ESO201A : THERMODYNAMICS
2021-22 Ist semester
IIT Kanpur

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Lecture 8

Specific heats of solids and liquids:

Solids and liquids are generally considered as incompressible substances. This is because the volume of a solid or a liquid does not change significantly upon application of pressure (i.e., compression)

Lets consider specific enthalpy of a substance $h = u + Pv$

Differentiating both sides, we get $dh = du + P\,dv + v\,dP$

Here we are considering differences between two equilibrium states. The above expression gives the difference in the value of h of the two states to the first order. Since change in v is negligible, Pdv term can be neglected in the above expression. Since specific volume of solids and liquids is small, vdP term can also be neglected.

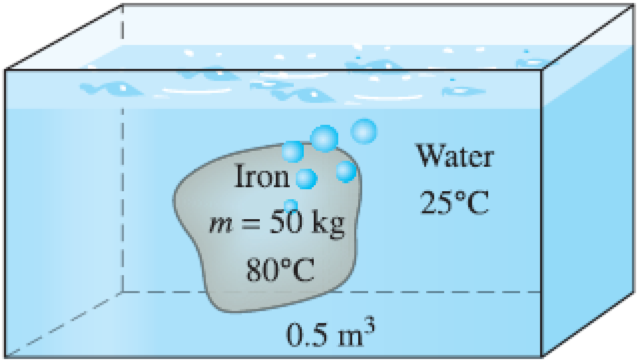
Thus $dh \approx du$

$$C_P \approx C_V = C$$

Thus the specific heats are nearly equal and therefore specific heat of a solid or liquid is denoted by symbol C.

Example :

A 50-kg iron block at 80°C is dropped into an insulated tank that contains 0.5 m^3 of liquid water at 25°C. Determine the temperature when thermal equilibrium is reached.

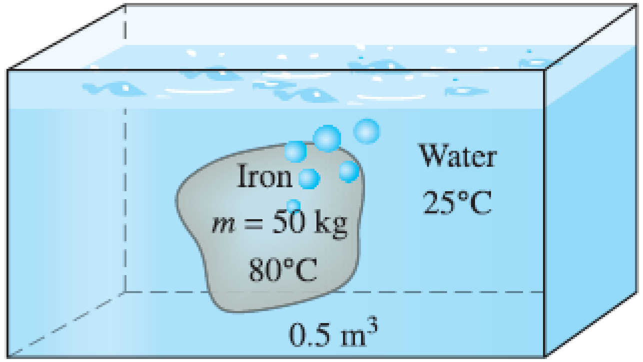


SOLUTION An iron block is dropped into water in an insulated tank. The final temperature when thermal equilibrium is reached is to be determined.

Assumptions **1** Both water and the iron block are incompressible substances. **2** Constant specific heats at room temperature can be used for water and the iron. **3** The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **4** There are no electrical, shaft, or other forms of work involved. **5** The system is well-insulated and thus there is no heat transfer.

Ref. Cengel and Boles, 8th Edition (2015)

Example :



We consider (Iron + Water) as our system (closed mass). Since system is enclosed in a rigid container (tank), there is **no work done** by surroundings on the sytem. Since container is insulated there is **no heat transfer** between system and surroundings. Hence according to first law total change in internal energy must be zero.

$$0 = \Delta U$$

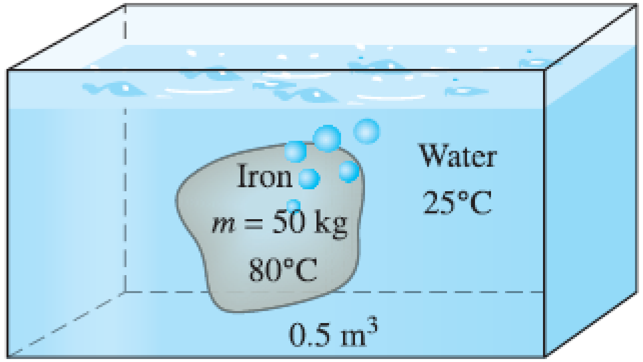
The total internal energy U is an extensive property, and therefore it can be expressed as the sum of the internal energies of the parts of the system. Then the total internal energy change of the system becomes

$$\Delta U_{sys} = \Delta U_{iron} + \Delta U_{water} = 0$$

$$[mc(T_2 - T_1)]_{iron} + [mc(T_2 - T_1)]_{water} = 0$$

Ref. Cengel and Boles, 8th Edition (2015)

