Welding Technology ME692

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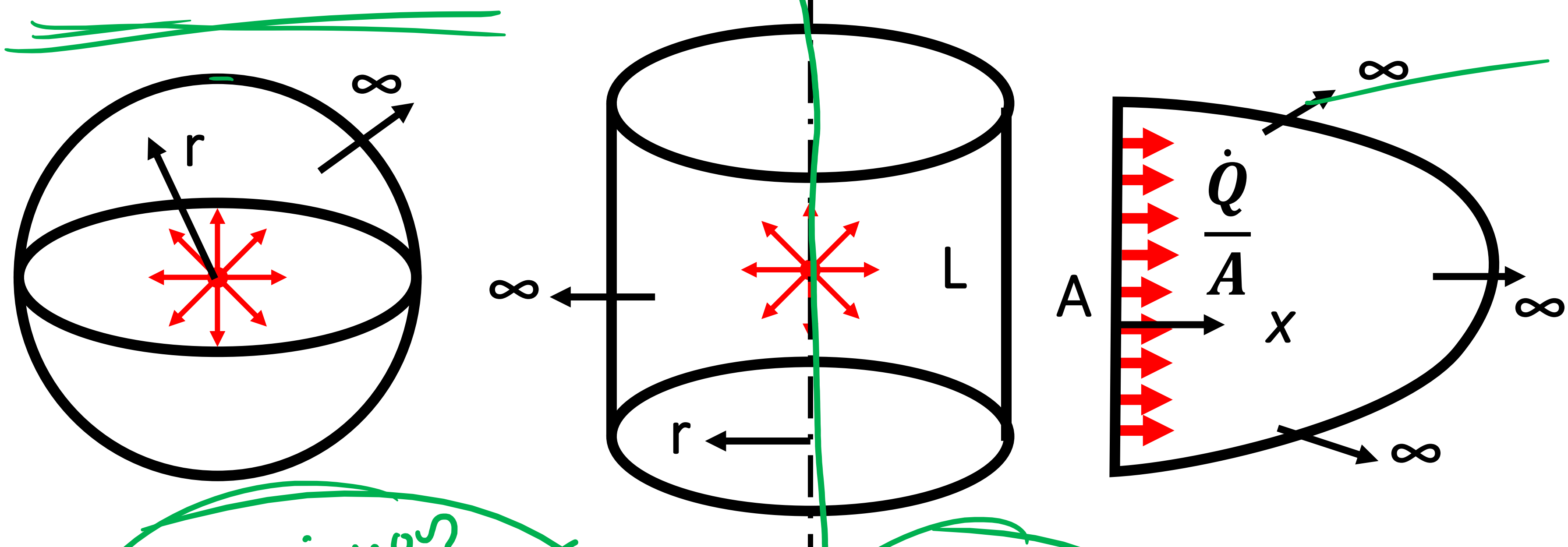
Thermal analysis in welding

Source/Sink in Welding

Heat transfer with heat sources or sinks

Sources of constant heat production rate:

(a) Point source (b) line source (c) plane source

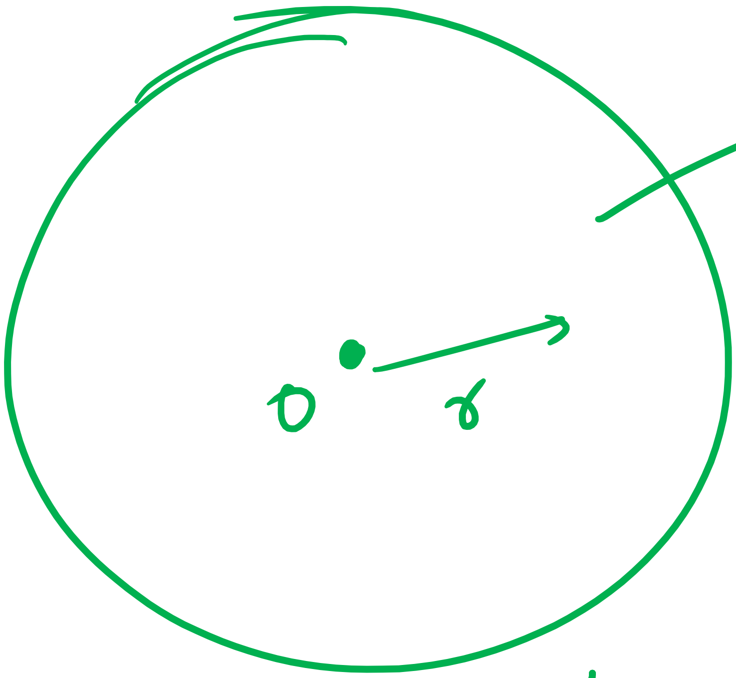


Continuous
• Nuclear waste → ocean

Explosion → ocean
• Bomb

Source/Sink in Welding: Point source

Point heat Source of Constant heat Production Rate



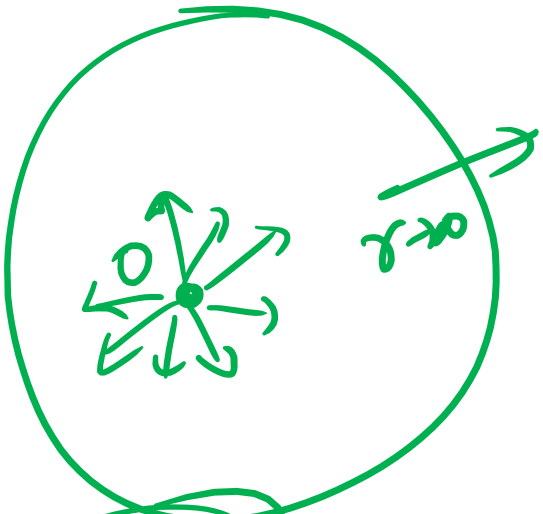
O is heating source which is producing const heat.

$r \to \infty$

Consider a body of large extent possessing a very small region (point) that produce heat continuously at a constant rate (Q, watt)

Source/Sink in Welding: Point source

Given that body was initially maintained at $T = T_\infty$



G.E.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad -(1)$$

B.C.

$r \to \infty$, $T = T_\infty$ — (2)

$$-KA\frac{dT}{dr} = Q$$

$r \to 0$, $-4\pi r^2 k \dfrac{dT}{dr} = Q$ — (3)

I.C. $\Rightarrow$ $t = 0$, $T = T_\infty$ — (4)

Source/Sink in Welding: Point source

$$\theta = T - T_\infty$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{--- } 1A$$

B.C. $\quad r \to 0, \quad -4\pi r^2 K \frac{\partial \theta}{\partial r} = Q \quad 2A$

$\quad r \to \infty, \quad \theta = 0 \quad \text{--- } 3A$

I.C. $\quad t = 0, \quad \theta = 0, \quad \text{--- } 4A$

$$\therefore \quad u = r(T - T_\infty) = r\theta$$

Source/Sink in Welding: Point source

$u = r\theta$

$$\frac{\partial u}{\partial t} = r\frac{\partial \theta}{\partial t} + \theta\frac{\partial r}{\partial t}$$

$$\frac{\partial r}{\partial t} \to 0$$

$$\boxed{\frac{\partial u}{\partial t} = \frac{r\,\partial \theta}{\partial t}} \quad \text{— 5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial u}{\partial t} \quad \text{— 5A}$$

$$\frac{\partial u}{\partial r} = r\frac{\partial \theta}{\partial r} + \theta \quad \text{— 6}$$

$$\frac{\partial \theta}{\partial r} = \frac{1}{r}\left(\frac{\partial u}{\partial r} - \theta\right) \quad \text{— 6A}$$

$$\frac{\partial^2 u}{\partial r^2} = r\frac{\partial^2 \theta}{\partial r^2} + \frac{\partial \theta}{\partial r} + \frac{\partial \theta}{\partial r}$$

$$= r\frac{\partial^2 \theta}{\partial r^2} + 2\frac{\partial \theta}{\partial r} \quad \text{— 7}$$

Source/Sink in Welding: Point source

1A $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial\theta}{\partial t}$$

$$\rightarrow \frac{1}{r^2}\left(r^2\frac{\partial^2\theta}{\partial r^2} + \frac{\partial\theta}{\partial r}\times 2r\right) = \frac{1}{\alpha}\frac{\partial\theta}{\partial t}$$

$$r\times\left(\frac{\partial^2\theta}{\partial r^2} + \frac{2}{r}\frac{\partial\theta}{\partial r}\right) = \left(\frac{1}{\alpha}\frac{\partial\theta}{\partial t}\right)\times r$$

$$\underbrace{r\frac{\partial^2\theta}{\partial r^2} + 2\frac{\partial\theta}{\partial r}} = \frac{r}{\alpha}\frac{\partial\theta}{\partial t}$$

$$\text{eq}^n\ 7 \longleftarrow \frac{\partial^2 u}{\partial r^2} = \frac{1}{\alpha}\frac{\partial u}{\partial t} \longrightarrow \text{eq}^n\ 5A$$

Source/Sink in Welding: Point source

$u = r\theta$
$= r(T - T_0)$

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{\alpha}\frac{\partial u}{\partial t} \quad \text{--- } 8$$

2A $\rightarrow$ $r \rightarrow 0$, $u \rightarrow 0$, $-4\pi r^2 K \frac{\partial T}{\partial r} = Q$

eqn 6A

$$-4\pi \cancel{r^2} K \frac{1}{\cancel{r}}\left(r\frac{\partial u}{\partial r} - \theta\right) = Q$$

$$-4\pi K\left(r\frac{\partial u}{\partial r} - r\theta\right) = Q$$

$$-4\pi K\left(r\frac{\partial u}{\partial r} - u\right) = Q \quad \text{--- } 9$$

Source/Sink in Welding: Point source

$u = r\theta$

3A $\rightarrow$ $r \to \infty, u \to \infty$, $\theta = 0$ — 10

4A $t = 0$, $u = 0$, $\theta = 0$ — 11

$$\eta = \frac{r}{\sqrt{4\alpha t}} \quad (12)$$

$$u = A + B\,\mathrm{erf}(\eta) \quad (13)$$

Source/Sink in Welding: Point source

$u = r\theta$

at $r \to 0$, $\eta \to 0$

$$-4\pi k\left(r\frac{\partial u}{\partial r} - u\right) = Q$$

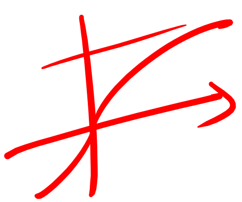
$$\eta = \frac{r}{\sqrt{4\alpha t}}$$

$$\frac{\partial u}{\partial r} = \frac{du}{d\eta}\frac{\partial \eta}{\partial r} = \frac{1}{\sqrt{4\alpha t}}\frac{du}{d\eta}$$

$$-4\pi k\left(r\frac{1}{\sqrt{4\alpha t}}\frac{du}{d\eta} - u\right) = Q$$

$$-4\pi k\left(\eta\frac{du}{d\eta} - u\right) = Q \quad \text{— 14A}$$

$-\cancel{4\pi k}($



$t = 0$, $r \to \infty$, $\eta \to \infty$, $u = 0$

$$u = A + B\,\mathrm{erf}(\eta) \Rightarrow 0 = A + B\,\mathrm{erf}(\infty)$$

$$\boxed{A + B = 0} \quad \text{— 14B}$$

Source/Sink in Welding: Point source

$r \to 0$ $\qquad -4\pi k\left(r\frac{du}{dr} - u\right) = Q$

$\to 0$

$-4\pi k\left(-A - Be^{\lambda(0)}\right) = Q$

$$A = \frac{Q}{4\pi k}$$

$$B = -\frac{Q}{4\pi k}$$

Source/Sink in Welding: Point source

$$u = A + B\,\mathrm{erf}(\eta) \qquad \eta = \frac{r}{\sqrt{4\alpha t}}$$

$u = r\theta$

$\theta = T - T_\infty$

$$u = \frac{Q}{4\pi k} - \frac{Q}{4\pi k}\,\mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)$$

$$r\theta = \frac{Q}{4\pi k}\left(1 - \mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)\right)$$

$$r\,\underline{(T - T_\infty)} = \frac{Q}{4\pi k}\underline{\left(1 - \mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)\right)}$$

Source/Sink in Welding: Point source

$$\boxed{T = T_\infty + \frac{Q}{4\pi k r}\left(1 - \mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right)\right)}$$

$t \to \infty$

$$\boxed{T = T_\infty + \frac{Q}{4\pi k r}}$$

Source/Sink in Welding: Point source

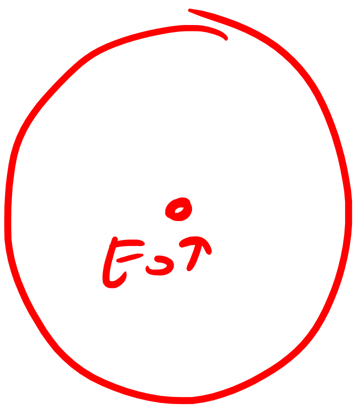
Source/Sink in Welding: Point source

Instantaneous heat source: Thermal Explosion at point region

Consider a body of large extent possessing a small region (point) that explosed E_0 energy at $t=0$ at centre.

