ESO201A : THERMODYNAMICS

2021-22 Ist semester

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Lecture 23

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Although both symbols "x" and "ϕ" are used to denote exergy per unit mass of a given control mass system, we will, henceforth, use the symbol "ϕ".

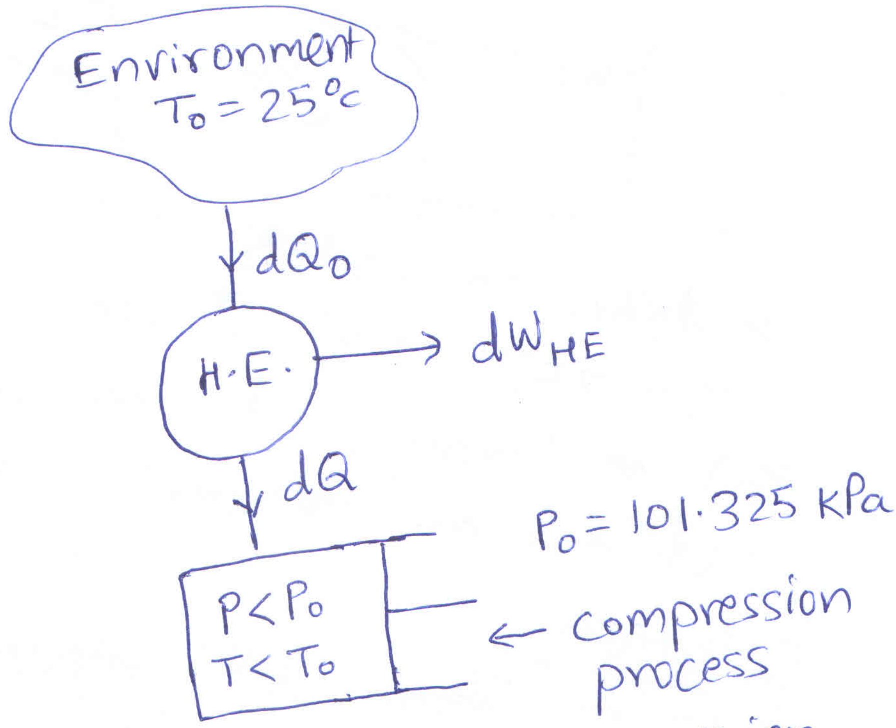
In lecture 22, we have seen that exergy per unit mass of a system at a given temperature T and pressure P, when the system is at rest, is given by

$$\phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$$

If the system possesses a certain kinetic energy and potential energy, then exergy per unit mass is given by

$$\phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \underbrace{\frac{V^2}{2}}_{\text{K.E per unit mass}} + \underbrace{gZ}_{\text{P.E. per unit mass}}$$

Note that even if temperature and pressure of the system are less than that of the environment ($T < T_o$, $P < P_o$), system still possesses a certain exergy (as given by the equation above). This is because we can extract work from the system by using a heat engine with environment as the source and the system as the sink



Note that during the compression ($P \rightarrow P_o$), the "inward" movement of piston can be used to lift a weight

Importance of exergy in a process between specified initial and final states

Let ΔU and ΔS be the change in internal energy and entropy of a control mass system in a process in which it exchanges heat with a single thermal energy reservoir at temperature T_0

In lecture 21, we have seen that actual (irreversible) work output in such a process is given by

$$W_{out} = T_0 \Delta S - \Delta U - T_0 \Delta S_T \quad \text{— A}$$

If ΔV is the charge in volume, then, $P_0 \Delta V$ = work done ON the environment

Therefore, useful work output is given by

$$W_{useful} = W_{out} - P_0 \Delta V \quad \text{— B}$$

Substituting A in B, we get 4

$$W_{useful} = T_o \Delta S - P_o \Delta V - \Delta U - T_o \Delta S_T \quad \text{— C}$$

The change in exergy in the process is

$$m \Delta \phi = \Delta U - T_o \Delta S + P_o \Delta V$$

OR $$\Phi_2 - \Phi_1 = \Delta U - T_o \Delta S + P_o \Delta V \quad \text{— D}$$

where

$m\phi_1 = \Phi_1 = X_1 =$ Exergy (or work potential) in the initial state '1'

$m\phi_2 = \Phi_2 = X_2 =$ Exergy (or work potential) in the final state '2'.

