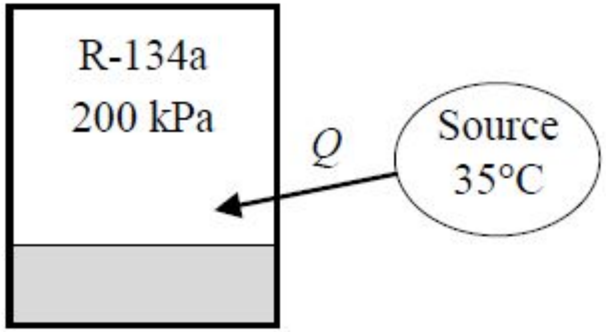
ESO201A

TUTORIAL 6: PROBLEMS AND SOLUTIONS

7-42 A 0.5m^3 rigid tank contains refrigerant-134a initially at 200 kPa and 40 percent quality. Heat is transferred now to the refrigerant from a source at 35°C until the pressure rises to 400 kPa. Determine (a) the entropy change of the refrigerant, (b) the entropy change of the heat source, and (c) the total entropy change for the process.



Solution:

A rigid tank is initially filled with a saturated mixture of R-134a. Heat is transferred to the tank from a source until the pressure inside rises to a specified value. The entropy change of the refrigerant, entropy change of the source, and the total entropy change for this process are to be determined.

Assumptions

1. The tank is stationary and thus the kinetic and potential energy changes are zero.
2. There are no work interactions.

Analysis

(*a*) From the refrigerant tables (Tables A-11 through A-13)

Saturated refrigerant-134a—Pressure table

		Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
Press., P kPa	Sat. temp., T_{sat} °C	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
200	−10.09	0.0007532	0.099951	38.26	186.25	224.51	38.41	206.09	244.50	0.15449	0.78339	0.93788
400	8.91	0.0007905	0.051266	63.61	171.49	235.10	63.92	191.68	255.61	0.24757	0.67954	0.92711

$$\left.\begin{aligned} P_1 &= 200 \text{ kPa} \\ x_1 &= 0.4 \end{aligned}\right\}$$

$$u_1 = u_f + x_1 u_{fg} = 38.26 + (0.4)(186.25) = 112.76 \text{ kJ/kg}$$

$$s_1 = s_f + x_1 s_{fg} = 0.15449 + (0.4)(0.78339) = 0.4678 \text{ kJ/kg} \cdot \text{K}$$

$$\boldsymbol{v}_1 = \boldsymbol{v}_f + x_1 \boldsymbol{v}_{fg} = 0.0007532 + (0.4)(0.099951 - 0.0007532) = 0.04043 \text{ m}^3/\text{kg}$$

$$\left.\begin{aligned} P_2 &= 400 \text{ kPa} \\ \boldsymbol{v}_2 &= \boldsymbol{v}_1 \end{aligned}\right\}$$

$$x_2 = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_f}{v_{fg}} = \frac{0.04043 - 0.0007905}{0.051266 - 0.0007905} = 0.7853$$

$$u_2 = u_f + x_2 u_{fg} = 63.61 + (0.7853)(171.49) = 198.29 \text{ kJ/kg}$$

$$s_2 = s_f + x_2 s_{fg} = 0.24757 + (0.7853)(0.67954) = 0.7813 \text{ kJ/kg} \cdot \text{K}$$

The mass of the refrigerant is

$$m = \frac{V}{v_1} = \frac{0.5\,\text{m}^3}{0.04043\,\text{m}^3/\text{kg}} = 12.37\,\text{kg}$$

Then the entropy change of the refrigerant becomes

$$\Delta S_{\text{system}} = m(s_2 - s_1) = (12.37\,\text{kg})(0.7813 - 0.4678)\,\text{kJ/kg}\cdot\text{K} = \mathbf{3.876\,kJ/K}$$

(b) We take the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$\boxed{\Delta S_{\text{source}} = -\frac{Q_{\text{source,out}}}{T_{\text{source}}}}$$

$$Q_{\text{in}} = \Delta U = m(u_2 - u_1)$$

Substituting,

$$Q_{\text{in}} = m(u_2 - u_1) = (12.37\,\text{kg})(198.29 - 112.76) = 1058\,\text{kJ}$$

