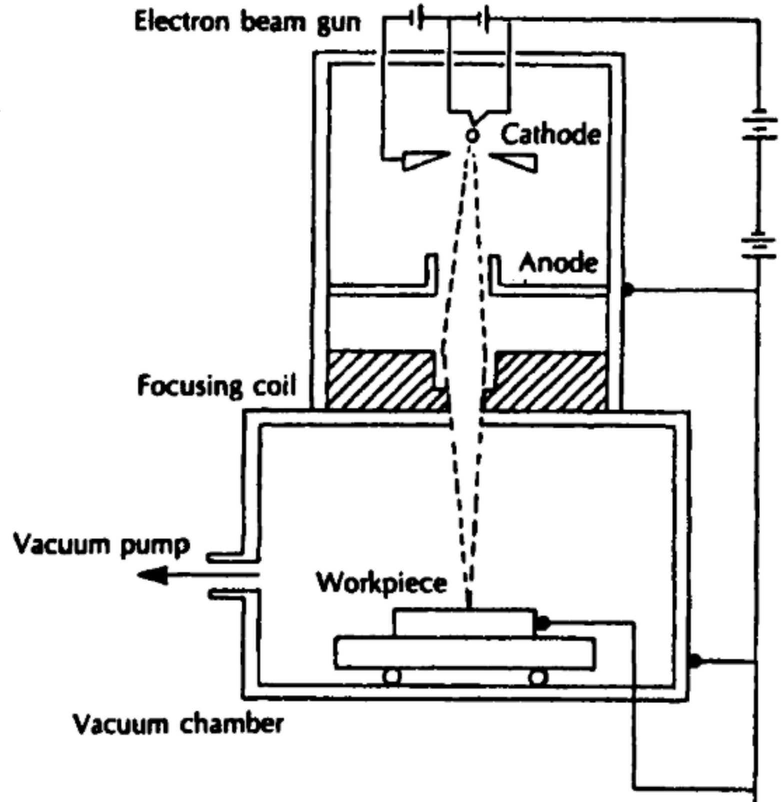


Energy Density: Type of Penetration

Process	Heat Source Intensity (Wm^{-2})	Condition	Fused Zone Profile
Flux-shielded arc welding	5×10^6 to 5×10^8		
Gas-shielded arc welding	5×10^6 to 5×10^8	Normal current	
		High current	
Plasma	5×10^6 to 5×10^{10}	Low current	
		High current	
Electron beam and laser	10^{10} to 10^{12}	Defocused beam	
		Focused beam	

Electron Beam Welding (EBW)

- ✓Electron beam is the energy source for melting the metals.
- ✓As the filament is negatively charged, it emits electrons which are accelerated by an electric field.
- ✓These electrons go through the anode and are focused (0.3 ~ 0.8 mm diam.) by an electromagnetic coil on the w/s surface with a high power density (~ 10^{10} W/m^2).
- ✓High-intensity beam forms a deep penetrating keyhole.

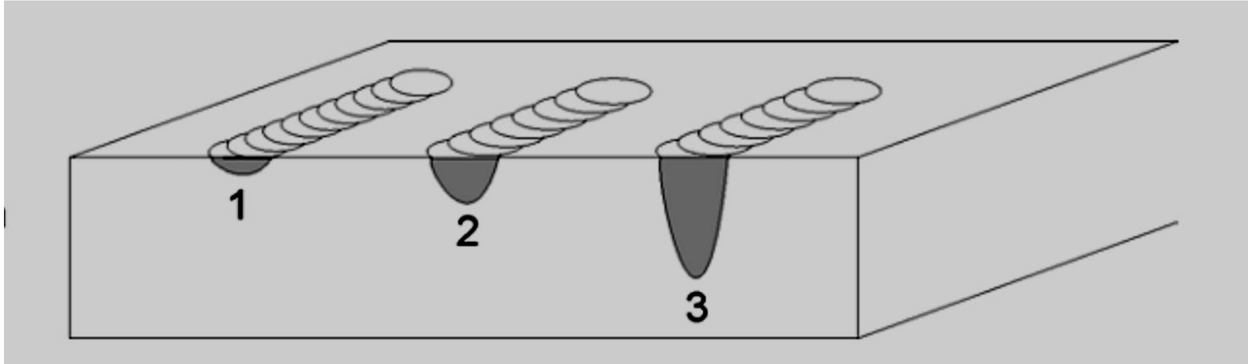


(a)

Laser Beam Welding (LBW)

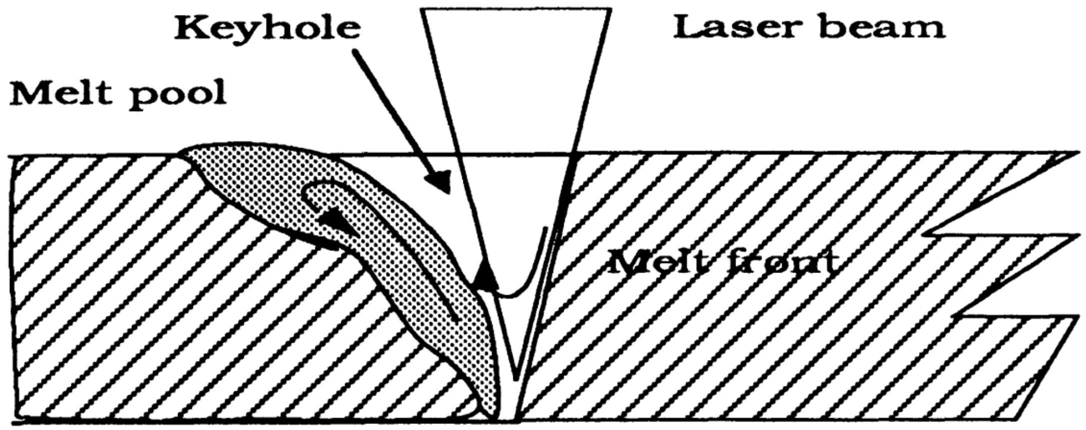
- ✓The weld is formed as the intense laser light rapidly heats the material-typically calculated in milli-seconds.

- ✓There are typically three types of welds:
  - ✓Conduction mode
  - ✓Conduction/penetration mode
  - ✓Penetration or keyhole mode.



