Rational Choice Theory

ECO 506: Behavioral Economics

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The Scope of Economics

- *Economics is "the science which studies human behavior as a relationship between ends and scarce means which have alternative uses."*— Lionel Robbins

The Scope of Economics

A more methodological definition:

- *Economics is the study of aggregate phenomena as (equilibrium) outcomes of individual choices.*

The Scope of Economics

The critical combination:

- Individual rationality
- Aggregate consistency

"Rationality"

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“Rationality”

- In economics, we want to model behavior that is “rational.” But, by itself this is a fairly vacuous claim—Why?
    - What is the substantive content of rationality?

Some Key Terms

What do the following terms mean to you?

- rationality
- utility
- preference
- choice

The Revealed Preference Revolution

- For much of the twentieth century, most mainstream economists prescribed to the following world-view:
  - The concept of choice is taken as the primitive
  - Preferences are essentially *revealed preferences*, i.e., preferences that are not directly observable but revealed by a decision maker's choice behavior
  - Rationality is about "internal consistency" of choice
  - In fact, it is this internal consistency that allows us to elicit a decision maker's (revealed) preferences from her choices

Bentham

- Jeremy Bentham brought the concept of utility to the forefront of discussion, at least in England
  - "But I have planted the tree of utility. I have planted it deep and spread it wide."
- Bentham thought of utility as a *psychophysical* concept that is *cardinally measurable*

Bentham

- In terms of economic analysis, Bentham was particularly concerned with the problem of inequality of income
- He used the assumption of utility being measurable to make interpersonal comparisons. Statements like the following pervade Bentham's writing:
    - Each portion of wealth has a corresponding portion of happiness
    - Of two individuals with unequal fortunes, he who has the most wealth has the most happiness
    - The excess in happiness of the richer will not be so great as the excess of his wealth

From Utility to Demand

- Bentham struggled right through out to provide foundations for utility as a measurable concept
- The concept of measurable utility persists in the works of Bentham's followers: Jevons, Menger and Walras
- Walras was particularly invested in the problem of deriving an individual's demand function from her utility
- Given that individuals demand multiple commodities, to express better the connection between consumption and utility, *utility functions* began to be used. At first, they took a simple additive form: eg., if x_1, x_2 and x_3 are the individual's consumption of three commodities 1, 2 and 3 respectively, then her utility was written (explicitly by Jevons and Walras, implicitly by Menger) as $f(x_1) + g(x_2) + h(x_3)$.

From Utility to Demand

- But Edgeworth questioned the simplicity of an additive form for utility and began to specify a general functional form, eg., $u(x_1, x_2) = \ln x_1 x_2$, for the utility function. In so doing, he laid the seeds of the eventual demise of the concept of measurable utility
- To illustrate graphically the relationship between consumption and utility, Edgeworth devised the concept of indifference curves

Measurable Utility

- It was Pareto who first cast serious doubts on utility as a measurable concept
- Pareto's position can be summarized thus:
  1. One need not assume anything about measurable utility to derive indifference curves, which can be elicited from market data or choice experiments
  2. There are an infinite number of utility functions consistent with a given set of indifference curves of an individual, i.e., the information contained in these indifference mappings is of an ordinal rather than cardinal nature
  3. The information about indifference curves is all that is needed to derive an individual's demand function

Modern Choice Theory

- By the 1930's measurable utility was a discredited concept in mainstream economics
- Positivistically minded economists thought that the difficulties of measuring a fundamental economic quantity such as utility could potentially discredit the whole of economics
- The goal was to build a choice theory that was founded exclusively on empirical grounds

Modern Choice Theory—Ordinalism

- John Hicks is usually credited with spreading ordinalism amongst economists
- The ordinalist position can be identified with two features:
  1. The substitution of old psychophysical utility with an index of preferences
  2. The proposition that, for the sake of consumer theory, we do not need to assume that the utility index is cardinal

Modern Choice Theory—Revealed Preference Theory

- Paul Samuelson introduced revealed preference theory in economics
- His work was motivated by his belief that ordinalism was not an adequate critique of measurable utility. Revealed preference theory was formulated to discard off "the last vestiges of utility analysis."
- Whereas ordinalism takes the concept of preferences as primitive, revealed preference theory views choices as primitives from which all other concepts like preferences and utility have to be *derived*

Interpersonal Comparisons of Utility

- With the advent of modern choice theory, the project of finding foundations for making interpersonal comparisons which had so occupied political economists of an earlier generation, lost its intellectual appeal, at least amongst economists
- Most influential voice in this regard is that of Lionel Robbins
    - His argument was not that such comparisons could not be made, but that since there is no way of objectively testing them we cannot rely on them to derive scientific statements in economics

Modeling Choice

- Modeling Goal: To provide a parsimonious way of capturing a decision maker's (DM's) choice behavior in any given environment of interest

Modeling Choice

- X: A finite set of alternatives
  - e.g. $X = \{Apple(a), Orange(o), Mango(m)\}$
- $\mathcal{X}$: Set of all non-empty subsets of X
  - If $X = \{Apple(a), Orange(o), Mango(m)\}$, what is $\mathcal{X}$?

