

09/11/24

* $Q_{\text{convection}} = hA(T_w - T_s)$

(Rate of heat convection)

(Newton's law of cooling)

* $Q_{\text{emit}} = \epsilon A T_w^4$

(Rate of heat radiation)

(Stefan Boltzmann law),

$$\epsilon = 5.67 \times 10^{-8}$$

T = Kelvin scale.

* $Q_{\text{cond}} = -kA \frac{dT}{dx}$.

* Solid converted to liquid, while welding.

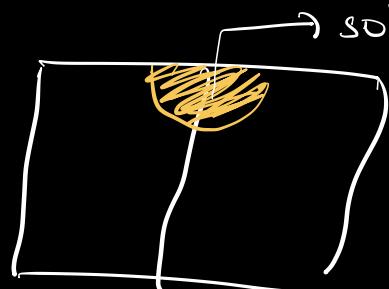
* $P = \frac{I U}{V} \rightarrow \text{noth}$
↳ current.

Heat input = $\frac{P}{V} = \frac{I U}{V} \rightarrow \text{velocity.}$

* Transfer efficiency

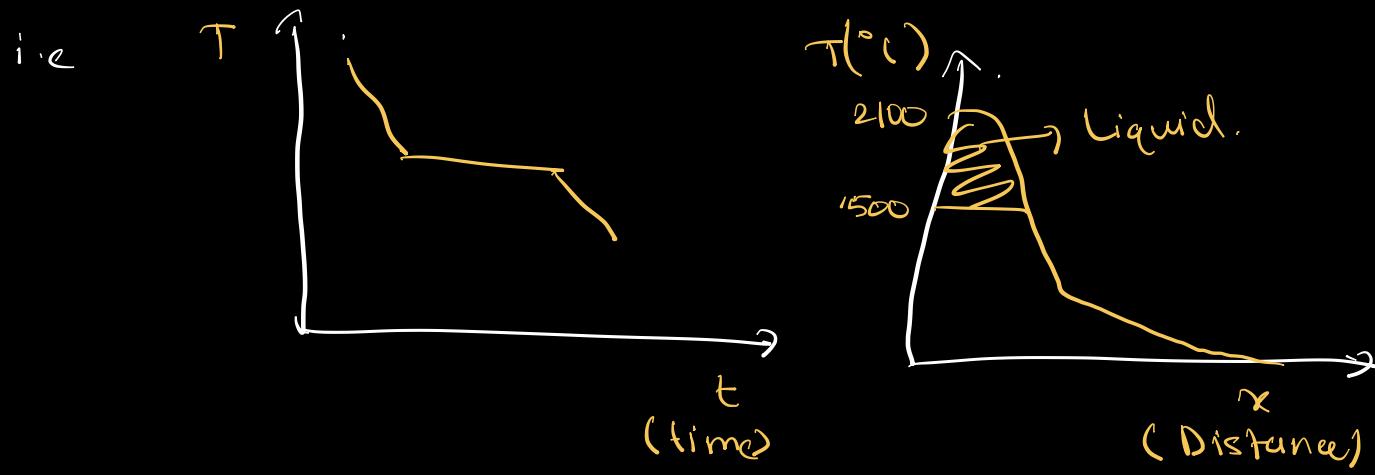
$$\eta = \frac{H_{\text{net}}}{H_{\text{input}}} = \frac{H_{\text{net}}}{(I U / V)}, \quad 0 < \eta < 1$$

* Submerged arc welding \rightarrow high efficiency
(90 to 98%)



solid converted
to liquid
(left alone) Then converts back
to solid (cooling).

$x=0$ (max temp) $\rightarrow 2100^\circ\text{C}$ for certain weld. process.



- * Without phase change, solid converts to gas or similar i.e. recrystallization temp. just change in microstructured, without phase change in base plate, (material property). → Heat affected zone.

- * $\sigma = \frac{1}{\sqrt{D}}$, D is grain diameter.
- * For spring, strength and resilience are needed.
- * Melting efficiency in fusion welding
 - To melt material (how much energy is needed). (H_{net})
 - $A_w = A_m + A_r$
 A_r = additional area.
 A_w = welding cross sectional area.

* $Q_{\text{required}} = \rho_m V$

$$= \rho_m (L + c_m(T_p - T_f) + c_s(T_p - T_0))$$

T

 T_p T_m T_l T_0 t

T_p — T_m T_l T_0 t

① sensible heat. ② latent heat. ③ liquid (sensible heat)

T_R T_R room temp.

L = Latent heat.

* Melting efficiency : $f = Q_{req} \times A_w H_{net}$

$$H_{net} = \eta \times H_{input}$$

$$= \eta \times \overline{IV}$$

$$\therefore f = Q_{req} \times A_w \times \eta \times \frac{\overline{IV}}{\sqrt{V}}$$

$$A_w = A_m + A_g$$

Solid state welding process

* Doing welding with use of pressure, load, deformation.

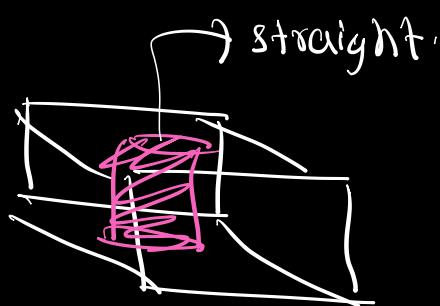
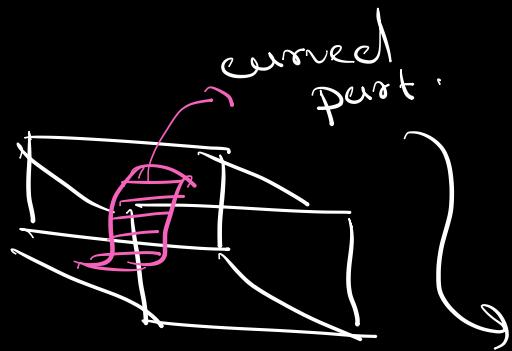
Not typical

~~S-S, L-S~~

* Material continuity using plastic deformation.

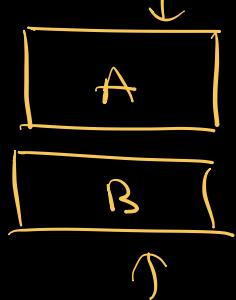
- Diffusion welding
- Friction welding
- Pressure welding.

Diffusion welding



For increased surface area, for better cooling.

- * Curved hollow parts can be made using Diffusion melting.
- * Applying pressure at elevated temperature. $(0.5 - 0.7 T_m) \rightarrow (\text{k})$.



Ultrasonic friction welding:

- * tool is less, frequency is high. (30 kHz).

conventional friction welding:

- * Amplitude of vibration is relatively large
 - Rotational
 - Angular reciprocating
 - Linear reciprocating.
- } Types of friction welding.



- * friction stir welding: $(0.8 T_m) \rightarrow \text{max Temp.}$

$$\boxed{\text{I}} < 5\text{ mm}$$

* Amount of heat generated.

$$Q_t = Q_{\text{tfr}} + Q_{\text{trot}}, \quad Q_{\text{tfr}} \leftarrow \text{Translational}$$

(Ignore)

$$Q_{\text{trot}} = \text{rotational}.$$

As $Q_{\text{tfr}} \ll Q_{\text{trot}}$

for flat surfaces (without any tapered surface)



$$Q_t = Q_{\text{trot}}$$

$$\therefore Q_t = Q_{\text{pt}} + Q_{\text{ps}} + Q_{\text{st}}$$

\downarrow \downarrow \downarrow
 probe probe solder
 tip surface tip

* $Q_t \propto \text{Torque}, \omega$

$$Q_t = \underbrace{\omega \times M_t}_{(\text{watt})} \text{ /min} = \frac{\text{N} \cdot \text{m}}{\text{sec}} = \text{watt}.$$

$$dQ_t = \omega \times dM_t$$

$$= \omega \times \sigma dF_t = \omega \times \tau \times T_{\text{contact}} \times dA.$$

$$\therefore dQ_t = \omega \times \tau \times T_{\text{contact}} \times dA.$$

dQ_t : infinitesimal amount of heat generation.

dF_t : infinitesimal amount of heat force

↳ flow stress of material or shear stress.

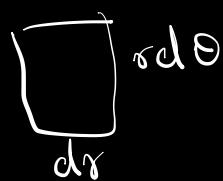
τ = distance from intin... part.

dA = Area of infinitesimal part.

T_{contact} = flow stress (or) shear const.,
stress

12/11/24

Amount of heat generated during FSW
(Q_t)



$$dA = r dr d\theta$$

$$dF = T_{\text{contact}} \cdot dA$$

$$= \tau_{\text{cont.}} \cdot r dr d\theta$$

$$dM_t = \tau \cdot dF$$

$$= r^2 dr d\theta \cdot \tau_{\text{cont.}}$$

$$dQ_{pt} = \omega \cdot dM_t$$

$$= r^2 \omega \cdot dr d\theta \tau_{\text{cont.}}$$

→ To find Q_{pt} .

$$\int dQ_{pt} = \int_0^{R_p} \int_0^{2\pi} \omega \cdot \tau_{\text{cont.}} r^2 dr d\theta$$

$$= \int_0^{2\pi} \omega \cdot \tau_{\text{cont.}} \cdot \frac{R_p^3}{3} \rho_r d\theta$$

$$Q_{pt} = \frac{2\pi}{3} \omega \tau_{\text{cont.}} R_p^3 \rho_r$$



$$\sigma = R_{px}$$

$$dA = \sigma d\theta dz$$

$$dF = \tau_{cont} \cdot dA$$

$$= \tau_{cont} \cdot \sigma d\theta dz$$

$$dM_t = \rho dF$$

$$= \sigma^2 \tau_{cont} d\theta dz$$

$$dQ_{ps} = \omega dM_t$$

$$= \omega \sigma^2 \tau_{cont} d\theta dz$$

$$Q_{ps} = \iint dQ_{ps}$$

$$= \iint_0^h \omega \sigma^2 d\theta dz \tau_{cont}, \quad \sigma = R_{px}$$

$$Q_{ps} = \omega R_{px}^2 \times 2\pi h \tau_{cont}$$

→ For shoulder tip:

$$dA = \sigma dr d\phi$$

$$dF = \tau_{cont} \cdot r dr d\phi$$

$$dM_t = \sigma^2 \tau_{cont} \cdot dr d\phi$$

$$dQ_{st} = \omega dM_t$$

$$= \omega \sigma^2 \tau_{cont} dr d\phi$$

$$Q_{st} = \iint_{R_{px}}^{R_{sh}} \omega \sigma^2 \tau_{cont} dr d\phi$$

$$= \frac{2\pi}{3} \omega (R_{sh}^3 - R_{fr}^3) \cdot T_{cont}$$

$$Q_f = Q_{fr} + Q_{ps} + Q_{sf}$$

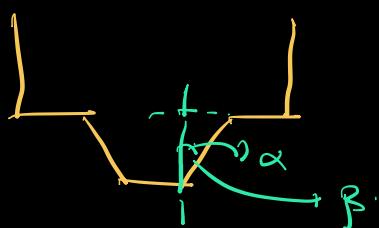
$$= \frac{2\pi}{3} \omega R_{fr}^3 T_{cont} + Q\pi \omega R_{fr}^3 T_{cont} + \frac{2\pi}{3} \omega (R_{sh}^3 - R_{fr}^3) T_{cont}$$

$$= 2\pi \omega T_{cont} \left(R_{fr}^3 + \frac{R_{sh}^3}{3} \right),$$

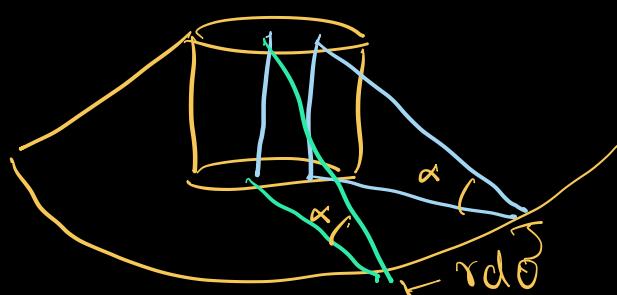
$$= \frac{2\pi}{3} \omega T_{cont} (3R_{fr}^3 + R_{sh}^3).$$

$$Q_f = \frac{2\pi}{3} \omega T_{cont} (3R_{fr}^3 + R_{sh}^3),$$

Tapered surface:



