ESO201A
Tutorial 10: Problems and Solutions

Relative Molar Volume : $\mathbf{V_r} = \frac{\tilde{\mathbf{V}}}{\tilde{\mathbf{V}}_{ref}}$

Relative Pressure : $\mathbf{P_r} = \frac{\mathbf{P}}{\mathbf{P}_{ref}}$

When we apply these two equations to an ideal gas that undergoes an isentropic process:

$$\boxed{\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{V_r(T_1)}{V_r(T_2)}}$$

$$\boxed{\frac{P_1}{P_2} = \frac{P_r(T_1)}{P_r(T_2)}}$$

The relative volume and relative pressure depend only on temperature.

Examples of how to use $\mathbf{V_r}$ and $\mathbf{P_r}$:

Given:	$\mathbf{T_1}$, $\mathbf{P_1}$ and $\mathbf{P_2}$	We can determine $\mathbf{T_2}$ using the equation on the right by interpolating on $\mathbf{P_r(T)}$.
Given:	$\mathbf{T_1}$, $\tilde{\mathbf{V}}_1$ and $\mathbf{T_2}$	We can determine $\tilde{\mathbf{V}}_2$ directly by looking up $\mathbf{V_r(T_1)}$ and $\mathbf{V_r(T_2)}$ and plugging them into the equation on the left.

9-33. An ideal Otto cycle has a compression ratio of 8. At the beginning the compression ,air is at 95kpa and 27ºC and 750 kJ/kg of heat is transferred to air during the constant volume heat addition process. Taking into account the variation of specific heats with temperature. determine (a)the pressure and the temperature at the end of the heat addition process . (b)the net work output. (c) the thermal efficiency and (d)the mean effective pressure for the cycle

Answer:

An ideal Otto cycle with air as the working fluid has a compression ratio of 8. The pressure and temperature at the end of the heat addition process, the net work output, the thermal efficiency, and the mean effective pressure for the cycle are to be determined.

Assumptions:

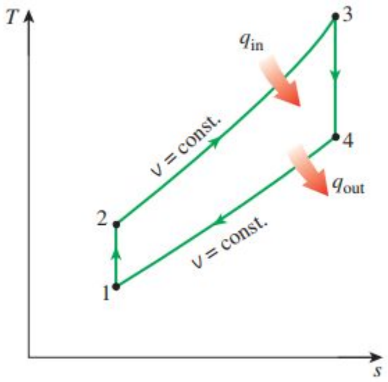
- The air-standard assumptions are applicable.
- Kinetic and potential energy changes are negligible.
- **Air is an ideal gas with variable specific heats**.

Properties :

The gas constant of air is R = 0.287 kJ/kg.K. The properties of air are given in Table A-17.

T K	h kJ/kg	P_r	u kJ/kg	v_r	s^o kJ/kg·K
300	300.19	1.3860	214.07	621.2	1.70203
760	778.18	39.27	560.01	55.54	2.66176
780	800.03	43.35	576.12	51.64	2.69013
670	681.14	24.46	488.81	78.61	2.52589
680	691.82	25.85	496.62	75.50	2.54175
1520	1660.23	636.5	1223.87	6.854	3.46120
1540	1684.51	672.8	1242.43	6.569	3.47712

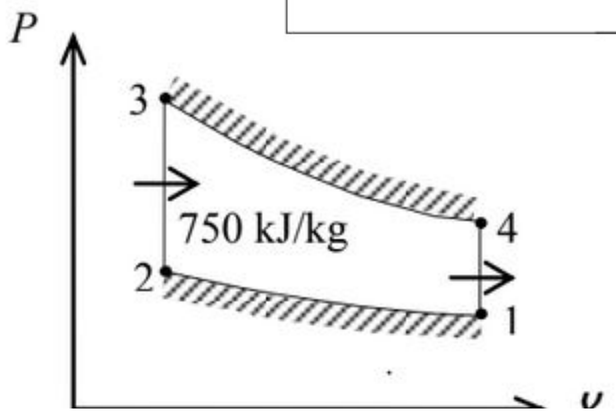
Analysis (a): ***start with: Draw the cycle diagram for the given problem**



