

# ESO201A : THERMODYNAMICS

## 2021-22 Ist semester

### IIT Kanpur

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## **Lecture 18**

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## Isoentropic processes :

In a quasi-static and adiabatic process final entropy is the same as initial entropy. A process in which final entropy is same as initial entropy is typically referred to as an “isoentropic process”.

Lets consider isoentropic process of an ideal gas. Assuming specific heats  $C_v$  and  $C_p$  to be constant, we get

$$s_2 - s_1 = C_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{V_2}{V_1} \right) = 0$$

Re-arranging the above equation,  $\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{V_2}{V_1} \right)^{-\left( \frac{R}{C_v} \right)}$

We denote ratio of specific heats as ‘k’ (recall lecture 7) :

$$k = \frac{C_p}{C_v}$$

## Isoentropic processes for ideal gas :

Recall that specific heats of ideal gas are related by the expression :

$$C_P - C_V = R$$

Substituting  $C_P = k C_V$  in the above equation, we get

$$(k - 1)C_V = R \quad (\text{OR}) \quad (k - 1) = \frac{R}{C_V}$$

Substituting above in the equation for the ratio of temperatures (see last slide), we get

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{V_1}{V_2}\right)^{(k-1)}$$

Substituting ideal gas equation of state in the above equation and rearranging, we get

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{\frac{(k-1)}{k}}$$

## Isoentropic processes for ideal gas :

If we do not consider specific heats to be constant, then we can use the following relation for isentropic process of an ideal gas :

$$s_2 - s_1 = [s^o(T_2) - s^o(T_1)] - R \ln \left( \frac{P_2}{P_1} \right) = 0$$

Re-arranging the above equation,

$$\left( \frac{P_2}{P_1} \right)_{s=\text{const.}} = \exp \left( \frac{s_2^o - s_1^o}{R} \right)$$

where,  $s_1^o = s^o(T_1)$  and  $s_2^o = s^o(T_2)$

In the ideal gas tables, the values of 'relative pressure' denoted as  $P_r$  are listed. The relative pressure is defined as :

$$P_r(T) = \exp \left( \frac{s^o(T)}{R} \right)$$

## Isoentropic processes for ideal gas :

Table A-17 to A-24 (ideal gas properties) list the values of  $P_r(T)$  at different T.

**TABLE A-17**

Ideal-gas properties of air

$T$ K	$h$ kJ/kg	$P_r$	$u$ kJ/kg	$v_r$	$s^\circ$ kJ/kg·K
200	199.97	0.3363	142.56	1707.0	1.29559
210	209.97	0.3987	149.69	1512.0	1.34444
220	219.97	0.4690	156.82	1346.0	1.39105
230	230.02	0.5477	164.00	1205.0	1.43557
240	240.02	0.6355	171.13	1084.0	1.47824

relative  
pressure ( $P_r$ )

relative  
volume ( $v_r$ )

In the ideal gas tables, the values of 'relative volume' (denoted as  $v_r$ ) is defined as :

$$v_r(T) = \frac{T}{P_r(T)}$$

## Isoentropic processes for ideal gas :

Using relative pressures listed in Table A-17 to A-24 (ideal gas properties) the pressure ratio in an isentropic process can be obtained as,

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

From the ideal gas equation of state, we get

$$\left(\frac{V_2}{V_1}\right)_{s=\text{const.}} = \left(\frac{T_2 P_1}{T_1 P_2}\right)_{s=\text{const.}} = \left(\frac{T_2 P_{r1}}{T_1 P_{r2}}\right)_{s=\text{const.}} = \left[\frac{(T_2/P_{r2})}{(T_1/P_{r1})}\right]_{s=\text{const.}}$$

Thus, we get the following simplified form :

$$\left(\frac{V_2}{V_1}\right)_{s=\text{const.}} = \frac{V_{r2}}{V_{r1}}$$

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio  $V_1/V_2$  of this engine is 8, determine the final temperature of the air.

