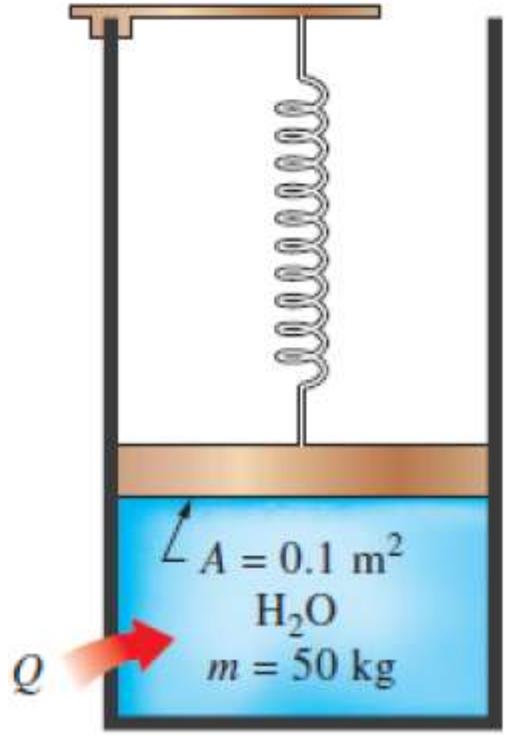
# ESO201A
# Tutorial 3: Problems and Solutions

**4-24.** A piston-cylinder device contains 50kg of water at 250kPa and 25°C. The cross-sectional area of the piston is 0.1 $m^2$. Heat is now transferred to the water, causing part of it to evaporate and expand. When the volume reaches 0.2 $m^3$, the piston reaches linear spring whose spring constant is 100kN/m. More heat is transferred to the water until the piston rises 20cm more. Determine (a) the final pressure and temperature and (b) the work done during this process. Also show the process on a P-V diagram.



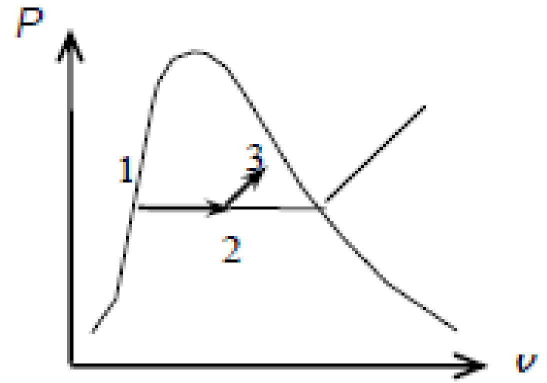

Read chapter 3

**Answer:**

Water in a cylinder equipped with a spring is heated and evaporated. The vapor expands until it compresses the spring 20cm. The final pressure and temperature, and the boundary work done are to be determined, and the process is to be shown on a *P*-V diagram.

***Assumptions:***

The process is quasi-equilibrium.




***Analysis:***

(*a*) The final pressure is determined from

$$P_3 = P_2 + \frac{F_s}{A} = P_2 + \frac{kx}{A} = (250 \text{ kPa}) + \frac{(100 \text{ kN/m})(0.2 \text{ m})}{0.1 \text{ m}^2}\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right) = \mathbf{450 \text{ kPa}}$$

The specific and total volumes at the three states are

$$\left.\begin{aligned} T_1 &= 25°\text{C} \\ P_1 &= 250 \text{ kPa} \end{aligned}\right\} \boldsymbol{v}_1 \cong \boldsymbol{v}_{f\,@25°\text{C}} = 0.001003 \text{ m}^3/\text{kg}$$

$$V_1 = m\mathbf{v}_1 = (50 \text{ kg})(0.001003 \text{ m}^3/\text{kg}) = 0.05 \text{ m}^3$$

$$V_2 = 0.2 \text{ m}^3$$

$$V_3 = V_2 + x_{23}A_p = (0.2 \text{ m}^3) + (0.2 \text{ m})(0.1 \text{ m}^2) = 0.22 \text{ m}^3$$

$$\mathbf{v}_3 = \frac{V_3}{m} = \frac{0.22 \text{ m}^3}{50 \text{ kg}} = 0.0044 \text{ m}^3/\text{kg}$$