Process 1-2: isentropic compression.

$$T_1 = 300\text{K} \longrightarrow \begin{array}{l} u_1 = 214.07\text{kJ/kg} \\ v_{r_1} = 621.2 \end{array}$$

From table A17



$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \frac{1}{r} v_{r_1} = \frac{1}{8}(621.2) = 77.65 \longrightarrow \begin{array}{l} T_2 = 673.1\text{K} \\ u_2 = 491.2 \text{ kJ/kg} \end{array}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \longrightarrow P_2 = \frac{v_1}{v_2}\frac{T_2}{T_1} P_1 = (8)\left(\frac{673.1\text{ K}}{300\text{ K}}\right)(95\text{ kPa}) = 1705\text{ kPa}$$

Process 2-3: v = constant heat addition.

$$r = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{V_{\text{BDC}}}{V_{\text{TDC}}}$$

$$q_{23,\text{in}} = u_3 - u_2 \longrightarrow u_3 = u_2 + q_{23,\text{in}} = 491.2 + 750 = 1241.2\text{ kJ/kg} \longrightarrow \begin{array}{l} T_3 = \mathbf{1539\text{ K}} \\ v_{r_3} = 6.588 \end{array}$$

From table A17 corresponding to u3 value

p∆v=0 (since constant volume)

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow P_3 = \frac{T_3}{T_2} P_2 = \left(\frac{1539\text{ K}}{673.1\text{ K}}\right)(1705\text{ kPa}) = \mathbf{3898\ kPa}$$

(b) Process 3-4: isentropic expansion.

$$v_{r_4} = \frac{v_1}{v_2} v_{r_3} = r v_{r_3} = (8)(6.588) = 52.70 \longrightarrow \begin{aligned} T_4 &= 774.5\text{ K} \\ u_4 &= 571.69\text{ kJ/kg} \end{aligned}$$

V3=v2
V4=V1

P**rocess 4-1:** v = constant heat rejection.

$$q_{out} = u_4 - u_1 = 571.69 - 214.07 = 357.62\text{ kJ/kg}$$

$$w_{net,out} = q_{in} - q_{out} = 750 - 357.62 = \mathbf{392.4\ kJ/kg}$$

(c) $$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \frac{392.4\text{ kJ/kg}}{750\text{ kJ/kg}} = \mathbf{52.3\%}$$

(d)

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(300\text{K})}{95\text{kPa}} = 0.906\text{m}^3/\text{kg} = v_{\max}$$

$$\text{MEP} = \frac{W_{\text{net}}}{V_{\max} - V_{\min}} = \frac{w_{\text{net}}}{v_{\max} - v_{\min}} \quad (\text{kPa})$$

$$v_{\min} = v_2 = \frac{v_{\max}}{r}$$

$$r = \frac{V_{\max}}{V_{\min}} = \frac{V_{\text{BDC}}}{V_{\text{TDC}}}$$

$$\text{MEP} = \frac{w_{\text{net,out}}}{v_1 - v_2} = \frac{w_{\text{net,out}}}{v_1(1-1/r)} = \frac{392.4\text{ kJ/kg}}{(0.906\text{ m}^3/\text{kg})(1-1/8)}\left(\frac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = \mathbf{495.0\ kPa}$$

9–57. A four-cylinder two-stroke 2.4-L diesel engine that operates on an ideal Diesel cycle has a compression ratio of 22 and a cutoff ratio of 1.8. Air is at 70ºC and 97 kPa at the beginning of the compression process. Using the cold-air standard assumptions, determine how much power the engine will deliver at 3500 rpm.

Answer:

A four-cylinder ideal diesel engine with air as the working fluid has a compression ratio of 22 and a cut off ratio of 1.8. The power the engine will deliver at 2300 rpm is to be determined.

Assumptions :

- The cold air-standard assumptions are applicable.
- Kinetic and potential energy changes are negligible.
- Air is an ideal gas with constant specific heats.

