

ESO201A : THERMODYNAMICS

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IIT Kanpur

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Lecture 22

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Example :

A 500-kg iron block shown in Fig. 8–11 is initially at 200°C and is allowed to cool to 27°C by transferring heat to the surrounding air at 27°C . Determine the reversible work and the irreversibility for this process.

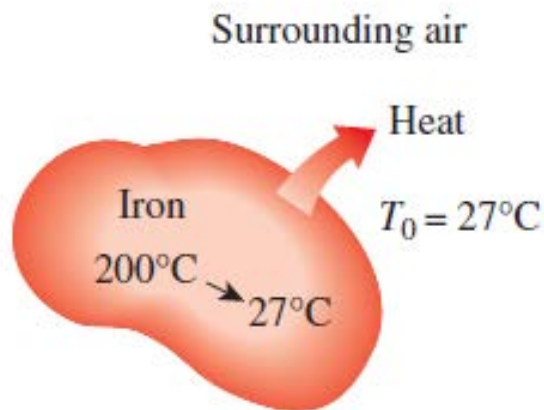


FIGURE 8–11

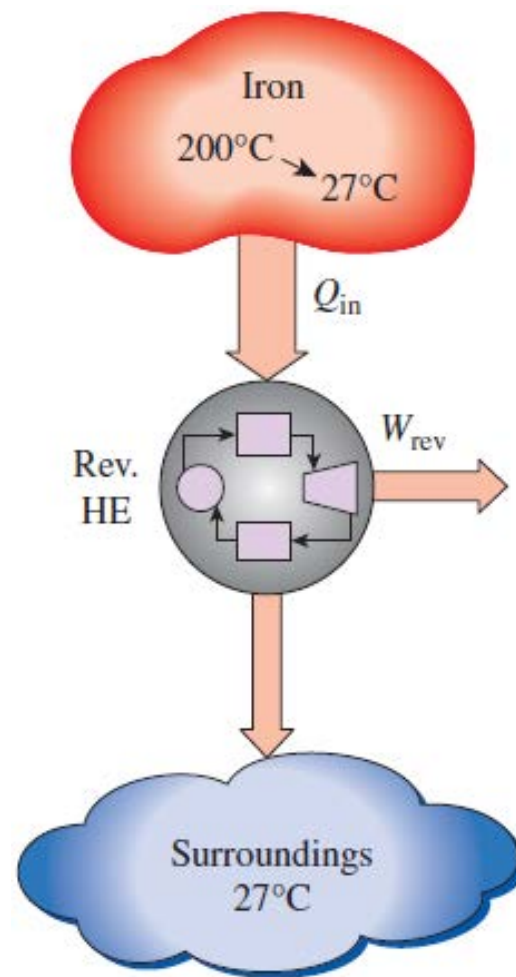


FIGURE 8–12

Analysis We take the *iron block* as the system. This is a *closed system* since no mass crosses the system boundary. We note that heat is lost from the system.

It probably came as a surprise to you that we are asking to find the “reversible work” for a process that does not involve any work interactions. Well, even if no attempt is made to produce work during this process, the potential to do work still exists, and the reversible work is a quantitative measure of this potential.

The reversible work in this case is determined by considering a series of imaginary reversible heat engines operating between the source (at a variable temperature T) and the sink (at a constant temperature T_0), as shown in Fig. 8–12. Summing their work output:

$$\delta W_{\text{rev}} = \eta_{\text{th,rev}} \delta Q_{\text{in}} = \left(1 - \frac{T_{\text{sink}}}{T_{\text{source}}}\right) \delta Q_{\text{in}} = \left(1 - \frac{T_0}{T}\right) \delta Q_{\text{in}}$$

and

$$W_{\text{rev}} = \int \left(1 - \frac{T_0}{T}\right) \delta Q_{\text{in}}$$

Example :

