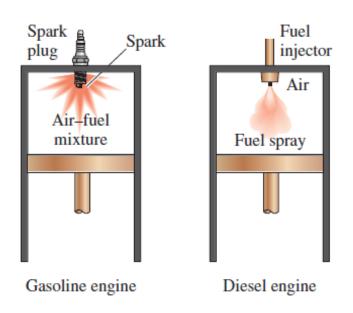
ESO201A: THERMODYNAMICS 2021-22 Ist semester IIT Kanpur

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Lecture 27

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DIESEL CYCLE: THE IDEAL CYCLE FOR COMPRESSION-IGNITION ENGINES



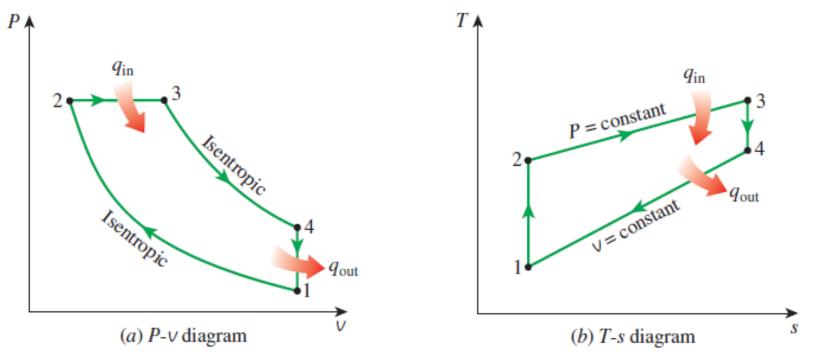
In spark-ignition engines (also known as *gasoline engines*), the air–fuel mixture is compressed to a temperature that is below the autoignition temperature of the fuel, and the combustion process is initiated by firing a spark plug. In CI engines (also known as *diesel engines*), the air is compressed to a temperature that is above the autoignition temperature of the fuel, and combustion starts on contact as the fuel is injected into this hot air. Therefore, the spark plug is replaced by a fuel injector in diesel engines

Ref. Cengel and Boles, 8th Edition (2015)

In gasoline engines, a mixture of air and fuel is compressed during the compression stroke, and the compression ratios are limited by the onset of autoignition or engine knock. In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of autoignition. Therefore, diesel engines can be designed to operate at much higher compression ratios, typically between 12 and 24. Not having to deal with the problem of autoignition has another benefit: many of the stringent requirements placed on the gasoline can now be removed, and fuels that are less refined (thus less expensive) can be used in diesel engines.

The fuel injection process in diesel engines starts when the piston approaches TDC and continues during the first part of the power stroke. Therefore, the combustion process in these engines takes place over a longer interval. Because of this longer duration, the combustion process in the ideal Diesel cycle is approximated as a constant-pressure heat-addition process. In fact, this is the only process where the Otto and the Diesel cycles differ. The remaining three processes are the same for both ideal cycles. That is, process 1-2 is isentropic compression, 2-3 is constant-pressure heat addition, 3-4 is isentropic expansion, and 4-1 is constant-volume heat rejection. The similarity between the two cycles is also apparent from the *P-V* and *T-s* diagrams of the Diesel cycle

Ref. Cengel and Boles, 8th Edition (2015)



$$q_{\text{in}} - w_{b,\text{out}} = u_3 - u_2 \rightarrow q_{\text{in}} = P_2(v_3 - v_2) + (u_3 - u_2)$$

= $h_3 - h_2 = c_p(T_3 - T_2)$

$$-q_{\text{out}} = u_1 - u_4 \rightarrow q_{\text{out}} = u_4 - u_1 = c_v(T_4 - T_1)$$