- * P_f and P_s will be same, so will be different (surface).



$$dA = dr \cdot r d\theta \cdot \tan \alpha$$

$$\tan \alpha = \frac{dz}{dx}$$

$$dA_z = dz \times d\theta$$

$$dA_r = r d\theta dr$$

$$dA_v = r dr d\theta \tan\alpha$$

$$dF_{st} = T_{cont} (r dr d\theta \tan\alpha + r dr d\theta)$$

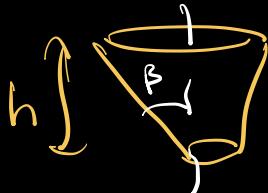
$$= T_{cont} r dr d\theta (1 + \tan\alpha)$$

$$Q_{st} = \frac{2\pi}{3} (R_{sn}^3 - R_{pr}^3) \times \omega \times T_{cont}$$

$(1 + \tan\alpha)$

↳ for taper surface.

Taper surface (Probe side).



$$Q_{ps} = 2\pi R_{pr}^2 \times h \omega T_{cont} (1 + \tan\beta)$$

Heat generation ratio

$$\eta_{pt} = \frac{Q_{pt}}{Q_t}, \quad \eta_{st} = \frac{Q_{st}}{Q_t}, \quad \eta_{ps} = \frac{Q_{ps}}{Q_t},$$

→ for flat surface,

$$Q_t = \frac{2\pi}{3} \omega T_{cont} (3R_{sn}^3 + R_{pr}^3)$$

$$\eta_{st} = \frac{Q_{st}}{Q_t} = \frac{\frac{2\pi}{3} \omega T_{cont} (3R_{sn}^3 - R_{pr}^3)}{\frac{2\pi}{3} \omega T_{cont} (3R_{sn}^3 + R_{pr}^3)}$$

Assume that

$$R_{Pr} = 3\text{mm}, \quad h = 5\text{mm}, \quad R_{Sh} = 10\text{mm}.$$

$$n_{st} = 85^\circ\text{7}' \quad \text{or} \quad 0^\circ 85A.$$

Now,

$$n_{Ps} = \frac{Q_{Ps}}{Q_f} = \frac{\frac{2\pi w}{3} R_{Ps}^2 h \gamma_{cont}}{\frac{2\pi}{3} w \gamma_{cont} (R_{Sh}^3 n + 3R_{Ps}^2 h)}$$

Now,

$$n_{Ps} = \frac{Q_{Pr}}{Q_f} = \frac{\frac{2\pi}{3} w \gamma_{cont} R_{Ps}^3}{\frac{2\pi}{3} w \gamma_{cont} (R_{Sh}^3 + 3R_{Ps}^2 h)} = \frac{R_{Ps}^3}{R_{Sh}^3 + 3R_{Ps}^2 h} = 2.37 = 0.023$$

*) Mainly heat generated due to shoulder tip (st), less due to Pr & Ps.

Amount of heat generated during FSW

Heat gen. Ratios

$$\eta_{sh} = \frac{\Phi_{st}}{\Phi_t}$$

$$= \frac{(\gamma_{sh}^3 - \gamma_{pr}^3)(1 + t_{max})}{(\gamma_{sh}^3 - \gamma_{pr}^3)(1 + t_{max}) + \gamma_{pr}^3 + 3\gamma_{pr}^2 h}$$

$\approx 86\%$

$\gamma_{sh} = 9 \text{ mm}, \gamma_{pr} = 3 \text{ mm}$

$h = 4 \text{ mm}, \alpha = 10^\circ$

$$\eta_{ps} = \frac{\Phi_{ps}}{\Phi_t} = 11\%$$

$$\eta_{pt} = \frac{\Phi_{pt}}{\Phi_t} = 03\%$$

Problems

Numerical Question: FSW

1. FSW process is carried out by a normal force of 15kN with a rotational speed of 650 RPM. The coefficient of friction is 0.2, and the weld area is 55 mm². Calculate the shear stress developed in FSW and condition whether the sticking friction or sliding friction holds good for Al alloys AA 2014.

AA 2014 alloy: $\sigma_0 = 414 \text{ MPa}$

Soln:

$$\text{Pressure, } P = \frac{F}{A} = \frac{15,000}{55} = 273 \text{ MPa}$$

$$\begin{aligned} \text{Sliding friction} &= T = \mu P \\ &= 0.2 \times 273 \\ &= 54.6 \text{ MPa} \end{aligned}$$

Shear stress developed = 55 MPa.

→ Distortion energy based theory:

$$\tau_d = \frac{\sigma_0}{\sqrt{3}} = \frac{414}{\sqrt{3}} = 240 \text{ MPa.}$$

* Shear stress in FSW \rightarrow (54.6 MPa)

Yielding condition / flow stress \rightarrow (240 MPa)

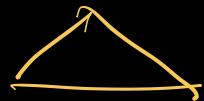
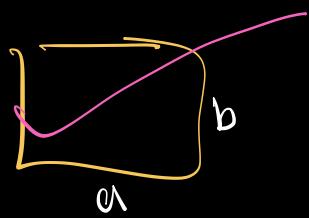
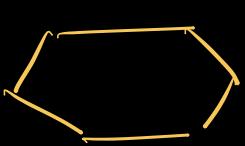
* If $\underbrace{\tau_{\text{shear in FSW}}}_{\text{Else sticking}} > \underbrace{\tau_{\text{flow/yielding}}}_{\text{sticking.}}$

→ We got sticking condition.

Problem 2?

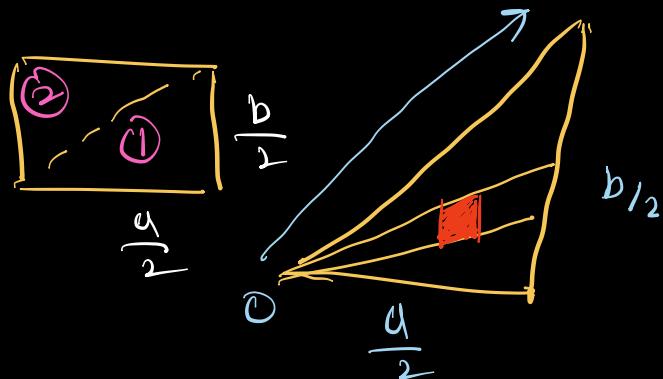
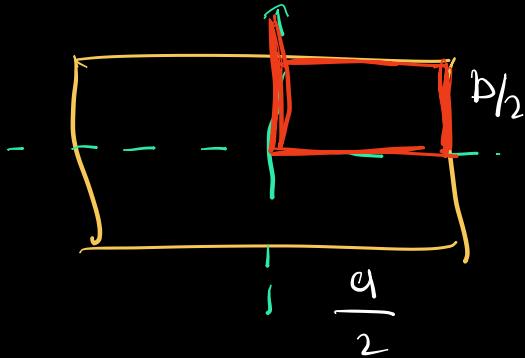
Numerical Question: FSW

2. For a friction stir welding process, the rotating tool has a rectangular cross-section with a side of a and b . If the shear contact stress between the workpiece and tool is σ_0 , the angular velocity of the tool is ω , and the coefficient of friction between the tool and workpiece is μ . Drive an expression using elemental analysis for the heat generated during the process during N rotations of the tool.



$$\sigma_0 = \tau_{\text{contact}}$$

→ Calculate Area, elemental analysis.



Prob 3

Numerical Problems

- Determine the net heat input for a butt welding job carried out at an arc voltage of 30V and a current of 200A at a welding speed of 300mm/min. Assume the heat transfer efficiency is 0.9.

SOLN

$$300 \text{ mm/min} = 5 \text{ mm/s}$$

$$\eta = 0.9 = \frac{H_{\text{net}}}{H_{\text{input}}} , \quad \frac{I \times V}{V} = \frac{200 \times 30}{5}$$

$$= 1200 \text{ J}$$

$$H_{\text{net}} = 0.9 \times 1200$$

$$= 1080 \text{ J/mm.}$$

Prob 4

Numerical Problems

2. Determine the melting efficiency for a butt welding job carried (area= 35 mm²) out at an arc voltage of 30V and a current of 200A at a welding speed of 300mm/min. Assume the heat transfer efficiency is 80%, and for melting 10 J/mm³ is required.

$$\eta = \frac{Q_{req} \times A}{H_{net}} , \quad H_{net} = \eta \times H_{input}$$
$$= \eta \times \frac{I \times V}{S}$$
$$= \frac{16 \times 35}{960}$$
$$= \frac{0.8 \times 30 \times 200}{5}$$
$$= 960$$

Prob 5

Numerical Problems

3. In a welding process under steady-state conditions, the voltage and current are measured at 18 V and 160 A, respectively. Heat loss during arc creation is 40% of heat input. Heat loss through conduction, convection, and radiation from the workpiece is 800W. The effective power is used to melt the workpiece. Calculate the melting efficiency.