Using variable specific heats (rigorous method)

Ideal-gas properties of air

$T$ K	$h$ kJ/kg	$P_r$	$u$ kJ/kg	$v_r$	$s^\circ$ kJ/kg·K
295	295.17	1.3068	210.49	647.9	1.68515
660	670.47	23.13	481.01	81.89	2.50985
670	681.14	24.46	488.81	78.61	2.52589

For closed systems:

$$\frac{V_2}{V_1} = \frac{v_2}{v_1}$$

At  $T_1 = 295$  K:

$$v_{r1} = 647.9$$

$$v_{r2} = v_{r1} \left( \frac{v_2}{v_1} \right) = (647.9) \left( \frac{1}{8} \right) = 80.99 \rightarrow T_2 = \mathbf{662.7 \text{ K}}$$

Using constant specific heats  
(approximate method)

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

We know that the temperature of the air will rise considerably during this adiabatic compression process, so we *guess* the average temperature to be about 450 K. The  $k$  value at this anticipated average temperature is determined from Table A-2b to be 1.391. Then, the final temperature of air becomes

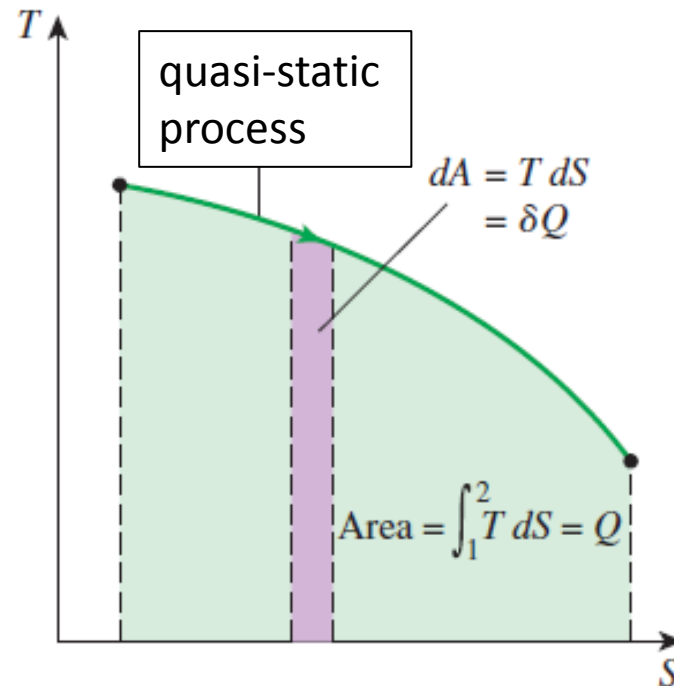
$$T_2 = (295 \text{ K})(8)^{1.391-1} = 665.2 \text{ K}$$

This gives an average temperature value of 480.1 K, which is sufficiently close to the assumed value of 450 K. Therefore, it is not necessary to repeat the calculations by using the  $k$  value at this average temperature.



