ESO201A : THERMODYNAMICS
2021-22 Ist semester
IIT Kanpur

Instructor : P.A.Apte

Lecture 21

Maximum work output and lost work :

Consider change in a closed (control mass) system from a specified initial state (1) to a specified final state (2). We consider that there are no changes in kinetic and potential energies between initial and final states. During the change system exchanges heat with a thermal energy reservoir at temperature T_o

Let ΔU and ΔS be the difference in energy and entropy of the two states. Applying first law, we have

$$\Delta U = Q_{in} - W_{out}$$

where Q_{in} and W_{out} is heat absorbed and work done by the system during a process between the two states. The system together with the reservoir form an adiabatically isolated system. Hence according to second law, the change in total entropy is either positive or zero.

$$\Delta S_T = \Delta S - \left(\frac{Q_{in}}{T_o}\right) \geq 0$$

Maximum work output :

Re-arranging entropy equation and substituting in the first law equation (see previous slide), we get

$$W_{out} = T_o \Delta S - \Delta U - T_o \Delta S_T$$

For a reversible process, $\Delta S_T = S_{gen} = 0$

The work done in a reversible process is : $W_{rev,out} = T_o \Delta S - \Delta U$

Substituting the work done in a reversible processs in the general equation for work done, we get

$$W_{out} = W_{rev,out} - T_o \Delta S_T$$

Thus, for an irreversible process, $W_{out} < W_{rev,out}$ since $\Delta S_T > 0$

This shows that : Among the processes between specified initial and final states during which there is a heat exchange with a single thermal energy reservoir at a specified temperature T_o, maximum work is done by the system in a reversible process.

Lost work or irreversibility :

In an irreversible process, lost work or Irreversibility is defined as the difference between maximum (reversible) work output and actual work output. The irreversibility is denoted by the symbol 'I'. Thus,

$$I = W_{rev,out} - W_{out} = T_o \Delta S_T$$

Maximum heat absorbed :

The heat absorbed in a reversible process is : $Q_{rev,in} = T_o \Delta S$

Thus we can relate the heat absorbed in irreversible and reversible processes :

$$Q_{in} = Q_{rev,in} - T_o \Delta S_T$$

Thus heat absorbed in an irreversible process is less than that in a reversible process $Q_{in} < Q_{rev,in}$

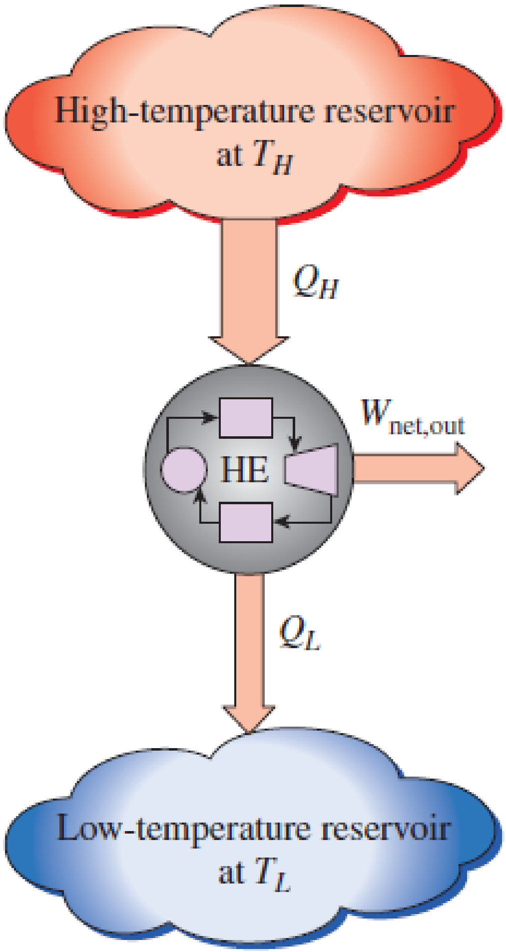
This shows that : Among the processes between specified initial and final states during which there is a heat exchange with a single thermal energy reservoir at a specified temperature T_o, maximum heat is absorbed by the system in a reversible process.

