Tutorial-9

① interpolate value at $x = 0.275$

$f(x) = \frac{1}{1+x^2}$

x	$f(x)$
0.1	0.990
0.2	0.961
0.3	0.917
0.4	0.862
0.5	0.800

a) Lagrange interpolation

$$P_n(x) = \sum_{j=0}^{n} \prod_{\substack{i=0 \\ i \neq j}}^{n} \frac{(x - x_i)}{(x_j - x_i)} y_j$$

for n^{th} order

We will give preference to the points near our given x

x_i	$f(x_i)$	$\lvert x - x_i \rvert$
0.1	0.990	0.175
0.2	0.961	0.075
0.3	0.917	0.025
0.4	0.862	0.125
0.5	0.800	0.225

; $x = 0.275$

Rearrange in ascending order of $|x - x_i|$

i	x_i	$f(x_i)$
0	0.3	0.917
1	0.2	0.961
2	0.4	0.862
3	0.1	0.990
4	0.5	0.800

i) First order Lagrange interpolation

$$P_1(x) = \frac{(x-x_1)}{(x_0-x_1)} \times y_0 + \frac{(x-x_0)}{(x_1-x_0)} \times y_1$$

$$= \frac{(x-0.2)}{(0.3-0.2)} \times 0.917 + \frac{(x-0.3)}{(0.2-0.3)} 0.9615$$

$$= -0.4411x + 1.0498$$

ii) Second order Lagrange interpolation

$$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \times y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \times y_2$$

$$= \frac{(x-0.2)(x-0.4)}{(0.3-0.2)(0.3-0.4)} \times 0.917$$

$$+ \frac{(x-0.3)(x-0.4)}{(0.2-0.3)(0.2-0.4)} \times 0.9615$$

$$+ \frac{(x-0.3)(x-0.2)}{(0.4-0.3)(0.4-0.2)} \times 0.862$$

$$= -0.562x^2 - 0.159x + 1.016$$

iii) Third order Lagrange interpolation:

$$P_3(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$= \frac{(x-0.2)(x-0.4)(x-0.1)}{(0.3-0.2)(0.3-0.4)(0.3-0.1)} \times 0.917$$

$$+ \frac{(x-0.3)(x-0.4)(x-0.1)}{(0.2-0.3)(0.2-0.4)(0.2-0.1)} \times 0.961$$

$$+ \frac{(x-0.3)(x-0.2)(x-0.1)}{(0.4-0.3)(0.4-0.2)(0.4-0.1)} \times 0.8621$$

$$+ \frac{(x-0.3)(x-0.2)(x-0.4)}{(0.1-0.3)(0.1-0.2)(0.1-0.4)} \times 0.990$$

$$= 0.716x^3 - 1.207x^2 + 0.0265x + 0.998$$

Lagrange polynomials @ $x = 0.275$; $f(0.275) = 0.92969$

order	$f(x)$	error
1	0.9284	0.133
2	0.9295	0.019
3	0.92968	0.001

b) Newtons divided difference interpolation

i) First order NDD

i	x_i	$f[x_i]$	$f[x_{i+1}, x_i]$
0	0.3	0.917	$\frac{0.917 - 0.961}{0.3 - 0.2} = -0.441$
1	0.2	0.961	

$$P_1(x) = y_0 + (x - x_0) f[x_1, x_0]$$

$$= 0.917 + (x - 0.3) \times -0.441$$

$$= -0.441x + 1.049$$

ii) Second order NDD

i	x	f[x_i]	f[x_{i+1}, x_i]	f[x_{i+2}, x_{i+1}, x_i]
0	0.3	0.917	$\frac{0.917 - 0.961}{0.3 - 0.2} = -0.441$	$\frac{-0.441 - (-0.497)}{0.3 - 0.4} = -0.562$
1	0.2	0.961	$\frac{0.961 - 0.862}{0.2 - 0.4} = -0.497$	
2	0.4	0.862		

i	x	f[x_i]	f[x_{i+1}, x_i]	f[x_{i+2}, x_{i+1}, x_i]
0	0.3	0.917		
			-0.441	
1	0.2	0.961		-0.562
			-0.497	
2	0.4	0.862		

$$f(x) = y_0 + (x - x_0) f[x_1, x_0] + (x - x_0)(x - x_1) f[x_2, x_1, x_0]$$

$$= 0.917 + (x - 0.3) - 0.441 + (x - 0.3)(x - 0.2) \times -0.562$$

$$= -0.562\, x^2 - 0.1599\, x + 1.016$$

Third order NDD

i	x_i	$f[x_i]$	$f[x_{i+1}, x_i]$	$f[x_{i+2}, x_{i+1}, x_i]$	$f[x_{i+3}, x_{i+2}, x_{i+1}, x_i]$
0	0.3	0.917			
			−0.441		
1	0.2	0.961		$\frac{-0.441 - 0.497}{0.3 - 0.4} = -0.562$	
			−0.497		$\frac{-0.562 - (-0.705)}{0.3 - 0.1} = 0.716$
2	0.4	0.862		$\frac{-0.497 - (-0.426)}{0.2 - 0.1} = -0.705$	
			−0.426		
3	0.1	0.990			

$$f(x) = y_0 + (x - x_0) f[x_1, x_0] + (x - x_0)(x - x_1) f[x_2, x_1, x_0] + (x - x_0)(x - x_1)(x - x_2) f[x_3, x_2, x_1, x_0]$$

$$= 0.917 + (x - 0.3) \times -0.441 + (x - 0.3)(x - 0.2) \times -0.562 + (x - 0.3)(x - 0.2)(x - 0.4) \times 0.716$$

$$= 0.716x^3 - 1.207x^2 + 0.0265x + 0.998$$

NDD polynomials @ $x = 0.275$

order	$f(x)$	error %
1	0.9284	0.133
2	0.9295	0.019
3	0.9296	0.001