ESO 208A: Computational Methods in Engineering

Tutorial 12

ODE: Initial value problem

1. Consider the following differential equation with initial condition $y(0)=1$.

$$\frac{dy}{dx} = yx^3 - 1.5y$$

Determine the solution over the interval $x=0$ to 1 by the following methods. Estimate the true error in the numerical results by comparing them against the analytical result.

a. Euler's method with $h=0.5$

b. Heun's method with no iterations and $h=0.5$

c. Midpoint method with $h=0.5$

d. Classical 4th order RK method with $h=0.5$

ODE: Boundary Value Problem

Consider the following differential equation

$$\frac{d^2y}{dx^2} - y = 0;\ x \in (0,2)$$

with boundary conditions $y(0) = 0$ and $y(2)=3.627$.

Determine the solution over the interval $x=0$ to 2 by the following methods. Estimate the true error in the numerical result by comparing it against the analytical result at $x=1.0$.

a. Shooting method with the Midpoint method and step size $h=0.5$. Assume first two approximations of $y'(0)$ as 1.0 and 2.0.

b. Finite difference method by approximating $\frac{d^2y}{dx^2}$ using a central difference formula of $O(h^2)$. Take the step size $h=1.0$.

c. Repeat (b) by taking the step size $h=0.5$.

d. Use Richardson extrapolation to improve the estimate of $y(1)$ obtained in (b) and (c).