ESO208A: Computational Methods in Engineering

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What are *Computational Methods* or *Numerical Methods* in Engineering?

Formulation of *mathematical problems* in such a way that *numerical answers* can be computed using *arithmetic operations* in a computer

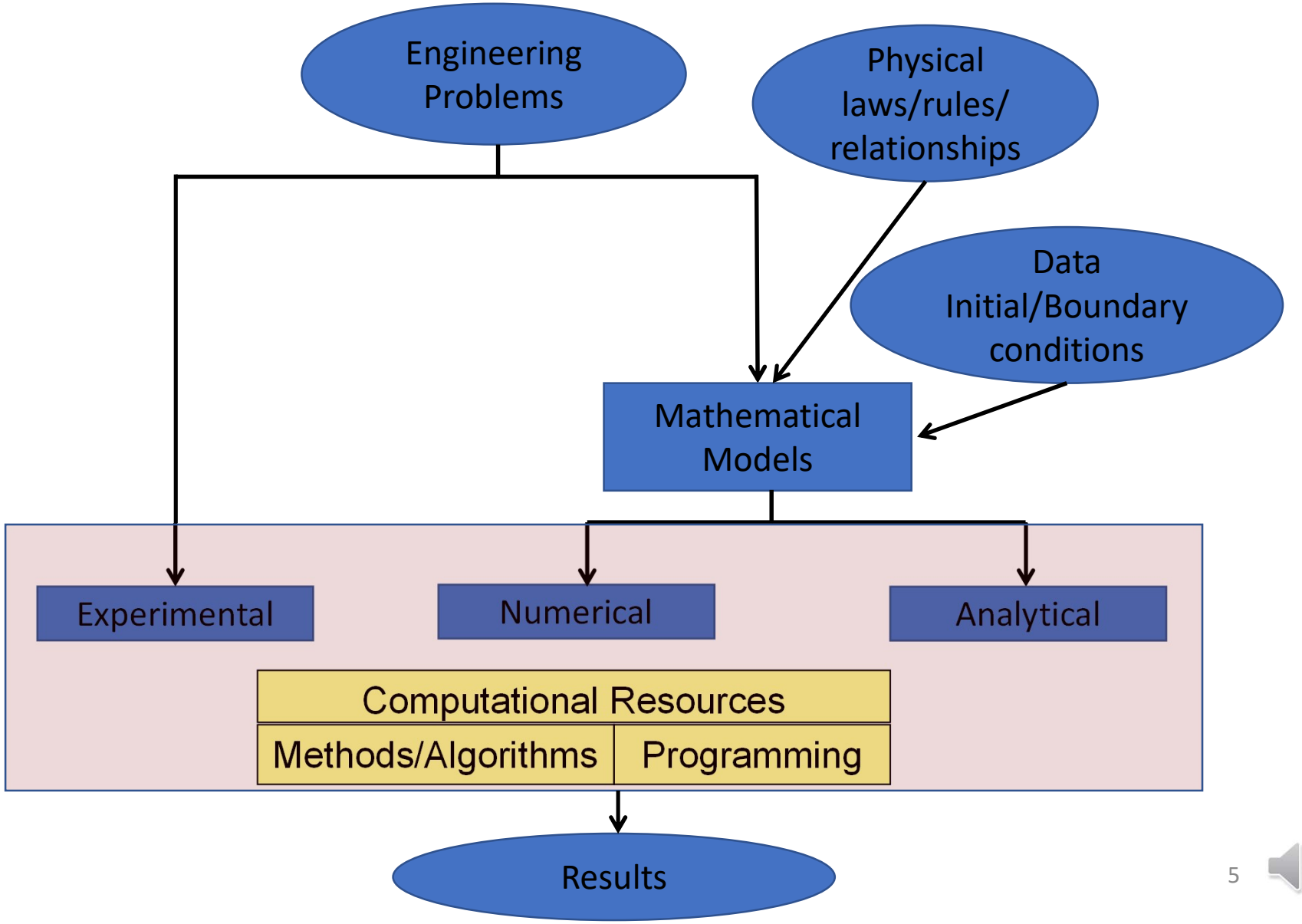
Computer can only perform: + - × /

All kinds of problems can be solved: linear and non-linear systems of equations; approximation and interpolation of data; differentiation and integration, ODE, PDE

Objectives of the course

- Introduce you to computational methods and algorithms for the solution of engineering problems.
- Familiarize you to the algorithms behind the software packages so that you don't use them as black boxes.
- Expose you to the analysis of these algorithms so that, if needed, you can modify an existing algorithm or develop your own algorithm for the problem at hand.

Scope of the course



Example

Example

To find t_H

GEq. $-\frac{d^2h}{dt^2} = g$

Data $t = 0 \quad h = 0$

$\frac{dh}{dt} = 0$

Solution $\frac{dh}{dt} = gt + a$

$h = \frac{gt^2}{2} + at + b$

Applying ICs $a = b = 0$

$h = \frac{gt^2}{2}$

$\boxed{t_H = \sqrt{\frac{2H}{g}}}$

$H = 666\ m$

$g = 9.81\ m/s^2$

$t_H = \sqrt{\frac{2 \times 666}{9.81}} = \sqrt{135.7982}$

Babolyian algorithm $x = \sqrt{a}$

1. x_0 – Guess value $\quad x_1 = \frac{a}{x_0}$
2. $x_1 = \frac{1}{2}\left(x_0 + \frac{a}{x}\right)$
3. :

$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$

Apply

$x_0 = 11$

$x_1 = \frac{1}{2}\left(11 + \frac{135.7982}{11}\right)$

$= 11.6718$

$x_2 = 11.6525$

$x_3 = 11.6525 \quad t_H = 11.6525_s$

Example

Based on the example, we need to answer several questions

- Do we need numerical methods to solve the problem?
  - Shall we solve the ODE directly using a numerical technique or go for analytical solutions and then utilise a numerical technique
- How many significant digits do we have? How many should we take? This is important as computer has finite space
- What algorithm should we take (here BA)?

✓ $x_{n+1} = \frac{2}{x_n}$

✓ $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$

✓ $x_{n+1} = \frac{x_n+2}{x_n+1}$

- We have selected an Algorithm. Is this algorithm going to converge? (Numerical Analysis)
  - What is the convergence rate (How frequently it converges?)

Example

- What is the error in the results?

  If we do experiments, will we get the same result?

  - Mainly because we have model error (air resistance is neglected, gravity is not constant)
  - Data error because of data, the height may not be correct, g may have error
  - Uncertainty propagates
  - Round off error because of finite nature of computer. Computer has an upper and lower limit of numbers.
  - Truncation error (again because of computer)

Condition Number or Stability helps us decide which algorithm we should select.

Number representation in Computer

- Binary digits 0 or 1

11010000

208

11 01 0000 . 101

$208 + 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3}$

$= 208.625$

1 bit

8 bit - byte

32-bit - word

64-bit - double word

- Digital computers

0 & 9 [Decimal]

118

$1\times10^{2} + 1\times10^{1} + 8\times10^{0}$

118.25

$1\times10^{2} + 1\times10^{1} + 8\times10^{0} + 2\times10^{-1} + 5\times10^{-2}$

I will leave this for you as a reading assignment

Summary

- What are computational methods? How they are used for solving engineering problems?
- The choice of computational method depends on the problem and intended use of the results.
- Example: Object falling from a building
  - Formulation of mathematical model
  - Choice of methods, convergence, convergence rate, errors [model, data, round-off and truncation], propagation of errors, stability & condition number.
- Number representation in computers
  - Binary and decimal representation