The source temperature T changes from $T_1 = 200^\circ\text{C} = 473 \text{ K}$ to $T_0 = 27^\circ\text{C} = 300 \text{ K}$ during this process. A relation for the differential heat transfer from the iron block can be obtained from the differential form of the energy balance applied on the iron block,

$$-\delta Q_{\text{out}} = dU = mc_{\text{avg}}dT$$

Then,

$$\delta Q_{\text{in,heat engine}} = \delta Q_{\text{out,system}} = -mc_{\text{avg}}dT$$

since heat transfers from the iron and to the heat engine are equal in magnitude and opposite in direction. Substituting and performing the integration, the reversible work is determined to be

$$\begin{aligned} W_{\text{rev}} &= \int_{T_1}^{T_0} \left(1 - \frac{T_0}{T} \right) (-mc_{\text{avg}} dT) = mc_{\text{avg}}(T_1 - T_0) - mc_{\text{avg}}T_0 \ln \frac{T_1}{T_0} \\ &= (500 \text{ kg})(0.45 \text{ kJ/kg}\cdot\text{K}) \left[(473 - 300) \text{ K} - (300 \text{ K}) \ln \frac{473 \text{ K}}{300 \text{ K}} \right] \\ &= \mathbf{8191 \text{ kJ}} \end{aligned}$$

Example :

where the specific heat value is obtained from Table A-3. The first term in the above equation [$Q = mc_{\text{avg}}(T_1 - T_0) = 38,925 \text{ kJ}$] is the total heat transfer from the iron block to the heat engine. The reversible work for this problem is found to be 8191 kJ, which means that 8191 (21 percent) of the 38,925 kJ of heat transferred from the iron block to the ambient air *could* have been converted to work. If the specified ambient temperature of 27°C is the lowest available environment temperature, the reversible work determined above also represents the exergy, which is the maximum work potential of the sensible energy contained in the iron block.

The irreversibility for this process is determined from its definition,

$$I = W_{\text{rev}} - W_u = 8191 - 0 = \mathbf{8191 \text{ kJ}}$$

Discussion Notice that the reversible work and irreversibility (the wasted work potential) are the same for this case since the entire work potential is wasted. The source of irreversibility in this process is the heat transfer through a finite temperature difference.

Exergy of a control mass (with no changes of composition) :

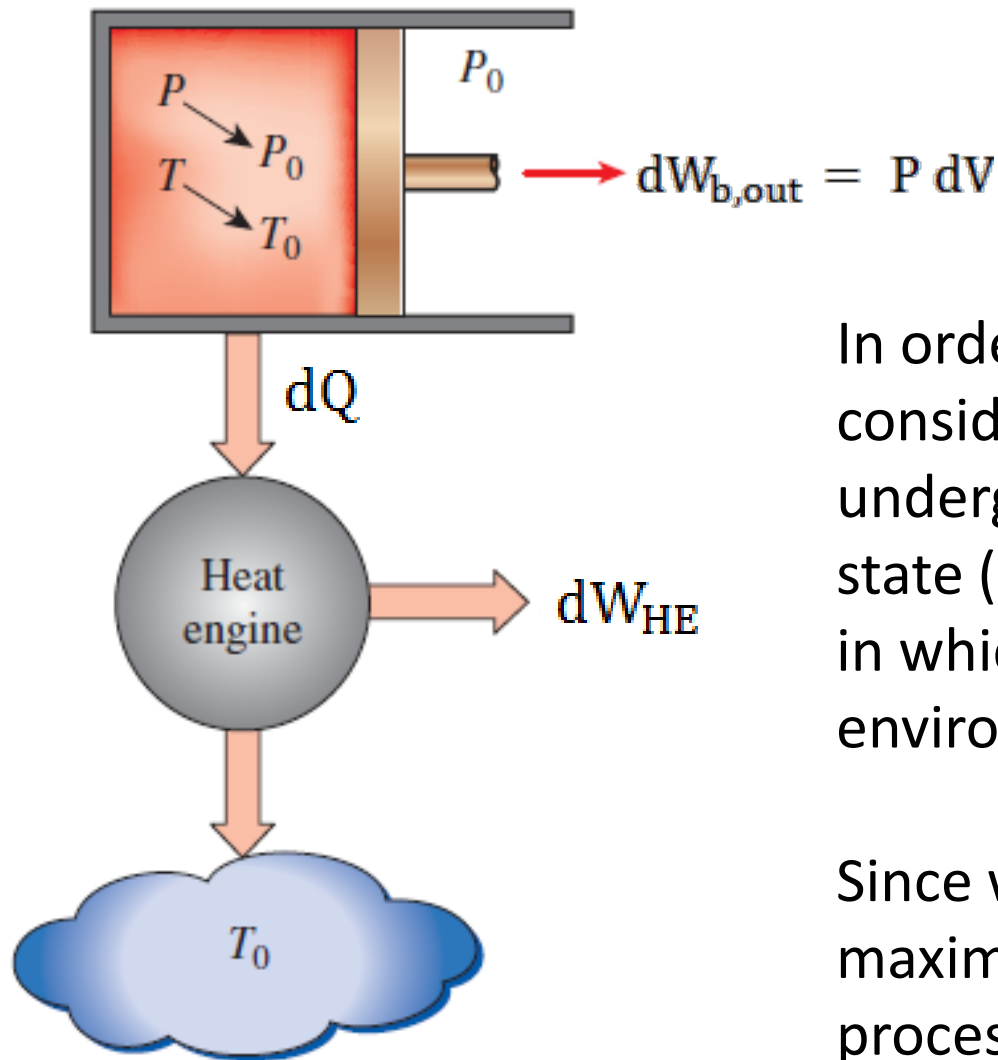
Consider a control mass system at a temperature T and pressure P . The temperature and pressure of the environment are T_o and P_o , respectively.

