Assignment 1

1a. $D(0,6m) = \frac{1}{1+\left(\frac{1.75}{100} \times 0.5\right)} = 0.99133$

$D(0,12m) = \frac{1}{1+\left(\frac{1.8}{100} \times 1\right)} = 0.98232$

$D(0,9m) = \frac{D(0,6m) + D(0,12m)}{2} = \frac{0.99133 + 0.98232}{2} = 0.98683$

$L(9m,12m) = \frac{1}{12m-9m} \frac{D(0,9m) - D(0,12m)}{D(0,12m)} = \frac{1}{0.25} \frac{0.98683 - 0.98232}{0.98232}$

$= \frac{1}{0.25} \times \frac{451}{98232}$

$= \frac{451}{24558} = 0.01836$

$\simeq 1.84\%$

b. Quarter = 3m

$D(0,3m) = \frac{1}{1+\left(\frac{1.65}{100} \times 0.25\right)} = 0.99589$

$S_{par} = \frac{1 - D(0,12m)}{0.25\left[D(0,3m) + D(0,6m) + D(0,9m) + D(0,12m)\right]}$

$= \frac{1 - 0.98232}{0.25\left(0.99589 + 0.99133 + 0.98683 + 0.98232\right)}$

$= 0.01787 = 1.79\%$

c. $D(0,T) = e^{-R(0,T)\,T}$

$\therefore -R(0,T) \times T = \ln(D(0,T))$

$\therefore R(0,T) = -\dfrac{\ln(D(0,T))}{T}$

$\therefore R(0,3m) = -\dfrac{\ln(D(0,3m))}{0.25} = -\dfrac{\ln(0.99589)}{0.25} = 1.65\%$ ✓

$\therefore R(0,6m) = -\dfrac{\ln(D(0,6m))}{0.5} = -\dfrac{\ln(0.99132)}{0.5} = 1.74\%$ ✓

$R(0,12m) = -\dfrac{\ln(D(0,12m))}{1} = -\ln(0.98232) = 1.78\%$ ✓

2. $\left(1 + r_{SGD} \cdot \Delta\right) = \dfrac{F}{S} \cdot \left(1 + r_{USD} \cdot \Delta\right)$

$S = 1.42$

$F = 1.39$

$r_{USD} = 1.5$

$T = 0.5$

$r_{SGD} = \dfrac{\left(\frac{F}{S} \cdot (1 + r_{USD} \cdot \Delta) - 1\right)}{\Delta} = \dfrac{\frac{1.39}{1.42}\left(1 + 0.015 \times 0.5\right) - 1}{0.5} = -0.02457 = -2.76\%$ ✓

3a. $D(0,6m) = \dfrac{1}{1 + (0.015 \times 0.5)} = 0.99256$

$PV_{fix} = 0.5 \times [D(0,6m) + D(0,1y)] \times (0.018)$

$PV_{flt} = D(0,6m) \times 0.5 \times 0.015 + D(0,1y) \times 0.5 \times \dfrac{1}{0.5} \dfrac{D(0,6m) - D(0,1y)}{D(0,1y)}$

$$= D(0,6m) \times 0.5 \times 0.015 + D(0,6m) - D(0,1y)$$
$$= D(0,6m)(1 + 0.5 \times 0.015) - D(0,1y)$$
$$= 1 - D(0,1y)$$

$PV_{fix} = PV_{flt}$

$$0.5[D(0,6m) + D(0,1y)] \times 0.018 = 1 - D(0,1y)$$
$$(0.5(0.99256) + 0.5\,D(0,1y))\,0.018 = 1 - D(0,1y)$$
$$0.00893304 + 0.009\,D(0,1y) = 1 - D(0,1y)$$
$$-0.99106696 = -1.009\,D(0,1y)$$
$$D(0,1y) = 0.98223$$

$PV_{fix} = PV_{flt}$

$$0.5 \cdot [D(0,6m) + D(0,1y) + D(0,1.5y) + D(0,2y)] \cdot 0.02 = 1 - D(0,2y)$$

$$D(0,1.5y) = \frac{D(0,1y) + D(0,2y)}{2} = \frac{0.98223}{2} + 0.5D(0,2y)$$
$$= 0.491115 + 0.5D(0,2y)$$

$$0.5 \times [0.99256 + 0.98223 + 0.491115 + 1.5\,D(0,2y)] \times 0.02 = 1 - D(0,2y)$$

$$0.02465905 + 0.015\,D(0,2y) = 1 - D(0,2y)$$

$$D(0,2y) = 0.96093$$

$$D(0,1.5y) = 0.491115 + 0.5(0.96093)$$
$$= 0.971615$$

$$S = \frac{1 - D(0,1.5y)}{0.5 \times [D(0,6m) + D(0,1y) + D(0,1.5y)]}$$

$$= \frac{1 - 0.971615}{0.5 \times (0.99256 + 0.98223 + 0.971615)}$$

$$= 0.01927$$

$$= 1.93\%$$

b. $D(0,2.5y) = \frac{D(0,2y) + D(0,3y)}{2} = \frac{0.96093}{2} + 0.5D(0,3y)$

$$= 0.680465 + 0.5D(0,3y)$$

$$PV_{fix} = 0.5[D(0,6m) + D(0,1y) + D(0,1.5y) + D(0,2y) + D(0,2.5y) + D(0,3y)] \times 0.0205$$

$$= 0.5(0.99256 + 0.98223 + 0.971615 + 0.96093 + 0.680465 + 1.5D(0,3y)) \times 0.0205$$

$$= 0.04497495 + 0.015345D(0,3y)$$

$$PV_{flt} = 1 - D(0,3y)$$

$$PV_{fix} = PV_{flt}$$

$$0.04497495 + 0.015345D(0,3y) = 1 - D(0,3y)$$

$$D(0,3y) = 0.94056$$

$$D(0,2.5y) = 0.680465 + 0.5D(0,3y)$$

$$= 0.95074$$

$$S = \frac{D(0,1y) - D(0,3y)}{0.5 \times [D(0,1.5y) + D(0,2y) + D(0,2.5y) + D(0,3y)]}$$

$$= \frac{0.98223 - 0.96056}{0.5 \times (0.971615 + 0.96093 + 0.95074 + 0.96056)}$$

$= 0.02149$

$= 2.18\%$