Substituting D in C, we get

$$W_{useful} = (\Phi_1 - \Phi_2) - T_o \Delta S_T \quad \text{— E}$$

For a reversible process, $\Delta S_T = 0$

$$\Rightarrow W^{rev}_{useful} = (\Phi_1 - \Phi_2) \quad \text{— F}$$

= Decrease in work potential (or exergy) during the process

Therefore,

Exergy (or work potential) destroyed

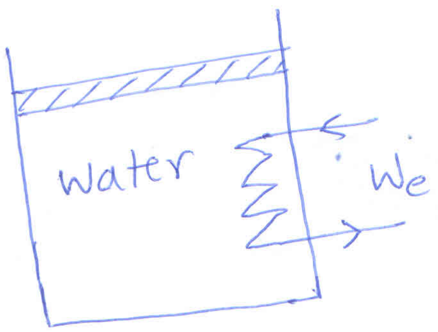
= (Decrease in work potential) − (actual useful work)

$= (\Phi_1 - \Phi_2) - W_{useful}$

using Eq. E (see last page)

$= T_0 \Delta S_T$

Example (8-33 of Textbook)



An insulated piston-cylinder device contains 8 L of saturated liq. water at 120 kPa. Electric work of 1400 kJ is done ON water. Considering $T_0 = 25°C$ and $P_0 = 100$ kPa, determine

(a) Minimum work required

(b) exergy (or work potential) destroyed.

Solution

(a) Although it is not explicitly mentioned, minimum work required or minimum work input for the process between specified initial and final state excludes work done by the environment in this particular problem.

Total work required in a process is negative of the total work done in a process, i.e.,

$$W_{in} = -W_{out}$$
$$= \Delta U - T_0 \Delta S + T_0 \Delta S_T \quad \text{— (G)}$$

↑ from Eq. (A)

Net work required in a process is equal to total work required minus the work done by environment:

[* see footnote below]

$$W_{useful,in} = \underbrace{W_{in}}_{\text{Total work required}} - \underbrace{(-P_0 \Delta V)}_{\text{work done by the environment ON the system}}$$

Using (G), we get

$$W_{useful,in} = \Delta U + P_0 \Delta V + T_0 \Delta S_T - T_0 \Delta S$$

or $$W_{useful,in} = (\Phi_2 - \Phi_1) + T_0 \Delta S_T \quad \text{— (H)}$$

since $\Phi_2 - \Phi_1 = \Delta U + P_0 \Delta V - T_0 \Delta S$

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* $W_{useful,in} = -W_{useful}$. Here W_{useful} is the useful work extracted from the system <u>excluding</u> work done on the environment

In a reversible process $\Delta S_T = 0$ and hence work required is minimum [see Eq. H]

Thus, minimum work required is given by

$$W^{rev}_{useful,in} = (\Phi_2 - \Phi_1) \quad \text{—— (I)}$$

$=$ Increase in exergy or work potential

In the given example, initial state is a saturated liquid @ 120 kPa

By linear interpolation

$$\begin{cases} u_1 = u_{f@120kPa} = 438.9 \text{ kJ/kg} \\ s_1 = s_{f@120kPa} = 1.356 \text{ kJ/kg·K} \\ v_1 = v_{f@120kPa} = 0.001047 \text{ m}^3\text{/kg} \end{cases}$$

We will assume that process is sufficiently slow so that mechanical equilibrium is maintained throughout the process.

