Tutorial - 04

$$f(x) = x^3 - 7x^2 + 16x - 12 = 0$$

use Bairstow's Method [initial guess $r = s = -1.0$]

Steps.

Step-01 determine $a_0, a_1 \ldots a_n$

$$P_n(x) = a_0 + a_1x + a_2x^2 + \ldots a_nx^n$$

$$f(x) = -12 + 16x - 7x^2 + x^3$$

$$a_0 = -12, \quad a_1 = 16, \quad a_2 = -7, \quad a_3 = 1$$

Step-02 compute $b_0, b_1 \ldots b_n$

$$b_n = a_n, \quad b_{n-1} = a_{n-1} + rb_n$$

$$b_i = a_i + rb_{i+1} + sb_{i+2} \quad i = (n-2) \ldots 2, 1, 0$$

$$(b_0, b_1, b_2, b_3)$$

$$b_3 = a_3 = 1$$

$$b_2 = a_2 + rb_3 = -7 + (-1)(1)$$

$$b_2 = -8$$

$$b_1 = a_1 + r b_2 + s b_3$$

$$b_1 = 16 + (-1)(-8) + (-1)(1)$$

$$b_1 = 23$$

$$b_0 = a_0 + r b_1 + s b_2$$

$$b_0 = -12 + (-1)(23) + (-1)(-8)$$

$$b_0 = -27$$

Step-03 compute $c_0, c_1 \dots c_n$

$[c_0, c_1, c_2, c_3] \longrightarrow 3^{rd}$ order polynomial.

$$C_n = b_n, \quad C_{n-1} = b_{n-1} + r C_n$$

$$c_i = b_i + r c_{i+1} + s c_{i+2} \Rightarrow i = (n-2) \dots 2, 1, 0$$

Scanned with CamScanner

$$c_3 = b_3 = 1$$

$$c_2 = b_2 + r c_3$$

$$c_2 = -8 + (-1)(1)$$

$$c_2 = -9$$

$$c_1 = b_1 + r c_2 + s c_3$$

$$= 23 + (-1)(-9) + (-1)(1)$$

$$c_1 = 31$$

$$c_0 = b_0 + r c_1 + s c_2$$

$$c_0 = -27 + (-1)(31) + (-1)(-9)$$

↳ not required for calculations.

Step-04 compute Δr & Δs using

$$\begin{bmatrix} c_1 & c_2 \\ c_2 & c_3 \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta s \end{bmatrix} = \begin{bmatrix} -b_0 \\ -b_1 \end{bmatrix}$$

Cramer's Rule

$$\Delta r = \frac{\begin{vmatrix} -b_0 & c_2 \\ -b_1 & c_3 \end{vmatrix}}{\begin{vmatrix} c_1 & c_2 \\ c_2 & c_3 \end{vmatrix}} \quad \cancel{\Delta s = \begin{vmatrix} -b_0 \\ -b_1 \end{vmatrix}}$$

$$\Delta s = \frac{\begin{vmatrix} c_1 & -b_0 \\ c_2 & -b_1 \end{vmatrix}}{\begin{vmatrix} c_1 & c_2 \\ c_2 & c_3 \end{vmatrix}}$$

$\hookrightarrow c_1 c_3 - c_2^2$

$$\Rightarrow \begin{bmatrix} 31 & -9 \\ -9 & 1 \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta s \end{bmatrix} = \begin{bmatrix} -(-27) \\ -23 \end{bmatrix}$$

$$\Delta r = \frac{\begin{vmatrix} 27 & -9 \\ -23 & 1 \end{vmatrix}}{\begin{vmatrix} 31 & -9 \\ -9 & 1 \end{vmatrix}}$$

$$\Delta r = \frac{-180}{-50}$$

$$\Delta r = 3.6$$

Similarly,

$$\Delta s = \frac{\begin{vmatrix} 31 & 27 \\ -9 & -23 \end{vmatrix}}{\begin{vmatrix} 31 & -9 \\ -9 & 1 \end{vmatrix}}$$