At 450 kPa, $v_f$ = 0.001088 m$^3$/kg and v$g$ = 0.41392 m$^3$/kg. Noting that $v_f < v_3 < v_g$ , the final state is a saturated mixture and thus the final temperature is

$$T_3 = T_{\text{sat@450 kPa}} = \mathbf{147.9°C}$$

($b$) The pressure remains constant during process 1-2 and changes linearly (a straight line) during process 2-3. Then the boundary work during this process is simply the total area under the process curve,

$$W_{b,\text{out}} = \text{Area} = P_1(V_2 - V_1) + \frac{P_2 + P_3}{2}(V_3 - V_2)$$

$$= \left( (250 \text{ kPa})(0.2 - 0.05)\text{m}^3 + \frac{(250 + 450) \text{ kPa}}{2}(0.22 - 0.2)\text{m}^3 \right)\left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) = \mathbf{44.5 \text{ kJ}}$$

***Discussion*** The positive sign indicates that work is done by the system (work output).

**4.54.** Determine the internal energy change $\Delta u$ of hydrogen in kJ/kg, as it is heated from 200 to 800k, using (a) the empirical specific heat equation as a function of temperature (table A-2c), (b) the $c_v$ value at the average temperature (table A-2b), and (c) the $c_v$ value at room temperature (table A-2a).

**Answer:**

The internal energy change of hydrogen gas during a heating process is to be determined using an empirical specific heat relation, constant specific heat at average temperature, and constant specific heat at room temperature.

***Analysis*:**

(*a*) Using the empirical relation for $c_p(T)$ from Table A-2c and relating it to $c_v(T)$,

$$\bar{c}_v(T) = \bar{c}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

where a = 29.11, b = -0.1916×10$^{-2}$, c = 0.4003×10$^{-5}$, and d = -0.8704×10$^{-9}$. Then,

$$\Delta\bar{u} = \int_1^2 \bar{c}_v(T)dT = \int_1^2 \left[(a - R_u) + bT + cT^2 + dT^3\right]dT$$

$$= (a - R_u)(T_2 - T_1) + \tfrac{1}{2}b(T_2^2 + T_1^2) + \tfrac{1}{3}c(T_2^3 - T_1^3) + \tfrac{1}{4}d(T_2^4 - T_1^4)$$

$$= (29.11 - 8.314)(800 - 200) - \tfrac{1}{2}(0.1961 \times 10^{-2})(800^2 - 200^2)$$

$$+ \tfrac{1}{3}(0.4003 \times 10^{-5})(800^3 - 200^3) - \tfrac{1}{4}(0.8704 \times 10^{-9})(800^4 - 200^4)$$

$$= 12{,}487 \text{ kJ/kmol}$$

$$\Delta u = \frac{\Delta\bar{u}}{M} = \frac{12{,}487 \text{ kJ/kmol}}{2.016 \text{ kg/kmol}} = \mathbf{6194\ kJ/kg}$$

$(b)$ Using a constant $c_v$ value from Table A-2b at the average temperature of 500K,

$$c_{v,\mathrm{avg}} = c_{v@500\mathrm{K}} = 10.389\ \mathrm{kJ/kg \cdot K}$$

$$\Delta u = c_{v,\mathrm{avg}}(T_2 - T_1) = (10.389\ \mathrm{kJ/kg \cdot K})(800 - 200)\mathrm{K} = \mathbf{6233\ kJ/kg}$$

$(c)$ Using a constant $c_v$ value from Table A-2a at the room temperature

$$c_{v,\mathrm{avg}} = c_{v@300\mathrm{K}} = 10.183\ \mathrm{kJ/kg \cdot K}$$

$$\Delta u = c_{v,\mathrm{avg}}(T_2 - T_1) = (10.183\ \mathrm{kJ/kg \cdot K})(800 - 200)\mathrm{K} = \mathbf{6110\ kJ/kg}$$

