ESO201A

Tutorial 5 : Problems & Solutions

6-88C It is well established that the thermal efficiency of a heat engine increases as the temperature T_L at which heat is rejected from the heat engine decreases. In an effort to increase the efficiency of a power plant, somebody suggests refrigerating the cooling water before it enters the condenser, where heat rejection takes place. Would you be in favour of this idea? Why?

Solution:

The answer is NO.

This is because, when everything is reversible, the increase in work produced will be equal to the work consumed by the refrigerator. In reality, the work consumed by the refrigerator will always be greater than the additional work produced, resulting in a decrease in the thermal efficiency of the power plant.
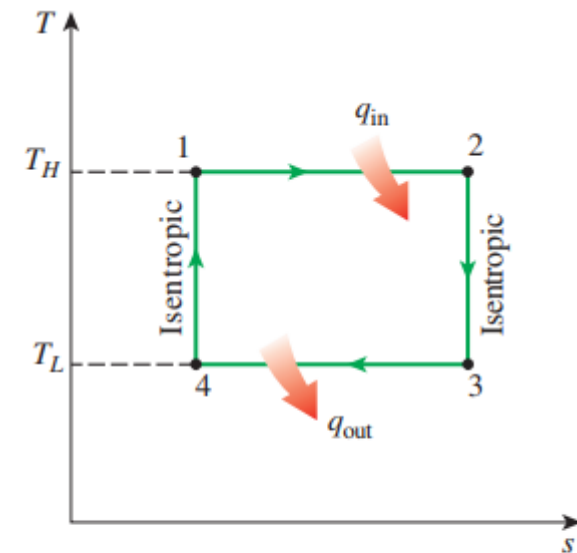
FIGURE

P-v and T-s diagrams of a Carnot cycle.

6-129 A Carnot heat engine receives heat at 900 K and rejects the waste heat to the environment at 300 K. The entire work output of the heat engine is used to drive a Carnot refrigerator that removes heat from the cooled space at -15°C at a rate of 250 kJ/min and rejects it to the same environment at 300 K. Determine (a) the rate of heat supplied to the heat engine and (b) the total rate of heat rejection to the environment.

Solution: A Carnot heat engine drives a **Carnot refrigerator** that removes heat from a cold medium at a specified rate. The rate of heat supply to the heat engine and the total rate of heat rejection to the environment are to be determined.

Analysis: (a) The coefficient of performance of the **Carnot refrigerator** is (page 284 and 300)

$$COP_R = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{\dot{W}_{net,in}}$$

$$COP_{R,C} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(300 \text{ K})/(258 \text{ K}) - 1} = 6.143$$

Then power input to the refrigerator becomes

$$\dot{W}_{net,in} = \frac{\dot{Q}_L}{COP_{R,C}} = \frac{250 \text{ kJ/min}}{6.143} = 40.7 \text{ kJ/min}$$
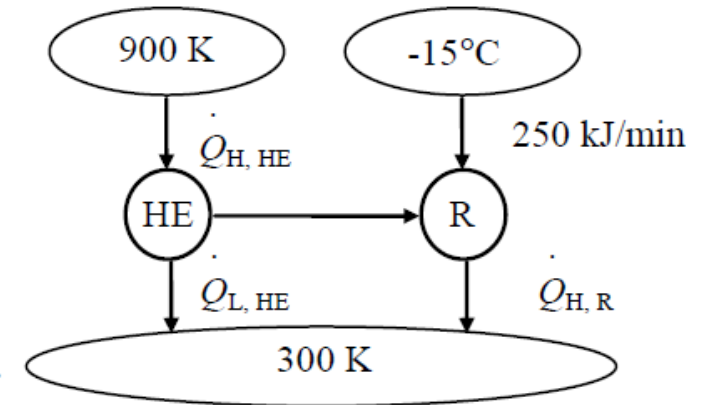
which is equal to the power output of the heat engine, $\dot{W}_{net,out}$.