Properties:

The properties of air at room temperature are cp = 1.005 kJ/kg·K, cv = 0.718 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.4

TABLE A–2

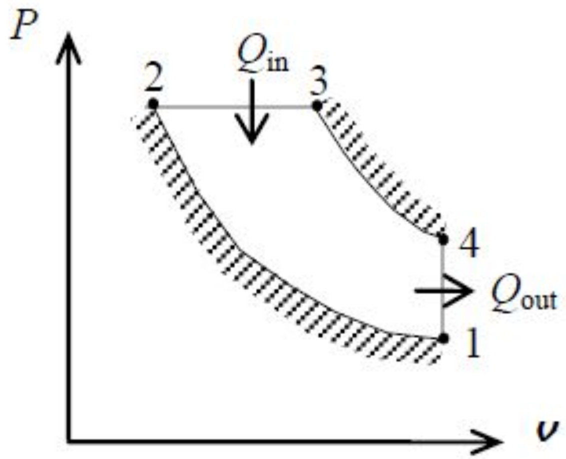
Ideal–gas specific heats of various common gases

(*a*) At 300 K

Gas	Formula	Gas constant, R kJ/kg·K	c_p kJ/kg·K	c_v kJ/kg·K	k
Air	—	0.2870	1.005	0.718	1.400

Analysis:

Process 1-2: isentropic compression.



$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (343\text{ K})(22)^{0.4} = 1181\text{ K}$$

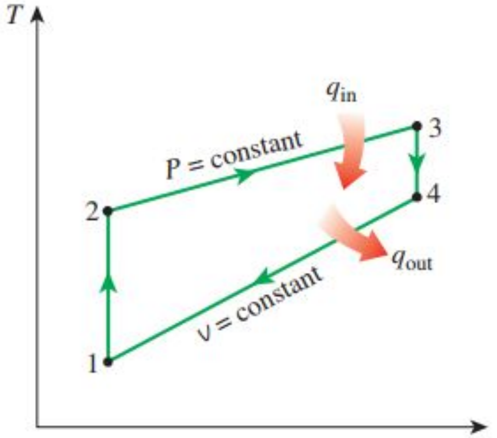
Process 2-3: P = constant heat addition.

Cut –off ratio

$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \longrightarrow T_3 = \frac{v_3}{v_2}T_2 = 1.8T_2 = (1.8)(1181\text{ K}) = 2126\text{ K}$$

Process 3-4: isentropic expansion.



(b) T-s diagram

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{2.2v_2}{v_4}\right)^{k-1} = T_3\left(\frac{2.2}{r}\right)^{k-1} = (2216\text{ K})\left(\frac{1.8}{22}\right)^{0.4} = 781\text{ K}$$

Its 1.8 not 2.2

For the cycle:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(97\ \text{kPa})(0.0024\ \text{m}^3)}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(343\ \text{K})} = 0.002365\ \text{kg}$$

$$\begin{aligned} Q_{\text{in}} &= m(h_3 - h_2) = mc_p(T_3 - T_2) \\ &= (0.002365\ \text{kg})(1.005\ \text{kJ/kg}\cdot\text{K})(2216 - 1181)\text{K} \\ &= 2.246\ \text{kJ} \end{aligned}$$

Heat addition in constant pressure and rejection in constant volume

$$\begin{aligned} Q_{\text{out}} &= m(u_4 - u_1) = mc_v(T_4 - T_1) \\ &= (0.002365\ \text{kg})(0.718\ \text{kJ/kg}\cdot\text{K})(781 - 343)\text{K} \\ &= 0.7438\ \text{kJ} \end{aligned}$$

