ESO201A : THERMODYNAMICS
2021-22 Ist semester
IIT Kanpur

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Lecture 20

Internally reversible steady flow equipment :

We will now derive an expression of work required in the internally reversible steady flow equipment with a single inlet and single outlet. By internally reversible, we mean that fluid flowing through the equipment undergoes a quasi-static process such that it is in internal equilibrium throughout.

We will consider that changes in kinetic and potential energies are negligible. Applying first law equation and dividing throughout by the mass flow rate, we get

$$w_{in} = (h_2 - h_1) + q_{out}$$

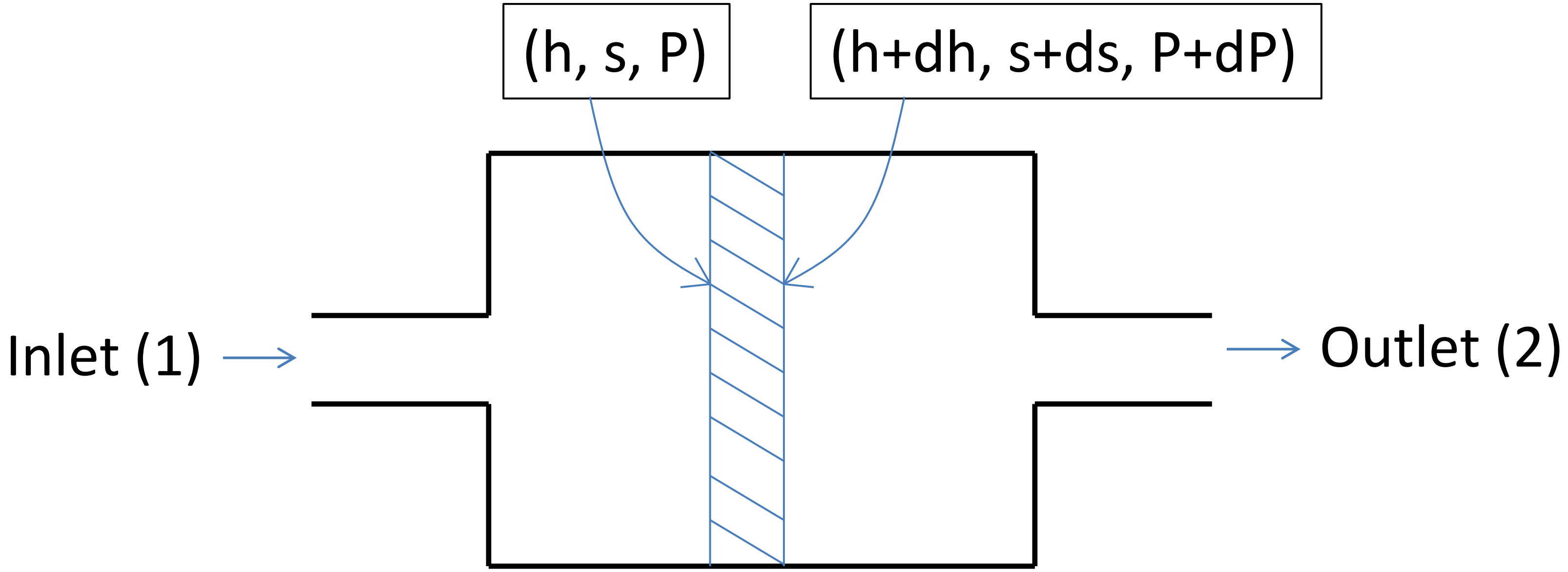
where w_{in} and q_{out} is the work required (input) and heat rejected per unit mass of the fluid flow :

$$w_{in} = \frac{\dot{W}_{in}}{\dot{m}} \quad \text{and} \quad q_{out} = \frac{\dot{Q}_{out}}{\dot{m}}$$

Internally reversible steady flow equipment :

Now we consider a differential volume (shaded region in the figure below) of the equipment.

