ESO201A : THERMODYNAMICS
2021-22 Ist semester
IIT Kanpur

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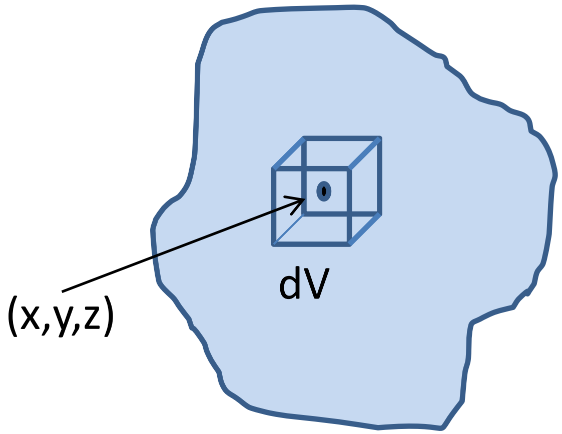
Lecture 10

Rate form of First law for a control mass :

Recall that the rate form of the first law as applied to a control mass is

$$\frac{dE_{CM}}{dt} = \dot{Q}_{in} - \dot{W}_{out,t}$$

Here : E_{CM} is the total energy of a given control mass system and it can be expressed as $E_{CM} = \int_{CM} e \, \rho \, dV$



Control mass (CM)

$\dot{Q}_{in}$ = Rate of energy transfer to the control mass as heat

$\dot{W}_{out,t}$ = Rate of energy transfer from the control mass to the surroundings as work

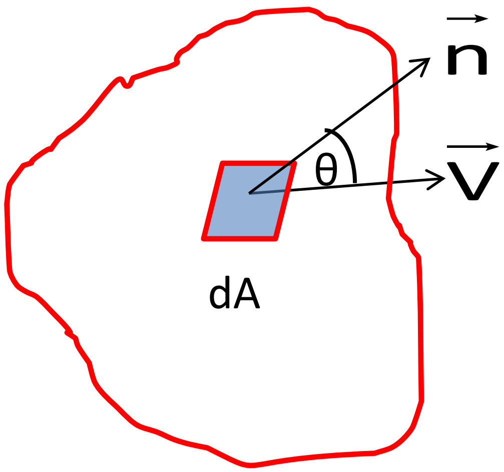
First law as applied to a control volume :

Applying Reynolds transport theorem to the left hand side of the rate form of first law (on previous slide) and splitting work term in two parts, we get

$$\frac{dE_{CV}}{dt} + \int_{CS} e\,\rho\,(\vec{V}.\vec{n})\,dA = \dot{Q}_{in} - \dot{W}_{out}^{non-flow} - \dot{W}_{out}^{flow}$$

Here : E_{CV} is the total energy of the fluid inside the control volume at a given time t, expressed as

$$E_{CV} = \int_{CV} e\,\rho\,dV$$



Control Volume (CV)

$$\dot{W}_{out,t} = \dot{W}_{out}^{non-flow} + \dot{W}_{out}^{flow}$$

Thus, the total work done is split into two parts: flow work and non-flow work. Non-flow work term includes shaft work, moving boundary work as well as non-mechanical forms of work

IMPORTANT : Note that the heat and work terms in the equation on the last slide can be interpreted as heat received and work done per unit time by the body of the fluid (control mass) that occupies the control volume at the given time t*. Thus,

$$\dot{Q}_{in} = \text{Heat input per unit time to the body of the fluid (control mass) that occupies the control volume at given time } t^*$$

$$\dot{W}_{out,t} = \dot{W}_{out}^{\text{non-flow}} + \dot{W}_{out}^{\text{flow}}$$

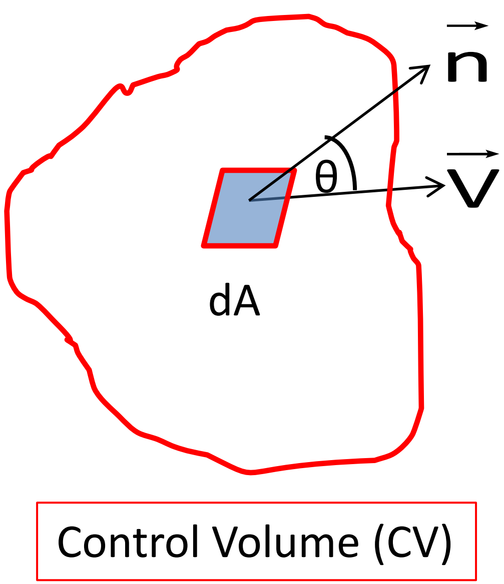
$$= \text{total work done per unit time by the body of the fluid (control mass) that occupies the control volume at a given time } t^*$$