Example :



The specific volume of liquid water at or about room temperature can be taken to be 0.001 m^3/kg. Then the mass of the water is

$$m_{\text{water}} = \frac{V}{v} = \frac{0.5\ \text{m}^3}{0.001\ \text{m}^3/\text{kg}} = 500\ \text{kg}$$

The specific heats of iron and liquid water are determined from Table A–3 to be $c_{\text{iron}} = 0.45$ kJ/kg·°C and $c_{\text{water}} = 4.18$ kJ/kg·°C. Substituting these values into the energy equation, we obtain

$$(50\ \text{kg})(0.45\ \text{kJ/kg·°C})(T_2 - 80°\text{C}) + (500\ \text{kg})(4.18\ \text{kJ/kg·°C})(T_2 - 25°\text{C}) = 0$$

$$T_2 = 25.6°\text{C}$$

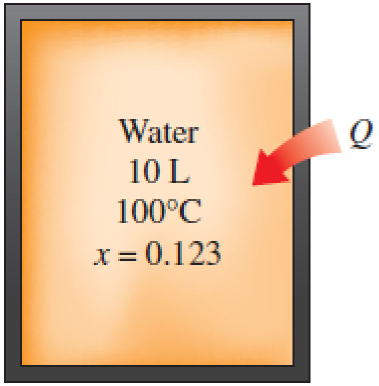
Therefore, when thermal equilibrium is established, both the water and iron will be at 25.6°C.

Discussion The small rise in water temperature is due to its large mass and large specific heat.

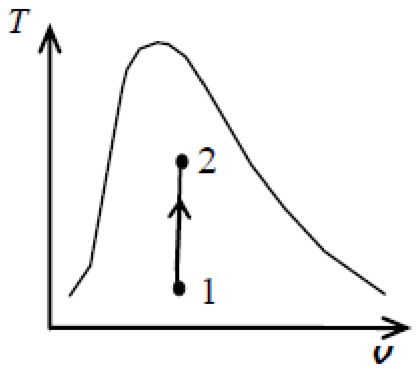
Ref. Cengel and Boles, 8th Edition (2015)

Example :

A rigid 10-L vessel initially contains a mixture of liquid water and vapor at 100°C with 12.3 percent quality. The mixture is then heated until its temperature is 150°C. Calculate the heat transfer required for this process.



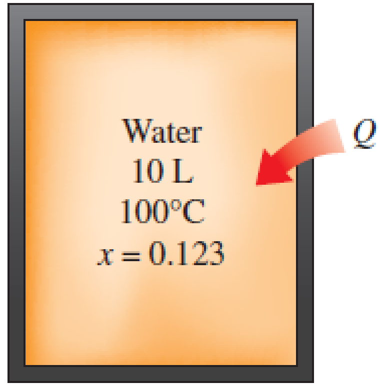
Solution : Since the system is stationary, changes in kinetic and potential energies are negligible. Since the vessel is rigid there is no work done on the system. Since the process occurs at constant volume and inititially contains a mixture of saturated liquid and saturated vapor, the process can be depicted pictorially as :

