Session 3: Multicurve Framework and OIS Discounting

Tee Chyng Wen

QF605 Fixed Income Securities

Overnight Index Swaps

Overnight Index Swaps (OIS) pay a fixed swap rate *vs.* a floating leg whose rate is compounded according to the daily overnight rate.

The swaps are quoted like IRS, by their par swap rate. Notes these swaps do not depend on LIBOR – they capture a different segment of the rates market.

OIS with maturity of $1y$ or less have a single payment on each leg. The compounding leg accrues daily, this is typically equal to the rate length.

overnight leg

$$\prod_{i=1}^{N}(1+\Delta_{i-1} f_o(t_{i-1},t_i)) = (1+\Delta S_o)$$

$$\prod_{i=1}^{N}\left(1+\cancel{\Delta_{i-1}}\cdot\frac{1}{\cancel{\Delta_{i-1}}}\cdot\frac{D_o(0,t_{i-1})-D_o(0,t_i)}{D_o(0,t_i)}\right) = (1+\Delta S_o)$$

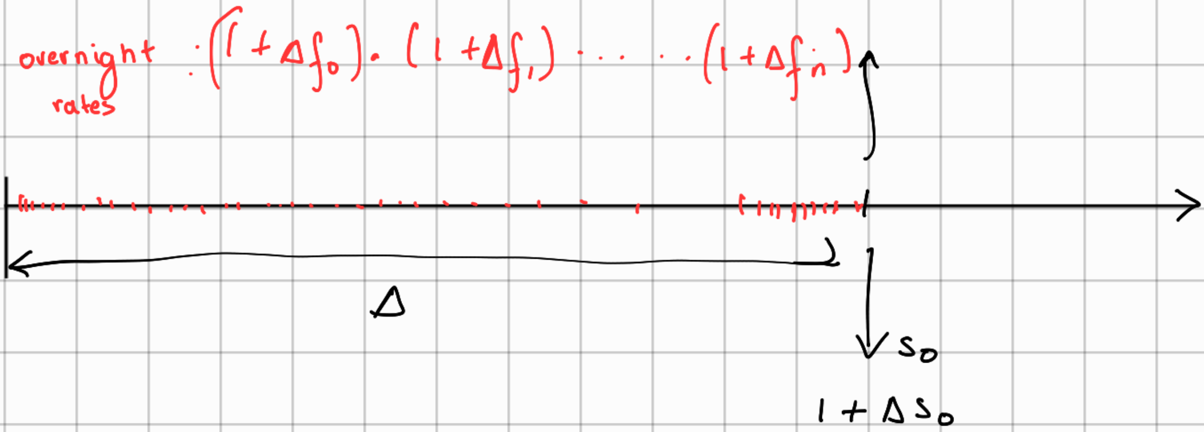
same formula as libor

$$= \frac{\cancel{D_o(0,t_i)} + D_o(0,t_{i-1}) - \cancel{D_o(0,t_i)}}{D_o(0,t_i)}$$

$$\prod_{i=1}^{N}\frac{D_o(0,t_{i-1})}{D_o(0,t_i)} = (1+\Delta S_o)$$

$$\frac{1}{D_o(0,t_N)} = (1+\Delta S_o)$$

formula sheet


overnight rates
$(1+\Delta f_0)\cdot(1+\Delta f_1)\cdots\cdots(1+\Delta f_n)$
Δ
S_0
$1+\Delta S_0$

Overnight Index Swaps

formula

Example We see the following instruments in the market:

Instrument	Quote
$1y$ OIS	1.5%
$12m$ LIBOR	2.75%

$D_o(0,1) = \frac{1}{1+0.015}$

OIS

$D(0,1) = \frac{1}{1+0.0275}$

Libor

Assuming 30/360 day count convention, calculate

1. The OIS discount factor for $1y$.
2. The $12m$-LIBOR discount factor for $1y$.

ans.: (1) $D_o(0,1y) = 0.9852,\ D(0,1y) = 0.9732.$

Types of Overnight Index

SOFR is an example of an overnight rate.

⇒ Based on transactions in the Treasury repurchase (**repo**) market where investors offer banks overnight loans backed by their bond assets.

Another important example is the **Effective Federal Funds Rate** (EFFR).

⇒ All US banks are required by law to hold reserves (a certain percentage of their deposits) with the banks that make up the Federal Reserve Bank system (US central bank).

⇒ There is incentive for these US banks to lend their excess money to other banks who might be short of reserves.

⇒ Every business day, US banks execute 1-day loan among themselves (hence the term "overnight"), and the interest rate charged is reported on the next day (by the New York Fed) as the weighted average called the **EFFR**.

⇒ These interbank transactions are known as the **Fed Funds market**.

Types of Overnight Index

The **Federal Open Market Committee** (FOMC) is responsible for implementing national monetary policy, and uses the EFFR to measure the success of their policy.