Applying first law to the process, we get

$$\Delta U = \cancel{Q_{in}}^{\,0} + W_{e,in} - \int_1^2 P_{ext}\, dV$$

(insulated)

Due to condition of mechanical equilibrium as mentioned above,

$P_{ext} = P$ (throughout the process)

Further, $P = 120\text{ kPa}$ is constant

$$\Rightarrow \quad W_{e,in} = \Delta U + P\Delta V$$

$$= \Delta(U + PV)$$

$$= \Delta H$$

$$h_1 = h_{f@120\text{ kPa}} = 439.02 \text{ kJ/kg}$$

$$m = \frac{V}{v_1} = \frac{0.008 \text{ m}^3}{(0.001047 \text{ m}^3/\text{kg})} = 7.641 \text{ kg}$$

$$W_{e,in} = 1400 \text{ kJ}$$

$$h_2 = \frac{m h_1 + W_{e,in}}{m} = 622.24 \frac{\text{kJ}}{\text{kg}}$$

$$h_{f@120\text{kPa}} < h_2 < h_{g@120\text{kPa}}$$

$$x_2 = \frac{h_2 - h_{f@120\text{kPa}}}{h_{fg@120\text{kPa}}} = 0.0817$$

$h_{fg@120\text{kPa}} \rightarrow$ (2243.96 kJ/kg by linear interpolation)

By linear interpolation
$$\left[\begin{aligned} v_{g@120kPa} &= 1.438 \text{ m}^3/\text{kg} \\ u_{fg@120kPa} &= 2072.6 \text{ kJ/kg} \\ s_{fg@120kPa} &= 5.939 \text{ kJ/kg·K} \end{aligned}\right.$$

$$\Rightarrow \quad u_2 = 608.23 \text{ kJ/kg}$$
$$s_2 = 1.841 \text{ kJ/kg·K}$$
$$v_2 = 0.1184 \text{ m}^3/\text{kg}$$

minimum work required [see Eq. I]

$$W^{rev}_{useful,in} = \Phi_2 - \Phi_1$$
$$= m\left[(u_2-u_1)+P_0(v_2-v_1)-T_0(s_2-s_1)\right]$$
$$=(7.641)(36.4626)$$
$$= \underline{\underline{278.6 \text{ kJ}}}$$ — [Ans. for part (a)]

(b) Exergy destroyed in the process is the difference between net work required (or) net work done on the system and the increase in the work potential (or exergy)

From Eqn. (H),

Exergy destroyed (or lost work) $= W_{useful,in} - (\Phi_2 - \Phi_1)$

$= T_0 \Delta S_T$

In the present example,

$$\Delta S_T = \Delta S + \Delta S_{environment}$$

Since process is adiabatic (which also means no friction), environment is not receiving or losing heat. Thus, according to Clausius equality, change in entropy of environment is zero (we are implicitely considering that environment is undergoing an internally reversible process), i.e;

$$\Delta S_{environment} = 0$$

$$\Rightarrow \Delta S_T = \Delta S = m(s_2 - s_1)$$
$$= (7.641)(1.841 - 1.356)$$
$$= 3.706 \text{ KJ/K}$$

Exergy (or work potential) destroyed

$= T_0 \Delta S_T = (298.15)(3.706)$

$= \underline{\underline{1105 \text{ KJ}}}$

[Ans to part (b)]

Alternative solution for part (b)

Total work required (or) total work done on the system is

$$W_{in} = W_{e,in} + W_{b,in}$$

$$= W_{e,in} + P\underbrace{(-\Delta V)}_{\text{Decrease in volume}}$$

$$= W_{e,in} + mP(v_1 - v_2)$$

$$= (1400\ kJ) + (7.641\ kg) \times (120\ kPa) \times (0.001047 - 0.1184)$$

$$= \underline{1292.4\ kJ}$$

Net work required (or) net work done on the system is

$$W_{useful,in} = W_{in} - P_0(-\Delta V)$$

$$= 1292.4 + (100\ kPa)(7.641)(0.1184 - 0.001047)$$

$$= 1382.1\ kJ$$

Exergy (or work potential) destroyed

$$= W_{useful,in} - (\Phi_2 - \Phi_1)$$

$$= (1382.1) - (278.6)$$

$$= \underline{1103.5\ kJ}$$

↳ This agrees closely with the value obtained on previous page