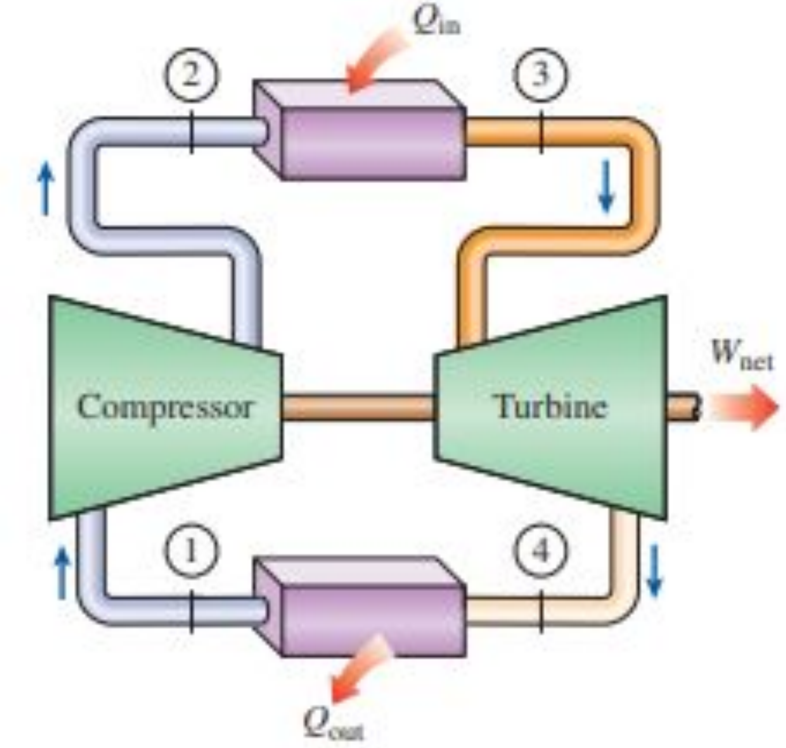
$$W_{\text{net,out}} = Q_{\text{in}} - Q_{\text{out}} = 2.246 - 0.7438 = 1.502\ \text{kJ/rev}$$

$$\dot{W}_{\text{net,out}} = \dot{n}W_{\text{net,out}} = (3500/60\ \text{rev/s})(1.502\ \text{kJ/rev}) = \mathbf{87.6\ kW}$$

***Discussion* Note that for 2-stroke engines, 1 thermodynamic cycle is equivalent to 1 mechanical cycle (and thus revolutions).**

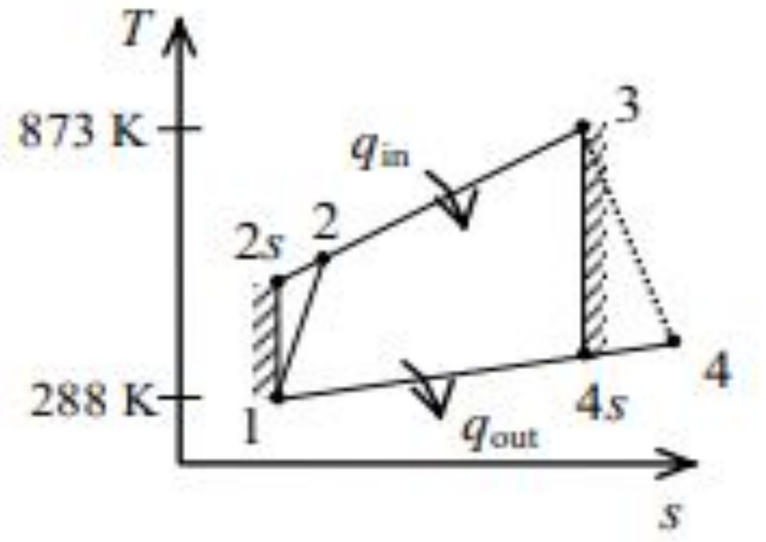
9–86. Consider a simple Brayton cycle using air as the working fluid; has a pressure ratio of 12; has a maximum cycle temperature of 600° C; and operates the compressor inlet at 100 kPa and 15°C. Which will have the greatest impact on the back-work ratio: a compressor isentropic efficiency of 80 percent or a turbine isentropic efficiency of 80 percent? Use constant specific heats at room temperature.