Modelling Choice

We want a convenient way of answering questions like the following:

- Suppose a DM is offered a non-empty subset A of X and told she can choose one alternative from that set. Which are the alternatives she will be happy to pick?

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  - Denote the answer to this question by the set $c(A) \subseteq A$

<u>*Example*</u>: Suppose $X = \{Apple(a), Orange(o), Mango(m)\}$, then the following are examples of such answers:

- $c(\{a\}) = \{a\}, c(\{o\}) = \{o\}, c(\{m\}) = \{m\}, c(\{a, o\}) = \{o\},$ $c(\{a, m\}) = \{m\}, c(\{o, m\}) = \{o, m\}, c(\{a, o, m\}) = \{o, m\}$
- $c'(\{a\}) = \{a\}, c'(\{o\}) = \{o\}, c'(\{m\}) = \{m\}, c'(\{a, o\}) = \{o\},$ $c'(\{a, m\}) = \{m\}, c'(\{o, m\}) = \{o, m\}, c'(\{a, o, m\}) = \{a\}$

Choice Correspondence

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Comment: Implicit Assumptions

- Unrestricted Domain
- Choice from any set is always non-empty

Choice Function

DEFINITION

A choice function on a set X is a function $c : \mathcal{X} \to X$ such that for all $A \in \mathcal{X}$, $c(A) \in A$

Interpretation: For any set A, the DM wants to choose $c(A)$ and nothing else

Example: Suppose $X = \{Apple(a), Orange(o), Mango(m)\}$, then the following is an example of a choice function

- $c(\{a\}) = a, c(\{o\}) = o, c(\{m\}) = m, c(\{a, o\}) = o,$ $c(\{a, m\}) = m, c(\{o, m\}) = m, c(\{a, o, m\}) = m$

Modeling Preferences

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Modeling Approach: That a description of preferences should fully specify DM's attitude towards each pair of alternatives

Modeling Preferences: Example

Suppose the set of relevant alternatives is the set of all professional football players. Consider two players from this set, Messi (M) and Ronaldo (R). The kind of question that the DM is expected to be able to answer under this approach is of the following type.

- Tick the statement or statements that apply to you:
  - ☐ I like Messi at least as much as Ronaldo (Statement $M \succcurlyeq R$)
  - ☐ I like Ronaldo at least as much as Messi (Statement $R \succcurlyeq M$)

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  - ☐ I like Ronaldo at least as much as Messi (Statement $R \succcurlyeq M$)

Question: What type of responses on the part of the DM are excluded under this approach?

Weak Preference Relation

- Preferences in economics are specified in terms of binary relations. A binary relation tells us whether any two elements in a set are related to one another by the relation specified under it
- If we define $\succcurlyeq$ to be the relation "is at least as good as", then we can pick any two elements from the set X and ask a DM: how are these elements related by this binary relation?

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- If we define $\succcurlyeq$ to be the relation "is at least as good as", then we can pick any two elements from the set X and ask a DM: how are these elements related by this binary relation?
- *Example*: Suppose $X = \{Apple(a), Orange(o), Mango(m)\}$. Then the following are examples of such answers
  - $a \succcurlyeq a, o \succcurlyeq o, m \succcurlyeq m, o \succcurlyeq a, m \succcurlyeq a, o \succcurlyeq m, m \succcurlyeq o$
  - $a \succcurlyeq' a, o \succcurlyeq' o, m \succcurlyeq' m, o \succcurlyeq' a, m \succcurlyeq' o, a \succcurlyeq' m$
- We refer to the binary relation $\succcurlyeq$ thus defined as the weak preference relation

Weak Preference Relation

- Consider as answer like $m \succcurlyeq a$. Since the order that m and a are placed in with respect to $\succcurlyeq$ matters, we can express the statement as an ordered pair (m, a); and to make it explicit that this order is in relation to $\succcurlyeq$, we can write $(m, a) \in \succcurlyeq$

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- This means that we can write the set of statements $a \succcurlyeq a$, $o \succcurlyeq o$, $m \succcurlyeq m$, $o \succcurlyeq a$, $m \succcurlyeq a$, $o \succcurlyeq m$, $m \succcurlyeq o$ also as:

$$\succcurlyeq = \{(a, a), (o, o), (m, m), (o, a), (m, a), (o, m), (m, o)\}$$

Weak Preference Relation

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$$\succcurlyeq = \{(a, a), (o, o), (m, m), (o, a), (m, a), (o, m), (m, o)\}$$