The heat transfer for the source is equal in magnitude but opposite in direction. Therefore,

$$Q_{\text{source, out}} = - Q_{\text{tank, in}} = - 1058 \text{ kJ}$$

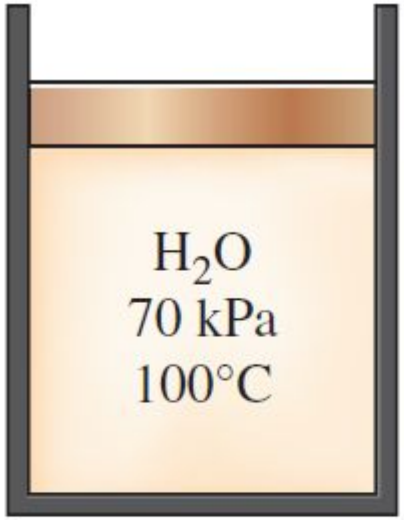
and

$$\Delta S_{\text{source}} = -\frac{Q_{\text{source,out}}}{T_{\text{source}}} = -\frac{1058 \text{ kJ}}{308 \text{ K}} = \mathbf{-3.434\,kJ/K}$$

(*c*) The total entropy change for this process is

$$\Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{source}} = 3.876 + (-3.434) = \mathbf{0.441\,kJ/K}$$

7-50 Water at 70 kPa and 100°C is compressed **isentropically** in a closed system to 4 MPa. Determine the final temperature of the water and the work required, **in kJ/kg**, for this compression.

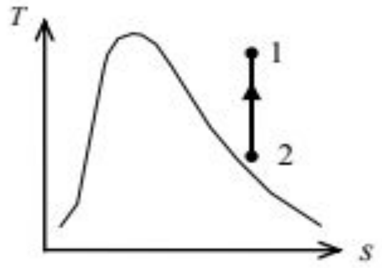


Solution:

Water is compressed in a closed system during which the entropy remains constant. The final temperature and the work required are to be determined.

Analysis

The initial state is superheated vapor and thus



$$\left.\begin{array}{l} P_1 = 70\text{ kPa} \\ T_1 = 100°\text{C} \end{array}\right\} \begin{array}{l} u_1 = 2509.4\text{ kJ/kg} \\ s_1 = 7.5344\text{ kJ/kg}\cdot\text{K} \end{array} \quad \text{(Table A - 6)}$$

TABLE A–6

Superheated water

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	P = 0.01 MPa (45.81°C)*				*P* = 0.05 MPa (81.32°C)				*P* = 0.10 MPa (99.61°C)			
Sat.†	14.670	2437.2	2583.9	8.1488	3.2403	2483.2	2645.2	7.5931	1.6941	2505.6	2675.0	7.3589
50	14.867	2443.3	2592.0	8.1741								
100	17.196	2515.5	2687.5	8.4489	3.4187	2511.5	2682.4	7.6953	1.6959	2506.2	2675.8	7.3611

In the above, we have used linear Interpolation for P = 70 KPa (0.07 MPa).

The entropy is constant during the process. The properties at the exit state are

$$\left.\begin{aligned} P_2 &= 4000\ \text{kPa} \\ s_2 = s_1 &= 7.5344\ \text{kJ/kg}\cdot\text{K} \end{aligned}\right\} \begin{aligned} u_2 &= 3396.5\ \text{kJ/kg} \\ T_2 &= \mathbf{664^\circ C} \end{aligned} \quad \text{(Table A - 6)}$$

TABLE A–6

Superheated water (*Continued*)

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
		P = 4.0 MPa (250.35°C)		
Sat.	0.04978	2601.7	2800.8	6.0696
275	0.05461	2668.9	2887.3	6.2312
300	0.05887	2726.2	2961.7	6.3639
350	0.06647	2827.4	3093.3	6.5843
400	0.07343	2920.8	3214.5	6.7714
450	0.08004	3011.0	3331.2	6.9386
500	0.08644	3100.3	3446.0	7.0922
600	0.09886	3279.4	3674.9	7.3706
700	0.11098	3462.4	3906.3	7.6214

$$\text{Linear Interpolation}(y) = y_1 + (x - x_1)\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