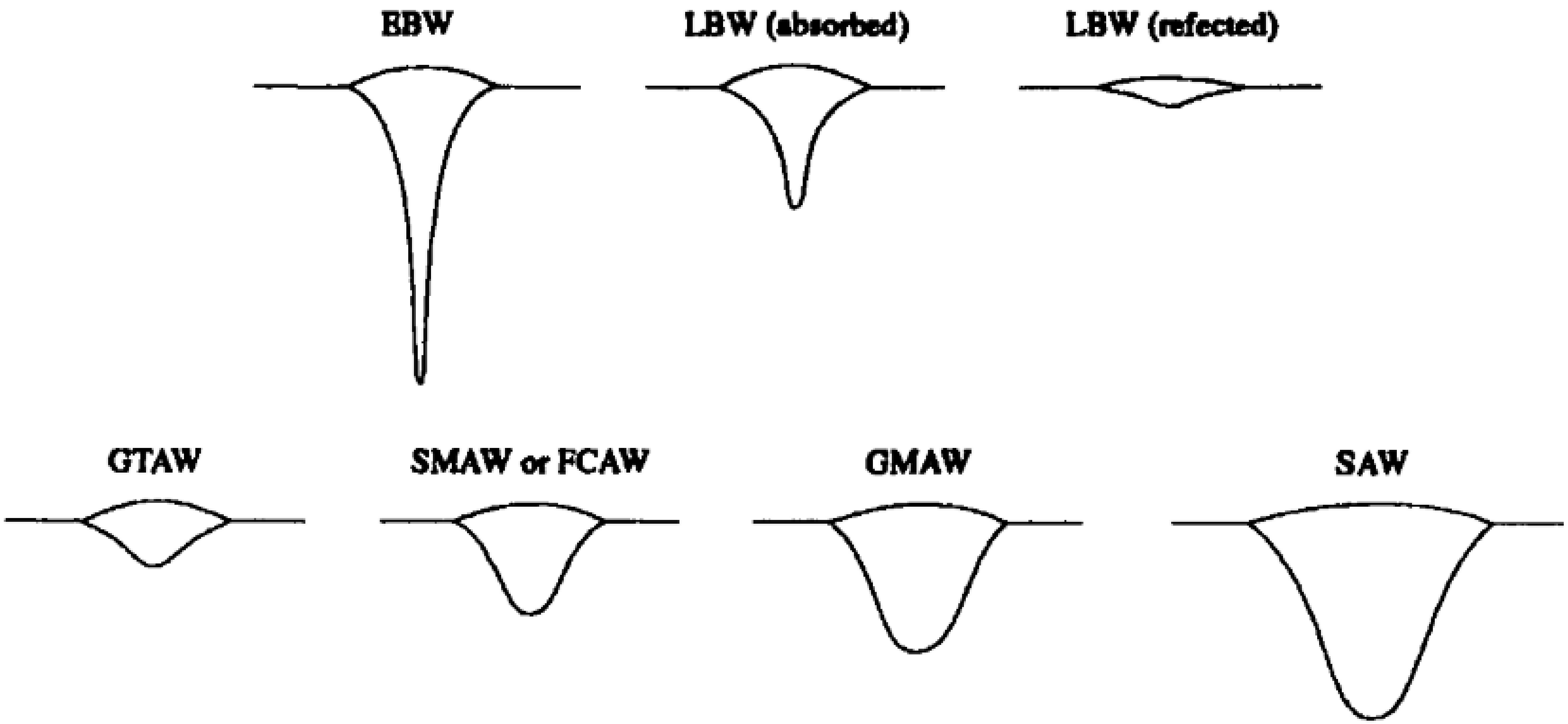
Laser Beam Welding (LBW)

- ✓ At high power densities, all materials will evaporate if the energy can be absorbed. Thus, when welding in this way, a hole is usually formed by evaporation.
- ✓ This "hole" is then traversed through the material with the molten walls sealing up behind it.
- ✓ The result is what is known as a "keyhole weld. This is characterized by its parallel-sided fusion zone and narrow width.



(a)

Comparison of typical EB, LB, and conventional arc



Chemical Fusion Welding

Chemical Fusion Welding

It is subset of fusion welding process where source of heat is exothermic chemical reaction.

Two types:

Thermit welding

employ a highly exothermic chemical reaction between solid-phase particulate materials (or solid particles and a gas).

$$3Fe_3O_4 + 8Al = 9Fe + 4Al_2O_3$$

(838 kJ/mol of oxide)

Oxyfuel gas welding

employ an exothermic chemical reaction involving the combustion of a fuel gas with oxygen

Oxyfuel gas welding

- ✓Exothermic Reaction between fuel gas with oxygen.

- ✓Fuel gas: Natural gas/methane, propane, propylene, butane, or other hydrocarbon gases, or even hydrogen

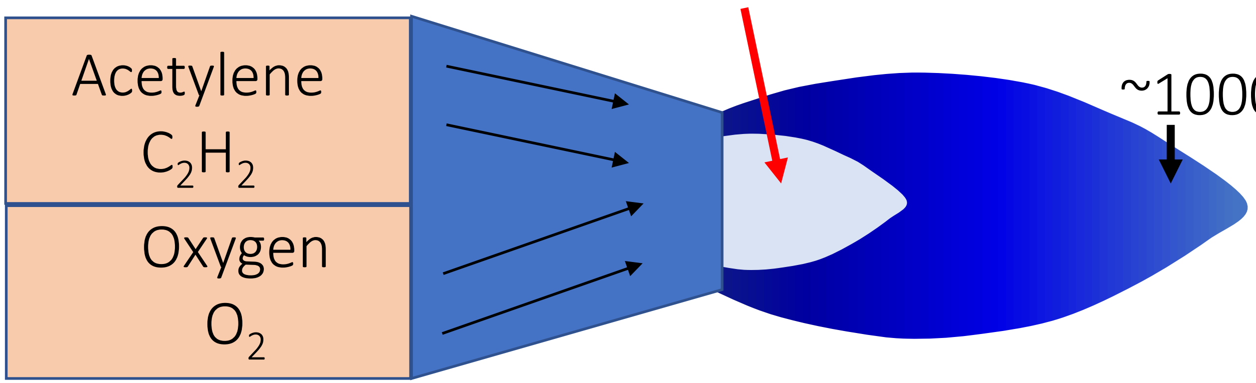
- ✓Oxyacetylene welding (OAW): Acetylene gas is used as the fuel due to its high flame temperature (i.e., intense source energy).

- ✓In combustion, two stages are there in OAW welding.
    - ✓Primary combustion zone
    - ✓Secondary combustion zone

Oxyacetylene welding (OAW)

Primary combustion zone:

Acetylene gas reacts with pressurized oxygen from the cylinder, and it forms carbon monoxide and hydrogen.



$$C_2H_2+O_2\,(\text{cylinder})=2CO+H_2$$

$$+448\ \text{kJ/mol of acetylene}$$

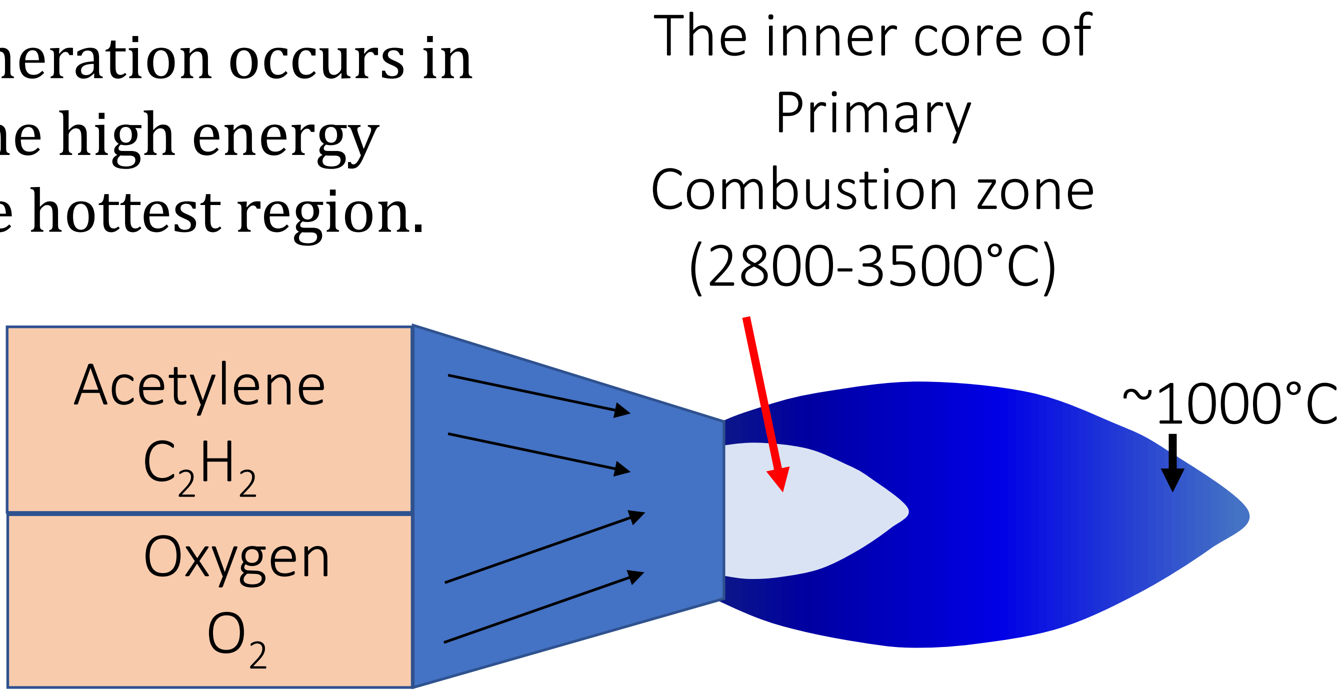
(1/3 of total heat generation OAW process)

High energy density region: The welder uses the inner flame to melt the workpiece.