Then the thermal efficiency of the ideal Diesel cycle under the cold-airstandard assumptions becomes

$$\eta_{\text{th,Diesel}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}$$

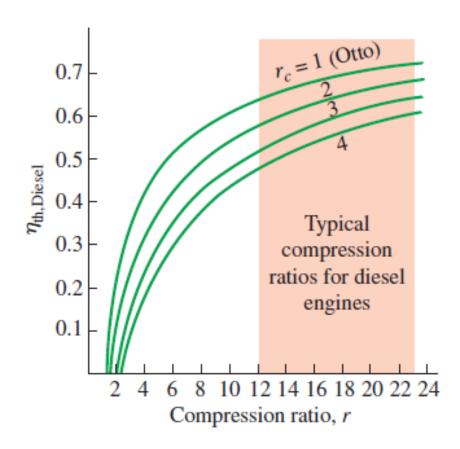
We now define a new quantity, the **cutoff ratio** r_c , as the ratio of the cylinder volumes after and before the combustion process:

$$r_c = \frac{V_3}{V_2} = \frac{V_3}{V_2}$$

Utilizing this definition and the isentropic ideal-gas relations for processes 1-2 and 3-4, we see that the thermal efficiency relation reduces to

$$\eta_{\text{th,Diesel}} = 1 - \frac{1}{r^{k-1}} \left[\frac{r_c^k - 1}{k(r_c - 1)} \right]$$

Ref. Cengel and Boles, 8th Edition (2015)



diesel engines operate at much higher compression ratios and thus are usually more efficient than the sparkignition (gasoline) engines. The diesel engines also burn the fuel more completely since they usually operate at lower revolutions per minute and the air–fuel mass ratio is much higher than spark-ignition engines. Thermal efficiencies of large diesel engines range from about 35 to 40 percent.

In modern high-speed compression ignition engines, fuel is injected into the combustion chamber much sooner compared to the early diesel engines. Fuel starts to ignite late in the compression stroke, and consequently part of the combustion occurs almost at constant volume. Fuel injection continues until the piston reaches the top dead center, and combustion of the fuel keeps the pressure high well into the expansion stroke. Thus, the entire combustion process can better be modeled as the combination of constant-volume and constant-pressure processes. The ideal cycle based on this concept is called the **dual cycle** and P-V diagram for The relative amounts of heat transferred durit is given in Fig. ing each process can be adjusted to approximate the actual cycle more closely. Note that both the Otto and the Diesel cycles can be obtained as special cases of the dual cycle. Dual cycle is a more realistic model than diesel cycle for representing modern, high-speed compression ignition engines.

EXAMPLE 9-3 The Ideal Diesel Cycle

An ideal Diesel cycle with air as the working fluid has a compression ratio of 18 and a cutoff ratio of 2. At the beginning of the compression process, the working fluid is at 14.7 psia, 80° F, and 117 in³. Utilizing the cold-air-standard assumptions, determine (a) the temperature and pressure of air at the end of each process, (b) the net work output and the thermal efficiency, and (c) the mean effective pressure.

SOLUTION An ideal Diesel cycle is considered. The temperature and pressure at the end of each process, the net work output, the thermal efficiency, and the mean effective pressure are to be determined.

Assumptions 1 The cold-air-standard assumptions are applicable and thus air can be assumed to have constant specific heats at room temperature. 2 Kinetic and potential energy changes are negligible.

Properties The gas constant of air is $R=0.3704~{\rm psia\cdot ft^3/lbm\cdot R}$ and its other properties at room temperature are $c_p=0.240~{\rm Btu/lbm\cdot R}$, $c_v=0.171~{\rm Btu/lbm\cdot R}$, and $k=1.4~{\rm (Table~A-2Ea)}$.

(a) The temperature and pressure values at the end of each process can be determined by utilizing the ideal-gas isentropic relations for processes 1-2 and 3-4. But first we determine the volumes at the end of each process from the definitions of the compression ratio and the cutoff ratio: $V_2 = \frac{V_1}{r} = \frac{117 \text{ in}^3}{19} = 6.5 \text{ in}^3$

$$V_3 = r_c V_2 = (2)(6.5 \text{ in}^3) = 13 \text{ in}^3$$
 $V_4 = V_1 = 117 \text{ in}^3$

 $T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{k-1} = (540 \text{ R})(18)^{1.4-1} = 1716 \text{ R}$

Process 1-2 (isentropic compression of an ideal gas, constant specific heats):

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^k = (14.7 \text{ psia})(18)^{1.4} = 841 \text{ psia}$$

Process 2-3 (constant-pressure heat addition to an ideal gas):

$$P_3 = P_2 = 841 \text{ psia}$$

$$P_{\alpha}V_{\alpha}$$
 $P_{\alpha}V_{\alpha}$ (V_{α})