Soln

$$H_{input} = 18 \times 160 = 2880 \text{ Watt}$$

$$\text{After arc creation} = 2880 \times 0.6 = 1728 \text{ Watt}$$

$$\text{After heat loss} = 1728 - 800 \\ = 928 \text{ Watt}$$

$$\text{Melting efficiency} = \frac{928 \times 100}{2880} = 32.2\%$$

ARC Welding

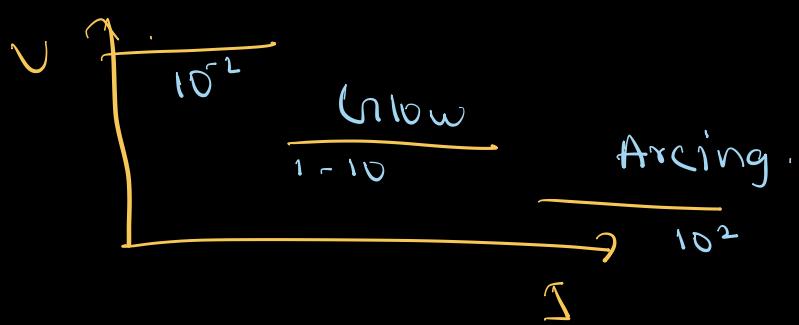
- phase change phenomenon happens.
- Electrode negative (DCEN)
Electrode positive (DCSP) Straight Polarity.
Electrode positive (DCRP) Reverse polarity.
- Thermionic emission (vibrations of electrons)
(with temperature).
cold emission (vibrations of electrons)
(without temperature).

$$\text{Energy} \rightarrow \frac{1}{2} mv^2, \quad V \rightarrow 100V \\ I \rightarrow 1A$$

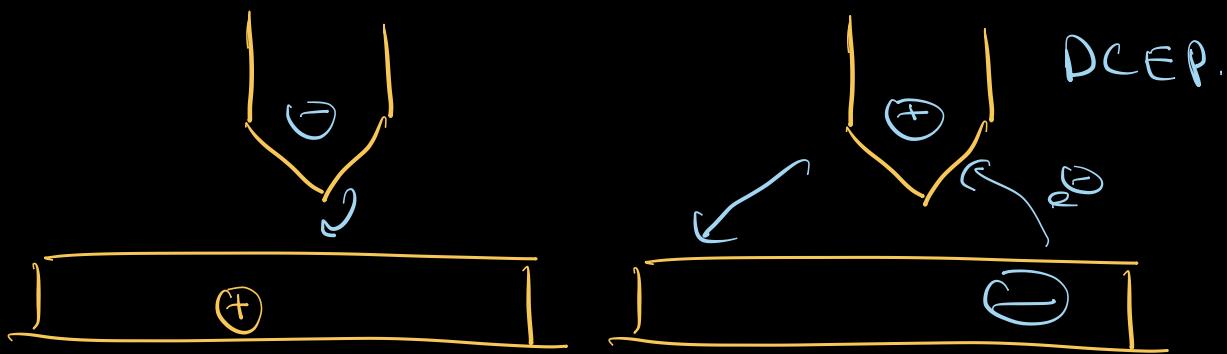
- Electron beam on H₂ producing H⁺ ions.
↳ Ionization.

$$\text{Total Energy} = n \times \frac{1}{2} mv^2 \rightarrow 10^{16} \approx 1000 \text{ J} \\ \underbrace{10^{18}}_{\text{ions}} \underbrace{10^{-31}}_{\text{size}}$$

So, This process is called discharge.



- * Ion has higher mass, electron number and velocity is high.



16/11/24 (Tuesday)

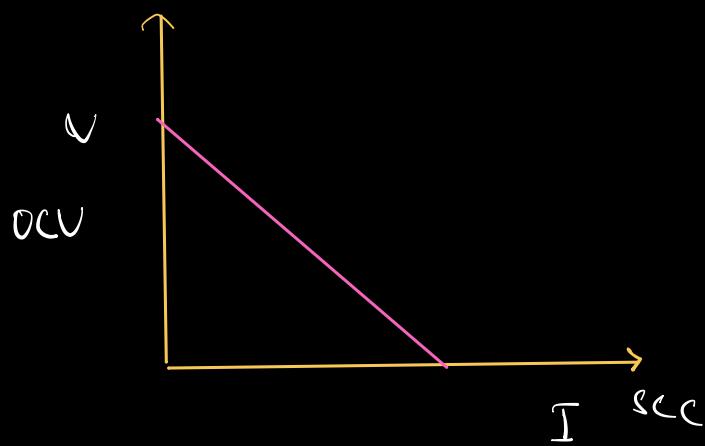
Basics in arc welding:

- * 1 Amp flow = 6.24×10^{23} electrons.
- * Thermionic emission.
- * Ionization: H_2 to H^+ and e^-
- * Discharge:

$\left| \begin{array}{l} \text{Townsend discharge} \\ \text{Glow discharge} \\ \text{arc discharge.} \end{array} \right.$

- * In DCEN, cathode drop zone
- * Arc column

* Electric arc Current model



V a \downarrow arc
column

$$V_{OCV} + \frac{I}{R} = I$$

$$4V + 3I = 120,$$

$$\frac{V}{30} + \frac{I}{40} = 1 \quad \text{---}$$

* Drooping condition

$\frac{dI}{dV}$ should be minimum.

Numerical Problems

2. Determine the maximum power for a $4V+2I=240$ DC power source.

$$4V + 2I = 240$$

Soln:

$$4V + 2I = 240,$$

$$\frac{V}{60} + \frac{I}{120} = 1,$$

$$V = \left(1 - \frac{I}{120}\right)^{60},$$

$$P = IV$$

$$P = I \left(1 - \frac{I}{120}\right)^{60}.$$

$$\frac{dP}{dI} = 0, \quad I = 60$$

$$\therefore P = VI = (30) 60 = 1800 \text{ Watt}$$

→) In hash and mullik book.

Numerical Problems

3. The voltage-length characteristic of a DC arc is given by $V=20+40l$ volts, where l is the length of the arc in cm. The power source characteristic is approximated by a straight line with an open circuit voltage = 80V and a short circuit current = 1000A. Determine the optimum arc length and the corresponding arc power.

(Ans 2 mm)

Soln: $V = 20 + 40l$, l in cm.

$$OCV = 80V$$

$$SSC = 1000A$$

$$\frac{V}{80} + \frac{I}{100} = 1, \quad I = \left(1 - \frac{V}{80}\right) 1000$$

$$\begin{aligned} P &= IV \\ &= \left(1 - \frac{V}{80}\right) V \times 1000 \end{aligned}$$

$$\frac{dP}{dl} = 0 \quad , \quad \text{optimum power}, \quad l = 0.5 \text{ cm}$$

*.) DC, Duty cycle = 60%, at 200A.

$$I^2 T = C \quad 60\% \rightarrow 6 \text{ min}$$

Duty Cycle

$$I^2 T = I_r^2 T_r$$

$$T^2 T = C$$

- Where T is % rated duty cycle, I is rated current
- T_r is required duty cycle, I_r is maximum required current output at desired duty cycle

Numerical problem:

- What will be the duty cycle with a 200A power supply rated at 60% duty cycle operated at the current 250A output?

20%

$$200^2 \times 0.6 = 250^2 \times 20\% \quad T$$

*) GMAW, MIGW
↳ metal, consumable electrode

*) PAW, Plasma

*) SAW, submerged.

$\eta = 0.55 - 0.99$, naval application.

Chemical fusion welding

*) Thermit welding - Track.

*) Oxyfuel gas welding - exothermic rxn in presence of O_2

$$*) Q(\omega) = 48 \times \frac{J}{L} \times V_{act} \times h/3600$$

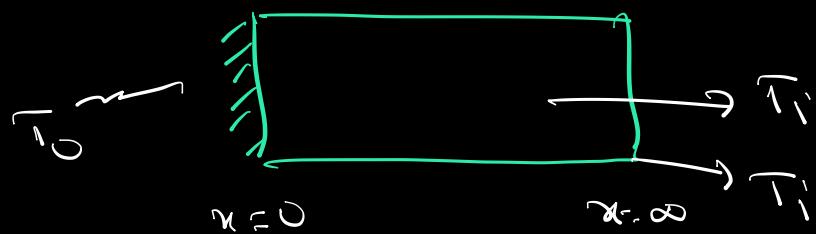
Heat of combustion

One class miss in between.

23/11/24 (Tuesday)

Semi-Infinite: Similarity Method

Consider a semi-infinite plate which is initially at a uniform temperature T_0 . The plate surface is suddenly maintained at T_i . The other surface is also maintained as initial temperature. Write the temperature distribution of body with x , and t .



Given, governing equation.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial x}$$

Boundary conditions ; $\alpha(0, t) = T_0$
 $\alpha(\infty, t) = T_i$

Initial conditions ; $T(x, 0) = T_i$

→ Solve using similarity method.

dependent variable = $T \rightarrow$ Temp.

Independent variable = (x, t)

$$\eta = \frac{x}{\sqrt{4\alpha t}}, \quad \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}.$$

$$\frac{\Delta T}{\Delta x^2} = \frac{1}{\alpha} \frac{\Delta T}{\Delta t}.$$

$$\eta = \sqrt{\frac{x^2}{\alpha t}} \Rightarrow \eta = \frac{x}{\sqrt{\alpha t}}.$$

$$\eta = \frac{x}{\sqrt{4\alpha t}}$$

* Non-dimension less temp

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \theta}{\partial t}, \quad \left. \begin{array}{l} \text{G.E.,} \\ \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}. \end{array} \right\}$$

B.C,

$$\theta(0, t) = 1.$$

$$\theta(\infty, t) = 0.$$

$$I.C, \quad \theta(x, 0) = 0.$$

$$T(0, t) = T_0$$

$$T(\infty, t) = T_i$$

I.C.

$$T(x, 0) = T_i$$

⇒ Similarity Variable : $\eta = \frac{x}{\sqrt{4\alpha t}}$

$$T(x, t), \theta(x, t)$$

$$\frac{\partial \vartheta}{\partial x} = \frac{\partial \vartheta}{\partial x} \times \frac{dx}{\partial x} = \frac{\partial \vartheta}{\partial x} \times \frac{1}{\sqrt{4\alpha t}},$$

$$\begin{aligned}\frac{\partial^2 \vartheta}{\partial x^2} &= \frac{\partial}{\partial x} \left[\frac{\partial \vartheta}{\partial x} \frac{1}{\sqrt{4\alpha t}} \right] \\ &= \frac{\partial}{\partial x} \left(\frac{\partial \vartheta}{\partial x} \frac{1}{\sqrt{4\alpha t}} \right) \times \frac{dx}{\partial x} \\ &= \frac{\partial^2 \vartheta}{\partial x^2} \times \frac{1}{\sqrt{4\alpha t}} \times \frac{1}{\sqrt{4\alpha t}} \\ &= \frac{1}{4\alpha t} \cdot \frac{\partial^2 \vartheta}{\partial x^2}\end{aligned}$$

a) $\frac{\partial \vartheta}{\partial t} = \frac{\partial \vartheta}{\partial x} \times \frac{\partial x}{\partial t}, \quad u = \frac{x}{\sqrt{4\alpha t}},$

$$= \frac{\partial \vartheta}{\partial x} \left[-\frac{1}{2} \frac{u}{\sqrt{4\alpha t}} \right] \quad \frac{\partial x}{\partial t} = -\frac{1}{2} \frac{u}{\sqrt{4\alpha t}},$$

$$= \frac{\partial \vartheta}{\partial x} \left(-\frac{1}{2} \frac{u}{t} \right), \quad \frac{\partial^2 \vartheta}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \vartheta}{\partial t},$$

$$\frac{1}{4\alpha t} \frac{\partial^2 \vartheta}{\partial x^2} = \frac{1}{\alpha} \times \frac{\partial \vartheta}{\partial x} \left(-\frac{1}{2} \frac{u}{t} \right).$$

$$\frac{\partial^2 \vartheta}{2\partial u^2} = -\frac{\partial \vartheta}{\partial x} \frac{u}{t},$$

$$\frac{\partial^2 \theta}{\partial y^2} = -\frac{\partial \theta}{\partial x} \left(\frac{2y}{t} \right)$$

$$*) u = \frac{x}{4xt}$$

$$\Theta(x,t) \rightarrow \Theta(u)$$

$$\Theta(0) = 1, \sim x \rightarrow 0, \quad y \rightarrow 0.$$

$$\Theta(\infty) = 0, \quad \sim x \rightarrow \infty, \quad y \rightarrow \infty.$$

$$*) \frac{\partial^2 \theta}{\partial x^2} = -2u \frac{\partial \theta}{\partial x}.$$

$$\frac{\partial}{\partial y} \left(\frac{\partial \theta}{\partial x} \right) = -2u \frac{\partial \theta}{\partial x},$$

$$\frac{\partial \left(\frac{\partial \theta}{\partial x} \right)}{\frac{\partial \theta}{\partial x}} = -2u \partial u.$$

