Q18- $f(x) = \sqrt{x+1} - \sqrt{x}$

(i) $C_p = \left| \frac{f'(x)x}{f(x)} \right| = \left| \frac{\frac{1}{2}\left[ 1/\sqrt{x+1} - 1/\sqrt{x} \right] x}{\sqrt{x+1} - \sqrt{x}} \right|$

$= \frac{1}{2} \frac{x}{\sqrt{x+1}\sqrt{x}}$ for large values of $x$

$C_p \simeq \frac{1}{2}$

$C_p < 1 \rightarrow$ well-conditioned

(ii) Relative error $f(x) = \sqrt{x+1} - \sqrt{x}$

$x = 208208$

$\tilde{f}(x) = 0.456299 \times 10^3 - 0.456290 \times 10^3$

$= 0.100000 \times 10^{-2}$

$f(x) = 0.109577 \times 10^{-2}$

$e_r = \frac{f(x) - \tilde{f}(x)}{f(x)} \times 100 = 8.74\%$

(iii) Modified Algorithm

Multiply & divide $f(x)$ by $\sqrt{x+1} + \sqrt{x}$

$$f(x) = \sqrt{x+1} - \sqrt{x} \times \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}}$$

$$f(x) = \frac{1}{\sqrt{x+1} + \sqrt{x}}$$

$$\tilde{f}(x) = \frac{1}{0.456299 \times 10^3 + 0.456298 \times 10^3}$$

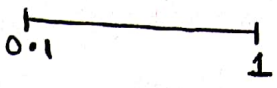
$$= 0.109577 \times 10^{-2}$$

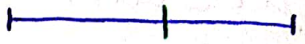
$$e_r = \frac{f(x) - \tilde{f}(x)}{f(x)} \times 100 = 0.4 \times 10^{-5}\%$$

Q28- $f(x) = 600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0$

$[0.1, 1]$

## BISECTION METHOD



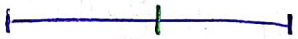
| $x$ | 0.1 | 0.55 | 1 |
|---|---|---|---|
| $f(x)$ | -1.19 | 11.897229 | |

### 1st Iteration

$$x_r = \frac{x_l + x_u}{2}$$

$$x_r = \frac{0.1 + 1}{2} = 0.55$$

$$e_a(\%) \sim$$

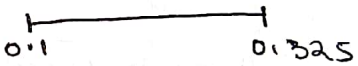
| $x$ | 0.1 | 0.325 | 0.55 |
|---|---|---|---|
| $f(x)$ | -1. | 1.4385 | 11.8975 |

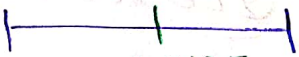
### 2nd Iteration

$$x_r = \frac{0.1 + 0.55}{2} = 0.325$$

$$e_a(\%) = \left|\frac{0.325 - 0.155}{0.325}\right|$$

$$= 69.231$$



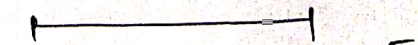
| $x$ | 0.1 | 0.2125 | 0.325 |
|---|---|---|---|
| $f(x)$ | -1.19 | -0.2729 | 1.4385 |

### 3rd Iteration

$$x_r = \frac{0.1 + 0.325}{2} = 0.2125$$

$$e_a(\%) = \left|\frac{0.2125 - 0.325}{0.2125}\right|$$

$$= 52.941$$




| $x$ | 0.2125 | 0.26875 | 0.325 |
|---|---|---|---|
| $f(x)$ | 0.52433 | 0.52433 | 1.438 |

### 4th Iteration

$x_r = 0.26875$

$e_a\% = 20.93$

(b) False position method

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

not @ scale

1st Iteration

$$x_r = 1 - \frac{229(0.1-1)}{-1.49 - 229}$$

$$= 0.10582$$

$$e_a(\%) = \sim .$$

| $x$ | 0.1 | 0.1058 | 1 |
|---|---|---|---|
| $f(x)$ | -1.49 | -1.4534 | 229 |

2nd Iteration

$$x_r = 1 - \frac{229(0.1058 - 1)}{-1.4534 - 229}$$

$$= 0.11146$$

$$e_a(\%) = \left| \frac{0.11146 - 0.10582}{0.11146} \right| = 5.059\%$$

$f(0.11146) = -1.4135$

| $x$ | 0.1058 | 0.11146 | 1 |
|---|---|---|---|
| $f(x)$ | -1.4534 | | 229 |

3rd Iteration

$$x_r = 1 - \frac{229(0.11146 - 1)}{-1.4135 - 229}$$

$$= 0.11691$$

$$e_a(\%) = \left| \frac{0.11691 - 0.11146}{0.11691} \right| = 4.663\%$$

| $x$ | 0.11146 | 0.11691 | 1 |
|---|---|---|---|
| $f(x)$ | -1.4135 | -1.37146 | 229 |

4th Iteration

$x_r = 0.12217$

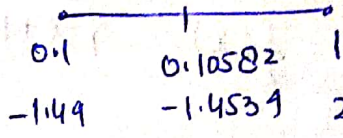
$e_a(\%)$ 4.303%

| $x$ | 0.11691 | 0.12217 | 1 |
|---|---|---|---|
| $f(x)$ | -1.37146 | | 229 |

# (c) Modified False Position method

## 1st Iteration

$$x_r = 1 - \frac{[229](0.1-1)}{-1.49-229} = 0.10582$$

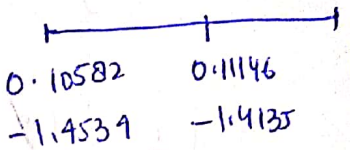



$\varepsilon_a(\%)$ ~

## 2nd Iteration

$x_r = 0.11146$

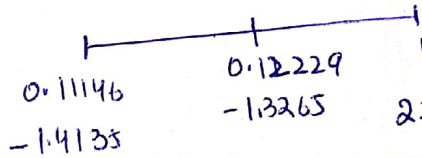
$\varepsilon_a(\%) = 5.059$




## 3rd Iteration

$$x_r = 1 - \frac{(114.5)(0.11146-1)}{-1.4135-1145} = 0.12229$$

$$\varepsilon_a(\%) = \left|\frac{0.12229-0.11146}{0.12229}\right| = 8.860\%$$

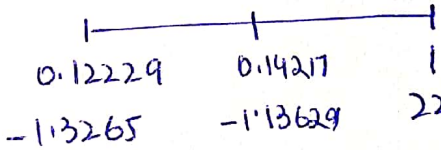



$\left(\frac{229}{2} = 114.5\right)$

## 4th Iteration

$$x_r = 1 - \frac{(57.25)(0.12229-1)}{-1.3265-57.25} = 0.14217$$

$$\varepsilon_a(\%) = \left|\frac{0.14217-0.12229}{0.14217}\right| = 13.98\%$$




$= \frac{114.5}{2} = 57.25$

## 5th Iteration

$$x_r = 1 - \frac{(28.625)(0.14217-1)}{-1.13629-28.625} = 0.17492$$

$\varepsilon_a(\%) = 10.724$