 $\frac{P_2V_2}{T_1} = \frac{P_3V_3}{T_2} \rightarrow T_3 = T_2\left(\frac{V_3}{V_2}\right) = (1716 \text{ R})(2) = 3432 \text{ R}$

Process 3-4 (isentropic expansion of an ideal gas, constant specific heats):

Process 3-4 (isentropic expansion of an ideal gas, constant specific heat
$$T_4 = T_3 \left(\frac{V_3}{V}\right)^{k-1} = (3432 \text{ R}) \left(\frac{13 \text{ in}^3}{117 \text{ in}^3}\right)^{1.4-1} = 1425 \text{ R}$$

 $P_4 = P_3 \left(\frac{V_3}{V_*}\right)^k = (841 \text{ psia}) \left(\frac{13 \text{ in}^3}{117 \text{ in}^3}\right)^{1.4} = 38.8 \text{ psia}$

(b) The net work for a cycle is equivalent to the net heat transfer. But first we find the mass of air:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(14.7 \text{ psia})(117 \text{ in}^3)}{(0.3704 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(540 \text{ R})} \left(\frac{1 \text{ ft}^3}{1728 \text{ in}^3}\right) = 0.00498 \text{ lbm}$$

Process 2-3 is a constant-pressure heat-addition process, for which the boundary work and Δu terms can be combined into Δh . Thus,

$$Q_{\text{in}} = m(h_3 - h_2) = mc_p(T_3 - T_2)$$

= (0.00498 lbm)(0.240 Btu/lbm·R)[(3432 - 1716) R]
= 2.051 Btu

Process 4-1 is a constant-volume heat-rejection process (it involves no work interactions), and the amount of heat rejected is

$$Q_{\text{out}} = m(u_4 - u_1) = mc_v(T_4 - T_1)$$

= $(0.00498 \text{ lbm})(0.171 \text{ Btu/lbm·R})[(1425 - 540) \text{ R}]$
= 0.754 Btu

Thus,

$$W_{\text{net}} = Q_{\text{in}} - Q_{\text{out}} = 2.051 - 0.754 = 1.297 \text{ Btu}$$

Then the thermal efficiency becomes

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{1.297 \text{ Btu}}{2.051 \text{ Btu}} = 0.632 \text{ or } 63.2\%$$

The thermal efficiency of this Diesel cycle under the cold-air-standard assumptions could also be determined from Eq. 9–12.

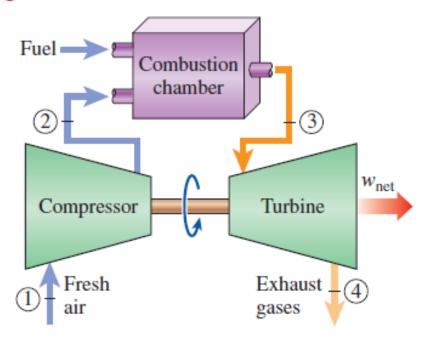
(c) The mean effective pressure is determined from its definition, Eq. 9-4:

MEP =
$$\frac{W_{\text{net}}}{V_{\text{max}} - V_{\text{min}}} = \frac{W_{\text{net}}}{V_1 - V_2} = \frac{1.297 \text{ Btu}}{(117 - 6.5) \text{ in}^3} \left(\frac{778.17 \text{ lbf·ft}}{1 \text{ Btu}}\right) \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)$$

= 110 psia

Discussion Note that a constant pressure of 110 psia during the power stroke would produce the same net work output as the entire Diesel cycle.

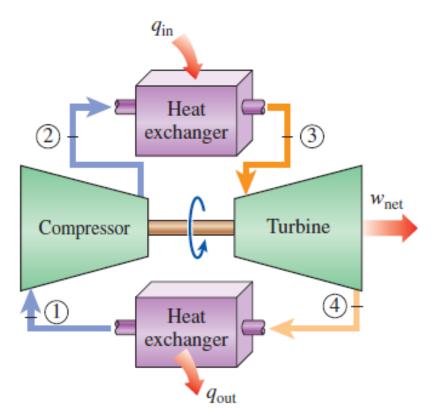
BRAYTON CYCLE: THE IDEAL CYCLE FOR GAS-TURBINE ENGINES



Gas turbines usually operate on an *open cycle*, as shown in Fig. Fresh air at ambient conditions is drawn into the compressor, where its temperature and pressure are raised. The high-pressure air proceeds into the combustion chamber, where the fuel is burned at constant pressure. The resulting high-temperature gases then enter the turbine, where they expand to the atmospheric pressure while producing power. The exhaust gases leaving the turbine are thrown out (not recirculated), causing the cycle to be classified as an open cycle.

