1a. effective annual rate

$m = 4$
↳ no. of times compounded

$$r_{EAR} = \left(1 + \frac{r_s}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.05}{4}\right)^4 - 1$$

$$= 0.0509453$$

$$= 5.09\%$$

b. $$r_{BEY} = \left[\left(1 + \frac{r_s}{2}\right)^{\frac{m}{2}} - 1\right] \times 2$$

$$= \left[\left(1 + \frac{0.05}{4}\right)^{\frac{4}{2}} - 1\right] \times 2$$

$$= 0.0503125$$

$$= 5.03\%$$

c. $$e^{r_c} = \left(1 + \frac{r_s}{m}\right)^m$$

$$r_c = \ln\left(1 + \frac{r_s}{m}\right)^m$$

$$r_c = \ln\left(1 + \frac{0.05}{4}\right)^4$$

$$= 0.04969$$

$$= 4.97\%$$

2a. Bond price $(B) = \sum_{i=1}^{N} \frac{C_i}{\left(1 + \frac{y}{m}\right)^{m \times T_i}}$

where: C_i = coupon amt ($5)

y = avg zero rates. In this question $y = \frac{5 \times 3}{3} = 5$

m = no. of times compounded. In this question: once annually.

T_i = year.

need to add the FV

$$\therefore B = \sum_{i=1}^{3} \frac{5}{\left(1+\frac{0.05}{1}\right)^{1\times T_i}} + \frac{100}{\left(1+\frac{0.05}{1}\right)^{1\times 3}}$$

$$= \frac{5}{(1+0.05)^1} + \frac{5}{(1+0.05)^2} + \frac{5}{(1+0.05)^3} + \frac{100}{(1+0.05)^3}$$

$$= 100$$

slide 17 and 18.

b. Bond price = face value

$\therefore$ at par

$\therefore$ par-yield = zero coupon rate = 5%.

3. $B = \sum_{i=1}^{N} c_i e^{-y \cdot T_i} = \sum_{i=1}^{N} \frac{c_i}{e^{yT_i}}$

$T_{i+1} - T_i = \Delta T$ ← multiply cuz continuously compounde

$$\therefore B = \frac{c}{e^{y \cdot 1\Delta T}} + \frac{c}{e^{y \cdot 2\Delta T}} + \frac{c}{e^{y \cdot 3\Delta T}} + \cdots \frac{c}{e^{y \cdot N\Delta T}} + \frac{100}{e^{y \cdot N\Delta T}}$$

$$\therefore B = \frac{c}{e^{y\Delta T}} \left[\frac{1 - \frac{1}{e^{yN\Delta T}}}{1 - \frac{1}{e^{y\Delta T}}} \right] + \frac{100}{e^{yN\Delta T}}$$

$$B = \left[\frac{c}{e^{y\Delta T}} \times \frac{1 - \frac{1}{e^{yN\Delta T}}}{1 - \frac{1}{e^{y\Delta T}}} \right] + \frac{100}{e^{yN\Delta T}}$$

$$B = \left[\frac{c \times 1 - \frac{1}{e^{yN\Delta T}}}{e^{y\Delta T} - \frac{e^{y\Delta T}}{e^{y\Delta T}}} \right] + \frac{100}{e^{yN\Delta T}}$$

$$= c \left[\frac{1 - \frac{1}{e^{yN\Delta T}}}{e^{y\Delta T} - 1} \right] + \frac{100}{e^{yN\Delta T}}$$

At par yield ; $\therefore B = 100$

$$100 = c \left[\frac{1 - \frac{1}{e^{y\Delta T N}}}{e^{y\Delta T} - 1} \right] + \frac{100}{e^{yN\Delta T}}$$

$$100 - \frac{100}{e^{y\Delta T N}} = C\left[\frac{1 - \frac{1}{e^{yN\Delta T}}}{e^{y\Delta T} - 1}\right]$$

$$\frac{100\, e^{y\Delta T N} - 100}{e^{y\Delta T N}} = C\left[\frac{1 - \frac{1}{e^{yN\Delta T}}}{e^{y\Delta T} - 1}\right]$$

$$100\left(\frac{e^{y\Delta T N} - 1}{e^{y\Delta T N}}\right) = C\left[\frac{1 - \frac{1}{e^{yN\Delta T}}}{e^{y\Delta T} - 1}\right]$$

$$100\left[1 - \frac{1}{e^{y\Delta T N}}\right] = C\left[\left(1 - \frac{1}{e^{y\Delta T N}}\right) \times \frac{1}{e^{y\Delta T} - 1}\right]$$

$$100 = \left[C \times \cancel{\left(1 - \frac{1}{e^{y\Delta T N}}\right)} \times \frac{1}{e^{y\Delta T} - 1}\right] \div \cancel{\left(1 - \frac{1}{e^{y\Delta T N}}\right)}$$

