Welding Technology ME692

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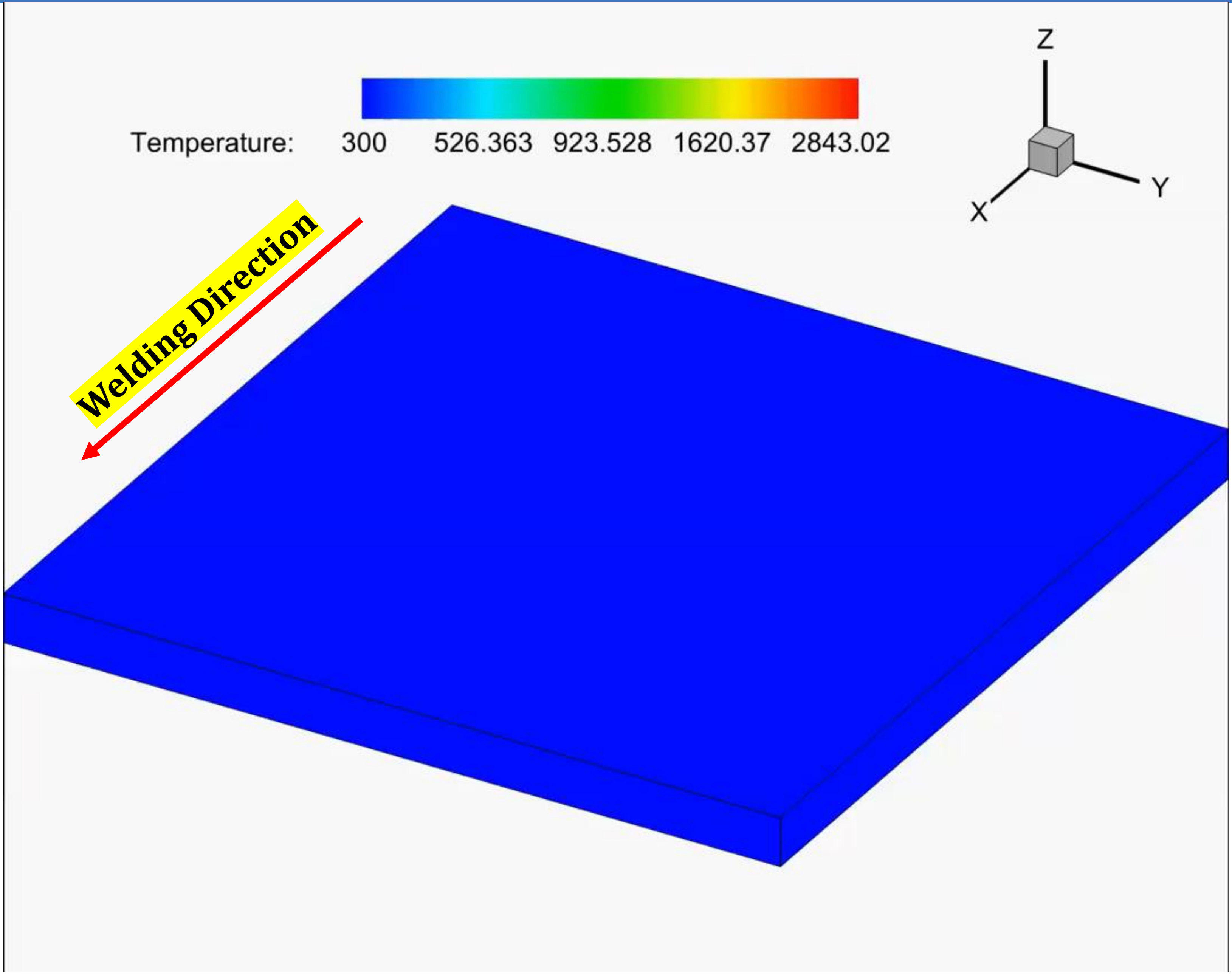
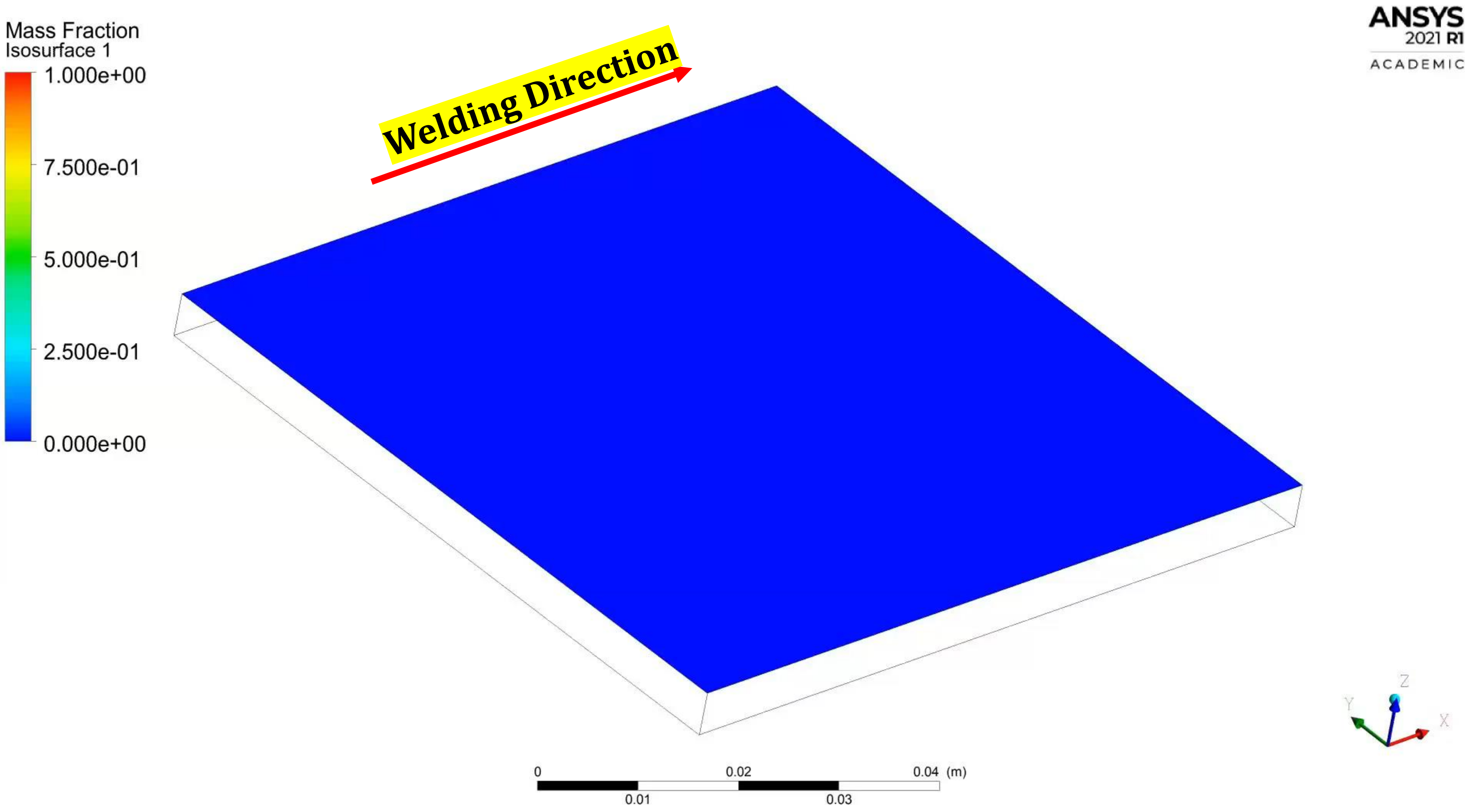
Thermal analysis in welding

