ESO201A : THERMODYNAMICS

2021-22 Ist semester

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Lecture 24

Flow exergy

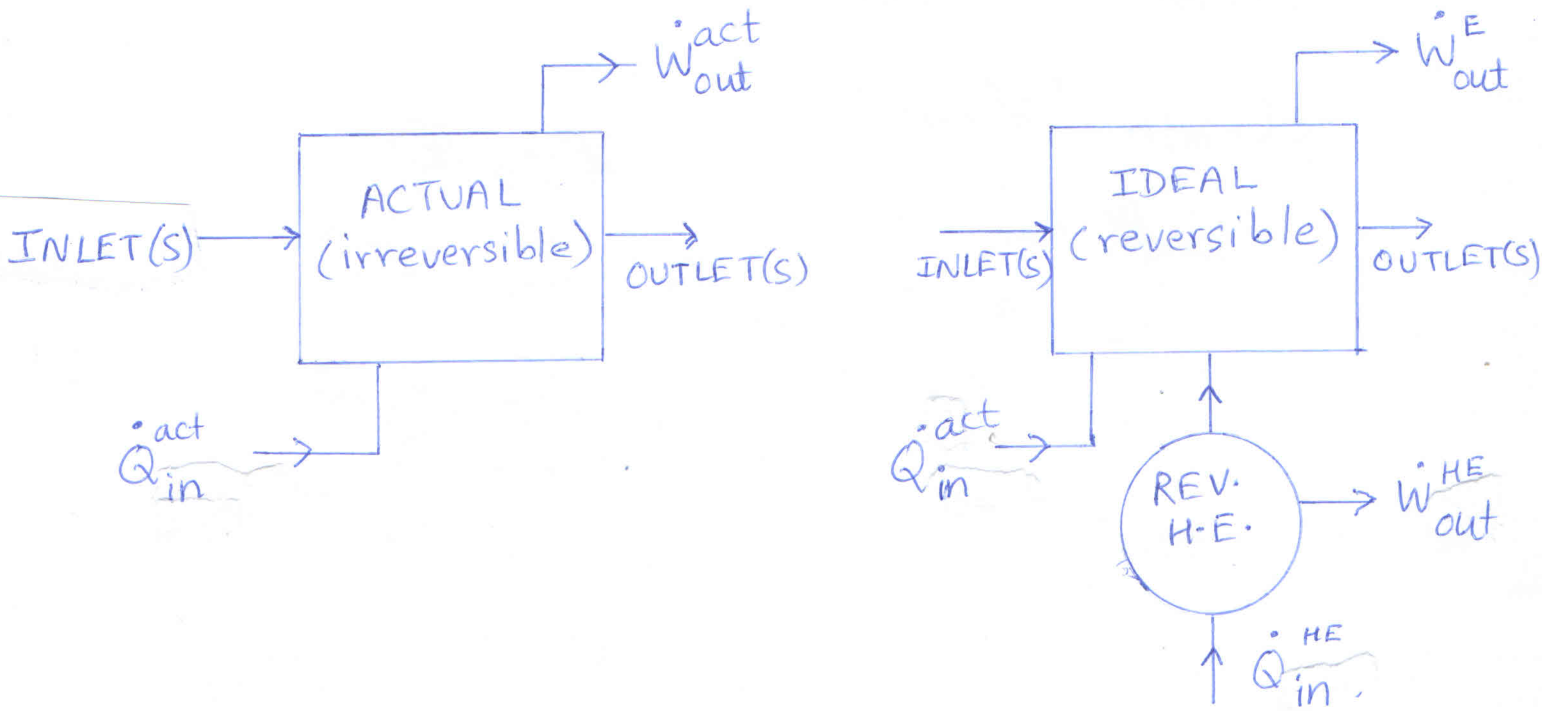
When dealing with control volume systems (flow equipments), we define a quantity known as "flow exergy per unit mass" as follows:

$$\psi = (h - h_o) - T_o(s - s_o) + \frac{V^2}{2} + gZ \qquad (1)$$

Here we are considering that there is heat exchange of the fluid in the control volume with ONLY one thermal energy reservoir, which is the environment at temperature T_o and pressure P_o. In Eq. (1), h_o and s_o are the specific enthalpy and specific entropy of fluid when it is in equilibrium with the environment.

Let's consider "actual (irreversible)" and "ideal (reversible)" flow equipments under steady-flow conditions.

The inlet and outlet flow conditions for both equipments are the SAME.



