09/11/24

*) $Q_{convection} = hA(T_w - T_s)$

(Rate of heat convection)

(Newton's law of cooling)

*) $Q_{emit} = \epsilon \sigma A T_w^4$

(Rate of heat radiation)

(stefan Boltzman - law),

$\sigma = 5.67 \times 10^{-8}$

T = kelvin scale.

*) $Q_{cond} = -kA \dfrac{dT}{dx}$.

*) Solid converted to liquid, while fusion welding.

*) $P = I \, U$ (U → volt; I → current).

Heat input $= \dfrac{P}{v} = \dfrac{IU}{v}$ (v → velocity).

*) Transfer efficiency

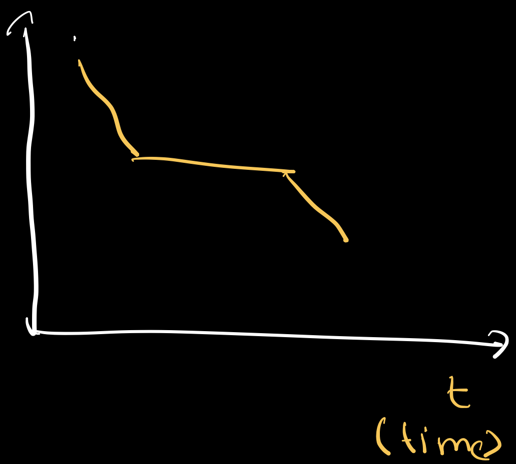
$$\eta = \frac{H_{net}}{H_{input}} = \frac{H_{net}}{(IU/v)}, \quad 0 < \eta < 1$$

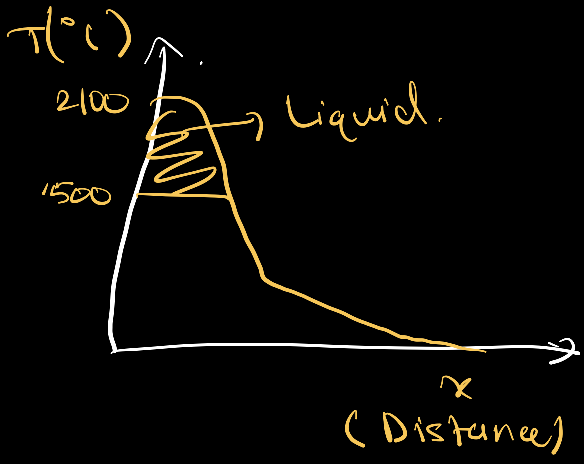
*) Submerged arc welding → high efficiency (90 to 98%)

→ solid converted to liquid (left alone) Then converts back to solid (cooling).

$x=0$ (max temp) → 2100°C for certain weld process.

i.e





*) Without phase change, solid converts to gas or similar i.e recrystalization temp. just change in microstructural, without phase Change in Base plate, (material property). ⟶ Heat affected zone.

*) $\sigma = \frac{1}{\sqrt{D}}$, D is grain diameter.

*) For spring, strength and resilance are needed.

*) <u>Melting effeiency</u>: <u>in fusion welding</u>

— To melt material (how much energy is needed) (H_{net})

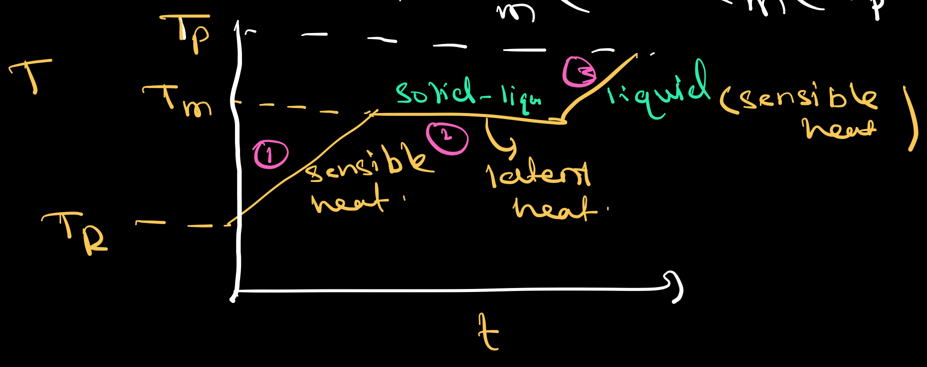
— $A_w = A_m + A_r$

A_r = additional area.

A_w = welding cross sectional area.

*) $Q_{required} = \rho_m V$

$= \rho_m (L + C_m (T_p - T_f) + C_s (T_f - T_0))$



T_R = room temp.

L = Latent heat.

*) Melting efficiency : $f = Q_{req} \times A_w H_{net}$

$$H_{net} = \eta \times H_{input}$$
$$= \eta \times \frac{IU}{V}$$

$$\therefore f = Q_{req} \times A_w \times \eta \times \frac{IU}{V}.$$

$$A_w = A_m + A_r.$$

Solid state welding process

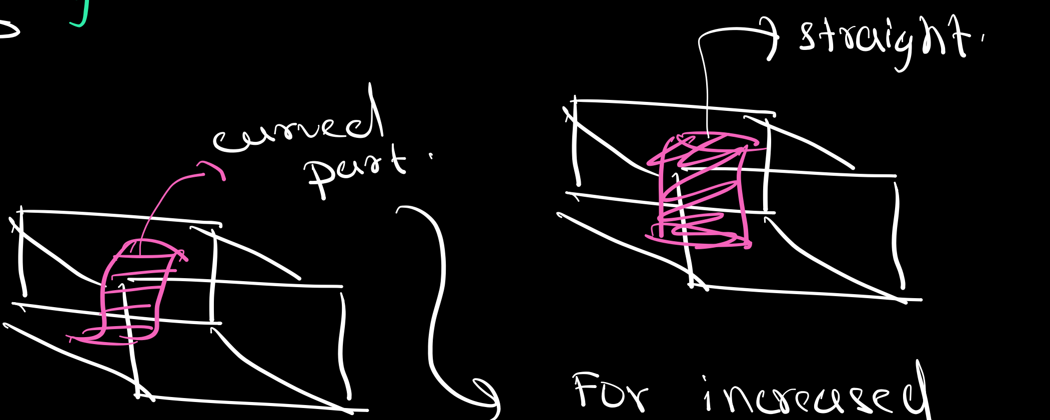
*) Doing welding with use of pressure, load, deformation.

Not typical ~~S – L, L – S~~

*) Material continuity using plastic deformation.

- Diffusion welding
- Friction welding
- Pressure welding.

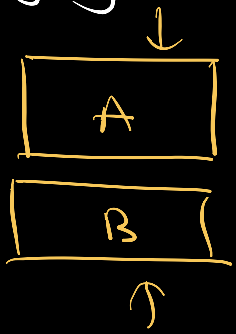
Diffusion welding



For increased surface area, for better cooling.

*) Curved hollow parts can be made using Diffusion welding.

*) Applying pressure at elevated temperature. $(0.5-0.7T_m) \longrightarrow (k)$.



Ultrasonic friction welding:

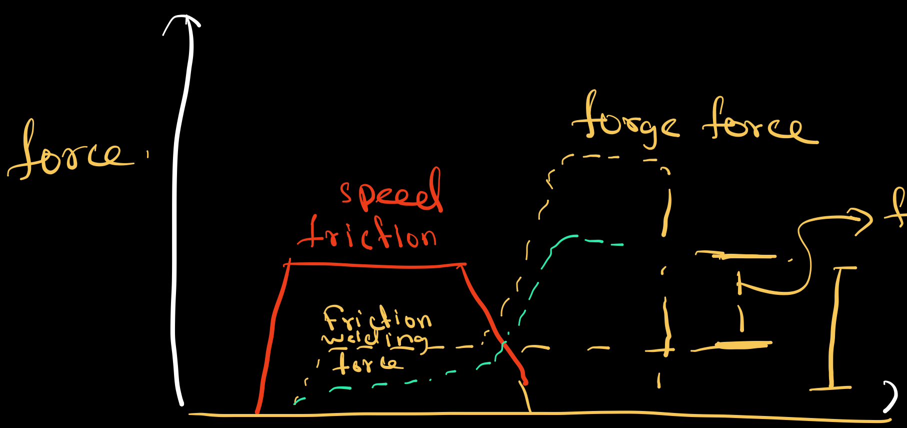
*) load is less, frequency is high. (30 kHz).

conventional friction welding:

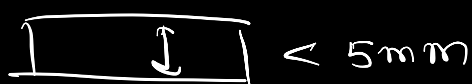
*) Amplitude of vibration is relatively large

- Rotational
- Angular reciprocating
- Linear reciprocating.

} Types of friction welding.



*) friction stir welding: $(0.8T_m) \rightarrow$ max Temp.



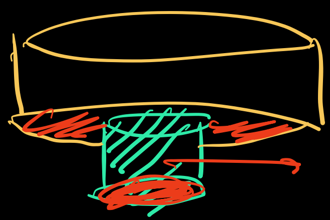
*) Amount of heat generated.

$$Q_t = \underbrace{\cancel{Q_{ttr}}}_{\text{(Ignore)}} + Q_{trot} \,, \quad Q_{ttr} = \text{Translational}$$

$$Q_{trot} = \text{rotational.}$$

As $Q_{ttr} <<< Q_{trot}$

for flat surfaces: (without any tapered surface)



$$Q_t = Q_{trot}$$

$$\therefore Q_t = \underset{\substack{\downarrow \\ \text{probe tip}}}{Q_{pt}} + \underset{\substack{\downarrow \\ \text{probe surface}}}{Q_{ps}} + \underset{\substack{\downarrow \\ \text{solder tip}}}{Q_{st}}$$

*) $Q_t \propto$ Torque, ω

$$\underbrace{Q_t}_{\text{(watt)}} = \overbrace{\omega}^{1/\min} \times \underbrace{M_t}_{N\cdot m} = \frac{N\cdot m}{sec} = \text{watt.}$$

$$dQ_t = \omega \times dM_t$$

$$= \omega \times r \, dF_t = \omega \times r \times \tau_{contact} \times dA.$$

$\tau_{contact}$ → flow stress of material or shear stress.

$$\therefore dQ_t = \omega \times r \times \tau_{contact} \times dA.$$

dQ_t = infinitesimal amount of heat generation.

dF_t = infinitesimal amount of heat force

r = distance from intin... part.

dA = Area of infinitesimal part.

$\tau_{contact}$ = flow stress (or) shear const., stress

12/1/24

Amount of heat generated during FSW (Q_t)

$$dA = r\,dr\,d\theta.$$

$$dF = \tau_{contact} \cdot dA$$

$$= \tau_{cont.}\, r\,dr\,d\theta.$$

$$dM_t = r \cdot dF.$$

$$= r^2\,dr\,d\theta \cdot \tau_{cont.}$$

$$dQ_{pt} = \omega \cdot dM_t$$

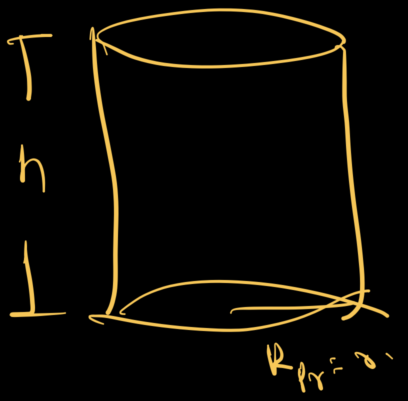
$$= r^2 \omega \cdot dr\,d\theta \cdot \tau_{cont.}$$

→ To find Q_{pt}.

$$\int dQ_{pt} = \int_0^{R_{pr}} \int_0^{2\pi} \omega \cdot \tau_{cont}\, r^2\,dr\,d\theta.$$

$$= \int_0^{2\pi} \omega \cdot \tau_{cont} \cdot \frac{R^3_{pr}}{3}\, d\theta$$

$$\boxed{Q_{pt} = \frac{2\pi}{3}\, \omega\, \tau_{cont} \cdot R^3_{pr}.}$$



$r = R_{pr}$.

$dA = r\, d\theta \cdot dz$.

$dF = \tau_{cont} \cdot dA$.

$= \tau_{cont} \cdot r\, d\theta\, dz$.

$dM_t = r\, dF$

$= r^2 \tau_{cont} \cdot d\theta\, dz$,

$dQ_{ps} = \omega\, dM_t$.

$= \omega r^2 \tau_{cont} \cdot d\theta\, dz$.

$Q_{ps} = \iint dQ_{ps}$

$= \int_0^h \int_0^{2\pi} \omega r^2 d\theta\, dz\; \tau_{cont}$, $r = R_{pr}$.

$$\boxed{Q_{ps} = \omega R_{pr}^2 \times 2\pi h\, \tau_{cont}}$$

→ For shoulder tip.

$dA = r\, dr\, d\theta$.

$dF = \tau_{cont} \cdot r\, dr\, d\theta$.

$dM_t = r^2 \tau_{cont} \cdot dr\, d\theta$.

$dQ_{st} = \omega\, dM_t$.

$= \omega r^2 \tau_{cont} \cdot dr\, d\theta$.

$Q_{st} = \int_{R_{pr}}^{R_{sh}} \int_0^{2\pi} \omega r^2 \tau_{cont} \cdot dr \cdot d\theta$

$$= \frac{2\pi}{3}\omega\left(R_{sh}^3 - R_{pr}^3\right)\cdot\tau_{cont}$$

$$Q_t = Q_{pt} + Q_{ps} + Q_{st}$$

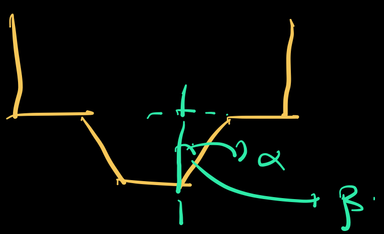
$$= \frac{2\pi}{3}\omega R_{pr}^3 \cancel{\tau_{cont}} + 2\pi\omega R_{pr}^3\tau_{cont} + \frac{2\pi}{3}\omega\left(R_{sh}^3 - \cancel{R_{pr}^3}\right)\tau_{cont}$$

$$= 2\pi\omega\tau_{cont}\left(R_{pr}^3 + \frac{R_{sh}^3}{3}\right)$$

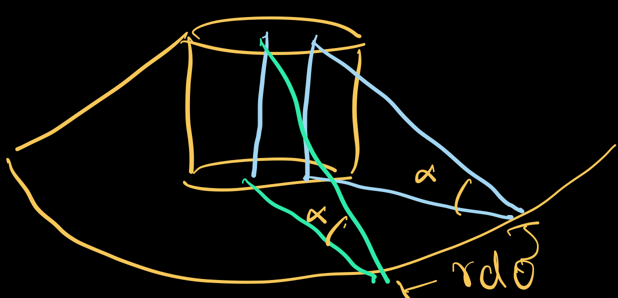
$$= \frac{2\pi}{3}\omega\tau_{cont}\left(3R_{pr}^3 + R_{sh}^3\right)$$

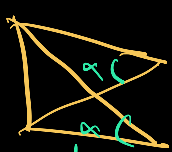
$$\boxed{Q_t = \frac{2\pi}{3}\omega\tau_{cont}\left(3R_{pr}^3 + R_{sh}^3\right)}$$

Tappered surface:



*) P_t and P_s will be same, s_t will be different (surface).





$$\tan\alpha = \frac{dz}{dr}$$

$$dA_r = dz \cdot r\, d\theta.$$

$$dA_h = r\, d\theta\, dr.$$

$$dA_v = r\, dr\, d\theta \tan\alpha.$$

$$dz = r \tan\alpha.$$

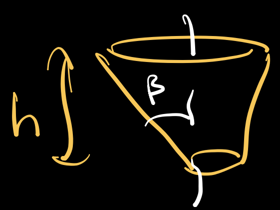
$$dF_{s1} = \tau_{cont}\left(r\, dr\, d\theta \tan\alpha + r\, dr\, d\theta \right)$$

$$= \tau_{cont}\, r\, dr\, d\theta \,(1 + \tan\alpha).$$

$$\boxed{Q_{st} = \frac{2\pi}{3}\left(R^3_{sh} - R^3_{pr}\right) \times \omega \times \tau_{cont}\,(1 + \tan\alpha)}$$

↳ for taper surface.