→ The amount of energy raises the temp. of body & it is independent of time.

→ Due to Diffusion



Instantaneous heat source: Thermal Explosion at point region

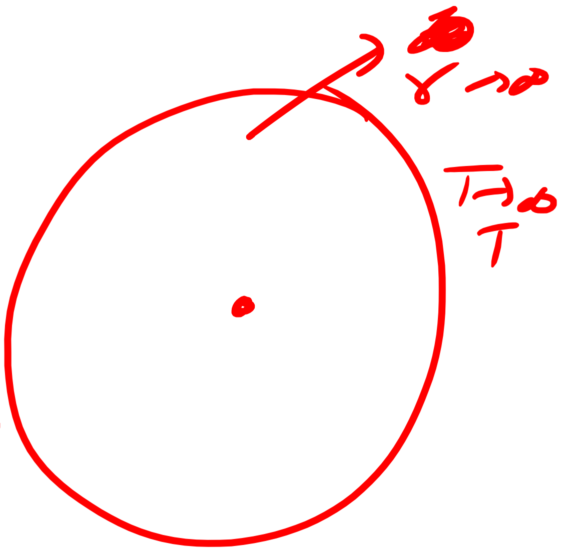
G.E. $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad — (1)$$

B.C. $$r \to \infty, \quad T \to T_\infty$$

Condition :- Energy Balance

$$\int_0^\infty 4\pi r^2 \rho C_p (T-T_\infty)\, dr = E_0$$

I.C = $$t = 0, \quad T = T_\infty$$



Instantaneous heat source: Thermal Explosion at point region

$\theta = T - T_\infty$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{— 1A}$$

$$\int_0^\infty 4\pi r^2 \rho c_p \theta \, dr = E_0 \quad \text{— 2A}$$

$$r \to \infty, \quad \theta = 0, \quad \text{— 3A}$$

$$t = 0, \quad \theta = 0, \quad \text{— 4A}$$

$$u = r\theta = r(T - T_\infty)$$

Instantaneous heat source: Thermal Explosion at point region

$u = r\theta$

$$\frac{\partial u}{\partial t} = r\frac{\partial \theta}{\partial t} \quad (5) \quad , \quad \frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial u}{\partial t} \quad \text{—} \; (5A)$$

$$\frac{\partial u}{\partial r} = r\frac{\partial \theta}{\partial r} + \theta \quad \text{—} \; (6) \quad , \quad \frac{\partial \theta}{\partial r} = \frac{1}{r}\left(\frac{\partial u}{\partial r} - \theta\right) \text{—} \; (6A)$$

$$\frac{\partial^2 u}{\partial r^2} = r\frac{\partial^2 \theta}{\partial r^2} + 2\frac{\partial \theta}{\partial r} \quad \text{—} \; (7)$$

Instantaneous heat source: Thermal Explosion at point region

1A

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial \theta}{\partial t}$$

$$\frac{\partial^2 u}{\partial r^2}=\frac{1}{\alpha}\frac{\partial u}{\partial t} \qquad (8)$$

$$\int_0^\infty 4\pi r^2 \rho C_p \,\theta\, dr = E_0$$

$r^2 \theta\, dr$

$r \cdot r\theta\, dr$

$r u\, dr$

$$4\pi \rho C_p \int_0^\infty u r\, dr = E_0 \qquad (9)$$

$r \to \infty, \ \theta = 0, \ u = 0, \qquad (10)$

$t = 0, \ \theta = 0, \ u = 0, \qquad (11)$

Instantaneous heat source: Thermal Explosion at point region

$$u = A + B\,\mathrm{erf}(\eta)$$

$$= A + B\,\mathrm{erf}\left(\frac{r}{\sqrt{4\alpha t}}\right) \quad (12)$$

$$u' = B\,\frac{\cancel{2}}{\sqrt{\pi}}\,e^{-\frac{r^2}{4\alpha t}}\left(\frac{1}{\cancel{\sqrt{4\alpha t}}}\right)$$

$$u' = B\,\frac{1}{\sqrt{\pi\alpha t}}\,e^{-\frac{r^2}{4\alpha t}} \quad (13)$$

Instantaneous heat source: Thermal Explosion at point region

$$u'' = \frac{B}{\sqrt{\pi \alpha t}} \, e^{\left(-\frac{r^2}{4\alpha t}\right)} \left(-\frac{2r}{4\alpha t}\right)$$

$$= \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad \text{--- } (14)$$

assuming that u'' in our sol^n

so' $u = u'' =$ eq^n (14)

Instantaneous heat source: Thermal Explosion at point region

eqn 9 $\quad 4\pi \rho C_p \int_0^\infty u\, r\, dr = E_0$

$$u = u'' = \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) \quad \text{--- } 14$$

$$4\pi \rho C_p \int_0^\infty \frac{-Br}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right) r\, dr = E_0 \quad \text{--- } 15$$

$$z^2 = \frac{r^2}{4\alpha t} \qquad 2z\,dz = \frac{2r\,dr}{4\alpha t}$$

$$z = \frac{r}{\sqrt{4\alpha t}}, \qquad dz = \frac{dr}{\sqrt{4\alpha t}}$$

Instantaneous heat source: Thermal Explosion at point region

$$\int_0^\infty \frac{-2Br^2}{(4\alpha t)\sqrt{\pi\alpha t}} \exp(-z^2)\, dr = \frac{E_0}{4\pi\rho C_p}$$

$$\int_0^\infty \frac{-2Br^2}{4\alpha t\sqrt{\pi\alpha t}} \exp(-z^2)\, \cancel{dz}\, \sqrt{\cancel{4\alpha t}}\, dz = \frac{E_0}{4\pi\rho C_p}$$

$$\int_0^\infty -\frac{4Bz^2}{\sqrt{\pi}} \exp(-z^2)\, dz = \frac{E_0}{4\pi\rho C_p}$$

Instantaneous heat source: Thermal Explosion at point region

$$-\frac{4B}{\sqrt{\pi}}\int_0^\infty z^2 \exp(-z^2)\,dz = \frac{E_0}{4\pi\rho c_p}$$

$$z^2 = w, \quad z = \sqrt{w}$$

$$dz = \frac{1}{2}(w)^{-1/2}\,dw$$

$$-\frac{\cancel{4}^{\,2}B}{\sqrt{\pi}}\int_0^\infty w\,\exp(-w)\,\frac{1}{\cancel{2}}\,w^{-1/2}\,dw = \frac{E_0}{4\pi\rho c_p}$$

$$-\frac{2B}{\sqrt{\pi}}\int_0^\infty w^{1/2}\exp(-w)\,dw = \frac{E_0}{4\pi\rho c_p}$$

Instantaneous heat source: Thermal Explosion at point region

Surdy

$$-\frac{2B}{\sqrt{\pi}}\,\Gamma^{3/2} = \frac{E_0}{4\pi\rho C_p}$$

$$\Gamma\frac{1}{2}+1 = \frac{1}{2}\,\Gamma^{1/2}$$

$$= \frac{1}{2}\times\sqrt{\pi}$$

$$= \frac{\sqrt{\pi}}{2}$$

$$-\frac{\cancel{2}B}{\cancel{\sqrt{\pi}}}\times\frac{\cancel{\sqrt{\pi}}}{\cancel{2}} = \frac{E_0}{4\pi\rho C_p}$$

$$B = -\frac{E_0}{4\pi\rho C_p} \quad 6$$

Instantaneous heat source: Thermal Explosion at point region

So, u'' is our sol

$$u = u'' = \frac{-B\, r}{2\sqrt{\pi}\,(\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

eq 14

(6) $B = -\dfrac{E_0}{4\pi\rho C_p}$

$$u = \frac{E_0\, r}{2\sqrt{\pi}\,(\alpha t)^{3/2} \times 4\pi\rho C_p} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$u = r\theta = r(T - T_0)$

$$u = \frac{E_0\, r}{8\,(\pi\alpha t)^{3/2}\rho C_p} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

Instantaneous heat source: Thermal Explosion at point region

$$\boxed{T = T_\infty + \frac{E_0}{8(\pi \alpha t)^{3/2} \rho c_p} \exp\left(\frac{-r^2}{4\alpha t}\right)}$$

Instantaneous heat source: Thermal Explosion at point region

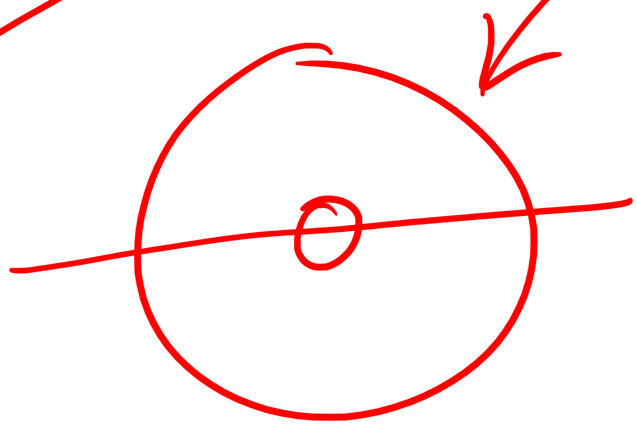
Instantaneous heat source: Thermal Explosion at point region

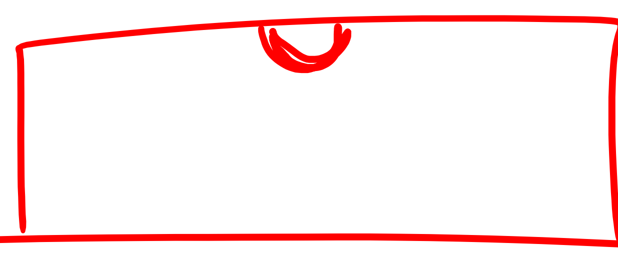
Instantaneous heat source: Thermal Explosion at point region

Arc strikes in Fusion welding

Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_0}{8\rho c_p (\pi \alpha t)^{3/2}} \exp\left(\frac{-r^2}{4\alpha t}\right)$$





$$\boxed{T = T_\infty + \frac{E_0}{4\rho c_p (\pi \alpha t)^{3/2}} \exp\left(\frac{-r^2}{4\alpha t}\right)}$$

Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_0}{4\rho c_p (\pi \alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

Assuming

$$\Theta = \frac{T - T_\infty}{T_c - T_\infty}$$

Non-dimensional temp.