Flow work :

$$\dot{W}_{out}^{flow} = \int_{CS} P\,(\vec{V}.\vec{n})\,dA$$

PdA = Force acting on the differential area element dA in the direction of $\vec{n}$

$\vec{V}.\vec{n}$ = displacement per unit time in the direction of $\vec{n}$



$\dot{W}_{out}^{flow}$ = Work done by the control mass (which is entirely occupying the control volume) in pushing the external mass of fluid at the surface of the control mass

First law as applied to control volumes :

$$\frac{dE_{CV}}{dt} + \int_{CS} e\,\rho\,(\vec{V}.\vec{n})\,dA = \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow} - \int_{CS} P\,(\vec{V}.\vec{n})\,dA$$

$$\frac{dE_{CV}}{dt} + \int_{CS} (e\,\rho + P)\,(\vec{V}.\vec{n})\,dA = \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow}$$

$$\frac{dE_{CV}}{dt} + \int_{CS} (e + Pv)\,\rho(\vec{V}.\vec{n})\,dA = \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow}$$

Choosing the control volume such that the control surface is perpendicular to the direction of flow, we have $\vec{V}.\vec{n}=V$ (the magnitude of the velocity) for outlets and $\vec{V}.\vec{n}= - V$ for inlets.

First law as applied to control volumes :

$$\frac{dE_{CV}}{dt} + \int_{CS(out)} (e + Pv)\rho V\, dA - \int_{CS(in)} (e + Pv)\rho V\, dA$$

$$= \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow}$$

Here CS(out) is that part of the control surface where fluid is exiting the control volume.

Similarly, CS(in) is that part of the control surface where fluid is entering the control volume.

Now, we assume that properties e, P, v, and $\rho(= 1 / v)$ are constant over CS(out) and CS(in). We can simplify the terms involving the area integration in the above expression.

First law as applied to control volumes :

Note that using the assumption made in the last slide, $\int_{CS(out)} (e+Pv)\, \rho\, V\, dA = \sum_{out} \left[(e+Pv)\, \rho \int_{out} V\, dA \right]$

$$= \sum_{out} (e+Pv)\, \dot{m}$$

Similarly integration over CS(in) can be simplified.

Thus,the first law expression can be written as

$$\frac{dE_{CV}}{dt} + \sum_{out} \dot{m}(e+Pv) - \sum_{in} \dot{m}(e+Pv)$$

$$= \dot{Q}_{in} - \dot{W}_{out}^{non\text{-}flow}$$

First law as applied to control volumes :

Note that

$$(e + Pv) = \left(u + \frac{1}{2}V^2 + gZ + Pv \right)$$

$$= \left(h + \frac{1}{2}V^2 + gZ \right)$$

Substituting this in the previous equation, we get

$$\boxed{\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \frac{1}{2}V^2 + gZ \right) - \sum_{out} \dot{m}\left(h + \frac{1}{2}V^2 + gZ \right) + \dot{Q}_{in} - \dot{W}_{out}}$$

where the superscript "non - flow" on work term is dropped for simplicity. Henceforth, it will be understood that the work term in the above expression is the non - flow work.

First law as applied to control volumes :

We shall use the first law expression in the box in the last slide for analysis of flow equipments.

Several equipments like pumps, compressors, diffusors, nozzles etc. operate under "steady-flow" conditions.

In "steady-flow" conditions, properties of the fluid may vary in space inside the control volume, however the properties at a given point in space are constant, i.e., do not change with time. For example, the velocity vector is a function of (x,y,z) but NOT time. Similarly, specific internal energy, specific enthalpy, specific volume and density of the fluid are also be functions of (x,y,z) (i.e., coordinates within the control volume) but NOT time.