**4-87.** If you ever slapped someone or got slapped yourself, you probably remember the burning sensation. Imagine you had the unfortunate occasion of being slapped by an angry person, which caused the temperature of the affected area of your face to rise by 2.4°C (ouch!). Assuming the slapping hand has a mass of 0.9 kg and about 0.150 kg of the tissue on the face and the hand is affected by the incident, estimate the velocity of the hand just before impact. Take the specific heat of the tissue to be 3.8 kJ/kg·K.

**Answer:**

The face of a person is slapped. For the specified temperature rise of the affected part, the impact velocity of the hand is to be determined.

***Assumptions***:

- The hand is brought to a complete stop after the impact.
- The face takes the blow without significant movement.
- No heat is transferred from the affected area to the surroundings, and thus the process is adiabatic.
- No work is done on or by the system.
- The potential energy change is zero, $\Delta PE = 0$ and $\Delta E = \Delta U + \Delta KE$.

***Analysis*:**

We analyze this incident in a professional manner without involving any emotions. First, we identify the system, draw a sketch of it, and state our observations about the specifics of the problem. We take the hand and the affected portion of the face as the system. This is a closed system since it involves a fixed amount of mass (no mass transfer). We observe that the kinetic energy of the hand decreases during the process, as evidenced by a decrease in velocity from initial value to zero, while the internal energy of the affected area increases, as evidenced by an increase in the temperature. There seems to be no significant energy transfer between the system and its surroundings during this process. Under the stated assumptions and observations, the energy balance on the system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$0 = \Delta U_{\text{affected tissue}} + \Delta \text{KE}_{\text{hand}}$$

$$0 = (mc\Delta T)_{\text{affected tissue}} + [m(0 - V^2)/2]_{\text{hand}}$$

That is, the decrease in the kinetic energy of the hand must be equal to the increase in the internal energy of the affected area. Solving for the velocity and substituting the given quantities, the impact velocity of the hand is determined to be

$$V_{\text{hand}} = \sqrt{\frac{2(mc\Delta T)_{\text{affected tissue}}}{m_{\text{hand}}}}$$

$$V_{\text{hand}} = \sqrt{\frac{2(0.15\ \text{kg})(3.8\ \text{kJ/kg}\cdot{}^\circ\text{C})(2.4^\circ\text{C})}{0.9\ \text{kg}}\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)}$$

$$= \mathbf{55.1\ m/s}\ (\text{or } 198\ \text{km/h})$$

***Discussion*** Reconstruction of events such as this by making appropriate assumptions are commonly used in forensic engineering.

**4-113.** In order to cool 1 ton of water at 20 $^0$C in an insulated tank, a person pours 80 kg of ice at -5$^0$C into the water. Determine the final equilibrium temperature in the tank. The melting temperature and the heat of fusion of ice at atmospheric pressure are 0$^0$C and 333.7$^0$C kJ/kg, respectively.

**Answer:**

A 1- ton (1000 kg) of water is to be cooled in a tank by pouring ice into it. The final equilibrium temperature in the tank is to be determined.

**Assumptions:**

- Thermal properties of the ice and water are constant.
- Heat transfer to the water tank is negligible.
- There is no stirring by hand or a mechanical device (it will add energy).

**Properties:**

- The specific heat of water at room temperature is $c = 4.18$ kJ/kg·°C, and the specific heat of ice at about $0^0$C is $C = 2.11$ kJ/kg·°C (Table A-3).
- The melting temperature and the heat of fusion of ice at 1 atm are $0^0$C and 333.7 kJ/kg.

Heat of fusion: Latent heat of fusion of ice is the amount of heat required to melt a unit mass of ice from the solid-state to the liquid state.