Oxyacetylene welding (OAW)

Primary combustion zone:

1/3 of total heat generation occurs in small volumes, so the high energy density region is the hottest region.



The welder uses the inner flame to melt the workpiece.

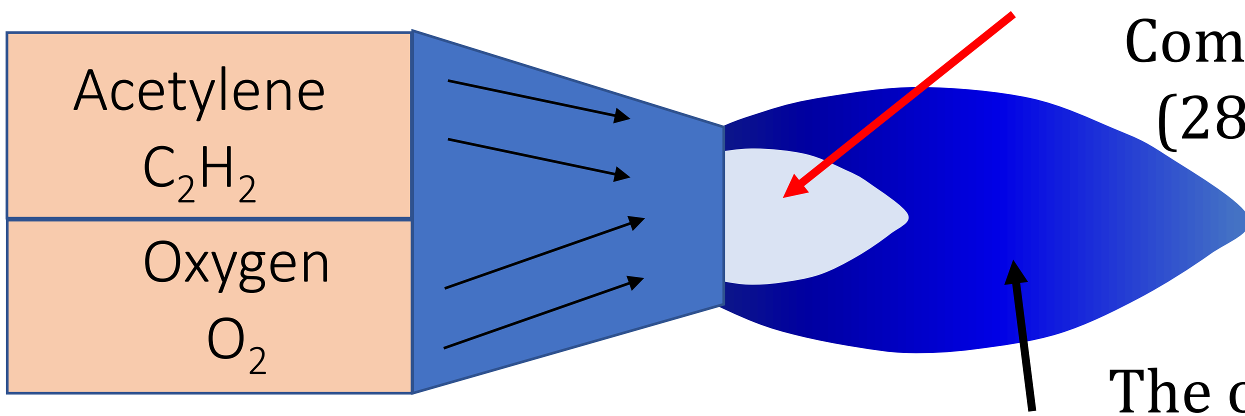
Oxyacetylene welding (OAW)

Secondary combustion zone:

Just after the primary combustion zone.

Monoxide (CO) from the primary combustion zone reacts with surrounding oxygen and forms Carbon dioxide (CO_2)

Primary Combustion zone (2800-3500°C)



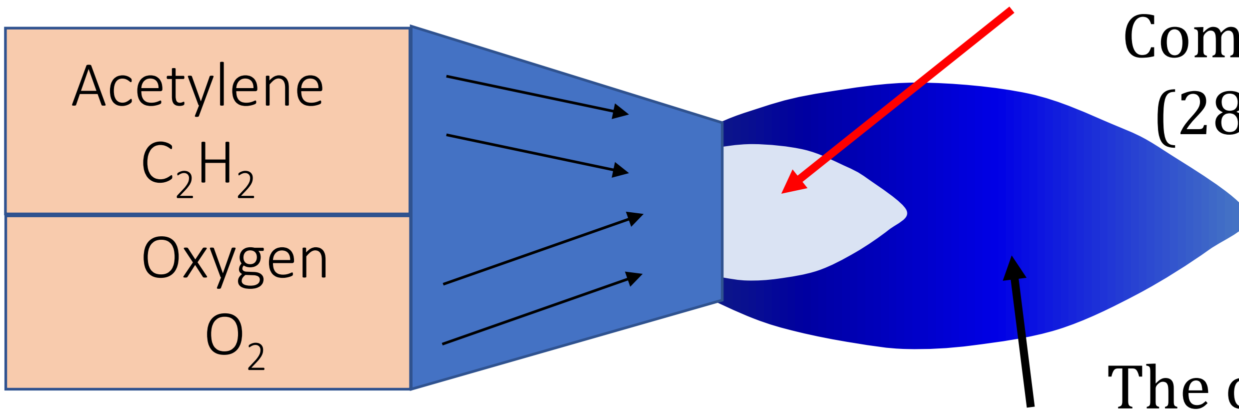
The outer flame of secondary Combustion zone (~2500°C)

$2CO+O_2$ (surrounding)= $2CO_2$ +570 kJ/mol of acetylene

Oxyacetylene welding (OAW)

Secondary combustion zone:

Hydrogen (H_2) from the primary combustion zone reacts with surrounding oxygen and forms water vapor(H2O).



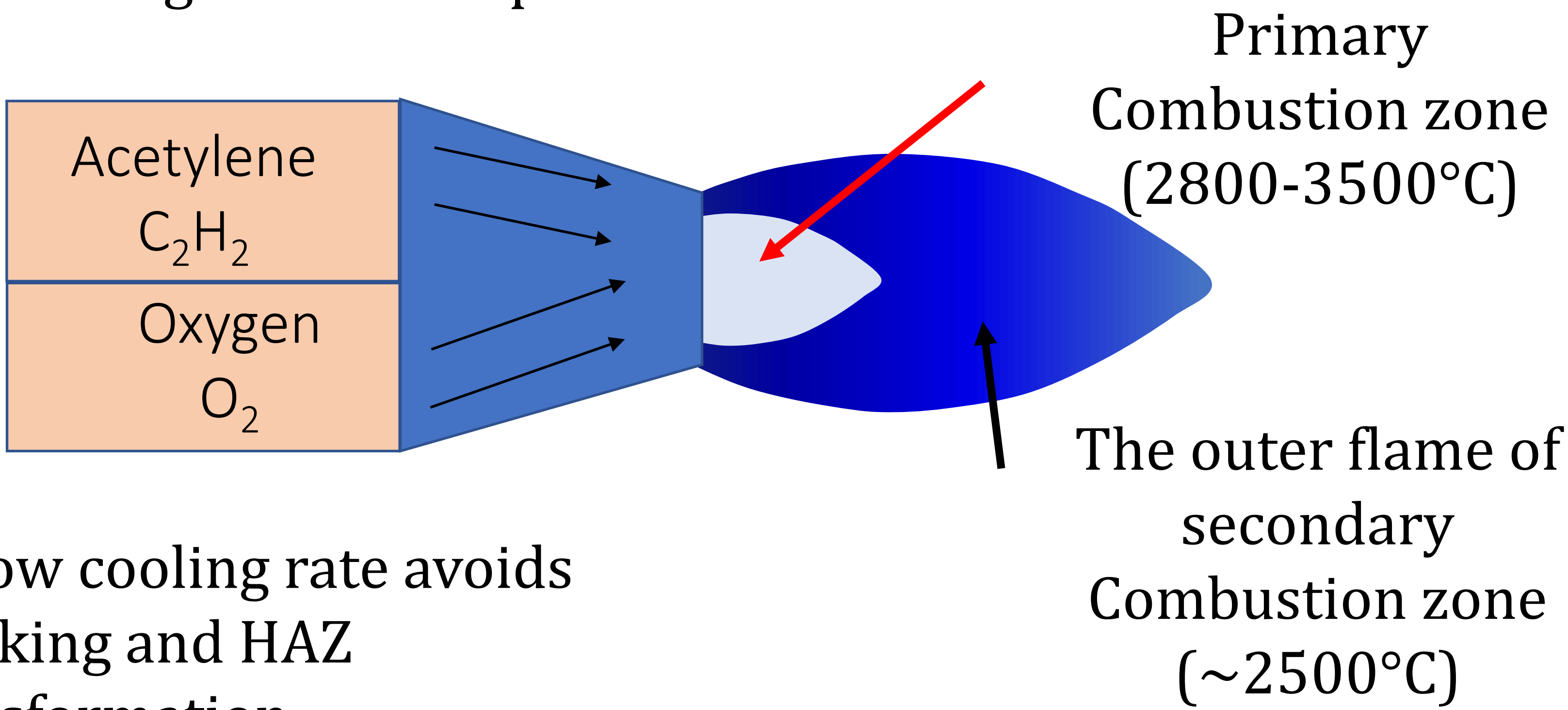
$$2H_2 + O_2 \text{ (surrounding)} = 2H_2O$$
$$+242 \text{ kJ/mol of acetylene}$$