For linear interpolation of temperature

- Take Temperature as y and entropy as x

For linear interpolation of internal energy

- Take u as y and entropy as x

Again, we have used linear Interpolation at 4 MPa to find Temperature T_2

To determine the work done, we take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as:

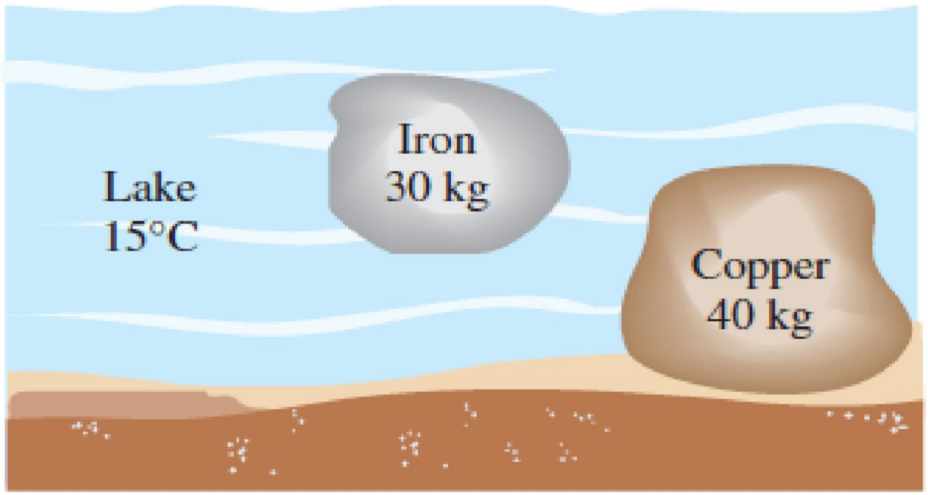
$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$w_{\text{in}} = \Delta u = u_2 - u_1 \quad (\text{since } Q = \text{KE} = \text{PE} = 0)$$

Substituting,

$$w_{\text{in}} = u_2 - u_1 = (3396.5 - 2509.4)\text{kJ/kg} = \mathbf{887.1 kJ/kg}$$

7-65 A 30-kg iron block and a 40-kg copper block, both initially at 80°C, are dropped into a large lake at 15°C. **Thermal equilibrium** is established after a while as a result as a result of heat transfer between the blocks and the lake water. Determine the total entropy change to this process.



Solution: An iron block and a copper block are dropped into a large lake. The total amount of entropy change when both blocks cool to the lake temperature is to be determined.

Assumptions

1. The water, the iron block and the copper block are incompressible substances with constant specific heats at room temperature.

2. Kinetic and potential energies are negligible.

Properties

The specific heats of iron and copper at room temperature are $c_{iron=}$ 0.45kJ/kg.°C and $c_{copper\ =}$ 0.386kJ/kg.°C (Table A-3).

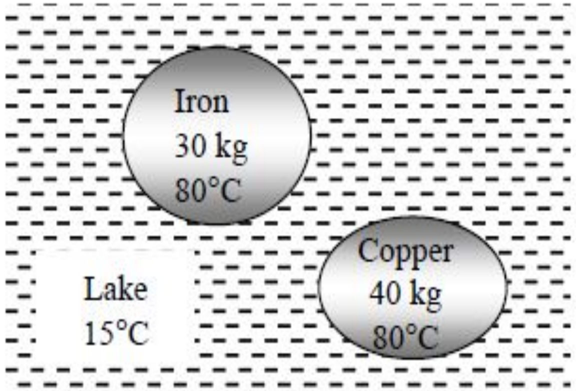
Substance	Density, ρ kg/m^3	Specific heat, c_p kJ/kg·K
Metals		
Copper		
−173°C		0.254
−100°C		0.342
−50°C		0.367
0°C		0.381
27°C	8,900	0.386
100°C		0.393
200°C		0.403
Iron	7,840	0.45