The thermal efficiency of the Carnot heat engine is determined from

$$\eta_{th,C} = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ K}}{900 \text{ K}} = 0.6667$$

Then the rate of heat input to this heat engine is determined from the definition of thermal efficiency to be

$$\dot{Q}_{H,HE} = \frac{\dot{W}_{net,out}}{\eta_{th,HE}} = \frac{40.7\ kJ/min}{0.6667} = \textbf{61.1kJ/min}$$

(b) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,HE}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,R}$).
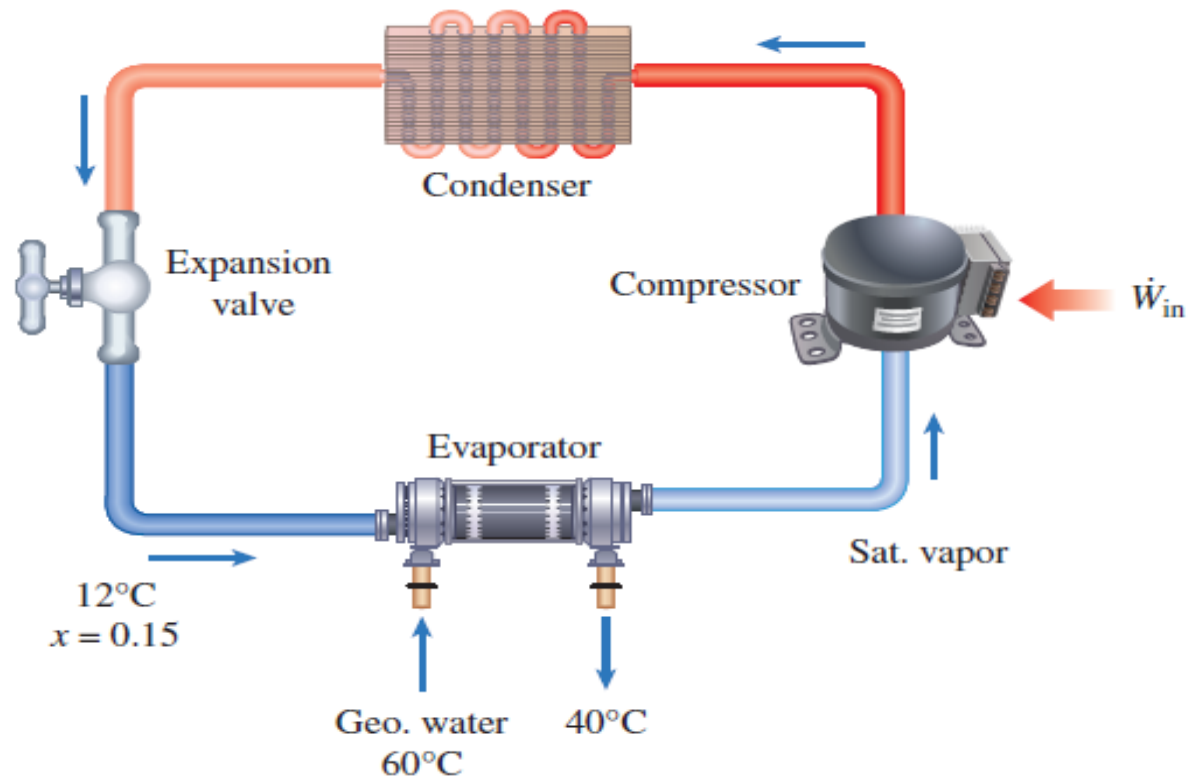
$$\dot{Q}_{L,HE} = \dot{Q}_{H,HE} - \dot{W}_{net,out} = 61.1 - 40.7 = 20.4\ kJ/min$$

$$\dot{Q}_{H,R} = \dot{Q}_{L,R} + \dot{W}_{net,in} = 250 + 40.7 = 290.7\ kJ/min$$

and

$$\dot{Q}_{Ambient} = \dot{Q}_{L,HE} + \dot{Q}_{H,R} = 20.4 + 290.7 = \textbf{311kJ/min}$$

6-149. A heat pump with refrigerant-134a as the working fluid is used to keep a space at 25°C by absorbing heat from geothermal water that enters the evaporator at 60°C at a rate of 0.065kg/s and leaves at 40°C. Refrigerant enters the evaporator at 12°C with a quality of 15 percent and leaves at the same pressure as saturated vapor. If the compressor consumes 1.6kW of power, determine (a) the mass flow rate of the refrigerant, (b) the rate of heat supply, (c) the COP, and (d) the minimum power input to the compressor for the same rate of heat supply.