On Integration,

$$\ln \frac{\partial \theta}{\partial u} = -u^2 + \ln k.$$

$$\frac{\partial \theta}{\partial u} = k e^{-u^2}$$

B.C.

$$T(0,t) = T_0$$

$$T(\infty, t) = T_i$$

T.C.

$$T(x,0) = T_i$$

$$\Theta(x,t) = \frac{T - T_i}{T_0 - T_i}$$

B.C

$$\Theta(0,t) = T_0 = 1$$

$$\Theta(\infty,t) = T_i = 0$$

T.C.

$$\Theta(x,0) = T_i = 0$$

$$\int_0^{\infty} e^{-y^2} dy = \frac{1}{2} \sqrt{\pi}$$

$$\Theta(\theta) - \Theta(0) = k \int_0^{\infty} e^{-y^2} dy$$

$$\Theta = 1 - \frac{k}{\sqrt{\pi}} \operatorname{erf}(y)$$

$$\operatorname{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2} dt$$

$$\Theta = 1 - \operatorname{erf}(y)$$



$$*) \quad \Theta = \frac{T - T_i}{T_0 - T_i} = 1 - \operatorname{erf}\left(\frac{x}{\sqrt{\Phi_{\text{ext}}}}\right)$$

$$\operatorname{erf}(0) = 0$$

$$\operatorname{erf}(\pm\infty) = \pm 1$$

$$T = T_i + (T_0 - T_i)(1 - \operatorname{erf}\left(\frac{x}{\sqrt{\Phi_{\text{ext}}}}\right))$$

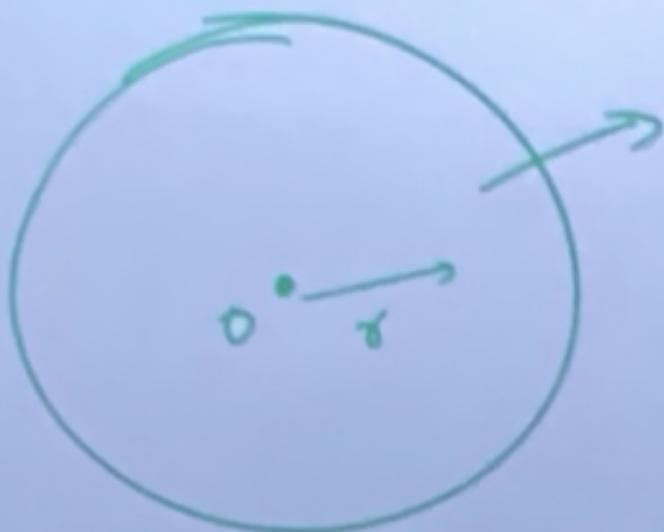
$$= T_i + (T_0 - T_i) \operatorname{erfc}\left(\frac{x}{\sqrt{\Phi_{\text{ext}}}}\right)$$

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$$

$$*) \quad \Theta(0) = 1, \quad \Theta(\infty) = 0$$

Source/Sink in Welding: Point source

Point heat Source of constant heat Production Rate



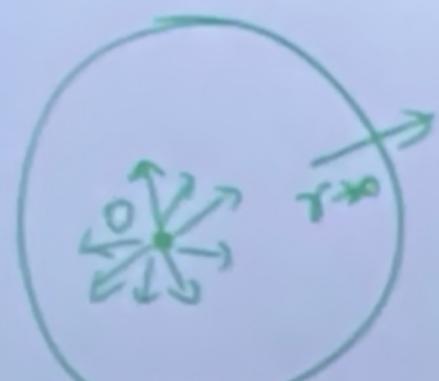
O is heating source which is producing const heat.

$$r \rightarrow \infty$$

Consider a body of large extent possessing a very small region (point) that produce heat continuously at a constant rate (Q_{heat})

Source/Sink in Welding: Point source

Given flat body was initially maintained at $T = T_{\infty}$



G.E.

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = -\frac{1}{\alpha} \frac{\partial T}{\partial t}$$

B.C.

$$r \rightarrow \infty, T = T_{\infty} \quad \text{--- } ②$$

$$r \rightarrow 0, -4\pi r^2 k \frac{dT}{dr} = Q \quad \text{--- } ③$$

$$T.C. \Rightarrow t = 0, T = T_{\infty} \quad \text{--- } ④$$

Source/Sink in Welding: Point source

$$\Theta = T - T_{\infty}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Theta}{\partial r} \right) = \frac{1}{r^2} \frac{\partial \Theta}{\partial t} \quad -1A$$

B.C. $r \rightarrow 0, -4\pi r^2 K \frac{\partial \Theta}{\partial r} = Q \quad 2A$

$r \rightarrow \infty, \Theta = 0 \quad 3A$

C.C. $t = 0, \Theta = 0, \quad 4A$

$$\therefore U = r(T - T_{\infty}) = r\Theta$$

Source/Sink in Welding: Point source

$$u = r\Theta$$

$$\frac{\partial u}{\partial r} = \frac{r}{\partial} \frac{\partial \Theta}{\partial r} + \Theta \frac{\partial r}{\partial r}$$

$$\frac{\partial \Theta}{\partial r} \rightarrow 0$$

$$\boxed{\frac{\partial u}{\partial r} = r \frac{\partial \Theta}{\partial r}} \quad -5$$

$$\frac{\partial \Theta}{\partial r} = \frac{1}{r} \left(\frac{\partial u}{\partial r} - \Theta \right) \quad -6A$$

$$\begin{aligned} \frac{\partial^2 u}{\partial r^2} &= r \frac{\partial^2 \Theta}{\partial r^2} + \frac{\partial \Theta}{\partial r} \\ &\quad + \frac{\partial \Theta}{\partial r} \\ &= r \frac{\partial^2 \Theta}{\partial r^2} + 2 \frac{\partial \Theta}{\partial r} \end{aligned} \quad -7$$

$$\boxed{\frac{\partial u}{\partial r} = r \frac{\partial \Theta}{\partial r} + \Theta} \quad -6$$

Source/Sink in Welding: Point source

(1A)

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Theta}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial \Theta}{\partial t}$$

\rightarrow

$$\frac{1}{r^2} \left(r^2 \frac{\partial^2 \Theta}{\partial r^2} + \frac{\partial \Theta}{\partial r} \times 2r \right) = \frac{1}{\alpha} \frac{\partial \Theta}{\partial t}$$

$$r \left(\frac{\partial^2 \Theta}{\partial r^2} + \frac{2}{r} \frac{\partial \Theta}{\partial r} \right) = \left(\frac{1}{\alpha} \frac{\partial \Theta}{\partial t} \right) \times r$$

$$\cancel{r} \frac{\partial^2 \Theta}{\partial r^2} + \cancel{2} \frac{\partial \Theta}{\partial r} = \frac{1}{\alpha} \frac{\partial \Theta}{\partial t}$$

$\frac{\partial^2 u}{\partial r^2} = \frac{1}{\alpha} \frac{\partial u}{\partial t} \rightarrow \text{eqn 5}$

Source/Sink in Welding: Point source

$\frac{\partial^2 u}{\partial r^2} = \frac{1}{\alpha} \frac{\partial u}{\partial t}$

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{\alpha} \frac{\partial u}{\partial t} \quad \text{--- eqn 5}$$

$$(2A) \rightarrow r \rightarrow 0, u \rightarrow 0, -4\pi r^2 K \frac{\partial^2 u}{\partial r^2} = Q \quad \text{--- eqn 6P}$$

$$-4\pi r^2 K \frac{1}{r} \left(\frac{\partial u}{\partial r} - 0 \right) = Q$$

$$-4\pi K \left(r \frac{\partial u}{\partial r} - u \right) = Q$$

$$-4\pi K \left(r \frac{\partial u}{\partial r} - u \right) = Q$$

--- ①

Source/Sink in Welding: Point source

$$U = r^{\theta}$$

$$(3A) \rightarrow r \rightarrow \infty, U \rightarrow \infty, \theta = 0 \longrightarrow 10$$

$$(4A) \quad \cancel{t=0}, \quad U = 0, \quad \theta = 0 \longrightarrow 11$$

$$n = \frac{r}{\sqrt{4\alpha t}}$$

$$12$$

$$U = A + B e^{rt(n)} \longrightarrow 13$$

Source/Sink in Welding: Point source

$$U = r^{\theta}$$

$$\text{at } r \rightarrow 0, n \rightarrow 0 \quad -4\pi k \left(r \frac{\partial u}{\partial r} - u \right) = Q$$

$$n = \frac{r}{\sqrt{4\alpha t}}$$

$$-4\pi k \left(r \frac{1}{\sqrt{4\alpha t}} \frac{du}{dn} - u \right) = Q$$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial n} \frac{\partial n}{\partial r}$$

$$= \frac{1}{\sqrt{4\alpha t}} \frac{du}{dn}$$

$$-4\pi k \left(n \frac{du}{dn} - u \right) = Q$$

~~at $r \rightarrow 0$~~

$$14A$$

$$t = 0, r \rightarrow \infty, n \rightarrow \infty, u = 0$$

$$u = A + B e^{rt(n)} \Rightarrow D = A + B e^{r(\infty)}$$

$$\boxed{A + B = 0} \longrightarrow 14B$$

~~at $r \rightarrow 0$~~

Source/Sink in Welding: Point source

$$\pi \rightarrow 0 \quad -\nu \pi k \left(\frac{\partial u}{\partial n} - u \right) = Q$$

$$-\nu \pi k (-A - B e^{\gamma t}) = Q$$

$$A = \frac{Q}{4\pi k}$$

$$B = -\frac{Q}{4\pi k}$$

Source/Sink In Welding: Point source

$$u = A + B e^{\gamma t} (y) \quad n = \cancel{\sqrt{u_{act}}}$$

$$u = \frac{Q}{4\pi k} - \frac{Q}{4\pi k} e^{\gamma t} \left(\frac{y}{\sqrt{u_{act}}} \right)$$

$$u(t) = \frac{Q}{4\pi k} \left(1 - e^{\gamma t} \left(\frac{y}{\sqrt{u_{act}}} \right) \right)$$

$$\gamma (T - T_{act}) = \frac{Q}{4\pi k} \left(1 - e^{\gamma t} \left(\frac{T}{\sqrt{u_{act}}} \right) \right)$$

Source/Sink in Welding: Point source

$$T = T_{\infty} + \frac{Q}{4\pi k r} \left(1 - e^{-rt/\alpha_r} \left(\frac{r}{\sqrt{\alpha_r t}} \right) \right)$$

$t \rightarrow \infty$

$$T = T_{\infty} + \frac{Q}{4\pi k r}$$

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* As $t \rightarrow \infty$

$$T = T_{\infty} + \frac{Q}{4\pi k r}$$

Instantaneous heat source: Thermal explosion at point region.

