Problem set 3

$$\prod_{i=1}^{90} x = x^{90}$$

1. $D(0,6m) = \frac{1}{1 + (0.012 \times 0.5)} = 0.99404$

$$D(0,1y) = \frac{1}{1 + (0.02 \times 1)} = 0.98039$$

$$D(0,2y) = \frac{1}{1 + (0.025 \times 2)} = 0.95238$$

a. $L(6m,1y) = \frac{1}{\Delta} \cdot \frac{D(0,6m) - D(0,1y)}{D(0,1y)}$

$$= \frac{1}{0.5} \times \frac{(.99404 - 0.98039)}{0.98039}$$

$$= 0.02785$$
$$= 2.785\%$$

b. $D(0,1y6m) = \frac{D(0,1y) + D(0,2y)}{2} = \frac{0.98039 + 0.95238}{2}$

$$= 0.966385$$

$$L(1y, 1y6m) = \frac{1}{D} \times \frac{D(0,1y) - D(0,1y6m)}{D(0,1y6m)}$$

$$= \frac{1}{0.5} \times \frac{0.98039 - 0.966385}{0.966385} = 0.028984$$

$$= 2.898\%$$

c. $L(1y6m, 2y) = \frac{1}{\Delta} \times \frac{D(0,1y6m) - D(0,2y)}{D(0,2y)}$

$= \frac{1}{0.5} \times \frac{0.966385 - 0.95238}{0.95238}$

$= 0.02941$

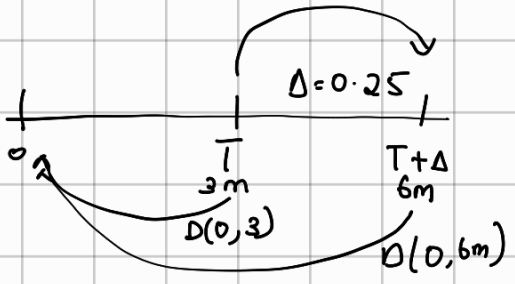
$= 2.941\%$

2a. $\tilde{D}(0,3m) = \frac{1}{1 + (0.0125 \times 0.25)} = 0.996885$

$D_0(0,3m) = \prod_{i=1}^{90} \frac{1}{1 + (0.007 \times \frac{1}{360})} = (0.999980555)^{90} = 0.998252$

b. $\tilde{D}(t,T) = (1 + \Delta \cdot L) \cdot \tilde{D}(t, T+\Delta)$

$1 + \Delta L = \frac{1}{\tilde{D}(3m,6m)}$



$\therefore \tilde{D}(0,3m) = \frac{1}{\tilde{D}(3m,6m)} \cdot \tilde{D}(0,6m)$

$\tilde{D}(3m,6m) = \frac{\tilde{D}(0,6m)}{\tilde{D}(0,3m)} = \frac{\frac{1}{1 + (0.0140 \times .5)}}{0.996885} = 0.996152$

$D_0(3m,6m) = \frac{1}{(1 + \Delta s_0)} = \frac{\prod_{i=1}^{N} D_0(0,6m)}{\prod_{i=1}^{N} D_0(0,3m)} = \frac{\prod_{i=1}^{180} \frac{1}{1 + (0.007 \times 1/360)}}{0.998252}$

$= \frac{0.996506}{0.998252}$

$= 0.998251$

c) $PV_{fix} = 0.25\left(\bar{D}(0,3m) + \bar{D}(0,6m) + \bar{D}(0,9m) + \bar{D}(0,12m)\right) \times 0.0145$

$= 0.25\left(0.99685 + 0.993049 + 0.988509 + 0.982801\right) \times 0.0145$

$= 0.01733$

$= 1.733\%$.

ii) $PV_{fix}^{collateral} = 0.25\left(D_0(0,3m) + D_0(0,6m) + D_0(0,9m) + D_0(0,12m)\right) \times 0.0145$

$= 0.0174236\,3084$

$= 1.742\%$.

d) $PV_{flt} = 1 - D(0,3m)$ (It is the 3m LIBOR so why $D(0,12m)$?)

$= 1 - 0.99685$

$= 0.00315$

$= 0.315\%$.

OK Ask Prof when to do $1 - D(0,T_n)$ and when $D(0,0) - D(0,T_n)$

ii) $\sum_{i=1}^{4} \Delta_{i-1} \times D_0(0,T_i) \cdot L(T_{i-1},T_i) = 0.25 \times \left(L(0,3m) \times D_0(0,3m) + L(3m,6m) D_0(0,6m) + L(6m,9m) D_0(0,9m) + L(9m,12m) D_0(0,12m)\right)$

$L(0,3m) = 0.0125$

$L(3m,6m) = \frac{1}{0.25} \times \frac{\bar{D}(0,3m) - \bar{D}(0,6m)}{\bar{D}(0,6m)} = 0.01545$

$L(6m,9m) = \frac{1}{0.25} \times \frac{\bar{D}(0,6m) - \bar{D}(0,9m)}{\bar{D}(0,9m)}$

$= \frac{1}{0.25} \times \frac{0.993049 - \bar{D}(0,9m)}{\bar{D}(0,9m)}$

$= 0.01837$

$$L(9m, 12m) = \frac{1}{0.25} \times \frac{\tilde{D}(0,9m) - \tilde{D}(0,12m)}{\tilde{D}(0,12m)}$$

$$= \frac{1}{0.25} \times \frac{0.9885086 - \tilde{D}(0,12m)}{\tilde{D}(0,12m)}$$

$$= 0.02323$$

$$PV = 0.25 \times \left(L(0,3m) \times D_0(0,3m) + L(3m,6m) D_0(0,6m) + L(6m,9m) D_0(0,9m) + L(9m,12m) D_0(0,12m) \right)$$

$$= 0.017386$$

$$= 1.7386\%$$