Note that (h,s,P) and (h+dh, s+ds, P+dP) are specific enthalpy, specific entropy, and pressure of the fluid at the inlet and outlet of the differential volume, respectively.



Internally reversible steady flow equipment

Internally reversible steady flow equipment :

We consider differential volume as our control volume. Applying the first law equation (on unit mass basis) for the differential volume, we get

$$dw_{in} = dh + dq_{out}$$

dw_{in} = the work done per unit mass on the fluid contained in the differential volume at a certain instant of time.

dq_{out} = the heat lost per unit mass by the fluid contained in the differential volume at a certain instant of time.

Note that we have neglected changes in kinetic and potential energies.

Since process is internally reversible, we can apply Clausius equality. Thus,

$$dq_{out} = -T\,ds$$

Here T is the temperature of the fluid contained in the differential volume. Temperature is considered to be uniform throughout the differential volume.

Internally reversible steady flow equipment :

Combining the two equations of the previous slides, we get

$$dw_{in} = dh - T\,ds$$

Note that right hand side consists of quantities which depend only upon the thermodynamic state at the inlet and outlet of the differential volume. Substituting the equation dh = T ds + v dP, in the aboe equation we get,

$$dw_{in} = v\,dP$$

Note that mass flow rate is the same throughout the entire flow equipment since it is operating at steady state. Therefore we can integrate above equation over the entire flow equipment from inlet to outlet. The result of this integration yields work required per unit mass Of the fluid in the entire flow equipment :

$$\boxed{w_{in} = \int_1^2 v\,dP}$$

Internally reversible steady flow compressor:

Lets consider an internally reversible steady flow compressor with a single inlet and single outlet. Consider that changes in kinetic and potential energies are negligible. Further we will consider that compressor handles ideal gas.

The state of the ideal gas (s_1, P_1) at the inlet of the compressor and outlet pressure (P_2) are fixed. However, outlet specific entropy (s_2) will vary according to the conditions in which compressor operates. The idea is that if we minimize specific volume (v), work input is minimized. There are three possibilities

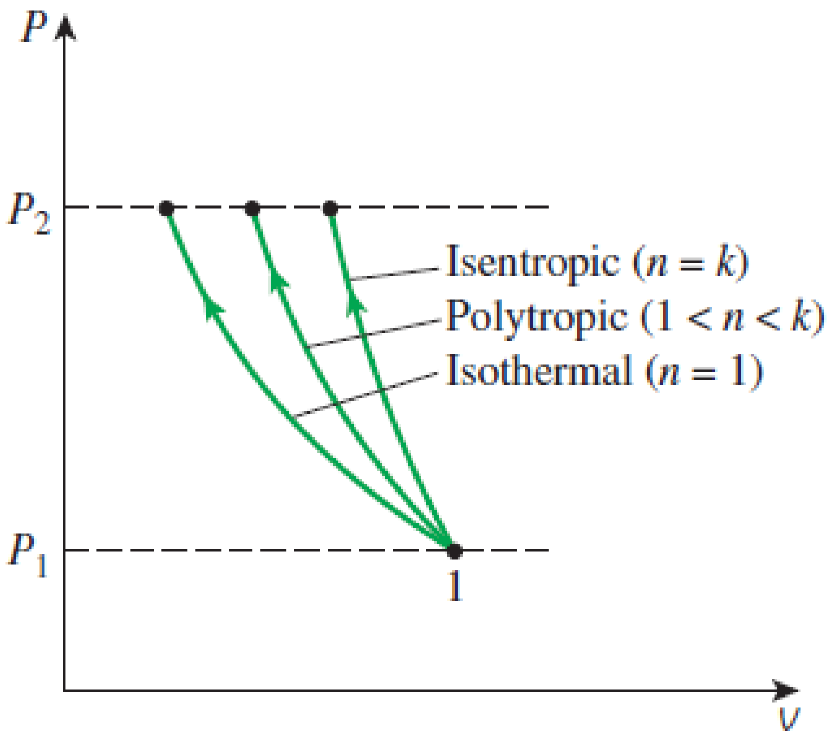
(1) Isoentropic
    (internally reversible with zero heat output)
(2) Polytropic
    (internally reversible with intermediate heat output)
(3) Isothermal
    (internally reversible with maximum heat output)