Oxyacetylene welding (OAW)

Secondary combustion zone:

Outer flame is used to provide shielding of molten liquid of weld by CO_2.

It can also be used to control the cooling rate and preheating of the workpiece.



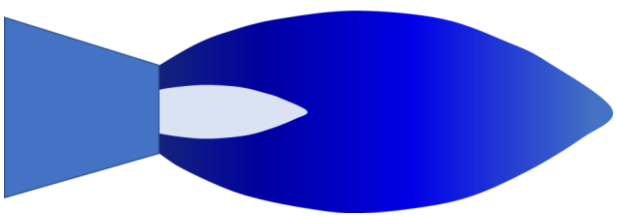
A slow cooling rate avoids cracking and HAZ transformation.

Types of flames in OAW

Chemically Neutral

Molar ratio of C_2H_2 to O_2 is 1:1.

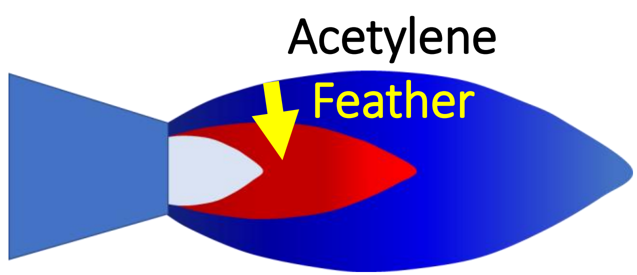
$C_2H_2+O_2$ (cylinder)$=2CO+H_2$



Chemically Reducing or carburizing flame

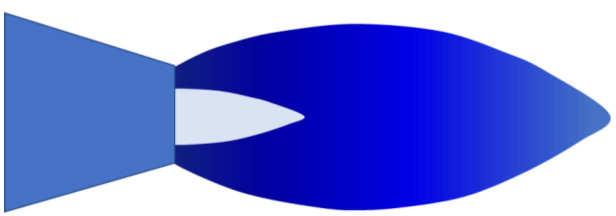
Supplying excess acetylene.

It burns with surrounding oxygen in the secondary zone and from an acetylene feather.



Chemically Oxidizing

The amount of O_2 is more, so some fraction is not used in the primary combustion zone.



Types of flames in OAW

Carburizing flame is good for the removal of oxides from metals (Al, Mg), and it prevents oxidation during welding.

Oxidizing flame: metal being welded to form an oxide. It prevents from loss of high vapor pressure components such as Zn out of brass by forming oxide skin.

$$\mathbf{Q\ (W) = 48\ kJ/L \times V_{acetylene} \times h/3600\ s}$$

$V_{acetylene}$ = the volumetric flow rate of acetylene in liters (L) per hour (h),

The heat of combustion of acetylene (C_2H_2) is 48 kJ/L at a standard state of 25°C, 1 atm pressure.

This equations apply to other oxyfuel gas combinations, simply substituting the heat of combustion for the particular fuel gas, in kilojoules per liter.

Joining Classification

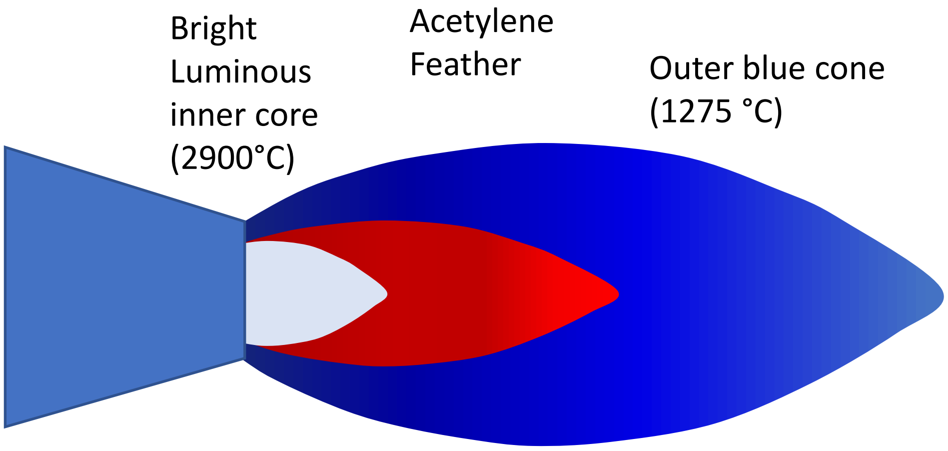
In an oxyacetylene flame, the available heat at the source

Q (W) = 48 kJ/L acetylene x $V_{acetylene}$ x h/3600 s

$V_{acetylene}$ = the volumetric flow rate of acetylene in liters (L) per hour (h),

The heat of combustion of acetylene (C_2H_2) is 48 kJ/L at a standard state of 25°C, 1 atm pressure.

This equations apply to other oxyfuel gas combinations, simply substituting the heat of combustion for the particular fuel gas, in kilojoules per liter.



Carburizing flame

https://www.youtube.com/watch?v=fs1UhhJH0E8&ab_channel=WeBuildStuff

Video

Thermit Welding

Generally, metal oxide reacts with Al. aluminum dust reduces the oxide of another metal due to the faster reactivity of aluminum.

$3Fe_3O_4 + 8Al = 9Fe + 4Al_2O_3$ (838 kJ/mol of oxide)

$Fe_2O_3 + 2Al = 2Fe + Al_2O_3$ (860 kJ/mol of oxide)