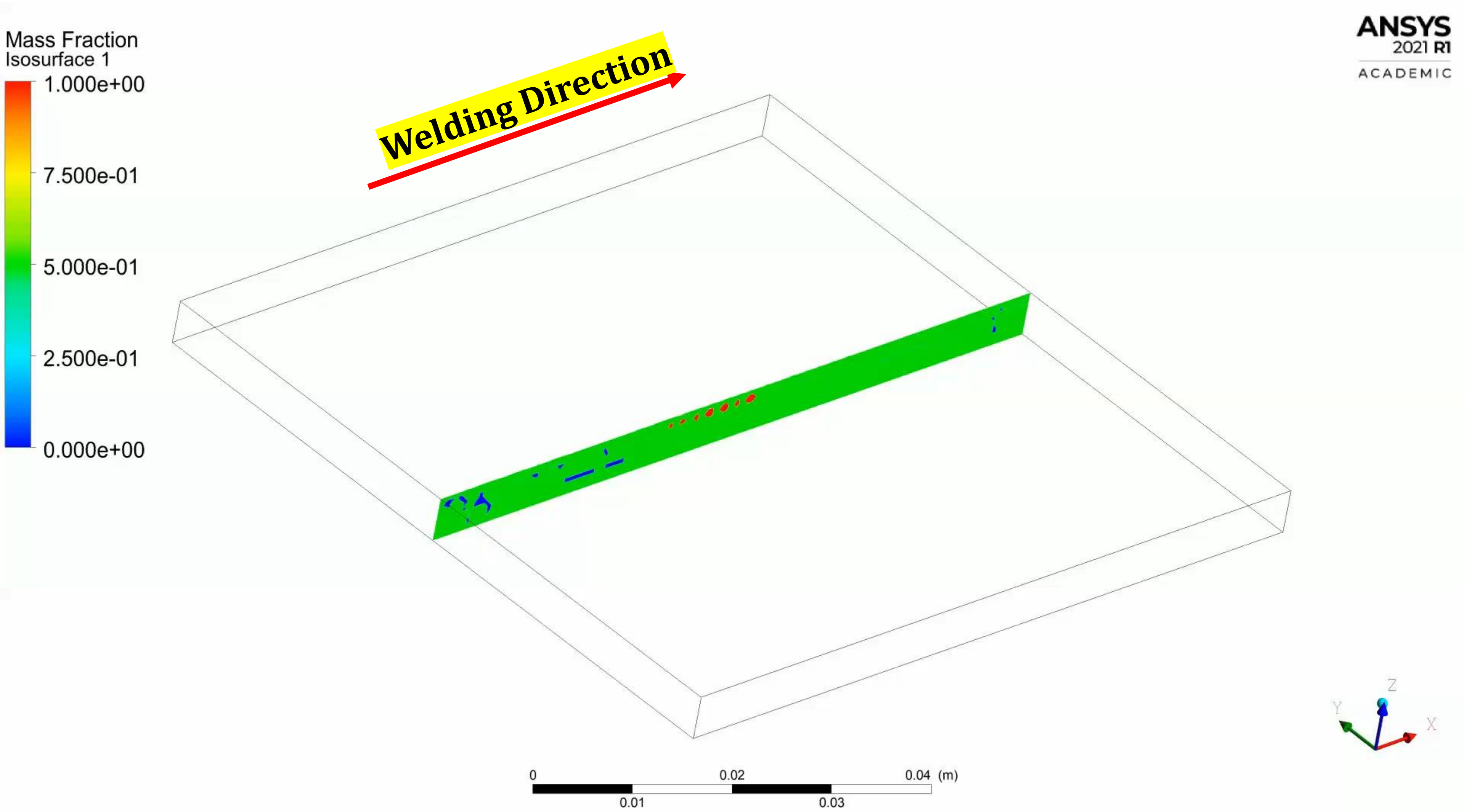
Welding simulation



Welding simulation



Welding simulation

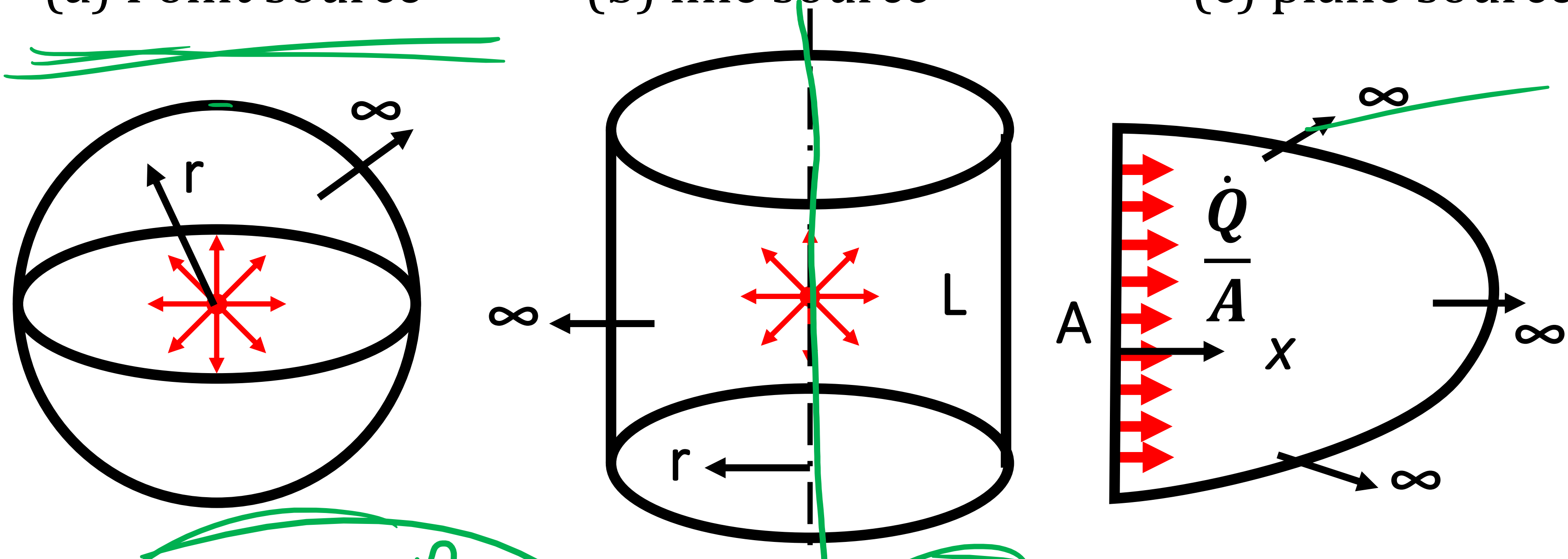


Source/Sink in Welding

Heat transfer with heat sources or sinks

Sources of constant heat production rate:

(a) Point source (b) line source (c) plane source

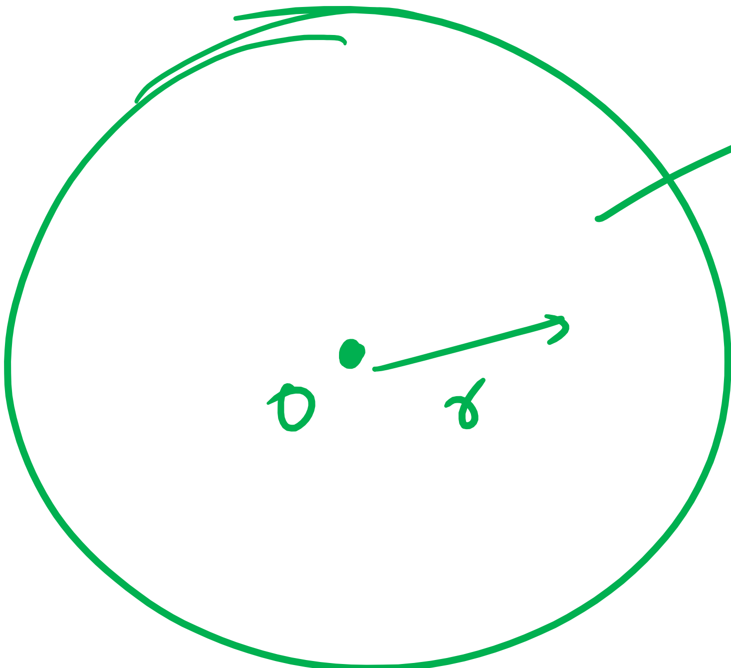


Continuous
• Nuclear waste → ocean

Explosion
• Bomb → ocean

Source/Sink in Welding: Point source

Point heat Source of Constant heat Production Rate



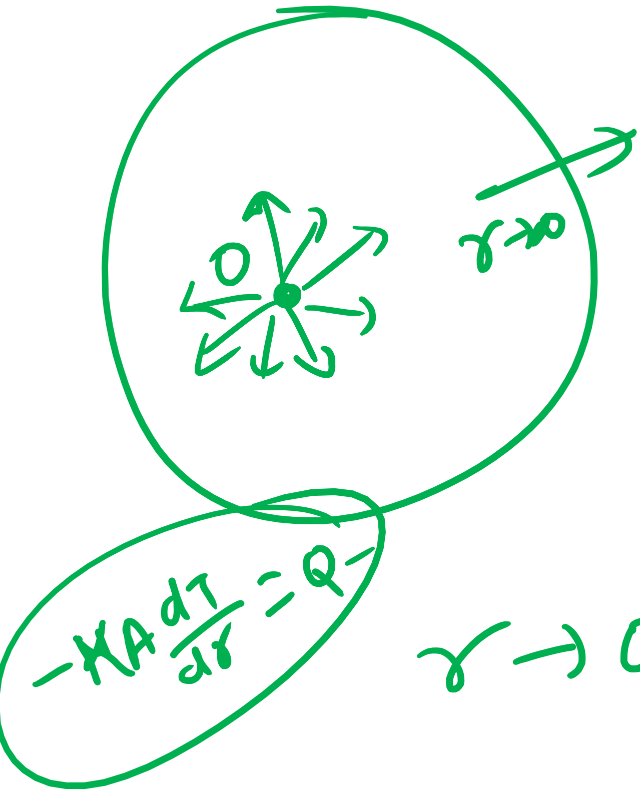
O is heating source which is producing const heat.

$r \to \infty$

Consider a body of large extent possessing a very small region (point) that produce heat continuously at a constant rate (Q, watt)

Source/Sink in Welding: Point source

Given that body was initially maintained at $T = T_\infty$



G.E.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{— (1)}$$

B.C.

$r \to \infty, \quad T = T_\infty$ — (2)

$r \to 0, \quad -4\pi r^2 K \frac{dT}{dr} = Q$ — (3)

I.C. $\Rightarrow$ $t = 0, \quad T = T_\infty$ — (4)

Source/Sink in Welding: Point source

$\theta = T - T_\infty$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{--- } 1A$$

B.C. $\quad r \to 0, \quad -4\pi r^2 K \frac{\partial \theta}{\partial r} = Q \quad 2A$

$r \to \infty, \quad \theta = 0 \quad \text{---} \; 3A$

I.C. $\quad t = 0, \quad \theta = 0, \quad \text{---} \; 4A$

$\therefore \quad u = r(T - T_\infty) = r\theta$

Source/Sink in Welding: Point source

$u = r\theta$

$$\frac{\partial u}{\partial t} = r\frac{\partial \theta}{\partial t} + \theta\frac{\partial r}{\partial t}$$

$$\frac{\partial r}{\partial t} \to 0$$

$$\boxed{\frac{\partial u}{\partial t} = r\frac{\partial \theta}{\partial t}} \quad \text{— 5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial u}{\partial t} \quad \text{— 5A}$$

$$\frac{\partial u}{\partial r} = r\frac{\partial \theta}{\partial r} + \theta \quad \text{— 6}$$

$$\frac{\partial \theta}{\partial r} = \frac{1}{r}\left(\frac{\partial u}{\partial r} - \theta\right) \quad \text{— 6A}$$

$$\frac{\partial^2 u}{\partial r^2} = r\frac{\partial^2 \theta}{\partial r^2} + \frac{\partial \theta}{\partial r} + \frac{\partial \theta}{\partial r}$$

$$= r\frac{\partial^2 \theta}{\partial r^2} + 2\frac{\partial \theta}{\partial r} \quad \text{— 7}$$

Source/Sink in Welding: Point source

1A $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial \theta}{\partial t}$$

$$\rightarrow \frac{1}{r^2}\left(r^2\frac{\partial^2 \theta}{\partial r^2}+\frac{\partial \theta}{\partial r}\times 2r\right)=\frac{1}{\alpha}\frac{\partial \theta}{\partial t}$$

$$r\times\left(\frac{\partial^2 \theta}{\partial r^2}+\frac{2}{r}\frac{\partial \theta}{\partial r}\right)=\left(\frac{1}{\alpha}\frac{\partial \theta}{\partial t}\right)\times r$$

$$\underbrace{r\frac{\partial^2 \theta}{\partial r^2}+2\frac{\partial \theta}{\partial r}}=\frac{r}{\alpha}\frac{\partial \theta}{\partial t}$$

eqn 7 $\leftarrow$ $$\frac{\partial^2 u}{\partial r^2}=\frac{1}{\alpha}\frac{\partial u}{\partial t}$$ $\rightarrow$ eqn 5A

Source/Sink in Welding: Point source

$u = r\theta$
$= r(T - T_0)$

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{\alpha}\frac{\partial u}{\partial t} \quad \text{--- } 8$$

$2A \rightarrow r \rightarrow 0, \quad u \rightarrow 0, \quad -4\pi r^2 K \frac{\partial T}{\partial r} = Q$

eqn 6A

$$-4\pi \cancel{r^2} K \frac{1}{\cancel{r}}\left(r\frac{\partial u}{\partial r} - \theta\right) = Q$$

$$-4\pi K\left(r\frac{\partial u}{\partial r} - r\theta\right) = Q$$

$$-4\pi K\left(r\frac{\partial u}{\partial r} - u\right) = Q \quad \text{--- } 9$$

