- Red for wrong or saw answers
- Blue for another way to solve

Additional Examples 2.

1a. $D(0,3m) = \dfrac{1}{1+(0.0125 \times 0.25)} = 0.99688$

b. $D(0,6m) = \dfrac{1}{1+(0.0140 \times 0.5)} = 0.993049$

$$D(3m,6m) = \frac{D(0,6m)}{D(0,3m)} = \frac{0.993049}{0.99688} = 0.99616$$

$\Delta = \frac{30}{360}$

c. $(1+\Delta_{2m}L_{2m})(1+\Delta_{7m}F(2m,9m)) = 1+\Delta_{9m}L_{9m}$

$$\therefore F(2m,9m) = \frac{1}{\Delta_{7m}} \frac{(1+\Delta_{9m}L_{9m}) - (1+\Delta_{2m}L_{2m})}{1+\Delta_{2m}L_{2m}}$$

$L(2m,9m) = \dfrac{1}{\frac{210}{360}} \dfrac{D(0,2m)-D(0,9m)}{D(0,9m)}$

$= \dfrac{360}{210} \dfrac{500}{50} -$

$= 0.016467$

$$= \frac{1}{\frac{210}{360}} \frac{\left(1+\frac{270}{360} \times 0.0155\right) - \left(1+\frac{60}{360} \times 0.0120\right)}{1+\frac{60}{360} \times 0.0120}$$

$$= \frac{1}{\frac{210}{360}} \left(\frac{1.011625 - 1.002}{1.002}\right)$$

$$= 0.016467$$

why did prof do this method?

d. 2×12 FRA

need to find this to prevent arbitrage

$F(2m, 12m)$

e I don't get this.

if 1m spot rate will remain unchanged a month later, we should short the 1x2 FRA since $F(1m,2m)>1.15\%$, we can borrow at 1.15% to deposit (lend) at $F(1m,2m)$ if we were right.

2a. $e^{F(0y,1y)\times 1} = e^{4\%\times 1}$

$e^{F(0y,1y)} = e^{4\%}$

$F(0_y,1_y) = 4\%$

$e^{F(0y,1y)\times 1} \times e^{F(1y,2y)\times(2-1)} = e^{4.5\%\times 2}$

$e^{4\%} \times e^{F(1y,2y)} = e^{9\%}$

$e^{4\% + F(1y,2y)} = e^{9\%}$

$F(1y,2y) = 9-4$

$F(1y,2y) = 5$

$e^{F(0y,1y)} \times e^{F(1y,2y)\cdot 1} \times e^{F(2y,3y)\cdot(3-2)} = e^{4.75\%\times 3}$

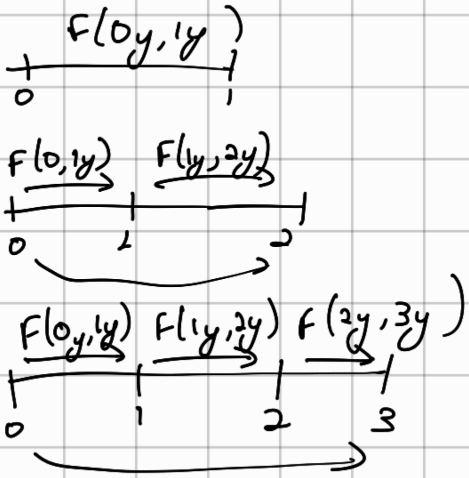
$e^{9\% + F(2y,3y)} = e^{14.25}$

$F(2y,3y) = 5.25\%$

b. $F(0y,1y) = 4\%$ $\qquad$ $1y = 4\%$

$F(1y,2y) = 5\%$ $\qquad$ $2y = 4.5\%$

$F(2y,3y) = 5.25\%$ $\qquad$ $3y = 4.75\%$



$\frac{F(0y,1y)}{1} = 4\%$ which is equal to 1y zero rate

$$\frac{F(0y,1y)+F(1y,2y)}{2} = \frac{4+5}{2} = 4.5\% = 2y \text{ zero rate}$$

$$\frac{F(0y,1y)+F(1y,2y)+F(2y,3y)}{3} = \frac{4+5+5.25}{3} = 4.75\%$$

|| equals to

3y zero rate

Zero rate at n year $= r$

$$\therefore e^{F(0,1y)\times 1} \times e^{F(1y,2y)\times 1} \cdots \times e^{F(n-1y,ny)\times 1} = e^{r\times n}$$

$$\therefore e^{F(0,1y)+F(1y,2y)+\dots+F(n-1y,ny)} = e^{rn}$$

$$\therefore \frac{F(0,1y)+F(1y,2y)+\dots+F(n-1y,ny)}{n} = r$$

3a. $FX_T = FX_0 \times \left(\frac{D_{USD}(0,T)}{D_{SGD}(0,T)}\right)$

$= 1.42 \times \left(\frac{0.964}{0.98}\right)$

$FX_T = 1.3968$

b. $FX_T = 1.3968$

if $FX_T = 1.42$

The arbitrage is the difference $0.02318

↳ At Time $t=0$ → long 1 USD bond by shorting SGD bond to get 0.964×1.42 $=1.36888$.