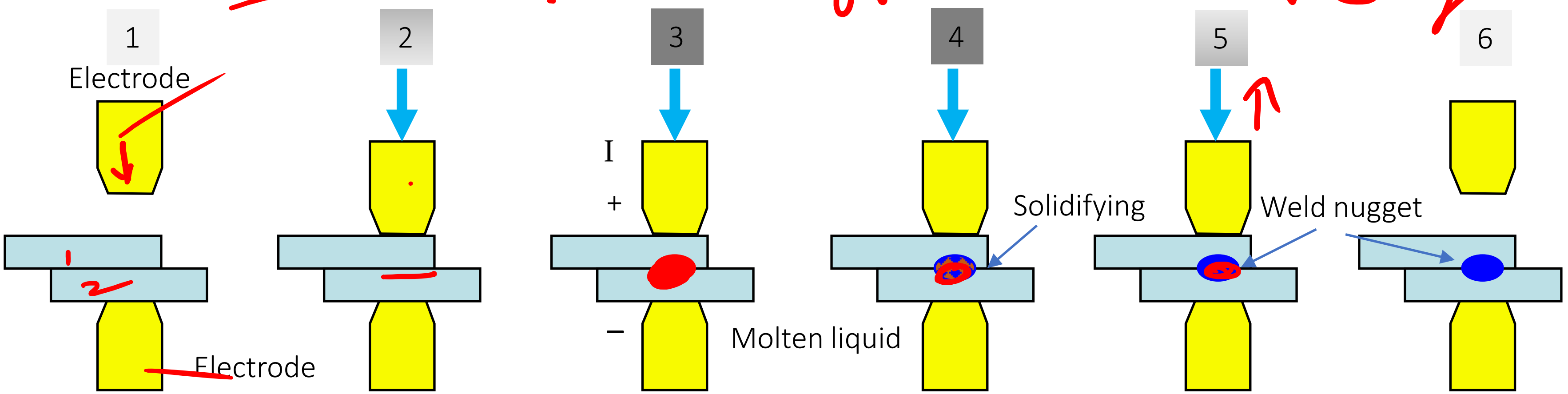
$3CuO + 2Al = 3Cu + Al_2O_3$ (1210 kJ/mol of oxide)

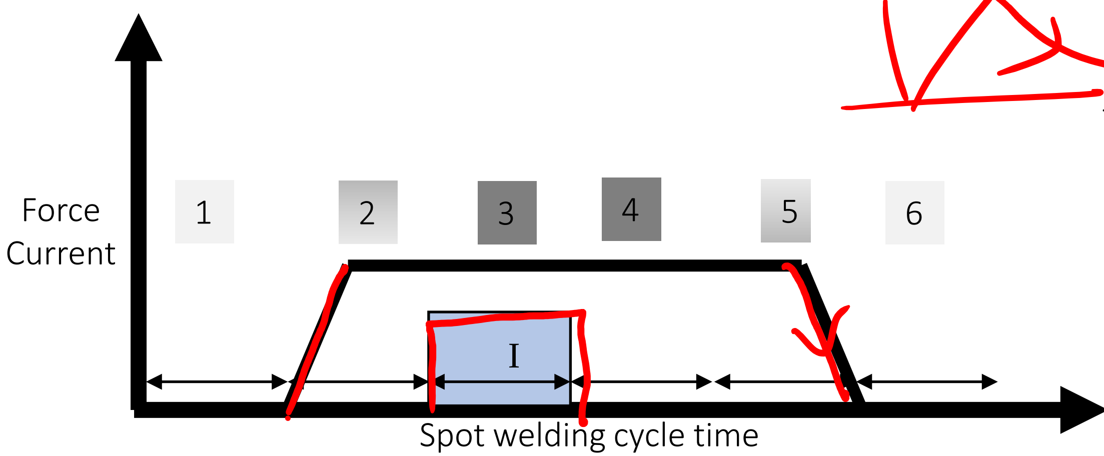
https://www.youtube.com/watch?v=5uxsFglz2ig&ab_channel=WolfgangLendner

Resistance Welding

Resistance Spot Welding

Spot welding Nugget Seam weld



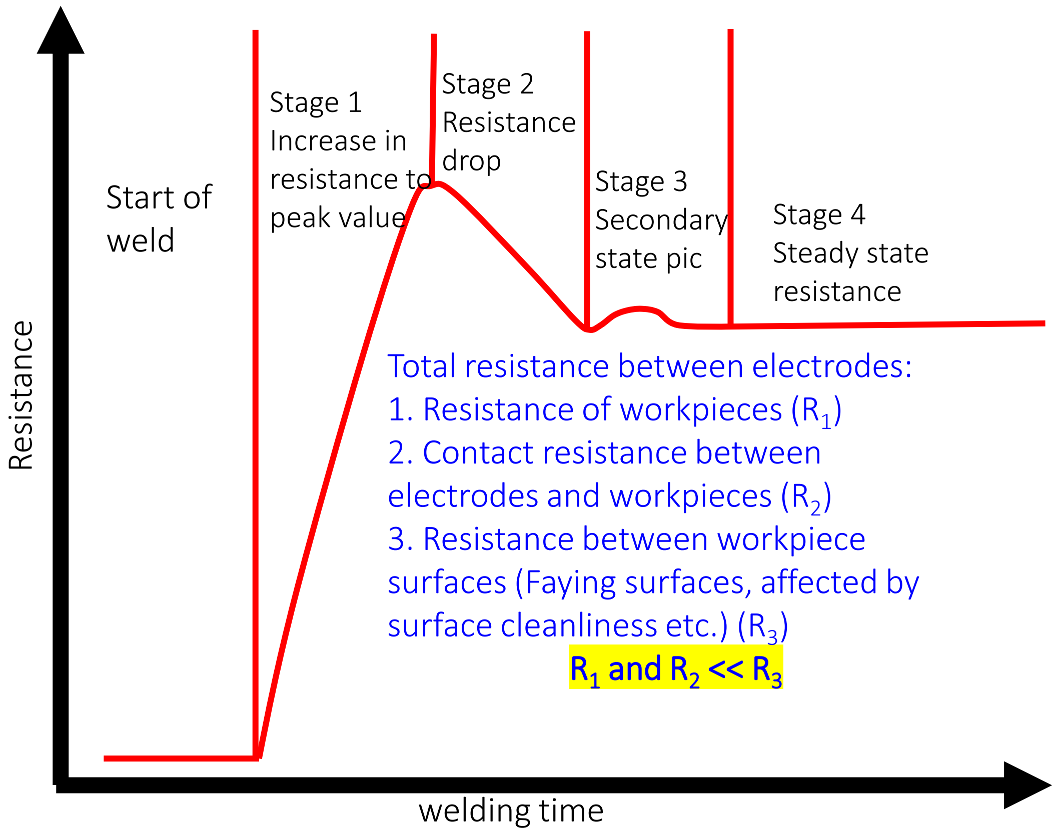


Plot of squeezing force and current during cycle. The sequence is:

(1) parts inserted between open electrodes
(2) Electrodes close and force is applied
(3) weld time current is switched on
(4) current is turned off but force is maintained or increased

I^2Rt

Nugget Growth Mechanism



At first the resistance increases in stage 1 do to the increased temperature (increased bulk resistance) outweighing the better surface contact.
With thinner materials, the peak occurs more rapidly.
Peak occurs earlier with thinner materials.

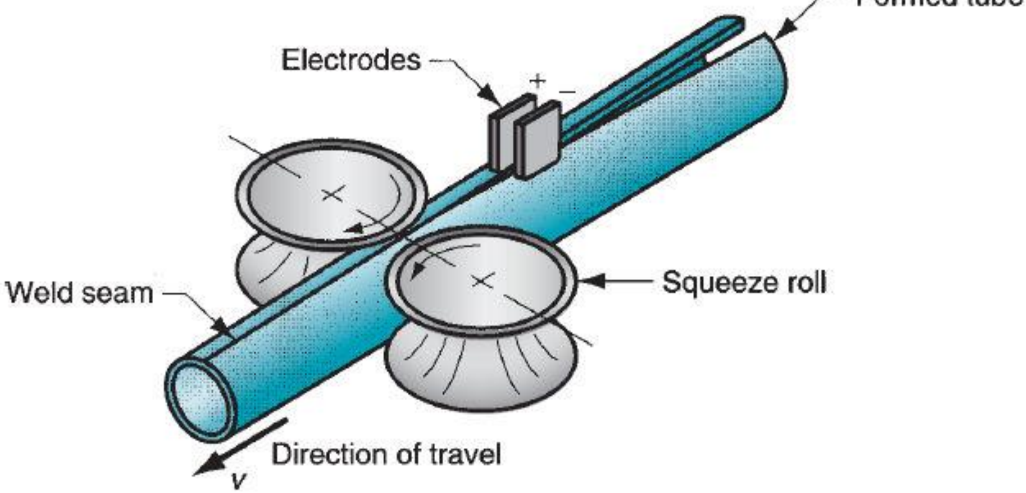
Stage 2 Drop in resistance because of current shunting into the mushy and solid state bonded region

Stage 3 instability in the current path as nugget approaching a steady state growth mode.