T_∞ = initial temp

T_c = Reference temp.

dimensionless time

$$\tau = \frac{t}{t_i}$$

$T_c = T_m$ = melting point

Arc strikes in Fusion welding

Dimension less radius factor $= \sigma_1 = \sqrt{\dfrac{\cancel{x}^2}{4\alpha t_i}}$

t_i = ignition time

Dimension less operating parameter

$$= n_1 = \frac{E_0}{4(\pi \alpha t_i)^{3/2} \rho C_p (T_c - T_\infty)}$$

$$\rho C_p (T_c - T_\infty) = \Delta H$$

$$= \frac{E_0}{4(\pi \alpha t_i)^{3/2} \Delta H}$$

Arc strikes in Fusion welding

q_0 Heat input $= \dfrac{E_0}{t_i} \rightarrow \dfrac{\text{Joule}}{\text{Sec}}$ watt

$= I \times V$

current $\times$ vol.

$$n_1 = \frac{E_0}{4(\pi\alpha)^{3/2} t_i^{3/2} \times H_i \, \Delta H} = \frac{q_0}{4(\pi\alpha)^{3/2} t_i^{1/2} \Delta H}$$

Arc strikes in Fusion welding

$$T = T_\infty + \frac{E_o}{4\rho c_p (\pi \alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\frac{T - T_\infty}{T_c - T_\infty} = \frac{E_o}{4\rho c_p (\pi \alpha t)^{3/2}} \frac{1}{T_c - T_\infty} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\theta = \frac{E_o}{4\rho c_p \left(\pi \alpha t \frac{t_i}{t_i}\right)^{3/2} (T_c - T_\infty)} \exp\left(-\frac{r^2}{4\alpha t \frac{t_i}{t_i}}\right)$$

$\rightarrow \Delta H$

Arc strikes in Fusion welding

$$\theta = \frac{E_0}{4\,\Delta H\left(\pi\alpha t_i \times \cancel{\frac{t}{t_i}}\right)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t_i \times \frac{t}{t_i}}\right)$$

$$r^2 \to \sigma_1^2 \qquad \frac{t}{t_i} \to \tau_1$$

$$= \frac{E_0}{4\,\Delta H\,(\pi\alpha t_i)^{3/2}\left(\frac{t}{t_i}\right)^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$$(\pi\alpha)^{3/2}\, t_i^{3/2} \times \cancel{t_i} \qquad \left(\frac{t}{t_i}\right)^{3/2} \to \tau_1^{3/2} \qquad \frac{E_0}{t_i} = q_0$$

$$= \frac{q_0}{4\,\Delta H\,(\pi\alpha)^{3/2}\, t_i^{1/2}}\,\tau_1^{3/2} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$$\frac{q_0}{4\,\Delta H\,(\pi\alpha)^{3/2}\, t_i^{1/2}} \to n_1$$

Arc strikes in Fusion welding

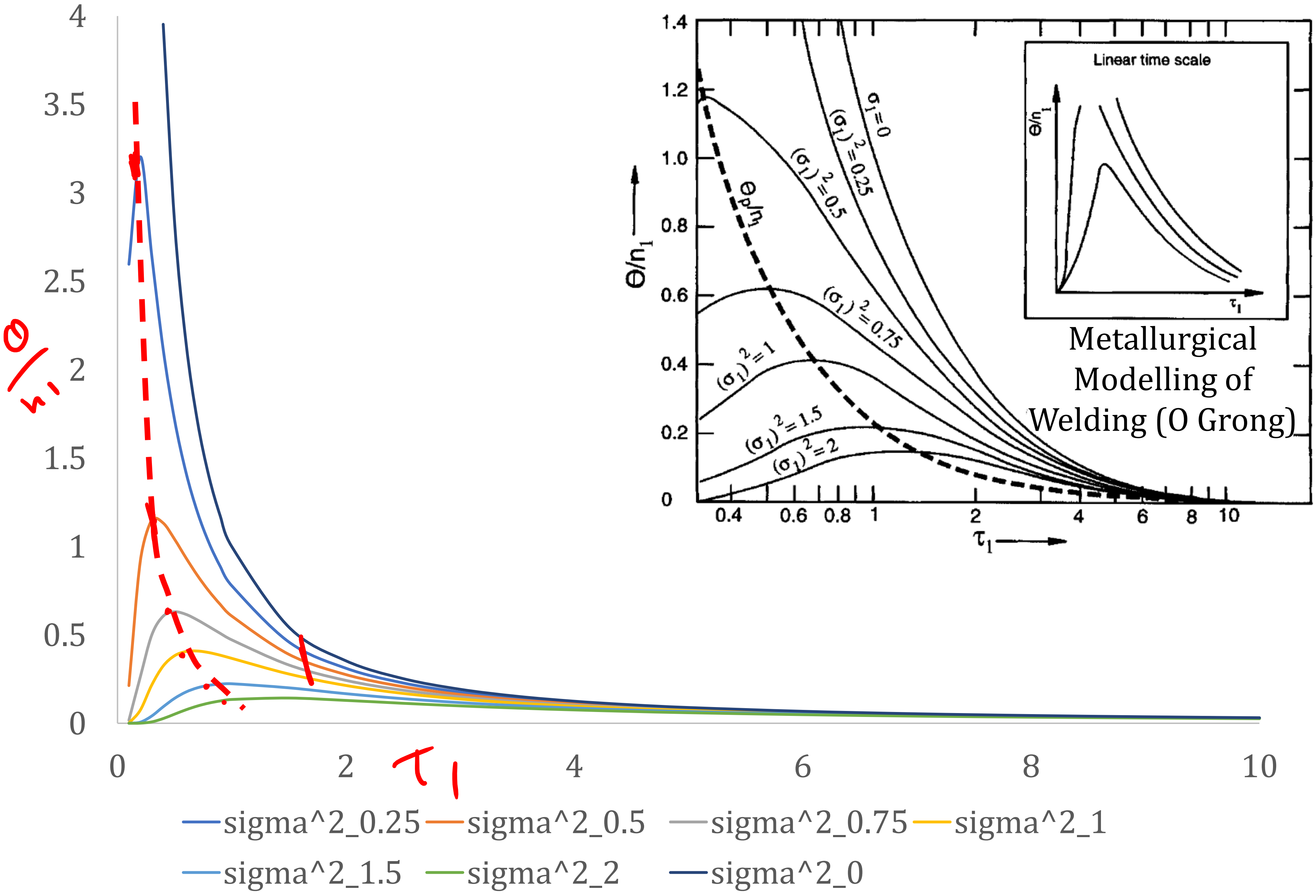
$$\boxed{\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)}$$

$$\boxed{\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)}$$

Fusion welding: thermal analysis

X_ti	sigma^2_0	sigma^2_0.25	sigma^2_0.5	sigma^2_0.75	sigma^2_1	sigma^2_1.5	sigma^2_2
0.1	31.6227766	2.595756	0.213073	0.01749	0.001436	9.67348E-06	6.51794E-08
0.2	11.18033989	3.203221	0.917738	0.262936	0.075333	0.006183671	0.000507587
0.3	6.085806195	2.64488	1.14946	0.499553	0.217105	0.04100584	0.007745003
0.4	3.952847075	2.115807	1.13251	0.606189	0.324469	0.092962053	0.026634074
0.5	2.828427125	1.715528	1.04052	0.631107	0.382786	0.140819095	0.05180445
0.6	2.151657415	1.41846	0.935106	0.61646	0.406396	0.176618796	0.076758212
0.7	1.707469442	1.194669	0.835877	0.58484	0.409197	0.200318891	0.098064442
0.8	1.397542486	1.022464	0.748051	0.547286	0.400403	0.214320082	0.114717273
0.9	1.171213948	0.887154	0.671988	0.509007	0.385555	0.221213741	0.12692214
1	1	0.778801	0.606531	0.472367	0.367879	0.22313016	0.135335283
1.5	0.544331054	0.460766	0.39003	0.330153	0.279469	0.200248204	0.143484108
2	0.353553391	0.31201	0.275348	0.242993	0.214441	0.167006796	0.130065024
2.5	0.252982213	0.228908	0.207124	0.187414	0.169579	0.138839582	0.113672236
3	0.19245009	0.177063	0.162905	0.14988	0.137897	0.11672688	0.098807171
3.5	0.15272071	0.142193	0.13239	0.123264	0.114766	0.099488235	0.086244152
4	0.125	0.117427	0.110312	0.103629	0.09735	0.08591116	0.075816332
4.5	0.10475656	0.099095	0.09374	0.088675	0.083882	0.075061355	0.067167852
5	0.089442719	0.085081	0.080931	0.076984	0.07323	0.066260796	0.059955248
6	0.068041382	0.065265	0.062601	0.060046	0.057596	0.052990681	0.04875378
7	0.053994925	0.052101	0.050273	0.048509	0.046807	0.043580262	0.04057596
8	0.044194174	0.042834	0.041517	0.040239	0.039001	0.036638257	0.034418457
9	0.037037037	0.036022	0.035036	0.034076	0.033142	0.031351175	0.029656941
10	0.031622777	0.030842	0.030081	0.029338	0.028613	0.027217976	0.02589054

Fusion welding: thermal analysis



Metallurgical Modelling of Welding (O Grong)

Arc strikes in Fusion welding

$$\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

for max^m temp or peak temp. condition

$$\frac{\partial(\theta/n_1)}{\partial \tau_1} = \tau_1^{-3/2} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)\left(+\frac{\sigma_1^2}{\tau^2}\right)$$

$$\frac{\theta}{n_1} = \frac{1}{\tau_1^{3/2}} \exp\left(\frac{-\sigma_1^2}{\tau_1}\right)$$

$$+ \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)\left(-\frac{3}{2}\right)\tau_1^{-5/2} = 0$$

$$\boxed{\sigma_{1m}^2 = \frac{3}{2}\tau_{1m}}$$

$$\boxed{\sigma_1^2 = \frac{3}{2}\tau_1}$$

Arc strikes in Fusion welding

$$\sigma_{1m}^2 = \frac{3}{2}\tau_{2m}$$

$$\frac{\Theta}{n_1} = \frac{1}{\tau_1^{3/2}} \times \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$$\frac{\Theta}{n_1} = \frac{1}{\tau_1^{3/2}} \exp\left(-\frac{3}{2}\right)$$

$$\boxed{\frac{\Theta_p}{n_1} = \frac{1}{(e\tau_1)^{3/2}}}$$

$$\boxed{\frac{\Theta_p}{n_1} = \frac{1}{\left(\frac{2e}{3}\right)^{3/2} \sigma_1^3}}$$

Arc strikes in Fusion welding

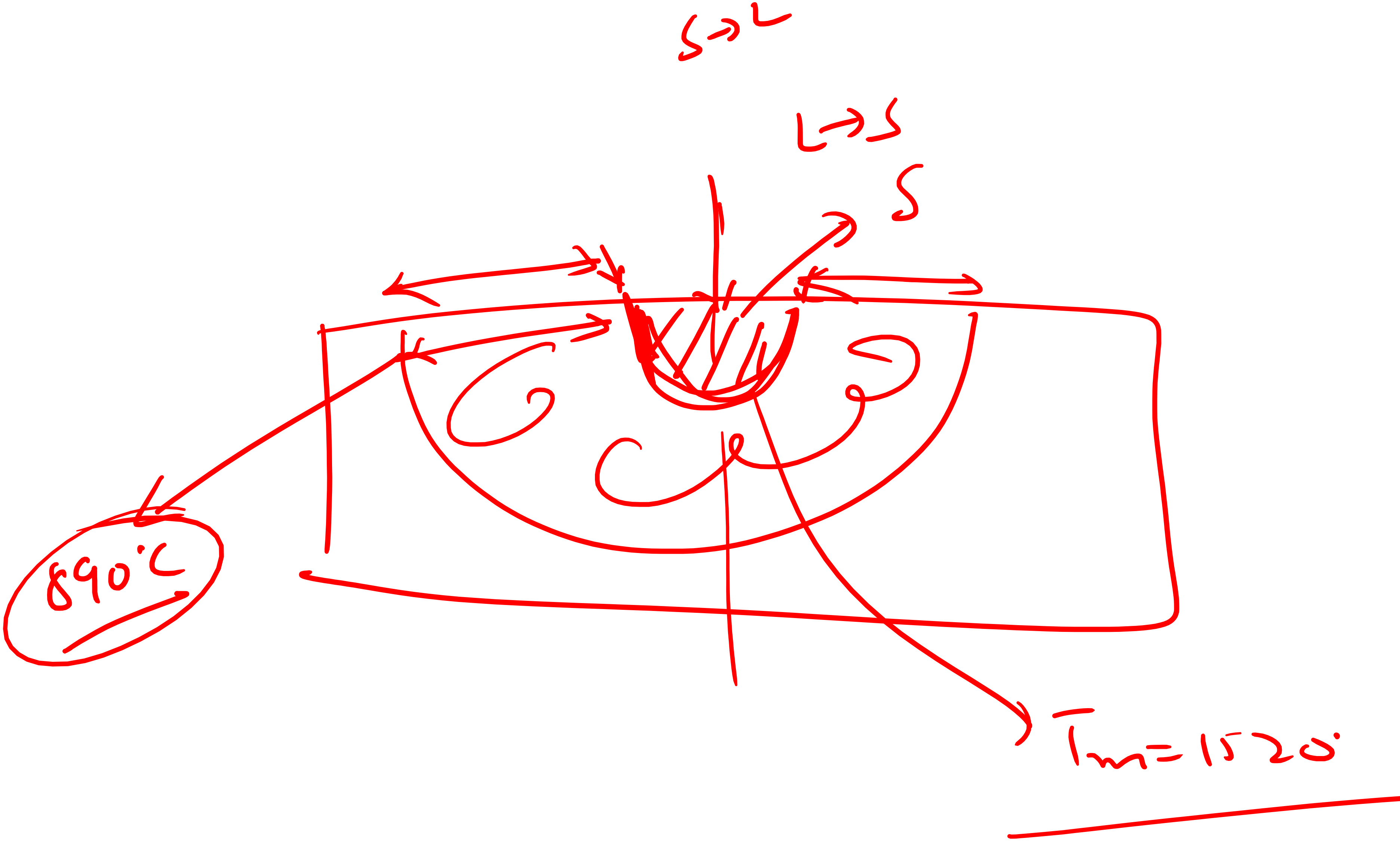
1

$$\boxed{\frac{\theta}{n_1} = \frac{1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)}$$