Analysis

The thermal-energy capacity of the lake is very large, and thus the temperatures of both the iron and the copper blocks will drop to the **lake temperature (15°C)** when the thermal equilibrium is established. Then the entropy changes of the blocks become

$$\Delta s_{iron} = mc_{avg}\ln\left(\frac{T_2}{T_1}\right) = (30\text{kg})(0.45\text{kJ/kg.K})\ln\left(\frac{288\ \text{K}}{353\ \text{K}}\right) = -2.746\ \text{kJ/K}$$

$$\Delta s_{copper} = mc_{avg}\ln\left(\frac{T_2}{T_1}\right) = (40\text{kg})(0.386\text{kJ/kg.K})\ln\left(\frac{288\ \text{K}}{353\ \text{K}}\right) = -3.141\ \text{kJ/K}$$



We take both the iron and copper blocks, as the system. This is a closed system since no mass crosses the system boundary during the process. The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

- $-Q_{out} = \Delta U = \Delta U_{iron} + \Delta U_{copper}$

or,

$$Q_{out} = [mC(T_1 - T_2)]_{iron} + [mC(T_1 - T_2)]_{copper}$$

Substituting,

$$Q_{out} = (30kg)(0.45kJ/kg.K)(353\text{-}288)K + (40kg)(0.386kJ/kg.K)(353\text{-}288)K$$
$$= 1881kJ$$

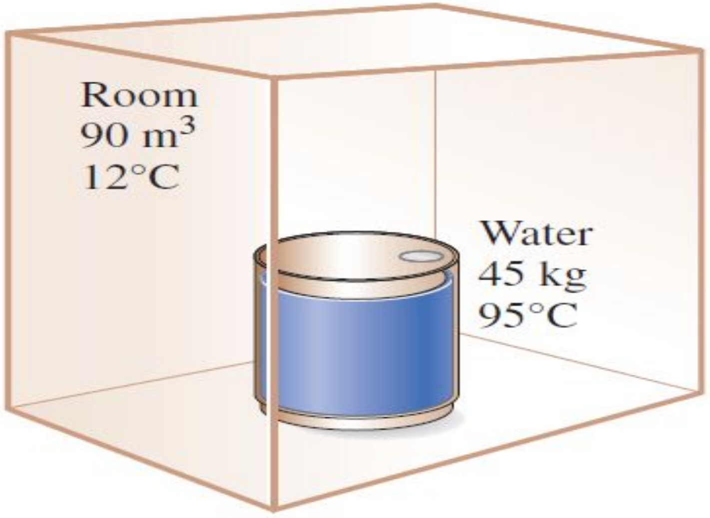
Thus,

$$\Delta S_{lake} = \left(\frac{Q_{lake,in}}{T_{lake}}\right) = \left(\frac{1881\ \text{kJ}}{288\ \text{K}}\right) = 6.528\ \text{kJ/K}$$

Then the total entropy change for this process is

$$\Delta S_{total} = \Delta S_{iron} + \Delta S_{copper} + \Delta S_{lake}$$
$$= (-2.746)+(-3.141)+6.528 = \mathbf{0.642\ kJ/K}$$

7-97 A container filled with 45 kg of liquid water at 95°C is placed in a 90-m^3 room that is initially at 12°C. Thermal equilibrium is established after a while as a result of heat transfer between the water and the air in the room. Using constant specific heats, determine (a) the final equilibrium temperature, (b) the amount of heat transfer between the water and the air in the room, and (c) the entropy generation. Assume the room is well sealed and heavily insulated.



Solution:

A container filled with liquid water is placed in a room and heat transfer takes place between the container and the air in the room until the thermal equilibrium is established. The final temperature, the amount of heat transfer between the water and the air, and the entropy generation are to be determined.

Assumptions

1. Kinetic and potential energy changes are negligible.

2. Air is an ideal gas with constant specific heats.

3. The room is well-sealed and there is no heat transfer from the room to the surroundings.

4. Sea level atmospheric pressure is assumed. P = 101.3 kPa.

Properties

The properties of air at room temperature are R = 0.287 kPa.m^3/kg.K, c_p = 1.005 kJ/kg.K, c_v = 0.718 kJ/kg.K. The specific heat of water at room temperature is c_w = 4.18 kJ/kg.K (Tables A-2, A-3).