**Analysis:**

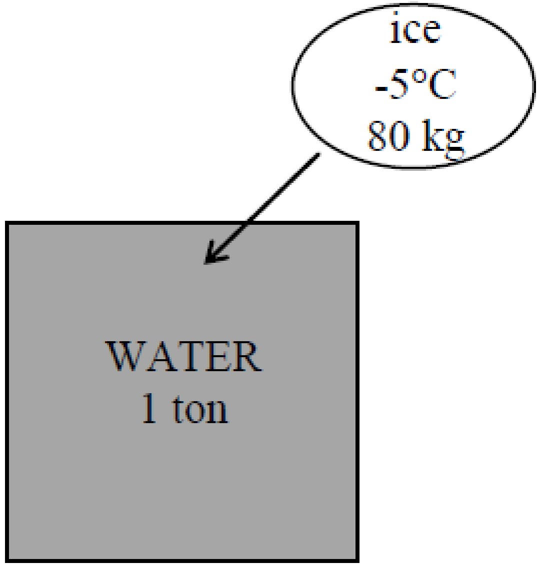
We take the ice and the water as our system, and disregard any heat transfer between the system and the surroundings. Then the energy balance for this process can be written as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$0 = \Delta U$$

$$0 = \Delta U_{\text{ice}} + \Delta U_{\text{water}}$$

$$[mc(0^\circ\text{C} - T_1)_{\text{solid}} + mh_{if} + mc(T_2 - 0^\circ\text{C})_{\text{liquid}}]_{\text{ice}} + [mc(T_2 - T_1)]_{\text{water}} = 0$$




Substituting,

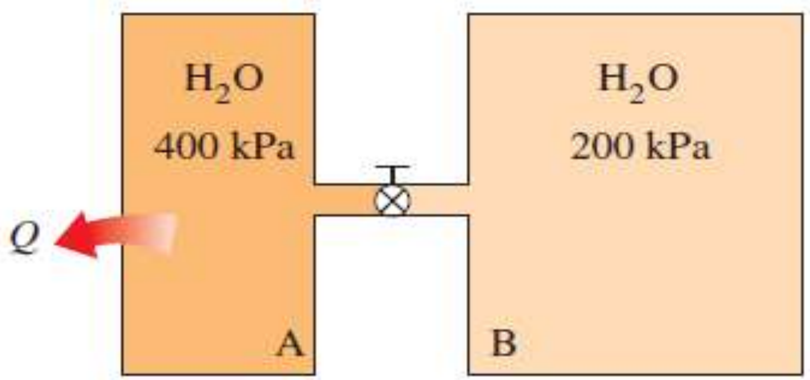
$$(80\ \text{kg})\{(2.11\ \text{kJ/kg}\cdot^\circ\text{C})[0 - (\text{-}5)]^\circ\text{C} + 333.7\ \text{kJ/kg} + (4.18\ \text{kJ/kg}\cdot^\circ\text{C})(T_2 - 0)^\circ\text{C}\}$$
$$+ (1000\ \text{kg})(4.18\ \text{kJ/kg}\cdot^\circ\text{C})(T_2 - 20)^\circ\text{C} = 0$$

It gives

$$T_2 = 12.4^oC$$

which is the final equilibrium temperature in the tank.

**4-131.**Two rigid tanks are connected by a valve. Tank A contains 0.2 $m^3$ of water at 400 kPa and 80 percent quality. Tank B contains 0.5 $m^3$ of water at 200 kPa and 250 $^0C$. The valve is now opened, and the two tanks eventually come to the same state. Determine the pressure and the amount of heat transfer when the system reaches **thermal equilibrium with the surroundings at 25 $^0$C.**



**Assumptions:**

- The tanks are stationary and thus the kinetic and potential energy changes are zero.
- There are no work interactions.
- State 1 → initial state; state 2→ final state (thermal equilibrium)