The ratio of the compressor work to the turbine work: back work ratio,



Solution:

A simple Brayton cycle with air as the working fluid operates between the specified temperature and pressure limits. The effects of non-isentropic compressor and turbine on the back-work ratio is to be compared.



Assumptions:

1. Steady operating conditions exist.
2. The air-standard assumptions are applicable.
3. Kinetic and potential energy changes are negligible.
4. Air is an ideal gas with constant specific heats.

Properties:

The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2a).

TABLE A–2

Ideal-gas specific heats of various common gases

(*a*) At 300 K

Gas	Formula	Gas constant, R kJ/kg·K	c_p kJ/kg·K	c_v kJ/kg·K	k
Air	—	0.2870	1.005	0.718	1.400

Analysis:

For the compression process,

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (288\text{ K})(12)^{0.4/1.4} = 585.8\text{ K}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)} \longrightarrow T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_C}$$

$$= 288 + \frac{585.8 - 288}{0.80}$$

$$= 660.2\text{ K}$$

For the expansion process,

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (873\text{ K})\left(\frac{1}{12}\right)^{0.4/1.4} = 429.2\text{ K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})} \longrightarrow T_4 = T_3 - \eta_T(T_3 - T_{4s})$$

$$= 873 - (0.80)(873 - 429.2)$$

$$= 518.0\text{ K}$$

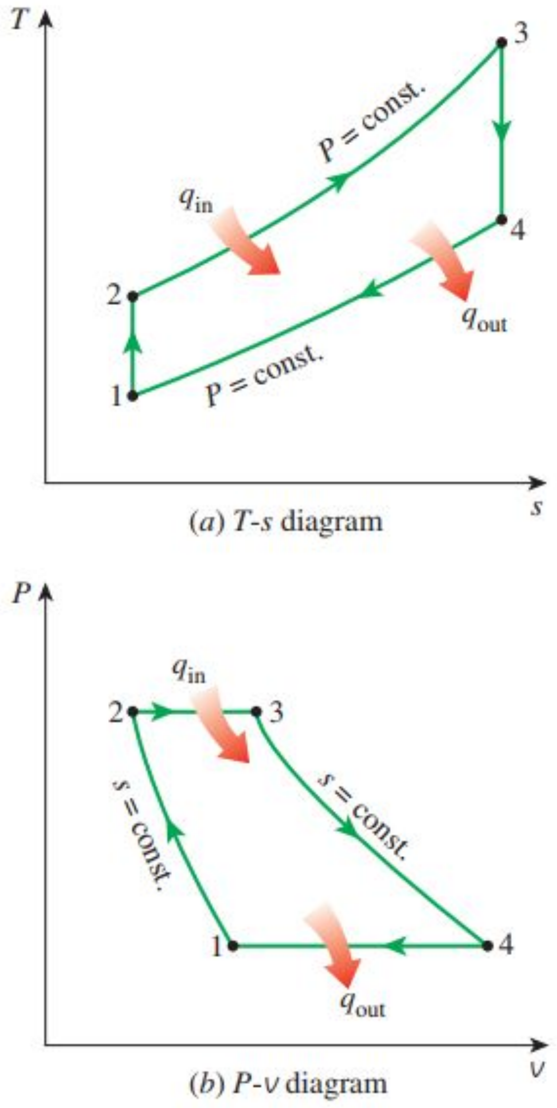


FIGURE 9–31

T-s and *P-v* diagrams for the ideal Brayton cycle.

The isentropic and actual work of compressor and turbine are

$$W_{\text{Comp},s} = c_p(T_{2s} - T_1) = (1.005\,\text{kJ/kg}\cdot\text{K})(585.8 - 288)\text{K} = 299.3\,\text{kJ/kg}$$

$$W_{\text{Comp}} = c_p(T_2 - T_1) = (1.005\,\text{kJ/kg}\cdot\text{K})(660.2 - 288)\text{K} = 374.1\,\text{kJ/kg}$$

$$W_{\text{Turb},s} = c_p(T_3 - T_{4s}) = (1.005\,\text{kJ/kg}\cdot\text{K})(873 - 429.2)\text{K} = 446.0\,\text{kJ/kg}$$

$$W_{\text{Turb}} = c_p(T_3 - T_4) = (1.005\,\text{kJ/kg}\cdot\text{K})(873 - 518.0)\text{K} = 356.8\,\text{kJ/kg}$$

