ESO201A : THERMODYNAMICS
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IIT Kanpur

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Lecture 15

Thermodynamic temperature scale :

In the last lecture, we had stated that the following relation can be derived for a reversible heat engine operating between two thermal reservoirs

$$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$$

where T_H and T_L are the temperatures of the high temperature and low temperature reservoirs defined according to Thermodynamic temperature scale (this scale is also known as Kelvin scale in SI units).

Now we consider derivation of the above equation.

Thermodynamic temperature scale :

According to Carnot Principle (2), all reversible heat engines have the same thermal efficiency when operating between the same two thermal reservoirs. Thus, the efficiency of a reversible heat engine does not depend upon the working fluid or the way of execution of the cyle or the type of the heat engine.

Since thermal reservoirs are characterized by the temperatures, the thermal efficiency or, equivalent, the ratio of Q_H to Q_L will depend only upon the temperatures of the reservoirs. Thus,

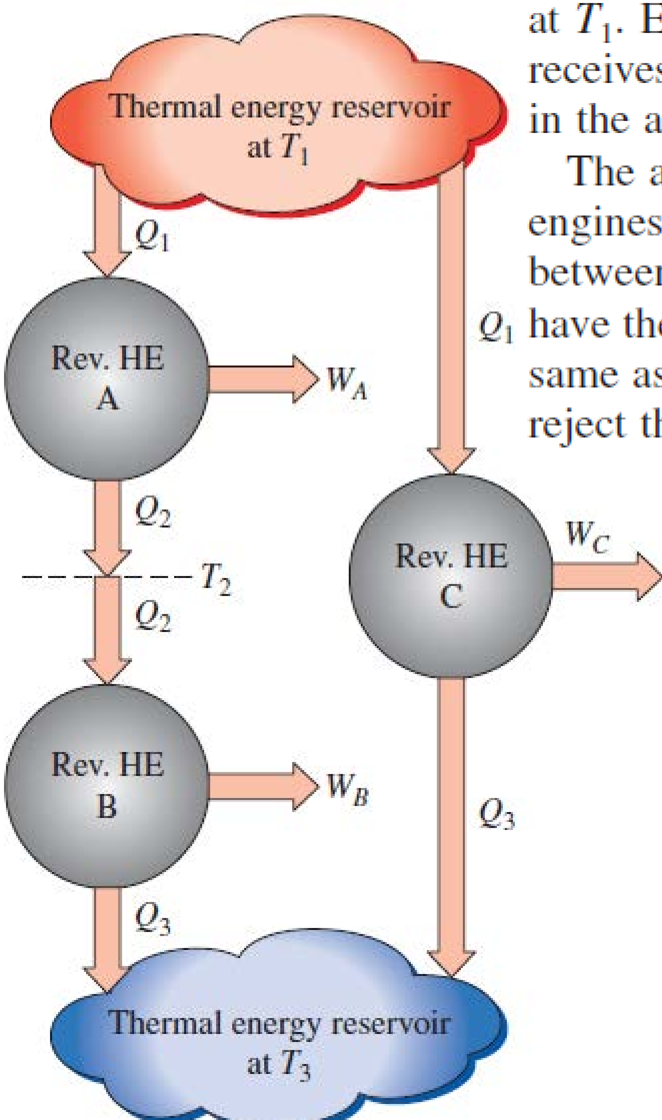
$$\eta_{\text{th,rev}} = g(T_H, T_L) \tag{A}$$

$$\frac{Q_H}{Q_L} = f(T_H, T_L) \tag{B}$$

The functional form can be developed by considering three reversible heat engines as shown on the next slide.

Ref. Cengel and Boles, 8th Edition (2015)

Thermodynamic temperature scale :



The functional form of $f(T_H, T_L)$ can be developed with the help of the three reversible heat engines shown in Fig. Engines A and C are supplied with the same amount of heat Q_1 from the high-temperature reservoir at T_1. Engine C rejects Q_3 to the low-temperature reservoir at T_3. Engine B receives the heat Q_2 rejected by engine A at temperature T_2 and rejects heat in the amount of Q_3 to a reservoir at T_3.

The amounts of heat rejected by engines B and C must be the same since engines A and B can be combined into one reversible engine operating between the same reservoirs as engine C and thus the combined engine will have the same efficiency as engine C. Since the heat input to engine C is the same as the heat input to the combined engines A and B, both systems must reject the same amount of heat.

Applying Eq. B (see last slide) to all three engines separately,

$$\frac{Q_1}{Q_2} = f(T_1, T_2), \quad \frac{Q_2}{Q_3} = f(T_2, T_3), \quad \text{and} \quad \frac{Q_1}{Q_3} = f(T_1, T_3)$$

Now consider the identity

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2}\frac{Q_2}{Q_3}$$

which corresponds to

$$f(T_1, T_3) = f(T_1, T_2) \cdot f(T_2, T_3)$$

Ref. Cengel and Boles, 8th Edition (2015)

Thermodynamic temperature scale :

The condition in the last equation of the previous slide can be satisfied ONLY if function ‘f’ has the following form

$$f(T_1, T_2) = \frac{\phi(T_1)}{\phi(T_2)} \quad \text{and} \quad f(T_2, T_3) = \frac{\phi(T_2)}{\phi(T_3)}$$

so that $\phi(T_2)$ will cancel from the product of $f(T_1, T_2)$ and $f(T_2, T_3)$, yielding

$$\frac{Q_1}{Q_3} = f(T_1, T_3) = \frac{\phi(T_1)}{\phi(T_3)} \qquad \text{(C)}$$

For a reversible heat engine operating between two reservoirs at temperatures T_H and T_L, Eq. (C) can be written as

$$\frac{Q_H}{Q_L} = \frac{\phi(T_H)}{\phi(T_L)}$$

Ref. Cengel and Boles, 8th Edition (2015)

Thermodynamic temperature scale :

This is the only requirement that the second law places on the ratio of heat transfers to and from the reversible heat engines. Several functions $\phi(T)$ satisfy this equation, and the choice is completely arbitrary. Lord Kelvin first proposed taking $\phi(T) = T$ to define a thermodynamic temperature scale as

$$\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{T_H}{T_L} \qquad \text{(D)}$$

The thermodynamic temperature scale is not completely defined by Eq. (D) since it gives us only a ratio of absolute temperatures. We also need to know the magnitude of a kelvin. At the International Conference on Weights and Measures held in 1954, the triple point of water (the state at which all three phases of water exist in equilibrium) was assigned the value 273.16 K The *magnitude of a kelvin* is defined as 1/273.16 of the temperature interval between absolute zero and the triple-point temperature of water. The magnitudes of temperature units on the Kelvin and Celsius scales are identical (1 K ≡ 1°C). The temperatures on these two scales differ by a constant 273.15:

$$T(°\text{C}) = T(\text{K}) - 273.15$$

Ref. Cengel and Boles, 8th Edition (2015)