

ESO 208A: Computational Methods in Engineering  
**Tutorial 8**

Least squares

1. Consider the equation  $y = ae^{bx}$  and the measured data of  $x$  and  $y$  given in the table below. Determine the constants  $a$  and  $b$  by using the method of least squares.

$x$	2	4	6	8	10
$y$	4.077	11.084	30.128	81.897	222.62

Orthogonal basis functions

2. (a) Estimate a quadratic polynomial approximation of the following function by using Legendre polynomials as the basis functions.

$$f(x) = \frac{1}{1+x^2}; \quad x \in (-1, 1)$$

Legendre Polynomials

$$P_0(x) = 1, P_1(x) = x, P_{n+1}(x) = \frac{2n+1}{n+1}xP_n(x) - \frac{n}{n+1}P_{n-1}(x); \quad \langle P_n, P_j \rangle = \begin{cases} 0 & \text{if } n \neq j \\ \frac{2}{2n+1} & \text{if } n = j \end{cases}$$

- (b) Perform the second order Taylor series approximation of  $f(x)$  at  $x = 0$ .
- (c) Estimate the true relative error in estimating  $f(x)$  by the above two approximations at  $x = -0.9, -0.5, 0$ , and  $0.5$ .