Condenser

Expansion valve

Compressor

$\dot{W}_{in}$

Evaporator

Sat. vapor

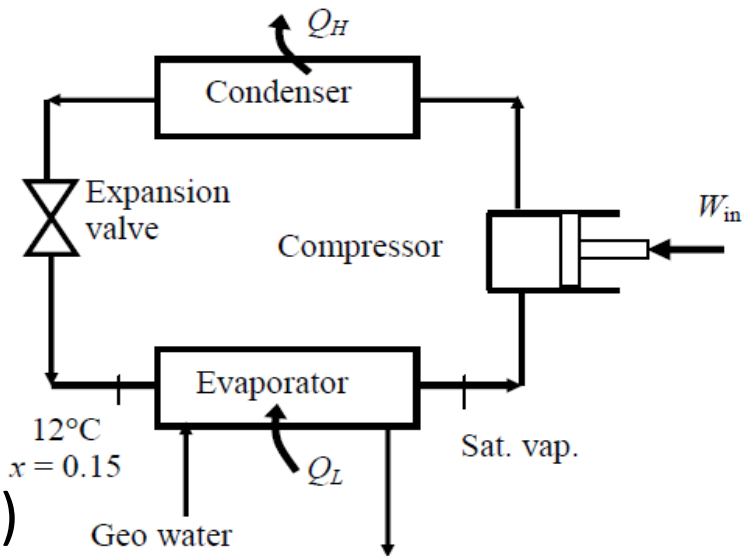12°C
x = 0.15

Geo. water   40°C
60°C

Solution: A geothermal heat pump with R-134a as the working fluid is considered. The evaporator inlet and exit states are specified. The mass flow rate of the refrigerant, the heating load, the COP, and the minimum power input to the compressor are to be determined.

Assumptions :

- The heat pump operates steadily.
- The kinetic and potential energy changes are zero.
- Steam properties are used for geothermal water.

Properties:

The properties of R-134a and water are (Steam and R-134a tables)

Saturated refrigerant-134a—Temperature table

Temp., T °C	Press., P_{sat} kPa	Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
12	443.31	0.0007975	0.046295	67.83	168.92	236.75	68.18	189.09	257.27	0.26246	0.66308	0.92554

$$\left. \begin{array}{l} T_1 = 12°C \\ x_1 = 0.15 \end{array} \right\} \begin{array}{l} h_1 = 96.54 \text{ kJ/kg} \\ P_1 = 443.3 \text{ kPa} \end{array} \qquad h_1 = h_f(1-x_1) + h_g(x_1)$$

$$\left. \begin{array}{l} P_2 = P_1 = 443.3 \text{ kPa} \\ x_2 = 1 \end{array} \right\} h_2 = 257.27$$

Saturated water—Temperature table

Temp., T °C	Sat. press., P_{sat} kPa	Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
40	7.3851	0.001008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6832	8.2556
60	19.947	0.001017	7.6670	251.16	2204.7	2455.9	251.18	2357.7	2608.8	0.8313	7.0769	7.9082

$$\left. \begin{array}{l} T_{w,1} = 60°C \\ x_{w,1} = 0 \end{array} \right\} h_{w,1} = 251.18 \text{ kJ/kg} \qquad \left. \begin{array}{l} T_{w,2} = 40°C \\ x_{w,2} = 0 \end{array} \right\} h_{w,2} = 167.53 \text{ kJ/kg}$$

Analysis:

(a) The rate of heat transferred from the water is the energy change of the water from inlet to exit

$$\dot{Q}_L = \dot{m}_w(h_{w,1} - h_{w,2}) = (0.065 \text{ kg/s})(251.18 - 167.53) \text{ kJ/kg} = 5.437 \text{ kW}$$

The energy increase of the refrigerant is equal to the energy decrease of the water in the evaporator. That is,

$$\dot{Q}_L = \dot{m}_R(h_2 - h_1) \longrightarrow \dot{m}_R = \frac{\dot{Q}_L}{h_2 - h_1} = \frac{5.437 kW}{(257.27 - 96.54)kJ/s} = 0.0338 \ kg/s$$

(b) The heating load is

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 5.437 + 1.6 = \textbf{7.04kW}$$

(c) The COP of the heat pump is determined from its definition,

$$COP = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{7.04 \text{ kW}}{1.6 \text{ kW}} = \textbf{4.40}$$

$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_H}{W_{net,in}}$$

(d) The COP of a reversible heat pump operating between the same temperature limits is

$$COP_{max} = \frac{1}{1 - T_L / T_H} = \frac{1}{1 - (25 + 273)/(60 + 273)} = 9.51$$

Then, the minimum power input to the compressor for the same refrigeration load would be

$$\dot{W}_{in,min} = \frac{\dot{Q}_H}{COP_{max}} = \frac{7.04 \text{ kW}}{9.51} = \textbf{0.740kW}$$