Thus in "steady flow", properties such as total energy E_{CV}, total mass M_{CV} are constant.

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“Steady-flow” conditions :

In “steady-flow” conditions, the first law expression simplifies to :

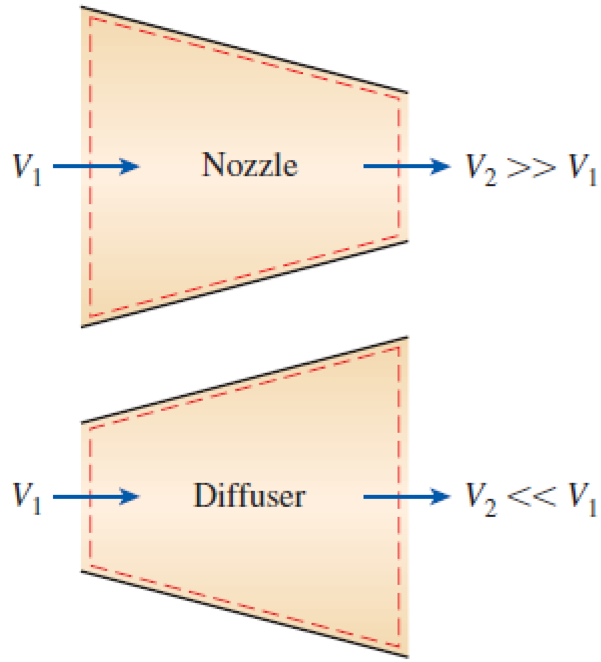
$$\boxed{\sum_{in} \dot{m}\left(h + \frac{1}{2}V^2 + gZ\right) - \sum_{out} \dot{m}\left(h + \frac{1}{2}V^2 + gZ\right) + \dot{Q}_{in} - \dot{W}_{out} = 0}$$

This is because under steady-flow :

$$\frac{dE_{CV}}{dt} = 0$$

Nozzles and diffusors:

Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses. A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure. A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down. That is, nozzles and diffusers perform opposite tasks.



Ref. Cengel and Boles, 8th Edition (2015)

Example: deceleration of air in a diffuser

Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m². The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine (*a*) the mass flow rate of the air and (*b*) the temperature of the air leaving the diffuser.

Solution : (a) To find mass flow rate, we assume that air is an ideal gas. The specific volume can then be obtained as:

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(283\ \text{K})}{80\ \text{kPa}} = 1.015\ \text{m}^3/\text{kg}$$

The volumetric flow rate is the velocity of air at the inlet multiplied by inlet area. Since both of these quantities are given, mass flow rate is obtained by dividing volumetric flow tate by specific volume :

$$\dot{m} = \frac{1}{v_1} V_1 A_1 = \frac{1}{1.015\ \text{m}^3/\text{kg}} (200\ \text{m/s})(0.4\ \text{m}^2) = \mathbf{78.8\ kg/s}$$

Ref. Cengel and Boles, 8th Edition (2015)

Example: deceleration of air in a diffuser

(b) we neglect changes in potential energy and heat transfer to the diffuser. Also, in the diffuser no work is done. The diffuser operates under steady state. The inlet is denoted as '1' and outlet as '2'.

Hence the first Law expression for steady-flow devices simplifies to :

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$

The exit velocity of a diffuser is usually small compared with the inlet velocity ($V_2 << V_1$); thus, the kinetic energy at the exit can be neglected. The enthalpy of air at the diffuser inlet is determined from the air table (Table A–17) to be

$$h_1 = h_{@\,283\,\text{K}} = 283.14 \text{ kJ/kg}$$

Substituting, we get

$$h_2 = 283.14 \text{ kJ/kg} - \frac{0 - (200 \text{ m/s})^2}{2}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right)$$
$$= 303.14 \text{ kJ/kg}$$

Ref. Cengel and Boles, 8th Edition (2015)

Example: deceleration of air in a diffuser

(b)

From Table A–17, the temperature corresponding to this enthalpy value is

$$T_2 = \mathbf{303\ K}$$

Discussion This result shows that the temperature of the air increases by about 20°C as it is slowed down in the diffuser. The temperature rise of the air is mainly due to the conversion of kinetic energy to internal energy.

Ref. Cengel and Boles, 8th Edition (2015)