$$100 = \frac{C}{e^{y\Delta T} - 1}$$

formula sheet

$$e^{y\Delta T} - 1 = \frac{C}{100}$$

$$e^{y\Delta T} = \frac{C}{100} + 1$$

$$y\Delta T = \ln\left(\frac{C}{100} + 1\right)$$

$$y = \frac{1}{\Delta T} \ln\left(\frac{C}{100} + 1\right)$$

4a. $B = \sum_{i=1}^{N} c_i e^{-yT_i}$

$c = 100 \times 8\% = 8 \qquad \Delta T = 1$

$\therefore B = 8e^{-.11\times1} + 8e^{-.11\times2} + 8e^{-.11\times3} + 8e^{-.11\times4} + 8e^{-.11\times5} + 100e^{-.11\times5}$

$= 8\left(e^{-.11} + e^{-.22} + e^{-.33} + e^{-.44} + e^{-.55}\right) + 100e^{-.11\times5}$

$= 86.801$

b. $D = \frac{1}{B} \sum_{i=1}^{n} t_i c_i e^{-yt_i}$

$\sum_{i=1}^{n} t_i c_i e^{-yt_i} = 1\times(8\times e^{-.11}) + 2(8\times e^{-.22}) + 3(8\times e^{-.33}) + 4(8\times e^{-.44})$
$+ 5(108\times e^{-.55})$
$= 369.423$

$D = \frac{1}{86.801} \times 369.423 = 4.256$

c. $\frac{\Delta B}{B} \simeq -D\Delta y \qquad \Delta y = -0.2\% = -0.002$

$\frac{\Delta B}{86.801} \simeq -4.256(-0.002)$

$\Delta B \simeq 86.801 \times (0.008512)$

$\Delta B \simeq 0.73885$

d. $C = \frac{1}{B} \sum_{i=1}^{n} t_i^2 c_i e^{-yt_i}$

$\sum_{i=1}^{n} t_i^2 c_i e^{-yt_i} = 1^2(8e^{-.11}) + 2^2(8e^{-.22}) + 3^2(8e^{-.33}) + 4^2(8e^{-.44}) + 5^2(108e^{-.55})$

$= 1724.810933$

$$\frac{1}{B}\sum_{i=1}^{n} t_i^2 c_i e^{-yt_i} = \frac{1724.810933}{86.801}$$

$$= 19.871$$

e. $B^* = \sum_{i=1}^{n} c_i e^{-y^* t_i}$ $\qquad y^* = 10.8\%$ $\qquad B^* \Rightarrow$ recalculated

$$= 8\left(e^{-0.108} + e^{-0.108\times2} + e^{-0.108\times3} + e^{-0.108\times4} + e^{-0.108\times5}\right) + 100e^{-0.108\times5}$$

$$= 87.5433914$$

$$\frac{\Delta B}{B} \approx -D\Delta y + \frac{1}{2}C\cdot(\Delta y)^2 \qquad \Delta y = 10.8 - 11 = -0.2\% = -0.002$$

$$\frac{\Delta B}{86.801} \approx -4.256(-0.002) + \frac{1}{2}\left(19.871 \times (-0.002)^2\right)$$

$$\Delta B \approx 0.00855142 \times 86.801$$

$$\Delta B \simeq 0.7422998$$

$$B^* - B = 87.5433914 - 86.801$$

$$= 0.7423914 \approx \Delta B$$

5. $D_\$ = B \times D$

$\therefore D_\$ = (1.5 \times 3.4) + (2 \times 2.8)$

$= 10.7$

$C_\$ = B \times C$

$\therefore C_\$ = (1.5 \times 20) + (2 \times 18)$

$= 66$

$$D_\$ + 2.9C + 1.4D = 0$$

$$C_\$ + 18C + 10D = 0$$

$\therefore \quad 10.7 + 2.9C + 1.4D = 0$

$2.9C + 1.4D = -10.7$

$1.4D = -10.7 - 2.9C$

$D = \dfrac{-10.7 - 2.9C}{1.4}$

$66 + 18C + 10D = 0$

$18C + 10D = -66$

$18C + 10\left(\dfrac{-10.7 - 2.9C}{1.4}\right) = -66$

$18C - \dfrac{107 - 29C}{1.4} = -66$

$25.2C - 107 - 29C = -92.4$

$25.2C - 29C = 14.6$

$-3.8C = 14.6$

$C = \$ -3.842 \text{ mil} \quad (\text{short})$

$D = \dfrac{-10.7 - 2.9(-3.842)}{1.4}$

$= \$0.3156 \text{ mil}$