$$\boxed{\sigma_{1m}^2 = \frac{3}{2}\tau_{1m}}$$

$$\boxed{\frac{\theta_p}{n_1} = \frac{1}{(e\tau_1)^{3/2}} = \frac{1}{\left(\frac{2e}{3}\right)^{3/2}\sigma_1^3}}$$

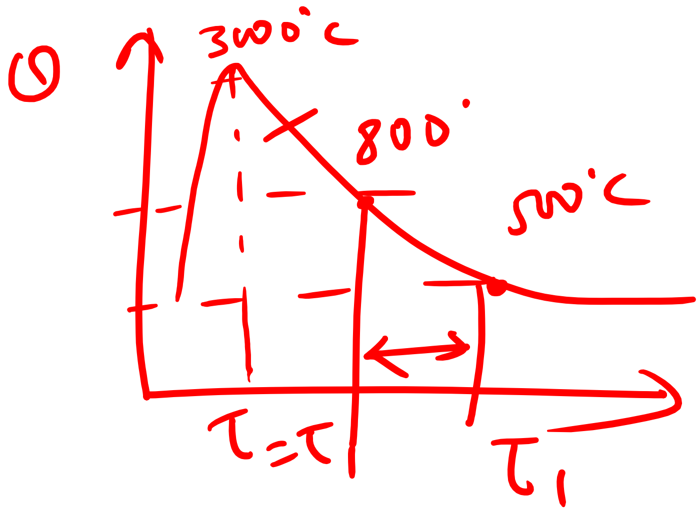
Arc strikes in Fusion welding



Arc strikes in Fusion welding

Cooling rate $= -\frac{dT}{dt}$

$\sigma_1 = 0$



$$\sigma_1^2 = \frac{r^2}{4\alpha t_i}$$

$r = 0$

Numerical: thermal analysis

Consider a small weld crater formed on a thick low alloy steel plate in arc welding.

1800°C 300°C

1. Calculate the cooling time from 800 to 500°C ($\Delta t_{800\text{-}500}$) at the center of the weld and the cooling rate (C.R.) at the onset of the austenite to ferrite transformation(475°C). 1000°C

2. Calculate the total width of the fully transformed region adjacent to the fusion boundary. Assuming that the transformation temperature is equal to 890°C for this particular steel.

The operational conditions are as follows:

I=80A, Voltage=35V, t_i=0.1 s, arc efficiency: 0.75, T_0=20°C, T_c=T_m= 1520°C, thermal diffusivity =5mm^2/s, ρC_p=0.005 J/mm^3.C, ΔH=7.5 J/mm^3

Numerical: thermal analysis

$$\theta = \frac{T - T_0}{T_c - T_0}$$

$T_0 = 20^\circ C$

$T_c = 1520^\circ C$

$$\theta = \frac{n_1}{t_1^{3/2}} \exp\left(-\frac{b_1^2}{t_1}\right)$$

$$\theta_{1520^\circ C} = \frac{1520 - 20}{1520 - 20} = 1$$

$$\theta_{890^\circ C} = \frac{890 - 20}{1500} = 0.58$$

$$\theta_{800^\circ C} = \frac{800 - 20}{1500} = 0.52$$

$$\theta_{500^\circ C} = \frac{500 - 20}{1500} = 0.32$$

$$\theta_{475^\circ C} = \frac{455}{1500} = 0.30$$

Numerical: thermal analysis

$$n_1 = \frac{q_0}{4\,\Delta H\,(\pi\alpha)^{3/2}\sqrt{t_i}}$$

$$q_0 = \eta I \times V$$

$$= \frac{0.75 \times 80 \times 35}{4 \times 7.5 \times (5 \times \pi)^{3/2}\sqrt{0.1}}$$

$$= \underline{3.56}$$

Numerical: thermal analysis

1 (9) at $r_1 = 0$

$$\frac{Q}{n_1} = \frac{1}{\tau_1^{3/2}} \cancel{\exp\left(\frac{-r_1^2}{\tau_1}\right)} \rightarrow 1$$

$$\tau_1^{3/2} = \frac{n_1}{Q}$$

$$\tau_1 = \left(\frac{n_1}{Q}\right)^{2/3}$$

$$\tau_{800^\circ C} = \left(\frac{n_1}{Q_{800}}\right)^{2/3}, \quad \tau_{500^\circ C} = \left(\frac{n_1}{Q_{500^\circ C}}\right)^{2/3}$$

$$\Delta\tau = \tau_{500} - \tau_{800}$$

Numerical: thermal analysis

$$\Delta\tau = \left(\frac{n_1}{\theta_{500}}\right)^{2/3} - \left(\frac{n_1}{\theta_{800}}\right)^{2/3}$$

$$= \left(\frac{3.56}{0.32}\right)^{2/3} - \left(\frac{3.56}{0.52}\right)^{2/3}$$

$$\Delta\tau = 1.38$$

$$\tau = \frac{t}{t_i}$$

$$\Delta\tau = \frac{\Delta t_{800-500^\circ C}}{t_i} = 1.38 \qquad \Delta\tau = \frac{\Delta t_{800-500^\circ C}}{t_i}$$

$$\Delta t_{800-500^\circ C} = 1.38 \times t_i = 0.138\ \text{sec}$$

$$=$$

Numerical: thermal analysis

1(b). Cooling rate at 475 °C

$\theta = 0.3$

$C.R. = -\frac{dT}{dt}$

$$\theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$$\frac{d\theta}{d\tau_1} = \cancel{\frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right) \times \left(\frac{\sigma_1^2}{\tau_1^2}\right)}$$

0

$$+ \exp\left(-\frac{\sigma_1^2}{\tau_1}\right) \frac{n_1}{\tau_1^{5/2}} \left(-3/2\right)$$

$\sigma_1 = 0$

Numerical: thermal analysis

$$\frac{\partial \theta}{\partial \tau_1} = \cancelto{1}{\exp\left(\frac{-\sigma_1^2}{\tau_1}\right)} \times \frac{n_1}{\tau_1^{5/2}} \left(-3/2\right)$$

$$\boxed{\frac{\partial \theta}{\partial \tau_1} = -\frac{3}{2}\,\frac{n_1}{\tau_1^{5/2}}}$$

$$\frac{\theta}{n_1} = \frac{1}{\tau_1^{3/2}}\,\cancelto{1}{\exp\left(\frac{-\sigma_1^2}{\tau_1}\right)}$$

$$\tau_1 = \left(\frac{n_1}{\theta}\right)^{2/3}$$

Numerical: thermal analysis

$$\frac{\partial \theta}{\partial \tau_1} = -\frac{3}{2}\frac{n_1}{\left(\left(\frac{n_1}{\theta}\right)^{2/3}\right)^{5/2}}$$

$$= -\frac{3}{2}\frac{\theta^{5/3}}{n_1^{2/3}}$$

$$\theta = \frac{T - T_\infty}{T_c - T_\infty}$$

$$d\theta = \frac{dT}{T_c - T_\infty}$$

$$C.R. = -\frac{dT}{dt}$$

$$\frac{d\theta}{d\tau_1} = \frac{\frac{dT}{T_c - T_\infty}}{\frac{dt}{t_i}} = \frac{t_i \, dT}{T_c - T_\infty \, dt}$$

$$\tau_1 = \frac{t}{t_i}$$

$$d\tau_1 = \frac{dt}{t_i}$$

Numerical: thermal analysis

$$\frac{t_i}{T_c - T_\infty} \frac{dT}{dt} = -\frac{3}{2} \frac{\theta^{5/3}}{n_1^{2/3}}$$

$$\frac{dT}{dt} = - \frac{T_c - T_\infty}{t_i} \frac{3}{2} \frac{\theta^{5/3}}{n_1^{2/3}}$$

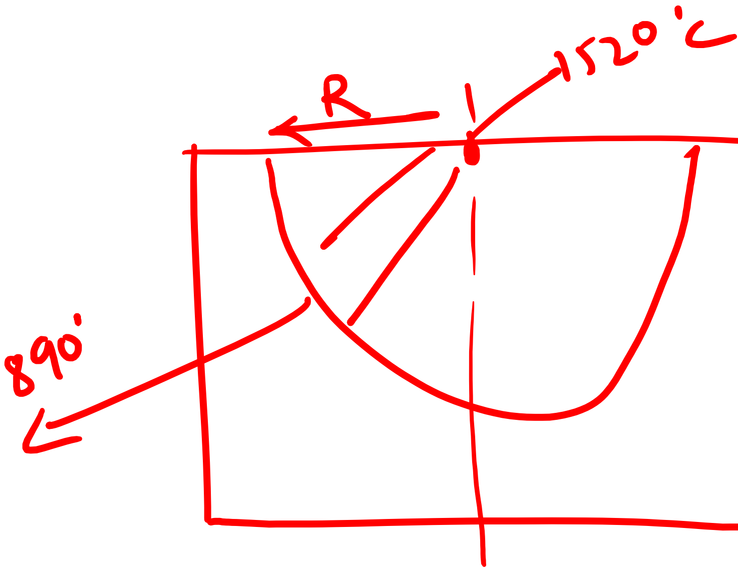
$$C.R. = -\frac{dT}{dt} = \frac{T_c - T_\infty}{t_i} \frac{3}{2} \frac{\theta^{5/3}}{n_1^{2/3}}$$