In an irreversible process, system absorbs less heat (by an amount $= T_o \Delta S_T$) and performs less work (by the SAME amount) as compared to reversible process (Note that initial and final states are the same for both reversible and irreversible processes)

Thus as the process approaches a reversible process ($\Delta S_T \rightarrow 0$), we can convert more heat into work ! → 'take-home' message for an engineer!!

Maximum work in case of heat exchange with two thermal energy reservoirs :

Consider a heat engine in which there is a exchange of heat between the working fluid and two thermal energy reservoirs as shown.



Ref. Cengel and Boles, 8th Edition (2015)

For one complete cycle, the initial and final states of the working fluid are completely specified and in fact these states are identical. Thus, for one complete cycle of the working fluid, $\Delta U = 0$ and $\Delta S = 0$. Hence, according to first law of thermodynamics,

$$W_{\text{net,out}} = Q_H - Q_L$$

Maximum work in case of heat exchange with two thermal energy reservoirs :

The entire system consisting of the thermal energy reservoirs, working fluid, and different components of the heat engine and the system on which work is done, together, form an adiabatic or thermally insulated (control mass) system. **Very importantly, we assume that all components of the heat engine (except for the working fluid) and the external system on which work is done are <u>purely mechanical</u> (the entropy of these components and the external system is zero).**

According to Second Law of thermodynamics the total entropy change of this entire system (reservoirs, working fluid and all components of heat engine and the external system on which work is done) will be positive (irreversible process) or zero (reversible process).

$$\Delta S_T = \cancelto{0}{\Delta S} + \left(\frac{-Q_H}{T_H}\right) + \left(\frac{Q_L}{T_L}\right) \geq 0$$

Maximum work in case of heat exchange with two thermal energy reservoirs :

Note that the second and third term in the equation on the previous slide represent entropy changes of high temperature and low temperature reservoirs, respectively. We are assuming that process is occuring sufficiently slowly (even if the overall process may be irreversible) such that each reservoir is in a state of internal equilibrium throughout the process, i.e., temperature is uniform throughout each thermal reservoir during the process. The entropy change of the high temperature reservoir is negative because it is losing heat. Note that both Q_H and Q_L are positive (see equation on previous slide). Re-arranging the entropy equation of previous slide, we get

$$Q_L = Q_H\left(\frac{T_L}{T_H}\right) + T_L \Delta S_T$$

Maximum work in case of heat exchange with two thermal energy reservoirs :

Substituting expression for Q_L on the previous slide in the first law equation, we get

$$W_{net,out} = Q_H\left(1 - \frac{T_L}{T_H}\right) - T_L \Delta S_T$$

In case of a reversible heat engine, the entire process is reversible. This means that total entropy change is zero : $\Delta S_T = S_{gen} = 0$

Thus the net work output of a reversible heat engine is :

$$W^{rev}_{net,out} = Q_H\left(1 - \frac{T_L}{T_H}\right)$$

Substituting in the work expression for an irreversible heat engine, we get

$$W_{net,out} = W^{rev}_{net,out} - T_L \Delta S_T$$

Maximum work in case of heat exchange with two thermal energy reservoirs :

From the last equation of the previous slide, we get

$$W_{net,out} < W_{net,out}^{rev} \text{ for a fixed value of } Q_H$$

Since $T_L \Delta S_T > 0$ for an irreversible process

Thus, we get maximum net work output for a reversible process. In a reversible process (see equation on slide 9), we have

$$Q_L^{rev} = Q_H \left(\frac{T_L}{T_H}\right)$$

Thus, $Q_L = Q_L^{rev} + T_L \Delta S_T$

Hence $Q_L > Q_L^{rev}$

Thus, less heat is rejected in a reversible heat engine. Thus reversible heat engine allows for conversion of more heat into work as compared to an irreversible heat engine.

