

Tutorial 11Numerical differentiation

1. The location of an object at various times was measured as follows:

Time (t ; s)	0	1	2	3	4	5	6	7	8	9
Distance (x ; m)	1	1.55	2.32	3.58	5.79	9.68	16.49	28.22	48.2	81.92

Estimate the speed and acceleration of the object at 5 seconds by using - (i) Forward difference, $O(h^2)$ (ii) Backward difference, $O(h^2)$ (iii) Central difference, $O(h^2)$ and (iv) Richardson extrapolation, $O(h^6)$ using three central differences of $O(h^2)$. Estimate the true percentage error if the object location is given by $x = e^{0.5t} - 0.1t^2$.

2. For the function $f(x) = \frac{\sin x}{x^3}$, obtain finite difference approximations of f' with first order backward difference, second order central difference and 4th order central difference. Evaluate f' by the three methods at 20 equally spaced points in the interval $[1, 2\pi]$. Also evaluate the true value of f' at the same points. Plot f' vs. x and graphically compare the true values with the three approximations you have obtained, all in the same plot. Show them by different styles of lines.