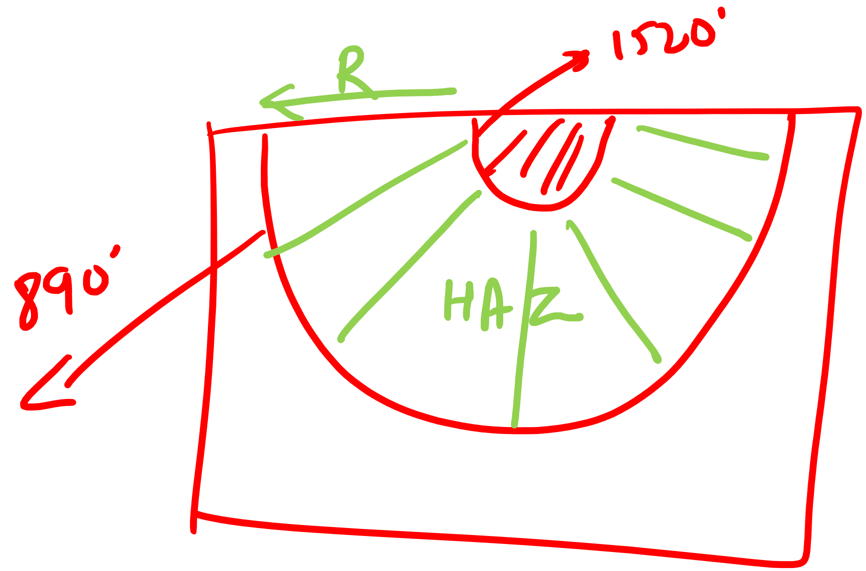
$\theta_{4\pi} = 0.3$

$$= \frac{100}{0.1} \times \frac{3}{2} \times \frac{0.3^{5/3}}{3.56^{2/3}} \ {}^\circ C/s$$

$$= 864 \ {}^\circ C/s$$

Numerical: thermal analysis





$\theta_{1520^\circ} = 1$

$\theta_{890} = 0.58$

Numerical: thermal analysis

$$\frac{\theta}{n_1} = \frac{1}{\left(\frac{2e}{3}\right)^{3/2}} \frac{1}{\sigma_1^3}$$

$$\sigma_1^3 = \frac{1}{\left(\frac{2e}{3}\right)^{3/2}} \frac{n_1}{\theta}$$

$$\sigma_1 = \frac{1}{\left(\frac{2e}{3}\right)^{1/2}} \left(\frac{n_1}{\theta}\right)^{1/3}$$

$\theta_{120} = 1$

$$\sigma_{1 \text{ at } 120^\circ} = \frac{1}{\left(\frac{2e}{3}\right)^{1/2}} \left(\frac{3.56}{1}\right)^{1/3}$$

Numerical: thermal analysis

$$\sigma_1 \text{ at } 890^\circ = \frac{1}{\left(\frac{2e}{3}\right)^{1/2}} \left(\frac{3.56}{0.58}\right)^{1/3}$$

$\theta_{890} = 0.58$

$$\Delta \sigma_1 = \frac{1}{\left(\frac{2e}{3}\right)^{1/2}} \left(\left(\frac{3.56}{0.58}\right)^{1/3} - \left(\frac{3.56}{1}\right)^{1/3} \right)$$

$$= 0.23$$

$$\sigma = \sqrt{\frac{R^2}{4\alpha t_i}}$$

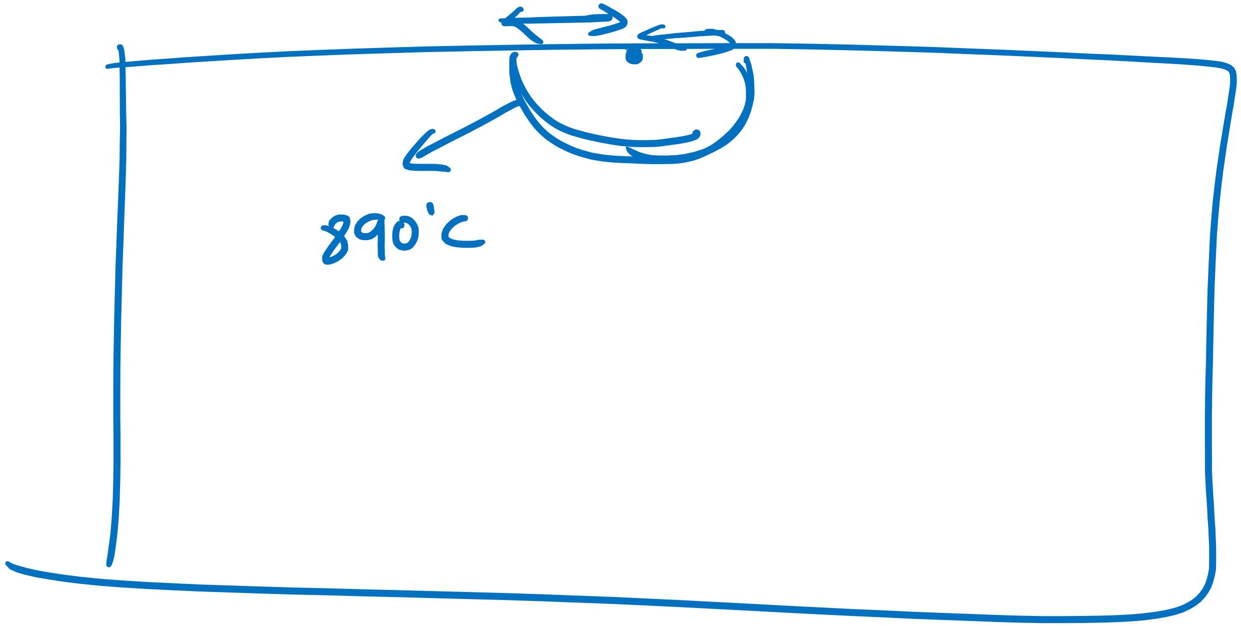
$$\Delta \sigma = \frac{\Delta R}{\sqrt{4\alpha t_i}}$$

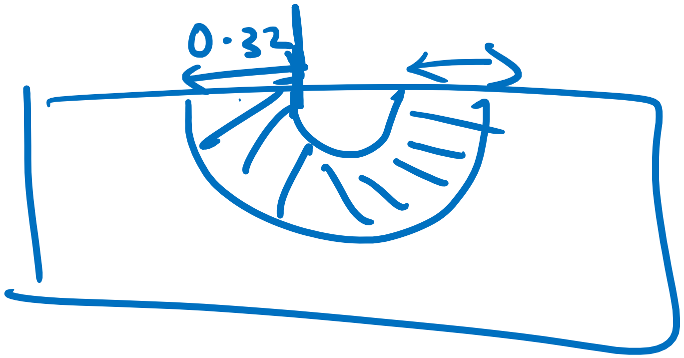
$$\Delta \sigma = \frac{\Delta R}{\sqrt{4\alpha t_i}} = 0.23$$

$$\Delta R = 0.23\sqrt{4 \times 5 \times 0.1}$$

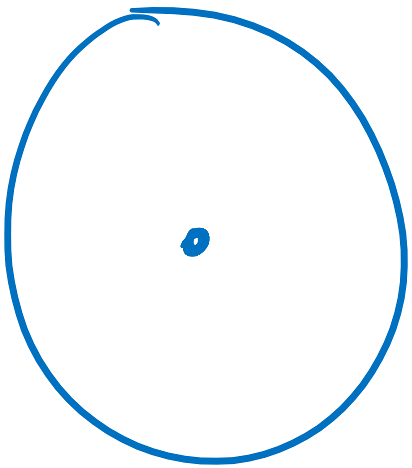
$$= 0.32 \text{ mm}$$

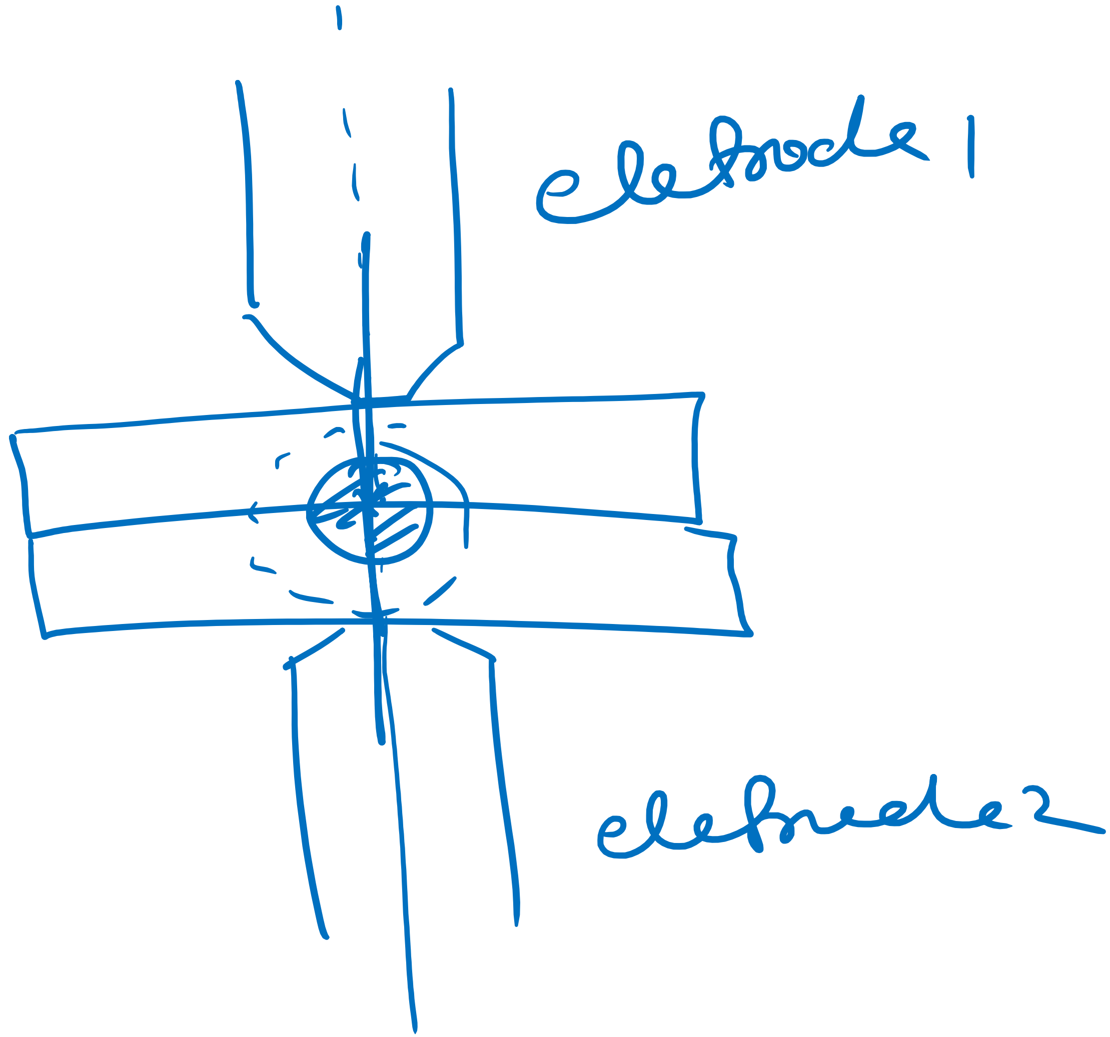
Numerical: thermal analysis



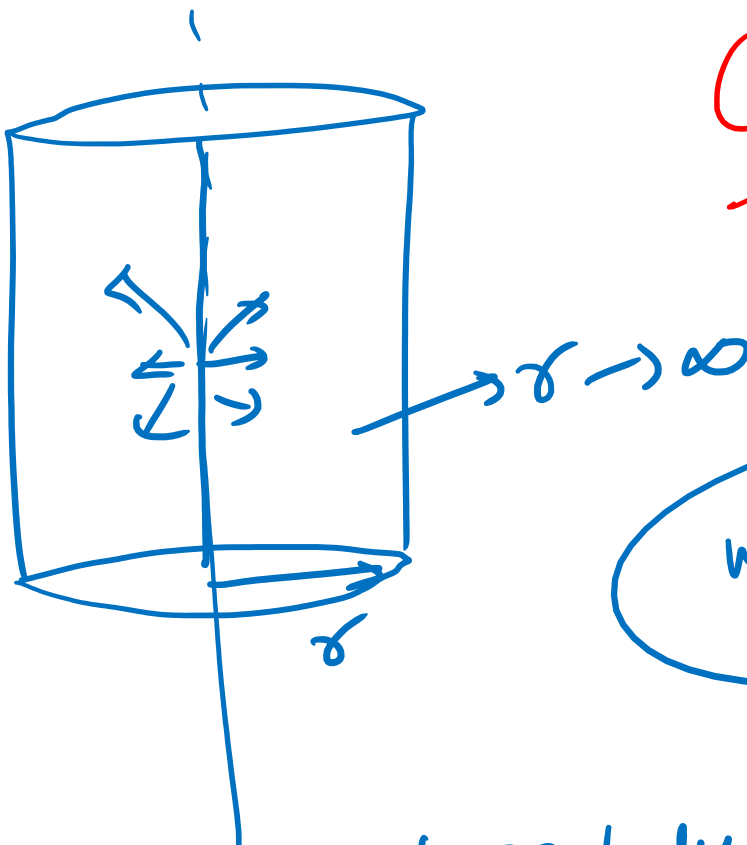


Numerical: thermal analysis





Source/Sink in Welding: Line source



$$\frac{q}{L}$$

$q = \text{watt}$

$q' = W/L$

$q'' = \frac{W}{m^2}$

$W/L = q'$

$q''' = \frac{W}{m^3}$

→ Consider a large body with a line heat source buried in that system at centre.