Consider a body of large extent possessing a small region (point) that expressed as E_0 energy at $t=0$ centre.

Soln: Governing equation

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{--- (1)}$$

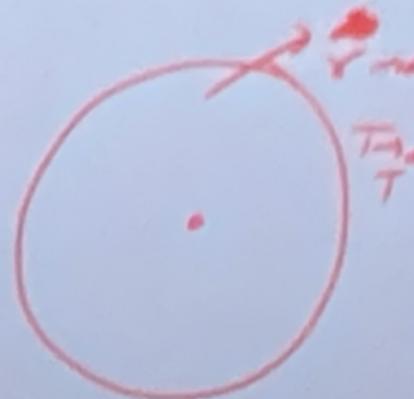
continuation



Instantaneous heat source: Thermal Explosion at point region

$$G.F. \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{--- (1)}$$

B.C. $r \rightarrow \infty, T \rightarrow T_\infty$



T.C. $t = 0, T = T_\infty$

Condition: Energy Balance

$$\int_0^\infty 4\pi r^2 \rho C_p (T - T_\infty) dr = E_0$$

Instantaneous heat source: Thermal Explosion at point region

$$\Theta = T - T_{\infty}$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Theta}{\partial r} \right) = \frac{1}{r^2} \frac{\partial \Theta}{\partial t} \quad \text{--- (1A)}$$

$$\int_0^{\infty} 4\pi r^2 \rho c \Theta dr = E_0 \quad \text{--- (2A)}$$

$$r \rightarrow \infty, \Theta = 0,$$

--- (3A)

$$t = 0, \Theta = 0,$$

--- (4A)

Instantaneous heat source: Thermal Explosion at point region

$$u = r^{\Theta}$$

$$\frac{\partial u}{\partial t} = r \frac{\partial \Theta}{\partial t} \quad \text{--- (5)} \quad \frac{\partial \Theta}{\partial t} = \frac{1}{r} \frac{\partial u}{\partial t} \quad \text{--- (5A)}$$

$$\frac{\partial u}{\partial r} = r \frac{\partial \Theta}{\partial r} + \Theta \quad \text{--- (6)} \quad \frac{\partial \Theta}{\partial r} = \frac{1}{r} \left(\frac{\partial u}{\partial r} - \Theta \right) \quad \text{--- (6A)}$$

$$\frac{\partial^2 u}{\partial r^2} = r \frac{\partial^2 \Theta}{\partial r^2} + 2 \frac{\partial \Theta}{\partial r} \quad \text{--- (7)}$$

Instantaneous heat source: Thermal Explosion at point region

(1A)

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \Theta \right) = \frac{1}{r^2} \frac{\partial u}{\partial t}$$

$$\frac{\partial^2 u}{\partial r^2} = \frac{1}{r^2} \frac{\partial u}{\partial t} \quad \text{--- (1)}$$

$$\int_0^\infty 4\pi r^2 \rho C_p \Theta dr = E_0 \quad \begin{cases} r^2 \Theta dr \\ r \cdot \Theta dr \\ r u dr \end{cases}$$

$$4\pi \rho C_p \int_0^\infty 4r dr = E_0 \quad \text{--- (2)}$$

$$\theta \rightarrow \infty, \Theta = 0, u = 0 \quad \text{--- (3)}$$

$$t = 0, \Theta = 0, u = 0 \quad \text{--- (4)}$$

Instantaneous heat source: Thermal Explosion at point region

~~$$U = A + B e^{rt} (n)$$~~

~~$$= A + B e^{rt} \left(\frac{r}{\sqrt{4\alpha t}} \right)$$~~

~~$$U' = B \frac{2}{\sqrt{\pi}} e^{-\frac{r^2}{4\alpha t}} \left(\frac{1}{\sqrt{4\alpha t}} \right)$$~~

$$\text{extra } \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-x^2} dx$$

Instantaneous heat source: Thermal Explosion at point region

$$U = A + B e^{\alpha t} \quad (n)$$

$$= A + B e^{\alpha t} \left(\frac{r}{\sqrt{4\alpha t}} \right) \quad - 12$$

$$U' = B \frac{1}{\sqrt{\pi}} e^{-\frac{r^2}{4\alpha t}} \left(\frac{1}{\sqrt{4\alpha t}} \right)$$

$$U' = B \frac{1}{\sqrt{\pi \alpha t}} e^{-\frac{r^2}{4\alpha t}} \quad - 13$$

Instantaneous heat source: Thermal Explosion at point region

$$\text{eq } 9 \quad 4\pi PC_p \int_0^\infty 4\tau dr = E_0$$

$$U = U' = \frac{-BY}{2\sqrt{\pi} (kH)^{3/2}} \exp\left(-\frac{Y^2}{4\alpha t}\right) \quad - 14$$

$$4\pi PC_p \int_0^\infty \frac{-BY}{2\sqrt{\pi} (kH)^{3/2}} \exp\left(-\frac{Y^2}{4\alpha t}\right) Y dr = E_0 \quad - 15$$

$$Z^2 = \frac{Y^2}{4\alpha t} \quad 2ZdZ = \frac{2Ydr}{4\alpha t}$$

$$Z = \frac{Y}{\sqrt{4\alpha t}} \quad dZ = \frac{dy}{\sqrt{4\alpha t}}$$

Instantaneous heat source: Thermal Explosion at point region

$$\int_0^{\infty} \frac{-2B\gamma^2}{4\alpha + \sqrt{4\alpha t}} \exp(-z^2) dz = \frac{E_0}{4\pi \rho C_p}$$

$$\int_0^{\infty} \frac{-2B\gamma^2}{4\alpha + \sqrt{4\alpha t}} \exp(-z^2) dz = \frac{E_0}{4\pi \rho C_p}$$

$$\int_0^{\infty} -\frac{4Bz^2}{\sqrt{\pi}} \exp(-z^2) dz = \frac{E_0}{4\pi \rho C_p}$$

Instantaneous heat source: Thermal Explosion at point region

$$\text{S.t. } -\frac{2B}{\sqrt{\pi}} \Gamma_{1/2}^{3/2} = \frac{E_0}{4\pi \rho C_p}$$

$$\begin{aligned} \Gamma_{1/2}^{3/2} &= \frac{1}{2} \Gamma_{1/2}^{1/2} \\ &= \frac{1}{2} \times \sqrt{\pi} \\ &= \frac{\sqrt{\pi}}{2} \end{aligned}$$

$$-\frac{2B}{\sqrt{\pi}} \times \frac{\sqrt{\pi}}{2} = \frac{E_0}{4\pi \rho C_p}$$

$$B = -\frac{E_0}{4\pi \rho C_p}$$

Instantaneous heat source: Thermal Explosion at point region

So, u'' is our soln

$$u = u'' = \frac{-B\gamma}{2\sqrt{\pi}(\alpha t)^{3/2}} \exp\left(-\frac{\gamma^2}{4\alpha t}\right)$$

eqn 14

$$u = \frac{E_0 \gamma}{2\sqrt{\pi}(\alpha t)^{3/2} \times 4\pi P_C} \exp\left(-\frac{\gamma^2}{4\alpha t}\right)$$

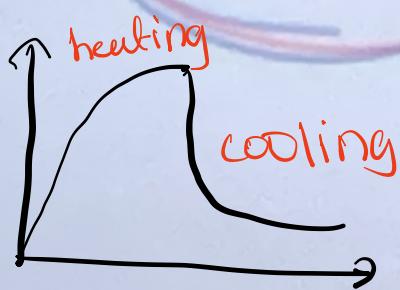
$$\textcircled{4} \quad B = \frac{E_0}{4\pi P_C}$$

$$\text{w/ } f(t) = \delta(t - T_0) \quad u = \frac{E_0 \gamma}{8(\pi \alpha t)^{3/2} P_C} \exp\left(-\frac{\gamma^2}{4\alpha t}\right)$$

Instantaneous heat source: Thermal Explosion at point region

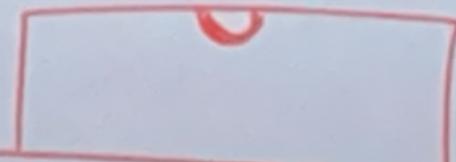
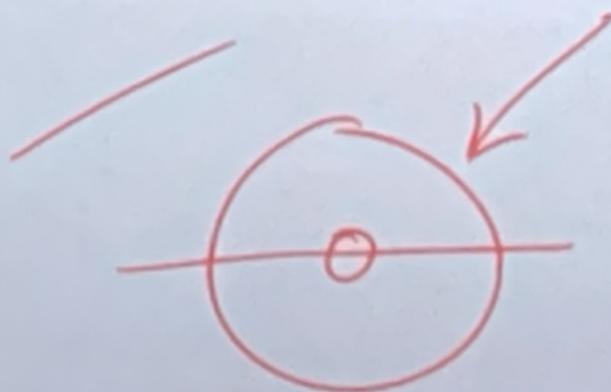
$$T = T_{\infty} + \frac{E_0}{8(\pi \alpha t)^{3/2} P_C} \exp\left(-\frac{\gamma^2}{4\alpha t}\right)$$

Arc strikes in Fusion welding



Arc strikes in Fusion welding

$$T = T_{\infty} + \frac{E_0}{8 \rho C_p (\pi \alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$



$$T = T_{\infty} + \frac{E_0}{4 \rho C_p (\pi \alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

Arc strikes in Fusion welding

$$T = T_{\infty} + \frac{E_0}{4\rho_q(\pi\alpha)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

Assum'g

$$\theta = \frac{T - T_{\infty}}{T_c - T_{\infty}}$$

Non-dimensional temp.

T_{∞} = initial
temp

T_c = Reference temp.