Question is how much maximum useful work we can extract from the system ? In other words, what is the exergy of the control mass ?

We will consider that no electric or magnetic work is involved. Also, no stress-strain work is involved. Also, consider that control mass does not undergo a change in the composition.

Further, we will consider that the system is at rest and hence does not undergo a change in kinetic energy or potential energy.

Exergy of a control mass (with no changes of composition) :



In order to calculate exergy, we consider a process in which system undergoes a change from the initial state (P, T) to the final state (P_0, T_0) in which it is in equilibrium with the environment.

Since we can are interested in maximum work that we can get in this process, we will consider that process is reversible.

Exergy of a control mass (with no changes of composition) :

Applying first law to the control mass, we get

$$dU = T dS - P dV \quad (A)$$

Since we are considering a reversible heat engine, work done by heat engine is given by

$$dW_{HE} = \left(1 - \frac{T_o}{T}\right) dQ \quad (B)$$

Since the process is reversible, we can apply Clausius equation to the change in entropy of the system :

$$dQ = -T dS \quad (C)$$

Substituting (C) in (B), we get

$$dW_{HE} = -T dS + T_o dS \quad (D)$$

Exergy of a control mass (with no changes of composition) :

Re-arranging first law equation, i.e., Equation (A), we get

$$T dS = dU + P dV \quad (E)$$

Substituting (E) in (D), we get $dW_{HE} = -dU - P dV + T_o dS \quad (F)$

Note that work done by the system on the environment ($P_o dV$) is NOT considered as useful work. Thus the total useful work done by the system is given by

$$dW_{\text{useful}} = dW_{HE} + PdV - P_o dV \quad (G)$$

Substituting (F) in (G), we get

$$dW_{\text{useful}} = -dU - P_o dV + T_o dS \quad (H)$$

Exergy of a control mass (with no changes of composition) :

Therefore, the useful work done in a reversible process from initial state to the final (dead) state is given by [using equation (H) of previous slide] :

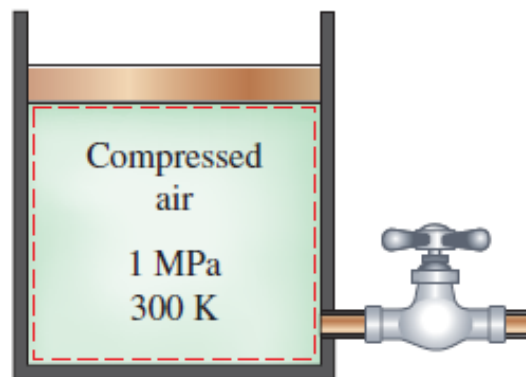
$$\begin{aligned} X &= \int_{\text{initial}}^{\text{final}} dW_{\text{useful}} \\ &= \int_{\text{initial}}^{\text{final}} (-dU - P_o dV + T_o dS) \\ &= -(U_o - U) - P_o(V_o - V) + T_o(S_o - S) \\ &= (U - U_o) + P_o(V - V_o) - T_o(S - S_o) \end{aligned}$$

Thus, the exergy or the work potential of the system in the initial state is

$$X = (U - U_o) + P_o(V - V_o) - T_o(S - S_o)$$

Example :

A 200-m³ rigid tank contains compressed air at 1 MPa and 300 K. Determine how much work can be obtained from this air if the environment conditions are 100 kPa and 300 K.