→ It is assumed that line source is producing heat at a constant rate per unit length. (watt/m)

Source/Sink in Welding: Line source

Body initial temp :- T_∞

for field temp = T_∞ $(r \to \infty)$

$$-kA\frac{dT}{dx} = Q$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{--- } 1$$

B.C. at $r \to 0$, $\quad -2\pi r k \frac{\partial T}{\partial r} = \frac{Q}{L} \quad$ --- 2

$r \to \infty$, $\quad T = T_\infty \quad$ --- 3

I.C, at $t = 0$, $\quad T = T_\infty$, --- 4

Source/Sink in Welding: Line source

assume $\theta = T - T_\infty$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{—} \quad 1A$$

B.C. $r \to 0$, $\quad -2\pi k r \dfrac{\partial \theta}{\partial r} = \dfrac{Q}{L} \quad$ — $2A$

$r \to \infty$, $\quad \theta = 0 \quad$ — $3A$

I.C. $t = 0$, $\quad \theta = 0 \quad$ — $4A$

Source/Sink in Welding: Line source

$$\eta = \frac{r^2}{4\alpha t} \quad \text{— (5)}$$

$$\frac{\partial \theta}{\partial r} = \frac{d\theta}{d\eta}\frac{\partial \eta}{\partial r} = \frac{d\theta}{d\eta}\frac{2r}{4\alpha t} \quad \text{— (5A)}$$

$$r\frac{\partial \theta}{\partial r} = \frac{2r^2}{4\alpha t}\frac{d\theta}{d\eta} \quad \text{— (5B)}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) = \frac{d}{d\eta}\left(\frac{d\theta}{d\eta}\frac{2r^2}{4\alpha t}\right)\frac{\partial \eta}{\partial r}$$

$$= \frac{d}{d\eta}\left(2\eta\frac{d\theta}{d\eta}\right)\frac{2r}{4\alpha t}$$

Source/Sink in Welding: Line source

$$= \left\{ 2\eta \frac{d^2\theta}{d\eta^2} + 2\frac{d\theta}{d\eta} \right\} \frac{2r}{4\alpha t} \quad \text{— 5C}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) = \left(2\eta \frac{d^2\theta}{d\eta^2} + 2\frac{d\theta}{d\eta}\right)\frac{2}{4\alpha t} \quad \text{— 5D}$$

$$\frac{\partial\theta}{\partial t} = \frac{d\theta}{d\eta}\frac{\partial\eta}{\partial t} = \frac{d\theta}{d\eta}\left(-\frac{r^2}{4\alpha t^2}\right) \quad \text{— 5E}$$

$$= \frac{d\theta}{d\eta}\left(-\frac{\eta}{t}\right) \quad \text{— 5F}$$

1A

$$\left(\cancel{2}\eta \frac{d^2\theta}{d\eta^2} + \cancel{2}\frac{d\theta}{d\eta}\right)\frac{\cancel{2}}{\cancel{4}\cancel{\alpha}\cancel{t}} = \frac{1}{\cancel{\alpha}}\left(-\frac{d\theta}{d\eta}\frac{\eta}{\cancel{t}}\right)$$

Source/Sink in Welding: Line source

$$\eta \frac{d^2\theta}{d\eta^2} + \frac{d\theta}{d\eta} = -\eta \frac{d\theta}{d\eta} \qquad \eta = \frac{r^2}{4\alpha t}$$

$$\eta \frac{d^2\theta}{d\eta^2} + (1+\eta)\frac{d\theta}{d\eta} = 0 \quad — \quad 6$$

5A

B.C. at $r \to 0$, $\eta \to 0$, $-2\pi k \frac{2r^2}{4\alpha t} \frac{d\theta}{d\eta} = \frac{Q}{L}$

$$\eta = \frac{r^2}{4\alpha t}$$

$$-2\pi k 2\eta \frac{d\theta}{d\eta} = \frac{Q}{L}$$

$$-4\pi k \eta \frac{d\theta}{d\eta} = \frac{Q}{L} \quad — \quad 6A$$

$t = 0$, $\eta \to \infty$, $r \to \infty$ $\eta \to \infty$ $\qquad \theta = 0 \quad — \quad 6B$

Constant heat production rate: Line source

$$\frac{d^2\theta}{dr^2} + \left(\frac{1+n}{r}\right)\frac{d\theta}{dr} = 0 \quad - \quad 6$$

$$\frac{d^2y}{dx^2} + f(x)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = p$$

$$\frac{d^2y}{dx^2} = \frac{dp}{dx}$$

$$\frac{dp}{dx} + f(x)\,p = 0$$

$$\frac{dp}{dx} = -f(x)\,p$$

$$\frac{dp}{p} = -f(x)\,dx$$

$$\ln p = -\int f(x)\,dx + C$$

$$\frac{dy}{dx} = p = A\,e^{-\int f(x)\,dx}$$

$$\boxed{y = A\int e^{-\int f(x)\,dx}\,dx + B}$$

Constant heat production rate: Line source

$$\Theta = A\int e^{-\int \frac{1+\eta}{\eta}\,d\eta}\,d\eta + B$$

$$\int \frac{1}{\eta} + 1$$

$$\ln\eta + \eta$$

$$\Theta = A\int e^{-(\ln\eta + \eta)}\,d\eta + B$$

$$\Theta = A\int \frac{e^{-\eta}}{\eta}\,d\eta + B$$

$$e^{\ln x} = x$$

Constant heat production rate: Line source

Sol $\left[\theta\right]_{\eta}^{\infty} = A\int_{\eta}^{\infty}\frac{e^{-\eta}}{\eta}\,d\eta$

$\theta(\infty) - \theta(\eta) = A\int_{\eta}^{\infty}\frac{e^{-\eta}}{\eta}\,d\eta$

$0 - \theta = A\int_{\eta}^{\infty}\frac{e^{-\eta}}{\eta}\,d\eta$

exponential integral fun: $-\int_{\eta}^{\infty}\frac{e^{-\lambda}}{\lambda}\,d\lambda$

$Ei(\eta) = $

$$\boxed{\theta = -A\,Ei(\eta)}$$

Property of $Ei(x)$

$Ei(x) = -Ei(-x)$

$Ei(\infty) = 0$

$Ei(0) = \infty$

Constant heat production rate: Line source

$$\theta = -A\,Ei(\eta)$$

$$\eta = \frac{r^2}{4\alpha t}$$

$\eta \to 0,$

$$-4\pi K \eta \frac{d\theta}{d\eta} = \frac{Q}{L}$$

$$\frac{d\theta}{d\eta} = \frac{Ae^{-\eta}}{\eta}$$

$$-4\pi K \cancel{\eta}\, \frac{A e^{-\eta}}{\cancel{\eta}} = \frac{Q}{L}$$

$$\boxed{A = \frac{-Q}{4\pi K L}}$$

Constant heat production rate: Line source

$$\theta = -A\,Ei(w) = \frac{Q}{4\pi K L} Ei\left(\frac{r^2}{4\alpha t}\right)$$

$T - T_\infty$

$$T = T_\infty + \frac{Q}{4\pi K L} Ei\left(\frac{r^2}{4\alpha t}\right)$$

$t \to \infty$

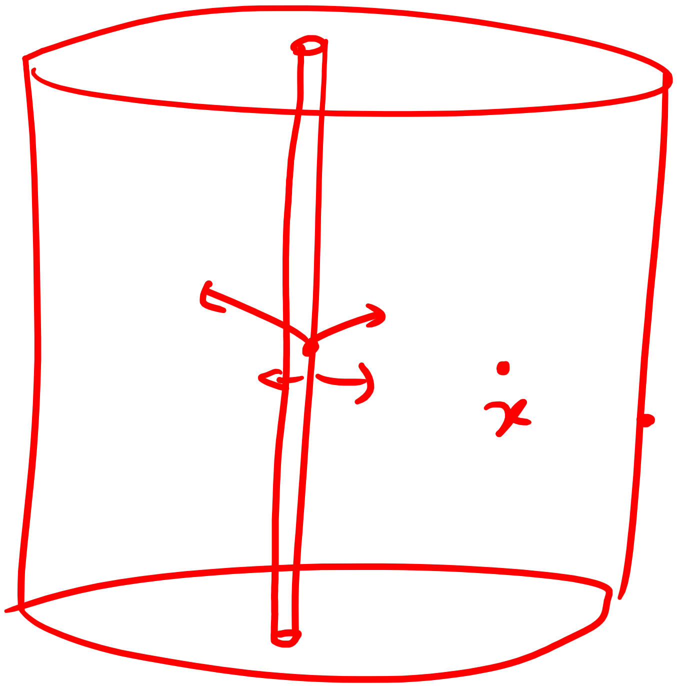
$Ei(0) \to \infty$

Constant heat production rate: Line source

Constant heat production rate: Line source

Constant heat production rate: Line source

Instantaneous heat source: Thermal Explosion at line region



condition

G.E.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Eo (1)

$r\to\infty,\ T=T_\infty$

$x+$

$$r\to 0\quad \int_0^\infty 2\pi r\, l\, \rho\, c_p (T-T_\infty)\, dr = E_0$$

$t=0,\ T=T_\infty$

Instantaneous heat source: Thermal Explosion at line region

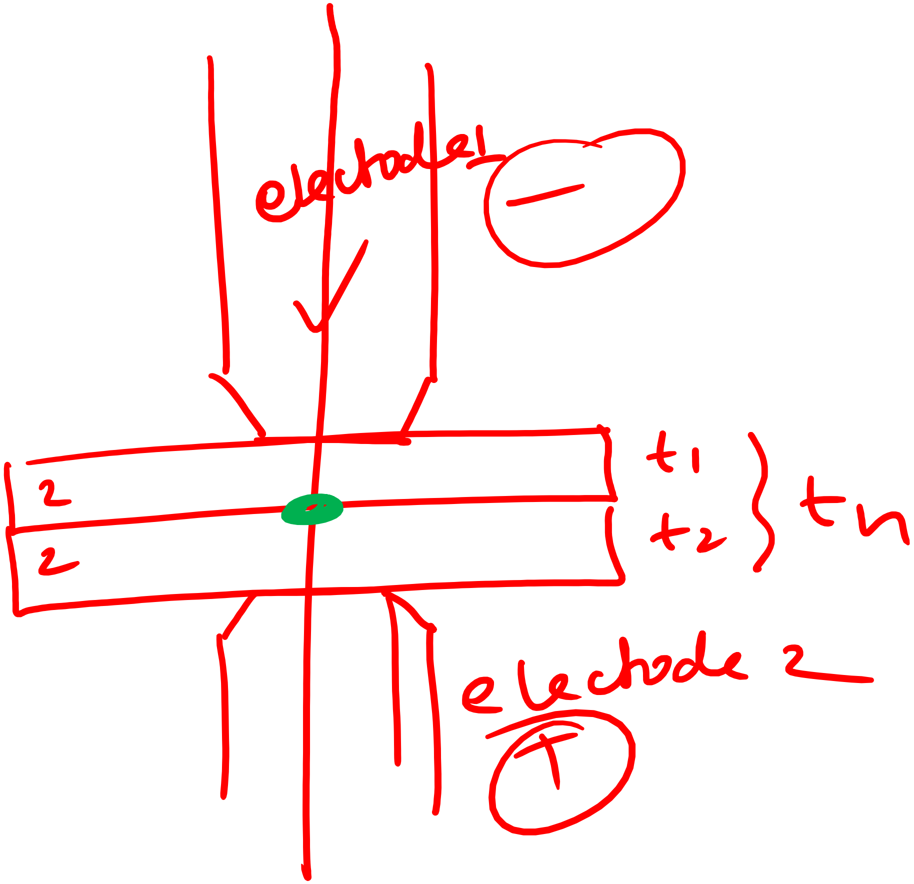
$$\boxed{T = T_\infty + \frac{E_0}{4\pi\alpha\rho C_p L t}\exp\left(-\frac{r^2}{4\alpha t}\right)}$$