Internally reversible steady flow compressor:

(1) Isoentropic (Pv^k = constant), $k = C_P/C_V$ is the ratio of specific heats
(internally reversible with zero heat output or no cooling)

(2) Polytropic (Pv^n = constant), ($1 < n < k$)
(internally reversible with intermediate cooling)

(3) Isothermal (Pv = constant)
(internally reversible with maximum cooling)



Ref. Cengel and Boles, 8th Edition (2015)

Area enclosed by dashed horizontal lines, the green curve, and P-axis yields the work input per unit mass

Area enclosed (and hence work input) is the least for isothermal curve.

Internally reversible steady flow compressor:

The work required for the three cases (see previous slide) can be obtained by integrating the following equation :

$$w_{comp,in} = \int_1^2 v\, dP$$

The results obtained are as follows :

Isentropic (Pv^k = constant):

$$w_{comp,in} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

Polytropic (Pv^n = constant):

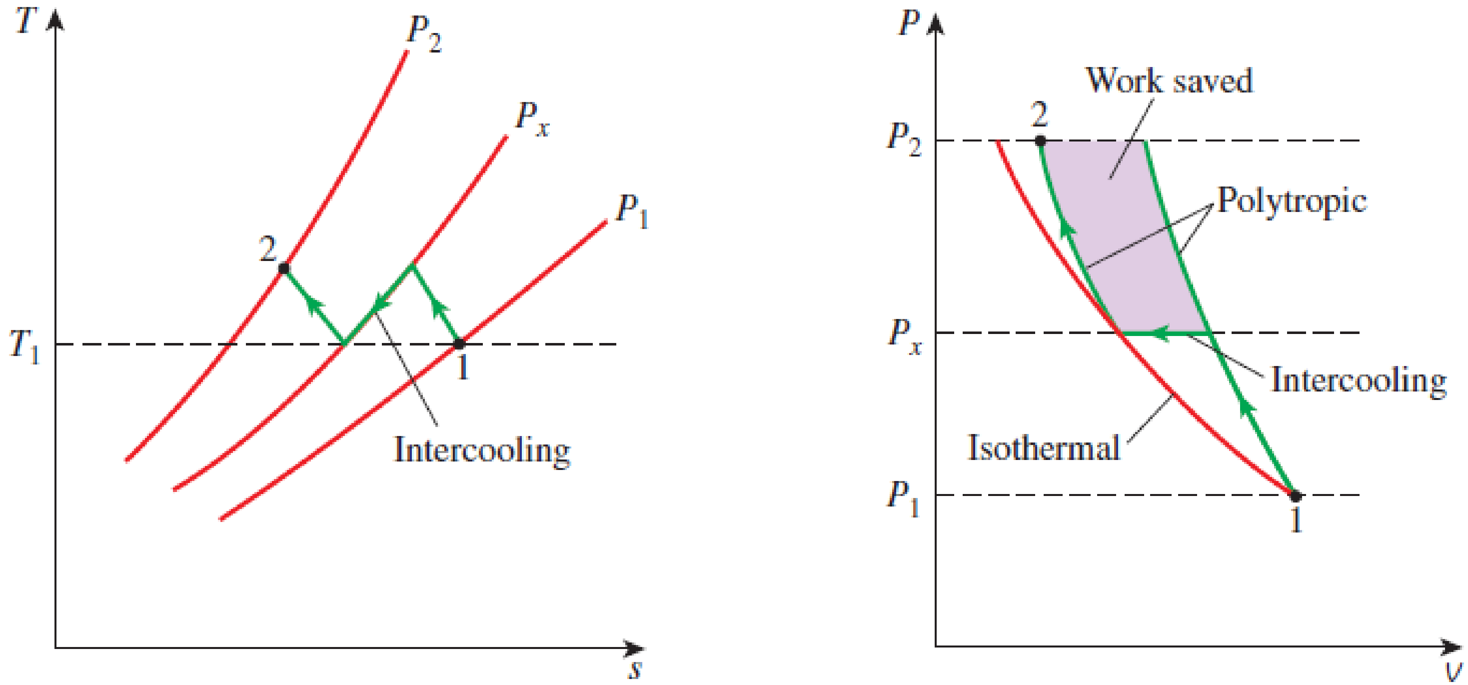
$$w_{comp,in} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Isothermal (Pv = constant):