Irreversibility in case of heat exchange with two thermal energy reservoirs :

The irreversibility or lost work of a heat engine operating between the two thermal reservoirs is given by

$$I = W_{net,out}^{rev} - W_{net,out}$$

Thus, irreversibility is the difference between maximum (reversible) work output and the actual work output for a given value of Q_H (i.e., heat absorbed from the high temperature reservoir).

Exergy :

If we consider any potential source of energy, for example, a geothermal well, one first needs to consider how much work we can extract from that source.

For example, if the temperature of the geothermal source is 300 ^{0}C, in which case work extracted will be more (a) environment at 25 ^{0}C or (b) environment at 5 ^{0}C ?

Considering that we extract work by means of a heat engine which rejects heat to the environment, efficiency of the engine will increase if the temperature of the environment is lower. Thus answer of the above question is (b) !

Thus work we can extract depends not only on the system, but also on the state of the environment.

Exergy :

Exergy is the maximum amount of useful work that can be obtained from the system as the system is brought to equilibrium with the environment.

It is denoted by the symbol ‘X’. Exergy per unit mass is denoted by ‘x’

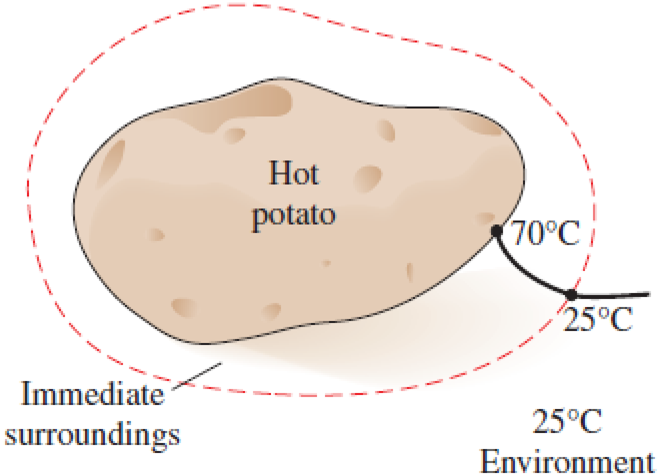
Exergy is also called “work potential” of a system in a specified environment. In some cases, the term “availability” is also used for exergy.

The state of the system which is in equilibrium with the environment is known as the “dead state”. This is because we cannot extract any work from the system once it reaches this state.

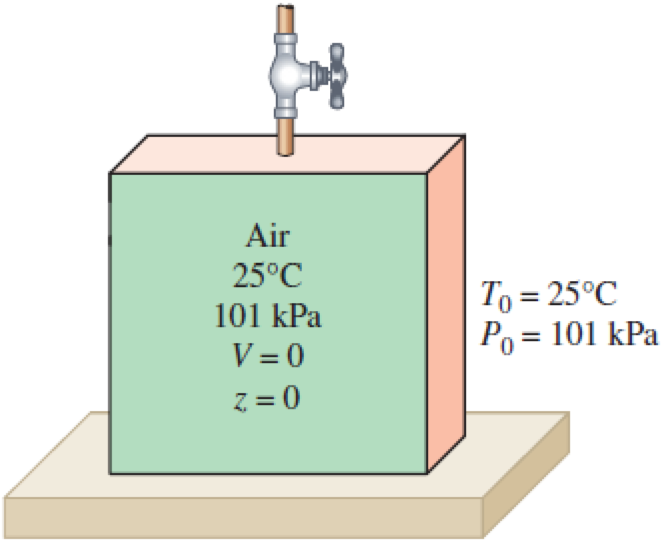
Exergy :

Ref. Cengel and Boles, 8th Edition (2015)

The atmosphere contains a tremendous amount of energy, but no exergy.



