# Additional Example 4

1 $\mathbb{P} \rightarrow dS_t = \mu S_t dt + \sigma S_t dW_t$

$S_T = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T^{\mathbb{P}}}$

$\mathbb{Q} \rightarrow dB_t = rB_t dt$

$B_T = B_0 e^{rT}$

a) $\mathbb{E}^{\mathbb{P}}[S_T] = \mathbb{E}^{\mathbb{P}}\left[S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T}\right]$

$= S_0 e^{(\mu - \frac{1}{2}\sigma^2)T}\, \mathbb{E}\left[e^{\sigma W_T}\right]$

$= S_0 e^{(\mu - \frac{1}{2}\sigma^2)T} \cdot e^{\frac{1}{2}\sigma^2 T}$

$= S_0 e^{\mu T}$

b). $\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left(-\kappa W_T - \frac{1}{2}\kappa^2 T\right)$ ; $\kappa = \frac{\mu - r}{\sigma}$

$dW_t^B = dW_t + \frac{\mu - r}{\sigma} dt$

$\mathbb{Q}$ $dS_t = \mu S_t dt + \sigma S_t dW_t$

$= \mu S_t dt + \sigma S_t \left(dW_t^B - \frac{\mu - r}{\sigma} dt\right)$

$= rS_t dt + \sigma S_t dW_t^B$

$\therefore \mathbb{E}^{\mathbb{Q}}[S_T] = S_0 e^{rT}$

2. $dL_i(t) = \sigma_i L_i(t) dW^{i+1} \leftarrow \mathbb{Q}^{i+1}$

$L_i(T) = L_i(0)^{\left(-\frac{\sigma_i^2 T}{2}\right) + \sigma_i W_T^{i+1}}$

Fixed income
↳ Lec 4 slide 9

$F_i(0) = D_{i+1}(0)\,\Delta_i\,\mathbb{E}^{i+1}\left[(K - L_i(T_i))^+\right]$

$= D_{i+1}(0)\,\Delta_i\left[K\Phi(-d_2) - L_i(0)\Phi(-d_1)\right]$ ← Derivatives Lec 2

$d_1 = \frac{\ln\left(\frac{L_i(0)}{K}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$ ; $d_2 = d_1 - \sigma\sqrt{T}$

3. $dS_{n,N}(t) = \sigma_{n,N} S_{n,N}(t)\, dW^{n+1,N}$

$$S_{n,N}(t) = S_{n,N}(0) \exp\left(-\frac{\sigma_{n,N}^2 T}{2} + \sigma_{n,N} W_T^{n+1,N}\right)$$

$$V^{rec}_{n,N}(0) = P_{n+1,N}\, \mathbb{E}^{n+1,N}\left[(K - S_{n,N}(T))^+\right]$$

$$= P_{n+1,N}(0) \cdot \left[K\Phi(-d_2) - S_{n,N}(0)\Phi(-d_1)\right]$$

← Ask prof if we can just write this and say this is $F_0$ valuation formula?