- The Federal Funds Rate is the target interest rate set by the Federal Open Market Committee (FOMC).
- FOMC cannot force banks to charge the exact Fed Fed Rate, but the Fed can adjust the money supply so that interest rates will move towards the target rate, either by increasing the amount of money so that interest rates fall or decreasing supply so interest rates rise.
- The FOMC meets eight times a year to set the target Fed Funds Rate based on prevailing economic conditions.

In summary, banks can borrow:

- **secured at SOFR** using Treasuries as collateral, or
- **unsecured at EFFR** without collateral.

Overnight Index Futures

SOFR and EFFR Futures are traded at CME. All contracts are cash settled, and for each contract month, trading ceases on the last day of the delivery month.

Used to hedge against unexpected shifts in short-term interest rates, or to speculate on interest rate movements.

$\Rightarrow$ Price Quote: $100 - F$, where F is the average daily overnight rate for the delivery month.

$\Rightarrow$ E.g. 98.5 means 1.5%

For the **current month** the contract price is equal to a weighted average of the actual overnight rates realized to date and the expected overnight rates for the remainder of the month.

The pricing for contracts in **deferred (future) months** is based on the average expected overnight rates for the contract month—based only on expected rates.

F : markets expectation of rates

F	1.5%	2.5%	3%	4%	5%
100 - F (quoted price)	98.5	97.5	97	96	95
	Feb	Mar	Apr	June	July

Futures Settlement

The overnight rate implied by the futures contract is equal to 100 minus the contract price.

$$\text{Settlement Price} = 100 - \frac{\sum_{i=1}^{n} f_i}{n}$$

where n is the number of calendar days in that month.

For current month contract, before the final settlement, the pricing becomes a weighted average of the realized and expected rates:

$$\text{Futures Price} = 100 - \left(\frac{\sum_{i=1}^{k} f_i}{n} + \frac{\sum_{i=k+1}^{n} \mathbb{E}[f_i]}{n}\right)$$

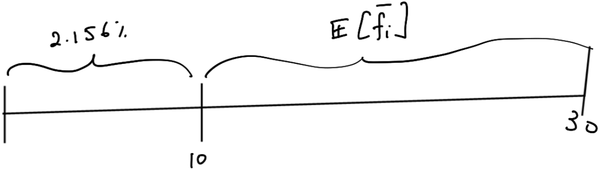
where k is the number of days to date and n is the number of calendar days in that month.

If market participants anticipate a rate change by the Fed, the market price of futures contracts will adjust to reflect the anticipated rate change.

Final Settlement

Example If we are 10 days into a month, and suppose there are 30 calendar days in this month, and the averaged realized rate is 2.156%, and the futures quotes is 97.75 (=2.25%), then

$$2.25 = \frac{10}{30} \times 2.156 + \frac{20}{30} \times \mathbb{E}\left[\frac{\sum_{i=k+1}^{n} f_i}{20}\right]$$
$$= \frac{10}{30} \times 2.156 + \frac{20}{30} \times \mathbb{E}[\bar{f}_i]$$
$$\mathbb{E}[\bar{f}_i] = 2.297$$



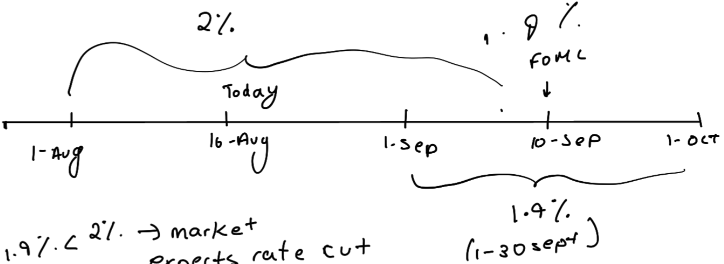
Implied Probability

Example Suppose today is 16-Aug, the Sep futures is trading at 98.1 (=1.9%). The prevailing Fed rate is 2%, we wish to work out the implied probability of a rate cut to 1.75% on 10-Sep, when an FOMC meeting is held.

⇒ We can work out the probability of a rate cut based on the futures quotes:

why these numbers

$$1.9 = \frac{10}{30} \times 2 + \frac{20}{30} \times \left[1.75 \times p + (1-p) \times 2\right]$$

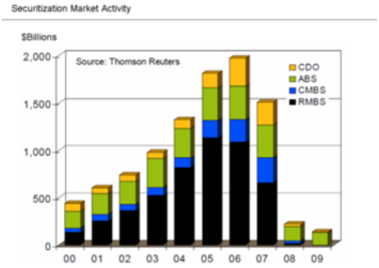


since 1.9% < 2% → market expects rate cut

OTC vs Exchange.