7-40 Refrigerant-134a at 320 kPa and 40°C undergoes an isothermal process in a closed system until its quality is 45 percent. On per unit mass basis, determine how much **work and heat transfer** required.
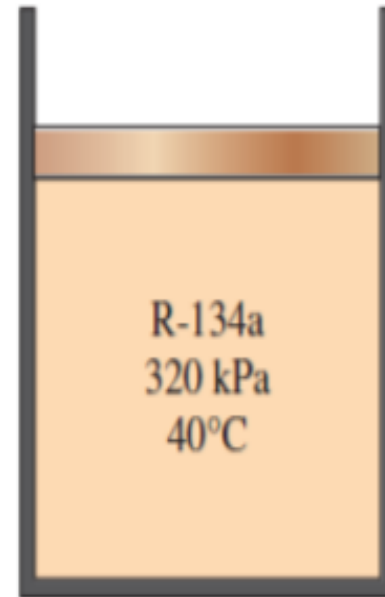
R-134a
320 kPa
40°C

Solution: R-134a undergoes an isothermal process in a closed system. The work and heat transfer are to be determined.

Assumptions:
1. The system is stationary and thus the kinetic and potential energy changes are zero.
2. There are no work interactions involved other than the boundary work.
3. The thermal energy stored in the cylinder itself is negligible.
4. The compression or expansion process is quasi-equilibrium.

Analysis: The energy balance for this system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$W_{in} - Q_{out} = \Delta U = m(u_2 - u_1)$$

R-134a
320 kPa
$T_1 = T_2 = 40°C$
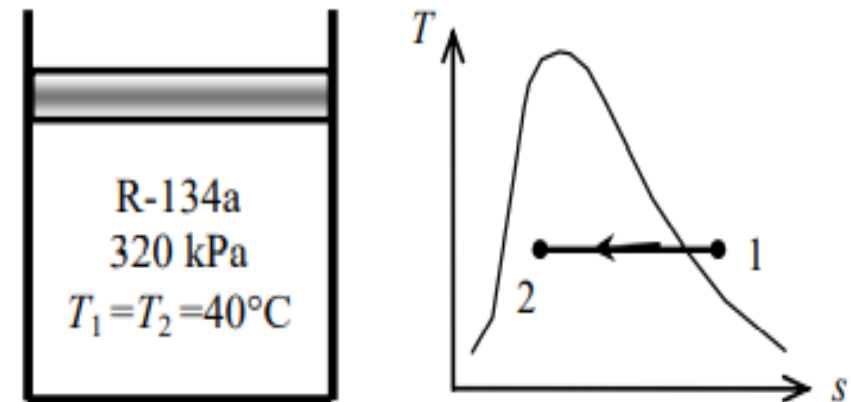
The initial state properties are

$$P_1 = 320\,\text{kPa}$$
$$T_1 = 40°C$$
$$\left.\begin{array}{l} u_1 = 261.62\,\text{kJ/kg} \\ s_1 = 1.0452\,\text{kJ/kg} \cdot \text{K} \end{array}\right\}$$
(Table A-13)

For this isothermal process, the final state properties are (Table A-11)

$$T_2 = T_1 = 40°C$$
$$x_2 = 0.45$$
$$\left.\begin{array}{l} u_2 = u_f + x_2 u_{fg} = 107.39 + (0.45)(143.61) = 172.02\,\text{kJ/kg} \\ s_2 = s_f + x_2 s_{fg} = 0.39493 + (0.45)(0.52059) = 0.62920\,\text{kJ/kg} \cdot \text{K} \end{array}\right\}$$

TABLE A-13

Superheated refrigerant-134a

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg-K
	P = 0.32 MPa (T_sat = 2.46°C)			
Sat.	0.06368	231.55	251.93	0.9303
0				
10	0.06609	237.56	258.70	0.9545
20	0.06925	245.51	267.67	0.9856
30	0.07231	253.52	276.66	1.0158
40	0.07530	261.62	285.72	1.0452

TABLE A-11

Saturated refrigerant-134a—Temperature table

Temp., T°C	Sat. press., P_sat kPa	Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_fg	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_fg	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_fg	Sat. vapor, s_g
38	963.68	0.0008657	0.021137	104.47	145.60	250.07	105.30	165.13	270.44	0.38554	0.53068	0.91622
40	1017.1	0.0008720	0.019968	107.39	143.61	251.00	108.28	163.03	271.31	0.39493	0.52059	0.91552
42	1072.8	0.0008786	0.018870	110.34	141.59	251.92	111.28	160.89	272.17	0.40432	0.51048	0.91480

