

Q18-

$$\frac{dy}{dx} = yx^3 - 1.5y$$

$$y(0) = 1$$

$x=0$ to 1 ; $h=0.5$

Euler's Method

→ Eulers

→ Heuns

→ Mid pt

→ 4th RK

$$\frac{dy}{dx} = f(x, y)$$

$$y_{i+1} = y_i + f(x_i, y_i) h$$

i	x	y	f(x, y)	e%
0	$x_0 = 0$	$y_0 = 1$	$f(x_0, y_0)$ = -1.5	0
1	$x_1 = 0.5$	$y_1 = y_0 + f(x_0, y_0) h$ = $1 + (-1.5) 0.5$ = 0.25	$f(x_1, y_1)$ = -0.34375	47.9
2	$x_2 = 1$	$y_2 = y_1 + f(x_1, y_1) h$ = $0.25 + (-0.34375) 0.5$ = 0.07013		72.7

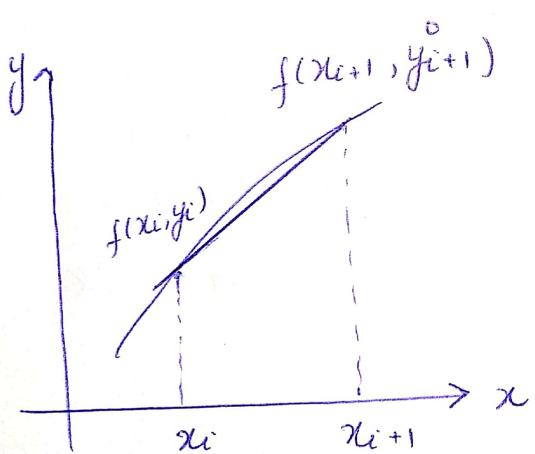
ERROR 8-

Analytical soln.

$$y = e^{-\frac{x^4}{4} - 1.5x}$$

x	y
0	1
0.5	0.4790
1	0.2865

HEUN'S METHOD



$$\hat{y}_{i+1}^{\circ} = y_i + f(x_i, y_i) \cdot h$$

$$y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, \hat{y}_{i+1})}{2} \cdot h$$

i	x	y	$f(x, y)$	$y_{i+1}^{\circ} = y_i + f(x_i, y_i)h$	$f(x_{\text{end}}, y_{\text{end}})$	$f_{\text{ang.}}$
0	$x_0 = 0$	$y_0 = 1$	$f(x_0, y_0) = -1.5$	$y_1^{\circ} = y_0 + f(x_0, y_0)h$ $= 1 + (-1.5) \cdot 0.5$ $= 0.25$	$f(x_1, y_1^{\circ})$ $= -0.34375$	$\frac{-1.5 - 0.34375}{2}$ $= -0.92188$
1	$x_1 = 0.5$	$y_1 = y_0 + f_{\text{ang.}} h$ $= 1 - 0.92188 \cdot 0.5$ $= 0.53906$	$f(x_1, y_1) = -0.741211$	$y_2^{\circ} = y_1 + f(x_1, y_1)h$ $= 0.53906 - 0.741211 \cdot 0.5$ $= 0.168457$	$f(x_2, y_2^{\circ})$ $= -0.084229$	$\frac{-0.741211 - 0.084229}{2}$ $= -0.412720$
2	$x_2 = 1$	$y_2 = y_1 + f_{\text{ang.}} h$ $= 0.53906 - 0.41272 \cdot 0.5$ $= 0.3327$				

$$f(x, y) = yx^3 - 1.5y$$

$\text{error } e_{\text{r.}}$	$y_{\text{estimated}}$	$y_{\text{analytical}}$
0	1	1
12.1	0.53906	0.47981
16.1	0.3327	0.2865

Midpoint method

$$y_{i+\frac{1}{2}} = y_i + f(x_i, y_i) \frac{h}{2}$$

$$y_{i+1} = y_i + f(x_{i+\frac{1}{2}}, y_{i+\frac{1}{2}}) h$$

i	x	y	$f(x, y)$	x_m	y_m	f_m $f(x_m, y_m)$
0	$x_0=0$	$y_0=1$	$f(x_0, y_0)$ = -1.5	0.25	$y_{0+\frac{1}{2}} = y_0 + f(x_0, y_0) \frac{h}{2}$ = $1 - 1.5 \times 0.25$ = 0.625	$f(0.25, 0.625)$ - 0.9277
1	$x_1=0.5$	$y_1 = y_0 + f(x_m, y_m) h$ = $1 - 0.9277 \times 0.5$ = 0.53613	$f(x_1, y_1)$ = -0.737	0.75	$y_{1+\frac{1}{2}} = y_1 + f(x_1, y_1) \frac{h}{2}$ = $0.53613 - 0.737 \times 0.25$ = 0.35108	$f(0.75, 0.35108)$ - 0.3793
2	$x_2=1$	$y_2 = y_1 + f(x_m, y_m) h$ = $0.53613 - 0.3793 \times 0.5$ = 0.34647				

