ESO201A : THERMODYNAMICS
2021-22 Ist semester
IIT Kanpur

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Lecture 19

Adiabatic steady flow equipment :

General equation of rate of change of entropy in a control volume :

$$\frac{d}{dt}(S_{CV}) = \sum_{in} \dot{m}\, s - \sum_{out} \dot{m}\, s + \sum_{k} \left(\frac{\dot{Q}_k}{T_k}\right) + \dot{S}_{gen}$$

For a steady-flow equipment,

$$\frac{d}{dt}(S_{CV}) = 0$$

Lets consider an <u>adiabatic</u> flow device with a single inlet and single outlet. The entropy equation reduced to :

$$\dot{m}(s_1 - s_2) + \dot{S}_{gen} = 0$$

Applying first law (in rate form) we get

$$\dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right] + \dot{W}_{in} = 0$$

Adiabatic steady flow equipment :

Re-arranging first law equation for adiabatic flow equipment, we get

$$w_{in} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

Here, w_{in} is the work done per unit mass of the fluid flowing through the equipment,

$$w_{in} = \frac{\dot{W}_{in}}{\dot{m}}$$

Integrating the relation dh = T ds + v dP, we get

$$h_2 - h_1 = \int_1^2 T\,ds + \int_1^2 v\,dP$$

Substituting the expression enthalpy difference in first law equation above, we get

$$w_{in} = \int_1^2 T\,ds + \int_1^2 v\,dP + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

Reversible – adiabatic (isoentropic) steady flow equipment :

If the flow equipment is reversible,

$$\dot{S}_{gen} = 0$$

Substituting in the entropy balance equation (see slide 2), we get

$$s_1 = s_2$$

Therefore the first term in the right hand side of the first law equation will be zero, i.e.,

$$\int_1^2 T\, ds = 0 \quad \text{since} \quad s_1 = s_2$$

Substituting in the first law equation (last equation of previous slide), we get

$$\boxed{w_{in}^{rev} = \int_1^2 v\, dP + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)}$$

The superscript ‘rev’ indicates that process is reversible.

Reversible – adiabatic (isoentropic) steady flow equipment :

If the equipment involves no work (for example in a pipe) and for liquid flow (considering specific volume constant), we get

$$\boxed{v\,(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0}$$

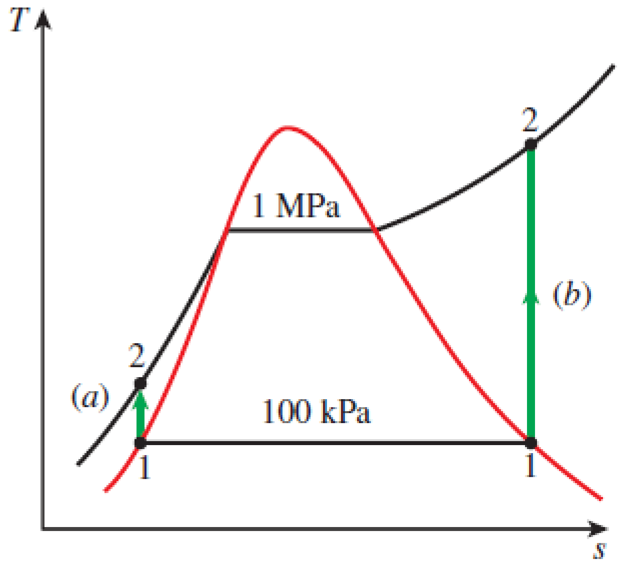
This equation is known as "Bernoulli equation" in fluid mechanics.

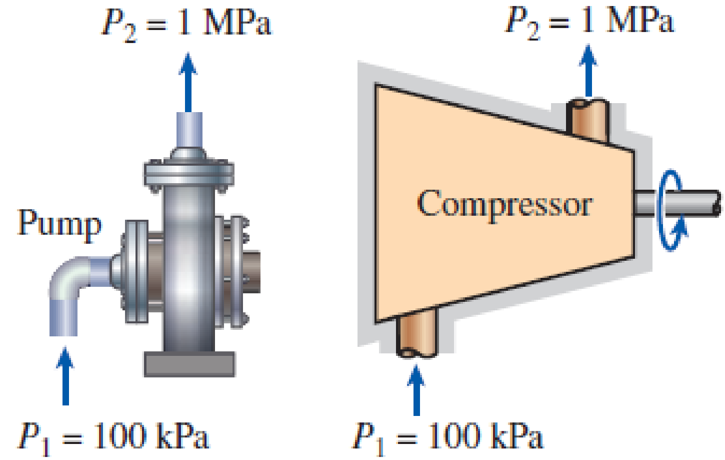
EXAMPLE 7–12 **Compressing a Substance in the Liquid versus Gas Phases**

Determine the compressor work input required to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (*a*) saturated liquid and (*b*) saturated vapor at the inlet state.