Background of Collateralization

- First sign on sub-prime lending issues began in mid 2007.
- Large amount of derivative contracts between banks and institutional investors.
- Banks began to worry about **counterparty's credit quality** (e.g. major investment banks vs. smaller financial institutions).
- Does **LIBOR discounting** still mean anything?
- Unique derivative price or counterparty specific valuation?



Importance of Collateralization

- Trades to be constantly **collateralized** to avoid financial loss in events of default.
- Portfolios are marked-to-market daily and **collaterals** posted.
- Either cleared through a **central clearing house** (e.g. London Clearing House LCH) or managed by operations at each banks.
- Interest will be paid on the posted collateral according to mutual agreement.
- **Credit Support Annex** (CSA) agreements are signed between counterparties to stipulate details of collateralizations.

Issues of Collateralization Discounting

- What can be posted as collateral?
- What is the interest to be paid on collateral?
- Should collateralization details enter the picture of derivative valuation?
- How wrong is LIBOR (uncollateralized) discounting?
- Does martingale valuation framework still work?

Collateralization

- Collateralization requires us to formulate a theory of swap valuation in the presence of **bilateral mark-to-market** (MTM).
- MTM requires that counterparties post collateral in the amount of the current MTM value of the contract.
- This generates an important departure from the traditional theory, which assumes that all cashflows exchanged between counterparties occur on the periodic swap dates.
- Since collateral is generally costly to post, these payments induce economic costs (benefits) to the payer (receiver).
- Given that these credit enhancements are part of the swap contract, they must be accounted for in valuation (**Credit Support Annex** to the **ISDA Master Swap Agreement**).

Collateralization

- An important feature of bilateral collateralization is that interest on collateral is often rebated.

- It is important to note that the posting of collateral, regardless of what or how it is posted, entails a cost and, for the other counterparty, a benefit.

- To appreciate this, we just need to observe that:
  - The receiver of collateral reduces or eliminates any losses conditional on **default**.
  - Collateral receivers, when allowed, typically reuse or **rehypothecate** the collateral for other purposes.
  - Even when interest is rebated, there is often a cost to posting collateral as the interest rebated is typically less than the payer's funding costs.
  - Most market participants borrow short term at rates higher than LIBOR, which generates an additional cost.

Matching Collateral & Payment Currency

Guideline 1: If a given portfolio has cashflows paid in a single currency, and if the collateral is paid in the same currency, and if the portfolio can be statically replicated by fixed payments. Then the portfolio should be discounted at the interest rate index paid on the collateral.

Reason: Consider the case where we receive 1 unit of currency in one day. Our aim is to solve for the present value of the cashflow – the collateral call amount – so that the next day's payment are net flat, i.e. there's no credit risk. The mechanics of the collateral are that

- We ask for the PV today: $D(t, t+\delta)$. day fraction usually $\frac{1}{360}$
- On the next day, we pay: $D(t, t+\delta)[1+\delta f(t, t+\delta)]$.

In order for there to be no credit risk, the amount received on the next day must be equal to the amount of collateral we hold, i.e.

$$1 = D(t, t+\delta)[1+\delta f(t, t+\delta)] \quad \Rightarrow \quad D(t, t+\delta) = \frac{1}{1+\delta f(t, t+\delta)}.$$

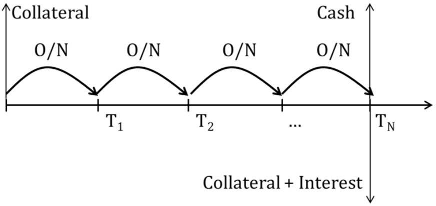
Matching Collateral & Payment Currency

The overnight discount factor is therefore based on the collateral interest rate.

Suppose now that the receivable is more than a day away, at time T in the future. Applying the above relationship recursively, we have

$$D(t,T) = \prod_{i=1}^{\frac{T-t}{\delta}} \frac{1}{1+\delta f(t+(i-1)\delta, t+i\delta)}$$

This formula is nothing more than the expression of term discount factors via the realised daily compound of overnight rates — the overnight index swap (OIS) market allows us to hedge the forwards at time t.



Matching Collateral & Payment Currency

Example You are expecting to receive a cashflow of $250,000 in $6m$'s time. We see the following quotes in the market:

Instrument	Quote
$6m$ OIS	1.05%
$6m$ LIBOR	2.1%

Using 30/360 day count convention, calculate the PV of this cashflow

1. if the trade is collateralized. → OIS or overnight
2. if the trade is uncollateralized. → LIBOR

OIS → same calc as Libor

ans.: (1) $PV = 248,694$, (2) $PV = 247,402$

overnight is the diff one

Different Collateral & Payment Currency

This is an important case – common collateral currencies are USD, EUR, GBP, JPY. For trades not denominated in one of those currencies, there is almost certainly going to be a mismatch between the payment currency and the collateral currency.