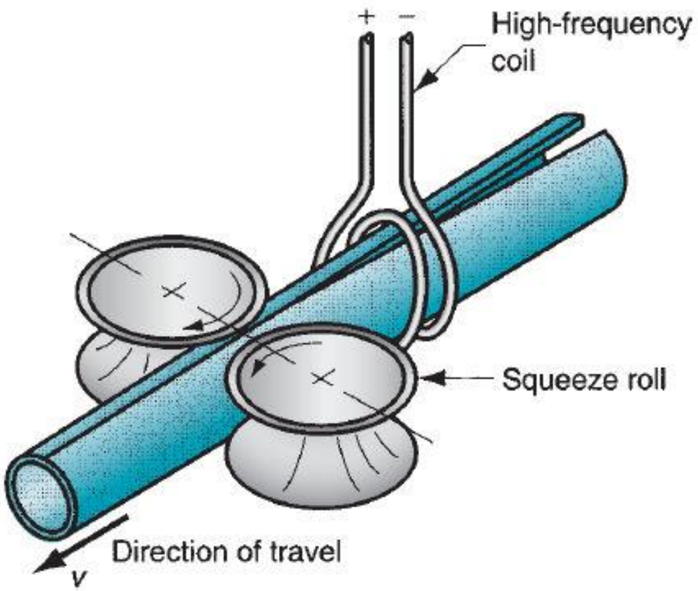
Stage 4 Steady State Growth of Nugget until either expulsion or current off time and new nugget formation

Resistance Welding

High-frequency resistance welding



High-frequency induction welding



https://www.youtube.com/watch?v=0x1uRR9Jb34&ab_channel=Tenaris

Thermal analysis in welding

Mode of Heat Transfer

Conduction mode of heat transfer

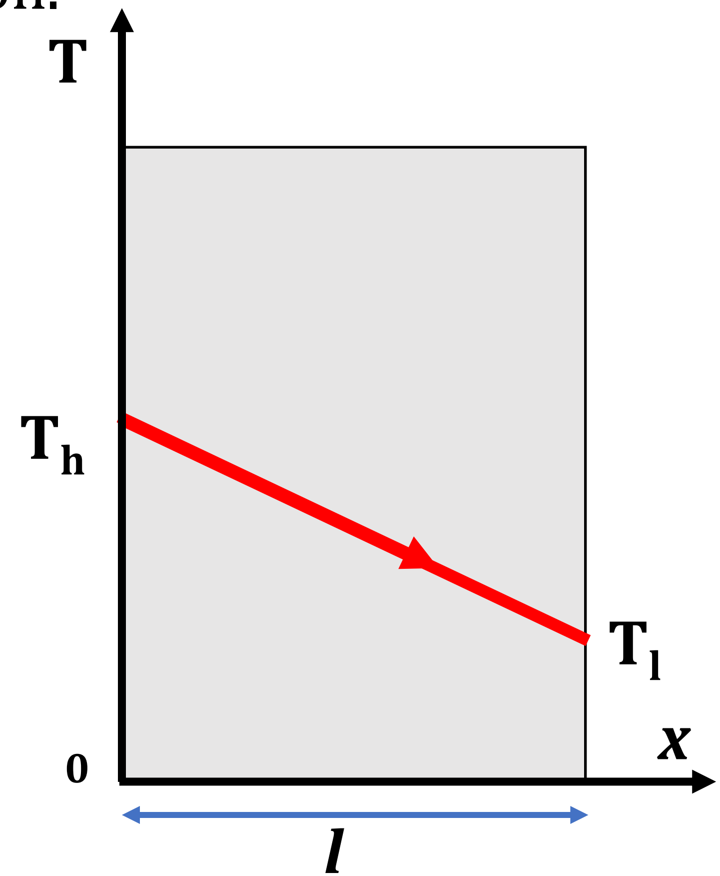
Conduction in solids: **lattice vibrations of the molecules** and **the movement of free electrons**.

In gases and liquids: **collisions and diffusion** of the molecules during their random motion.

Fourier's law of heat conduction

Rate of heat conduction:

$$Q_{cond} = -kA\frac{dT}{dx}$$

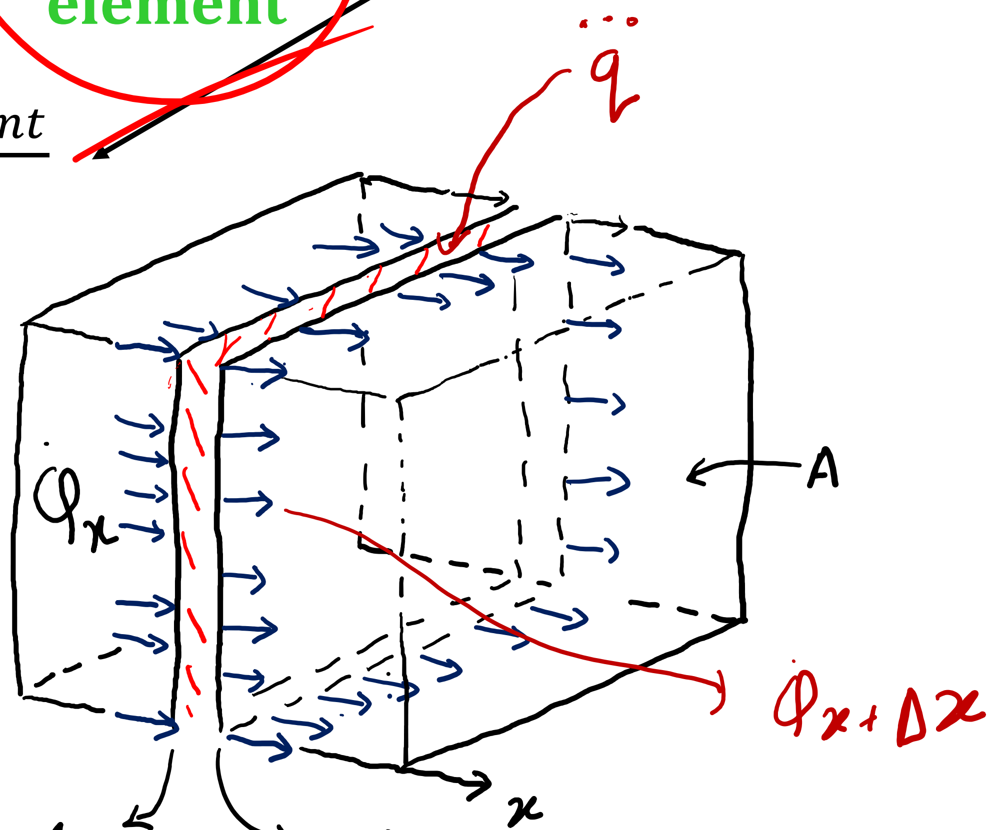


Conservation of Energy: 1-D

1D Heat Conduction Equation - Plane Wall

(**Rate of energy added at *x***) - (**Rate of energy removed at *x+Dx***) + (**Rate of heat generation inside the element**) = (**Rate of energy change within the element**)

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dddot{q} = \frac{\Delta E_{element}}{\Delta t}$$



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dddot{q} = \rho c\frac{\partial T}{\partial t}$$

$$k\frac{\partial^2 T}{\partial x^2} + \dddot{q} = \rho c\frac{\partial T}{\partial t}$$