$$w_{comp,in} = RT \ln\frac{P_2}{P_1}$$

Ref. Cengel and Boles, 8th Edition (2015)

Multi-stage compression with inter-cooling :



$$w_{\text{comp,in}} = w_{\text{comp I,in}} + w_{\text{comp II,in}}$$

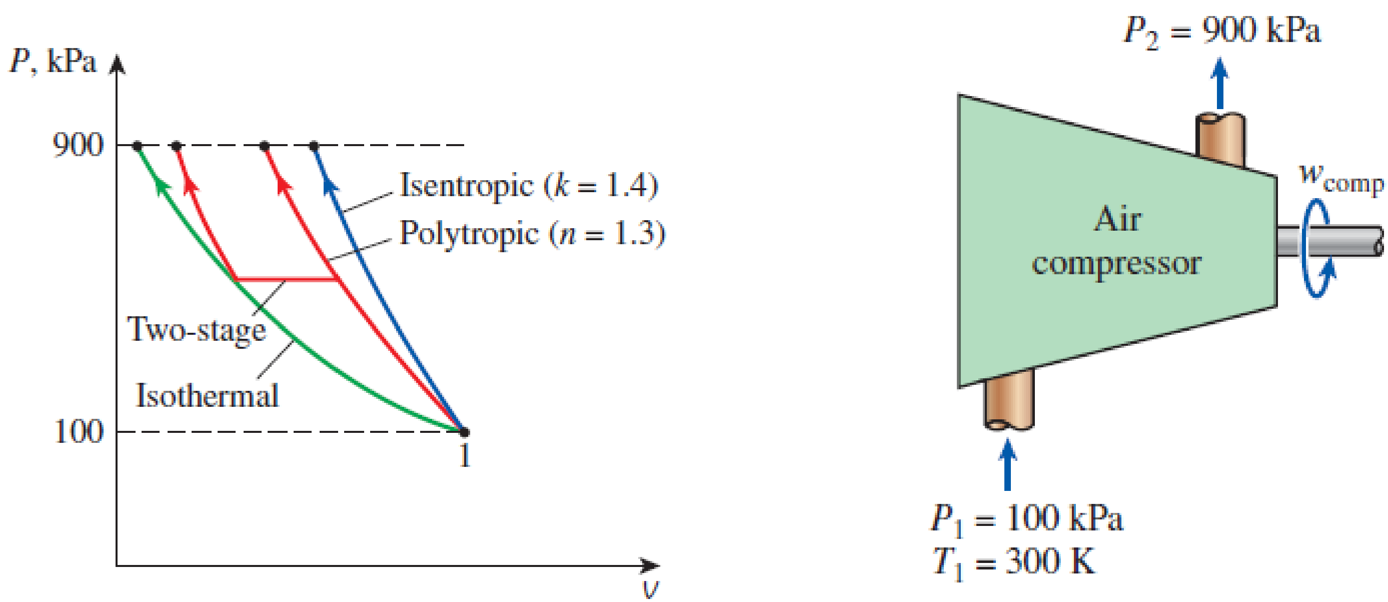
$$= \frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_x}\right)^{(n-1)/n} - 1\right]$$