$$\Delta s = \frac{-470}{-50}$$

$$\Delta s = 9.4$$

Step-05 $r_{new} = r + \Delta r$

$s_{new} = s + \Delta s$

$r_{new} = -1 + 3.6$

$r_{new} = 2.6$

$S_{new} = -1 + 9.4$

$S_{new} = 8.4$

Step-06 check for convergence

$$e_r = \left| \frac{r_{new} - r_{old}}{r_{new}} \right| \times 100$$

$$= \left| \frac{2.6 - (-1)}{2.6} \right| \times 100$$

$e_r = 138.46\ \%$

$$e_s = \left| \frac{S_{new} - S_{old}}{S_{new}} \right| \times 100$$

$$= \left| \frac{8.4 - (-1)}{8.4} \right| \times 100$$

$e_s = 111.9048\ \%$

convergence criterion not satisfied

Iteration - 02

$r = r_{new} = 2.6$

$s = s_{new} = 8.4$

continue from step - 02 ....

After 8 iterations

$r = 4.0$

$s = -4.0$

Step - 08 $f(x) = (x^2 - rx - s)(b_3 x - b_2) = 0$

$f(x) = (x^2 - 4x + 4)(x - 3) = 0$

$x^2 - 4x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{(4)^2 - 4 \times 4}}{2}$$

$\boxed{x = 2, 2}$

$x - 3 = 0$

$\boxed{x = 3}$

Question - 02

Gauss Elimination Method

$$10x + 2y - z = 27$$
$$-3x - 6x + 2z = -61.5$$
$$x + y + 5z = -21.5$$

$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

$\underline{b} \neq 0$ Non- homogenous system of Equations

Coefficient Matrix

$$\underline{\underline{A}} = \begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 27 \\ -61.5 \\ -21.5 \end{bmatrix}$$

$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

Augumented Matrix $[\underline{\underline{A}} \,|\, \underline{b}]$

$$\tilde{A} = \left[\begin{array}{ccc|c} 10 & 2 & -1 & 27 \\ -3 & -6 & 2 & -61.5 \\ 1 & 1 & 5 & -21.5 \end{array}\right]$$

Only Row operations allowed

$$l_{21} = \frac{a_{21}}{a_{11}} = \frac{-3}{10}$$

$$l_{31} = \frac{a_{31}}{a_{11}} = \frac{1}{10}$$

$$R_2 = R_2 - l_{21} R_1$$

$$R_2 = R_2 + \frac{3}{10} R_1$$

Similarly,

$$R_3 = R_3 - l_{31} R_1$$

$$R_3 = R_3 - \frac{1}{10} R_1$$

$$\tilde{A} = \left[\begin{array}{ccc|c} 10 & 2 & -1 & 27 \\ 0 & -5.4 & 1.7 & -53.4 \\ 0 & 0.8 & 5.1 & -24.2 \end{array}\right]$$

$$l_{32} = \frac{a_{32}}{a_{22}} = \frac{0.8}{-5.4}$$

$$R_3 = R_3 - l_{32} R_2$$

$$\tilde{A} = \left[\begin{array}{ccc|c} 10 & 2 & -1 & 27 \\ 0 & -5.4 & 1.7 & -53.4 \\ 0 & 0 & 5.1 & -32.1111 \end{array}\right]$$

New system of Eq^n

$$\left.\begin{array}{r} 10x + 2y - 1z = 27 \\ -5.4y + 1.7z = -53.4 \\ 5.1z = -32.1111 \end{array}\right\}$$

$$z = \frac{-32.1111}{5.3518}$$

$$\boxed{z = -6}$$

$$-5.4y + 1.7z = -53.4$$

$$-5.4y = -53.4 - 1.7(-6)$$

$$y = \frac{-43.2}{-5.4}$$

$$\boxed{y = 8}$$

$$10x + 2y - z = 27$$

$$10x + 2 \times 8 + 6 = 27$$

$$10x = 5$$

$$\boxed{x = 0.5}$$

if you want to confirm the solution put x, y & z in system of equations & check if L.H.S. = R.H.S.