Taper surface (Probe side).



$$Q_{ps} = 2\pi R^2_{pr} \times h\, \omega\, \tau_{cont}\,(1 + \tan\beta)$$

Heat generation ratio

$$\eta_{pt} = \frac{Q_{pt}}{Q_t}, \quad \eta_{st} = \frac{Q_{st}}{Q_t}, \quad \eta_{ps} = \frac{Q_{ps}}{Q_t},$$

→ for flat surface.

$$Q_t = \frac{2\pi}{3}\, \omega\, \tau_{cont}\left(3R^3_{sn} + R^3_{pr}\right)$$

$$\boxed{\eta_{st} = \frac{Q_{st}}{Q_t} = \frac{\frac{2\pi}{3}\, \omega\, \tau_{cont}\left(3R^3_{sn} - R^3_{pr}\right)}{\frac{2\pi}{3}\, \omega\, \tau_{cont}\left(3R^3_{sn} + R^3_{pr}\right)}}$$

Assume that

$R_{pr} = 3mm$, $h = 5mm$, $R_{sh} = 10mm$.

$\eta_{st} = 85.7\%$ or 0.857.

Now,

$$\eta_{ps} = \frac{Q_{ps}}{Q_T} = \frac{2\pi\omega R_{pr}^2 h \tau_{cont}}{\frac{2\pi}{3}\omega\tau_{cont}(R_{sh}^3 + 3R_{pr}^2 h)}$$

Now,

$$\eta_{pr} = \frac{Q_{pr}}{Q_T} = \frac{\cancel{\frac{2\pi}{3}\omega\tau_{cont}} R_{pr}^3}{\cancel{\frac{2\pi}{3}\omega\tau_{cont}}(R_{sh}^3 + 3R_{pr}^2 h)}$$

$$= \frac{R_{pr}^3}{R_{sh}^3 + 3_{pr}^2 h} = 2.37\% = 0.023$$

*) Mainly heat generated due to shoulder tip (st), less due to Pr & Ps.

Amount of heat generated during FSW

Heat gen. Ratio

$$\eta_{sh} = \frac{Q_{st}}{Q_t}$$

$$= \frac{(r_{sh}^3 - r_{pr}^3)(1+\tan\alpha)}{(r_{sh}^3 - r_{pr}^3)(1+\tan\alpha) + r_{pr}^3 + 3r_{pr}^2 h}$$

$(1+\tan\beta)$

$\simeq 86\%$

$r_{sh} = 9\text{mm}, \; r_{pr} = 3\text{mm}$

$h = 4\text{mm}, \; \alpha = 10^\circ$

$$\eta_{ps} = \frac{Q_{ps}}{Q_t} = 11\%$$

$$\eta_{pt} = \frac{Q_{pt}}{Q_t} = 03\%$$

Problems

Numerical Question: FSW

1. FSW process is carried out by a normal force of 15kN with a rotational speed of 650 RPM. The coefficient of friction is 0.2, and the weld area is 55 mm². Calculate the shear stress developed in FSW and condition whether the sticking friction or sliding friction holds good for Al alloys AA 2014.

AA 2014 alloy: σ_0=414 MPa

Soln:

Pressure, $P = \frac{F}{A} = \frac{15{,}000}{55} = 273 \text{ MPa}$

Sliding friction $= \tau = \mu P$

$= 0.2 \times 273$

$= 54.6 \text{ MPa}$.

Shear stress developed = 55 MPa.

→ Distortion energy based theory:

$$\tau_0 = \frac{\sigma_0}{\sqrt{3}} = \frac{414}{\sqrt{3}} = 240 \text{ MPa}.$$

*) Shear stress in FSW → (54.6 MPa)

Yielding condition / flow stress → (240 MPa)

*) if $\tau_{\text{shear in FSW}} > \tau_{\text{flow/yielding}}$

↳ sticking.

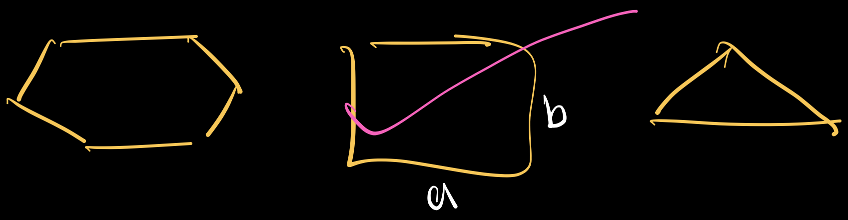
Else sliding

→ We got sliding condition.

Problem 2:

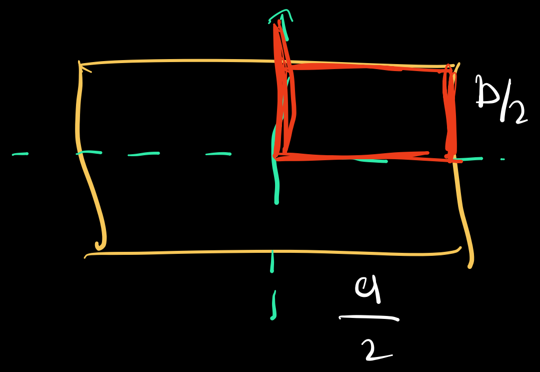
Numerical Question: FSW

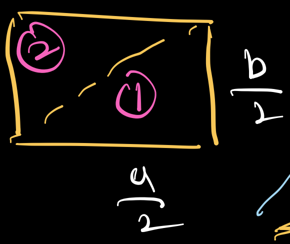
2. For a friction stir welding process, the rotating tool has a rectangular cross-section with a side of **a** and **b**. If the shear contact stress between the workpiece and tool is σ_0, the angular velocity of the tool is ω, and the coefficient of friction between the tool and workpiece is μ. Drive an expression using elemental analysis for the heat generated during the process during *N* rotations of the tool.

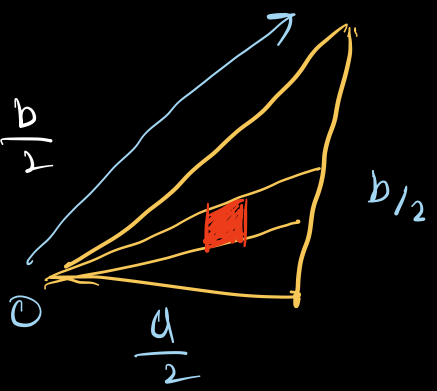


$\tau_0 = \tau_{contact}$

→ Calculate Area, elemental analysis.







Prob 3

Numerical Problems

1. Determine the net heat input for a butt welding job carried out at an arc voltage of 30V and a current of 200A at a welding speed of 300mm/min. Assume the heat transfer efficiency is 0.9.

Soln

300 mm/min = 5 mm/s

$$\eta = 0.9 = \frac{H_{net}}{H_{input}}, \quad \frac{I \times V}{v} = \frac{200 \times 30}{5}$$

$$= 1200\,J$$

$$H_{net} = 0.9 \times 1200$$

$$= 1080\ J/mm.$$

Prob 4

Numerical Problems

2. Determine the melting efficiency for a butt welding job carried (area= 35 mm^2) out at an arc voltage of 30V and a current of 200A at a welding speed of 300mm/min. Assume the heat transfer efficiency is 80%, and for melting 10 J/mm^3 is required.

$$f = \frac{Q_{req} \times A}{H_{net}}, \quad H_{net} = \eta \times H_{input}$$

$$= \eta \times \frac{I \times U}{v}$$

$$= \frac{10 \times 35}{960}$$

$$= \frac{0.8 \times 30 \times 200}{5}$$

$$= 960$$

Prob 5

Numerical Problems

3. In a welding process under steady-state conditions, the voltage and current are measured at 18 V and 160 A, respectively. Heat loss during arc creation is 40% of heat input. Heat loss through conduction, convection, and radiation from the workpiece is 800W. The effective power is used to melt the workpiece. Calculate the melting efficiency.

soln

$$H_{input} = 18 \times 160 = 2880 \text{ Watt}$$

After arc creation $= 2880 \times 0.6 = 1728$ Watt.

After heat loss = 1728 - 800

= 928 Watt.

Melting efficiency = $\frac{928 \times 100}{2880}$ = 32.2%

ARC Welding:

→ phase change phenomenon happens.

→ Electrode negative (DCEN)

Electrode positive. (DCSP) Straight Polarity.

Electrode positive (DCRP) Reverse polarity.

→ Thermionic emission (vibrations of electrons) (with temperature).

cold emssion (vibrations of electrons) (without temperature).

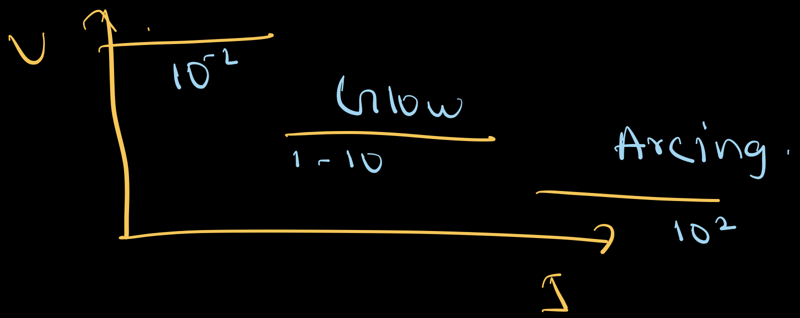
Energy → $\frac{1}{2}mv^2$, $V \to 100V$, $I \to 1A$.

→ Electron beam on H_2 produces H^+ ions.

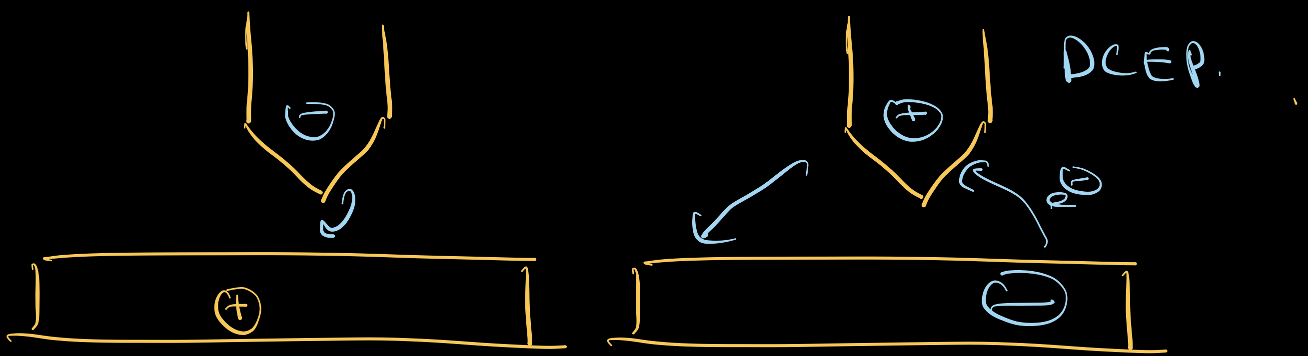
↳ Ionization.

Total Energy = $\underbrace{n}_{10^{18}} \times \frac{1}{2} \underbrace{m}_{10^{-31}} v^2 \longrightarrow 10^{16} \approx 1000\,J$

So, This process is called discharge.



*) Ion has higher mass, electron number and velocity is high.



16/1/24 (Tuesday)

Basics in arc welding:

*) 1 Amp flow = 6.24×10^{23} electrons.

*) Thermionic emission:

*) Ionization: H_2 to H^+ and e^-

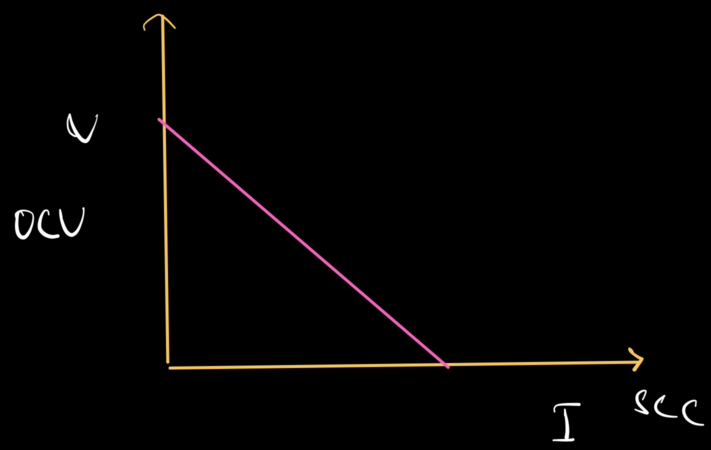
*) Discharge:

- Townsend discharge
- Glow discharge
- arc discharge.

* In DCEN, cathode drop zone

* Arc column

*) Electric arc: Current mode.



$V \propto l \rightarrow$ arc column

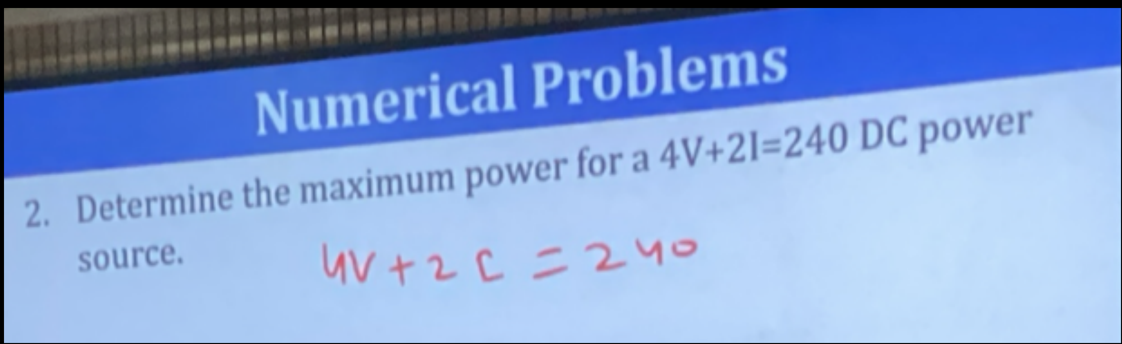
$$\frac{V}{V_{OCV}} + \frac{I}{I_{SCC}} = 1$$

$$4V + 3I = 120.$$

$$\frac{V}{30} + \frac{I}{40} = 1 \quad **.$$

*) Drooping condition

$\frac{dI}{dV}$ should be minimum.



<u>Soln:</u>

$$4V + 2I = 240.$$

$$\frac{V}{60} + \frac{I}{120} = 1.$$

$$V = \left(1 - \frac{I}{120}\right)60 , \qquad P = IV$$

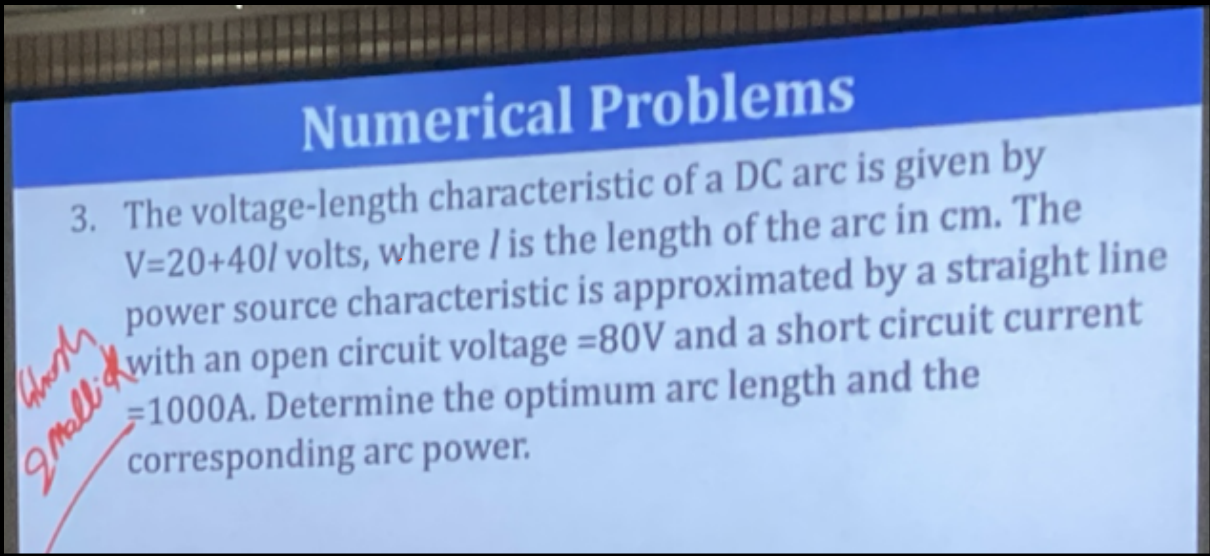
$$P = I\left(1 - \frac{I}{120}\right)60.$$

$$\frac{dP}{dI} = 0 , \qquad I = 60$$

$\therefore \quad P = VI = (30)\,60$

$= 1800$ Watt

*) Ghosh and mallick book.