Assumptions **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible. **3** The process is given to be isentropic.

Ref. Cengel and Boles , 8th Edition (2015)





(*a*) Compressing a liquid

(*b*) Compressing a vapor

(*a*) In this case, steam is a saturated liquid initially, and its specific volume is

$$v_1 = v_{f\,@\,100\text{ kPa}} = 0.001043\ \text{m}^3/\text{kg} \qquad \text{(Table A–5)}$$

which remains essentially constant during the process. Thus,

$$w_{\text{rev,in}} = \int_1^2 v\,dP \cong v_1(P_2 - P_1)$$

$$= (0.001043\ \text{m}^3/\text{kg})[(1000 - 100)\ \text{kPa}]\left(\frac{1\ \text{kJ}}{1\ \text{kPa·m}^3}\right)$$

$$= \mathbf{0.94\ kJ/kg}$$

Ref. Cengel and Boles , 8th Edition (2015)

(*b*) This time, steam is a saturated vapor initially and remains a vapor during the entire compression process. Since the specific volume of a gas changes considerably during a compression process, we need to know how v varies with P to perform the integration   This relation, in general, is not readily available. But for an isentropic process, it is easily obtained from the second $T\,ds$ relation by setting $ds = 0$:

Thus,

$$w_{\text{rev,in}} = \int_1^2 v\,dP = \int_1^2 dh = h_2 - h_1$$

This result could also be obtained from the energy balance relation for an isentropic steady-flow process. Next we determine the enthalpies:

$$\textit{State 1:} \quad \left.\begin{array}{l} P_1 = 100\ \text{kPa} \\ \text{(sat. vapor)} \end{array}\right\} \begin{array}{l} h_1 = 2675.0\ \text{kJ/kg} \\ s_1 = 7.3589\ \text{kJ/kg·K} \end{array} \quad \text{(Table A–5)}$$

$$\textit{State 2:} \quad \left.\begin{array}{l} P_2 = 1\ \text{MPa} \\ s_2 = s_1 \end{array}\right\} \quad h_2 = 3194.5\ \text{kJ/kg} \quad \text{(Table A–6)}$$

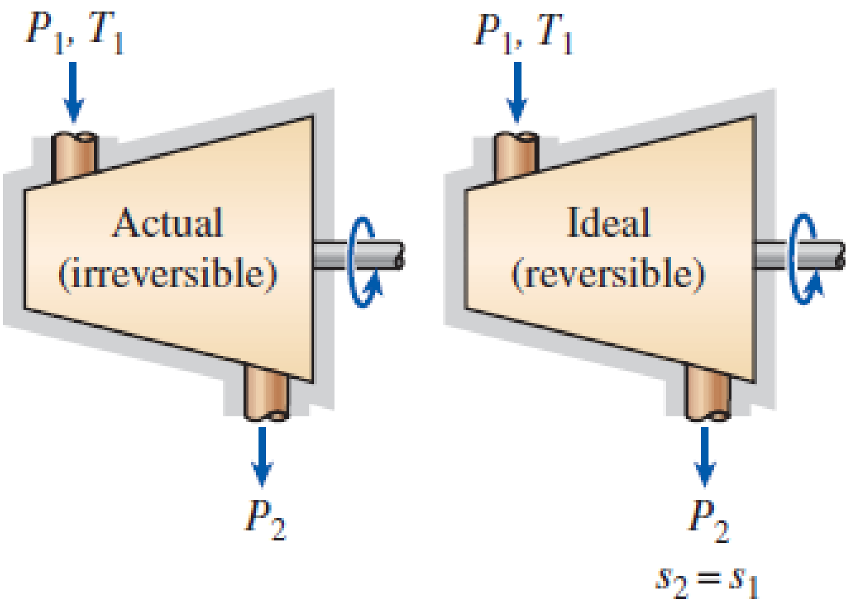
Thus,

$$w_{\text{rev,in}} = (3194.5 - 2675.0)\ \text{kJ/kg} = \mathbf{519.5\ kJ/kg}$$

Discussion Note that compressing steam in the vapor form would require over 500 times more work than compressing it in the liquid form between the same pressure limits.

Isoentropic efficiency of a turbine :

It is the ratio of work output of an actual turbine to that of a reversible turbine with the same outlet pressure (inlet conditions are considered same).