\tilde{y}
 $e^{t(\%)}$
 0
 11.7
 0.53613
 20.9
 0.34647

(d) Fourth order RK Method

$$y_{i+1} = y_i + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$

$$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$$

$$k_4 = f(x_i + h, y_i + k_3 h)$$

$$x_0 = 0; y_0 = 1$$

for y_1 :-

$$y_1 = y_0 + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) h$$

$$k_1 = f(x_0, y_0) = -1.5$$

$$k_2 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1 h\right)$$

$$= f(0.25, 0.625) = -0.927$$

$$k_3 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2 h\right)$$

$$= f(0.25, 0.768) = -1.1401$$

$$k_4 = f(x_0 + h, y_0 + k_3 h)$$

$$= f(0.5, 0.42995) = -0.15912$$

$$y_1 = 0.481096$$

$$x_1 = 0.5 ; \quad y_1 = 0.981096$$

$$y_2 = ?$$

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$k_1 = f(x_1, y_1) = -0.66151$$

$$k_2 = f(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}k_1 h) = -0.34030$$

$$k_3 = f(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}k_2 h) = -0.4269$$

$$k_4 = f(x_1 + h, y_1 + k_3 h) = -0.1338$$

$$y_2 = 0.28693$$

Q28

$$\frac{d^2y}{dx^2} = y$$

$$\text{BC; } y(0) = 0$$

$$y(2) = 3.627$$

x	$y(x) = \sinh(x)$
0	0
0.5	0.5211
1	1.1752
1.5	2.1293
2	3.6269

$$y' = z$$

$$\frac{dy}{dx} = z$$

$$y(0) = 0$$

$$y(2) = 3.627$$

$$\frac{dz}{dx} = y$$

$$y'(0) = z(0) = 1$$

$$y'(0) = z(0) = 2$$

} guesses.

$$\frac{dy}{dx} = z \quad \& \quad \frac{dz}{dx} = y$$

$$y(0) = 0$$

$$z(0) = 1$$

$$\left. \begin{array}{l} \frac{dy}{dx} = z \quad \& \quad \frac{dz}{dx} = y \\ y(0) = 0 \quad \quad \quad z(0) = 2 \end{array} \right\}$$

$$\frac{dy}{dx} = z = f$$

$$x=0 \\ y=0$$

$$h = \underline{\underline{0.15}}$$

$$\frac{dz}{dx} = y = g$$

$$x=0 \\ z=1.$$

x	y	f	$y_i + f h y_2$	f_m	x	z	g	$z_i + f h y_2$	z_m	g_m
0	0	1	0.25	1	0	1	0	1	0.25	
0.5	$y = y_0 + f_m h$ 0.5	1.1250	$0.15 + 1.125 \times 0.25$ $= 0.7813$	1.25	0.5	$z_0 + g_m h$ 1.1250	0.5	$1.1250 + 0.15 \times 0.25$ $= 1.25$	0.7813	
1	$y_2 = y_1 + f_m h$ 0.5 + 1.25 × 0.5 1.125	1.5156	$1.125 + 1.5156 \times 0.25$ $= 1.5039$	1.7969	1	$z_1 + g_m h$ $= 1.1250 + 0.7813 \times 0.5$ $= 1.5156$	1.125	$1.5156 + 1.125 \times 0.25$ $= 1.7969$	1.5039	
1.5	$y_3 = y_2 + f_m h$ 2.0234	2.2676	$2.0234 + 2.2675 \times 0.25$ $= 2.5903$	2.7739	1.5	$z_2 + g_m h$ 2.2675	2.0234	$2.2675 + 2.0234 \times 0.25$ $= 2.7734$	2.5903	
2	$y_4 = y_3 + f_m h$ 3.4102				2	$z_4 = z_3 + g_m h$ 3.5627				