$T_m = T_m$ = melting point

Arc strikes in Fusion welding

$$\text{Dimension less radius factor} = \sigma_i = \sqrt{\frac{R^2}{4\alpha t_i}}$$

t_i = ignition time

Dimension less operating parameter

$$= n_i = \frac{E_0}{4(\pi\alpha t_i)^{3/2} \rho C_p (T_c - T_{\infty})}$$

$$\rho C_p (T_c - T_{\infty}) = \frac{\Delta H}{\sigma_i}$$

$$= \frac{E_0}{4(\pi\alpha t_i)^{3/2} \Delta H}$$

$$\boxed{\gamma = t / t_i}$$

Arc strikes in Fusion welding

$$Q_0 = \text{Heat input} = \frac{E_0}{t_i} \rightarrow \frac{T_{\text{out}}}{s \cdot m} \text{ watt}$$

$$= I \times V$$

current \times vol.

$$n_1 = \frac{E_0}{4(\pi\alpha)^{3/2} t_i^{1/2} \times H} \Delta H = \frac{q_0}{4(\pi\alpha)^{3/2} t_i^{1/2} \Delta H}$$

Arc strikes in Fusion welding

$$T = T_{\infty} + \frac{E_0}{4\rho C_p (\pi\alpha t)^{3/2}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\frac{T - T_{\infty}}{T_c - T_{\infty}} = \frac{E_0}{4\rho C_p (\pi\alpha t)^{3/2}} \frac{1}{T_c - T_{\infty}} \exp\left(-\frac{r^2}{4\alpha t}\right)$$

$$\theta = \frac{E_0}{4\rho C_p (\pi\alpha t)^{3/2} (T_c - T_{\infty})} \exp\left(-\frac{r^2}{4\alpha t}\right) \Delta H$$

t_i

Arc strikes in Fusion welding

$$\Theta = \frac{E_0}{4\Delta H} \left(\frac{\pi \alpha t_i \times t}{t_i} \right)^{3/2} \exp \left(-\frac{\sigma_i^2}{4\alpha t_i \times t + \frac{E_0}{t_i}} \right)$$

$\frac{E_0}{t_i} = E_0$

$$= \frac{E_0}{4\Delta H} \left(\frac{\pi \alpha t_i}{t_i} \right)^{3/2} \left(\frac{t}{t_i} \right)^{3/2} \exp \left(-\frac{\sigma_i^2}{t_i} \right)$$
$$= \frac{n_0}{4\Delta H} \left(\frac{\pi \alpha}{t_i} \right)^{3/2} \left(\frac{t}{t_i} \right)^{3/2} \exp \left(-\frac{\sigma_i^2}{t_i} \right)$$

Arc strikes in Fusion welding

$$\Theta = \frac{n_1}{t_i^{3/2}} \exp \left(-\frac{\sigma_i^2}{t_i} \right)$$

$$\Theta = \frac{n_1}{t_i^{3/2}} \exp \left(-\frac{\sigma_i^2}{t_i} \right)$$

21/12/24 (Friday)

$$*) \quad \theta = \frac{n_1}{\tau_1^{3/2}} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

⇒ for maximum temp or peak temp

$$\frac{\partial \theta}{\partial \tau_1} = 0$$

or

$$\frac{\partial \left(\frac{\theta}{n_1}\right)}{\partial \tau_1} = \tau_1^{-3/2} \exp\left(-\frac{\sigma_1^2}{\tau_1}\right) \left(+ \frac{\sigma_1^2}{\tau_1^2}\right)$$

$$+ \exp\left(-\frac{\sigma_1^2}{\tau_1}\right) \left(-\frac{3}{2}\right) \tau_1^{-5/2}$$

$$= 0.$$

$$\therefore \sigma^2 = \frac{3}{2} \tau_1$$

→ for max temp /
peak temp

$$\rightarrow \sigma_{1m}^2 = \frac{3}{2} \tau_{1m}$$

$$*) \quad \frac{\theta}{n_1} = \frac{1}{\tau_1^{3/2}} \times \exp\left(-\frac{\sigma_1^2}{\tau_1}\right)$$

$$\frac{\theta_1}{n_1} = \frac{1}{\tau_1^{3/2}} \times \exp\left(\frac{3}{2}\right).$$

$$\Rightarrow \frac{\theta}{n_1} = \frac{1}{(e \tau_1)^{3/2}}$$

$$\frac{\theta_1}{n_1} = \frac{1}{\left(\frac{2}{3} e\right)^{3/2} \sigma_1^3}$$

locus of
peak condition.

$$*\frac{\partial}{\partial t} = \frac{1}{\gamma_1^{3/2}} \exp\left(-\frac{c_1^2}{T_1}\right)$$

$$*) c_{im}^2 = \frac{3}{2} T_{im}$$

$$*) \frac{\partial p}{\partial t} = \frac{1}{(e T_1)^{3/2}} = \frac{1}{\left(\frac{\partial e}{3}\right)^{3/2}} \sigma_1^3$$

Numericals

Numerical: thermal analysis

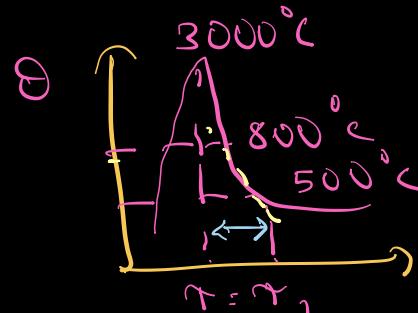
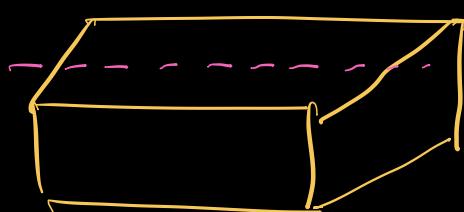
Consider a small weld crater formed on a thick low alloy steel plate in arc welding.

1. Calculate the cooling time from 800 to 500°C ($\Delta t_{800-500}$) at the center of the weld and the cooling rate (C.R.) at the onset of the austenite to ferrite transformation (475°C).
2. Calculate the total width of the fully transformed region adjacent to the fusion boundary. Assuming that the transformation temperature is equal to 890°C for this particular steel.

The operational conditions are as follows:

$I=80A$, Voltage=35V, $t_i=0.1$ s, arc efficiency: 0.75, $T_0=20^\circ C$, $T_c=T_m=1520^\circ C$, thermal diffusivity =5mm²/s, $\rho C_p=0.005$ J/mm³.C, $\Delta H=7.5$ J/mm³

1) Soln:



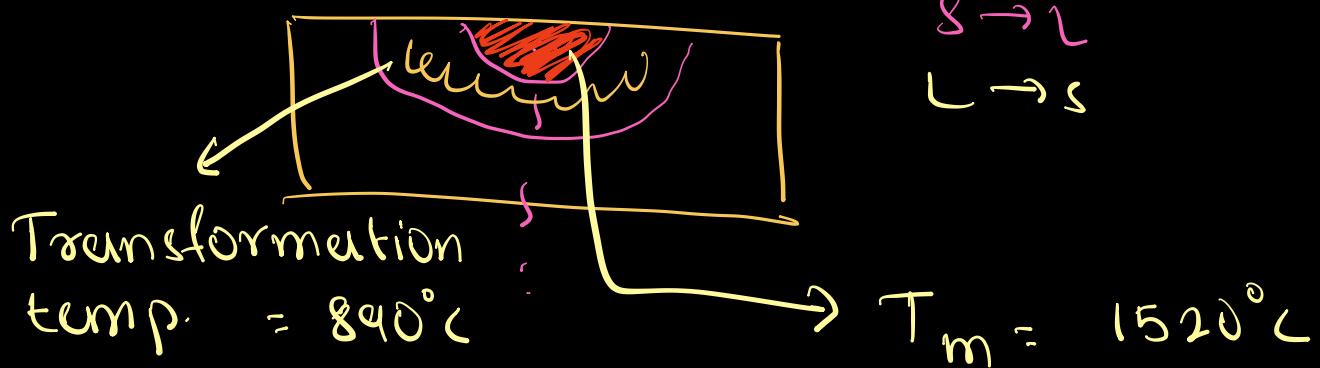
Calculate where $\sigma_r^2 = 0$ (At center)

From 800°C to 500°C

$$\text{cooling rate} = -\frac{dT}{dt}$$

= slope at given temp.

2) Soln:



$$I = 80\text{A}$$

$$V = 35\text{V}$$

$$t_i = 0.1\text{ sec}$$

$$\eta = 0.75 \rightarrow \text{Arc efficiency}$$

$$T_0 = 20^\circ\text{C}$$

α = Thermal diffusivity

Formally (soln)

$$\Theta_{1520} = \frac{T - T_0}{T_c - T_0} = \frac{1520 - 20}{1520 - 20} = 1.$$

$$\Theta_{840} = \frac{840 - 20}{1500} = 0.58$$

$$\Theta_{800} = \frac{800 - 20}{1500} = 0.52$$

$$\Theta_{500} = \frac{500 - 20}{1500} = 0.32$$

$$\Theta_{475} = \frac{475 - 20}{1500} = 0.303.$$

$$U_1 = \frac{q_0}{4 \Delta H (\pi \alpha)^{3/2} \sqrt{t_i}} \quad q_0 = U \cdot I \cup$$

$$= \frac{0.75 \times 80 \times 35}{4 \times 1.5 \times (5 \times \pi)^{3/2} \times \sqrt{0.1}}$$

$$= 3.56$$

① (a) At $\sigma_1 = 0$

$$\frac{\Theta}{U_1} = \frac{1}{\pi^{3/2}} \exp \left(\frac{-\alpha_1^2}{\sigma_1} \right)$$

$$\pi_1^{3/2} = \frac{U_1}{\Theta} \Rightarrow \pi_1 = \left(\frac{U_1}{\Theta} \right)^{2/3}$$

$$\text{so, } \pi_{800^\circ C} = \left(\frac{U_1}{\Theta_{800}} \right)^{2/3}$$

$$\pi_{500^\circ C} = \left(\frac{U_1}{\Theta_{500}} \right)^{2/3}$$

$$\Delta T = \pi_{500^\circ C} - \pi_{800^\circ C}$$

$$\text{Now, } \Delta T = \left(\frac{U_1}{\Theta_{500}} \right)^{2/3} - \left(\frac{U_1}{\Theta_{800}} \right)^{2/3}$$

$$= \left(\frac{3.56}{0.32} \right)^{2/3} - \left(\frac{3.56}{0.52} \right)^{2/3}$$

$$= 1.38$$

$$\Delta t_{800-500} = 1.38 \times t_i$$

$$\approx 0.14 \text{ secs}$$

$$\gamma = \frac{t}{t_i}$$

$$\Delta T = \frac{\Delta t_{800-500}}{t_i}$$

$$= 0.138 \text{ secs}$$

①(b) $\Theta_{4+5} = 0.303 \approx 0.3$

Cooling rate at 475

$$= -\frac{dT}{dt}$$

$$\Theta = \frac{n_1}{T_1^{3/2}} \exp\left(-\frac{c_1^2}{\tau_1}\right)$$

$$\frac{\partial \Theta}{\partial \tau_1} = n_1 \left[\frac{1}{T_1^{3/2}} \exp\left(-\frac{c_1^2}{\tau_1}\right) \left(\frac{c_1^2}{\tau_p}\right) - \frac{3}{2} \frac{1}{T_1^{5/2}} \exp\left(-\frac{c_1^2}{\tau_1}\right) \right]$$

$$\text{At } \tau = 0, c_1 = 0.$$

$$\frac{\partial \Theta}{\partial \tau_1} = n_1 \left[0 - \frac{3}{2} \frac{1}{T_1^{5/2}} \exp\left(-\frac{c_1^2}{\tau_1}\right) \right].$$

$$\frac{\partial \Theta}{\partial T_1} = n_1 \left[-\frac{3}{2} \frac{1}{T_1^{5/2}} \right] - \frac{3}{2} \frac{n_1}{T_1^{5/2}}.$$

$$\Rightarrow \frac{1}{T_1^{3/2}} \left[\exp\left(-\frac{c_1^2}{\tau_1}\right) \right], c_1 = 0.$$

$$T_1 = \left(\frac{n_1}{\delta}\right)^{2/3}$$

$$\frac{\partial \Theta}{\partial T_1} = -\frac{3}{2} \frac{n_1}{\left(\left(\frac{n_1}{\delta}\right)^{2/3}\right)^{5/2}} = -\frac{3}{2} \frac{\Theta^{5/3}}{n_1^{2/3}}$$