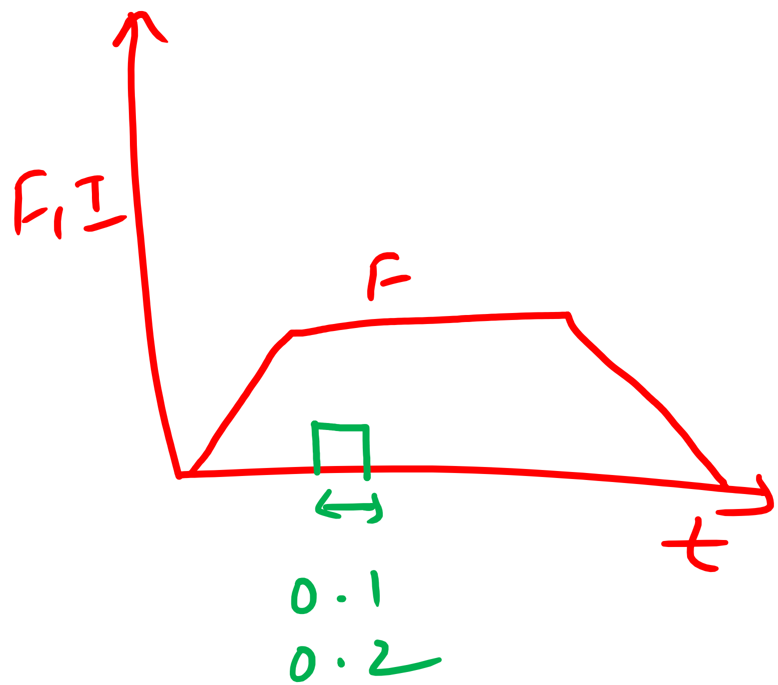
Instantaneous heat source: Thermal Explosion at line region

Instantaneous heat source: Thermal Explosion at line region

Spot welding

Resistance welding





I^2Rt

Spot welding

$$T = T_\infty + \frac{E_0}{\rho C_p (4\pi\alpha t) L} e^{-\frac{r^2}{4\alpha t}}$$

(1) Dimensionless temp :- $\theta = \dfrac{T - T_\infty}{T_c - T_\infty}$

Refe. temp $= T_m$

(II) Dimensionless time factor $= \dfrac{t}{t_h}$

Heating time
or current on time
or pulse time.

Spot welding

(iii) Dimensionless radius factor

$$\sigma_2 = \sqrt{\frac{r^2}{4\alpha t_n}}$$

ϕ $\frac{E_0}{t_n}$

$q_0 =$ watt

(iv) Dimensionless process parameter

$E_0 \rightarrow$ Joule

$$\eta_2 = \frac{E_0}{(4\pi\alpha t_n)\, L\, \underbrace{\rho c(T_c - T_\infty)}}$$

L = total thickness of plate

$\rho c(T_c - T_\infty) = H_c - H_\infty$

$= \Delta H$

$$= \frac{q_0}{\Delta H\, 4\pi\alpha L}$$

Spot welding

$$T = T_\infty + \frac{E_0}{4\pi\alpha t L \rho c_p} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

θ

$$\frac{T - T_\infty}{T_c - T_\infty} = \frac{E_0}{4\pi\alpha L \rho c_p t} \exp\left(-\frac{r^2}{4\alpha t}\right) \frac{1}{T_c - T_\infty}$$

ΔH

q_0 $\qquad$ σ_2^2

$$\theta = \frac{E_0}{4\pi\alpha L \Delta H\, t \times \frac{t_h}{t_h}} \exp\left(-\frac{r^2}{4\alpha t}\frac{t_h}{t_h}\right)$$

$\rightarrow \tau_2$ $\qquad$ $\rightarrow \tau_2$

n_2

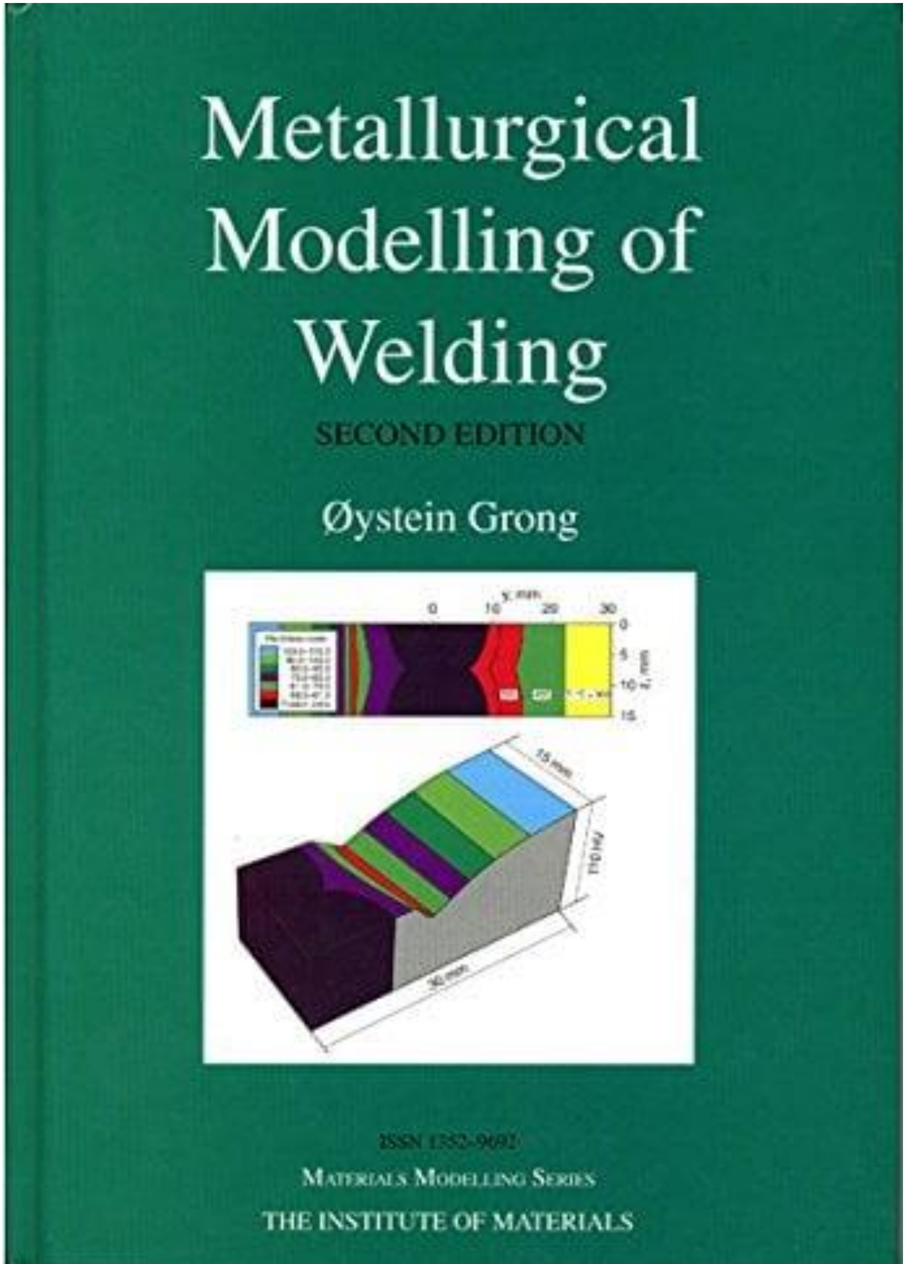
$$\theta = \frac{q_0}{4\pi\alpha L \Delta H \tau_2} \exp\left(-\frac{\sigma_2^2}{\tau_2}\right)$$

Spot welding

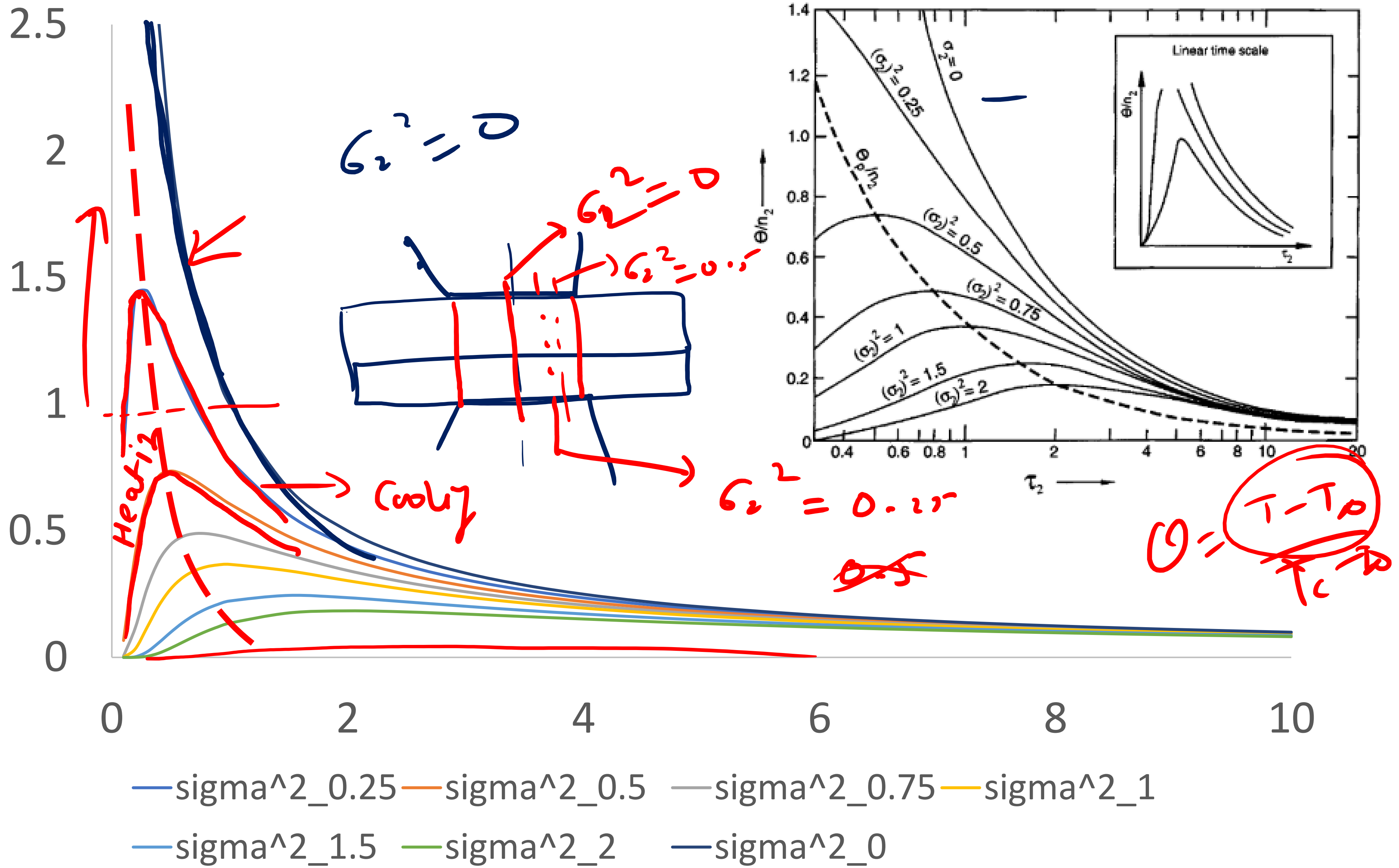
$$\boxed{\theta = \frac{n_2}{\tau_2} \exp\left(-\frac{\xi_2^2}{\tau_2}\right)}$$