Here we are considering a fluid for which thermodynamic state can be completely specified by two independent, intensive variables (recall 'state postulate' of Lecture 3)

Ref. Cengel and Boles , 8th Edition (2015)

Work output of actual turbine :

Neglecting changes in kinetic and potential energies, the work output of an adiabatic turbine is given as (see first equation on slide 3)

$$w_{out} = h_1 - h_{2a}$$

Note that 'a' in the subscript of h_{2a} indicates that enthalpy at the outlet is for the actual (irreversible) turbine. Since dh = T dS + v dP, the enthalpy difference in above equation can also be expressed as :

$$h_1 - h_{2a} = -\int_1^{2a} T\,ds - \int_1^{2} v\,dP$$

From the entropy balance, we have $\dot{m}(s_1 - s_{2a}) + \dot{S}_{gen} = 0$

For an irreversible turbine, we have $\dot{S}_{gen} > 0$

Therefore, $s_{2a} > s_1$

Work output of isoentropic turbine :

Neglecting changes in kinetic and potential energies, the work output of an reversible - adiabatic (isoentropic) turbine is given as

$$w_{out}^{rev} = h_1 - h_{2s}$$

Note that 's' in the subscript of h_{2s} indicates that enthalpy at the outlet is for the reversible (isoentropic) turbine. Integrating dh = T ds + v dP,

$$h_1 - h_{2s} = -\int_1^{2s} T\,ds - \int_1^{2} v\,dP$$

From the entropy balance, we have $\dot{m}(s_1 - s_{2s}) = 0$

This is because for an reversible turbine, we have $\dot{S}_{gen} = 0$

Therefore, $\displaystyle\int_1^{2s} T\,ds = 0$

Work output of isoentropic turbine :

Hence the work output of isoentropic turbine is given by

$$w_{out}^{rev} = h_1 - h_{2s} = -\int_1^2 v\, dP$$

Comparing work output of reversible and irreversible turbines, we have

$$w_{out}^{rev} = w_{out} + \underbrace{\int_1^{2a} T\, ds}_{> 0}$$

The integral on the right hand side of above equation is positive. Therefore,

$$w_{out}^{rev} > w_{out}$$

Isoentropic efficiency of an adiabatic turbine :

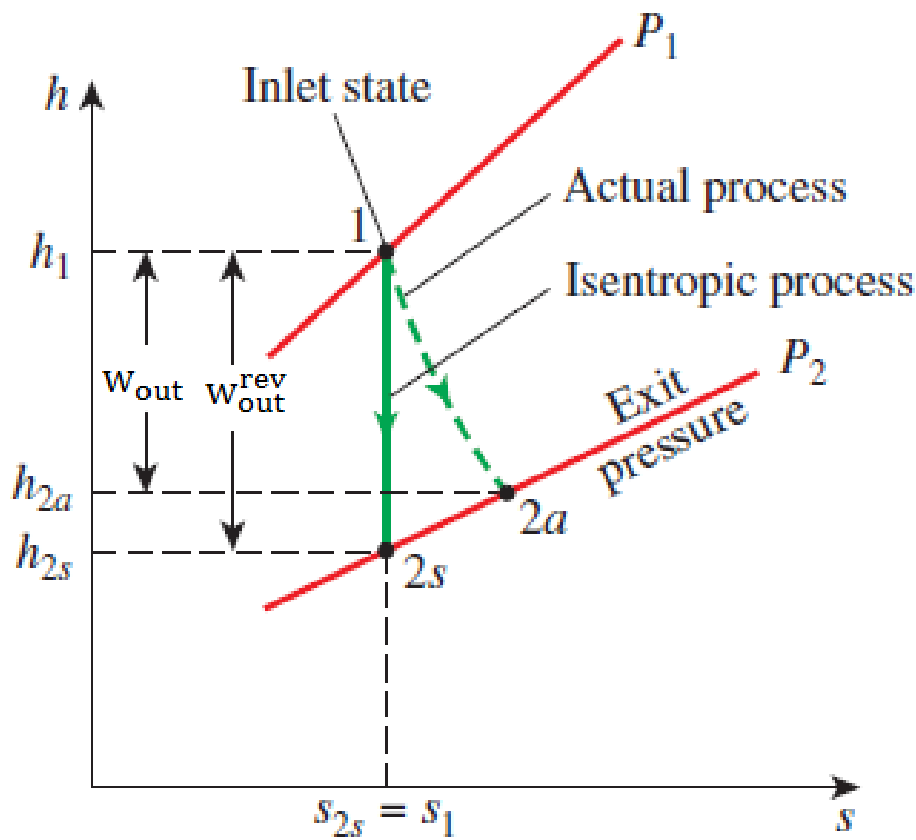
Thus, the isoentropic efficiency of an adiabatic turbine is given by

$$\eta_T = \frac{w_{out}}{w_{out}^{rev}} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Since $w_{out}^{rev} > w_{out}$, $\eta_T < 1$

In general, h - s plot is known as "<u>Mollier diagram</u>"

η_T typically varies from 70% to 90%



Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine (*a*) the isentropic efficiency of the turbine and (*b*) the mass flow rate of the steam flowing through the turbine.