Cooling rate = $-\frac{dT}{dt}$

$$= \frac{\frac{dT}{dt}}{\frac{T_c - T_0}{t_i}}$$

$$\Theta = \frac{T - T_0}{T_c - T_0}$$

$$\partial \Theta = \frac{dT}{T_c - T_0}$$

$$T_1 = \frac{t}{t_i}$$

$$\partial \gamma_1 = \frac{\partial t}{t_i}$$

so, $\frac{t_i}{T_c - T_0} \frac{dT}{dt} = -\frac{3}{2} \frac{\Theta^{5/3}}{n_1^{2/3}}$

$$\frac{dT}{dt} = -\frac{T_c - T_0}{t_i} \frac{3}{2} \frac{\Theta^{5/3}}{n_1^{2/3}}$$

$$C.R = -\frac{dT}{dt} = \frac{T_c - T_0}{t_i} \frac{3}{2} \frac{\Theta^{5/3}}{n_1^{2/3}}$$

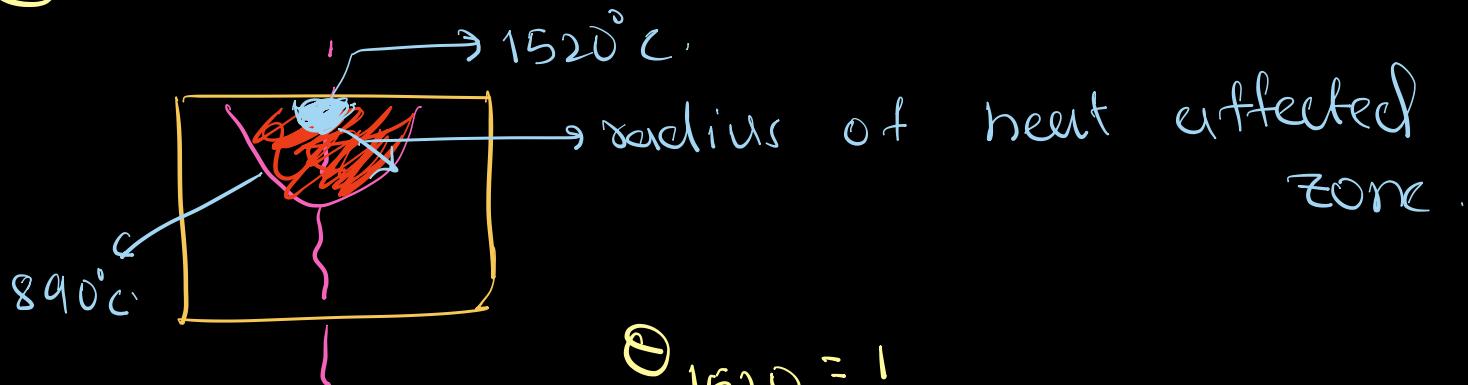
$$\Theta_{475} = 0.3$$

$$= \frac{100}{0.1} \times \frac{3}{2} \times \frac{0.3^{5/3}}{(3.56)^{2/3}} \text{ Oe/s}$$

or K/s.

= $864^\circ\text{C}/\text{s}$ (apron shayad)
 Prof khud ko nahi
 Pata.

② soln continuation



$$\Theta_{1520} = 1$$

$$\Theta_{890} = 0.58$$

We use peak temp. conditions.

$$\frac{\delta p}{n_1} = \left(\frac{1}{\left(\frac{2e}{3} \right)^{3/2}} \right) c_1^3$$

$$\text{So, } (c_1)^3 = \frac{1}{\left(\frac{2e}{3} \right)^{3/2}} \cdot \frac{n_1}{\delta p}$$

$$c_1 = \left(\frac{1}{\left(\frac{2e}{3} \right)^{1/2}} \cdot \left(\frac{n_1}{\delta p} \right) \right)^{1/3}$$

$$c_1 \Big|_{890^\circ\text{C}} = \left(\frac{1}{\left(\frac{2e}{3} \right)^{1/2}} \cdot \left(\frac{3.56}{0.58} \right) \right)^{1/3}$$

$$c_1 \Big|_{1520} = - - -$$

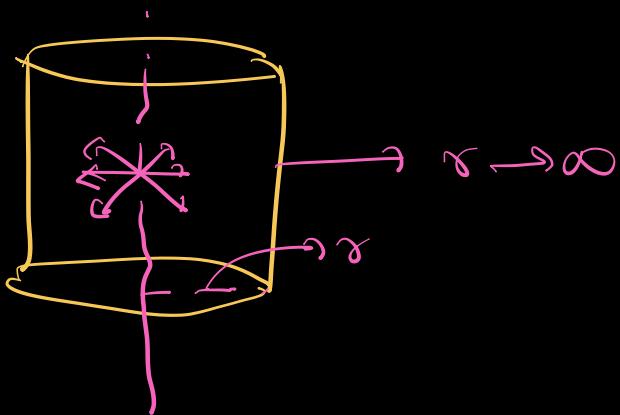
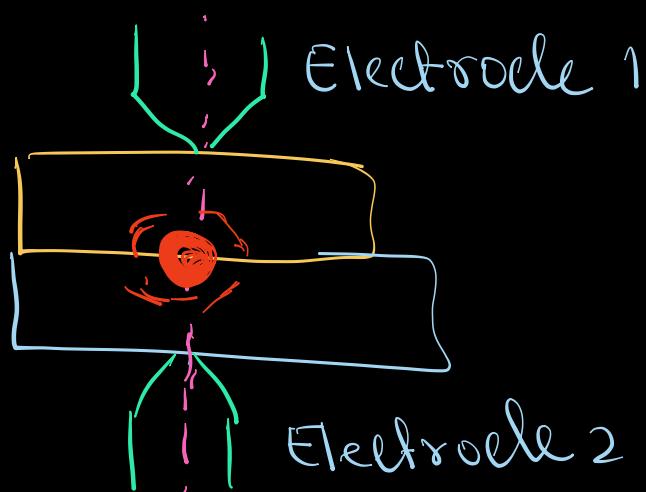
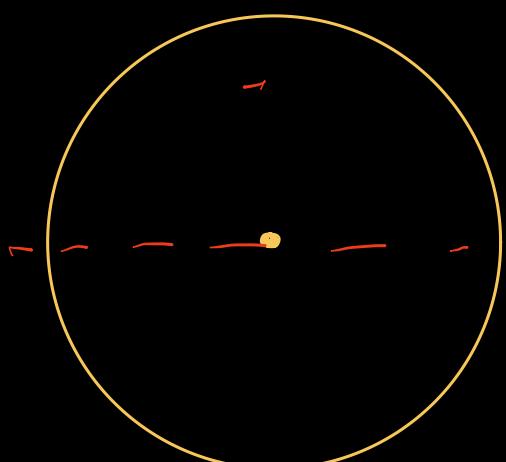
$$\Delta c_1 = c_{890^\circ\text{C}} - c_{1520}$$

$$= \frac{1}{\left(\frac{2\alpha}{3}\right)^{1/2}} \left[\left(\frac{3 \cdot 56}{0 \cdot 58}\right)^{1/3} - \left(\frac{3 \cdot 56}{1}\right)^{1/3} \right]$$

$$= 0.23.$$

$$\sigma = \sqrt{\frac{R^2}{4\alpha H}}, \quad \Delta \alpha = \frac{\Delta R}{\sqrt{4\alpha H}} = 0.23.$$

$$\therefore \Delta R = 0.23 \times \sqrt{4 \times 5 \times 0.1} = 0.32 \text{ mm}$$



$$\left. \begin{aligned} q &= \text{watt} \\ q' &= \text{W/L (per length)} \\ q'' &= \frac{w}{m^2} (\text{per area}) \\ q''' &= \frac{w}{m^3} (\text{per Vol}) \end{aligned} \right\}$$

*) Consider a large body with a line heat source buried in that system at center.

* It is assumed that line source is producing heat at a constant rate per unit length (Watt/m)

* Body initial temp : T_0

Temp = T_∞ ($r \rightarrow \infty$)

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial T}{\partial r} \right] = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{--- (1)}$$

Boundary conditions

$$\text{At } r=0; \quad -2\pi r K \frac{\partial T}{\partial r} = \frac{Q}{L}$$

$$\text{As } r \rightarrow \infty, \quad T = T_\infty \quad \text{--- (3)} \quad \hookrightarrow \text{--- (2)}$$

Initial condition

$$\text{At } t=0, \quad T = T_0 = T_\infty \quad \text{--- (4)}$$

Assume $\theta = T - T_\infty$

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial \theta}{\partial r} \right] = \frac{1}{\alpha} \frac{\partial \theta}{\partial t} \quad \text{--- (1A)}$$

BC Replace T with θ

$$\eta = \frac{r^2}{4\alpha t} \quad \text{--- (5)}$$

$$\frac{\partial \theta}{\partial r} = \frac{\partial \theta}{\partial \eta} \cdot \frac{\partial \eta}{\partial r} = \frac{\partial \theta}{\partial \eta} \cdot \left(\frac{2r}{4\alpha t} \right) \quad \text{--- (5A)}$$

$$\gamma \frac{\partial \phi}{\partial r} = \frac{2\gamma^2}{4\alpha t} - \frac{\partial \phi}{\partial u}$$

$$\frac{\partial}{\partial x} \left[\gamma \frac{\partial \phi}{\partial r} \right] = \frac{\partial}{\partial r} \left[\frac{2\gamma^2}{4\alpha t} \frac{\partial \phi}{\partial u} \right].$$

$$= \frac{\partial}{\partial u} \left[2u \frac{\partial \phi}{\partial u} \right] \frac{2\gamma}{4\alpha t}$$

$$= \left[2u \frac{\partial^2 \phi}{\partial u^2} + \frac{\partial \phi}{\partial u} \right] \frac{2\gamma}{4\alpha t}. \quad (5c)$$

$$\therefore \frac{1}{r} \frac{1}{\partial r} \left[\gamma \frac{\partial \phi}{\partial r} \right] = \left(2u \frac{\partial^2 \phi}{\partial u^2} + \frac{\partial \phi}{\partial u} \right) \frac{2}{4\alpha t}$$

(5D)

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial u} \frac{\partial u}{\partial t} = \frac{\partial \phi}{\partial u} \left[\frac{-r^2}{4\alpha t^2} \right] \quad (5E)$$

$$= \frac{\partial \phi}{\partial u} \left[-\frac{u}{t} \right]. \quad (5F)$$

$$(1A) \quad \left(2u \frac{\partial^2 \phi}{\partial u^2} + \frac{\partial \phi}{\partial u} \right) \cancel{\frac{2}{4\alpha t}} = \frac{1}{\alpha} \left(-\frac{\partial \phi}{\partial u} \frac{u}{t} \right)$$

$$n \frac{\partial^2 \phi}{\partial u^2} + \frac{\partial \phi}{\partial u} = -n \frac{\partial \phi}{\partial u}.$$

$$\text{i.e. } n \frac{\partial^2 \theta}{\partial y^2} + (1+n) \frac{\partial \theta}{\partial n} = 0 \quad \text{--- (6)}$$

