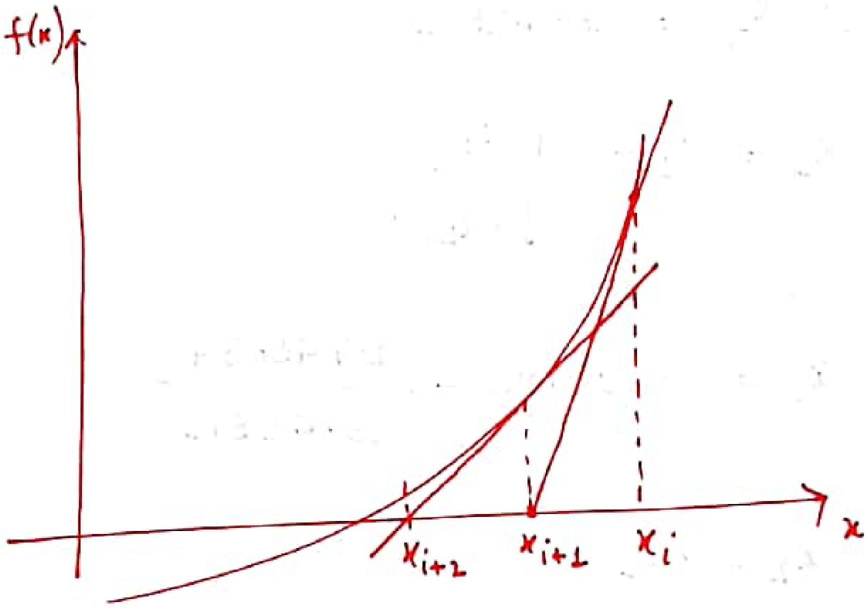
Tutorial - 03

$$f(x) = 600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0$$

(a) Newton - Raphson Method (0.5 as guess)

$$f'(x) = 2400x^3 - 1650x^2 + 400x - 20$$



$$f'(x_i) = \frac{-f(x_i)}{x_{i+1} - x_i}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Iteration - 01 $[x_0 = 0.5]$

$$f(x = 0.5) = 7.75$$

$$f'(x = 0.5) = 67.5$$

$$x_1 = 0.5 - \frac{7.75}{67.5}$$

$$x_1 = 0.385185$$

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$$\text{Error} = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$= \left| \frac{0.38518 - 0.5}{0.38518} \right| \times 100$$

$$\boxed{\text{Error} = 29.80769\ \%}$$

(b.) Secant Method $[x_0 = 0.1 \;\&\; x_1 = 1.0]$

same as Newton - Raphson except we do not have to calculate the derivative of given function.

$$f(x) = 600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0$$

$$\text{Approximate } f'(x) \approx \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

$$x_{k+1} = x_k - f(x_k)\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$$

Iteration - 01 $[x_0 = 0.1 \quad x_1 = 1.0]$

$$f(x = x_0) = -1.49$$

$$f(x = x_1) = 229$$

$$\cancel{x_3 = x_2 - \frac{f(x_2)}{x}}$$

$$x_2 = x_1 - f(x_1)\frac{x_0 - x_1}{f(x_0) - f(x_1)}$$

$$x_2 = 1.0 - 229\frac{(0.1 - 1.0)}{-1.49 - 229}$$

$$\boxed{x_2 = 0.10582}$$

$$\text{Error} = \left|\frac{0.10582 - 1.0}{0.10582}\right| \times 100$$

$$\boxed{\text{Error} = 845.018\,\%}$$

Solution of system of Nonlinear Equation

2.