The back work ratio for 80% efficient compressor and isentropic turbine case is

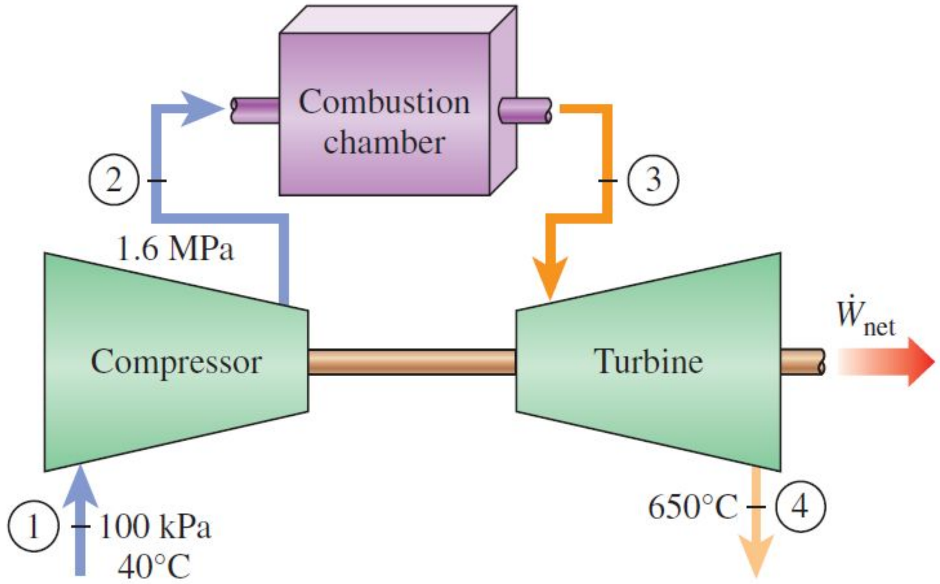
$$r_{\text{bw}} = \frac{W_{\text{Comp}}}{W_{\text{Turb},s}} = \frac{374.1\,\text{kJ/kg}}{446.0\,\text{kJ/kg}} = \mathbf{0.8387}$$

The back work ratio for 80% efficient turbine and isentropic compressor case is

$$r_{\text{bw}} = \frac{W_{\text{Comp},s}}{W_{\text{Turb}}} = \frac{299.3\,\text{kJ/kg}}{356.8\,\text{kJ/kg}} = \mathbf{0.8387}$$

The two results are identical.

9-90 A gas-turbine power plant operates on the simple Brayton cycle between the pressure limits of 100 and 1600 kPa. The working fluid is air, which enters the compressor at 40°C at a rate of 850 m^3/min and leaves the turbine at 650°C. Using **variable specific heats** for air and assuming a compressor isentropic efficiency of 85 percent and a turbine isentropic efficiency of 88 percent, determine (a) the net power output, (b) the back work ratio, and (c) the thermal efficiency.



Solution

A gas-turbine plant operates on the simple Brayton cycle. The net power output, the back work ratio, and the thermal efficiency are to be determined.

Assumptions

- The air-standard assumptions are applicable.
- Kinetic and potential energy changes are negligible.
- Air is an ideal gas with variable specific heats.