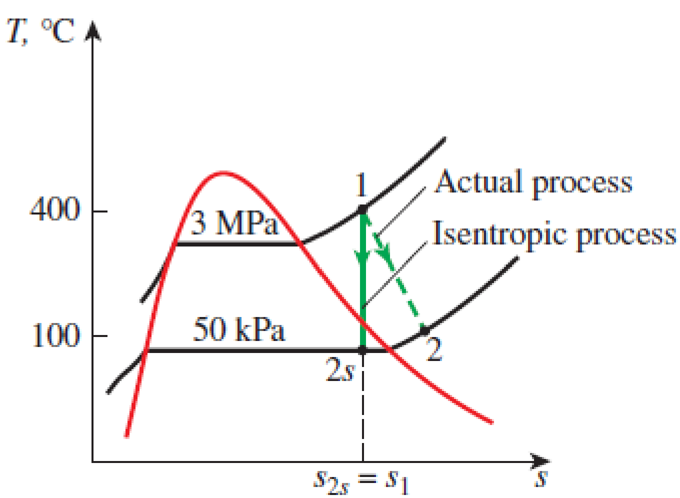
Assumptions **1** Steady operating conditions exist. **2** The changes in kinetic and potential energies are negligible.

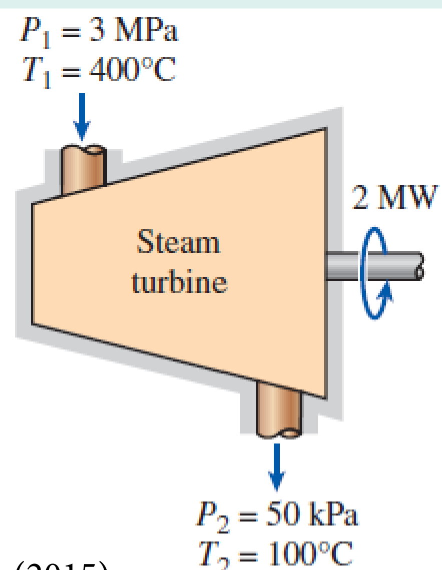
Analysis A sketch of the system and the *T-s* diagram of the process are given in Fig. 7–49.

(*a*) The enthalpies at various states are

$$\textit{State 1:}\quad \left.\begin{array}{l} P_1 = 3\ \text{MPA} \\ T_1 = 400°\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3231.7\ \text{kJ/kg} \\ s_1 = 6.9235\ \text{kJ/kg·K} \end{array} \quad \text{(Table A–6)}$$

$$\textit{State 2a:}\quad \left.\begin{array}{l} P_{2a} = 50\ \text{kPa} \\ T_{2a} = 100°\text{C} \end{array}\right\} \ h_{2a} = 2682.4\ \text{kJ/kg} \quad \text{(Table A–6)}$$





Ref. Cengel and Boles , 8th Edition (2015)

The exit enthalpy of the steam for the isentropic process h_{2s} is determined from the requirement that the entropy of the steam remain constant ($s_{2s} = s_1$):

$$\textit{State 2s:}\quad \begin{array}{l} P_{2s} = 50 \text{ kPa} \\ (s_{2s} = s_1) \end{array} \longrightarrow \begin{array}{l} s_f = 1.0912 \text{ kJ/kg·K} \\ s_g = 7.5931 \text{ kJ/kg·K} \end{array} \quad \text{(Table A–5)}$$

Obviously, at the end of the isentropic process steam exists as a saturated mixture since $s_f < s_{2s} < s_g$. Thus, we need to find the quality at state 2*s* first:

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897$$

$$h_{2s} = h_f + x_{2s} h_{fg} = 340.54 + 0.897(2304.7) = 2407.9 \text{ kJ/kg}$$

By substituting these enthalpy values into Eq. 7–61, the isentropic efficiency of this turbine is determined to be

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = \mathbf{0.667} \text{ (or 66.7\%)}$$

(*b*) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m} h_1 = \dot{W}_{a,\text{out}} + \dot{m} h_{2a}$$

$$\dot{W}_{a,\text{out}} = \dot{m}(h_1 - h_{2a})$$

$$2 \text{ MW}\left(\frac{1000 \text{ kJ/s}}{1 \text{ MW}}\right) = \dot{m}(3231.7 - 2682.4) \text{ kJ/kg}$$

$$\dot{m} = \mathbf{3.64 \text{ kg/s}}$$

Ref. Cengel and Boles , 8th Edition (2015)

Work input of an adiabatic compressor :

We consider an adiabatic turbine in which inlet conditions are specified and outlet pressure is specified.