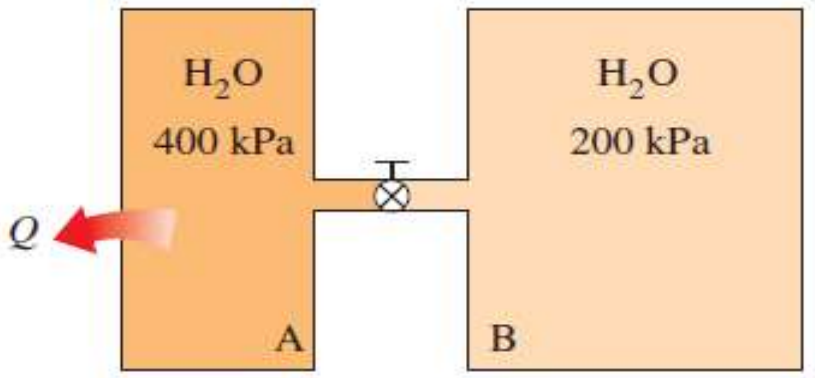
**Analysis:**

We take the entire contents of the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work. The energy balance for this stationary closed system can be expressed as:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$-Q_{\text{out}} = \Delta U = (\Delta U)_A + (\Delta U)_B \quad (\text{since } W = \text{KE} = \text{PE} = 0)$$

$$Q_{\text{out}} = -[U_{2,A+B} - U_{1,A} - U_{1,B}]$$
$$= -[m_{2,\text{total}} u_2 - (m_1 u_1)_A - (m_1 u_1)_B]$$



The properties of water in each tank are (Tables A-4 through A-6)

Tank A:

$$\left.\begin{array}{l} P_1 = 400 \text{ kPa} \\ x_1 = 0.80 \end{array}\right\} \begin{array}{ll} v_f = 0.001084, & v_g = 0.46242 \text{ m}^3/\text{kg} \\ u_f = 604.22, & u_{fg} = 1948.9 \text{ kJ/kg} \end{array}$$

$$v_{1,A} = v_f + x_1 v_{fg} = 0.001084 + [0.8 \times (0.46242 - 0.001084)] = 0.37015 \text{ m}^3/\text{kg}$$

See Page 127

$$u_{1,A} = u_f + x_1 u_{fg} = 604.22 + (0.8 \times 1948.9) = 2163.3 \text{ kJ/kg}$$

Tank B:

$$\left.\begin{array}{l} P_1 = 200 \text{ kPa} \\ T_1 = 250°\text{C} \end{array}\right\} \begin{array}{l} v_{1,B} = 1.1989 \text{ m}^3/\text{kg} \\ u_{1,B} = 2731.4 \text{ kJ/kg} \end{array}$$

$$m_{1,A} = \frac{V_A}{v_{1,A}} = \frac{0.2 \text{ m}^3}{0.37015 \text{ m}^3/\text{kg}} = 0.5403 \text{ kg}$$

$$m_{1,B} = \frac{V_B}{v_{1,B}} = \frac{0.5 \text{ m}^3}{1.1989 \text{ m}^3/\text{kg}} = 0.4170 \text{ kg}$$

$$m_t = m_{1,A} + m_{1,B} = 0.5403 + 0.4170 = 0.9573 \text{ kg}$$

Therefore, specific volume at final state, $v_2 = (V_A + V_B)/m_t = (0.2+0.5)/0.9573 = 0.73117$

Thus at the final state the system will be a saturated liquid-vapor mixture since $v_f < v_2 < v_g$. Then the final pressure must be

$$P_2 = P_{sat\ @25C} = \mathbf{3.17\ kPa}$$

Also,

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.73117 - 0.001}{43.340 - 0.001} = 0.01685$$

where, vfg=vg-vf

$$u_2 = u_f + x_2 u_{fg} = 104.83 + (0.01685 \times 2304.3) = 143.65 \text{ kJ/kg}$$

Substituting, $$Q_{out} = -[(0.9573)(143.65) - (0.5403)(2163.3) - (0.4170)(2731.4)] = \mathbf{2170\ kJ}$$

**See solved example 4-6 also**