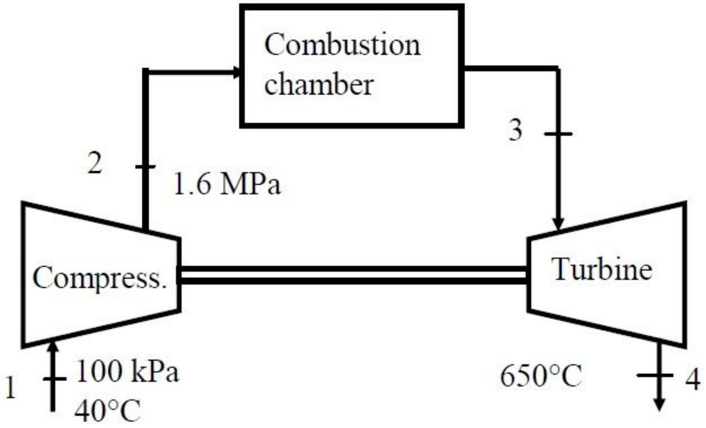
Properties

The gas constant of air is R = 0.287 kJ/kg·K (Table A-1).

Substance	Formula	Molar mass, M kg/kmol	Gas constant, R kJ/kg·K*
Air	—	28.97	0.2870

Analysis

(a) Remember that for an ideal gas, enthalpy is a function of temperature only whereas entropy is functions of both temperature and pressure.



Process 1-2: Compression

From Table A-17 (refer to partially reproduced table),

h_1 = 313.6 kJ/kg and P_{r1} = 1.6163 @ T_1 = 313 K

(this is obtained by interpolation)

$P_{r2} = P_{r1}$ (P2/P1) = 1.6163*(1600/100) = 25.86

From Table A-17 (refer to partially reproduced table),

h_{2s} = 691.9 @ P_{r2} = 25.86

(this is again obtained by linear interpolation)

Now we can obtain h_2 from the following formula:

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow 0.85 = \frac{691.9 - 313.6}{h_2 - 313.6} \longrightarrow h_2 = 758.6\,\text{kJ/kg}$$

TABLE A–17

Ideal-gas properties of air

T K	*h* kJ/kg	P_r
310	310.24	1.5546
315	315.27	1.6442
670	681.14	24.46
680	691.82	25.85
690	702.52	27.29

Process 3-4: Expansion $T_4 = 650°\text{C} \longrightarrow h_4 = 959.2 \text{ kJ/kg}$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow 0.88 = \frac{h_3 - 959.2}{h_3 - h_{4s}}$$

.Again, from A17
.T4 is the actual exit temp

To find h_3, we follow trial-error (iterative) procedure : Step 1. Assume T_3 Step 2: Find h_3 (Table A-17) and calculate $P_{r4} = P_{r3}(P_4/P_3)$ Step 3. Find h_{4s}(@ P_{r4} from Table A17). Step 4 : Calculate η_T . If η_T is not equal to 0.88, go back to step 1. Using this procedure, we get

$$h_3 = 1790 \text{ kJ/kg}, T_3 = 1353°\text{C}$$

The mass flow rate is determined from

$$\dot{m} = \frac{P_1\dot{V}_1}{RT_1} = \frac{(100 \text{ kPa})(850/60 \text{ m}^3/\text{s})}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(40+273 \text{ K})} = 15.77 \text{ kg/s}$$

The net power output is

$$\dot{W}_{\text{C,in}} = \dot{m}(h_2 - h_1) = (15.77 \text{ kg/s})(758.6 - 313.6)\text{kJ/kg} = 7017 \text{ kW}$$

$$\dot{W}_{\text{T,out}} = \dot{m}(h_3 - h_4) = (15.77 \text{ kg/s})(1790 - 959.2)\text{kJ/kg} = 13{,}098 \text{ kW}$$

$$\dot{W}_{\text{net}} = \dot{W}_{\text{T,out}} - \dot{W}_{\text{C,in}} = 13{,}098 - 7017 = \mathbf{6081 kW}$$

(b) The back work ratio is

$$r_{\text{bw}} = \frac{\dot{W}_{\text{C,in}}}{\dot{W}_{\text{T,out}}} = \frac{7017\ \text{kW}}{13{,}098\ \text{kW}} = \mathbf{0.536}$$

(c) The Thermal efficiency is

$$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_2) = (15.77\ \text{kg/s})(1790 - 758.6)\text{kJ/kg} = 16{,}262\ \text{kW}$$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{6081\ \text{kW}}{16{,}262\ \text{kW}} = 0.374 = \mathbf{37.4\%}$$

9-138. Determine the total energy destruction associated with the Otto cycle described in Problem 9-33**, assuming a source temperature of 2000K and a sink temperature of 300K. Also, determine the energy at the end of the power stroke.