The state of the fluid is completely determined by two independent, intensive variables.

Work input per unit mass of the fluid in an adiabatic turbine (neglecting changes in kinetic and potential energies) is given by,

$$w_{in} = h_{2a} - h_1 = \int_1^{2a} T\,ds + \int_1^{2} v\,dP$$

Note that '2a' represents outlet conditions for actual (irreversible) compressor. Note that outlet pressure P_2 is the same for both reversible and irreversible compressors.

Isoentropic efficiency of an adiabatic compressor :

On the other hand, work input per unit mass of the fluid in a reversible-adiabatic (isoentropic) compressor is

$$w_{in} = h_{2a} - h_1 = \int_1^{2a} T\,ds + \int_1^{2} v\,dP$$

$$w_{in}^{rev} = h_{2s} - h_1 = \int_1^{2} v\,dP$$

The derivation of the above two equations is similar to that in case of adiabatic and reversible-adiabatic turbine (see previous slides). From these equations, we get

$$w_{in} = w_{in}^{rev} + \int_1^{2a} T\,ds$$

Note that $\int_1^{2a} T\,ds > 0$ since $s_{2a} > s_1$

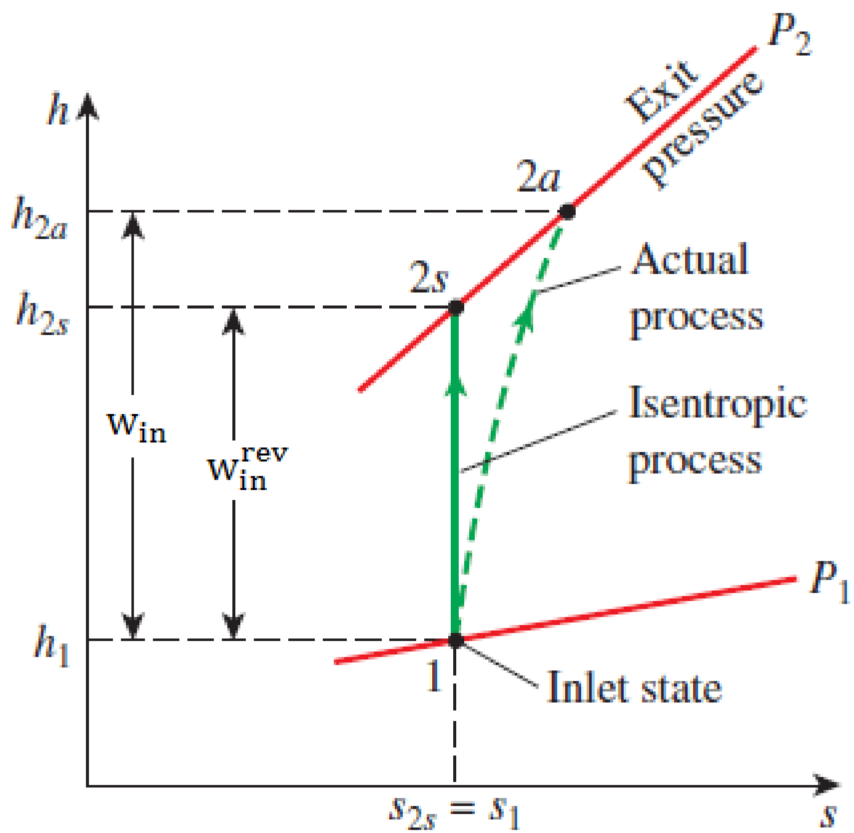
Therefore, $w_{in} > w_{in}^{rev}$

Isoentropic efficiency of an adiabatic compressor :

Thus, isoentropic efficiency of adiabatic compressor is given by

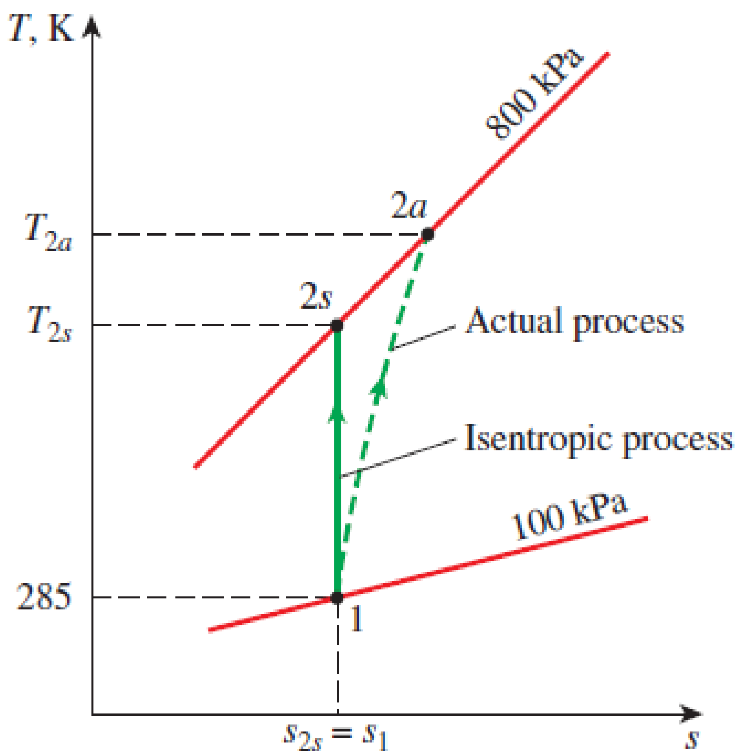
$$\eta_C = \frac{w_{in}^{rev}}{w_{in}} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