$$u(x,y) = x^2 - x + y - 0.75 = 0$$

$$v(x,y) = x^2 - 5xy - y = 0$$

$$x_0 = 1.2 \quad , \quad y_0 = 1.2$$

$$x_{i+1} = g_1(x_i, y_i)$$

$$x_{i+1} = \sqrt{x - y + 0.75}$$

$$x^2 - y(5x+1) = 0$$

$$y = x^2/(5x+1)$$

$$y_{i+1} = \frac{(x_{i+1})^2}{(5x_{i+1} + 1)}$$

$$E_x = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

$$E_y = \left| \frac{y_{i+1} - y_i}{y_{i+1}} \right| \times 100$$

Iteration - 01

$$x_{i+1} = x_1 = \sqrt{x_0 - y_0 + 0.75}$$

$$= \sqrt{1.2 - 1.2 + 0.75}$$

$$x_1 = 0.86603$$

$$y_1 = \frac{x_1^2}{5x_1+1}$$

$$y_1 = \frac{(0.86603)^2}{5 \times 0.86603+1}$$

$$\boxed{y_1 = 0.14071}$$

$$E_x = \left| \frac{0.86603 - 1.2}{0.86603} \right| \times 100$$

$$\boxed{E_x = 38.5641\%}$$

$$E_y = \left| \frac{0.14071 - 1.2}{0.14071} \right| \times 100$$

$$\boxed{E_y = 752.82\%}$$

(b.) Newton-Raphson Method

$$x_0 = 1.2 \quad \& \quad y_0 = 1.2$$

$$u(x,y) = x^2 - x + y - 0.75 = 0$$

$$v(x,y) = x^2 - 5xy - y = 0$$

$$\left.\begin{aligned} x_{i+1} &= x_i - \frac{u_i \frac{\partial v_i}{\partial y} - v_i \frac{\partial u_i}{\partial y}}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}} \\ y_{i+1} &= y_i - \frac{v_i \frac{\partial u_i}{\partial x} - u_i \frac{\partial v_i}{\partial x}}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}} \end{aligned}\right\} \text{can be derived using Taylor Series Approximation}$$

denominator $\rightarrow$ determinant of Jacobian

$$u = x^2 - x + y - 0.75$$

$$\left.\frac{\partial u}{\partial x}\right|_y = 2x - 1$$

$$\left.\frac{\partial u}{\partial y}\right|_x = 1$$

$$v = x^2 - 5xy - y$$

$$\left.\frac{\partial v}{\partial x}\right|_y = 2x - 5y$$

$$\left.\frac{\partial v}{\partial y}\right|_x = -5x - 1$$

Iteration-01 $[x_0 = 1.2, \; y_0 = 1.2]$

$u(x_0, y_0) = 0.69$

$v(x_0, y_0) = -6.96$

Denominator (D) $= \frac{\partial u_i}{\partial x} \frac{\partial v_i}{\partial y} - \frac{\partial u_i}{\partial y} \frac{\partial v_i}{\partial x}$

$\left.\frac{\partial u_i}{\partial x}\right|_y = 2x_0 - 1 = 2 \times 1.2 - 1$

$= 1.4$

$\left.\frac{\partial u_i}{\partial y}\right|_x = 1$

$\left.\frac{\partial v_i}{\partial x}\right|_y = 2x_0 - 5y_0$

$= 2 \times 1.2 - 5 \times 1.2$

$= -3.6$

$\left.\frac{\partial v_i}{\partial y}\right|_x = -5x_0 - 1$

$= -5 \times 1.2 - 1$

$= -7$

$D = 1.4 \times (-7) - (1) \times (-3.6)$

$D = -6.2$

$$x_{i+1} = x_i - \frac{u_i \frac{\partial v_i}{\partial y} - v_i \frac{\partial u_i}{\partial y}}{D}$$

$$x_1 = x_0 - \frac{u_0 \frac{\partial v_0}{\partial y} - v_0 \frac{\partial u_0}{\partial y}}{D}$$

Similarly

$$y_{\cancel{0}1} = y_0 - \frac{v_0 \frac{\partial u_0}{\partial x} - u_0 \frac{\partial v_0}{\partial x}}{D}$$

$$x_1 = 1.2 - \frac{0.69(-7) - (-6.96)(1)}{-6.2}$$

$$\boxed{x_1 = 1.54355}$$

$$y_1 = 1.2 - \frac{(-6.96)(1.4) - 0.69(-3.6)}{-6.2}$$

$$\boxed{y_1 = 0.02903}$$

$$E_x = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$= \left| \frac{1.54355 - 1.2}{1.54355} \right| \times 100 = 22.2571\%$$

$$E_y = \left| \frac{0.02903 - 1.2}{0.02903} \right| \times 100$$

$$E_y = 4033.33\%$$

Multiple Roots.

3 $f(x) = x^3 - 7x^2 + 16x - 12 = 0$

$x_0 = 1.0$

(a.) Newton - Raphson Method

$f'(x) = 3x^2 - 14x + 16$

& $x_{i+1} = x_i - f(x_i)/f'(x_i)$

Iteration-01

$f(x=1.0) = -2$

$f'(x=1.0) = 3 - 14 + 16$

$= 5$

$x_1 = x_0 - (-2)/5$

$= 1 + 2/5$

$x_1 = 1.4$

$$\text{Error} = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$= \left| \frac{1.4 - 1.0}{1.0} \right| \times 100$$

$$\boxed{E = 28.57\ \%}$$

(b.) First Modification of Newton-Raphson Method [m = 2]

$$f(x) = x^3 - 7x^2 + 16x - 12 = 0$$

$$x_0 = 1.0$$

first Modification

$$x_{i+1} = x_i - m \frac{f(x_i)}{f'(x_i)}$$

multiplicity = 2

$$f'(x) = 3x^2 - 14x + 16$$

Iteration-01

$$\left.\begin{aligned} f(x=1.0) &= -2 \\ f'(x=1.0) &= 5 \end{aligned}\right\} \text{from Previous Part}$$

$$x_1 = x_0 - 2 \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1.0 - 2\frac{(-2)}{5}$$