- That is $\succcurlyeq \subseteq X \times X$, which is precisely what a binary relation is mathematically

Strict Preference Relation

- Suppose the DM told you: "m is at least as good as a, but a is not at least as good as m," i.e., $m \succcurlyeq a$ but $\neg[a \succcurlyeq m]$. You would probably agree that this means that, for her, m is *strictly* preferred to a
- We define the *strict preference relation*, denote it by $\succ$, from the weak preference relation $\succcurlyeq$ by:

$$x \succ y \text{ is } x \succcurlyeq y \text{ and } \neg[y \succcurlyeq x]$$

Strict Preference Relation

- If $\succcurlyeq = \{(a,a),(o,o),(m,m),(o,a),(m,a),(o,m),(m,o)\}$, then what is the strict preference relation $\succ$ defined from it?
- If $\succcurlyeq' = \{(a,a),(o,o),(m,m),(o,a),(m,o),(a,m)\}$, then what is the strict preference relation $\succ'$ defined from it?
- Note: $\succ \subseteq X \times X$ and $\succ' \subseteq X \times X$, i.e., they are binary relations

Indifference Relation

- Suppose the DM told you: "m is at least as good as o and o is at least as good as m," i.e., $m \succcurlyeq o$ and $o \succcurlyeq m$. You would agree that this means that she is indifferent between m and o
- We define the *indifference relation*, denote it by $\sim$, from the weak preference relation $\succcurlyeq$ by:

$$x \sim y \text{ is } x \succcurlyeq y \text{ and } y \succcurlyeq x$$

Indifference Relation

- If $\succcurlyeq = \{(a,a),(o,o),(m,m),(o,a),(m,a),(o,m),(m,o)\}$, then what is the indifference relation $\sim$ defined from it?
- If $\succcurlyeq' = \{(a,a),(o,o),(m,m),(o,a),(m,o),(a,m)\}$, then what is the indifference relation $\sim'$ defined from it?
- Note: $\sim \subseteq X \times X$ and $\sim' \subseteq X \times X$, i.e., they are binary relations

Modeling Preference

- What is $\sim \cup \succ$?
- What is $\sim \cap \succ$?

Completeness of Preferences

DEFINITION

A preference relation $\succsim$ on X is complete if for any $x, y \in X$, either $x \succsim y$ or $y \succsim x$.

What do you think about this property?

Transitivity of Preferences

DEFINITION

A preference relation $\succsim$ on X is transitive if for any $x, y, z \in X$, $x \succsim y$ and $y \succsim z$ implies $x \succsim z$

What do you think about this property?

Transitivity of Preferences

PROPOSITION

If the weak preference relation $\succcurlyeq$ *is transitive, then so is the strict preference relation* $\succ$ *and the indifference relation* $\sim$ *that are defined from* $\succcurlyeq$.

Discussion on Transitivity

- Descriptive Violations
- Money Pumps and a Normative Defense

Revealed Preferences

- In the Samuelsonian world view, choice is taken as the primitive concept as it is observable. From a DM's choices we elicit what her preferences are. But, what is the guarantee that this can be done?
- As it turns out, if the DM's choices are "internally consistent," then we can indeed elicit her preferences from her choices.
- . . . and this is what we mean by *internal consistency of choices*: If the DM is happy to choose an alternative x in some set where another alternative y is also available, then in any other set in which x is available, if she is happy to choose y, she must be happy to choose x as well

Revealed Preferences

AXIOM (Weak Axiom of Revealed Preference (WARP))

Let $c : \mathcal{X} \to \mathcal{X}$ be a choice correspondence on the set X. Then c satisfies WARP if for any A, $B \in \mathcal{X}$ and $x, y \in A \cap B$,

$$[x \in c(A) \text{ and } y \in c(B)] \Longrightarrow x \in c(B).$$

Do the following choice correspondences defined on the set $\{a, o, m\}$ satisfy WARP?

- $c(\{a\}) = \{a\}, c(\{o\}) = \{o\}, c(\{m\}) = \{m\}, c(\{a, o\}) = \{o\},$
  $c(\{a, m\}) = \{m\}, c(\{o, m\}) = \{o, m\}, c(\{a, o, m\}) = \{o, m\}$
- $c'(\{a\}) = \{a\}, c'(\{o\}) = \{o\}, c'(\{m\}) = \{m\}, c'(\{a, o\}) = \{o\},$
  $c'(\{a, m\}) = \{m\}, c'(\{o, m\}) = \{o, m\}, c'(\{a, o, m\}) = \{a\}$

Revealed Preferences

THEOREM

Let $c : \mathcal{X} \to \mathcal{X}$ be a choice correspondence (on the set X) that satisfies WARP. Then there exists a complete and transitive preference relation $\succcurlyeq$ on X such that for any set $A \in \mathcal{X}$,

$$c(A) = \{x \in A : x \succcurlyeq y \text{ for all } y \in A\}$$

Interpretation: The preference relation $\succcurlyeq$ *rationalizes* the choice. correspondence c. We refer to it as the *revealed preference relation.*

PROOF: Define the binary relation $\succcurlyeq_c$ on X by: $x \succcurlyeq_c y$ if $x \in c(\{x, y\})$.

