Tutorial-05

(1.) use Gauss Elimination Method & four-digit floating point arithmetic

(a.) without pivoting

$$0.003x + 59.14y = 59.17$$

$$5.291x - 6.130y = 46.78$$

$$Ax = b$$

A → Coefficient matrix

$[A|b] \Rightarrow$ Augumented matrix

$$[A|b] = \left[\begin{array}{cc|c} 3.000 \times 10^{-3} & 5.914 \times 10^{1} & 5.917\times10^{1} \\ 5.291 \times 10^{0} & -6.130 \times 10^{0} & 4.678\times10^{1} \end{array}\right]$$

forward Elimination

$$l_{21} = \frac{a_{21}}{a_{11}} = \frac{5.291 \times 10^{0}}{3.000 \times 10^{-3}}$$

$$l_{21} = 1.764 \times 10^{3}$$

$$R_2 = R_2 - R_1 l_{21}$$

$$[A|b] = \left[\begin{array}{cc|c} 3.000\times10^{-3} & 5.914\times10^{1} & 5.917\times10^{1} \\ 0 & -1.043\times10^{5} & -1.044\times10^{5} \end{array}\right]$$

Backward Substitution

$$R_2 \Rightarrow \quad -1.043\times10^{5}\, y = -1.044\times10^{5}$$

$$\boxed{y = 1.001\times10^{0}}$$

$$R_1 \Rightarrow \quad 3.000\times10^{-3} x + 5.914\times10^{1} y = 5.917\times10^{1}$$

$$\boxed{x = -1.000\times10^{1}}$$

(b.) Use partial pivoting and four-digit rounding arithmetic

[Exchange R_1 & R_2 b/c ~~relative~~ magnitude of a_{11} is very small compared to a_{12}]

$$[A|b] = \left[\begin{array}{cc|c} 5.291\times10^{0} & -6.130\times10^{0} & 4.678\times10^{1} \\ 3.000\times10^{-3} & 5.914\times10^{1} & 5.917\times10^{1} \end{array}\right]$$

$$l_{21} = \frac{a_{21}}{a_{11}} = \frac{3.000\times10^{-3}}{5.291\times10^{0}}$$

$$l_{21} = 5.670\times10^{-4}$$

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$R_2 \rightarrow R_2 - l_{21} R_1$

$$[A|b] = \left[\begin{array}{cc|c} 5.291 \times 10^0 & -6.130 \times 10^0 & 4.678 \times 10^1 \\ 0 & 5.914 \times 10^1 & 5.914 \times 10^1 \end{array}\right]$$

Backward substitution

$R_2 \rightarrow \quad 5.914 \times 10^1 y = 5.914 \times 10^1$

$$\boxed{y = 1.000 \times 10^0}$$

$\cancel{R_1 \rightarrow 5.291 \times 10^0 x + 5.}$

$R_1 \rightarrow 5.291 \times 10^0 x - 6.130 \times 10^0 y = 4.678 \times 10^1$

$$\boxed{x = 1.000 \times 10^1}$$

correct Answer $\Rightarrow \left.\begin{array}{l} X = 10 \\ Y = 1 \end{array}\right\}$

with pivoting $\quad X = 10$

$Y = 1$

without pivoting $\quad X = -10$

$Y = 1$

Matrix Decomposition

(i). ~~Gauss~~ Elimination

$$5x_1 + x_2 + 0x_3 = 7$$
$$x_1 + 5x_2 + x_3 = 14$$
$$0x_1 + x_2 + 5x_3 = 17$$

$$Ax = b$$

if $|A| \neq 0$

then $A \rightarrow LU$

L: Lower triangular matrix

U: upper triangular matrix

$$A \rightarrow \begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 1 \\ 0 & 1 & 5 \end{bmatrix}$$

$$|A| = 115 \neq 0$$

$A \longrightarrow LU$ [Possible]

(i) Gauss Elimination

all l_{ij} becomes the coefficients of LTM

$$A = \begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 1 \\ 0 & 1 & 5 \end{bmatrix}$$