## T-S diagram:

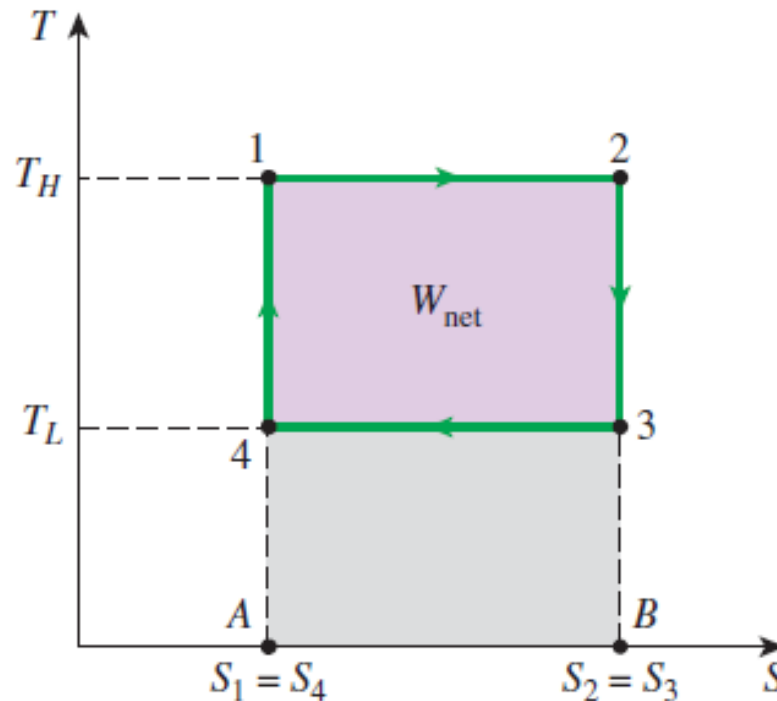
As a visual aid in understanding the process, it is sometimes helpful to plot  $T$  as a function of  $S$  (entropy). Such a diagram is shown in the figure. If the process is quasi-static (in which system is in internal equilibrium), then area under the curve is equal to the heat absorbed by the system (recall Clausius equality).



# T-S diagram for a Carnot cycle :

Show the Carnot cycle on a  $T$ - $S$  diagram and indicate the areas that represent the heat supplied  $Q_H$ , heat rejected  $Q_L$ , and the net work output  $W_{\text{net,out}}$  on this diagram.

T-S diagram for Carnot cycle is shown below. The area under the curve is equal to heat absorbed if  $S$  increase during the process. The area under the curve is equal to heat rejected if  $S$  decrease during the process.



$$W_{\text{net,out}} = Q_H - Q_L$$

## Entropy change for adiabatic process :

Note that system together with surroundings (with which energy exchange in the form of heat transfer occurs) undergo an adiabatic process and hence the Second law takes the form (recall fifth statement of Second law for adiabatic process) :

$$(dS_{\text{system}} + dS_{\text{surr.}}) \geq 0$$

Assuming that surrounding undergo quasi-static processes such that surroundings are in internal equilibrium throughout the process and temperature is uniform throughout the process, we can Apply Clausius equation to surroundings. If  $dQ_{\text{in}}$  is the amount of heat transferred to the system from the surroundings, then

$$dS_{\text{surr.}} = - \left( \frac{dQ_{\text{in}}}{T} \right)$$

Temperature of the surrounding (considered to be at a uniform temperature) .

This is NOT necessarily equal to the system temperature.

## Entropy change for adiabatic process :

Substituting entropy change of surroundings in the expression for total entropy change (see last slide), we get

$$dS_{\text{system}} - \left( \frac{dQ_{\text{in}}}{T} \right) \geq 0$$

$$\text{(OR)} \quad dS_{\text{system}} \geq \left( \frac{dQ_{\text{in}}}{T} \right)$$

Here the equality applies to a reversible process. For the entire adiabatically isolated system which consists of system + surroundings (see last slide) the total entropy change is positive (irreversible process) or zero (reversible process). The total entropy change is also known as entropy generated which is positive or zero. Thus,

$$dS_{\text{gen}} = (dS_{\text{system}} + dS_{\text{surr.}}) \geq 0$$

## Entropy change for adiabatic process :

In case the surrounding consists of multiple thermal energy reservoirs, then

$$dS_{\text{surr.}} = - \sum_k \left( \frac{dQ_k}{T_k} \right)$$

Here  $dQ_k$  is the heat transferred to the system from the thermal energy reservoir at a uniform temperature  $T_k$ . It is assumed that process is sufficiently slow so that all of the thermal energy reservoirs are in a state of internal equilibrium throughout the process.