The heat transfer is determined from

$$q_{in} = T_0(s_2 - s_1) = (313\,\text{K})(0.62920 - 1.0452)\,\text{kJ/kg} \cdot \text{K} = -130.2\,\text{kJ/kg}$$

The negative sign shows that the heat is actually transferred from the system. That is,

$$q_{out} = \mathbf{130.2kJ/kg}$$

The work required is determined from the energy balance to be

$$w_{in} = q_{out} + (u_2 - u_1) = 130.2\,\text{kJ/kg} + (172.02 - 261.62)\,\text{kJ/kg} = \mathbf{40.6kJ/kg}$$

7-52 2-kg of saturated water vapor at 600 kPa are contained in a piston-cylinder device. The water expands adiabatically until the pressure is 100 kPa and is said to produce 700 kJ of work output.
(a) Determine the entropy change of the water, in kJ/kg.K.
(b) Is this process realistic? Using the *T*-s diagram for the process and the concepts of second law, support your answer.

Solution: Water vapor is expanded adiabatically in a piston-cylinder device. The entropy change is to be determined and it is to be discussed if this process is realistic.

Analysis: *(a)* The properties at the initial state are

$$P_1 = 600 \text{ kPa} \\ x_1 = 1 \left.\begin{array}{l} \\ \\ \end{array}\right\} \begin{array}{l} u_1 = 2566.8 \text{ kJ/kg} \\ s_1 = 6.7593 \text{ kJ/kg} \cdot \text{K} \end{array} \quad \text{(Table A - 5)}$$
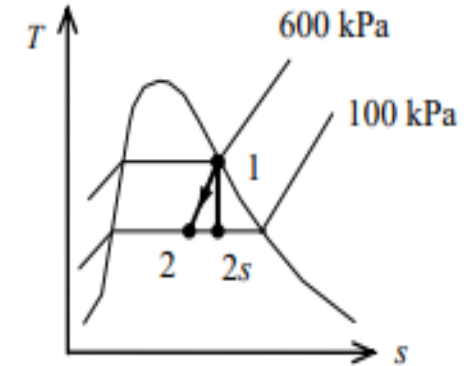
TABLE A–5

Saturated water—Pressure table

Press., P kPa	Sat. temp., T_{sat} °C	Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
101.325	99.97	0.001043	1.6734	418.95	2087.0	2506.0	419.06	2256.5	2675.6	1.3069	6.0476	7.3545
550	155.46	0.001097	0.34261	655.16	1908.8	2563.9	655.77	2096.6	2752.4	1.8970	4.8916	6.7886
600	158.83	0.001101	0.31560	669.72	1897.1	2566.8	670.38	2085.8	2756.2	1.9308	4.8285	6.7593
650	161.98	0.001104	0.29260	683.37	1886.1	2569.4	684.08	2075.5	2759.6	1.9623	4.7699	6.7322

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$-W_{out} = \Delta U = m(u_2 - u_1) \quad (\text{since } Q = KE = PE = 0)$$

Solving for the final state internal energy,

$$u_2 = u_1 + \frac{W_{out}}{m} = 2566.8 \text{ kJ/kg} + \frac{700 \text{ kJ}}{2 \text{ kg}} = 2216.8 \text{ kJ/kg}$$

The entropy at the final state is (from Table A-5)

$$\left. \begin{array}{l} P_2 = 100 \text{ kPa} \\ u_2 = 2216.8 \text{ kJ/kg} \end{array} \right\}$$

$$x_2 = \frac{u_2 - u_f}{u_{fg}} = \frac{2216.8 - 417.40}{2088.2} = 0.8617$$

$$s_2 = s_f + x s_{fg} = 1.3028 + 0.8617 \times 6.0562 = 6.5215 \text{ kJ/kg} \cdot \text{K}$$

The entropy change is

$$\Delta s = s_2 - s_1 = 6.5215 - 6.7593 = \mathbf{-0.238 kJ/kg \cdot K}$$

(b) The process is not realistic since entropy cannot decrease during an adiabatic process. In the limiting case of a reversible (and adiabatic) process, the entropy would remain constant.

7-77 An insulated piston-cylinder device initially contains 300 L of air at 120 kPa and 17°C. Air is now heated for 15 min by a 200-W resistance heater placed inside the cylinder. The pressure of air is maintained constant during this process. Determine the entropy change of air, assuming (a) constant specific heats and (b) variable specific heats.

Assumptions: At specified conditions, air can be treated as an ideal gas.