Soln:

$V = 20 + 40l$, l in cm.

$OCV = 80V$

$SSC = 1000A$.

$\frac{V}{80} + \frac{I}{1000} = 1$, $\quad I = \left(1 - \frac{V}{80}\right) 1000$

$P = IV$ $\quad \rightarrow 20 + 40l$.

$= \left(1 - \frac{V}{80}\right) V \times 1000$

$\frac{dP}{dl} = 0$, optimum power, $l = 0.5$cm

---

*) DC, Duty cycle = 60% at 200A.

60% $\rightarrow$ 6 min

$I^2T = C$

Duty Cycle

- $I^2T = I_r^2 T_r$

$I^2 T = C$

- Where T is % rated duty cycle, I is rated current
- T_r is required duty cycle, I_r is maximum required current output at desired duty cycle

Numerical problem:

1. What will be the duty cycle with a 200A power supply rated at 60% duty cycle operated at the current 250A output?

200

*) $200^2 \times 0.6 = 250^2 \times 2c$.

---

*) GMAW, MIGW — 90's
→ metal, consumable electrode

*) PAW, Plasma

*) SAW, submerged.
$\eta = 0.55 - 0.99$, naval application.

Chemical fusion welding:

*) Thermit welding – Tracks.

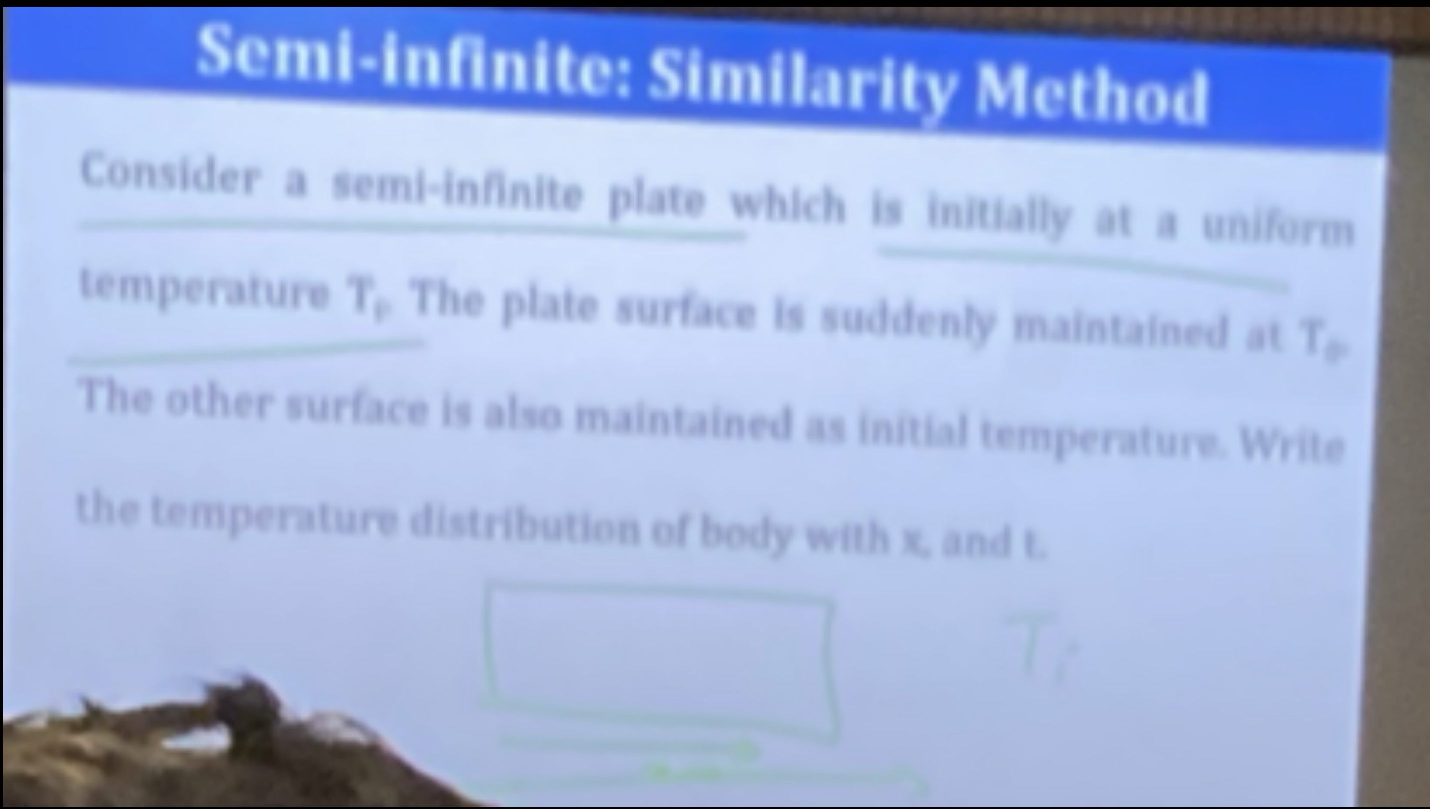
*) Oxyfuel gas welding – exothermic rxn in presence of O_2

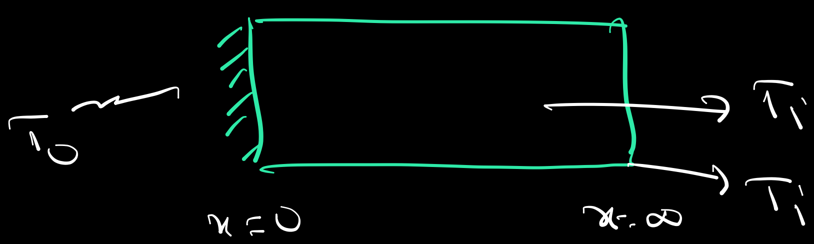
*) $Q(W) = 48\ kJ/L \times V_{act} \times h/3600$

heat of combustion

One class miss in between.

23/11/24 (Tuesday)





Given, Governing equation.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial x}$$

Boundary conditions ; $x(0,t) = T_0$

$x(\infty,t) = T_i$

Initial conditions ; $T(x,0) = T_i$

→ Solve using similarity method.

dependent variable $= T \rightarrow$ Temp.

Independent variable $= (x, t)$

$$\eta = \frac{x}{\sqrt{4\alpha t}}, \quad \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

$$\frac{\Delta T}{\Delta x^2} = \frac{1}{\alpha}\frac{\Delta T}{\Delta t}.$$

$$\eta = \sqrt{\frac{x^2}{\alpha t}} \Rightarrow \eta = \frac{x}{\sqrt{\alpha t}}.$$

$$\eta = \frac{x}{\sqrt{4\alpha t}}$$

*) Non-dimension less temp

$$= \frac{T - T_i}{T_0 - T_i}$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial t}.$$

B.C,

$\theta(0,t) = 1.$

$\theta(\infty,t) = 0.$

I.C, $\theta(x,0) = 0.$

G.E,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

B.C,

$T(0,t) = T_0$

$T(\infty,t) = T_i$

I.C.

$T(x,0) = T_i$

⇒ Similarity variable : $\eta = \frac{x}{\sqrt{4\alpha t}}$

$T(x,t)$, $\theta(x,t)$

$$\frac{\partial \theta}{\partial x} = \frac{\partial \theta}{d\eta} \times \frac{d\eta}{\partial x} = \frac{\partial \theta}{\partial x} \times \frac{1}{\sqrt{4\alpha t}},$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial \theta}{\partial x} \frac{1}{\sqrt{4\alpha t}}\right]$$

$$= \frac{d}{\partial x}\left(\frac{\partial \theta}{\partial x} \frac{1}{\sqrt{4\alpha t}}\right) \times \frac{d\eta}{\partial x}.$$

$$= \frac{\partial^2 \theta}{\partial x^2} \times \frac{1}{\sqrt{4\alpha t}} \times \frac{1}{\sqrt{4\alpha t}}$$

$$= \frac{1}{4\alpha t} \cdot \frac{\partial^2 \theta}{\partial x^2}$$

A) $\frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial x} \times \frac{\partial x}{\partial t}$, $\quad \eta = \frac{x}{\sqrt{4\alpha t}}$,

$= \frac{\partial \theta}{\partial x}\left[-\frac{1}{2} \frac{\eta}{\sqrt{4\alpha} t^{3/2}}\right]$ $\quad \frac{\partial \eta}{\partial t} = -\frac{1}{2} \frac{x}{\sqrt{4\alpha}\, t^{3/2}}$.

$= \frac{\partial \theta}{\partial x}\left(-\frac{1}{2} \frac{\eta}{t}\right)$. $\quad , \quad \frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \theta}{\partial t}$.

$$\frac{1}{\cancel{4}\cancel{\alpha} t} \frac{\partial^2 \theta}{\partial \eta^2} = \frac{1}{\cancel{\alpha}} \times \frac{\partial \theta}{\partial x}\left(-\frac{1}{\cancel{2}} \frac{\eta}{t}\right).$$

$$\frac{\partial^2 \theta}{2\, \partial \eta^2} = -\frac{\partial \theta}{\partial x} \frac{\eta}{t}.$$

$$\frac{\partial^2\theta}{\partial\eta^2} = -\frac{\partial\theta}{\partial x}\left(\frac{2\eta}{t}\right)$$

*) $\eta = \frac{x}{4\alpha t}$

$\theta(x,t) \to \theta(\eta)$

$\theta(0) = 1 \rightsquigarrow x \to 0$, $\eta \to 0$

$\theta(\infty) = 0 \rightsquigarrow x \to \infty$, $\eta \to \infty$

B.C,

$T(0,t) = T_0$

$T(\infty,t) = T_i$

I.C,

$T(x,0) = T_i$

$\theta(x,t) = \frac{T - T_i}{T_0 - T_i}$

B.C

$\theta(0,t) = T_0 = 1$

$\theta(\infty,t) = T_i = 0$

I.C.

$\theta(x,0) = T_i = 0$

*) $$\frac{d^2\theta}{dx^2} = -2\eta\frac{\partial\theta}{\partial x}$$

$$\frac{\partial}{\partial\eta}\left(\frac{\partial\theta}{\partial x}\right) = -2\eta\frac{\partial\theta}{\partial x}$$

$$\frac{d\left(\frac{\partial\theta}{\partial x}\right)}{\frac{\partial\theta}{\partial x}} = -2\eta\,\partial\eta$$

On Integration,

$$\ln\frac{\partial\theta}{\partial\eta} = -\eta^2 + \ln k$$

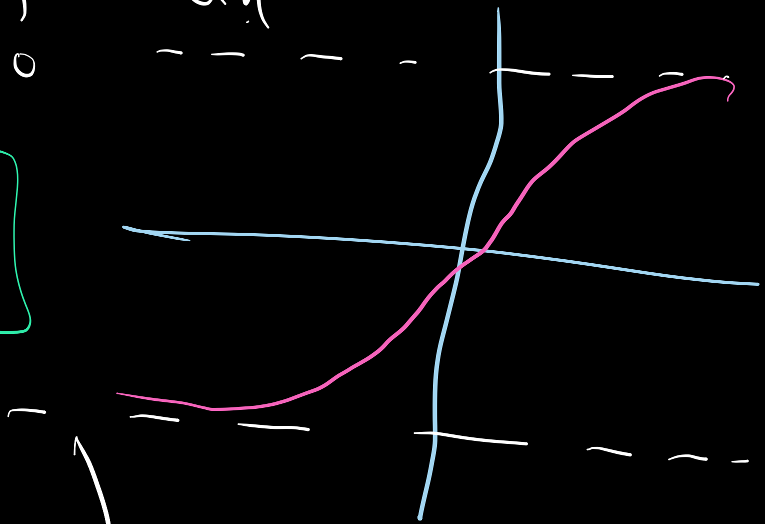
$$\frac{\partial\theta}{\partial\eta} = ke^{-\eta^2}$$

$$\int_0^{\theta} \partial\theta = \int_0^{\eta} k e^{-\eta^2} \partial\eta$$

$$\theta(\theta) - \theta(0) = k \int_0^{\eta} e^{-\eta^2} d\eta$$

$$\theta - 1 = k \frac{\sqrt{\pi}}{2} \operatorname{erf}(\eta)$$

$$\operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-\eta^2} d\eta$$



$$\therefore \boxed{\theta = 1 - \operatorname{erf}(\eta)}$$

*) $$\theta = \frac{T - T_i}{T_0 - T_i} = 1 - \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

$$\operatorname{erf}(0) = 0$$

$$\operatorname{erf}(\pm\infty) = \pm 1$$

$$T = T_i + (T_0 - T_i)\left(1 - \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right)\right)$$

$$= T_i + (T_0 - T_i)\left(\operatorname{erfc}\left(\frac{x}{\sqrt{4\alpha t}}\right)\right)$$

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$$

*) $\theta(0) = 1$, $\theta(\infty) = 1$

Source/Sink in Welding: Point source

Point heat Source of constant heat Production Rate

O is heating source which is producing const heat.

$r \to \infty$

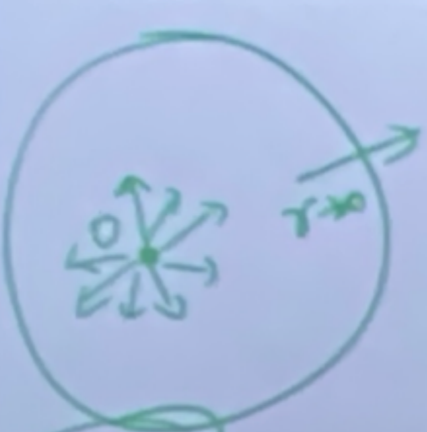
Consider a body of large extent possessing a very small region (point) that produce heat continuously at a constant rate (Q, watt)

Source/Sink in Welding: Point source

Given that body was initially maintained at $T = T_\infty$

G.E.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{— } 1$$



B.C.