Substituting above in the last equation of previous slide, we get

$$dS_{\text{gen}} = \left[ dS_{\text{system}} - \sum_k \left( \frac{dQ_k}{T_k} \right) \right] \geq 0$$

## Entropy changes in rate processes :

If we are considering changes with respect to time, we can differentiate the last equation of the previous slide to obtain

$$\dot{S}_{\text{gen}} = \left[ \dot{S}_{\text{system}} - \sum_k \left( \frac{\dot{Q}_k}{T_k} \right) \right] \geq 0$$

In the above equation,

$\dot{S}_{\text{system}} = \frac{d}{dt} (S_{\text{system}})$  = Rate of increase of entropy of the (control mass) system

$\dot{Q}_k$  = Rate at which heat is transferred to the system from the thermal energy reservoir at a uniform temperature  $T_k$

$\dot{S}_{\text{gen}}$  = Rate at which entropy of the (system+surroundings) increases. Note that surroundings include all of the thermal energy reservoirs with which the system exchanges heat.

## Entropy changes in rate processes :

Rearranging the equation on the last slide, we get

$$\dot{S}_{\text{system}} = \sum_k \left( \frac{\dot{Q}_k}{T_k} \right) + \dot{S}_{\text{gen}}$$

The entropy of the system at a given time can be expressed as:

$$S_{\text{system}}(t) = \int_{\text{CM}} s \rho \, dV$$

In the above equation,  $s$  and  $\rho$  are the specific entropy and mass density at a given point within the control mass (CM).

## Rates of change of entropy of the control mass and control volume:

The entropy of a control volume (this could be an equipment through which fluid flows) is defined as the entropy of the mass of the fluid within the CV at any given instant of time  $t$ .

$$S_{cv}(t) = \int_{cv} s \rho \, dV$$

According to Reynolds transport theorem (see Lecture 8), the rates of change entropy of the control mass and the control volume satisfy the following equation:

$$\frac{d}{dt}(S_{system}) = \frac{d}{dt}(S_{cv}) + \int_{cs} s \rho (\vec{V} \cdot \vec{n}) \, dA$$

Here the integral on the right hand side is over the control surface (CS) that bounds the control volume (see also Lecture 8)



## Rates of change of entropy of the control volume:

Substituting the right hand side of the last equation (see previous slide) in the expression of the rate of change of entropy of the (control mass) system, we get

$$\frac{d}{dt}(S_{cv}) + \int_{cs} s \rho (\vec{V} \cdot \vec{n}) dA = \sum_k \left( \frac{\dot{Q}_k}{T_k} \right) + \dot{S}_{gen}$$

The control volume is so chosen that the direction of the flow at the inlet or outlet is perpendicular to the control surface. If we assume the properties  $s$  and  $\rho$  to be constant over the area of an inlet or outlet, the product  $(s \rho)$  in the integral on the left hand side can be taken outside of the integral sign. In that case, we get the following simplified form.

$$\frac{d}{dt}(S_{cv}) + \sum_{out} \dot{m} s - \sum_{in} \dot{m} s = \sum_k \left( \frac{\dot{Q}_k}{T_k} \right) + \dot{S}_{gen}$$

## Rates of change of entropy of the control volume:

Rearranging the last equation in the previous slide, we get

$$\frac{d}{dt}(S_{cv}) = \sum_{in} \dot{m} s - \sum_{out} \dot{m} s + \sum_k \left( \frac{\dot{Q}_k}{T_k} \right) + \dot{S}_{gen}$$

Here the first and second sums on the right hand side are over inlets and outlets respectively. The third sum on the right hand side is over all the thermal energy reservoirs with which there is a exchange of heat of the fluid in the control volume. The last term is the rate of change of entropy of (fluid in the control volume + surroundings). This is also called rate of generation of entropy. It is positive for irreversible processes and zero for a reversible process.

For a steady-flow equipment,

$$\frac{d}{dt}(S_{cv}) = 0$$