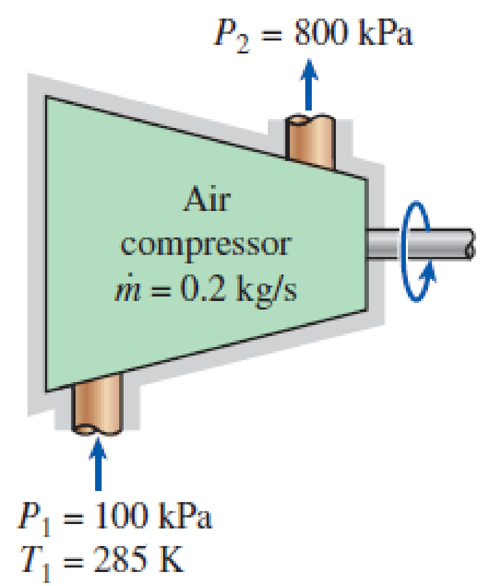
η_C typically varies from 80% to 90%



Ref. Cengel and Boles , 8th Edition (2015)

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine (*a*) the exit temperature of air and (*b*) the required power input to the compressor.





Ref. Cengel and Boles , 8th Edition (2015)

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The changes in kinetic and potential energies are negligible.

Analysis A sketch of the system and the *T-s* diagram of the process are given in Fig. 7–52.

(*a*) We know only one property (pressure) at the exit state, and we need to know one more to fix the state and thus determine the exit temperature. The property that can be determined with minimal effort in this case is h_{2a} since the isentropic efficiency of the compressor is given. At the compressor inlet,

$$T_1 = 285 \text{ K} \quad \rightarrow \quad h_1 = 285.14 \text{ kJ/kg} \qquad \text{(Table A–17)}$$
$$(P_{r1} = 1.1584)$$

The enthalpy of the air at the end of the isentropic compression process is determined by using one of the isentropic relations of ideal gases,

$$P_{r2} = P_{r1}\left(\frac{P_2}{P_1}\right) = 1.1584\left(\frac{800 \text{ kPa}}{100 \text{ kPa}}\right) = 9.2672$$
$$P_{r2} = 9.2672 \quad \rightarrow \quad h_{2s} = 517.05 \text{ kJ/kg}$$

Substituting the known quantities into the isentropic efficiency relation, we have

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \rightarrow \quad 0.80 = \frac{(517.05 - 285.14) \text{ kJ/kg}}{(h_{2a} - 285.14) \text{ kJ/kg}}$$

Thus,

$$h_{2a} = 575.03 \text{ kJ/kg} \quad \rightarrow \quad T_{2a} = \mathbf{569.5 \text{ K}}$$

Ref. Cengel and Boles , 8th Edition (2015)

(*b*) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 + \dot{W}_{a,in} = \dot{m}h_{2a}$$

$$\dot{W}_{a,in} = \dot{m}(h_{2a} - h_1)$$

$$= (0.2 \text{ kg/s})[(575.03 - 285.14) \text{ kJ/kg}]$$

$$= \mathbf{58.0 \text{ kW}}$$

Discussion Notice that in determining the power input to the compressor, we used h_{2a} instead of h_{2s} since h_{2a} is the actual enthalpy of the air as it exits the compressor. The quantity h_{2s} is a hypothetical enthalpy value that the air would have if the process were isentropic.

Isoentropic efficiency of an adiabatic pump :

Although the purpose of both pumps and compressor is to increase fluid pressure, the main difference is that compressor handle gases while the pump handle liquids.