Properties: The gas constant of air is $R = 0.287$ kJ/kg.K (Table A-1).

Substance	Formula	Molar mass, M kg/kmol	Gas constant, R kJ/kg·K*
Air	—	28.97	0.2870
Ammonia	NH_3	17.03	0.4882

Analysis: The mass of the air and the electrical work done during this process are

$$m = \frac{P_1 V_1}{RT_1} = \frac{(120 \text{ kPa})(0.3 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(290 \text{ K})} = 0.4325 \text{ kg}$$
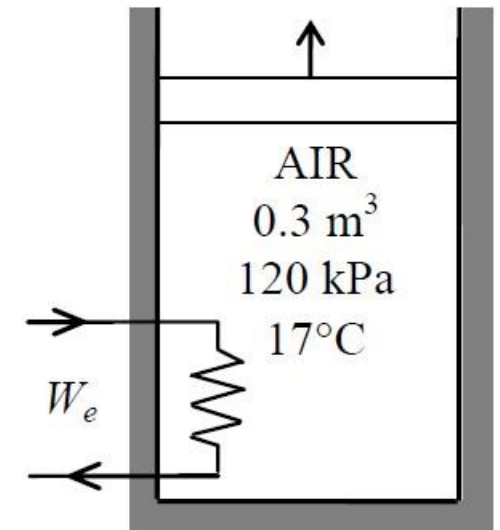
$$W_{e,in} = \dot{W}_{e,in}\Delta t = (0.2 \text{ kJ/s})(15 \times 60 \text{ s}) = 180 \text{ kJ}$$

The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$W_{e,in} - W_{b,out} = \Delta U \longrightarrow W_{e,in} = m(h_2 - h_1) \cong c_p(T_2 - T_1)$$

since $\Delta U + W_b = \Delta H$ during a constant pressure quasi-equilibrium process.

(a) Using a constant c_p value at the anticipated average temperature of 450 K, the final temperature becomes

Thus,
$$T_2 = T_1 + \frac{W_{e,in}}{mc_p} = 290 \text{ K} + \frac{180 \text{ kJ}}{(0.4325 \text{ kg})(1.02 \text{ kJ/kg} \cdot \text{K})} = 698 \text{ K}$$

Then the entropy change becomes

$$\Delta S_{sys} = m(s_2 - s_1) = m\left(c_{p,avg} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}^{\nearrow 0} \right) = mc_{p,avg} \ln\frac{T_2}{T_1}$$

$$= (0.4325 \text{ kg})(1.020 \text{ kJ/kg} \cdot \text{K})\ln\left(\frac{698 \text{ K}}{290 \text{ K}}\right) = \mathbf{0.387\ kJ/K}$$

T	h		u		$s°$
K	kJ/kg	P_r	kJ/kg	v_r	kJ/kg·K
580	586.04	14.38	419.55	115.7	2.37348
590	596.52	15.31	427.15	110.6	2.39140
600	607.02	16.28	434.78	105.8	2.40902
610	617.53	17.30	442.42	101.2	2.42644
620	628.07	18.36	450.09	96.92	2.44356
630	638.63	19.84	457.78	92.84	2.46048
640	649.22	20.64	465.50	88.99	2.47716
650	659.84	21.86	473.25	85.34	2.49364
660	670.47	23.13	481.01	81.89	2.50985
670	681.14	24.46	488.81	78.61	2.52589
680	691.82	25.85	496.62	75.50	2.54175
690	702.52	27.29	504.45	72.56	2.55731
700	713.27	28.80	512.33	69.76	2.57277
710	724.04	30.38	520.23	67.07	2.58810

(b) Assuming variable specific heats, (use table A-17)

$$W_{e,in} = m(h_2 - h_1) \longrightarrow h_2 = h_1 + \frac{W_{e,in}}{m} = 290.16 \text{ kJ/kg} + \frac{180 \text{ kJ}}{0.4325 \text{ kg}} = 706.34 \text{ kJ/kg}$$

(h1 calculated from A17 @T1=290)

From the air table (Table A-17), we read $s_2° = 2.5628$ kJ/kg·K corresponding to this h_2 value (use interpolation to find $s_2°$ from Table A-17).

Then,

$$\Delta S_{sys} = m\left(s_2° - s_1° + R\ln\frac{P_2}{P_1}^{\nearrow 0} \right) = m(s_2° - s_1°) = (0.4325 \text{ kg})(2.5628 - 1.66802)\text{kJ/kg} \cdot \text{K} = \mathbf{0.387\ kJ/K}$$