The open gas-turbine cycle described above can be modeled as a *closed cycle*, as shown in Fig. , by utilizing the air-standard assumptions. Here the compression and expansion processes remain the same, but the combustion process is replaced by a constant-pressure heat-addition process from an external source, and the exhaust process is replaced by a constant-pressure heat-rejection process to the ambient air. The ideal cycle that the working fluid undergoes in this closed loop is the **Brayton cycle**, which is made up of four internally reversible processes:

- 1-2 Isentropic compression (in a compressor)
- 2-3 Constant-pressure heat addition
- 3-4 Isentropic expansion (in a turbine)
- 4-1 Constant-pressure heat rejection



Ref. Cengel and Boles, 8th Edition (2015)

$$q_{\rm in} = h_3 - h_2 = c_p(T_3 - T_2)$$

and

$$q_{\text{out}} = h_4 - h_1 = c_p(T_4 - T_1)$$

Then the thermal efficiency of the ideal Brayton cycle under the cold-airstandard assumptions becomes

$$\eta_{\text{th,Brayton}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

Processes 1-2 and 3-4 are isentropic, and $P_2 = P_3$ and $P_4 = P_1$. Thus,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

$$q_{\text{in}}$$

$$q_{\text{in}}$$

$$q_{\text{out}}$$

$$\eta_{\text{th,Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

$$r_p = \frac{P_2}{P_1}$$

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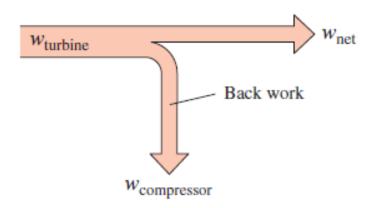
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In gas-turbine power plants, the ratio of the compressor work to the turbine work, called the **back work ratio**, is very high Usually more than one-half of the turbine work output is used to drive the compressor. The situation is even worse when the isentropic efficiencies of the compressor and the turbine are low. This is quite in contrast to steam power plants, where the back work ratio is only a few percent. This is not surprising, however, since a liquid is compressed in steam power plants instead of a gas, and the steady-flow work is proportional to the specific volume of the working fluid.

A power plant with a high back work ratio requires a larger turbine to provide the additional power requirements of the compressor. Therefore, the turbines used in gas-turbine power plants are larger than those used in steam power plants of the same net power output.

EXAMPLE 9-5 The Simple Ideal Brayton Cycle

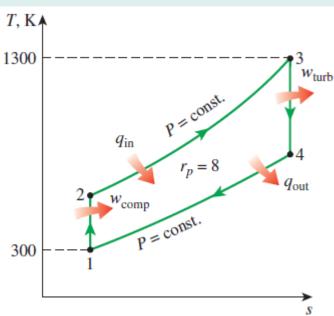
A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Utilizing the air-standard assumptions, determine (a) the gas temperature at the exits of the compressor and the turbine, (b) the back work ratio, and (c) the thermal efficiency.

SOLUTION A power plant operating on the ideal Brayton cycle is considered. The compressor and turbine exit temperatures, back work ratio, and the thermal efficiency are to be determined.

Assumptions 1 Steady operating conditions exist. 2 The air-standard assumptions are applicable. 3 Kinetic and potential energy changes are negligible.

4 The variation of specific heats with temperature is to be considered.

Analysis The *T-s* diagram of the ideal Brayton cycle described is shown in Fig. We note that the components involved in the Brayton cycle are steady-flow devices.