Isoentropic efficiency of an adiabatic pump is defined similar to that of an adiabatic compressor :

$$\eta_P = \frac{w_{in}^{rev}}{w_{in}} = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{\int_1^2 v\, dP}{h_{2a} - h_1}$$

Since pumps handles liquids which are incompressible, v can be taken as constant to a good degree of approximation. In that case,

$$\eta_P = \frac{w_{in}^{rev}}{w_{in}} \approx \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

Isoentropic efficiency of a nozzle :

The purpose of nozzle is to accelerate a fluid. Nozzles are generally considered to be adiabatic devices and involve no work.

Lets consider that inlet state and outlet pressure of a nozzle are fixed. Key question is how much is the maximum kinetic energy we can obtain if nozzle were to behave ideally (i.e., reversible adiabatic or isoentropic nozzle). Thus the isoentropic efficiency of a nozzle is defined as

$$\eta_N = \frac{\frac{1}{2}V_{2a}^2}{\frac{1}{2}V_{2s}^2} = \frac{V_{2a}^2}{V_{2s}^2}$$

η_N typically varies between 90 % to 95 %. Applying first law equation (neglecting changes in potential energies) we get

$$h_1 + \frac{1}{2}V_1^2 = h_{2a} + \frac{1}{2}V_{2a}^2$$

Isoentropic efficiency of a nozzle :

The inlet velocity is generally significantly smaller than outlet velocity. In that case, inlet kinetic energy can be neglected as compared to outlet kinetic energy. Thus,

$$\frac{1}{2}V_{2a}^2 \approx h_1 - h_{2a}$$

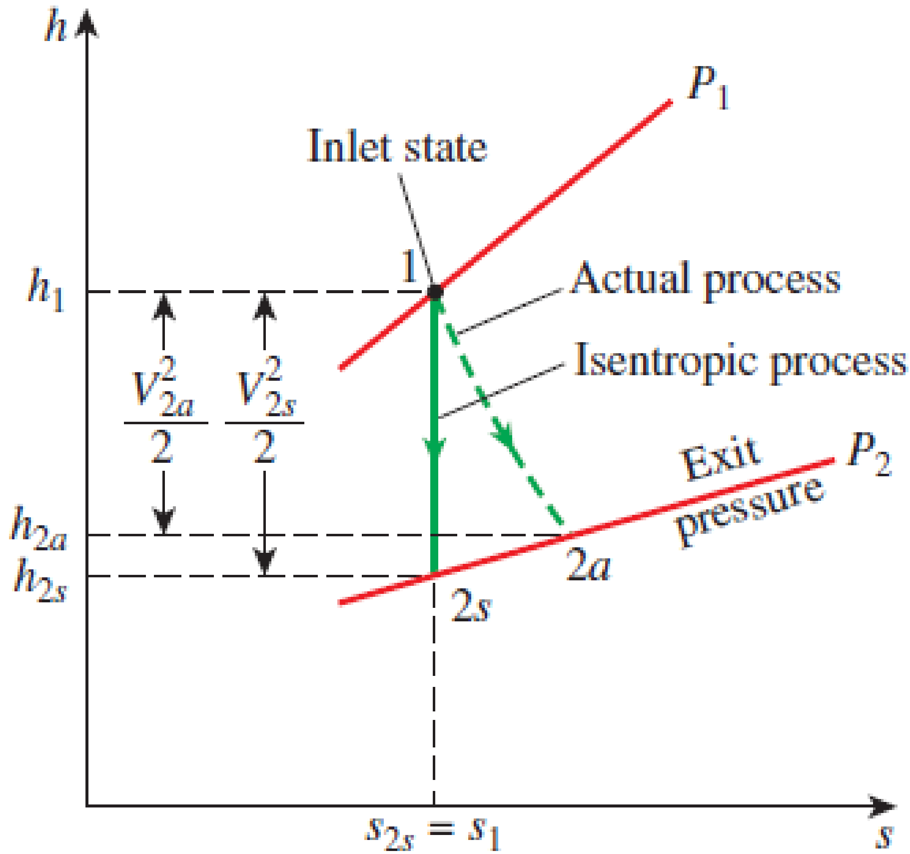
For the isoentropic nozzle with the same outlet pressure as actual adiabatic (irreversible) nozzle, first law equation yields

$$\frac{1}{2}V_{2s}^2 \approx h_1 - h_{2s}$$

Substituting these equations in the expression for isoentropic efficiency, we get

$$\eta_N \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Isoentropic efficiency of a nozzle :



$$\eta_N \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

Ref. Cengel and Boles , 8th Edition (2015)

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 110 kPa. If the isentropic efficiency of the nozzle is 92 percent, determine (*a*) the maximum possible exit velocity, (*b*) the exit temperature, and (*c*) the actual exit velocity of the air. Assume constant specific heats for air.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The inlet kinetic energy is negligible.