For a two-stage compression with intercooling, minimum work is obtained when intermediate pressure (P_x) satisfies the following relation

$$P_x = (P_1P_2)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

Ref. Cengel and Boles, 8th Edition (2015)

Air is compressed steadily by a reversible compressor from an inlet state of 100 kPa and 300 K to an exit pressure of 900 kPa. Determine the compressor work per unit mass for (*a*) isentropic compression with $k = 1.4$, (*b*) polytropic compression with $n = 1.3$, (*c*) isothermal compression, and (*d*) ideal two-stage compression with intercooling with a polytropic exponent of 1.3.



Ref. Cengel and Boles, 8th Edition (2015)

(*a*) Isentropic compression with $k = 1.4$:

$$w_{\text{comp,in}} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

$$= \frac{(1.4)(0.287\text{ kJ/kg·K})(300\text{ K})}{1.4-1}\left[\left(\frac{900\text{ kPa}}{100\text{ kPa}}\right)^{(1.4-1)/1.4} - 1\right]$$

$$= \mathbf{263.2\text{ kJ/kg}}$$

(*b*) Polytropic compression with $n = 1.3$:

$$w_{\text{comp,in}} = \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

$$= \frac{(1.3)(0.287\text{ kJ/kg·K})(300\text{ K})}{1.3-1}\left[\left(\frac{900\text{ kPa}}{100\text{ kPa}}\right)^{(1.3-1)/1.3} - 1\right]$$

$$= \mathbf{246.4\text{ kJ/kg}}$$

(*c*) Isothermal compression:

$$w_{\text{comp,in}} = RT\ln\frac{P_2}{P_1} = (0.287\text{ kJ/kg·K})(300\text{ K})\ln\frac{900\text{ kPa}}{100\text{ kPa}}$$

$$= \mathbf{189.2\text{ kJ/kg}}$$

Ref. Cengel and Boles, 8th Edition (2015)

(*d*) Ideal two-stage compression with intercooling ($n = 1.3$): In this case, the pressure ratio across each stage is the same, and its value is

$$P_x = (P_1 P_2)^{1/2} = [(100\text{ kPa})(900\text{ kPa})]^{1/2} = 300\text{ kPa}$$

The compressor work across each stage is also the same. Thus the total compressor work is twice the compression work for a single stage:

$$w_{\text{comp,in}} = 2w_{\text{comp I,in}} = 2\frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right]$$

$$= \frac{2(1.3)(0.287\text{ kJ/kg·K})(300\text{ K})}{1.3-1}\left[\left(\frac{300\text{ kPa}}{100\text{ kPa}}\right)^{(1.3-1)/1.3} - 1\right]$$