TABLE A–2

Ideal–gas specific heats of various common gases

(*a*) At 300 K

Gas	Formula	Gas constant, *R* kJ/kg·K	c_p kJ/kg·K	c_v kJ/kg·K	*k*
Air	—	0.2870	1.005	0.718	1.400
Argon	Ar	0.2081	0.5203	0.3122	1.667
Butane	C_4H_{10}	0.1433	1.7164	1.5734	1.091
Carbon dioxide	CO_2	0.1889	0.846	0.657	1.289

TABLE A–3

Properties of common liquids, solids, and foods

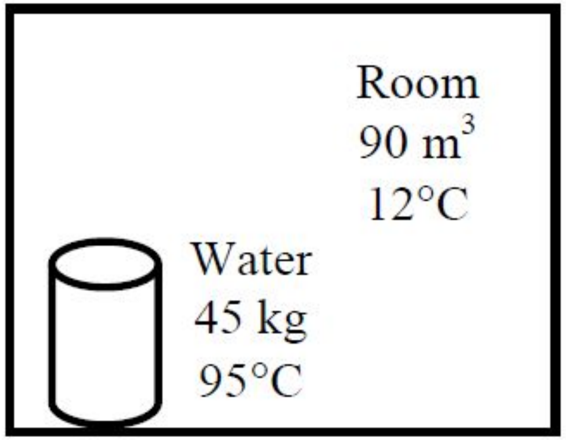
(*a*) Liquids

	Boiling data at 1 atm		*Freezing data*		*Liquid properties*		
Substance	Normal boiling point, °C	Latent heat of vaporization h_{fg}, kJ/kg	Freezing point, °C	Latent heat of fusion h_{if}, kJ/kg	Temperature, °C	Density ρ, kg/m^3	Specific heat c_p, kJ/kg·K
Water	100	2257	0.0	333.7	0	1000	4.22
					25	997	4.18
					50	988	4.18
					75	975	4.19
					100	958	4.22

Analysis

(a) The mass of the air in the room is

$$m_a = \frac{PV}{RT_{a1}} = \frac{(101.3\,\text{kPa})(90\,\text{m}^3)}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(12+273\,\text{K})} = 111.5\,\text{kg}$$



An energy balance on the system that consists of the water in the container and the air in the room gives the final equilibrium temperature

$$0 = m_w c_w (T_2 - T_{w1}) + m_a c_v (T_2 - T_{a1})$$

$$0 = (45\,\text{kg})(4.18\,\text{kJ/kg.K})(T_2 - 95) + (111.5\,\text{kg})(0.718\,\text{kJ/kg.K})(T_2 - 12) \longrightarrow T_2 = \mathbf{70.2^\circ C}$$

(b) The heat transfer to the air is (or can also calculate heat loss from water)

$$Q = m_a c_v (T_2 - T_{a1}) = (111.5\,\text{kg})(0.718\,\text{kJ/kg.K})(70.2 - 12) = \mathbf{4660kJ}$$

(c) The entropy generation associated with this heat transfer process may be obtained by calculating total entropy change, which is the sum of the entropy changes of water and the air.

$$\Delta S_w = m_w c_w \ln\frac{T_2}{T_{w1}} = (45\,\text{kg})(4.18\,\text{kJ/kg.K})\ln\frac{(70.2+273)\,\text{K}}{(95+273)\,\text{K}} = -13.11\,\text{kJ/K}$$