$A_x = A_{x+\Delta x} = A$

Conservation of Energy: 1-D

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

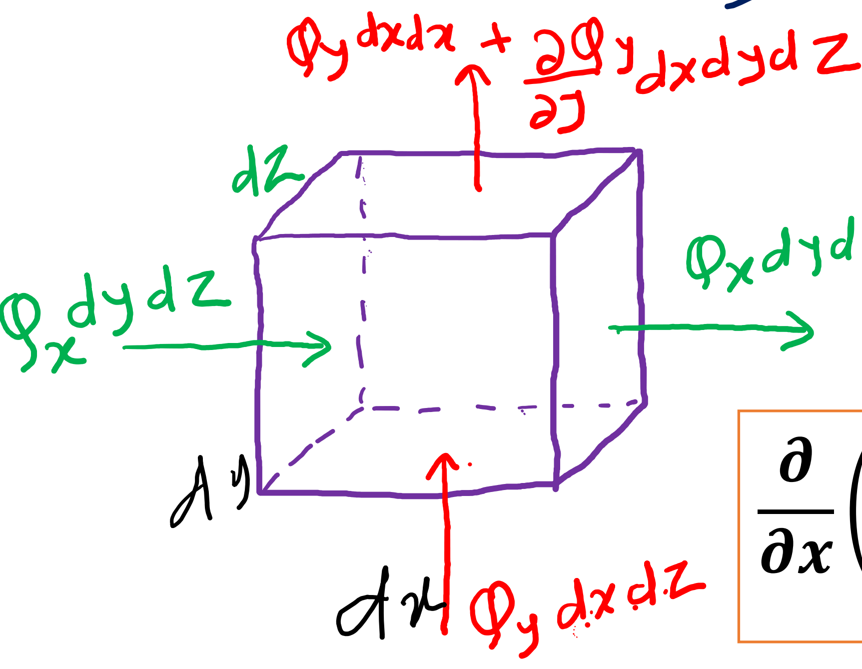
Thermal diffusivity $$\alpha = \frac{k}{\rho C_p}$$

- Thermal diffusivity (α, unit m^2/s) is ability of a material to conduct thermal energy relative to its ability to store thermal energy.
- Diffusivity is an important parameter for understanding diffusion (rate) processes.
- Thermal diffusivity gives a measure of "time scale (t (s)$=L^2/\alpha$)" of heat diffusion or how fast the heat would flow between two temperatures.
- For e.g. if you hold an iron rod ($\alpha=2.3\times10^{-5}$ m^2/s) in the left hand and an aluminum rod ($\alpha=9.7\times10^{-5}$ m^2/s) in the right hand and place the other two ends in a fire-place, the right hand will sense the heat much quicker.

Conservation of Energy: 2-D

2D Heat Conduction Equation

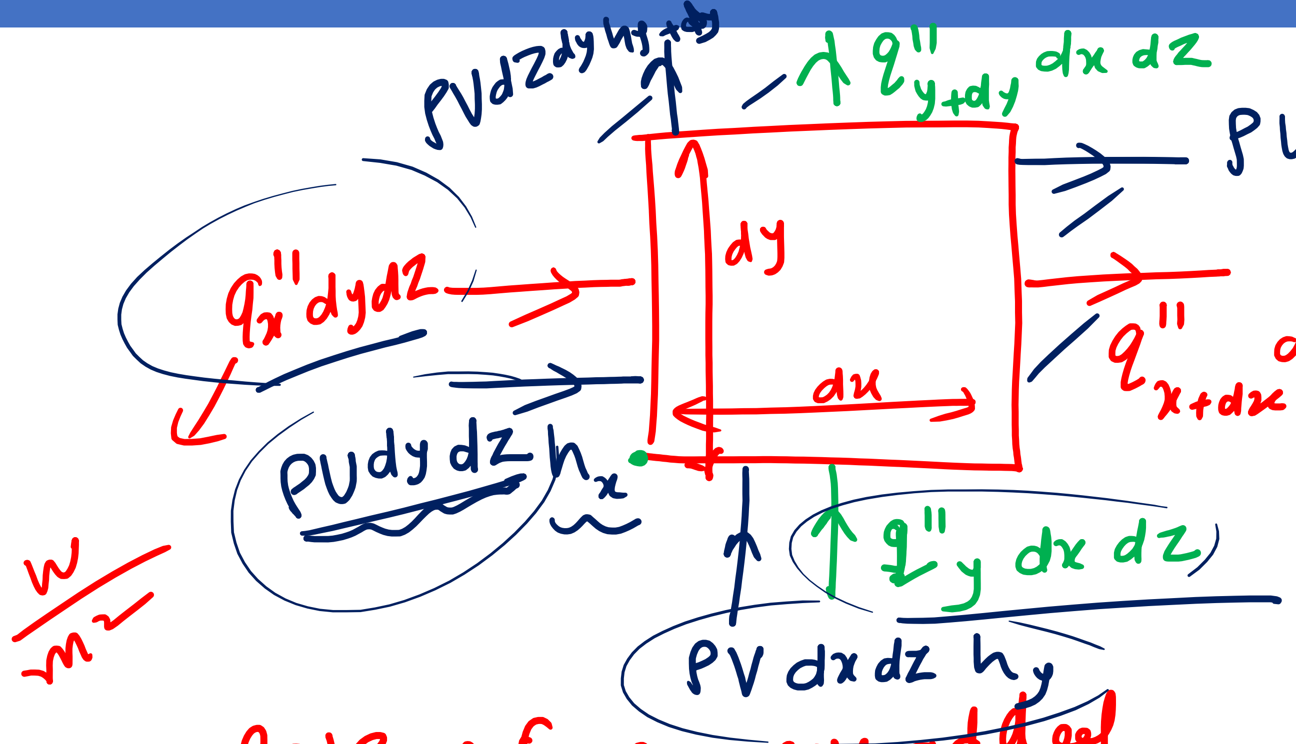
[Rate of energy added] - [Rate of energy removed] + [Rate of heat generation inside the element] = [Rate of energy change within the element]



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\ddot{q}=\rho c\frac{\partial T}{\partial t}$$

$$k\frac{\partial^2 T}{\partial x^2}+k\frac{\partial^2 T}{\partial y^2}+\ddot{q}=\rho c\frac{\partial T}{\partial t}$$

Conservation of Energy



$q''' = \frac{W}{m^3}$

$q' = W/m$

Rate of energy added

− Rate of energy removed + Rate of heat gen.

