ESO 208A: Computational Methods in Engineering
Tutorial 6

Matrix Decomposition

1. Consider the following set of linear equations:

$$d_1x + by + cz = f_1$$
$$bx + d_2 y + az = f_2$$
$$cx + ay + d_3z = f_3$$

where, the mean ± standard deviation of the values of the constants are:

$d_1 = 4.34 \pm 0.05$, $d_2 = 7.8 \pm 0.10$, $d_3 = 4.2 \pm 0.07$,
$b = 2.1 \pm 0.02$, $a = 1.8 \pm 0.01$, $c = -2.4 \pm 0.11$,
$f_1 = 87.65 \pm 0.56$, $f_2 = 121.76 \pm 1.80$ $f_3 = -2.0 \pm 0.03$.

a) Obtain a *LU* decomposition of the coefficient matrix using *Cholesky's* method by considering the mean values of the constants.

b) Obtain the solution vector using the *LU* decomposition of (a) and the mean values for the constantsin the right hand side vector.

c) Compute the inverse of the coefficient matrix using the *LU* decomposition in (a).

d) Derive an analytical expression for the maximum norm of relative error in the solution vector for small perturbations in both, coefficient matrix and the right hand side vectors.

e) Using the results of (c) and (d), obtain the maximum norm of the relative error in the solution vector for one standard deviation perturbations in all the constants (in both coefficient matrix and right handside vector) of the set of equations. Use *column sum norm* for the matrices and L_∞ -*norm* for the vectors.

2. Solve the following system of equations using Thomas algorithm:

$$\begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -2 \end{bmatrix}$$

Indirect methods for solving system of linear equations

3. Solve the following system of linear equations using Jacobi and Gauss-Seidel methods. Use initial guess as zero for all the variables and compare the number of iterations required to achieve approximate relative error less than 0.1%.

$$6x - 2y + z = 11$$
$$-2x + 7y + 2z = 5$$
$$x + 2y - 5z = -1$$