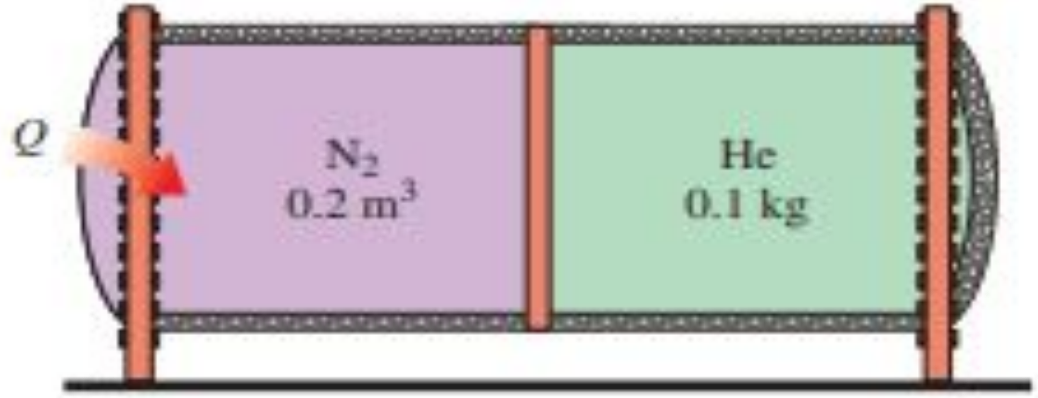
$$P_2 = \frac{m_a R T_2}{V} = \frac{(111.5\,\text{kg})(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(70.2+273\,\text{K})}{(90\,\text{m}^3)} = 122\,\text{kPa}$$

$$\Delta S_a = m_a\left(c_p \ln\frac{T_2}{T_{a1}} - R\ln\frac{P_2}{P_1}\right)$$

$$= (111.5\,\text{kg})\left[(1.005\,\text{kJ/kg.K})\ln\frac{(70.2+273)\,\text{K}}{(12+273)\,\text{K}} - (0.287\,\text{kJ/kg.K})\ln\frac{122\,\text{kPa}}{101.3\,\text{kPa}}\right] = 14.88\,\text{kJ/K}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_w + \Delta S_a = -13.11 + 14.88 = \mathbf{1.77kJ/K}$$

7-175 A horizontal cylinder is separated into two compartments by an adiabatic, frictionless piston. One side contains 0.2 m³ of nitrogen and the other side contains 0.1 kg of helium, both initially at 20°C and 95 kPa. The sides of the cylinder and the helium end are insulated. Now heat is added to the nitrogen side from a reservoir at 500°C until the pressure of the helium rises to 120 kPa. Determine (a) the final temperature of the helium, (b) the final volume of the nitrogen, (c) the heat transferred to the nitrogen, and (d) the entropy generation during this process.

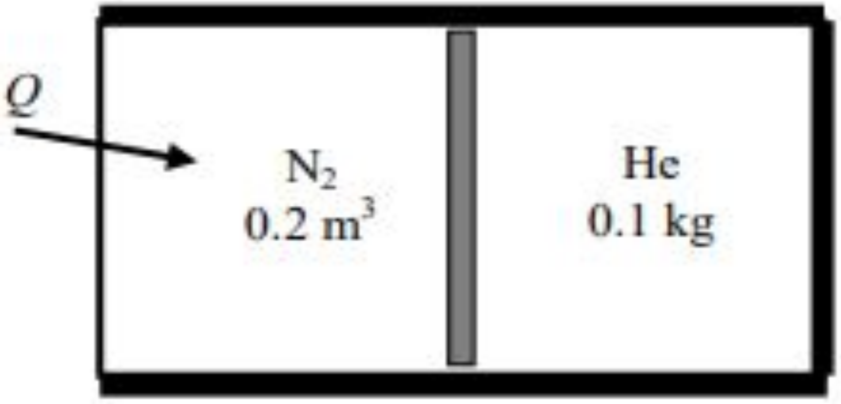


Solution:

A horizontal cylinder is separated into two compartments by a piston, one side containing nitrogen and the other side containing helium. Heat is added to the nitrogen side. The final temperature of the helium, the final volume of the nitrogen, the heat transferred to the nitrogen, and the entropy generation during this process are to be determined.

Assumptions

1. Kinetic and potential energy changes are negligible.
2. Nitrogen and helium are ideal gases with constant specific heats at room temperature.
3. The piston is adiabatic and frictionless.



Properties

The properties of nitrogen at room temperature are R = 0.2968 kPa.m³/kg.K, c_p = 1.039 kJ/kg.K, c_v = 0.743 kJ/kg.K, k = 1.4. The properties for helium are R = 2.0769 kPa.m³/kg.K, c_p = 5.1926 kJ/kg.K, c_v = 3.1156 kJ/kg.K, k = 1.667 (Table A-2).