Source/Sink in Welding: Point source

$u = r\theta$

3A $\rightarrow$ $r \rightarrow \infty, u \rightarrow \infty$, $\theta = 0$ — 10

4A $\cancel{\alpha}$ $t = 0$, $u = 0$, $\theta = 0$ — 11

$$\eta = \frac{r}{\sqrt{4\alpha t}}$$ — 12

$$u = A + B\,\mathrm{erf}(\eta)$$ — 13

Source/Sink in Welding: Point source

$u = r\theta$

at $r \to 0,\ \eta \to 0$

$$-4\pi k\left(r\frac{\partial u}{\partial r} - u\right) = Q$$

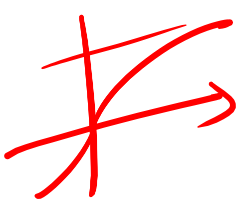
$$\eta = \frac{r}{\sqrt{4\alpha t}}$$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial r} = \frac{1}{\sqrt{4\alpha t}}\frac{du}{d\eta}$$

$$-4\pi k\left(r\frac{1}{\sqrt{4\alpha t}}\frac{du}{d\eta} - u\right) = Q$$

$$-4\pi k\left(\eta\frac{du}{d\eta} - u\right) = Q \quad \text{— 14A}$$

$\cancel{-4\pi k}\,($



$t = 0,\ r \to \infty,\ \eta \to \infty,\ u = 0$

$$u = A + B\,\mathrm{erf}(\eta) \Rightarrow 0 = A + B\,\mathrm{erf}(\infty)$$

$$\boxed{A + B = 0} \quad \text{— 14B}$$

Source/Sink in Welding: Point source

$r \to 0 \quad -4\pi K\left(r\frac{dy}{dr} - y\right) = Q$

$-4\pi K\left(-A - B\,\mathrm{erf}(0)\right) = Q$

$$A = \frac{Q}{4\pi K}$$

$$B = -\frac{Q}{4\pi K}$$

Source/Sink in Welding: Point source

$$u = A + B\,\mathrm{erf}(\eta) \qquad \eta = \frac{r}{\sqrt{4\alpha t}}$$

$u = r\theta$

$\theta = T - T_\infty$

$$u = \frac{Q}{4\pi k} - \frac{Q}{4\pi k}\,\mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)$$

$$r\theta = \frac{Q}{4\pi k}\left(1 - \mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)\right)$$

$$r\,\underline{(T - T_\infty)} = \frac{Q}{4\pi k}\underline{\left(1 - \mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)\right)}$$

Source/Sink in Welding: Point source

$$\boxed{T = T_\infty + \frac{Q}{4\pi k r}\left(1 - \mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)\right)}$$

$t \to \infty$

$$\boxed{T = T_\infty + \frac{Q}{4\pi k r}}$$

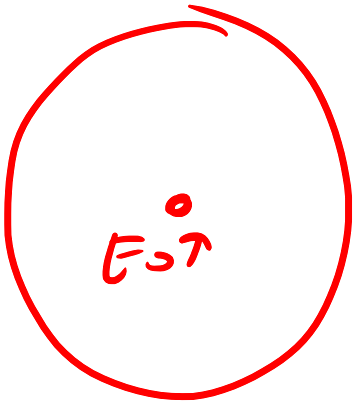
Source/Sink in Welding: Point source

Instantaneous heat source: Thermal Explosion at point region

Consider a body of large extent possessing a small region (point) that exploded E_0 energy at $t=0$ at centre.

→ The amount of energy raises the temp. of body & it is independent of time.

→ Due to Diffusion



Instantaneous heat source: Thermal Explosion at point region

G.E. $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad — (1)$$

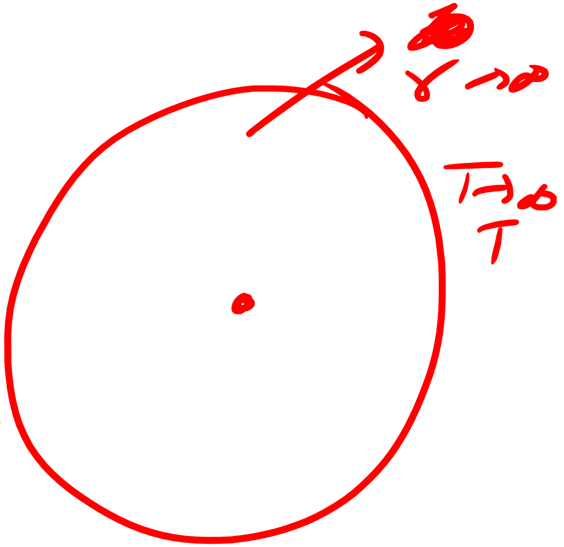
B.C.

$$r \to \infty, \quad T \to T_\infty$$

Condition :- Energy Balance

$$\int_0^\infty 4\pi r^2 \rho C_p (T - T_\infty)\, dr = E_0$$

I.C = $$t = 0, \quad T = T_\infty$$



Instantaneous heat source: Thermal Explosion at point region

$$\theta = T - T_\infty$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial\theta}{\partial t} \quad \text{—} \quad 1A$$

$$\int_0^\infty 4\pi r^2 \rho c_p \theta \, dr = E_0 \quad \text{—} \quad 2A$$

$$r \to \infty, \quad \theta = 0, \quad \text{—} \quad 3A$$

$$t = 0, \quad \theta = 0, \quad \text{—} \quad 4A$$

$$u = r\theta = r(T - T_\infty)$$

Instantaneous heat source: Thermal Explosion at point region

$u = r\theta$

$$\frac{\partial u}{\partial t} = r\frac{\partial \theta}{\partial t} \quad (5) \quad , \quad \frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial u}{\partial t} \quad - (5A)$$

$$\frac{\partial u}{\partial r} = r\frac{\partial \theta}{\partial r} + \theta \quad - (6) \quad , \quad \frac{\partial \theta}{\partial r} = \frac{1}{r}\left(\frac{\partial u}{\partial r} - \theta\right) - (6A)$$

$$\frac{\partial^2 u}{\partial r^2} = r\frac{\partial^2 \theta}{\partial r^2} + 2\frac{\partial \theta}{\partial r} \quad - (7)$$