Is $\succcurlyeq_c$ a complete and transitive preference relation?

- Is $\succcurlyeq_c$ complete?

  Yes, since for any x, $y \in X$, either x or y (or both) are in $c(\{x, y\})$

- Is $\succcurlyeq_c$ transitive?

  Yes. To see this, let $x \succcurlyeq_c y$ and $y \succcurlyeq_c z$, i.e., $x \in c(\{x, y\})$ and $y \in c(\{y, z\})$. We need to show that $x \in c(\{x, z\})$. To do so, first note that either (i) $y \in c(\{x, y, z\})$ or (ii) $y \notin c(\{x, y, z\})$. If case (i) holds, then by WARP, $x \in c(\{x, y, z\})$. Now, if $z \in c(\{x, z\})$, then by WARP, $x \in c(\{x, z\})$. Further, if $z \notin c(\{x, z\})$, then by definition of choice correspondence, $x \in c(\{x, z\})$. On the other hand, if case (ii) holds, then by WARP $z \notin c(\{x, y, z\})$, which implies $x \in c(\{x, y, z\})$. Then, by a similar argument as above, $x \in c(\{x, z\})$.

PROOF: Define $\succcurlyeq_c$ on X by: $x \succcurlyeq_c y$ if $x \in c(\{x, y\})$.

Is $c(A) = \{x \in A : x \succcurlyeq_c y \text{ for all } y \in A\} := A^*$?

- Is $c(A) \subseteq A^*$?
  Let $x \in c(A)$. Pick any $y \in A$. By WARP, $x \in c(\{x, y\})$. So, $x \succcurlyeq_c y$ for all $y \in A$, i.e., $x \in A^*$.

- Is $A^* \subseteq c(A)$?
  Let $x \in A^*$. We know that there exists $y' \in c(A)$. If $y' = x$, we are done. If $y' \neq x$, then, since $x \in c(\{x, y'\})$, by WARP it follows that $x \in c(A)$. $\square$

Revealed Preferences

Definition (WARP for Choice Functions)

A choice function $c : \mathcal{X} \to X$ satisfies WARP if dor any $A, B \in \mathcal{X}$,

$$[x = c(A), y \in A, x \in B] \implies y \neq c(B)$$

Theorem

If a choice function $c : \mathcal{X} \to X$ satisfies WARP then there exists a complete, transitive and asymmetric binary relation $\succ$ on X (i.e., a strict preference relation on X) such that for any $A \in \mathcal{X}$,

$$c(A) = \{x \in A : x \succ y, \text{ for all } y \in A, y \neq x\}$$

Utility Representation

DEFINITION

*Let $\succcurlyeq$ be a preference relation on X. An **ordinal utility representation** of $\succcurlyeq$ is a function $U\colon X \to \mathbb{R}$ such that for all x, $y \in X$, $x \succcurlyeq y$ if and only if $U(x) \geq U(y)$.*

Utility Representation

THEOREM

If X is a finite set, then the preference relation $\succcurlyeq$ on X has an ordinal utility representation if and only if it is complete and transitive.

Comment: If u is a utility representation of $\succcurlyeq$, then so is any increasing transformation of it

Utility Representation: A Caveat

The theorem above assumes finiteness and this is important.

Example: Lexicographic Preferences

Consider a consumer whose preferences over bundles of two goods, 1 and 2, are given by the following:

$$(x_1, x_2) \succcurlyeq (y_1, y_2) \text{ if either (i) } x_1 > y_1 \text{ or (ii) } x_1 = y_1 \text{ and } x_2 \geq y_2$$

Question: Plot the family of indifference curves for these preferences

On Rationality in Neo-classical Economics

If WARP holds, then for any $A \subseteq X$,

$$\begin{aligned} c(A) &= \{x \in A : x \succcurlyeq y, \text{ for all } y \in A\} \\ &= \{x \in A : u(x) \geq u(y), \text{ for all } y \in A\} \end{aligned}$$

On Rationality in Neo-classical Economics

A criticism often heard from social scientists outside economics is that the utility maximization paradigm in economics shows just how far disconnected mathematical models in economics are from reality—after all, the critics would ask, do you know of anyone in the real world who makes decisions by optimizing a utility function!

Based on our analysis here, how would you respond to this criticism?

Interpretation

The “as if” approach