M 361K Homework 4

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6.3

13. Try to use L'Hospital's Rule to find the limit of $\frac{\tan x}{\sec x}$ as $x \to (\pi/2)-$. Then evaluate directly by changing to sines and cosines.

14. Show that if $c > 0$, then $\lim_{x\to c} \frac{x^c - c^x}{x^x - c^c} = \frac{1-\ln c}{1+\ln c}$.

6.4

11. If $x \in [0, 1]$ and $n \in \mathbb{N}$, show that

$$\left|\ln(1+x) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1}\frac{x^n}{n}\right)\right| < \frac{x^{n+1}}{n+1}.$$

Use this to approximate $\ln 1.5$ with an error less than 0.01. Less than 0.001.

13. Calculate e correct to 7 decimal places.

7.1

2. If $f(x) := x^2$ for $x \in [0, 4]$, calculate the following Riemann sums, where $\dot{\mathcal{P}}_i$ has the same partition points as in Exercise 1, and the tags are selected as indicated. $\mathcal{P}_2 := (0, 2, 3, 4)$.

- $\dot{\mathcal{P}}_2$ with the tags at the left endpoints of the subintervals.
- $\dot{\mathcal{P}}_2$ with the tags at the right endpoints of the subintervals.

6b. Let $h(x) := 2$ if $0 \le x < 1, h(1) := 3$ and $h(x) := 1$ if $1 < x \le 2$. Show that $h \in \mathcal{R}[0, 2]$ and evaluate its integral.

8. If $f \in \mathcal{R}[a, b]$ and $|f(x)| \le M$ for all $x \in [a, b]$, show that $\left|\int_a^b f\right| \le M(b - a)$.

10. Let $g(x) := 0$ if $x \in [0,1]$ is rational and $g(x) := 1/x$ if $x \in [0,1]$ is irrational. Explain why $g \notin \mathcal{R}[0,1]$. However, show that there exists a sequence $\left(\dot{\mathcal{P}}_n\right)$ of tagged partitions of $[a,b]$ such that $\left\|\dot{\mathcal{P}}_n\right\| \to 0$ and $\lim_n S\left(g; \dot{\mathcal{P}}_n\right)$ exists.