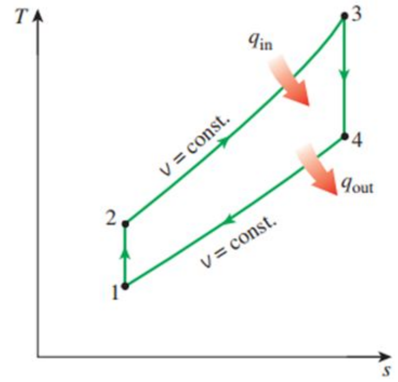
** Problem 9-33 is the first problem of this tutorial

Answer:

The total exergy destruction associated with the Otto cycle described in Prob. 9-33 and the exergy at the end of the power stroke are to be determined.

Analysis

From Prob. 9-33,



$$q_{in} = 750,\ q_{out} = 357.62\ \text{kJ/kg},\ T_1 = 300\ \text{K, and } T_4 = 774.5\ \text{K.}$$

The total exergy destruction associated with this Otto cycle is determined from

$$x_{\text{destroyed}} = T_0\left(\frac{q_{out}}{T_L} - \frac{q_{in}}{T_H}\right) = (300\ \text{K})\left(\frac{357.62\ \text{kJ/kg}}{300\ \text{K}} - \frac{750\ \text{kJ/kg}}{2000\ \text{K}}\right) = \mathbf{245.1\ kJ/kg}$$

Noting that state 4 is identical to the state of the surroundings, the exergy at the end of the power stroke (state 4) is determined from

$$\phi_4 = (u_4 - u_0) - T_0(s_4 - s_0) + P_0(\boldsymbol{v}_4 - \boldsymbol{v}_0)$$

where

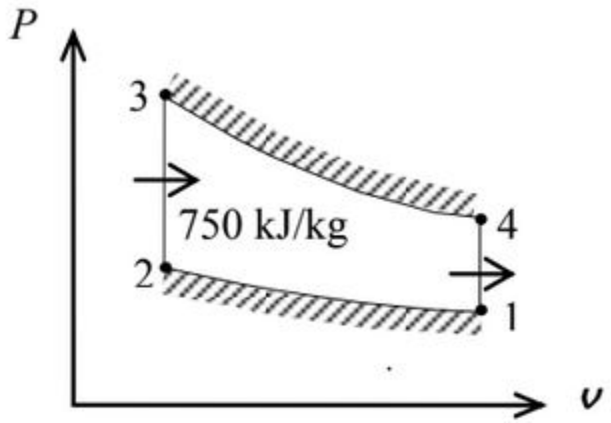
$$u_4 - u_0 = u_4 - u_1 = q_{\text{out}} = 357.62 \text{kJ/kg}$$

$$\boldsymbol{v}_4 - \boldsymbol{v}_0 = \boldsymbol{v}_4 - \boldsymbol{v}_1 = 0$$

$$s_4 - s_0 = s_4 - s_1 = s_4^\circ - s_1^\circ - R\ln\frac{P_4}{P_1} = s_4^\circ - s_1^\circ - R\ln\frac{T_4 \boldsymbol{v}_1}{T_1 \boldsymbol{v}_4} = s_4^\circ - s_1^\circ - R\ln\frac{T_4}{T_1}$$

$$= 2.6823 - 1.70203 - (0.287 \text{ kJ/kg}\cdot\text{K})\ln\frac{774.5\text{ K}}{300\text{ K}} = 0.7081 \text{ kJ/kg}\cdot\text{K}$$

P=RT/v:
ideal gas



Thus,

$$\phi_4 = (357.62 \text{ kJ/kg}) - (300 \text{ K})(0.7081 \text{ kJ/kg}\cdot\text{K}) + 0 = \mathbf{145.2\ \ kJ/kg}$$

thank you