Check

Boundary condition as $y \rightarrow 0, n \rightarrow 0$

$$\text{i.e. } -2\pi k \left(\frac{2y^2}{4\alpha t} \right) = \frac{\theta}{L}$$

↳ from (5A)

$$-2\pi k^2 n \frac{\partial \theta}{\partial n} = \frac{\theta}{L}$$

$$-4\pi k n \frac{\partial \theta}{\partial y} = \frac{\theta}{L} \quad \text{--- (6A)}$$

At $t=0, n \rightarrow \infty$

$t=r, n \rightarrow \infty$

06/2/24

* Metallurgical modelling of welding
↳ Chapter 1

$$\frac{d^2 \theta}{dn^2} + \left(\frac{1+n}{n} \right) \frac{d\theta}{dn} = 0 \quad \text{--- (6)}$$

$$\text{For } \frac{d^2 y}{dx^2} + f(x) \frac{dy}{dx} = 0$$

$$\text{Assume } \frac{dy}{dx} = p, \quad \frac{d^2 y}{dx^2} = \frac{dp}{dx}$$

$$\frac{dp}{dx} + f(x) p(x) = 0.$$

$$\frac{dp}{dx} = -f(x) p.$$

$$\frac{dp}{p} = -f(x) dx \Rightarrow \ln p = - \int f(x) dx + C.$$

$$\frac{dy}{dx} = p = a e^{- \int f(x) dx}.$$

$$\therefore y = a \int e^{- \int f(x) dx} + B.$$

$$\text{Here } f(x) = \frac{1+u}{u},$$

$$\text{So, } \Theta = A \int (e^{- \int \frac{1+u}{u} du}) dy + B.$$

$$\Theta = A \int e^{-(\ln u + u)} dy + B.$$

$$\boxed{\Theta = A \int \frac{e^{-u}}{u} dy + B.}$$

$$\Rightarrow \text{So, } [\Theta]_n^\infty = A \int_n^\infty \frac{e^{-u}}{u} dy$$

$$\Theta(\infty) - \Theta(n) = A \int_n^\infty \frac{e^{-u}}{u} dy$$

$$0 - \theta = A \int_{\infty}^{\infty} \frac{e^{-y}}{y} dy$$

Exponential integral function:

$$E_i(u) = \int_u^{\infty} \frac{e^{-y}}{y} dy$$

$$\therefore \theta = -A E_i(u)$$

As $u \rightarrow 0$,

$$u = \frac{r^2}{4\alpha t}$$

$$-4\pi k u \frac{d\theta}{du} = \frac{Q}{L}$$

$$-4\pi k u \left(A \frac{e^{-u}}{u} \right) = \frac{Q}{L}$$

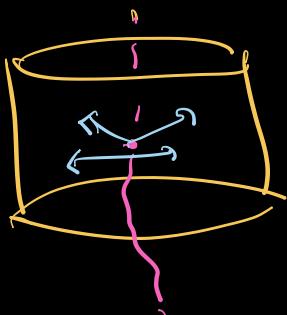
$$A = -\frac{Q}{4\pi k} e^u$$

$$T \rightarrow T_{\infty}, \quad \theta = -A E_i(u)$$

$$= \frac{Q}{4\pi k L} E_i\left(\frac{r^2}{4\alpha t}\right)$$

$$T = T_{\infty} + \frac{Q}{4\pi k L} E_i\left(\frac{r^2}{4\alpha t}\right)$$

$$t \rightarrow \infty, \quad E_i(\theta) \rightarrow \infty.$$

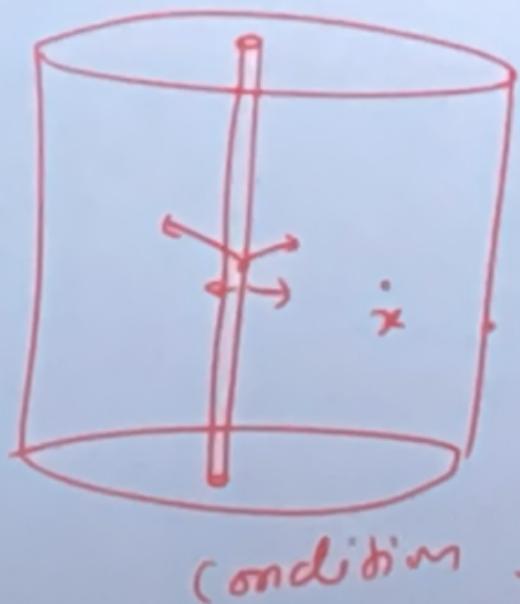


Governing Eqn:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{\partial T}{\partial r} \right] = \frac{1}{\alpha} \frac{\partial T}{\partial t}.$$

$$r \rightarrow \infty, \quad T = T_0.$$

Instantaneous heat source: Thermal Explosion at line region



Gr. E.

$$E_0 \quad \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (1)$$

$$r \rightarrow \infty, T = T_\infty$$

$$r \rightarrow 0 \quad \int_{\infty}^0 2\pi r L P(\rho(T-T_\infty)) dr = E_0$$

$$t = 0, T = T_\infty$$

$$\text{Now, } T = T_\infty + \frac{E_0}{4\pi\alpha\rho c_p L t} \exp\left(-\frac{r^2}{4\alpha t}\right).$$

* In resistance welding, $t = 0.1, 0.2 \text{ sec}$

$$\Rightarrow T = T_\infty + \frac{E_0}{\rho c_p (4\pi\alpha t)^2} e^{-\frac{r^2}{4\alpha t}}$$

* Dimensionless temp:

$$\Theta = \frac{T - T_\infty}{T_c - T_\infty}$$

Rate temp \downarrow $= T_m$

* Dimensionless time factor. $= \frac{t}{\sqrt{t_n}}$
 (heating time, pulse time)

* Dimensionless radius factor.

$$\sigma_2 = \sqrt{\frac{\sigma^2}{4\alpha t_n}}$$

* Dimensionless process parameter.

$$\gamma_2 = \frac{E_0}{(4\pi\alpha t) L P C (T_c - T_\infty)} \rightarrow \text{Joule}$$

$$L = \text{total thickness of plate.}$$

$$P C (T_c - T_\infty) = H_c - H_g \\ = \Delta H,$$

$$= \frac{q_0}{\Delta H 4\pi\alpha L} \rightarrow \text{watt.}$$

* $T = T_\infty + \frac{E_0}{4\pi\alpha L P C_p} \exp\left(-\frac{\sigma^2}{4\alpha t}\right)$

$$\text{Left) } \frac{T - T_{\infty}}{T_c - T_{\infty}} = \Theta = \frac{\frac{E_0}{4\pi\alpha L D H \times t} \exp\left(-\frac{\sigma^2}{4\alpha t}\right)}{x \frac{1}{T_c - T_{\infty}}}$$

$$\Theta = \frac{E_0}{4\pi\alpha L D H \times t} \exp\left(-\frac{\sigma^2}{4\alpha t}\right)$$

$$\Theta = \frac{q_0}{4\pi\alpha L D H \gamma_2} \exp\left(-\frac{\sigma_2^2}{\gamma_2}\right)$$

$$\Theta = \frac{w_2}{T_2} \exp\left(-\frac{\sigma_2^2}{T_2^2}\right)$$

Spot welding (graph analysis)

Peak temperature

$$\Theta = \frac{n_2}{\zeta_2} \exp\left(-\frac{\sigma_2 v}{\zeta_2}\right)$$

$$\frac{\partial \Theta}{\partial \zeta_2} = 0$$

$$\frac{\Theta_p}{n_2} =$$

$$\frac{\partial \Theta}{\partial \zeta_2}$$

$$= -\frac{n_2}{\zeta_2 v} \exp\left(-\frac{\sigma_2 v}{\zeta_2}\right)$$

$$+ \frac{n_2}{\zeta_2} \exp\left(-\frac{\sigma_2 v}{\zeta_2}\right) \frac{\sigma_2 v}{\zeta_2} = 0$$

$$\Rightarrow \boxed{\sigma_2^v = z_2}$$

$$\theta_p = \frac{n}{e^{z_2}} \quad \sigma_2^v = z_2$$

$$\theta_p = \frac{n}{e^{\sigma_2^v}}$$

$$\theta = \frac{n_2}{\bar{z}_2} \exp\left(-\frac{\sigma_2^v}{z_2}\right)$$

$$\sigma_2^v = z_2$$

$$\boxed{\theta_p = \frac{n_2}{e^{z_2}} = \frac{n_2}{e^{\sigma_2^v}}}$$

Problem: *

Numerical: thermal analysis: SW

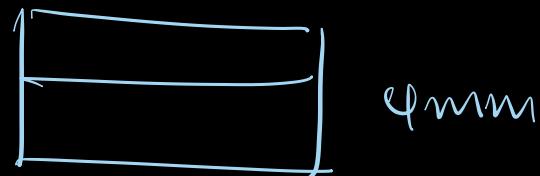
Consider spot welding of 2 mm plates of low alloy steel under the following operational conditions:

$I=8000\text{A}$, Total voltage drop between the electrodes is $=1.6 \text{ V}$, $t_h=0.3 \text{ s}$, Transfer efficiency: 0.5, $T_0=20^\circ\text{C}$, $T_c=T_m= 1520^\circ\text{C}$, $\alpha=5 \text{ mm}^2/\text{s}$, $\Delta H=7.5\text{J/mm}^3$

1. Calculate the cooling time from 800 to 500°C ($\Delta t_{800-500}$) at the center of the weld and the cooling rate (C.R.) at the onset of the austenite to ferrite transformation(475°C).
2. Calculate the total width of the fully transformed region adjacent to the fusion boundary. Assuming that the transformation temperature is equal to 890°C for this particular steel.

Soln:

Thickness = 4mm



$$\Theta = \frac{T - T_\alpha}{T_c - T_\alpha} \approx \frac{T - 20}{1520}$$

$$\Theta_{1520} = 1$$

$$\Theta_{500} = 0.52$$

$$\Theta_{890} = 0.58$$

$$\Theta_{500} = 0.32$$

$$\theta = \frac{u}{t} \times \exp\left(-\frac{u^2}{t}\right).$$

At $\sigma = 0$,

$$\theta = \frac{u}{t} \Rightarrow t = \frac{u}{\theta}$$

$$T_{800} = \frac{u}{\theta_{800}}, \quad T_{500} = \frac{u}{\theta_{500}}$$

$$T_{470} = \dots$$

$$\Delta T = \frac{\Delta t}{t}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}.$$

$$\boxed{\theta = \frac{u_2}{e \sigma_2^2}}$$

Use this

B.C's $x=0, -kA \frac{dT}{dx} = Q$

$$x \rightarrow \infty, T = T_\infty$$

T.C's $t=0, T = T_\infty$

$$T = T_\infty + \frac{Q_x}{kA} \left[\frac{\sqrt{4\alpha t}}{x \sqrt{\pi}} e^{-\frac{x^2}{4\alpha t}} \right]$$

$$= \exp\left(-\frac{x}{\sqrt{4\alpha t}}\right)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial x}$$

$$x \rightarrow \infty, \quad T = T_\infty$$

$$t = 0, \quad T = T_\infty$$

$$x \rightarrow 0, \quad \int_0^\infty A \rho C_p (T - T_\infty) dx = E_0.$$

$$T = T_\infty + \frac{E_0}{\rho C_p A \sqrt{4\pi \alpha t}} \exp\left(-\frac{x^2}{4\alpha t}\right)$$