The temperature of air will drop during this acceleration process because some of its internal energy is converted to kinetic energy. This problem can be solved accurately by using property data from the air table. But we will assume constant specific heats (thus sacrifice some accuracy) to demonstrate their use. Let us guess the average temperature of the air to be about 850 K. Then, the average values of c_p and k at this anticipated average temperature are determined from Table A–2*b* to be $c_p = 1.11$ kJ/kg·K and $k = 1.349$.

(*a*) The exit velocity of the air will be a maximum when the process in the nozzle involves no irreversibilities. The exit velocity in this case is determined from the steady-flow energy equation. However, first we need to determine the exit temperature. For the isentropic process of an ideal gas we have:

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k}$$

Ref. Cengel and Boles , 8th Edition (2015)

or

$$T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k} = (950\text{ K})\left(\frac{110\text{ kPa}}{200\text{ kPa}}\right)^{0.349/1.349} = 814\text{ K}$$

This gives an average temperature of 882 K, which is somewhat higher than the assumed average temperature (850 K). This result could be refined by reevaluating the *k* value at 882 K and repeating the calculations, however, it is not warranted since the two average temperatures are sufficiently close (doing so would change the temperature by only 0.6 K, which is not significant).

Now we can determine the isentropic exit velocity of the air from the energy balance for this isentropic steady-flow process:

$$e_{\text{in}} = e_{\text{out}}$$

$$h_1 + \frac{V_1^2}{2} = h_{2s} + \frac{V_{2s}^2}{2}$$

or

$$V_{2s} = \sqrt{2(h_1 - h_{2s})} = \sqrt{2c_{p,\text{avg}}(T_1 - T_{2s})}$$

$$= \sqrt{2(1.11\text{ kJ/kg·K})[(950 - 814)\text{ K}]\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

$$= \mathbf{549\ m/s}$$

Ref. Cengel and Boles , 8th Edition (2015)

or

$$T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{(k-1)/k} = (950\text{ K})\left(\frac{110\text{ kPa}}{200\text{ kPa}}\right)^{0.349/1.349} = 814\text{ K}$$

This gives an average temperature of 882 K, which is somewhat higher than the assumed average temperature (850 K). This result could be refined by reevaluating the *k* value at 882 K and repeating the calculations, however, it is not warranted since the two average temperatures are sufficiently close (doing so would change the temperature by only 0.6 K, which is not significant).

Now we can determine the isentropic exit velocity of the air from the energy balance for this isentropic steady-flow process:

$$e_{\text{in}} = e_{\text{out}}$$

$$h_1 + \frac{V_1^2}{2} = h_{2s} + \frac{V_{2s}^2}{2}$$

or

$$V_{2s} = \sqrt{2(h_1 - h_{2s})} = \sqrt{2c_{p,\text{avg}}(T_1 - T_{2s})}$$

$$= \sqrt{2(1.11\text{ kJ/kg·K})[(950 - 814)\text{ K}]\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)}$$

$$= \mathbf{549\text{ m/s}}$$

Ref. Cengel and Boles , 8th Edition (2015)

(*b*) The actual exit temperature of the air is higher than the isentropic exit temperature evaluated above and is determined from

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{c_{p,\text{avg}}(T_1 - T_{2a})}{c_{p,\text{avg}}(T_1 - T_{2s})}$$

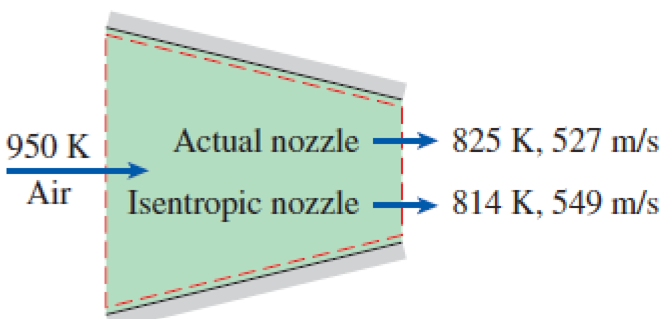
or

$$0.92 = \frac{950 - T_{2a}}{950 - 814} \rightarrow T_{2a} = \mathbf{825\ K}$$

That is, the temperature is 11 K higher at the exit of the actual nozzle as a result of irreversibilities such as friction. It represents a loss since this rise in temperature comes at the expense of kinetic energy (Fig. 7–55).

(*c*) The actual exit velocity of air can be determined from the definition of isentropic efficiency of a nozzle,

$$\eta_N = \frac{V_{2a}^2}{V_{2s}^2} \rightarrow V_{2a} = \sqrt{\eta_N V_{2s}^2} = \sqrt{0.92(549\ \text{m/s})^2} = \mathbf{527\ m/s}$$



Thus, fluid leaves the nozzle at a higher temperature and lower velocity (as compared to isoentropic nozzle) due to friction