Analysis We take the air in the rigid tank as the system. This is a *closed system* since no mass crosses the system boundary during the process. Here the question is the work potential of a fixed mass, which is the nonflow exergy by definition.

Taking the state of the air in the tank to be state 1 and noting that $T_1 = T_0 = 300$ K, the mass of air in the tank is

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(1000 \text{ kPa})(200 \text{ m}^3)}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300 \text{ K})} = 2323 \text{ kg}$$

Example :

The exergy content of the compressed air can be determined from

$$\begin{aligned} X_1 &= m\phi_1 \\ &= m \left[(u_1 - u_0)^{\nearrow 0} + P_0(v_1 - v_0) - T_0(s_1 - s_0) + \frac{V_1^2}{2}^{\nearrow 0} + gz_1^{\nearrow 0} \right] \\ &= m[P_0(v_1 - v_0) - T_0(s_1 - s_0)] \end{aligned}$$

We note that

$$P_0(v_1 - v_0) = P_0 \left(\frac{RT_1}{P_1} - \frac{RT_0}{P_0} \right) = RT_0 \left(\frac{P_0}{P_1} - 1 \right) \quad (\text{since } T_1 = T_0)$$

$$T_0(s_1 - s_0) = T_0 \left(c_p \ln \frac{T_1}{T_0} - R \ln \frac{P_1}{P_0} \right) = -RT_0 \ln \frac{P_1}{P_0} \quad (\text{since } T_1 = T_0)$$

Example :

Therefore,

$$\begin{aligned}\phi_1 &= RT_0 \left(\frac{P_0}{P_1} - 1 \right) + RT_0 \ln \frac{P_1}{P_0} = RT_0 \left(\ln \frac{P_1}{P_0} + \frac{P_0}{P_1} - 1 \right) \\ &= (0.287 \text{ kJ/kg}\cdot\text{K})(300 \text{ K}) \left(\ln \frac{1000 \text{ kPa}}{100 \text{ kPa}} + \frac{100 \text{ kPa}}{1000 \text{ kPa}} - 1 \right) \\ &= 120.76 \text{ kJ/kg}\end{aligned}$$

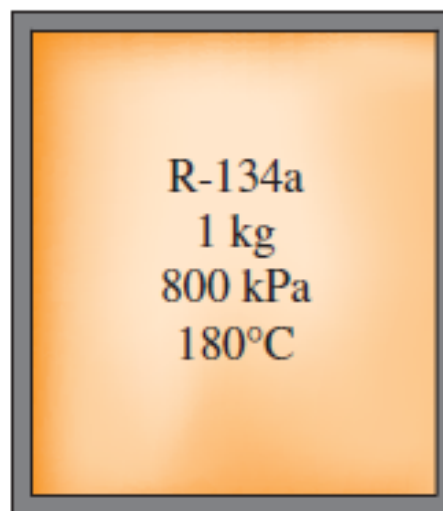
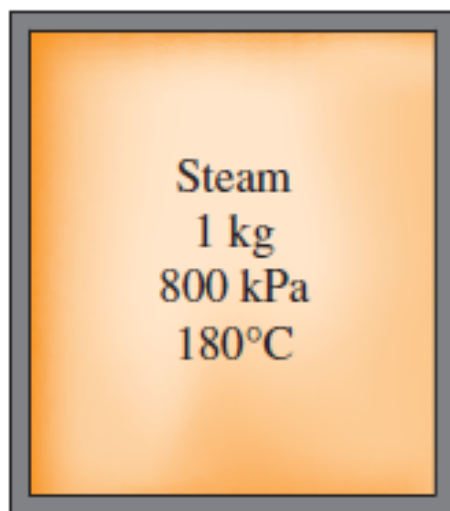
and

$$X_1 = m_1 \phi_1 = (2323 \text{ kg})(120.76 \text{ kJ/kg}) = 280,525 \text{ kJ} \cong \mathbf{281 \text{ MJ}}$$

Discussion The work potential of the system is 281 MJ, and thus a maximum of 281 MJ of useful work can be obtained from the compressed air stored in the tank in the specified environment.