Ref. Cengel and Boles, 8th Edition (2015)

(a) The air temperatures at the compressor and turbine exits are determine from isentropic relations:

Process 1–2 (isentropic compression of an ideal gas):

$$T_1 = 300 \text{ K} \rightarrow h_1 = 300.19 \text{ kJ/kg}$$

 $P_{r1} = 1.386$

$$P_{r2} = \frac{P_2}{P_1} P_{r1} = (8)(1.386) = 11.09 \rightarrow T_2 = 540 \text{ K}$$
 (at compressor exit)
 $h_2 = 544.35 \text{ kJ/kg}$

Process 3-4 (isentropic expansion of an ideal gas):

$$T_3=1300~{
m K}
ightarrow h_3=1395.97~{
m kJ/kg}$$

$$P_{r3}=330.9$$

$$P_{r4}=\frac{P_4}{P_3}P_{r3}=\left(\frac{1}{8}\right)\!(330.9)=41.36
ightarrow T_4=770~{
m K} \qquad ({
m at turbine \ exit})$$

$$h_4=789.37~{
m kJ/kg}$$

(b) To find the back work ratio, we need to find the work input to the compressor and the work output of the turbine:

$$w_{\text{comp,in}} = h_2 - h_1 = 544.35 - 300.19 = 244.16 \text{ kJ/kg}$$

 $w_{\text{turb,out}} = h_3 - h_4 = 1395.97 - 789.37 = 606.60 \text{ kJ/kg}$

Thus,

$$r_{\text{bw}} = \frac{w_{\text{comp,in}}}{w_{\text{turb,out}}} = \frac{244.16 \text{ kJ/kg}}{606.60 \text{ kJ/kg}} = 0.403$$

That is, 40.3 percent of the turbine work output is used just to drive the compressor.

(c) The thermal efficiency of the cycle is the ratio of the net power output to the total heat input:

$$q_{\text{in}} = h_3 - h_2 = 1395.97 - 544.35 = 851.62 \text{ kJ/kg}$$

 $w_{\text{net}} = w_{\text{out}} - w_{\text{in}} = 606.60 - 244.16 = 362.4 \text{ kJ/kg}$

Thus,

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{362.4 \text{ kJ/kg}}{851.62 \text{ kJ/kg}} = 0.426 \text{ or } 42.6\%$$

The thermal efficiency could also be determined from

$$\eta_{\rm th} = 1 - \frac{q_{\rm out}}{q_{\rm in}}$$

where

$$q_{\text{out}} = h_4 - h_1 = 789.37 - 300.19 = 489.2 \text{ kJ/kg}$$

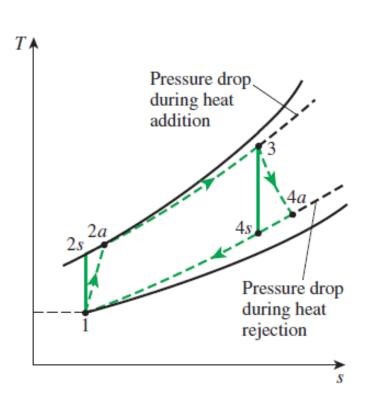
Discussion Under the cold-air-standard assumptions (constant specific heat values at room temperature), the thermal efficiency would be,

$$\eta_{\text{th,Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}} = 1 - \frac{1}{8^{(1.4-1)/1.4}} = 0.448 \text{ or } 44.8\%$$

which is sufficiently close to the value obtained by accounting for the variation of specific heats with temperature.

Deviation of Actual Gas-Turbine Cycles from Idealized Ones

The actual gas-turbine cycle differs from the ideal Brayton cycle on several accounts. For one thing, some pressure drop during the heat-addition and heat-rejection processes is inevitable. More importantly, the actual work input to the compressor is more, and the actual work output from the turbine is less because of irreversibilities. The deviation of actual compressor and turbine behavior from the idealized isentropic behavior can be accurately accounted for by utilizing the isentropic efficiencies of the turbine and compressor as



$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \tag{9-19}$$

$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \tag{9-20}$$

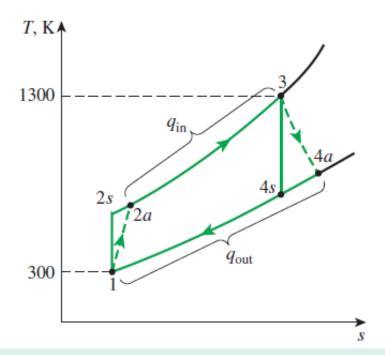
EXAMPLE 9-6 An Actual Gas-Turbine Cycle

Assuming a compressor efficiency of 80 percent and a turbine efficiency of 85 percent, determine (a) the back work ratio, (b) the thermal efficiency, and (c) the turbine exit temperature of the gas-turbine cycle discussed in Example 9–5.