$$x_1 = 1.8$$

$$\text{Error} = \left|\frac{1.8 - 1.0}{1.8}\right| \times 100$$

$$\boxed{E = 44.44\%}$$

(c.) Second modification of Newton - Raphson Method

$$x_{i+1} = x_i - \frac{f(x_i)\,f'(x_i)}{[f'(x_i)]^2 - f(x_i)\,f''(x_i)}$$

$$f(x) = x^3 - 7x^2 + 16x - 12 = 0$$

$$f'(x) = 3x^2 - 14x + 16$$

$$f''(x) = 6x - 14$$

Iteration - 01 $[x_0 = 1.0]$

$$\left.\begin{aligned} f(x=1.0) &= -2 \\ f'(x=1.0) &= 5 \end{aligned}\right\} \text{from first part}$$

$$f''(x=1.0) = 6\times 1 - 14$$
$$= -8$$

$$x_1 = x_0 - \frac{f(x_0)\, f'(x_0)}{[f'(x_0)]^2 - f(x_0)\, f''(x_0)}$$

$$x_1 = 1.0 - \frac{(-2)\,5}{(5)^2 - (-2)(-8)}$$

$$\boxed{x_1 = 2.11111}$$

$$\text{Error} = \left| \frac{x_1 - x_0}{x_1} \right| \times 100$$

$$= \left| \frac{2.1111 - 1.0}{2.1111} \right| \times 100$$

$$\boxed{E = 52.63\,\%}$$

For the given problem Newton - Raphson Method gives smallest convergence rate while first and second modification give similar convergence [Second modification performs slightly better than first modification]

Roots of polynomial

4 $\Rightarrow$ Use. Muller's method

$\Rightarrow$ $x_0 = 0.0$

$x_1 = 1.25$

$x_2 = 3.25$

Formulas

(Muller's method tries to obtain the root by projecting a parabola to the x-axis)

$$h_0 = x_1 - x_0 \qquad h_1 = x_2 - x_1$$

$$\delta_0 = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \qquad \delta_1 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$a = \frac{\delta_1 - \delta_0}{h_1 + h_0} \qquad b = ah_1 + \delta_1 \qquad c = f(x_2)$$

$$x_3 = x_2 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

Sign of $\pm$ term is chosen to agree with b. So that we have a large denominator which will give a root closest to x_2.

$$x_0, \; x_1, \; x_2 \xrightarrow[\text{calculate}]{\text{Iteration 1}} x_3$$

$$x_1, \; x_2, \; x_3 \xrightarrow{\text{Iteration-2}} x_4$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x_0 & x_1 & x_2 & x_3 \end{array}$$

Iteration-01

$$f(x) = x^3 - 7x^2 + 16x - 12 = 0$$

$x_0 = 0.0 \qquad f(x_0) = -12$

$x_1 = 1.25 \qquad f(x_1) = -0.98438$

$x_2 = 3.25 \qquad f(x_2) = 0.39063$

$$h_0 = 1.25 - 0.0$$
$$= 1.25$$

$$h_1 = x_2 - x_1 = 3.25 - 1.25$$
$$= 2.0$$

$$\delta_0 = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{h_0}$$

$$= \frac{-0.98438 - (-12)}{1.25 - 0.0}$$

$$\delta_0 = 8.8125$$

$$\delta_1 = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{h_1}$$

$$= \frac{0.39063 - (-0.98438)}{3.25 - 1.25}$$

$$\delta_1 = 0.6875$$

$$a = \frac{\delta_1 - \delta_0}{h_1 + h_0} = \frac{0.6875 - 0.8125}{1.25 + 2.0}$$

$$a = -2.5$$

$$b = ah_1 + \delta_1$$

$$= (-2.5)\,2.0 + 0.6875$$

$$b = -4.3125$$

$$c = f(x_2) = 0.39063$$

$$x_3 = x_2 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

$b \Rightarrow \ominus ve$

$\pm\sqrt{} \Rightarrow \ominus ve$

$$x_3 = 3.25 - \frac{2(0.39063)}{-4.3125 - \sqrt{(4.3125)^2 - 4(-2.5) \times 0.39063}}$$

$$\boxed{x_3 = 3.33627}$$

x_0	x_1	x_2	$\longrightarrow$	x_3
0.0	1.25	3.25		3.33627

4 –

for second iteration

$x_0 = 1.25$ $\quad x_1 = 3.25$ $\quad x_2 = 3.33625$

$\Rightarrow \quad x_3 = 3.07344$

$$\text{Error} = \left| \frac{x_3^{new} - x_3^{old}}{x_3^{new}} \right| \times 100$$

$$= \cancel{0.5} \left| \frac{3.073 - 3.336}{3.073} \right| \times 100$$

$$\boxed{E = 8.552\%}$$

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