↳ At Time $t=T$ → USD matures and get back 1 USD. convert to SGD ($\underline{1.42\$}$)

→ Short SGD $= -1.36888 \times \frac{1}{0.98} = \underline{1.3968 \text{ USD}}$

→ Diff is the arbitrage

4a. k = fixed rate = 1.75%

$$\Delta = \frac{\cancel{30}^{0.5}}{\cancel{360}_{\cancel{60}}} \times \cancel{6}^{1} = 0.5$$

$$PV_{fix} = \sum_{i=1}^{4} D(0,T_i) \times 1.75\% \times 0.5$$

$$= 0.5 \times \big(D(0,6m) + D(0,12m) + D(0,18m) + D(0,24m)\big) \times 0.0175$$

$$D(0,6m) = \frac{1}{1+(0.0140 \times 0.5)} = 0.99305$$

$$D(0,12m) = \frac{1}{1+(0.0175 \times 1)} = 0.982801$$

$$D(0,18m) = \frac{1}{1+(0.0190 \times 1.5)} = 0.97229$$

$$D(0,24m) = \frac{1}{1+(0.02 \times 2)} = 0.96154$$

$$\therefore PV_{fix} = 0.5(0.99305 + 0.982801 + 0.97229 + 0.96154) \times 0.0175$$

$$= 0.0342097087 5$$

$$= 0.0342$$

b. $PV_{fix} = 1 - D(0,6m)$

$$= 1 - 0.99305$$

$$= 0.00695$$

c. $$S_{par} = \frac{1 - D(0,24m)}{0.5\big(D(0,6m) + D(0,12m) + D(0,18m) + D(0,24m)\big)}$$

$$= \frac{1 - 0.96154}{PV_{fix}/0.0175} = 0.01967$$

d. Before 3m

6m — 12m — 18m — 24m

After 3m

3m — 9m — 15m — 21m

Let R_i be the zero rates for the respective 'i's.

$\therefore$ $R_{3m} = 1.20\%$

$$R_{9m} = \frac{R_{6m} + R_{12m}}{2} = \frac{1.50 + 1.85}{2} = 1.675\%$$

$$R_{15m} = \frac{R_{12m} + R_{18m}}{2} = \frac{1.85 + 1.95}{2} = 1.9\%$$

$$R_{21m} = \frac{R_{18m} + R_{24m}}{2} = \frac{1.95 + 2.05}{2} = 2\%$$

$$D(0, 3m) = \frac{1}{1 + (0.0120 \times 0.25)} = 0.99700$$

$$D(0, 9m) = \frac{1}{1 + (0.01675 \times 9/12)} = 0.98754$$

$$D(0, 15m) = \frac{1}{1 + (0.019 \times 15/12)} = 0.976801$$

$$D(0, 21m) = \frac{1}{1 + (0.02 \times 21/12)} = 0.96618$$

$PV_{fix} = 0.5\,(0.99400 + 0.98454 + 0.976801 + 0.96618) \times 0.01967$
$= 0.038627660 79$

this won't work cause need Libor rate

$PV_{flt} = 1 - 0.96618$
$= 0.03382$

why is this method wrong?

$V = PV_{fix} - PV_{flt} = 0.005$

Question 5 I still have to see

6.

B	D	C
1	3.2	16
2.5	4	24

a. $D_\$ = (1 \times 3.2) + (2.5 \times 4)$
$= 13.2$ mil
$C_\$ = (1 \times 16) + (2.5 \times 24)$
$= 76.$ mil

b. $\Delta V \simeq -D_\$ \Delta y + \frac{C_\$}{2} (\Delta y)^2$ $\qquad \Delta y = 0.1\%$
$= 0.001$

$= -13.2(+0.001) + \frac{76}{2}(0.001)^2$

$= -0.013162$ mil

$V^{New} \simeq V + \Delta V = 1 + 2.5 - 0.013162 = 3.486838$ mil

c. B_3 and B_4

$$13.2 + 1.6B_3 + 3.2B_4 = 0$$

$$76 + 12B_3 + 20B_4 = 0$$

$$1.6B_3 + 3.2B_4 = 13.2$$

$$B_3 = \frac{-13.2 - 3.2B_4}{1.6}$$

$$76 + 12\left(\frac{-13.2 - 3.2B_4}{1.6}\right) + 20B_4 : 0$$

$$121.6 - 158.4 - 38.4B_4 + 32B_4 = 0$$

$$-36.8 \quad -6.48B_4 = 0$$

$$B_4 = -5.68 \text{ mil}$$

$$B_3 = 3.11 \text{ mil}$$