New B-

$$y(0) = 0$$

$$z(0) = 2.$$

x	y	z	f	g	g_m	Z_m	f_m	g_m
0	0	2	2	0	0.15	2.0	2	0.15
0.5	1	2.25	2.25	1	1.5625	2.5	2.5	1.5625
1	2.25	3.0313	3.0313	2.25	3.0070	3.593	3.5938	3.0070
1.5	4.0469	4.5357	4.5352	4.046	5.1807	5.5469	5.5469	5.1807
2	6.0203							

$$z(0) = 1$$

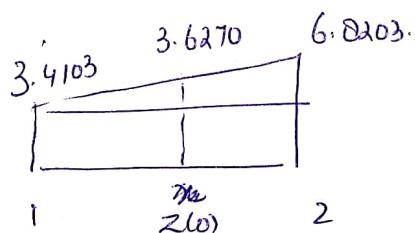
$$y(2) = 3.4102.$$

$$z(0) = 2$$

$$y(2) = 6.0203$$

$$z(0) = ?$$

$$y(2) = 3.6270$$



α -

$$z(0) = 1.0636$$

$$y(0) = 0$$

$$z(0) = 1.0636$$

$$\frac{dy}{dx} = z = f$$

$$x=0 \\ y=0$$

$$\frac{dz}{dx} = y = g$$

$$x=0 \\ z=1.0636$$

x	y	z	f	g	y_m	z_m	f_m	g_m
0	0	1.0636	1.0636	0.10	0.265	1.063	1.063	0.265
0.5	0.5318	1.1965	1.1965	0.531	0.830	1.329	1.329	0.830
1	<u>1.1965</u>	1.6120	1.6120	1.196	1.599	1.911	1.911	1.599
1.5	2.1521	2.4110	2.4110	2.152	2.755	2.949	2.949	2.755
2	3.6270	3.7893						

$$x=1 \quad \tilde{y} = 1.1965$$

$$y = 1.1752$$

$$e = 1.02\%$$

$$Q_2(b) \quad \frac{d^2y}{dx^2} - y = 0$$

$$y(0) = 0$$

$$y(2) = 3.627$$

$$h = 1$$

$h=1$

$$y_0 = 0 ; \quad y_2 = 3.627 ; \quad y_1 = ?$$

$$\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} - y_i = 0$$

$$\frac{y_0 - 2y_1 + y_2}{1} - y_1 = 0$$

$$y_1 = \frac{y_0 + y_2}{3} = 1.209.$$

$$\tilde{y} = 1.209$$

$$y = 1.1752$$

$$er = 2.00\%.$$

$$(c) \quad h = 0.5$$

$$y_0, y_1, y_2, y_3, y_4$$

$$\begin{cases} x = 0 - 2 \\ x = 0, 0.5, 1, 1.5, 2 \end{cases}$$

$$y_0 = 0$$

$$y_4 = 3.627$$

$$\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} - y_i = 0 \quad i = 1, 2, 3$$

$$y_{i-1} - 2.25 y_i + y_{i+1} = 0$$

for y_1

$$y_0 - 2.25 y_1 + y_2 = 0$$

$$-2.25 y_1 + 1 \cdot y_2 + 0 \cdot y_3 = 0$$

for y_2

$$y_1 - 2.25 y_2 + y_3 = 0$$

$$y_1 - 2.25 y_2 + 1 \cdot y_3 = 0$$

for y_3

$$y_2 - 2.25 y_3 + y_4 = 0$$

$$0 \quad 1 \cdot y_2 - 2.25 y_3 = -3.627$$

$$Ax = B$$

$$\begin{bmatrix} -2.25 & 1 & 0 \\ 0 & -2.25 & 1 \\ 0 & 1 & -2.25 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3.627 \end{bmatrix}$$

$$y_1 = 0.52637$$

$$y_2 = 1.18433$$

$$y_3 = 2.13037$$

i	x_i	y_i	
0	0.5	0.52637	
1	1	1.18433	$\rightarrow \tilde{y} = 1.18433;$
2	1	2.13037	$y = 1.1752$
3	1.5		$e = 0.78\%$

(d) Richardson extrapolation

$$x_i = 1 \quad ; \quad y_i = 1.209 \quad (h=1)$$

$$x_i = 1 \quad ; \quad y_i = 1.18433 \quad h = 0.3$$

for REx

$$\begin{aligned}y_i &= \frac{4D_{h/2} - D_h}{3} \\&= \frac{4 \times 1.18433 - 1.209}{3} \\&= 1.1761\end{aligned}$$

$$y(x=1) = 1.1752$$

$$\tilde{y} = 1.1761$$

$$e = 0.077\%$$