Example :

8–28 Which has the capability to produce the most work in a closed system – 1 kg of steam at 800 kPa and 180°C or 1 kg of R–134a at 800 kPa and 180°C? Take $T_0 = 25^\circ\text{C}$ and $P_0 = 100 \text{ kPa}$. *Answers: 623 kJ, 5.0 kJ*



Example :

Analysis The properties of water at the given state and at the dead state are

$$\left. \begin{array}{l} P = 800 \text{ kPa} \\ T = 180^\circ\text{C} \end{array} \right\} \begin{array}{l} u = 2594.7 \text{ kJ/kg} \\ \nu = 0.24720 \text{ m}^3/\text{kg} \\ s = 6.7155 \text{ kJ/kg}\cdot\text{K} \end{array} \quad (\text{Table A - 6})$$

$$\left. \begin{array}{l} T_0 = 25^\circ\text{C} \\ P_0 = 100 \text{ kPa} \end{array} \right\} \begin{array}{l} u_0 \cong u_f @ 25^\circ\text{C} = 104.83 \text{ kJ/kg} \\ \nu_0 \cong \nu_f @ 25^\circ\text{C} = 0.001003 \text{ m}^3/\text{kg} \\ s_0 \cong s_f @ 25^\circ\text{C} = 0.3672 \text{ kJ/kg}\cdot\text{K} \end{array} \quad (\text{Table A - 4})$$

Steam 1 kg 800 kPa 180°C

The exergy of steam is

$$\begin{aligned} \Phi &= m[u - u_0 + P_0(\nu - \nu_0) - T_0(s - s_0)] \\ &= (1 \text{ kg}) \left[(2594.7 - 104.83) \text{ kJ/kg} + (100 \text{ kPa})(0.24720 - 0.001003) \text{ m}^3/\text{kg} \left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3} \right) \right. \\ &\quad \left. - (298 \text{ K})(6.7155 - 0.3672) \text{ kJ/kg}\cdot\text{K} \right] \\ &= \mathbf{622.7 \text{ kJ}} \end{aligned}$$

Example :

For R-134a;

$$\left. \begin{array}{l} P = 800 \text{ kPa} \\ T = 180^\circ\text{C} \end{array} \right\} \begin{array}{l} u = 386.99 \text{ kJ/kg} \\ \nu = 0.044554 \text{ m}^3/\text{kg} \\ s = 1.3327 \text{ kJ/kg}\cdot\text{K} \end{array} \quad (\text{Table A - 13})$$

$$\left. \begin{array}{l} T_0 = 25^\circ\text{C} \\ P_0 = 100 \text{ kPa} \end{array} \right\} \begin{array}{l} u_0 \cong u_f @ 25^\circ\text{C} = 85.85 \text{ kJ/kg} \\ \nu_0 \cong \nu_f @ 25^\circ\text{C} = 0.0008286 \text{ m}^3/\text{kg} \\ s_0 \cong s_f @ 25^\circ\text{C} = 0.32432 \text{ kJ/kg}\cdot\text{K} \end{array} \quad (\text{Table A - 11})$$

R-134a
1 kg
800 kPa
180°C

$$\begin{aligned} \Phi &= m[u - u_0 + P_0(\nu - \nu_0) - T_0(s - s_0)] \\ &= (1 \text{ kg}) \left[(386.99 - 85.85) \text{ kJ/kg} + (100 \text{ kPa})(0.044554 - 0.0008286) \text{ m}^3/\text{kg} \left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3} \right) \right. \\ &\quad \left. - (298 \text{ K})(1.3327 - 0.32432) \text{ kJ/kg}\cdot\text{K} \right] \\ &= \mathbf{5.02 \text{ kJ}} \end{aligned}$$

The steam can therefore has more work potential than the R-134a.