Instantaneous heat source: Thermal Explosion at point region

(1A) $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \theta}{\partial t}$$

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{\alpha}\frac{\partial u}{\partial t} \quad \text{—— (8)}$$

$$\int_0^\infty 4\pi r^2 \rho C_p \, \theta \, dr = E_0$$

$$4\pi \rho C_p \int_0^\infty u r \, dr = E_0 \quad \text{—— (9)}$$

$r^2 \, \theta \, dr$

$r \cdot r \theta \, dr$

$r \, u \, dr$

$r \to \infty, \; \theta = 0, \; u = 0, \quad \text{—— (10)}$

$t = 0, \; \theta = 0, \; u = 0, \quad \text{—— (11)}$

Instantaneous heat source: Thermal Explosion at point region

$$U = A + B\,\mathrm{erf}(\eta)$$

$$= A + B\,\mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right) \quad — \quad (12)$$

$$u' = B\,\frac{\cancel{2}}{\sqrt{\pi}}\,e^{-\frac{r^2}{4\alpha t}}\left(\frac{1}{\sqrt{\cancel{4}\alpha t}}\right)$$

$$u' = B\,\frac{1}{\sqrt{\pi\alpha t}}\,e^{-\frac{r^2}{4\alpha t}} \quad — \quad (13)$$

Instantaneous heat source: Thermal Explosion at point region

$$u'' = \frac{B}{\sqrt{\pi \alpha t}} e^{\left(-\frac{r^2}{4\alpha t}\right)} \left(-\frac{2r}{4\alpha t}\right)$$

$$= \frac{-Br}{2\sqrt{\pi}(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad \text{--- } (14)$$

assuming that u'' in our sol^n

so $u = u'' =$ eq^n (14)

Instantaneous heat source: Thermal Explosion at point region

eq^n (9) $4\pi\rho C_p \int_0^\infty u\, r\, dr = E_0$

$$u = u'' = \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad — (14)$$

$$4\pi\rho C_p \int_0^\infty \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) r\, dr = E_0 \quad — (15)$$

$$z^2 = \frac{r^2}{4\alpha t} \qquad 2z\,dz = \frac{2r\,dr}{4\alpha t}$$

$$z = \frac{r}{\sqrt{4\alpha t}}, \qquad dz = \frac{dr}{\sqrt{4\alpha t}}$$

Instantaneous heat source: Thermal Explosion at point region

$$\int_0^\infty \frac{-2Br^2}{(4\alpha t)\sqrt{\pi\alpha t}} \exp(-z^2)\, dr = \frac{E_0}{4\pi\rho C_p}$$

$$\int^\infty \frac{-2Br^2}{4\alpha t\sqrt{\pi\alpha t}} \exp(-z^2)\, 2\sqrt{\cancel{4\alpha t}}\, dz = \frac{E_0}{4\pi\rho C_p}$$

$$\int^\infty -\frac{4Bz^2}{\sqrt{\pi}} \exp(-z^2)\, dz = \frac{E_0}{4\pi\rho C_p}$$

Instantaneous heat source: Thermal Explosion at point region

$$-\frac{4B}{\sqrt{\pi}}\int_0^\infty z^2 \exp(-z^2)\,dz = \frac{E_0}{4\pi\rho c_p}$$

$$z^2 = w \,, \quad z = \sqrt{w}$$

$$dz = \frac{1}{2}(w)^{-1/2}\,dw$$

$$-\frac{\cancel{4}^{\,2} B}{\sqrt{\pi}}\int_0^\infty w \exp(-w)\,\frac{1}{\cancel{2}}\,w^{-1/2}\,dw = \frac{E_0}{4\pi\rho c_p}$$

$$-\frac{2B}{\sqrt{\pi}}\int_0^\infty w^{1/2} \exp(-w)\,dw = \frac{E_0}{4\pi\rho c_p}$$

Instantaneous heat source: Thermal Explosion at point region

Surdy

$$-\frac{2B}{\sqrt{\pi}}\,\Gamma^{3/2} = \frac{E_0}{4\pi\rho C_p}$$

$$\Gamma\tfrac{1}{2}+1 = \tfrac{1}{2}\,\Gamma\tfrac{1}{2}$$

$$= \tfrac{1}{2} \times \sqrt{\pi}$$

$$= \frac{\sqrt{\pi}}{2}$$

$$-\frac{\cancel{2}B}{\cancel{\sqrt{\pi}}} \times \frac{\cancel{\sqrt{\pi}}}{\cancel{2}} = \frac{E_0}{4\pi\rho C_p}$$

$$B = -\frac{E_0}{4\pi\rho C_p} \quad (6)$$

Instantaneous heat source: Thermal Explosion at point region

So, u'' is our sol^n

$$u = u'' = \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

eq 14

$$u = \frac{E_0\, r}{2\sqrt{\pi}\,(\alpha t)^{3/2} \times 4\pi \rho C_p} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