$r \to \infty, \quad T = T_\infty \quad \text{— } 2$

$-KA\frac{dT}{dr} = Q$

$r \to 0, \quad -4\pi r^2 K \frac{dT}{dr} = Q \quad \text{— } 3$

I.C. ⇒ $t = 0, \quad T = T_\infty \quad \text{— } 4$

Source/Sink in Welding: Point source

$$\theta = T - T_\infty$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{— (1A)}$$

B.C. $r \to 0, \; -4\pi r^2 K \frac{\partial \theta}{\partial r} = Q$ (2A)

$r \to \infty, \quad \theta = 0$ ——— 3(A)

I.C. $t = 0, \quad \theta = 0,$ ——— (4A)

$\because \; u = r(T - T_\infty) = r\theta$

Source/Sink in Welding: Point source

$u = r\theta$

$$\frac{\partial u}{\partial t} = r\frac{\partial \theta}{\partial t} + \theta \frac{\partial r}{\partial t}$$

$$\frac{\partial r}{\partial t} \to 0$$

$$\boxed{\frac{\partial u}{\partial t} = r\frac{\partial \theta}{\partial t}} \quad \text{— (5)}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial u}{\partial t} \quad \text{— 5A}$$

$$\frac{\partial u}{\partial r} = r\frac{\partial \theta}{\partial r} + \theta \quad \text{— (6)}$$

$$\frac{\partial \theta}{\partial r} = \frac{1}{r}\left(\frac{\partial u}{\partial r} - \theta\right) \quad \text{— (6 A)}$$

$$\frac{\partial^2 u}{\partial r^2} = r\frac{\partial^2 \theta}{\partial r^2} + \frac{\partial \theta}{\partial r} + \frac{\partial \theta}{\partial r}$$

$$= r\frac{\partial^2 \theta}{\partial r^2} + 2\frac{\partial \theta}{\partial r} \quad \text{— (7)}$$

Source/Sink in Welding: Point source

(1A) $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial\theta}{\partial t}$$

$$\rightarrow \quad \frac{1}{r^2}\left(r^2\frac{\partial^2\theta}{\partial r^2}+\frac{\partial\theta}{\partial r}\times 2r\right)=\frac{1}{\alpha}\frac{\partial\theta}{\partial t}$$

$$r\times\left(\frac{\partial^2\theta}{\partial r^2}+\frac{2}{r}\frac{\partial\theta}{\partial r}\right)=\left(\frac{1}{\alpha}\frac{\partial\theta}{\partial t}\right)\times r$$

$$\underbrace{r\frac{\partial^2\theta}{\partial r^2}+2\frac{\partial\theta}{\partial r}}_{\text{eqn } (7)}=\frac{r}{\alpha}\frac{\partial\theta}{\partial t}$$

$$\frac{\partial^2 u}{\partial r^2}=\frac{1}{\alpha}\frac{\partial u}{\partial t} \longrightarrow \text{eqn } (5)$$

Source/Sink in Welding: Point source

$u = r\theta = r(T-T_0)$

$$\frac{\partial^2 u}{\partial r^2}=\frac{1}{\alpha}\frac{\partial u}{\partial t} \quad \text{—— } (8)$$

(2A) $\rightarrow r\rightarrow 0,\ u\rightarrow 0,\ -4\pi r^2 k\frac{\partial\theta}{\partial r}=Q$ eqn (6A)

$$-4\pi \cancel{r^2}\, k\,\frac{1}{\cancel{r}}\left(\frac{\partial u}{\partial r}-\theta\right)=Q$$

$$-4\pi k\left(r\frac{\partial u}{\partial r}-r\theta\right)=Q$$

$$-4\pi k\left(r\frac{\partial u}{\partial r}-u\right)=Q \quad \text{—— } (9)$$

Source/Sink in Welding: Point source

$u = r\theta$

(3A) $\rightarrow r \to \infty,\ u \to \infty,\ \theta = 0$ — (10)

(4A) $t = 0,\ u = 0,\ \theta = 0$ — (11)

$$\eta = \frac{r}{\sqrt{4\alpha t}} \quad \text{— (12)}$$

$$u = A + B\,\mathrm{erf}(\eta) \quad \text{— (13)}$$

Source/Sink in Welding: Point source

$u = r\theta$

at $r \to 0,\ \eta \to 0$ $\quad -4\pi k\left(r\frac{\partial u}{\partial r} - u\right) = Q$

$$\eta = \frac{r}{\sqrt{4\alpha t}}$$

$$\frac{\partial u}{\partial r} = \frac{du}{d\eta}\frac{\partial \eta}{\partial r} = \frac{1}{\sqrt{4\alpha t}}\frac{du}{d\eta}$$

$$-4\pi k\left(r\frac{1}{\sqrt{4\alpha t}}\frac{du}{d\eta} - u\right) = Q$$

$$-4\pi k\left(\eta\frac{du}{d\eta} - u\right) = Q \quad \text{— (14A)}$$

$t = 0,\ r \to \infty,\ \eta \to \infty,\ u = 0$

$$u = A + B\,\mathrm{erf}(\eta) \Rightarrow 0 = A + B\,\mathrm{erf}(\infty)$$

$$\boxed{A + B = 0} \quad \text{— (14B)}$$

Source/Sink in Welding: Point source

$r \to 0$ $\quad -4\pi k \left(\cancel{r\frac{du}{dr}} - u \right) = Q$

$$-4\pi k \left(-A - \cancel{B\,\mathrm{erf}(0)} \right) = Q$$

$$A = \frac{Q}{4\pi k}$$

$$B = -\frac{Q}{4\pi k}$$

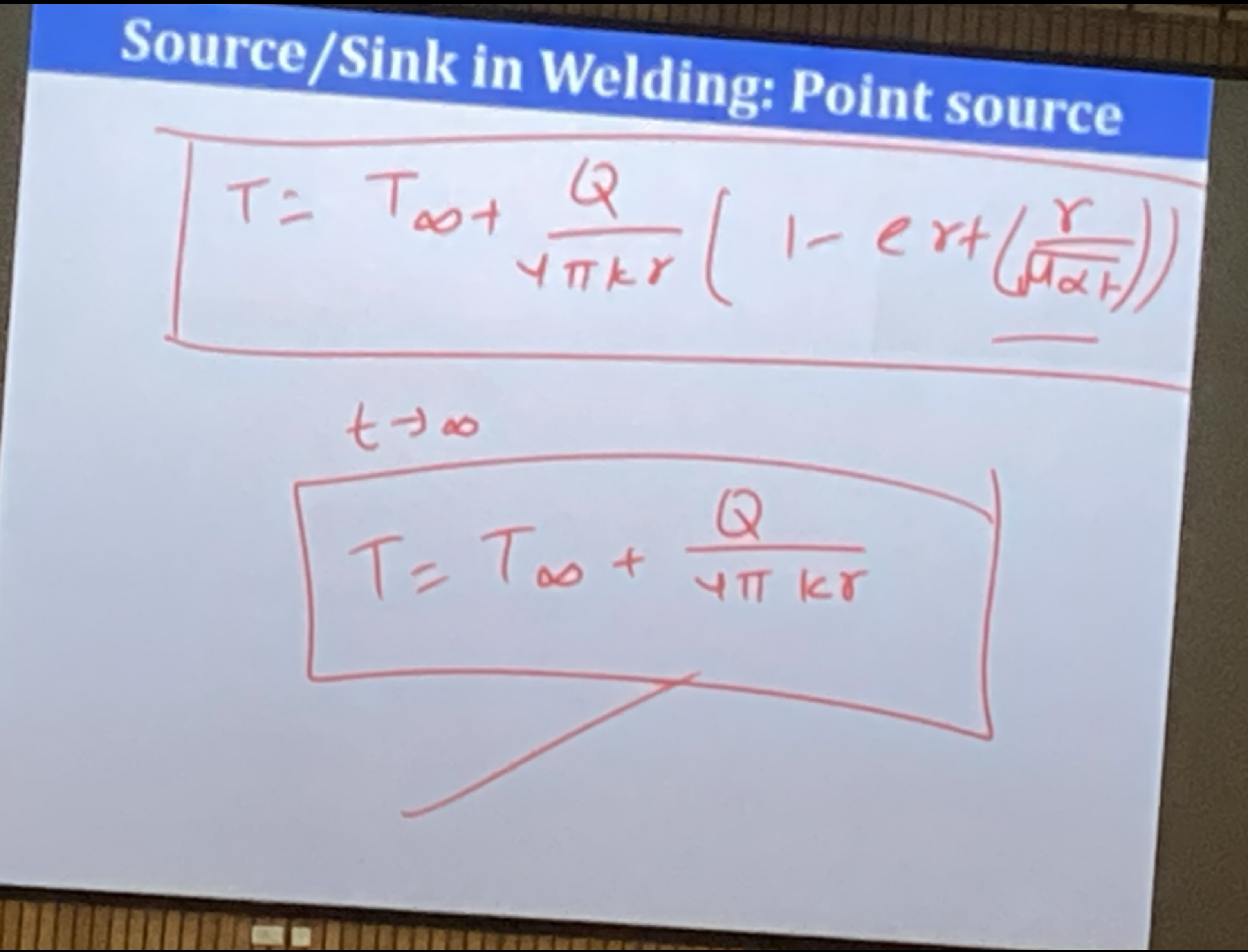
Source/Sink in Welding: Point source

$$u = A + B\,\mathrm{erf}(\eta) \qquad \eta = \frac{r}{\sqrt{4\alpha t}}$$

$$u = \frac{Q}{4\pi k} - \frac{Q}{4\pi k}\,\mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)$$

$$r\theta = \frac{Q}{4\pi k}\left(1 - \mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)\right)$$

$$r(T - T_\infty) = \frac{Q}{4\pi k}\left(1 - \mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)\right)$$



30/1/24

*) As $t \to \infty$

$$T = T_\infty + \frac{Q}{4\pi k r}.$$

Instantaneous heat source: Thermal explosion at point region.

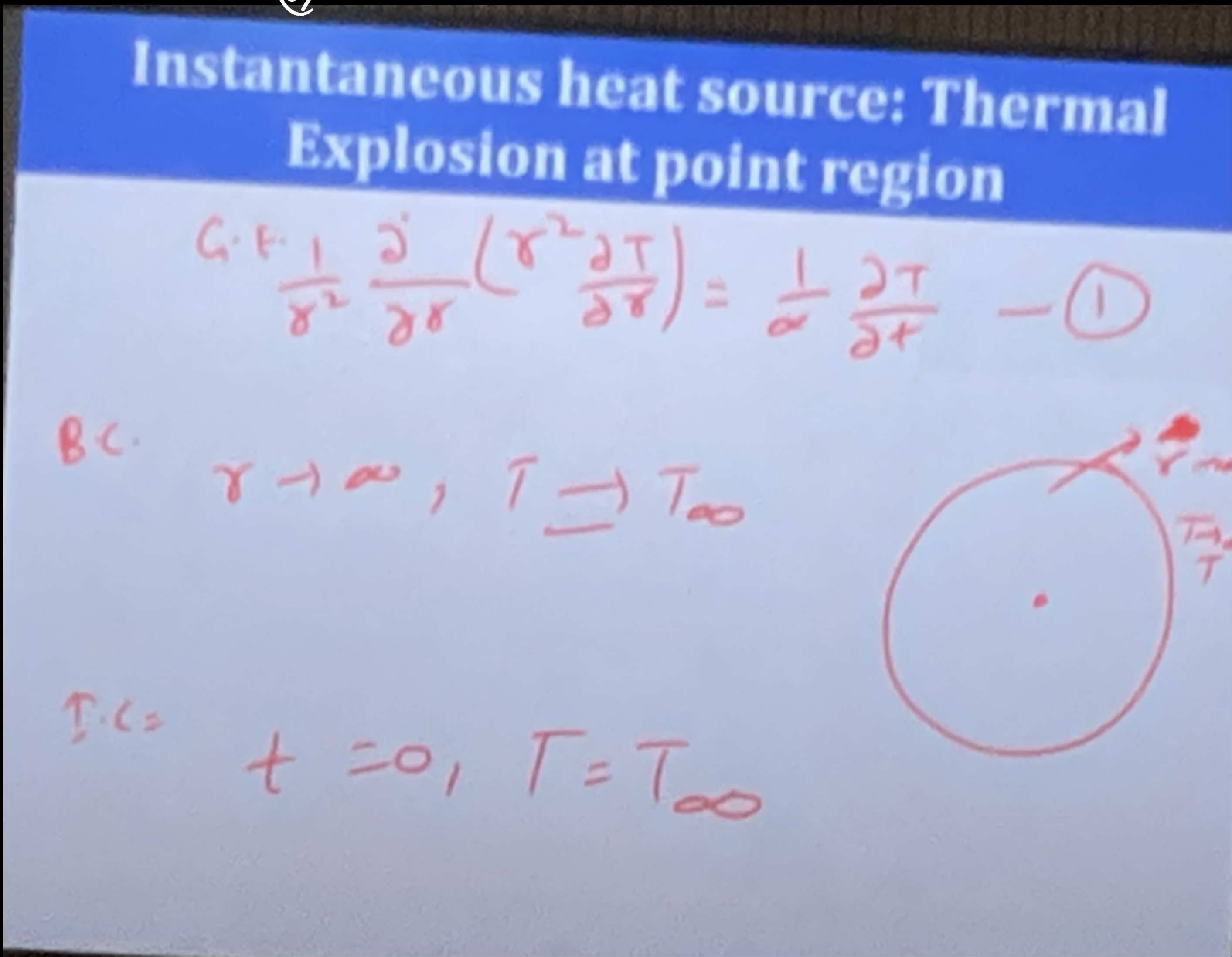
Consider a body of large extent processing a small region (point) that expressed as E_0 energy at $t=0$ centre.

Soln: Governing equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad — (1).$$

Continuation

↓



Condition: Energy Balance

$$\int_0^\infty 4\pi r^2 \rho c_p (T - T_\infty)\, dr = E_0$$

Instantaneous heat source: Thermal Explosion at point region

$\Theta = T - T_\infty$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \Theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \Theta}{\partial t} \quad \text{— (1A)}$$

$$\int_0^\infty 4\pi r^2 \rho c_p \Theta \, dr = E_0 \quad \text{— (2A)}$$

$$r \to \infty, \quad \Theta = 0, \quad \text{— (3A)}$$

$$t = 0, \quad \Theta = 0, \quad \text{— (4A)}$$

Instantaneous heat source: Thermal Explosion at point region

$u = r\Theta$

$$\frac{\partial u}{\partial t} = r\frac{\partial \Theta}{\partial t} \quad (5), \qquad \frac{\partial \Theta}{\partial t} = \frac{1}{r}\frac{\partial u}{\partial t} \quad \text{— (5A)}$$

$$\frac{\partial u}{\partial r} = r\frac{\partial \Theta}{\partial r} + \Theta \quad \text{— (6)}, \qquad \frac{\partial \Theta}{\partial r} = \frac{1}{r}\left(\frac{\partial u}{\partial r} - \Theta\right) \quad \text{— (6A)}$$

$$\frac{\partial^2 u}{\partial r^2} = r\frac{\partial^2 \Theta}{\partial r^2} + 2\frac{\partial \Theta}{\partial r} \quad \text{— (7)}$$

Instantaneous heat source: Thermal Explosion at point region

(1A) $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \theta}{\partial t}$$

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{\alpha}\frac{\partial u}{\partial t} \quad \text{— (8)}$$

$$\int_0^\infty 4\pi r^2 \rho C_p\, \theta\, dr = E_0$$

$r^2\, \theta\, dr$

$r \cdot r\, \theta\, dr$

$r\, u\, dr$

$$4\pi \rho C_p \int_0^\infty u r\, dr = E_0 \quad \text{— (9)}$$

$$r \to \infty,\ \theta = 0,\ u = 0 \quad \text{— (10)}$$

$$t = 0,\ \theta = 0,\ u = 0 \quad \text{— (11)}$$

Instantaneous heat source: Thermal Explosion at point region

$$u = A + B\,\mathrm{erf}(\eta)$$
$$= A + B\,\mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)$$

$$u' = B\,\frac{2}{\sqrt{\pi}}\, e^{-\frac{r^2}{4\alpha t}} \left(\frac{1}{\sqrt{4\alpha t}}\right)$$

$$\mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-\eta^2}\, d\eta$$

Instantaneous heat source: Thermal Explosion at point region

$$u = A + B\,\mathrm{erf}(\eta)$$

$$= A + B\,\mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right) \quad \text{—} \; (12)$$

$$u' = B\,\frac{\cancel{2}}{\sqrt{\pi}}\, e^{-\frac{r^2}{4\alpha t}} \left(\frac{1}{\sqrt{\cancel{4}\alpha t}}\right)$$

$$u' = B\,\frac{1}{\sqrt{\pi\alpha t}}\, e^{-\frac{r^2}{4\alpha t}} \quad \text{—} \; (13)$$

