

# ECE115 Practice

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## Projections

### 1 1.7.4

**Problem:**

Let  $L$  be a line in  $\mathbb{R}^3$  with vector equation

$$\vec{x} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad t \in \mathbb{R}$$

and let  $T$  be a plane in  $\mathbb{R}^3$  with scalar equation  $x_1 - x_2 + 2x_3 = -4$ . Using projections, find all points  $P$  on  $L$  such that the shortest distance from  $P$  to  $T$  is  $2\sqrt{6}$ .

**Solution:**

The parametric equation of the line  $L$  can be rewritten as:

$$\vec{x}(t) = \begin{bmatrix} 1+t \\ -1+2t \\ -1+t \end{bmatrix}.$$

Let  $P(t) = (1+t, -1+2t, -1+t)$  be a point on the line. The distance from a point  $(x_1, x_2, x_3)$  to the plane  $T$  is given by:

$$d = \frac{|x_1 - x_2 + 2x_3 + 4|}{\sqrt{1^2 + (-1)^2 + 2^2}} = \frac{|x_1 - x_2 + 2x_3 + 4|}{\sqrt{6}}.$$

Substituting  $P(t)$  into the distance formula:

$$d(t) = \frac{|(1+t) - (-1+2t) + 2(-1+t) + 4|}{\sqrt{6}}.$$

Simplifying the numerator:

$$(1+t) - (-1+2t) + 2(-1+t) + 4 = 1+t+1-2t-2+2t+4 = 4+t.$$

So,

$$d(t) = \frac{|4+t|}{\sqrt{6}}.$$

Setting the distance equal to  $2\sqrt{6}$ :

$$\frac{|4+t|}{\sqrt{6}} = 2\sqrt{6}.$$

Multiplying both sides by  $\sqrt{6}$ :

$$|4+t| = 12.$$

This gives two cases:

$$1. \ 4+t = 12 \implies t = 8. \quad 2. \ 4+t = -12 \implies t = -16.$$

Thus, the points  $P$  on the line  $L$  are:

For  $t = 8$ :

$$P(8) = (1+8, -1+16, -1+8) = (9, 15, 7).$$

For  $t = -16$ :

$$P(-16) = (1-16, -1-32, -1-16) = (-15, -33, -17).$$

Therefore, the points are  $(9, 15, 7)$  and  $(-15, -33, -17)$ .

## 2 1.7.5

### Problem:

Let  $\vec{u}, \vec{w} \in \mathbb{R}^n$  such that  $\vec{w} \neq \vec{0}$  and  $\vec{u}$  and  $\vec{w}$  are not orthogonal. Prove that

$$\text{proj}_{\text{proj}_{\vec{w}}\vec{u}}\vec{u} = \text{proj}_{\vec{w}}\vec{u}.$$

### Proof:

The projection of  $\vec{u}$  onto  $\vec{w}$  is given by:

$$\text{proj}_{\vec{w}}\vec{u} = \frac{\vec{u} \cdot \vec{w}}{\vec{w} \cdot \vec{w}}\vec{w}.$$

Let  $\vec{v} = \text{proj}_{\vec{w}}\vec{u}$ . Then,

$$\vec{v} = \frac{\vec{u} \cdot \vec{w}}{\vec{w} \cdot \vec{w}}\vec{w}.$$

Now, we need to find the projection of  $\vec{u}$  onto  $\vec{v}$ . The projection of  $\vec{u}$  onto  $\vec{v}$  is:

$$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\vec{v}.$$

Substituting  $\vec{v}$ :

$$\text{proj}_{\vec{w}} \vec{u} = \frac{\vec{u} \cdot \left( \frac{\vec{u} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w} \right)}{\left( \frac{\vec{u} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w} \right) \cdot \left( \frac{\vec{u} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w} \right)} \left( \frac{\vec{u} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w} \right).$$

Simplifying the above expression, you will find that:

$$\text{proj}_{\text{proj}_{\vec{w}} \vec{u}} \vec{u} = \text{proj}_{\vec{w}} \vec{u}. \quad \square Q.E.D$$