= Rate of energy change within element

Conservation of Energy

Rate of energy added $E_{in} = q_x'' dy\,dz + q_y'' dx\,dz$

$+ \rho u\, dy\, dz\, h + \rho V\, dx\, dz\, h$

Rate of energy removal $= E_{out}$

$= q''_{x+dx}\, dy\, dz + q''_{y+dy}\, dx\, dz$

$+ \rho u\, dy\, dz\, h_{x+dx} + \rho V\, dx\, dz\, h_{y+dy}$

$\Rightarrow q''_{x+dx} = q_x'' + \frac{\partial}{\partial x}(q_x'')\, dx$

$q''_{y+dy} = q_y'' + \frac{\partial}{\partial y}(q_y'')\, dy$

Conservation of Energy

$$h_{x+dx} = h + \frac{\partial h}{\partial x} dx$$

$$h_{y+dy} = h + \frac{\partial h}{\partial y} dy$$

$$E_{out} = \left(q_x'' + \frac{\partial q_x''}{\partial x} dx\right) dy\, dz$$

$$+ \left(q_y'' + \frac{\partial q_y''}{\partial y} dy\right) dx\, dz$$

$$+ \rho u\, dy\, dz \left(h + \frac{\partial h}{\partial x} dx\right)$$

$$+ \rho v\, dx\, dz \left(h + \frac{\partial h}{\partial y} dy\right)$$

Conservation of Energy

W/m^3

Rate of ~~Eng~~ Energy gen. $= q''' \, dx\, dy\, dz$

Rate of energy ~~heat~~ change within element $= \dot{E}$

$$= \rho \frac{\partial \hat{u}}{\partial t} \, dx\, dy\, dz$$

$\cancel{\rho V C_p \Delta T}$

Conservation of Energy

E_{in}

$$\cancel{q_x''}\,dy\,dz + \cancel{q_y''}\,dx\,dz + \rho u \cancel{h}\,dy\,dz + \rho v \cancel{h}\,dx\,dz$$

$$-\left(\cancel{q_x''} + \frac{\partial q_x''}{\partial x}dx\right)dy\,dz + \left(\cancel{q_y''} + \frac{\partial q_y''}{\partial y}dy\right)dx\,dz$$

$$-\rho u\left(\cancel{h} + \frac{\partial h}{\partial x}dx\right)dy\,dz - \rho v\left(\cancel{h} + \frac{\partial h}{\partial y}dy\right)dx\,dz$$

$$+\,q'''\,dx\,dy\,dz = \rho\frac{du}{dt}\,dx\,dy\,dz$$

$$q_x'' = -k\frac{dT}{dx}$$

$$q_y'' = -k\frac{dT}{dy}$$

Conservation of Energy

$$-\frac{\partial q''_x}{\partial x}\cancel{dx\,dy\,dz} - \frac{\partial q''_y}{\partial y}\cancel{dx\,dy\,dz} - \rho u \frac{\partial h}{\partial x}\cancel{dx\,dy\,dz}$$

$$-\rho v \frac{\partial h}{\partial y}\cancel{dx\,dy\,dz} + q'''\cancel{dx\,dy\,dz} = \rho \frac{\partial u}{\partial t}\cancel{dx\,dy\,dz}$$

$$-\frac{\partial q''_x}{\partial x} - \frac{\partial q''_y}{\partial y} - \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \rho \frac{\partial u}{\partial t}$$

$$+\,q'''$$

Conservation of Energy

$$-\frac{\partial q_x''}{\partial x} - \frac{\partial q_y''}{\partial y} + q''' = \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y}$$

$$h = u + \frac{p}{\rho}$$

$$dh = du + (\quad)$$

assumptions

∴ 1. uniform velocity
2. Constant Pressure, density
3. Negligible change in potential Energy
4. No viscous dissipation / No work done by viscous forces.

Conservation of Energy

$$-\frac{\partial q_x''}{\partial x} - \frac{\partial q_y''}{\partial y} + \dot{q}''' = \rho\frac{\partial h}{\partial t} + \rho u\frac{\partial h}{\partial x} + \rho v\frac{\partial h}{\partial y}$$

$\partial u = \partial h$

$$dh = c_p\, dT$$

$$q_x'' = -k\frac{dT}{dx}$$

$$q_y'' = -k\frac{dT}{dy}$$

$$= \rho c_p \frac{\partial T}{\partial t} + \rho u c_p \frac{dT}{dx} + \rho v c_p \frac{dT}{dy}$$

$$\frac{\partial q_x''}{\partial x} = -\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)$$

$$= \rho c_p\left\{\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

$$\frac{\partial q_y''}{\partial y} = -\frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial y}\right)$$

Conservation of Energy

$$\frac{\partial\left(k\frac{\partial T}{\partial x}\right)}{\partial x} + \frac{\partial\left(k\frac{\partial T}{\partial y}\right)}{\partial y} + q''' = \rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

If k = constant

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + q''' = \rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q'''}{k} = \frac{\rho C_p}{k}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

If no Heat gen. $q''' = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

Conservation of Energy

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + q''' = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right)$$

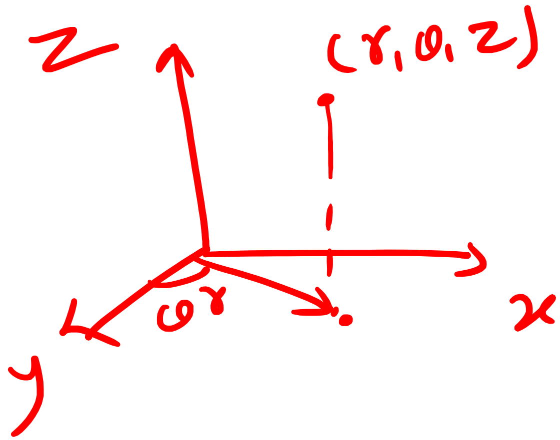
No advection

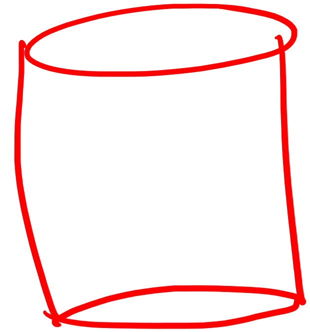
No Heat gen

steady state

Conservation of Energy

for cylindrical co-ordinate.





$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{K} =$$

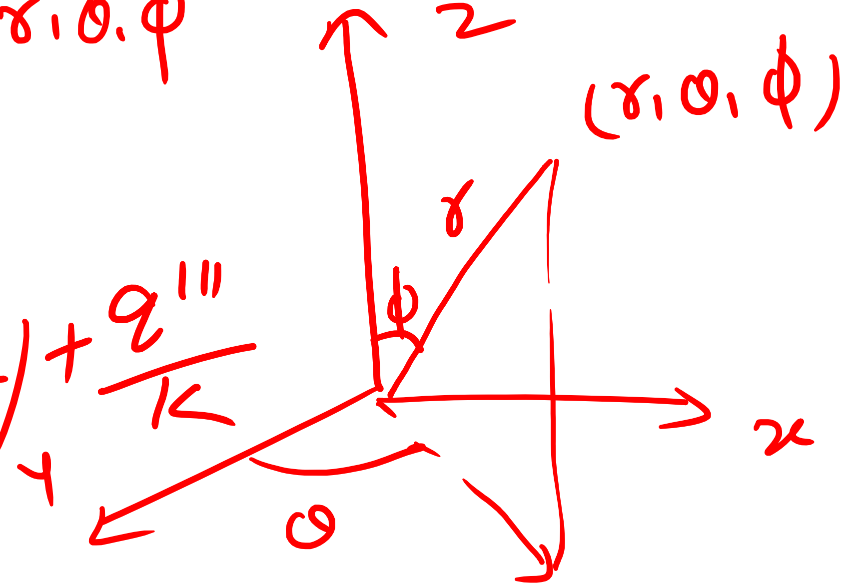
$$\frac{1}{\alpha}\left(\frac{\partial T}{\partial t} + V_r\frac{\partial T}{\partial r} + \frac{V_\theta}{r}\frac{\partial T}{\partial \theta} + V_z\frac{\partial T}{\partial z}\right)$$

Conservation of Energy

Spherical co-ordinate r, θ, ϕ

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\phi}\frac{\partial^2 \theta}{\partial \theta^2}$$

$$+ \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial T}{\partial \phi}\right) + \frac{q'''}{K}$$



$$= \frac{1}{\alpha}\left(\frac{\partial T}{\partial t} + V_r\frac{\partial T}{\partial r} + \frac{V_\phi}{r}\frac{\partial T}{\partial \phi} + \frac{V_\theta}{r\sin\phi}\frac{\partial T}{\partial \theta}\right)$$