Instantaneous heat source: Thermal Explosion at point region

eq^n (9) $\quad 4\pi\rho C_p \int_0^\infty u\, r\, dr = E_0$

$$u = u'' = \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad \text{—} \; (14)$$

$$4\pi\rho C_p \int_0^\infty \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) r\, dr = E_0 \quad \text{—} \; (15)$$

$$z^2 = \frac{r^2}{4\alpha t}, \qquad 2z\,dz = \frac{2r\,dr}{4\alpha t}$$

$$z = \frac{r}{\sqrt{4\alpha t}}, \qquad dz = \frac{dr}{\sqrt{4\alpha t}}$$

Instantaneous heat source: Thermal Explosion at point region

$$\int_0^{\infty} \frac{-2Br^2}{(4\alpha t)\sqrt{\pi\alpha t}} \exp(-z^2)\, dr = \frac{E_0}{4\pi\rho C_p}$$

$$\int_0^{\infty} \frac{-2B\cancel{r^2}}{\cancel{4\alpha t\sqrt{\pi\alpha t}}} \exp(-z^2)\, \sqrt{\cancel{4\alpha t}}\, dz = \frac{E_0}{4\pi\rho C_p}$$

$$\int_0^{\infty} -\frac{4BZ^2}{\sqrt{\pi}} \exp(-z^2)\, dz = \frac{E_0}{4\pi\rho C_p}$$

Instantaneous heat source: Thermal Explosion at point region

$$-\frac{2B}{\sqrt{\pi}} \Gamma 3/2 = \frac{E_0}{4\pi\rho C_p}$$

$$\Gamma\tfrac{1}{2}+1 = \tfrac{1}{2}\Gamma 1/2$$
$$= \tfrac{1}{2} \times \sqrt{\pi}$$
$$= \frac{\sqrt{\pi}}{2}$$

$$-\frac{\cancel{2}B}{\cancel{\sqrt{\pi}}} \times \frac{\cancel{\sqrt{\pi}}}{\cancel{2}} = \frac{E_0}{4\pi\rho C_p}$$

$$B = -\frac{E_0}{4\pi\rho C_p}$$

Instantaneous heat source: Thermal Explosion at point region

So, u'' is our sol^n

$$u = u'' = \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

eq^n 14

(6) $B = -\frac{E_0}{4\pi\rho C_P}$

$$u = \frac{E_0\, r}{2\sqrt{\pi}\,(\alpha t)^{3/2} \times 4\pi\rho C_P} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

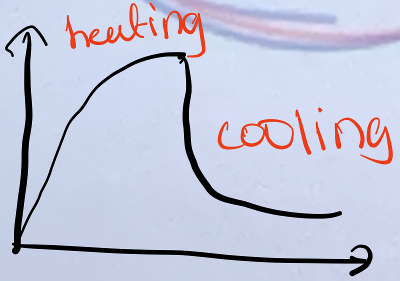
$u = r\theta = r(T - T_\infty)$

$$u = \frac{E_0\, r}{8\,(\pi\alpha t)^{3/2}\rho C_P} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

Instantaneous heat source: Thermal Explosion at point region

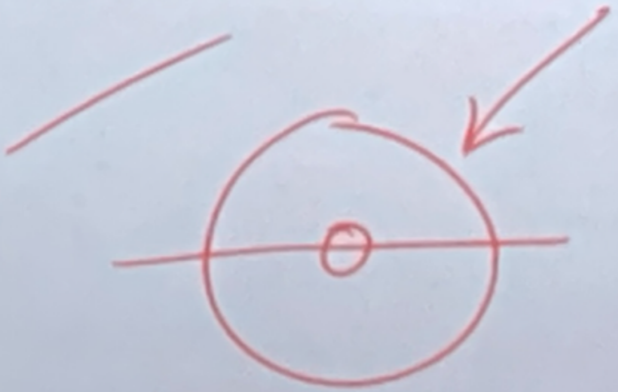
$$\boxed{T = T_\infty + \frac{E_0}{8\,(\pi\alpha t)^{3/2}\rho C_P} \exp\left(\frac{-r^2}{4\alpha t}\right)}$$

Arc strikes in Fusion welding



Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_0}{8 \rho C_p (\pi \alpha t)^{3/2}} \exp\left(\frac{-r^2}{4 \alpha t}\right)$$



$$\boxed{T = T_\infty + \frac{E_0}{4 \rho C_p (\pi \alpha t)^{3/2}} \exp\left(\frac{-r^2}{4 \alpha t}\right)}$$

Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_0}{4\rho c_p(\pi\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

Assuming

$$\theta = \frac{T - T_\infty}{T_c - T_\infty}$$

Non-dimensional temp.

T_∞ = initial temp

T_c = Reference temp.

$T_c = T_m$ = melting point

Arc strikes in Fusion welding

Dimension less radius factor $= \sigma_1 = \sqrt{\frac{R^2}{4\alpha t_i}}$

t_i = ignition time

Dimension less operating parameter

$$= n_1 = \frac{E_0}{4(\pi\alpha t_i)^{3/2}\rho c_p (T_c - T_\infty)}$$

$$\rho c_p (T_c - T_\infty) = \Delta H$$

$$= \frac{E_0}{4(\pi\alpha t_i)^{3/2}\Delta H}$$

$$\boxed{\tau = t/t_i}$$

Arc strikes in Fusion welding

q_0 Heat input = $\frac{E_0}{t_i} \rightarrow \frac{Joule}{Sec}$ watt

$= I \times V$

current × vol.

$$n_1 = \frac{E_0}{4(\pi\alpha)^{3/2}\, t_i^{1/2} \times t_i\, \Delta H} = \frac{q_0}{4(\pi\alpha)^{3/2}\, t_i^{1/2}\, \Delta H}$$

Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_0}{4\rho C_p(\pi\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\frac{T - T_\infty}{T_c - T_\infty} = \frac{E_0}{4\rho C_p(\pi\alpha t)^{3/2}} \frac{1}{T_c - T_\infty} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\theta = \frac{E_0}{4\,\underbrace{\rho C_p (\pi\alpha t)^{3/2} (T_c - T_\infty)}_{\Delta H}} \exp\left(\frac{-r^2}{4\alpha \frac{t}{t_i}}\right)$$

τ

Arc strikes in Fusion welding

$$\theta = \frac{E_0}{4\Delta H\left(\pi\alpha t_i \times \frac{t}{t_i}\right)^{3/2}} \exp\left(\frac{-r^2}{4\alpha t_i \times \frac{t}{t_i}}\right)$$

$r^2 \to \sigma_1^2$, $\frac{t}{t_i} \to \tau_1$

$$= \frac{E_0}{4\Delta H(\pi\alpha t_i)^{3/2}\left(\frac{t}{t_i}\right)^{3/2}} \exp\left(\frac{-\sigma_1^2}{\tau_1}\right)$$

$(\pi\alpha)^{3/2}\, t_i^{3/2}\, t_i \to \tau_1^{3/2}$

$\frac{E_0}{t_i} = q_0$

$$= \frac{q_0}{4\Delta H(\pi\alpha)^{3/2}\, t_i^{1/2}\, \tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$\to n_1$

Arc strikes in Fusion welding

$$\boxed{\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)}$$

$$\boxed{\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)}$$

2/2/24 (Friday)

*) $\theta = \dfrac{\eta_1}{\tau_1^{3/2}} \exp\left(-\dfrac{\sigma_1^2}{\tau_1}\right)$

$\Rightarrow$ for maximum temp or peak temp

$$\frac{\partial \theta}{\partial \tau_1} = 0$$

(or) $$\frac{\partial\left(\frac{\theta}{\eta_1}\right)}{\partial \tau_1} = \tau_1^{-3/2} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)\left(+\frac{\sigma_1^2}{\tau^2}\right) + \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)\left(-\frac{3}{2}\right)\tau_1^{-5/2}$$

$$= 0.$$

$$\boxed{\therefore \; \sigma^2 = \frac{3}{2}\tau_1}$$ $\rightarrow$ for max temp./ peak temp.

$\rightarrow \sigma_{1m}^2 = \dfrac{3}{2}\tau_{1m}$

*) $$\frac{\theta}{\eta_1} = \frac{1}{\tau_1^{3/2}} \times \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$$\frac{\theta_1}{\eta_1} = \frac{1}{\tau_1^{3/2}} \times \exp\left(\frac{3}{2}\right).$$

$$\Rightarrow \frac{\theta}{\eta_1} = \frac{1}{(e\tau_1)^{3/2}}$$

$$\frac{\theta_p}{\eta_1} = \frac{1}{\left(\frac{2}{3}e\right)^{3/2}\sigma_1^3}$$

Locus of peak condition.

$$* \quad \frac{\theta}{n_1} = \frac{1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$$*) \quad \sigma_{1m}^2 = \frac{3}{2} \tau_{1m}$$

$$*) \quad \frac{\theta_p}{n_1} = \frac{1}{(e\tau_1)^{3/2}} = \frac{1}{\left(\frac{2e}{3}\right)^{3/2} \sigma_1^3}$$

Numericals.

Numerical: thermal analysis

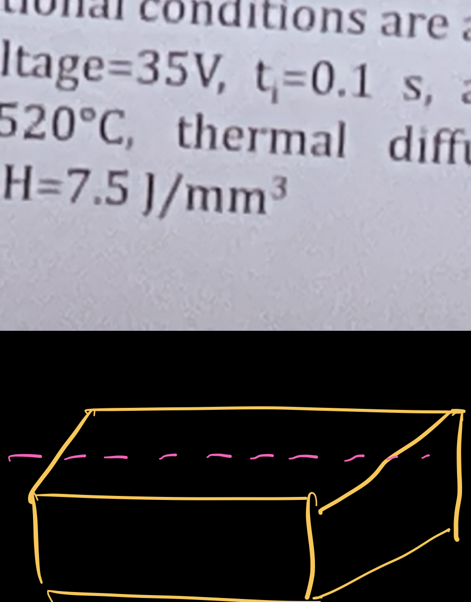
Consider a small weld crater formed on a thick low alloy steel plate in arc welding.

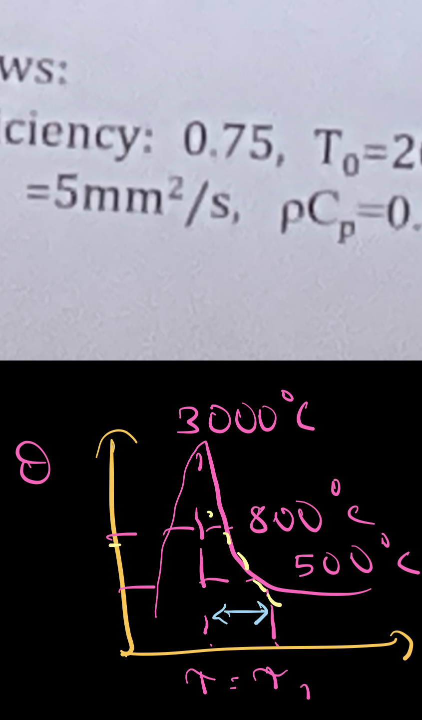
1. Calculate the cooling time from 800 to 500°C ($\Delta t_{800\text{-}500}$) at the center of the weld and the cooling rate (C.R.) at the onset of the austenite to ferrite transformation(475°C).

2. Calculate the total width of the fully transformed region adjacent to the fusion boundary. Assuming that the transformation temperature is equal to 890°C for this particular steel.

The operational conditions are as follows:
I=80A, Voltage=35V, t_i=0.1 s, arc efficiency: 0.75, T_0=20°C, T_c=T_m= 1520°C, thermal diffusivity =5mm²/s, ρC_p=0.005 J/mm³.C, ΔH=7.5 J/mm³

1) Soln:





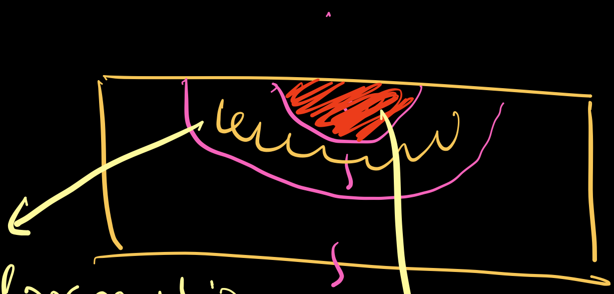
Calculate where $\sigma_1^2 = 0$ (At center)

From 800°C to 500°C

cooling rate $= \dfrac{-dT}{dt}$

= slope at given temp.

2) soln:



$S \to L$

$L \to S$

Transformation temp. = 840°C

$T_m = 1520°C$

$= T_c$

$I = 80A$

$V = 35V$

$t_i = 0.1$ sec

$\eta = 0.75 \longrightarrow$ Arc efficiency

$\alpha =$ Thermal diffusivity

$T_0 = 20°C$

Formally (soln)

$$\theta_{1520} = \frac{T - T_0}{T_c - T_0} = \frac{1520-20}{1520-20} = 1.$$

$$\theta_{840} = \frac{840-20}{1500} = 0.58$$

$$\theta_{800} = \frac{800-20}{1500} = 0.52$$

$$\theta_{500} = \frac{500-20}{1500} = 0.32$$

$$\theta_{475} = \frac{475-20}{1500} = 0.303$$

$$\eta_1 = \frac{q_0}{4\,\Delta H\,(\pi\alpha)^{3/2}\sqrt{t_1}} \qquad q_0 = \eta \cdot IU$$

$$= \frac{0.75 \times 80 \times 35}{4 \times 7.5 \times (5 \times \pi)^{3/2} \times \sqrt{0.1}}$$

$$= 3.56$$

① (a) At $\sigma_1 = 0$

$$\frac{\theta}{\eta_1} = \frac{1}{\tau_1^{3/2}} \exp\left(-\cancel{\frac{\sigma_1^2}{\tau_1}}\right) = 1$$

$$\tau_1^{3/2} = \frac{\eta_1}{\theta} \Rightarrow \tau_1 = \left(\frac{\eta_1}{\theta}\right)^{2/3}$$

So, $$\tau_{800^\circ C} = \left(\frac{\eta_1}{\theta_{800}}\right)^{2/3}$$

$$\tau_{500^\circ C} = \left(\frac{\eta_1}{\theta_{500}}\right)^{2/3}$$

$$\Delta T = \tau_{500^\circ C} - \tau_{800^\circ C}$$

Now, $$\Delta T = \left(\frac{\eta_1}{\theta_{500}}\right)^{2/3} - \left(\frac{\eta_1}{\theta_{800}}\right)^{2/3}$$

$$= \left(\frac{3.56}{0.32}\right)^{2/3} - \left(\frac{3.56}{0.52}\right)^{2/3}$$

$$= 1.38$$

$$\tau = \frac{t}{t_i}$$

$$\Delta t_{800-500} = 1.38 \times t_i$$

$$\approx 0.14 \text{ secs}$$

$$\Delta T = \frac{\Delta t_{800-500}}{t_i}$$

$$= 0.138 \text{ secs}$$

1 b

$$\theta_{475} = 0.303 \approx 0.3$$

Cooling rate at 475

$$= -\frac{dT}{dt}.$$

$$\theta = \frac{u_1}{T_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$$\frac{\partial \theta}{\partial \tau_1} = u_1 \left[\frac{1}{T_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)\left(\frac{\sigma_1^2}{\tau_1^2}\right) - \frac{3}{2}\frac{1}{\tau_1^{5/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right) \right]$$

At $\tau = 0$, $\sigma_1 = 0$.