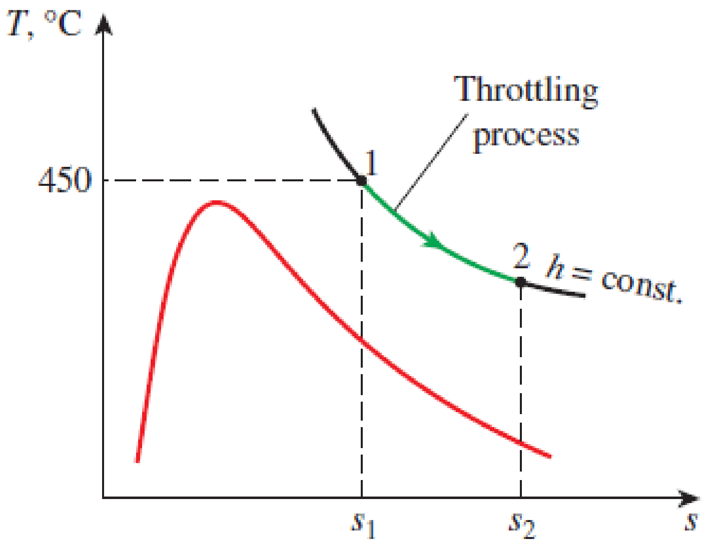
$$= \mathbf{215.3\text{ kJ/kg}}$$

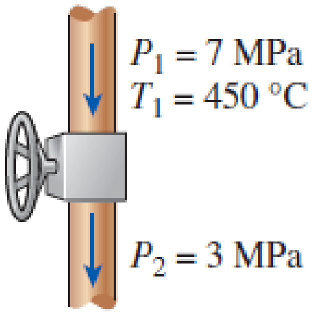
Discussion Of all four cases considered, the isothermal compression requires the minimum work and the isentropic compression the maximum. The compressor work is decreased when two stages of polytropic compression are utilized instead of just one. As the number of compressor stages is increased, the compressor work approaches the value obtained for the isothermal case.

Ref. Cengel and Boles, 8th Edition (2015)

Problems on entropy balance in flow equipments :

Steam at 7 MPa and 450°C is throttled in a valve to a pressure of 3 MPa during a steady-flow process. Determine the entropy generated during this process and check if the increase of entropy principle is satisfied.





Analysis We take the throttling valve as the *system* This is a *control volume* since mass crosses the system boundary during the process. We note that there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Also, the enthalpy of a fluid remains nearly constant during a throttling process and thus $h_2 \cong h_1$.

Ref. Cengel and Boles, 8th Edition (2015)

$$\text{State 1:} \quad \left.\begin{aligned} P_1 &= 7\ \text{MPa} \\ T_1 &= 450^\circ\text{C} \end{aligned}\right\} \quad \begin{aligned} h_1 &= 3288.3\ \text{kJ/kg} \\ s_1 &= 6.6353\ \text{kJ/kg·K} \end{aligned}$$

$$\text{State 2:} \quad \left.\begin{aligned} P_2 &= 3\ \text{MPa} \\ h_2 &= h_1 \end{aligned}\right\} \quad s_2 = 7.0046\ \text{kJ/kg·K}$$

$$\dot{m}s_1 - \dot{m}s_2 + S_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_2 - s_1)$$

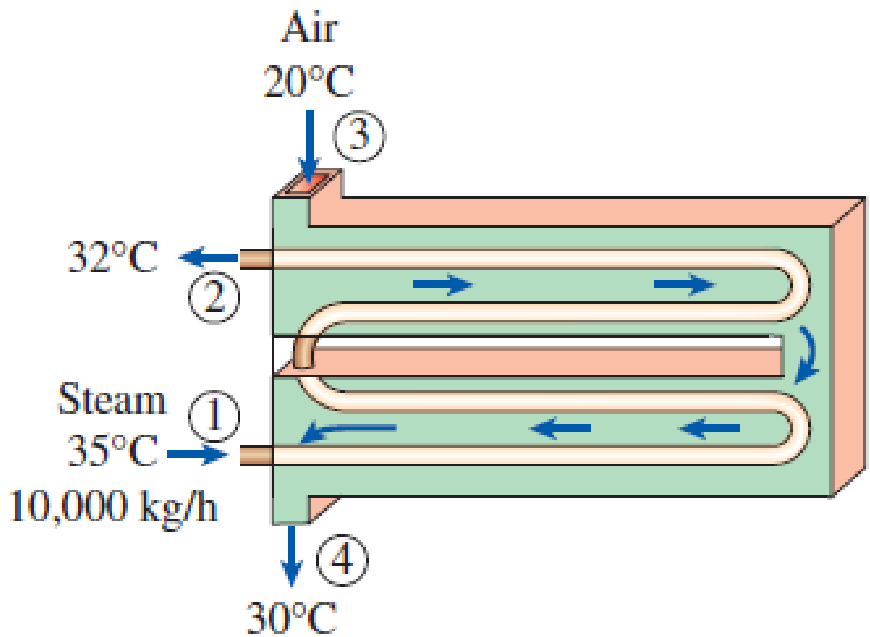
Dividing by mass flow rate and substituting gives