Work can be extracted from the Hot potato since it is at a higher temperature than the environment

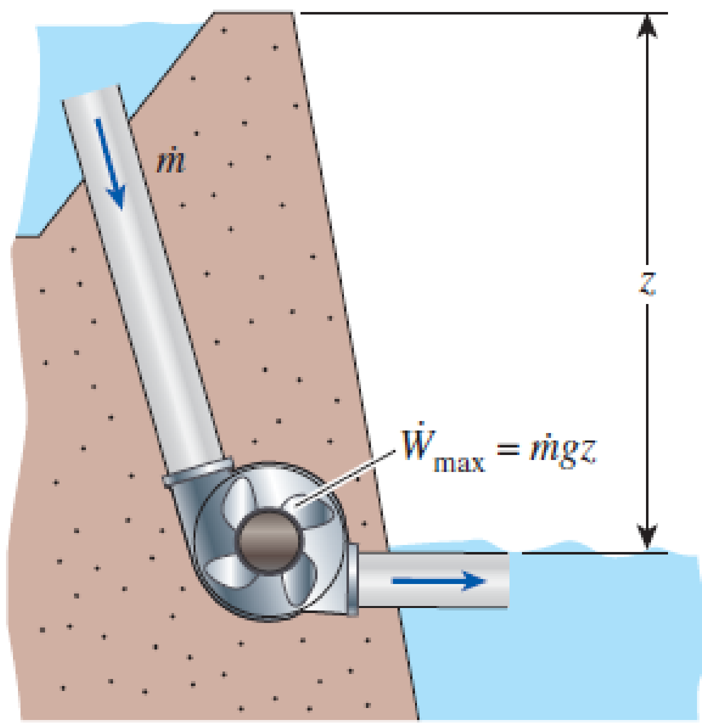


Air in the tank is in a ‘dead’ state

Exergy :

Exergy associated with kinetic and potential energies : Kinetic or potential energy of a given (control mass) system can be completely converted into useful work. Hence exergy associated with kinetic energy is equal to kinetic energy itself. Same is the case with potential energy.

Thus $x_{ke} = \frac{1}{2}V^2$ and $x_{pe} = g\,Z$

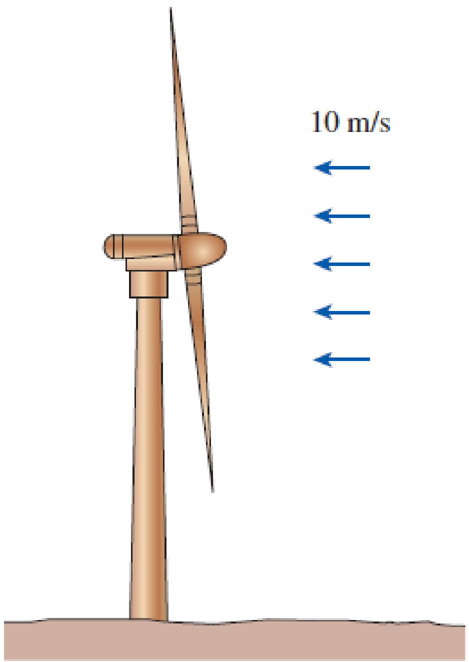


The *work potential* or *exergy* of potential energy is equal to the potential energy itself.

Ref. Cengel and Boles, 8th Edition (2015)

Example on exergy :

A wind turbine with a 12-m-diameter rotor, as shown in Fig. 8–5, is to be installed at a location where the wind is blowing steadily at an average velocity of 10 m/s. Determine the maximum power that can be generated by the wind turbine.



Assumptions Air is at standard conditions of 1 atm and 25°C, and thus its density is 1.18 kg/m^3.