The ideal set-up consists of a reversible heat engine in addition to the flow equipment. The entire ideal set-up operates reversibly. The total heat input and the total work output of the ideal set-up are:

$$\dot{Q}^{rev}_{in} = \dot{Q}^{act}_{in} + \dot{Q}^{HE}_{in} \quad \text{— (2)}$$

$$\dot{W}^{rev}_{out} = \dot{W}^{E}_{out} + \dot{W}^{HE}_{out}$$

Here $\dot{Q}_{in}^{act}$ is the heat input required ③ in the actual flow equipment.

Applying first law (in rate form) to the ideal set-up,

$$\sum_{out} \dot{m}\left(h+\frac{V^2}{2}+gZ\right) - \sum_{in} \dot{m}\left(h+\frac{V^2}{2}+gZ\right) = \dot{Q}_{in}^{rev} - \dot{W}_{out}^{rev}$$

$\llcorner$ ③

Applying second law,

$$\sum_{out} \dot{m}s - \sum_{in} \dot{m}s + \left(\frac{-\dot{Q}_{in}^{rev}}{T_0}\right) = \cancel{\dot{S}_{gen}}^{\,0}$$

$\llcorner$ ④

Note that $\dot{S}_{gen}=0$ since it is a reversible process. Substituting ④ in ③, we get

$$\dot{W}_{out}^{rev} = \sum_{in} \dot{m}\left(h+\frac{V^2}{2}+gZ\right) - \sum_{out} \dot{m}\left(h+\frac{V^2}{2}+gZ\right)$$

$$-T_0\left(\sum_{in} \dot{m}s - \sum_{out} \dot{m}s\right)$$

$$+\sum_{out} \dot{m}(h_0 - T_0 s_0) - \sum_{in} \dot{m}(h_0 - T_0 s_0)$$

$\llcorner$ ⑤

We note that in Eq. 5 we have deliberately added the last two terms because we want to write the right hand side of Eq. 5 in terms flow exergy. Moreover, the last two terms of Eq. 5 add up to zero as shown below:

sum of last two terms of Eq. 5

$$= \left(\sum_{out} \dot{m} - \sum_{in} \dot{m}\right)(h_0 - T_0 s_0)$$

$$= 0 \quad \text{since } \sum_{out} \dot{m} = \sum_{in} \dot{m}$$

due to steady-flow conditions.

We can write Eqn 5 in terms of flow exergies after combining last two terms on the right hand side with the other terms. Thus,

$$\dot{W}^{rev}_{out} = \sum_{in} \dot{m}\psi - \sum_{out} \dot{m}\psi \qquad (6)$$

Thus work output is equal to decrease of flow exergy across the flow equipment

Now let's consider the actual (irreversible) equipment. Applying first law, we get

$$\sum_{out} \dot{m}\left(h+\frac{V^2}{2}+gZ\right)-\sum_{in} \dot{m}\left(h+\frac{V^2}{2}+gZ\right) = \dot{Q}_{in}^{act} - \dot{W}_{out}^{act} \quad (7)$$

Note that the left hand side of Eqs. (3) & (7) are the same. Therefore,

if $\quad \dot{W}_{out}^{rev} > \dot{W}_{out}^{act}$

then $\quad \dot{Q}_{in}^{rev} > \dot{Q}_{in}^{act}$

Applying Second law to actual equipment, we get

$$\sum_{out} \dot{m} s - \sum_{in} \dot{m} s + \left(\frac{-\dot{Q}_{in}^{act}}{T_0}\right) = \dot{S}_{gen} \quad (8)$$

Substituting for $\dot{Q}_{in}^{act}$ in Eqn (7) using Eqn (8), we get

$$\dot{W}^{act}_{out} = \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

$$- T_0\left(\sum_{in} \dot{m}s - \sum_{out} \dot{m}s\right) - T_0\dot{S}_{gen}$$

$$+ \sum_{out} \dot{m}\left(h_0 - T_0 s_0\right) - \sum_{in} \dot{m}\left(h_0 - T_0 s_0\right) \qquad (9)$$

Just like in Eqn. (5), we have added last two terms in Eqn. (9) [which add up to zero] so that right hand side of (9) can be written in terms of flow exergies. Thus, from (9), we get

$$\dot{W}^{act}_{out} = \sum_{in} \dot{m}\psi - \sum_{out} \dot{m}\psi - T_0\dot{S}_{gen} \qquad (10)$$