(6) $B = -\frac{E_0}{4\pi \rho C_p}$

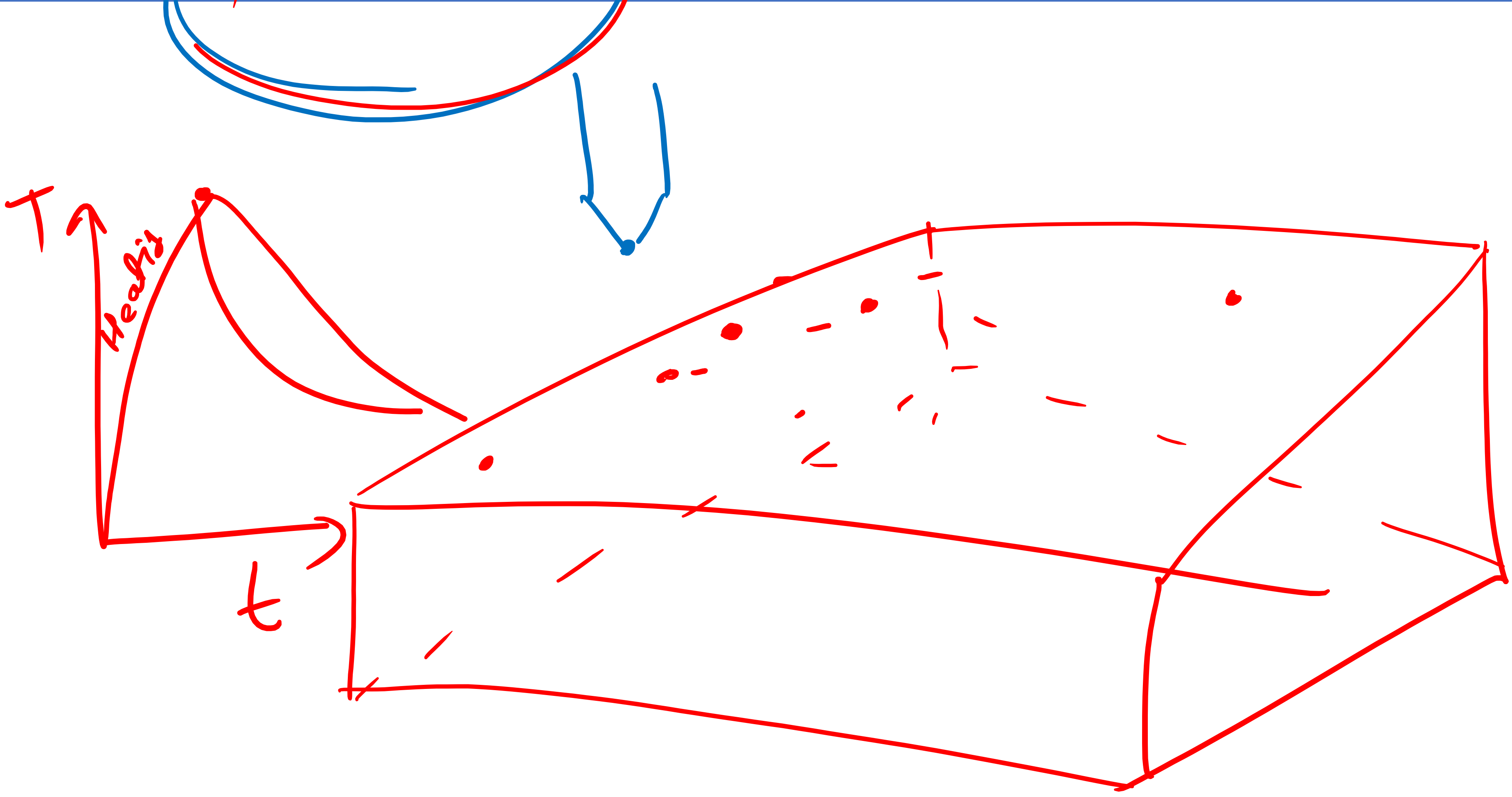
$$u = \frac{E_0\, r}{8\,(\pi \alpha t)^{3/2} \rho C_p} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$u = r\theta = r(T - T_0)$

Instantaneous heat source: Thermal Explosion at point region

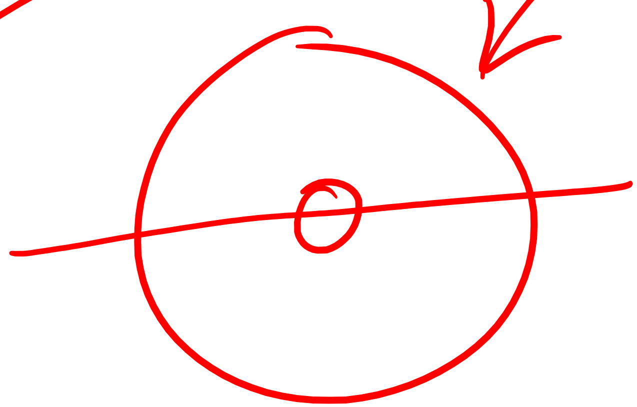
$$\boxed{T = T_\infty + \frac{E_0}{8(\pi\alpha t)^{3/2}\rho c_p}\exp\left(\frac{-r^2}{4\alpha t}\right)}$$

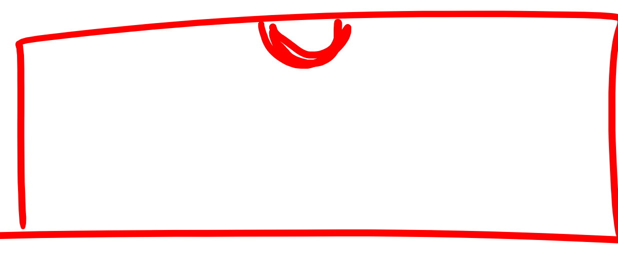
Arc strikes in Fusion welding



Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_0}{8\rho C_p (\pi \alpha t)^{3/2}} \exp\left(\frac{-r^2}{4\alpha t}\right)$$





$$\boxed{T = T_\infty + \frac{E_0}{4\rho C_p (\pi \alpha t)^{3/2}} \exp\left(\frac{-r^2}{4\alpha t}\right)}$$

Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_0}{4\rho c_p (\pi \alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

Assuming

$$\Theta = \frac{T - T_\infty}{T_c - T_\infty}$$

Non-dimensional temp.