Guideline 2: If a given portfolio has cashflows paid in a single currency x, and the collateral paid in another currency y, and the portfolio can be statically replicated by fixed payments. Then the portfolio should be discounted at

$$D_{x,y}(t,T) = D_y(t,T)\frac{FX_{x,y}(t,T)}{FX_{x,y}(t,t)}.$$

Reason: Again considering the case of single fixed receivable cashflow:

- The currency x's PV is $D_{x,y}(t,t+\delta)$.
- We convert to currency y using the spot FX rate – this is the amount we require in currency y: $FX_{x,y}(t,t)D_{x,y}(t,t+\delta)$.

Different Collateral & Payment Currency

We put on an overnight FX forward hedge, so that we can assume that the $1d$ forward rate $f(t,t+\delta)$ is realised – this can be done at zero cost.

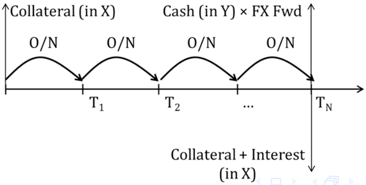
On the next day, we pay (note that we have accrued interest in y)

$$FX_{x,y}(t,t)D_{x,y}(t,t+\delta)[1+\delta f_y(t,t+\delta)]$$

Also on the next day, the counterparty pays one unit of currency x. In currency y this is worth $FX_{x,y}(t,t+\delta)$ as a result of our FX hedge.

As before, we equate the 2 payments and solve for the cross-currency discount factor

$$D_{x,y}(t,t+\delta) = \frac{1}{1+\delta f_y(t,t+\delta)} \frac{FX_{x,y}(t,t+\delta)}{FX_{x,y}(t,t)} = \underbrace{D_y(t,t+\delta)}_{=\frac{1}{(1+\delta\cdot f_y)}} \frac{FX_{x,y}(t,t+\delta)}{FX_{x,y}(t,t)}.$$



Different Collateral & Payment Currency

To extend to the case where the receivable is paid at time T in the future, we note that

- We are solving for the currency x present value $D_{x,y}(t,T)$.
- We convert to currency y using the spot rate (this is the amount we require in y): $FX_{x,y}(t,t)D_{x,y}(t,T)$.
- We put on an FX forward hedge to time T, so that at time T, when the counterparty pays 1 unit of currency x, we have, in currency y, thanks to our FX hedge: $FX_{x,y}(t,T)$.
- Now consider the full period, we pay at time T:

$$FX_{x,y}(t,t)D_{x,y}(t,T)\prod_{i=1}^{\frac{T-t}{\delta}}[1+\delta f_y(t+(i-1)\delta,t+i\delta)].$$

Different Collateral & Payment Currency

We can enter into a currency y overnight index swap (OIS) such that we are effectively paying

$$\frac{FX_{x,y}(t,t)D_{x,y}(t,T)}{D_y(t,T)}.$$

Equating pay and receive leg, and solving, we obtain

$$\begin{aligned} FX_{x,y}(t,T) &= \frac{FX_{x,y}(t,t)D_{x,y}(t,T)}{D_y(t,T)} \\ \Rightarrow \quad D_{x,y}(t,T) &= \frac{FX_{x,y}(t,T)D_y(t,T)}{FX_{x,y}(t,t)}. \end{aligned}$$

IRS Example

Example Consider the following market quotes of *collateralized* interest rate swaps (annual payment):

fixed rate

Maturity	Instrument	Rate
$1y$	IRS	2%
$2y$	IRS	2.5%

Determine the forward LIBOR rate $L(1y, 2y)$. Assume that collateral are posted in cash of the same denomination of the swap and that the daily overnight compounding rate $f(t_{i-1}, t_i)$ is flat at 0.5%. Use 30/360 day count convention.

→ assignment ques 3. $PV_{fix} = PV_{flt}$

$\therefore D_0(0,1) \times 2\% = D_0(0,1) \times L(0,1)$

$\therefore L(0,1) = 2\%$.

ans.: $L(0, 1y) = 2\%$, $L(1y, 2y) = 3\%$.

formulasheet

$\left(D_0(0,1) + D_0(0,2)\right) \times 2.5 = D_0(0,1) \times 2\% + D_0(0,2) \times L(1,2)$

Liquidity Value Adjustment

- Liquidity Value Adjustment (LVA) is the present value of the difference between the risk free rate vs. the collateral rate paid/rebated on the collateral received/posted.
- This corresponds to the cashflow generated daily due to the mark-to-market process between 2 counterparties with mutual collateralization agreement.
- From the perspective of the trading desk, as pricing and valuation architecture for collateral agreements has been rolled out, it is no longer meaningful to calculate LVA as a separate adjustment.
- All discounting should takes collateral agreement into account (instead of assuming risk free discounting and implementing a correction step later).