TABLE A-2

Ideal-gas specific heats of various common gases

(a) At 300 K

Gas	Formula	Gas constant, R kJ/kg·K	c_p kJ/kg·K	c_v kJ/kg·K	k
Helium	He	2.0769	5.1926	3.1156	1.667
Hydrogen	H_2	4.1240	14.307	10.183	1.405
Methane	CH_4	0.5182	2.2537	1.7354	1.299
Neon	Ne	0.4119	1.0299	0.6179	1.667
Nitrogen	N_2	0.2968	1.039	0.743	1.400

Analysis

(a) Helium undergoes an isentropic compression process, and thus the final helium temperature is determined from

$$T_{\mathrm{He},2} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (20+273)\mathrm{K}\left(\frac{120\,\mathrm{kPa}}{95\,\mathrm{kPa}}\right)^{(1.667-1)/1.667}$$

$$= \mathbf{321.7K}$$

(b) The initial and final volumes of the helium are

$$\mathbf{V}_{\mathrm{He},1} = \frac{mRT_1}{P_1} = \frac{(0.1\,\mathrm{kg})(2.0769\,\mathrm{kPa\cdot m^3/kg\cdot K})(20+273\,\mathrm{K})}{95\,\mathrm{kPa}} = 0.6406\,\mathrm{m^3}$$

$$\mathbf{V}_{\mathrm{He},2} = \frac{mRT_2}{P_2} = \frac{(0.1\,\mathrm{kg})(2.0769\,\mathrm{kPa\cdot m^3/kg\cdot K})(321.7\,\mathrm{K})}{120\,\mathrm{kPa}} = 0.5568\,\mathrm{m^3}$$

Then, the final volume of nitrogen becomes

$$\mathbf{V}_{\mathrm{N2},2} = \mathbf{V}_{\mathrm{N2},1} + \mathbf{V}_{\mathrm{He},1} - \mathbf{V}_{\mathrm{He},2} = 0.2 + 0.6406 - 0.5568 = \mathbf{0.2838m^3}$$

(c) The mass and final temperature of nitrogen are

$$m_{\mathrm{N2}} = \frac{P_1\mathbf{V}_1}{RT_1} = \frac{(95\,\mathrm{kPa})(0.2\,\mathrm{m^3})}{(0.2968\,\mathrm{kPa\cdot m^3/kg\cdot K})(20+273\,\mathrm{K})} = 0.2185\,\mathrm{kg}$$

$$T_{\mathrm{N2},2} = \frac{P_2\mathbf{V}_2}{mR} = \frac{(120\,\mathrm{kPa})(0.2838\,\mathrm{m^3})}{(0.2185\,\mathrm{kg})(0.2968\,\mathrm{kPa\cdot m^3/kg\cdot K})} = 525.1\,\mathrm{K}$$

The heat transferred to the nitrogen is determined from an energy balance

$$\begin{aligned} Q_{\text{in}} &= \Delta U_{\text{N2}} + \Delta U_{\text{He}} \\ &= \left[m c_v (T_2 - T_1) \right]_{\text{N2}} + \left[m c_v (T_2 - T_1) \right]_{\text{He}} \\ &= (0.2185\ \text{kg})(0.743\ \text{kJ/kg.K})(525.1 - 293) + (0.1\ \text{kg})(3.1156\ \text{kJ/kg.K})(321.7 - 293) \\ &= \mathbf{46.6kJ} \end{aligned}$$

(d) Noting that helium undergoes an isentropic process, the entropy generation is determined to be

$$S_{\text{gen}} = \Delta S_{\text{N2}} + \Delta S_{\text{surr}} = m_{\text{N2}} \left(c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right) + \frac{-Q_{\text{in}}}{T_{\text{R}}}$$

ΔS_{He} =0

$$\begin{aligned} &= (0.2185\ \text{kg}) \left[(1.039\ \text{kJ/kg.K}) \ln \frac{525.1\ \text{K}}{293\ \text{K}} - (0.2968\ \text{kJ/kg.K}) \ln \frac{120\ \text{kPa}}{95\ \text{kPa}} \right] + \frac{-46.6\ \text{kJ}}{(500 + 273)\ \text{K}} \\ &= \mathbf{0.057kJ/K} \end{aligned}$$

THANK YOU