$$\frac{\partial \theta}{\partial \tau_1} = u_1 \left[0 - \frac{3}{2}\frac{1}{\tau_1^{5/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right) \right].$$

$$\frac{\partial \theta}{\partial T_1} = u_1 \left[-\frac{3}{2}\frac{1}{\tau_1^{5/2}} \right] = -\frac{3}{2}\frac{u_1}{\tau_1^{5/2}}.$$

$$\Rightarrow \frac{1}{\tau_1^{3/2}}\left[\exp\left(-\frac{\sigma_1^2}{\tau_1}\right)\right], \quad \sigma_1 = 0.$$

$$T_1 = \left(\frac{n_1}{\theta}\right)^{2/3}$$

$$\frac{\partial \theta}{\partial T_1} = -\frac{3}{2} \frac{n_1}{\left(\left(\frac{n_1}{\theta}\right)^{2/3}\right)^{5/2}} = -\frac{3}{2} \frac{\theta^{5/3}}{n_1^{2/3}}$$

cooling rate $= -\frac{dT}{\partial t}$

$$= \frac{\frac{dT}{T_c - T_0}}{\frac{dt}{t_1}}$$

$$\theta = \frac{T - T_0}{T_c - T_0}$$

$$\partial\theta = \frac{dT}{T_c - T_0}$$

$$\tau_1 = \frac{t}{t_i}$$

$$\partial\tau_1 = \frac{\partial t}{t_i}$$

so, $\frac{t_i}{T_c - T_0} \frac{dT}{\partial t} = -\frac{3}{2} \frac{\theta^{5/3}}{n_1^{2/3}}$.

$$\frac{dT}{dt} = -\frac{T_c - T_0}{t_i} \frac{3}{2} \frac{\theta^{5/3}}{n_1^{2/3}}.$$

$$C.R = -\frac{dT}{dt} = \frac{T_c - T_\infty}{t_i} \frac{3}{2} \frac{\theta^{5/3}}{n_1^{2/3}}.$$

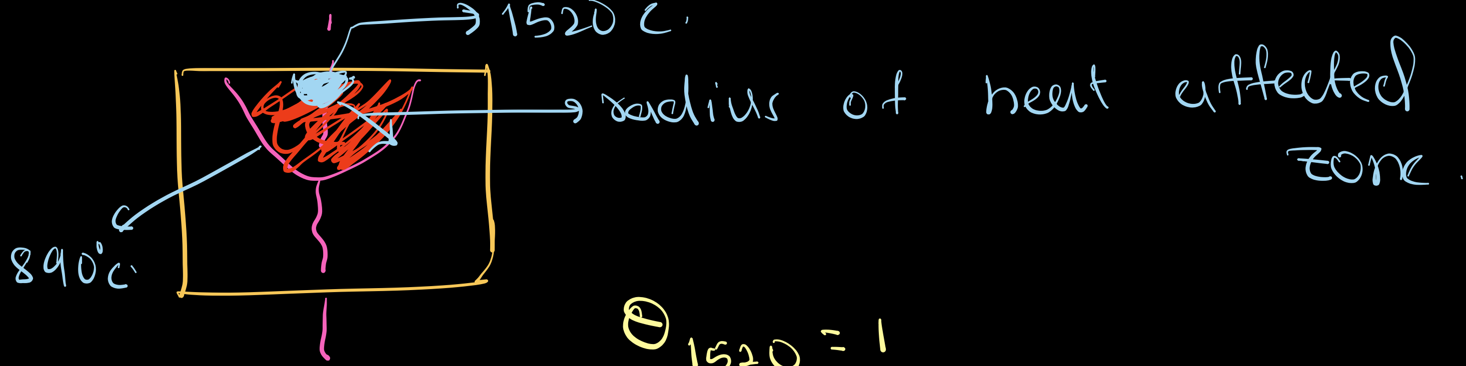
$$\theta_{475} = 0.3.$$

$$= \frac{100}{0.1} \times \frac{3}{2} \times \frac{0.3^{5/3}}{(3.56)^{2/3}} \quad {}^\circ C/s$$

or K/s.

$= 864\ ^\circ C/s$ (approx shayad)
Prof khud ko nahi pata.

② soln continuation



$$\theta_{1520} = 1$$

$$\theta_{890} = 0.58$$

We use peak temp. conditions.

$$\frac{\theta_p}{n_1} = \frac{1}{\left(\frac{2e}{3}\right)^{3/2} \sigma_1^3}$$

So, $$(\sigma_1)^3 = \frac{1}{\left(\frac{2e}{3}\right)^{3/2}} \cdot \frac{n_1}{\theta_p}$$

$$\sigma_1 = \frac{1}{\left(\frac{2e}{3}\right)^{1/2}} \cdot \left(\frac{n_1}{\theta_p}\right)^{1/3}$$

$$\sigma_1\Big|_{890^\circ C} = \frac{1}{\left(\frac{2e}{3}\right)^{1/2}} \cdot \left(\frac{3.56}{0.58}\right)^{1/3}$$

$$\sigma_1\Big|_{1520} = - - - .$$

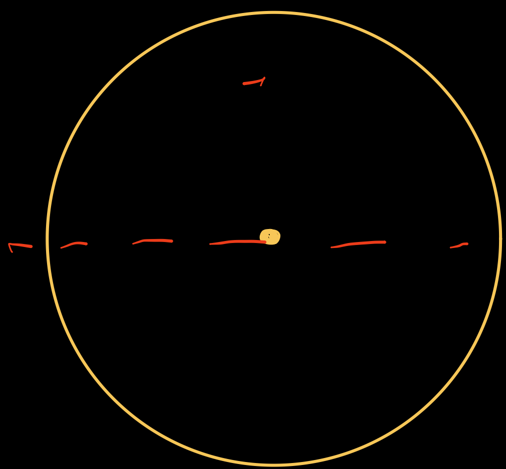
$$\Delta\sigma_1 = \sigma_{890^\circ} - \sigma_{1520}$$

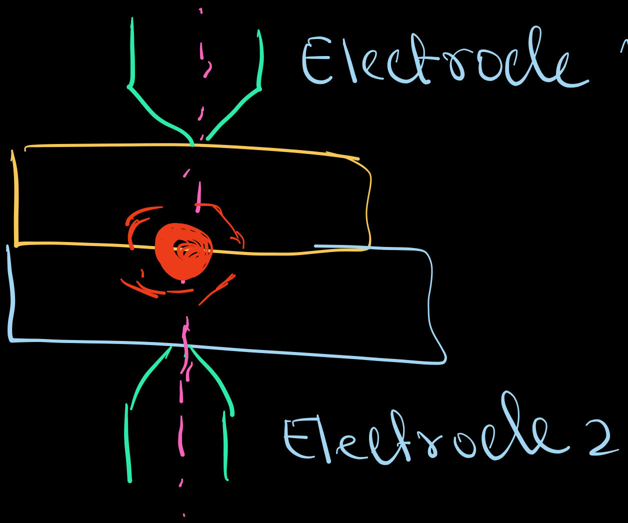
$$= \frac{1}{\left(\frac{2e}{3}\right)^{1/2}} \left[\left(\frac{3.56}{0.58}\right)^{1/3} - \left(\frac{3.56}{1}\right)^{1/3} \right]$$

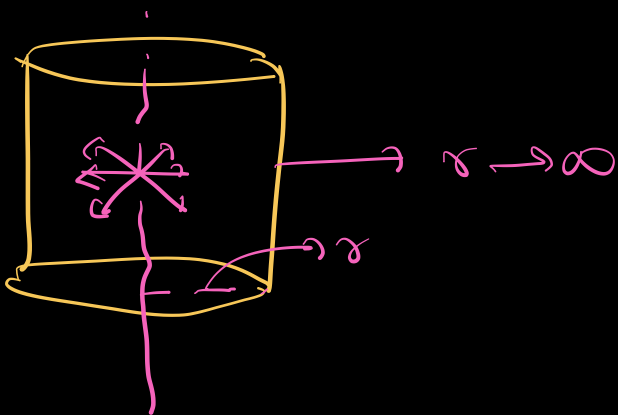
$$= 0.23.$$

$$\sigma = \sqrt{\frac{R^2}{4\alpha H}} \quad , \quad \Delta\alpha = \frac{\Delta R}{\sqrt{4\alpha H}} = 0.23.$$

$$\therefore \Delta R = 0.23 \times \sqrt{4 \times 5 \times 0.1} = 0.32\,mm$$







q = watt

$q' = W/L$ (per length

$q'' = \frac{W}{m^2}$ (per area)

$q''' = \frac{W}{m^3}$ (per vol)

*) Consider a large body with a line heat source buried in that system at center.

*) It is assumed that line source is producing heat at a constant state per unit length (Watt/m)

*) Body initial temp : T_0

Temp = T_∞ $(r \to \infty)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{— (1)}$$

Boundary conditions

At $r = 0$; $\quad -2\pi r k \frac{\partial T}{\partial r} = \frac{Q}{L}$ — (2)

As $r \to \infty$, $T = T_\infty$ — (3)

Initial condition

At $t = 0$, $T = T_0 = T_\infty$ — (4)

Assume $\theta = T - T_\infty$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{— (1A)}$$

<u>B.C</u> Replace T with θ

$$\eta = \frac{r^2}{4\alpha t} \quad \text{— (5)}$$

$$\frac{\partial \theta}{\partial r} = \frac{\partial \theta}{\partial \eta}\cdot\frac{\partial \eta}{\partial r} = \frac{\partial \theta}{\partial \eta}\cdot\left(\frac{2r}{4\alpha t}\right) \quad \text{— (5A)}$$

$$r\frac{\partial\theta}{\partial r} = \frac{2r^2}{4\alpha t}\frac{\partial\theta}{\partial\eta}$$

$$\frac{\partial}{\partial r}\left[r\frac{\partial\theta}{\partial r}\right] = \frac{\partial}{\partial r}\left(\frac{2r^2}{4\alpha t}\frac{\partial\theta}{\partial\eta}\right)$$

$$= \frac{\partial}{\partial\eta}\left[2\eta\frac{\partial\theta}{\partial\eta}\right]\frac{2r}{4\alpha t}$$

$$= \left(2\eta\frac{\partial^2\theta}{\partial\eta^2} + \frac{2\partial\theta}{\partial\eta}\right)\frac{2r}{4\alpha t} \quad \text{— 5C}$$

$$\therefore \frac{1}{r}\frac{1}{\partial r}\left[r\frac{\partial\theta}{\partial r}\right] = \left(2\eta\frac{\partial^2\theta}{\partial\eta^2} + \frac{2\partial\theta}{\partial\eta}\right)\frac{2}{4\alpha t}$$

— 5D

$$\frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial t} = \frac{\partial\theta}{\partial\eta}\left[\frac{-r^2}{4\alpha t^2}\right] \quad \text{— 5E}$$

$$= \frac{\partial\theta}{\partial\eta}\left[-\frac{\eta}{t}\right] \quad \text{— 5F}$$

1 A

$$\left(\cancel{2}\eta\frac{\partial^2\theta}{\partial\eta^2} + \cancel{2}\frac{\partial\theta}{\partial\eta}\right)\frac{\cancel{2}}{\cancel{4}\cancel{\alpha}\cancel{t}} = \frac{1}{\cancel{\alpha}}\left[-\frac{\partial\theta}{\partial\eta}\frac{\eta}{\cancel{t}}\right]$$

$$\eta\frac{\partial^2\theta}{\partial\eta^2} + \frac{\partial\theta}{\partial\eta} = -\eta\frac{\partial\theta}{\partial\eta}$$

i.e $$\eta \frac{\partial^2 \theta}{\partial \eta^2} + (1+\eta)\frac{\partial \theta}{\partial \eta} = 0 \quad \text{—— } (6)$$

Check

Boundary condition as $r \to 0$, $\eta \to 0$

i.e $$-2\pi k\left(\frac{2r^2}{4\alpha t}\right) = \frac{\theta}{L}$$

↳ From (5A)

$$-2\pi \cancel{k} 2\eta \frac{\partial \theta}{\partial \eta} = \frac{\theta}{L}$$

$$-4\pi k\eta \frac{\partial \theta}{\partial \eta} = \frac{\theta}{L} \quad \text{—— } (6A).$$

At $t = 0$, $\eta \to \infty$

$t = r$, $\eta \to \infty$

06/2/24

*) Metallurgical modelling of welding

↳ Chapter 1

$$\frac{d^2\theta}{d\eta^2} + \left(\frac{1+\eta}{\eta}\right)\frac{d\theta}{d\eta} = 0 \quad \text{—— } (6).$$

For $$\frac{d^2y}{dx^2} + f(x)\frac{dy}{dx} = 0$$

Assume $\frac{dy}{dx} = p$, $\frac{d^2y}{dx^2} = \frac{dp}{dx}$.

$$\frac{dp}{dx} + f(x)\,p(x) = 0.$$

$$\frac{dp}{dx} = -f(x)\,p.$$

$$\frac{dp}{p} = -f(x)\,dx. \implies \ln p = -\int f(x)\,dx + c.$$

$$\frac{dy}{dx} = p = a\,e^{-\int f(x)\,dx}.$$

$$\therefore \quad y = a\int e^{-f(x)dx} + B.$$

Here $f(x) = \frac{1+\eta}{\eta}$.

So, $\Theta = A\int \left(e^{-\int \frac{1+\eta}{\eta}d\eta}\right) d\eta + B.$

$$\Theta = A\int e^{-(\ln \eta + \eta)}\, d\eta + B.$$

$$\boxed{\Theta = A\int \frac{e^{-\eta}}{\eta}\, d\eta + B.}$$

$$\Rightarrow \text{ So, } [\Theta]_{\eta}^{\infty} = A\int_{\eta}^{\infty} \frac{e^{-\eta}}{\eta}\, d\eta$$

$$\Theta(\infty) - \Theta(\eta) = A\int_{\eta}^{\infty} \frac{e^{-\eta}}{\eta}\, d\eta$$

$$0 - \theta = A \int_{\eta}^{\infty} \frac{e^{-\eta}}{\eta} d\eta$$

Exponential integral function:

$$E_i(\eta) = \int_{\eta}^{\infty} \frac{e^{-\eta}}{\eta} d\eta$$

$$\therefore \theta = -A E_i(\eta).$$

$$\eta = \frac{r^2}{4\alpha t}.$$

As $\eta \to 0$,

$$-4\pi k \eta \frac{d\theta}{d\eta} = \frac{Q}{L}$$

$$-4\pi k \cancel{\eta} \left(A \frac{e^{-\eta}}{\cancel{\eta}} \right) = \frac{Q}{L}.$$

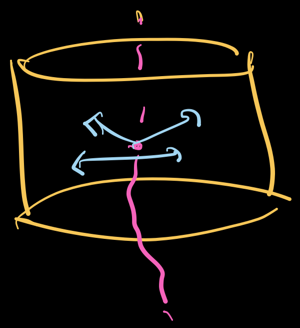
$$A = -\frac{Q}{4\pi k} e^{\eta}$$

$T \to T_\infty$, $\quad \theta = -A E_i(\eta)$

$$= \frac{Q}{4\pi k L} E_i\left(\frac{r^2}{4\alpha t}\right).$$

$$T = T_\infty + \frac{Q}{4\pi k L} E_i\left(\frac{r^2}{4\alpha t}\right).$$

$t \to \infty$. $\quad E_i(0) \to \infty$.

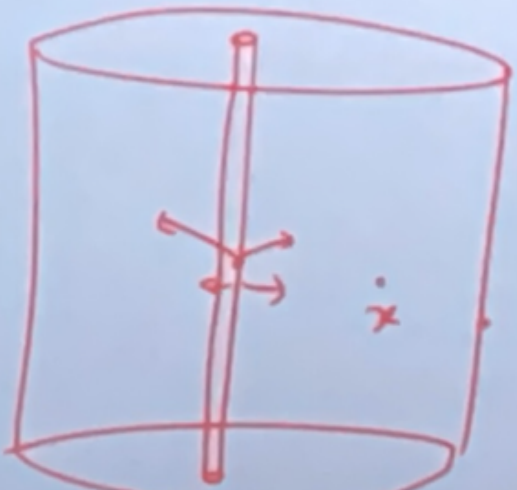


Governing Eqn:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

$r \to \infty \, , \quad T = T_0.$

Instantaneous heat source: Thermal Explosion at line region



G.E.