$$\boxed{\frac{\theta}{n_2}, \quad \tau_2}$$ plot

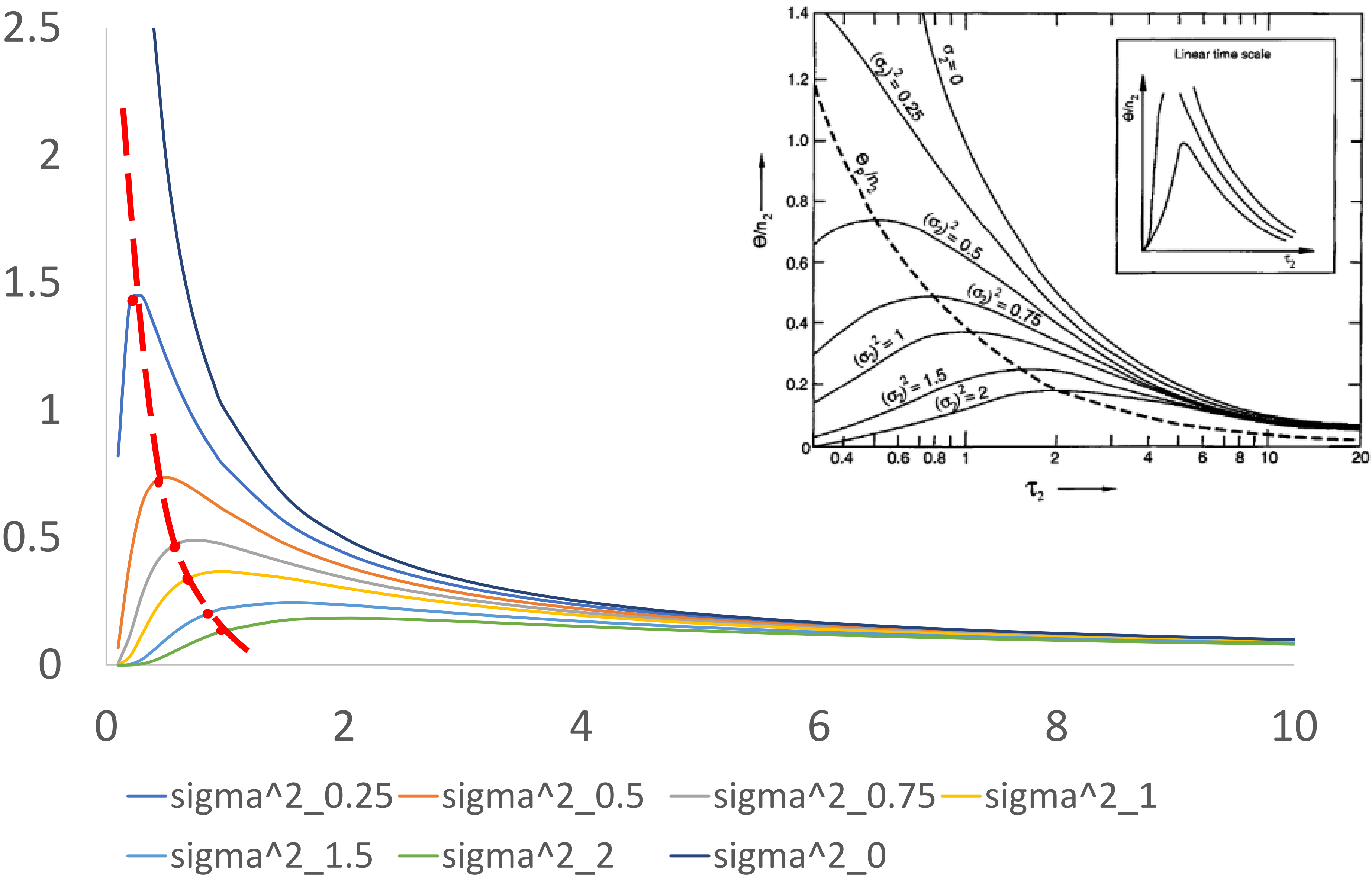
Spot welding



Spot welding



Spot welding



Spot welding

$$\theta = \frac{n_2}{\tau_2} \exp\left(-\frac{\sigma_2^2}{\tau_2}\right)$$

$$\frac{\partial\theta}{\partial\tau_2} = -\frac{n_2}{\tau_2^2} \exp\left(-\frac{\sigma_2^2}{\tau_2}\right)$$

$$+ \frac{n_2}{\tau_2} \exp\left(-\frac{\sigma_2^2}{\tau_2}\right) \frac{\sigma_2^2}{\tau_2^2} = 0$$

$$\boxed{\sigma_2^2 = \tau_2}$$

$$\cancel{\sigma_2^2 = 3\tau_2}$$

Spot welding

$$\boxed{\theta_p = \frac{n_2}{e\,\tau_2}}$$

put $\sigma_2^2 = \tau_2$

$$\boxed{\theta_p = \frac{n_2}{e\,\sigma_2^2}}$$

$$\boxed{\theta = \frac{n_2}{\tau_2} \exp\left(-\frac{\sigma_2^2}{\tau_2}\right)}$$

$\sigma_2^2 = \tau_2$

$$\theta_p = \frac{n_2}{e\,\tau_2} = \frac{n_2}{e\,\sigma_2^2}$$

$$\theta = \frac{n_2}{e\,\sigma_2^2}$$

Spot welding

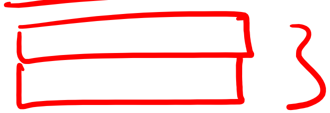
Spot welding

Spot welding

Numerical: thermal analysis: SW

Consider spot welding of 2 mm plates of low alloy steel under the following operational conditions:

I=8000A, Total voltage drop between the electrodes is =1.6 V, t_h=0.3 s, ~~Transfer efficiency:~~ 0.5, T_0=20°C, T_c=T_m= 1520°C, α=5 mm^2/s, ΔH=7.5J/mm^3



4 mm

1. Calculate the cooling time from 800 to 500°C ($\Delta t_{800\text{-}500}$) at the center of the weld and the cooling rate (C.R.) at the onset of the austenite to ferrite transformation(475°C).

2. Calculate the total width of the fully transformed region adjacent to the fusion boundary. Assuming that the transformation temperature is equal to 890°C for this particular steel.

Numerical: thermal analysis: SW

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = \frac{T - 20}{T_{500}}$$

$\theta_{1520} = 1$

$\theta_{800} = 0.52$

$\theta_{475} = 0.3$

$\theta_{890} = 0.58$

$\theta_{5W} < = 0.32$

Numerical: thermal analysis: SW

1 a) $\theta = \frac{\eta}{t} \cancel{x} \exp\left(\frac{-\delta^2}{t}\right)$

at $\delta = 0$

$\theta = \frac{\eta}{t}$

$t = \frac{\eta}{\theta}$

$t_{800} = \frac{\eta}{\theta_{800}}$, $t_{500} = \frac{\eta}{\theta_{500}}$

$\Delta t = t_{500} - t_{800} =$

Numerical: thermal analysis: SW

$$\Delta \tau = \frac{\Delta t}{t_n}$$

Numerical: thermal analysis: SW

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Numerical: thermal analysis: SW

Constant heat production rate: plane source

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$x = 0, \quad -KA\frac{dT}{dx} = Q$

$x \to \infty, \quad T = T_\infty$

$t = 0, \quad T = T_\infty$

Friction welding

$$T = T_\infty + \frac{Qx}{KA}\left[\frac{\sqrt{4\alpha t}}{x\sqrt{\pi}}\, e^{-\frac{x^2}{4\alpha t}} - \mathrm{erfc}\left(\frac{x}{\sqrt{4\alpha t}}\right)\right]$$

Constant heat production rate: plane source

Instantaneous heat source: Thermal Explosion at ~~line~~ plane region

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

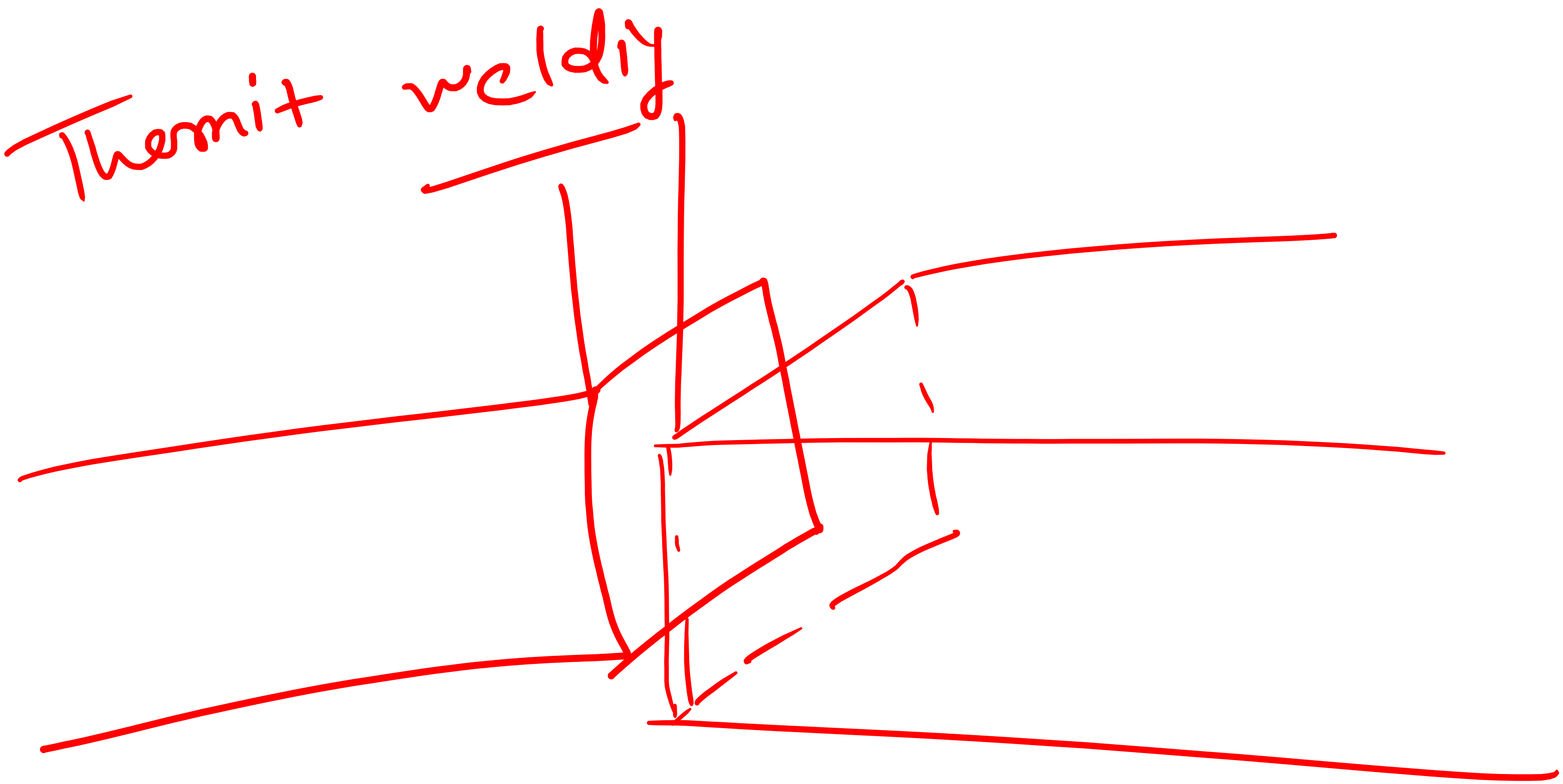
$$x \to \infty, \quad T = T_\infty$$

$$t = 0, \quad T = T_\infty$$

$$x \to 0, \quad \int_0^\infty A \rho c_p (T - T_\infty)\, dx = E_0$$

$$\boxed{T = T_\infty + \frac{E_0}{\rho c_p A \sqrt{4\pi\alpha t}} \exp\left(-\frac{x^2}{4\alpha t}\right)}$$

Instantaneous heat source: Thermal Explosion at ~~line~~ plane region



Instantaneous heat source: Thermal Explosion at line region