Substituting (6) in (10), we get

$$\dot{W}^{act}_{out} = \dot{W}^{rev}_{out} - T_0\dot{S}_{gen} \qquad (11)$$

According to Second law,

$$\dot{S}_{gen} > 0$$

for the actual (irreversible) equipment. Hence, from 11 we get:

$$\dot{W}_{out}^{act} < \dot{W}_{out}^{rev}$$

Therefore, lost work or exergy (work potential) destroyed

$$= \dot{W}_{out}^{rev} - \dot{W}_{out}^{act}$$

$$= \underbrace{\left(\sum_{in} \dot{m}\psi - \sum_{out} \dot{m}\psi\right)}_{\text{from } 6} - \dot{W}_{out}^{act}$$

$$= \underbrace{T_o \dot{S}_{gen}}_{\text{from } 11} \quad \text{— } 12$$

Note that

$$\left(\sum_{in} \dot{m}\psi - \sum_{out} \dot{m}\psi\right) = \text{Exergy supplied to the equipment}$$

Second law efficiency of the actual (irreversible) flow equipment is given by

$$\eta_{II} = \frac{\dot{W}_{out}^{act}}{\dot{W}_{out}^{rev}} = \frac{\dot{W}_{out}^{rev} - T_0 \dot{S}_{gen}}{\dot{W}_{out}^{rev}}$$

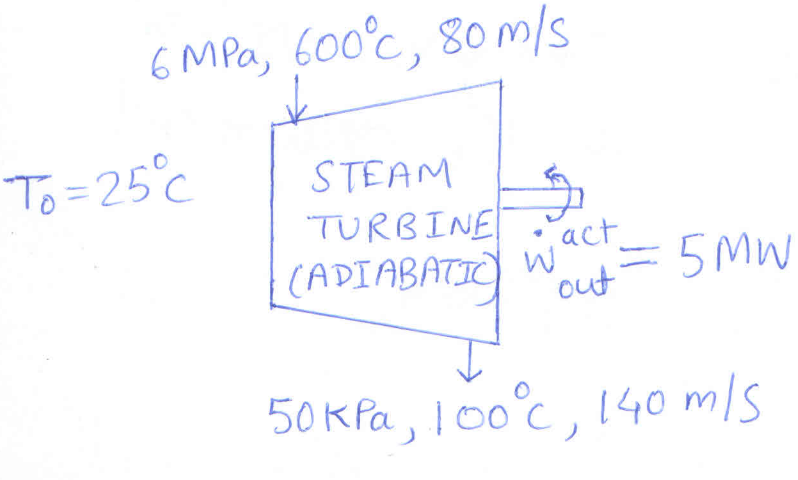
$$= \frac{\left(\sum_{in} \dot{m}\psi - \sum_{out} \dot{m}\psi\right) - T_0 \dot{S}_{gen}}{\left(\sum_{in} \dot{m}\psi - \sum_{out} \dot{m}\psi\right)}$$

$$= \frac{\text{Exergy supplied} - \text{Exergy destroyed}}{\text{Exergy supplied}}$$

$$= \frac{\text{Exergy recovered}}{\text{Exergy supplied}}$$

L 13

Example (8-58 from Textbook)



Inlet and outlet conditions of a steam turbine are as shown. Power output of turbine is 5 MW. Find
(a) Reversible power output
(b) Second law efficiency

(a) Reversible work output is given by Eqn. 6 :

$$\dot{W}_{out}^{rev} = \dot{m}(\psi_1 - \psi_2)$$

To calculate $\dot{m}$, we apply first law to actual turbine.

$$\dot{W}_{out}^{act} = \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

Note that turbine is adiabatic. Hence there is no heat transfer. Also, we neglect change in potential energy across turbine.

From property tables,

$h_1 = 3658.8$ kJ/kg $\quad$ $h_2 = 2682.4$ kJ/kg

$s_1 = 7.1693$ kJ/kg·K $\quad$ $s_2 = 7.6953$ kJ/kg·K

$\dot{W}_{out}^{act} = 5000$ kJ/s

Substituting values in above eqn; we get $\quad \dot{m} = 5.156$ kg/s.

$$\Rightarrow \dot{W}_{out}^{rev} = \dot{m}(\psi_1 - \psi_2)$$
$$= \dot{m}\left[(h_1 - h_2) - T_0(s_1 - s_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

$$\dot{W}^{rev}_{out} = (5.156)\left[(3658.8 - 2682.4) - (298.15)(7.1693 - 7.6953) + \frac{(80)^2 - (140)^2}{2(1000)}\right]$$

$$= \underline{\underline{5809 \text{ kJ/s}}}$$

Ans. to part (a)

(b) $$\eta_{II} = \frac{\dot{W}^{act}_{out}}{\dot{W}^{rev}_{out}} = \frac{5000}{5809} = \underline{0.861} \text{ or } \underline{86.1\%}$$

Ans. to part (b)