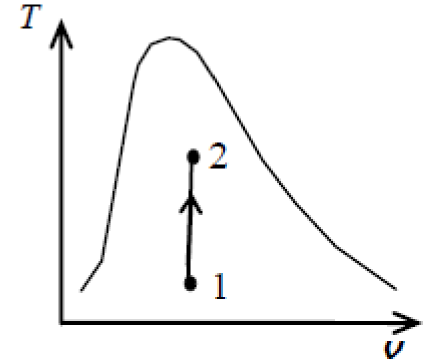


Ref. Cengel and Boles, 8th Edition (2015)

Example (continued):

Ref. Cengel and Boles, 8th Edition (2015)





Due to absence of work term, the first law takes the form : $Q_{\text{in}} = \Delta U = m(u_2 - u_1)$

The properties at the initial and final states are (Table A-4)

$$\left.\begin{array}{l} T_1 = 100°\text{C} \\ x_1 = 0.123 \end{array}\right\} \begin{array}{l} v_1 = v_f + x v_{fg} = 0.001043 + (0.123)(1.6720 - 0.001043) = 0.2066\,\text{m}^3/\text{kg} \\ u_1 = u_f + x u_{fg} = 419.06 + (0.123)(2087.0) = 675.76\,\text{kJ/kg} \end{array}$$

$$\left.\begin{array}{l} T_2 = 150°\text{C} \\ v_2 = v_1 = 0.2066\,\text{m}^3/\text{kg} \end{array}\right\} \begin{array}{l} x_2 = \dfrac{v_2 - v_f}{v_{fg}} = \dfrac{0.2066\text{-}0.001091}{0.39248\text{-}0.001091} = 0.5250 \\ u_2 = u_f + x_2 u_{fg} \\ \quad = 631.66 + (0.5250)(1927.4) = 1643.5\,\text{kJ/kg} \end{array}$$

The mass in the system is

$$m = \frac{V_1}{v_1} = \frac{0.100\,\text{m}^3}{0.2066\,\text{m}^3/\text{kg}} = 0.04841\,\text{kg}$$

Substituting,

$$Q_{\text{in}} = m(u_2 - u_1) = (0.04841\,\text{kg})(1643.5 - 675.76)\,\text{kJ/kg} = \mathbf{46.9kJ}$$

Mass balance and first law analysis for Control volume:

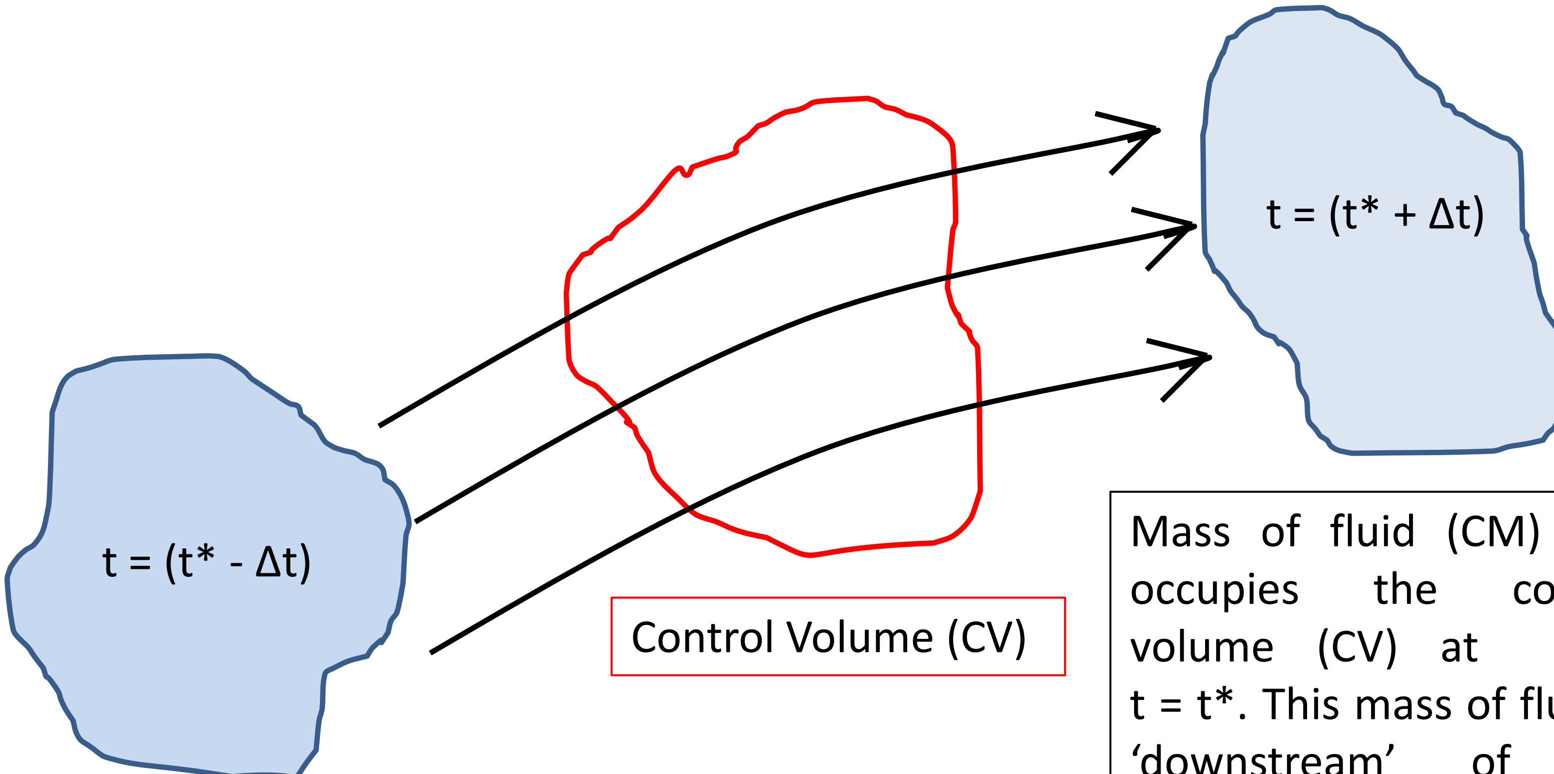
So far we have considered closed system or a control mass, in which no mass can enter or leave the system.

In many cases we need to deal with equipments such as pumps, compressors, turbines, nozzles etc. These are open systems, i.e., there is a flow of fluid across the equipments.

We cannot apply the first law equation for control mass to such open systems.

In order to apply the first law , we need to make use of Reynolds Transport Theorem.

Reynolds Transport Theorem:



Mass of fluid (CM) that occupies the control volume (CV) at time t = t*. This mass of fluid is 'upstream' of the control volume at t = (t* - Δt), i.e., it has not yet entered the CV at t = (t* - Δt).

Mass of fluid (CM) that occupies the control volume (CV) at time t = t*. This mass of fluid is 'downstream' of the control volume at t = (t* + Δt), i.e., it has now come out of the CV. Note that it has a different shape and volume than what it had at t = (t* - Δt)

Reynolds Transport Theorem:

Consider a scalar quantity b which is a function of coordinates and time, i.e., b=b(x,y,z,t). As an example, this scalar quantity can be density of the fluid. Then we define two functions of time f(t) and g(t) as follows :