SOLUTION The Brayton cycle discussed in Example 9–5 is reconsidered. For specified turbine and compressor efficiencies, the back work ratio, the thermal efficiency, and the turbine exit temperature are to be determined. *Analysis* (a) The *T-s* diagram of the cycle is shown in Fig. 9–37. The actual compressor work and turbine work are determined by using the definitions of compressor and turbine efficiencies, Eqs. 9–19 and 9–20:

Compressor:
$$w_{\text{comp,in}} = \frac{w_s}{\eta_C} = \frac{244.16 \text{ kJ/kg}}{0.80} = 305.20 \text{ kJ/kg}$$

Turbine:
$$w_{\text{turb,out}} = \eta_T w_s = (0.85)(606.60 \text{ kJ/kg}) = 515.61 \text{ kJ/kg}$$



$$r_{\text{bw}} = \frac{w_{\text{comp,in}}}{w_{\text{turb,out}}} = \frac{305.20 \text{ kJ/kg}}{515.61 \text{ kJ/kg}} = 0.592$$

That is, the compressor is now consuming 59.2 percent of the work produced by the turbine (up from 40.3 percent). This increase is due to the irreversibilities that occur within the compressor and the turbine.

(b) In this case, air leaves the compressor at a higher temperature and enthalpy, which are determined to be

$$w_{\text{comp,in}} = h_{2a} - h_1 \rightarrow h_{2a} = h_1 + w_{\text{comp,in}}$$

= 300.19 + 305.20
= 605.39 kJ/kg (and T_{2a} = 598 K)

$$q_{\text{in}} = h_3 - h_{2a} = 1395.97 - 605.39 = 790.58 \text{ kJ/kg}$$

 $w_{\text{net}} = w_{\text{out}} - w_{\text{in}} = 515.61 - 305.20 = 210.41 \text{ kJ/kg}$

and

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{210.41 \text{ kJ/kg}}{790.58 \text{ kJ/kg}} = 0.266 \text{ or } 26.6\%$$

That is, the irreversibilities occurring within the turbine and compressor caused the thermal efficiency of the gas turbine cycle to drop from 42.6 to 26.6 percent. This example shows how sensitive the performance of a gasturbine power plant is to the efficiencies of the compressor and the turbine. (c) The air temperature at the turbine exit is determined from an energy balance on the turbine:

$$w_{\text{turb,out}} = h_3 - h_{4a} \rightarrow h_{4a} = h_3 - w_{\text{turb,out}}$$

= 1395.97 - 515.61
= 880.36 kJ/kg

Then, from Table A-17,

$$T_{4a} = 853 \text{ K}$$

Discussion The temperature at turbine exit is considerably higher than that at the compressor exit ($T_{2a} = 598$ K), which suggests the use of regeneration to reduce fuel cost.

Ref. Cengel and Boles, 8th Edition (2015)

SECOND-LAW ANALYSIS OF GAS POWER CYCLES

$$\begin{split} X_{\rm dest} &= T_0 S_{\rm gen} = T_0 (\Delta S_{\rm sys} - S_{\rm in} + S_{\rm out}) \\ &= T_0 \bigg[(S_2 - S_1)_{\rm sys} - \frac{Q_{\rm in}}{T_{b,\rm in}} + \frac{Q_{\rm out}}{T_{b,\rm out}} \bigg] \end{split}$$

For a cycle that involves heat transfer only with a source at T_H and a sink at T_L , the exergy destruction becomes

$$x_{\text{dest}} = T_0 \left(\frac{q_{\text{out}}}{T_L} - \frac{q_{\text{in}}}{T_H} \right)$$

The exergies of a closed system ϕ and a fluid stream ψ at any state can be determined from

$$\phi = (u - u_0) - T_0(s - s_0) + P_0(v - v_0) + \frac{V^2}{2} + gz \quad \text{(kJ/kg)}$$

and

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$
 (kJ/kg)

where subscript "0" denotes the state of the surroundings.

Ref. Cengel and Boles, 8th Edition (2015)