T_∞ = initial temp

T_c = Reference temp.

dimension less time

$$\tau = \frac{t}{t_i}$$

$T_c = T_m$ = melting point

Arc strikes in Fusion welding

Dimension less radius factor $= \sigma_1 = \sqrt{\dfrac{x^2}{4\alpha t_i}}$

t_i = ignition time

Dimension less operating parameter

$$= n_1 = \frac{E_0}{4(\pi\alpha t_i)^{3/2}\rho C_p (T_c - T_\infty)}$$

$$\rho C_p (T_c - T_\infty) = \Delta H$$

$$= \frac{E_0}{4(\pi\alpha t_i)^{3/2}\Delta H}$$

Arc strikes in Fusion welding

q_0 = Heat input = $\dfrac{E_0}{t_i} \rightarrow \dfrac{\text{Joule}}{\text{Sec}}$ watt

$= I \times V$

current × vol.

$$n_1 = \frac{E_0}{4(\pi\alpha)^{3/2}\, t_i^{3/2} \times H'\, \Delta H} = \frac{q_0}{4(\pi\alpha)^{3/2}\, t_i^{1/2}\, \Delta H}$$

Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_0}{4\rho C_p (\pi \alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\frac{T - T_\infty}{T_c - T_\infty} = \frac{E_0}{4\rho C_p (\pi \alpha t)^{3/2}} \frac{1}{T_c - T_\infty} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\theta = \frac{E_0}{4\rho C_p \left(\pi \alpha t \frac{t_i}{t_i}\right)^{3/2} (T_c - T_\infty)} \exp\left(-\frac{r^2}{4\alpha t \frac{t_i}{t_i}}\right)$$

$\rightarrow \Delta H$

Arc strikes in Fusion welding

$$\theta = \frac{E_0}{4\Delta H\left(\pi\alpha t_i \times \frac{t}{t_i}\right)^{3/2}} \exp\left(\frac{-r^2}{4\alpha t_i \times \frac{t}{t_i}}\right)$$

$r^2 \to \sigma_1^2$

$\frac{t}{t_i} \to \tau_1$

$$= \frac{E_0}{4\Delta H(\pi\alpha t_i)^{3/2}\left(\frac{t}{t_i}\right)^{3/2}} \exp\left(\frac{-\sigma_1^2}{\tau_1}\right)$$

$\left(\frac{t}{t_i}\right)^{3/2} \to \tau_1^{3/2}$

$(\pi\alpha)^{3/2} t_i^{3/2} \times t_i$

$\frac{E_0}{t_i} = q_0$

$$= \frac{q_0}{4\Delta H(\pi\alpha)^{3/2} t_i^{1/2}\,\tau_1^{3/2}} \exp\left(\frac{-\sigma_1^2}{\tau_1}\right)$$

$\frac{q_0}{4\Delta H(\pi\alpha)^{3/2} t_i^{1/2}} \to n_1$

Arc strikes in Fusion welding

$$\boxed{\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)}$$

$$\boxed{\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)}$$

Fusion welding: thermal analysis

$\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right) \to 0$

X_ti	sigma^2_0	sigma^2_0.25	sigma^2_0.5	sigma^2_0.75	sigma^2_1	sigma^2_1.5	sigma^2_2
0.1	31.6227766	2.595756	0.213073	0.01749	0.001436	9.67348E-06	6.51794E-08
0.2	11.18033989	3.203221	0.917738	0.262936	0.075333	0.006183671	0.000507587
0.3	6.085806195	2.64488	1.14946	0.499553	0.217105	0.04100584	0.007745003
0.4	3.952847075	2.115807	1.13251	0.606189	0.324469	0.092962053	0.026634074
0.5	2.828427125	1.715528	1.04052	0.631107	0.382786	0.140819095	0.05180445
0.6	2.151657415	1.41846	0.935106	0.61646	0.406396	0.176618796	0.076758212
0.7	1.707469442	1.194669	0.835877	0.58484	0.409197	0.200318891	0.098064442
0.8	1.397542486	1.022464	0.748051	0.547286	0.400403	0.214320082	0.114717273
0.9	1.171213948	0.887154	0.671988	0.509007	0.385555	0.221213741	0.12692214
1	1	0.778801	0.606531	0.472367	0.367879	0.22313016	0.135335283
1.5	0.544331054	0.460766	0.39003	0.330153	0.279469	0.200248204	0.143484108
2	0.353553391	0.31201	0.275348	0.242993	0.214441	0.167006796	0.130065024
2.5	0.252982213	0.228908	0.207124	0.187414	0.169579	0.138839582	0.113672236
3	0.19245009	0.177063	0.162905	0.14988	0.137897	0.11672688	0.098807171
3.5	0.15272071	0.142193	0.13239	0.123264	0.114766	0.099488235	0.086244152
4	0.125	0.117427	0.110312	0.103629	0.09735	0.08591116	0.075816332
4.5	0.10475656	0.099095	0.09374	0.088675	0.083882	0.075061355	0.067167852
5	0.089442719	0.085081	0.080931	0.076984	0.07323	0.066260796	0.059955248
6	0.068041382	0.065265	0.062601	0.060046	0.057596	0.052990681	0.04875378
7	0.053994925	0.052101	0.050273	0.048509	0.046807	0.043580262	0.04057596
8	0.044194174	0.042834	0.041517	0.040239	0.039001	0.036638257	0.034418457
9	0.037037037	0.036022	0.035036	0.034076	0.033142	0.031351175	0.029656941
10	0.031622777	0.030842	0.030081	0.029338	0.028613	0.027217976	0.02589054

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Fusion welding: thermal analysis