$$f(t) = \int_{CV} b(x,y,z,t)dV$$

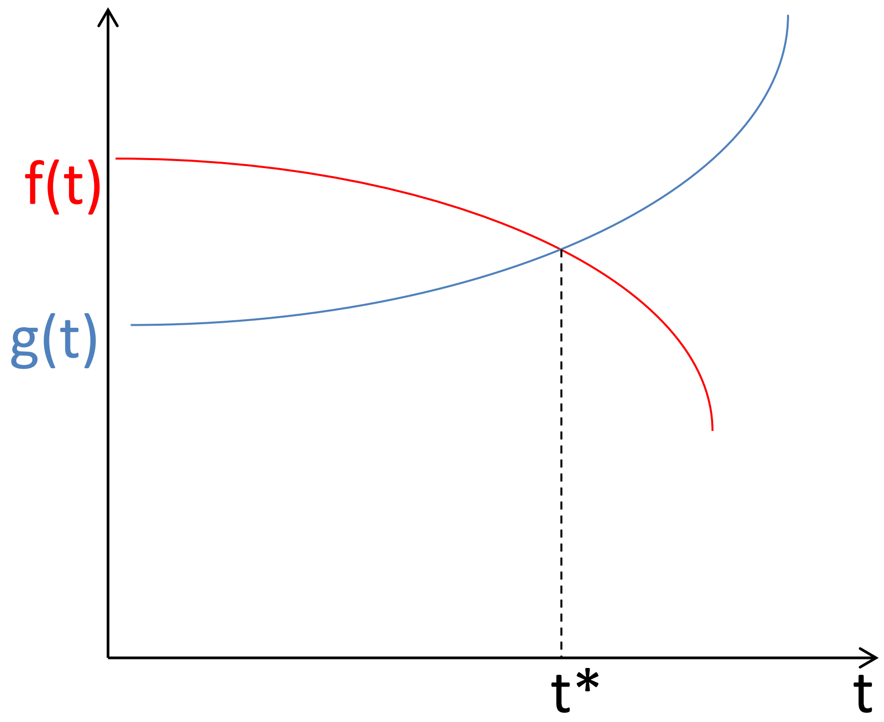
$$g(t) = \int_{CM} b(x,y,z,t)dV$$

In defining f(t), integration is taken over the control volume (CV) [see figure in the last slide]

In defining g(t), integration is taken over the mass of fluid (CM) that occupies the control volume at t = t* [see last slide]

Reynolds Transport Theorem:

The two functions of time can be plotted on a graph and the graph might look as depicted below.



Since the mass of the fluid (CM) occupies the control volume CV

At t = t*, the two functions of time will have the same value :

$$f(t^*) = g(t^*)$$

Thus the two curves interesect at t = t* as seen in above plot.

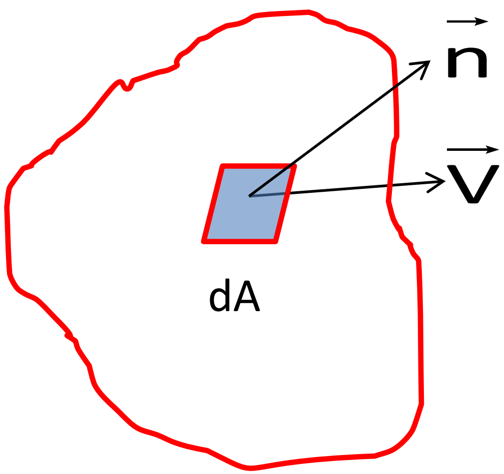
Reynolds Transport Theorem:

However the derivatives of the functions f(t) and g(t) at t = t* will not be equal. This can also be seen by the fact that slopes of the functions f(t) and g(t) plotted on last slide are not the same at t = t *. This condition is mathematically expressed as :

$$\left(\frac{dg}{dt}\right)_{t=t^*} \neq \left(\frac{df}{dt}\right)_{t=t^*}$$

The difference between the values of the two derivatives at t = t* is given by **Reynolds Transport Theorem** as follows :

$$\boxed{\left(\frac{dg}{dt}\right)_{t=t^*} = \left(\frac{df}{dt}\right)_{t=t^*} + \int_{CS} b\,(\vec{V}\,.\,\vec{n})\,dA}$$



The integral in the above equation is taken over the entire surface (CS) bounding the control volume (CV). The integration is performed at t = t*.

$\vec{n}$= normal vector (directed outwards) to the differential area 'dA' on the surface of the control volume

$\vec{V}$= Velocity vector at the center of the differential area 'dA'