L U

Step 01 $\quad l_{21} = \frac{a_{21}}{a_{11}} = \frac{1}{5} = 0.2$

$$l_{31} = \frac{a_{31}}{a_{11}} = \frac{0}{5} = 0$$

$$A' \Rightarrow \begin{cases} R_2 \rightarrow R_2 - l_{21}R_1 \\ R_3 \rightarrow R_3 - l_{31}R_1 \end{cases}$$

$$A' = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 4.8 & 1 \\ 0 & 1 & 5 \end{bmatrix}$$

$$l_{32} = \frac{a_{32}}{a_{22}} = \frac{1}{4.8} = 0.2083$$

$$R_3 \rightarrow R_3 - l_{32}R_2$$

$$A' = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 4.8 & 1 \\ 0 & 0 & 4.792 \end{bmatrix} = UTM$$

$$LTM = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 0 & 0.2083 & 1 \end{bmatrix}$$

original system of Eq^{n}s $Ax = b$

$$LUx = b$$

$$Ly = b \qquad [Ux = y]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 0 & 0.2083 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 14 \\ 17 \end{bmatrix}$$

forward substitution

$R_1 \longrightarrow \boxed{y_1 = 7}$

$R_2 \longrightarrow 0.2y_1 + y_2 = 14$

$$y_2 = 14 - 1.4$$

$$\boxed{y_2 = 12.6}$$

$R_3 \longrightarrow 0.2083y_2 + y_3 = 17$

$$\boxed{y_3 = 14.375}$$

$$Ux = y$$

$$\begin{bmatrix} 5 & 1 & 0 \\ 0 & 4.8 & 1 \\ 0 & 0 & 4.792 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 12.6 \\ 14.375 \end{bmatrix}$$

backward substitution

$R_1 \rightarrow \quad 4.792\, x_3 = 14.375$

$$\boxed{x_3 = 3} \quad (\text{Approx})$$

$R_2 \rightarrow \quad 4.8x_2 + x_3 = 12.6$

$$\boxed{x_2 = 2}$$

$R_1 \rightarrow \quad 5x_1 + x_2 = 7$

$$x_1 = \frac{(7-2)}{5}$$

$$\boxed{x_1 = 1}$$

Use doolittle decomposition

$\Downarrow$

Lower triangular Matrix should have only 1 ~~at~~ as its diagonal elements.

$$\begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 1 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

compare

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$$\left.\begin{aligned} u_{11} &= 5 \\ u_{12} &= 1 \\ u_{13} &= 0 \end{aligned}\right\}$$

$l_{21}u_{11} = 1 \Rightarrow l_{21} \times 5 = 1 \Rightarrow \boxed{l_{21} = 0.2}$

$l_{21}u_{12} + u_{22} = 5 \Rightarrow 0.2 \times 1 + u_{22} = 5$

$\boxed{u_{22} = 4.8}$

$l_{21}u_{13} + u_{23} = 1 \Rightarrow 0.2 \times 0 + u_{23} = 1$

$\boxed{u_{23} = 1}$

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$l_{31} u_{11} = 0$

$l_{31} \times 5 = 0$

$\boxed{l_{31} = 0}$

$l_{31} u_{12} + l_{32} u_{22} = 1 \Rightarrow 0 \times 1 + l_{32} \times 4.8$

$\boxed{l_{32} = 0.2083}$

$l_{31} u_{13} + l_{32} u_{23} + u_{33} = 5$

$\boxed{u_{33} = 4.791}$

$Ax = b$

$LUx = b$

$Ly = b$

$\hookrightarrow$ get y

$Ux = y$

$x \longrightarrow$ final Answer

these step have already been performed in previous part (Gauss Elimination)

Use Crout Decomposition

$$\begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 1 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

$$\boxed{l_{11} = 5}$$

$$l_{11}u_{12} = 1 \Rightarrow \boxed{u_{12} = 0.2}$$

$$l_{11}u_{13} = 0 \Rightarrow \boxed{u_{13} = 0}$$

$$\boxed{l_{21} = 1}$$

$$l_{21}u_{12} + l_{22} = 5$$

$$1 \times 0.2 + l_{22} = 5$$

$$\boxed{l_{22} = 4.8}$$

$$l_{21}u_{13} + l_{22}u_{23} = 1$$

$$\boxed{u_{23} = 0.2083}$$

$$Ux = y$$

$$\begin{bmatrix} 1 & 0.2 & 0 \\ 0 & 1 & 0.2083 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 2.625 \\ 3 \end{bmatrix}$$

for backward substitution

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

use crout's decomposition

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{ij} \rightarrow$ any no. from your Roll. No.

determine L and U