Analysis The air flowing with the wind has the same properties as the stagnant atmospheric air except that it possesses a velocity and thus some kinetic energy. This air will reach the dead state when it is brought to a complete stop. Therefore, the exergy of the blowing air is simply the kinetic energy it possesses:

$$\text{ke} = \frac{V^2}{2} = \frac{(10\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = 0.05\text{ kJ/kg}$$

Ref. Cengel and Boles, 8th Edition (2015)

Example on exergy :

That is, every unit mass of air flowing at a velocity of 10 m/s has a work potential of 0.05 kJ/kg. In other words, a perfect wind turbine will bring the air to a complete stop and capture that 0.05 kJ/kg of work potential. To determine the maximum power, we need to know the amount of air passing through the rotor of the wind turbine per unit time, that is, the mass flow rate, which is determined to be

$$\dot{m} = \rho A V = \rho \frac{\pi D^2}{4} V = (1.18 \text{ kg/m}^3) \frac{\pi (12 \text{ m})^2}{4} (10 \text{ m/s}) = 1335 \text{ kg/s}$$

Thus,

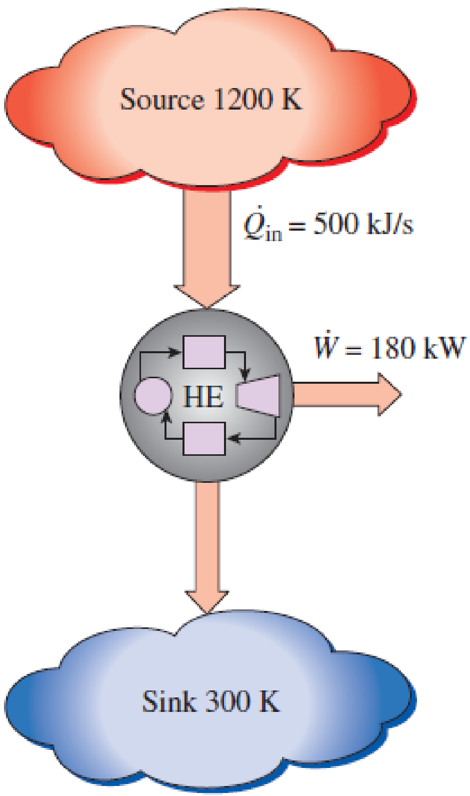
$$\text{Maximum power} = \dot{m}(\text{ke}) = (1335 \text{ kg/s})(0.05 \text{ kJ/kg}) = \mathbf{66.8\ kW}$$

This is the maximum power available to the wind turbine. Assuming a conversion efficiency of 30 percent, an actual wind turbine will convert 20.0 kW to electricity. Notice that the work potential for this case is equal to the entire kinetic energy of the air.

Ref. Cengel and Boles, 8th Edition (2015)

Example on irreversibility or lost work or exergy destroyed:

A heat engine receives heat from a source at 1200 K at a rate of 500 kJ/s and rejects the waste heat to a medium at 300 K The power output of the heat engine is 180 kW. Determine the reversible power and the irreversibility rate for this process.



Analysis The reversible power for this process is the amount of power that a reversible heat engine, such as a Carnot heat engine, would produce when operating between the same temperature limits, and is determined to be:

$$\dot{W}_{\text{rev,out}} = \eta_{\text{th,rev}}\,\dot{Q}_{\text{in}} = \left(1 - \frac{T_{\text{sink}}}{T_{\text{source}}}\right)\dot{Q}_{\text{in}} = \left(1 - \frac{300\text{ K}}{1200\text{ K}}\right)(500\text{ kW}) = \mathbf{375\ kW}$$

This is the maximum power that can be produced by a heat engine operating between the specified temperature limits and receiving heat at the specified rate. This would also represent the *available power* if 300 K were the lowest temperature available for heat rejection.

The irreversibility rate is the difference between the reversible power (maximum power that could have been produced) and the useful power output:

$$\dot{I} = \dot{W}_{\text{rev,out}} - \dot{W}_{u\text{,out}} = 375 - 180 = \mathbf{195\ kW}$$

Note that irreversibility is also known as "exergy destroyed".

Ref. Cengel and Boles, 8th Edition (2015)