$$s_{\text{gen}} = s_2 - s_1 = 7.0046 - 6.6353 = \mathbf{0.3693\ kJ/kg·K}$$

This is the amount of entropy generated per unit mass of steam as it is throttled from the inlet state to the final pressure, and it is caused by unrestrained expansion. The increase of entropy principle is obviously satisfied during this process since the entropy generation is positive.

Ref. Cengel and Boles, 8th Edition (2015)

Air in a large building is kept warm by heating it with steam in a heat exchanger (Fig. 7–67). Saturated water vapor enters this unit at 35°C at a rate of 10,000 kg/h and leaves as saturated liquid at 32°C. Air at 1-atm pressure enters the unit at 20°C and leaves at 30°C at about the same pressure. Determine the rate of entropy generation associated with this process.



Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Air is an ideal gas with constant specific heats at room temperature. **5** The pressure of air remains constant.

Ref. Cengel and Boles, 8th Edition (2015)

$$\dot{m}_{steam}\, s_1 + \dot{m}_{air}\, s_3 - \dot{m}_{steam}\, s_2 - \dot{m}_{air}\, s_4 + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \dot{m}_{steam}(s_2 - s_1) + \dot{m}_{air}(s_4 - s_3)$$

The specific heat of air at room temperature is $c_p = 1.005$ kJ/kg·°C (Table A–2*a*). The properties of the steam at the inlet and exit states are

$$\left.\begin{array}{l} T_1 = 35°\text{C} \\ x_1 = 1 \end{array}\right\} \begin{array}{l} h_1 = 2564.6 \text{ kJ/kg} \\ s_1 = 8.3517 \text{ kJ/kg·K} \end{array} \quad \text{(Table A–4)}$$

$$\left.\begin{array}{l} T_2 = 32°\text{C} \\ x_2 = 0 \end{array}\right\} \begin{array}{l} h_2 = 134.10 \text{ kJ/kg} \\ s_2 = 0.4641 \text{ kJ/kg·K} \end{array} \quad \text{(Table A–4)}$$

From an energy balance the heat transferred from steam is equal to the heat transferred to the air. Then, the mass flow rate of air is determined to be

$$\dot{Q} = \dot{m}_{steam}(h_1 - h_2) = (10{,}000/3600 \text{ kg/s})(2564.6 - 134.10) \text{ kJ/kg} = 6751 \text{ kW}$$

$$\dot{m}_{air} = \frac{\dot{Q}}{c_p(T_4 - T_3)} = \frac{6751 \text{ kW}}{(1.005 \text{ kJ/kg·°C})(30 - 20)°\text{C}} = 671.7 \text{ kg/s}$$

Ref. Cengel and Boles, 8th Edition (2015)

Substituting into the entropy balance relation, the rate of entropy generation becomes

$$
\begin{aligned}
\dot{S}_{\text{gen}} &= \dot{m}_{\text{steam}}(s_2 - s_1) + \dot{m}_{\text{air}}(s_4 - s_3) \\
&= \dot{m}_{\text{steam}}(s_2 - s_1) + \dot{m}_{\text{air}} c_p \ln\frac{T_4}{T_3} \\
&= (10{,}000/3600\ \text{kg/s})(0.4641 - 8.3517)\ \text{kJ/kg·K} \\
&\quad + (671.7\ \text{kg/s})(1.005\ \text{kJ/kg·K}) \ln\frac{303\ \text{K}}{293\ \text{K}} \\
&= \mathbf{0.745\ kW/K}
\end{aligned}
$$

Discussion Note that the pressure of air remains nearly constant as it flows through the heat exchanger, and thus the pressure term is not included in the entropy change expression for air.