$E_0 \qquad \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (1)$

$r \to \infty, \; T = T_\infty$

$x+$ condition

$$\boxed{r \to 0 \quad \int_0^\infty 2\pi r l \rho c_p (T - T_\infty)\, dr = E_0}$$

$t = 0, \; T = T_\infty$

Now, $$T = T_\infty + \frac{E_0}{4\pi\alpha \rho c_p L t} \exp\left(-\frac{r^2}{4\alpha t}\right).$$

*) In resistance welding, $t = 0.1, 0.2$ secs

$$\Rightarrow \quad \underline{T = T_\infty + \frac{E_0}{\rho c_p (4\pi\alpha t)^2} e^{-\frac{r^2}{4\alpha t}}}$$

*) Dimensionless temp:

$$\theta = \frac{T - T_\infty}{T_c - T_\infty}$$

Rate temp $= T_m$

*) Dimensionless time factor. $= \frac{t}{t_\eta}$.

(Heating time, pulse time)

*) Dimensionless radius factor.

$$\sigma_2 = \sqrt{\frac{r^2}{4\alpha t_h}}$$

*) Dimensionless process parameter.

$$\eta_2 = \frac{E_0 \rightsquigarrow \text{Joule}}{(4\pi\alpha t) L \rho c (T_c - T_\infty)}$$

$L =$ total thickness of plate.

$$\rho c (T_c - T_\infty) = H_c - H_8$$

$$= \Delta H.$$

$$= \frac{q_0 \rightsquigarrow \text{watt}}{\Delta H 4\pi\alpha L}$$

*) $$T = T_\infty + \frac{E_0}{4\pi\alpha L \rho c_p} \exp\left(\frac{-r^2}{4\alpha t}\right)$$

$$\Rightarrow \quad \frac{T - T_\infty}{T_c - T_\infty} = \theta = \frac{E_0}{4\pi \alpha L \rho c_p} \exp\left(\frac{-r^2}{4\alpha t}\right) \times \frac{1}{T_c - T_\infty}$$

$$\theta = \frac{E_0}{4\pi \alpha L \Delta H \times t} \exp\left(\frac{-r^2}{4\alpha t}\right)$$

$$\theta = \frac{q_0}{4\pi \alpha L \Delta H\, \tau_2} \exp\left(-\frac{\sigma_2^2}{\tau_2}\right)$$

$$\theta = \frac{n_2}{\tau_2} \exp\left(\frac{-\sigma_2^2}{\tau_2^2}\right)$$

Spot welding (graph analysis)

Peak temperature

$$\theta = \frac{n_2}{\tau_2} \exp\left(\frac{-\sigma_2^2}{\tau_2}\right)$$

$$\frac{\partial \theta}{\partial \tau_2} = 0$$

$$\frac{\theta_p}{n_2} =$$

$$\frac{\partial \theta}{\partial \tau_2} = \frac{-n_2}{-\tau_2^2} \exp\left(\frac{-\sigma_2^2}{\tau_2}\right) + \frac{n_2}{\tau_2} \exp\left(\frac{-\sigma_2^2}{\tau_2}\right) \frac{\sigma_2^2}{\tau_2^2} = 0$$

$$\Rightarrow \boxed{\sigma_2 \nu = \tau_2}$$

$$\theta_p = \frac{n_2}{e\tau_2} \qquad \sigma_2 \nu = \tau_2$$

$$\theta_p = \frac{n_2}{e\sigma_2 \nu}$$

$$\theta = \frac{n_2}{\tau_2} \exp\left(-\frac{\sigma_2 \nu}{\tau_2}\right)$$

$$\sigma_2 \nu = \tau_2$$

$$\boxed{\theta_p = \frac{n_2}{e\tau_2} = \frac{n_2}{e\sigma_2 \nu}}$$

Problem:

Numerical: thermal analysis: SW

Consider spot welding of 2 mm plates of low alloy steel under the following operational conditions:

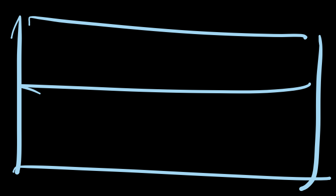
I=8000A, Total voltage drop between the electrodes is =1.6 V, t_h=0.3 s, Transfer efficiency: 0.5, T_0=20°C, T_c=T_m= 1520°C, α=5 mm^2/s, ΔH=7.5J/mm^3

1. Calculate the cooling time from 800 to 500°C ($\Delta t_{800\text{-}500}$) at the center of the weld and the cooling rate (C.R.) at the onset of the austenite to ferrite transformation(475°C).

2. Calculate the total width of the fully transformed region adjacent to the fusion boundary. Assuming that the transformation temperature is equal to 890°C for this particular steel.

Soln:

Thickness = 4mm



4mm

$$\theta = \frac{T - T_\infty}{T_c - T_\infty} = \frac{T - 20}{1520}.$$

$\theta_{1520} = 1$

$\theta_{500} = 0.52$

$\theta_{890} = 0.58$

$\theta_{500} = 0.32$

$$\theta = \frac{u}{t} \times \exp\left(-\frac{\sigma^2}{\tau}\right).$$

At $\sigma = 0$.

$$\theta = \frac{u}{t} \Longrightarrow t = \frac{u}{\theta}.$$

$$\tau_{800} = \frac{u}{\theta_{800}}, \quad \tau_{500} = \frac{u}{\theta_{500}}$$

$\tau_{470} = \ldots$

$$\Delta T = \frac{\Delta t}{}$$

$$\boxed{\theta = \frac{u_2}{e\,\sigma_2^{\,2}}}$$

Use this

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

B.C's $\quad x = 0, \quad -kA\frac{dT}{dx} = Q$

$\quad x \to \infty, \quad T = T_\infty$

I.C's $\quad t = 0, \quad T = T_\infty.$

$$T = T_\infty + \frac{Qx}{kA}\left[\frac{\sqrt{4\alpha t}}{x\sqrt{\pi}}\, e^{-\frac{x^2}{4\alpha t}} - \operatorname{erfc}\left(\frac{x}{\sqrt{4\alpha t}}\right)\right]$$

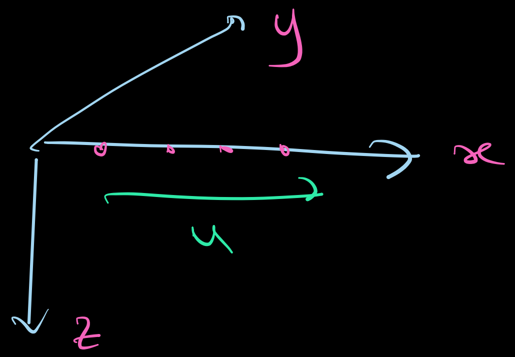
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial x}$$

$x \to \infty$, $T = T_\infty$

$t = 0$, $T = T_\infty$

$x \to 0$, $\int_0^\infty A\rho c_p (T - T_\infty)\, dx = E_0$.

$$\boxed{T = T_\infty + \frac{E_0}{\rho c_p A \sqrt{4\pi\alpha t}} \exp\left(\frac{-x^2}{4\alpha t}\right)}$$



9/2/24

*) Moving heat sources: (MHS)

→ Change of origin ($O \to O'$)

- Initial co-ordinate system (x, y, z) shifted to (ξ, y, z)

so, $\xi = x - ut$

u = velocity.

To solve,

Assumptions:

1. All physical properties are const. independent of temp.
2. Uniform velocity of source.
3. Rate heat input = const.
4. $\alpha = \frac{1}{2\lambda}$, where α = Thermal diffusivity.

Governing Eqn:

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} = 2\lambda \frac{\partial T}{\partial t}}$$

- By changing co-ordinates.

$$\xi = x - ut.$$

$T(\xi, y, z, t) \longrightarrow$ In this form.

$$\frac{\partial T}{\partial t}(\xi, y, z, t) = \frac{\partial T}{\partial \xi} \times \frac{\partial \xi}{\partial t} + \cancel{\frac{\partial T}{\partial y} \times \frac{\partial y}{\partial t}}^{\,0} + \cancel{\frac{\partial T}{\partial z} \times \frac{\partial z}{\partial t}}^{\,0}$$

no velocity in y & z

So, $\frac{\partial T}{\partial t}(\xi, y, z, t) = \frac{\partial T}{\partial \xi} \times \frac{\partial \xi}{\partial t}$.

$$0 = -u \frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial t}$$

$$\therefore \frac{\partial T}{\partial x} = \frac{\partial T}{\partial \xi} \frac{\partial \xi}{\partial x} + \frac{\partial T}{\partial t} \frac{\partial t}{\partial x} \quad \text{— (1A)}.$$

$\xi = x - ut$.

$\frac{\partial \xi}{\partial x} = 1$. $\Longrightarrow$ $\frac{\partial^2 T}{\partial x^2} = \frac{\partial^2 T}{\partial \xi^2}$

$\hookrightarrow$ (2A)

*) $T(\xi, y, z, t)$

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 2\lambda \left\{ -u \frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial t} \right\}.$$

$\hookrightarrow$ (2).

*) Quasi-steady state $\longrightarrow$ $\frac{\partial T}{\partial t} = 0$
(or)
pseudo-steady state.

So, Assuming that state, $\frac{\partial T}{\partial t} = 0$.

So, G.E becomes.

$$\frac{\partial^2 T}{\partial \xi^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -2\lambda u \frac{\partial T}{\partial \xi} \quad \text{---} (3).$$

*) if we know y & z,

$$T \propto e^{-\lambda u z} \phi(\xi, y, z).$$

Assuming heat.

$$T = T_0 + e^{-\lambda u \xi} \phi(\xi, y, z)$$

$$\frac{\partial T}{\partial \xi} = e^{-\lambda u \xi} \frac{\partial \phi}{\partial \xi} + \phi e^{-\lambda u \xi}(-\lambda u)$$

$$\frac{\partial^2 T}{\partial \xi^2} = e^{-\lambda u \xi} \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} e^{-\lambda u \xi}(-\lambda u)$$

$$+ (-\lambda u)\left[e^{-\lambda u \xi} \frac{\partial \phi}{\partial \xi} + \phi e^{-\lambda u \xi}(-\lambda u)\right]$$

$$= e^{-\lambda u \xi} \frac{\partial^2 \phi}{\partial \xi^2} + 2\lambda u \frac{\partial \phi}{\partial \xi} e^{-\lambda u \xi}$$

$$+ (\lambda u)^2 \phi e^{-\lambda u \xi}.$$

*) $$\frac{\partial^2 T}{\partial y^2} = e^{-\lambda u \xi} \frac{\partial^2 \phi}{\partial y^2}$$

*) $\dfrac{\partial^2 T}{\partial z^2} = e^{-\lambda u \xi} \dfrac{\partial^2 \phi}{\partial z^2}$

*) $-2\lambda u \dfrac{\partial T}{\partial \xi} = -2\lambda u \left(e^{-\lambda u \xi} \dfrac{\partial \phi}{\partial \xi} + \phi e^{-\lambda u \xi} \right).$

*)

$$e^{-\lambda u \xi} \dfrac{\partial^2 \phi}{\partial \xi^2} - \cancel{2\lambda u \dfrac{\partial \phi}{\partial \xi} e^{-\lambda u \xi}} + (\lambda u)^2 \phi e^{-\lambda u \xi}$$

$$+ e^{-\lambda u \xi} \dfrac{\partial^2 \phi}{\partial y^2} + e^{-\lambda u \xi} \dfrac{\partial^2 \phi}{\partial z^2}$$

$$= \cancel{-2\lambda u e^{-\lambda u \xi} \dfrac{\partial \phi}{\partial \xi}} - 2\lambda u e^{-\lambda u \xi} (-\lambda u).$$

*) $\dfrac{\partial^2 \phi}{\partial \xi^2} + \dfrac{\partial^2 \phi}{\partial y^2} + \dfrac{\partial^2 \phi}{\partial z^2} = (\lambda u)^2 \phi$

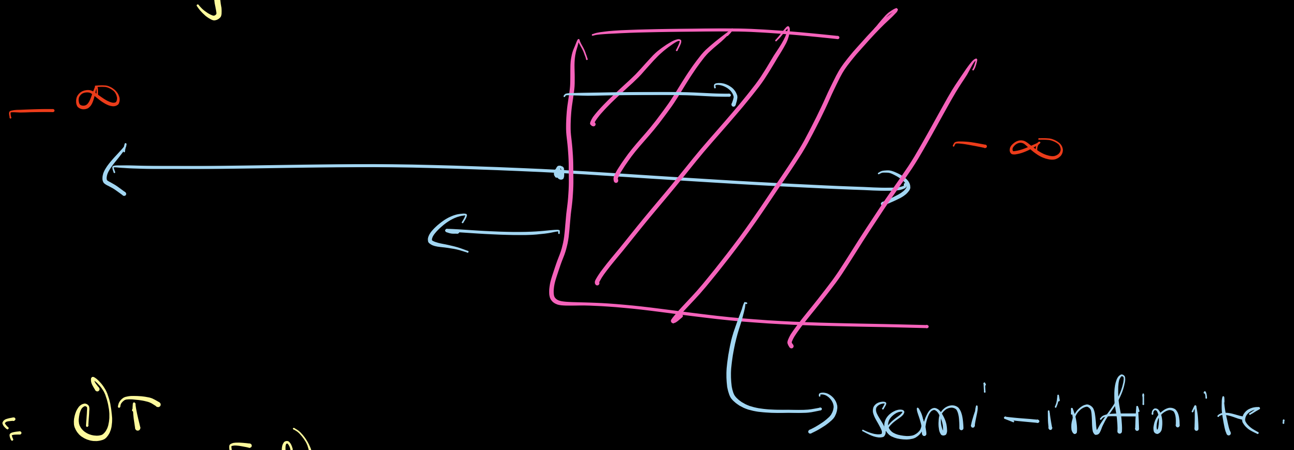
↳ final expressi

*) $\dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} + \dfrac{\partial^2 \phi}{\partial z^2} - (\lambda u)^2 \phi = 0.$

$$\nabla^2 \phi - (\lambda u)^2 \phi = 0.$$

<u>case (i)</u> for infinite (or) semi-infinite

body with linear flow of heat.

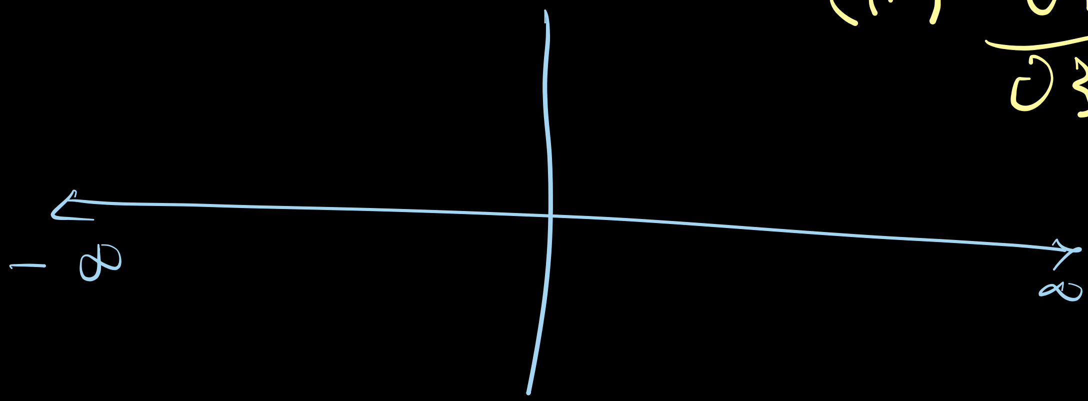


$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0.$$

$$\frac{\partial^2 \phi}{\partial \xi^2} - (\lambda u)^2 \phi = 0.$$

Boundary cond: (i) $\xi \to +\infty$, $-\infty$

(ii) $\frac{\partial T}{\partial \xi} = 0.$



$\xi = 0$ (Temp. continuity should maintain).

$\xi \to 0$, $-k\frac{\partial T}{\partial \xi} = q''$

*) $$\frac{\partial^2 \phi}{\partial \xi^2} - (\lambda u)^2 \phi = 0.$$

Soln is.

$$\phi = c_1 e^{-ux} + c_2 e^{ux}.$$

so, $\phi = c_1 e^{-\lambda u \xi} + c_2 e^{\lambda u \xi}$

$\therefore \; T = T_0 + e^{-\lambda u \xi} \phi(\xi, y, z).$

$T = T_0 + e^{-2\lambda u \xi} \phi(\xi, y, z)$

$\therefore \; T = T_0 + c_1 e^{-2\lambda u \xi} \phi(\xi, y, z) + c_2$

$\xi < 0$ | $\xi > 0$.

$\dfrac{\partial T}{\partial \xi} \to 0$ | $\dfrac{\partial T}{\partial \xi} \to 0.$

$\xi \to -\infty$ | $\xi \to \infty.$

*) $\dfrac{\partial T}{\partial \xi} = c_1(-2\lambda u) e^{-2\lambda u \xi}.$

for $\dfrac{\partial T}{\partial \xi} = 0$, $c_1 = 0.$

Similarly $c_2 = 0.$

$\therefore$ for $\xi < 0$, $c_1 = 0$ | $\xi > 0$, $c_2 = 0.$

Then, $T = T_0 + c_2 e^{-2\lambda u \xi}$ | $T = T_0 + c_1 e^{-2\lambda u \xi}$

At $\xi = 0$, $\xi < 0$, $\xi > 0$, All are equal.

$$\therefore c_1 e^{-2\lambda u \xi} = c_2 e^{-2\lambda u \xi}$$

$$\boxed{\therefore c_1 = c_2} \rightarrow \text{At } \xi = 0.$$

*) Using second BC,

$$T = T_0 + c_1 e^{-2\lambda u \xi} + \cancel{c_2} \rightarrow 0.$$

for $\xi > 0$.

$$\frac{\partial T}{\partial \xi} = -c_1(-2\lambda u)\, e^{-2\lambda u \xi}.$$

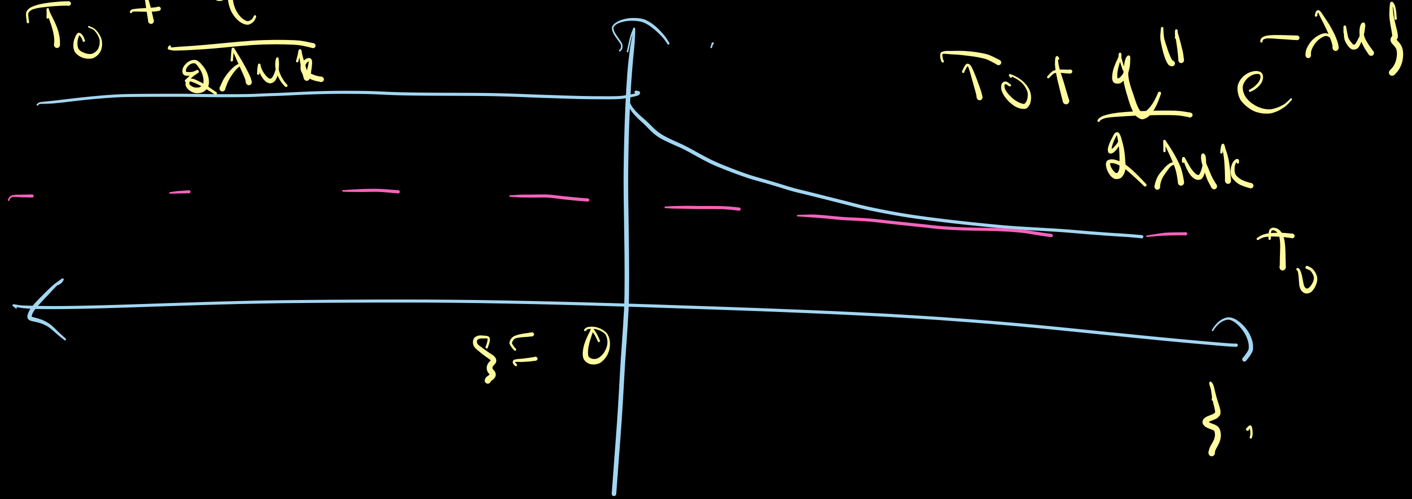
$$-k \frac{\partial T}{\partial \xi}\bigg|_{\xi=0} = q''$$

$$k c_1 (2\lambda u) = q''.$$

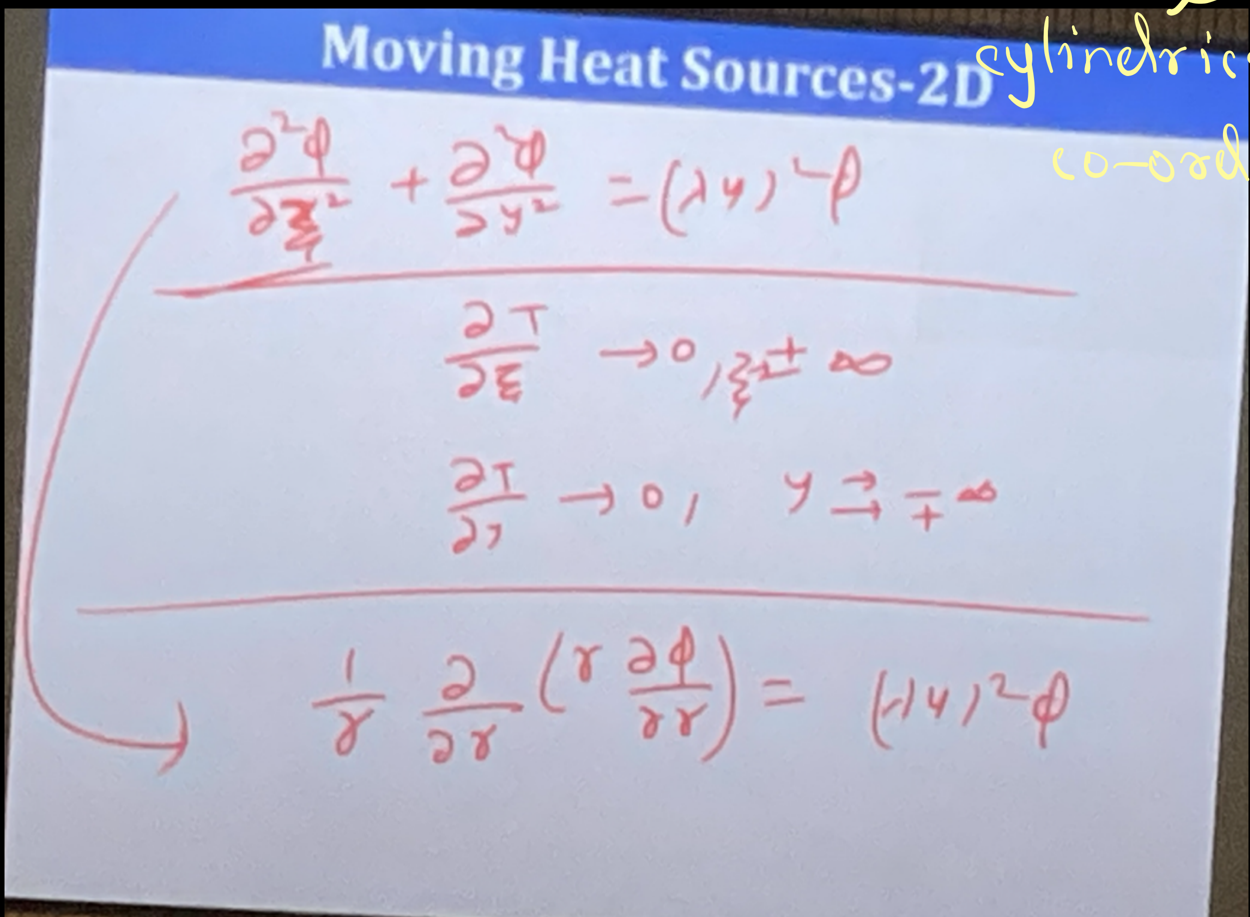
$$c_1 = \frac{q''}{2\lambda u k},$$

*) $$T = T_0 + \frac{q''}{2\lambda u k}, \quad \xi < 0.$$

$$T = T_0 + \frac{q''}{2\lambda u k} e^{-\lambda u \xi}, \quad \xi > 0$$



*) $$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = (\lambda u)^2 \phi$$



cylindrical co-ordinates.

$T \propto K_0(\lambda u r) \propto -\ln(\lambda u \xi)$.

$\hookrightarrow$ 2nd kind (or) zero order.

$T = c K_0(\lambda u \xi)$.

B.C's ; $-2\pi r k \dfrac{\partial T}{\partial r} = q'$.

$$c = \frac{q''}{2\pi k}.$$

*) $T = T_0 + \dfrac{q''}{2\pi k} e^{-\lambda u \xi} K_0(\lambda u \xi)$.

*) $$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = (\lambda u)^2 \phi.$$

$$\frac{\partial T}{\partial \xi} \to 0 \quad ; \quad \xi \to \pm\infty$$

$$\frac{\partial T}{\partial y} \to 0 \quad ;$$

$$\frac{1}{R^2} \frac{\partial}{\partial R}\left[R^2 \frac{\partial \phi}{\partial R} \right] = (\lambda u)^2 \phi$$

$$R = \sqrt{\xi^2 + y^2 + z^2}$$

$$-4\pi R^2 k \frac{\partial T}{\partial R} = q \quad , \quad R \to 0.$$

$$\rightarrow R\phi = c_1 e^{-\lambda u R}$$

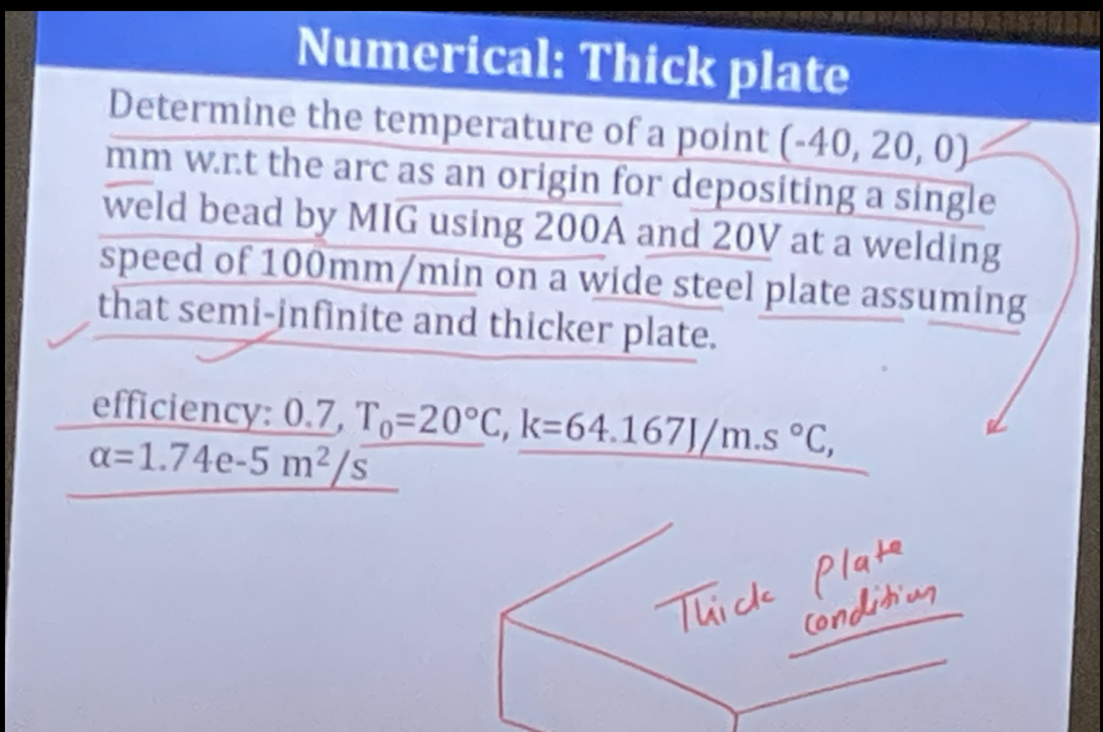
$$\phi = \frac{c_1}{R} e^{-\lambda u R}$$

*) $$\boxed{T = T_0 + \frac{q}{4\pi k} e^{-\lambda u \xi} \frac{e^{-\lambda u R}}{R}}$$

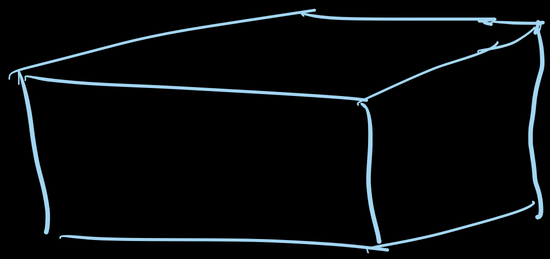
Assume $2\lambda = \frac{1}{\alpha}$, $\lambda = \frac{1}{2\alpha}$.

- $$T = T_0 + \frac{q}{2\pi k} e^{-\frac{u\xi}{2\alpha}} \frac{e^{-\frac{uR}{2\alpha}}}{R}.$$

Problems!



Soln:



$$T = T_0 + \frac{q}{2\pi k} e^{-\frac{u\xi}{2\alpha}} \frac{e^{-\frac{4R}{2\alpha}}}{R}$$

$T(-40, 20, 0)$

$\phi(\xi, y, z)$

$$R = \sqrt{\xi^2 + y^2 + z^2}$$

$$= \sqrt{40^2 + 20^2 + 0^2}$$

$$= 0.4472\,m.$$

$$q_0 = \eta I V$$

$$= 0.7 \times 200 \times 20$$

$$= 2800 \text{ Watt.}$$

$$v = 100\ mm/min = \frac{100}{60}\ mm/sec.$$

$$= \frac{10^{-1}}{60}\ m/s = 0.00167\ m/s$$

$$\lambda = \frac{1}{2\alpha} = 28,800\ s/m^2.$$

Substitute in main eqn.

$$\therefore -\lambda u\xi = -28,800 \times 0.00167 \times \frac{-40}{100}$$

$$= 1.92.$$

$$e^{-\lambda u \xi} = e^{1.92} = 6.82.$$

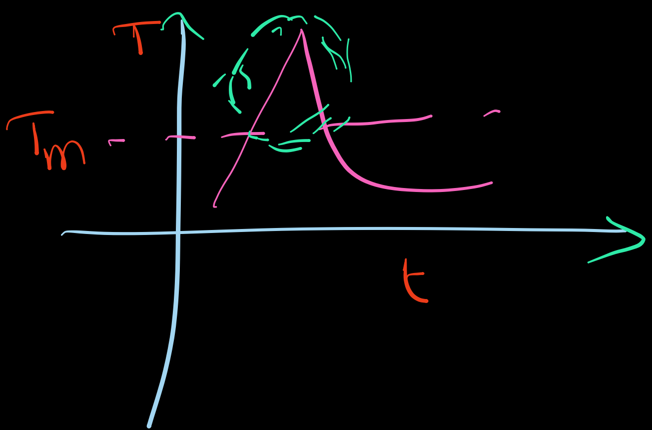
$$\frac{e^{-\lambda u R}}{R} = \frac{e^{-28,800 \times 0.00167 \times 0.0472}}{0.0472}$$

$$= 12.77.$$

$$\therefore \quad T - T_0 = \frac{2800}{2 \times 3.14 \times 64.16 \times 12.77 \times 6.82}$$

$$\Rightarrow \boxed{T - T_0 = 605.82^\circ C.}$$

*) Temp. plot in welding.



$\Longrightarrow$ cooling rate.

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial x} \times \frac{\partial x}{\partial t}$$

Growth rate.

Thermal Gradient

$$q'' = -k \frac{\partial T}{\partial x}.$$