QF605 Fixed-Income Securities
Solutions to Assignment 1

1. (a) Since the market is uncollateralized, all discounting will be based on LIBOR. The $6m$ and $12m$ spot discount factors are:

$$\begin{aligned} D(0,6m) &= \frac{1}{1+0.5\times 0.0175} = 0.991326 \\ D(0,12m) &= \frac{1}{1+1.0\times 0.018} = 0.982318 \end{aligned}$$

The $9m$ spot discount factor is linearly interpolated:

$$D(0,9m) = \frac{D(0,6m)+D(0,12m)}{2} = 0.986822$$

Hence, the forward rate for the 9×12 FRA should be

$$L(9m,12m) = \frac{1}{0.25}\times\frac{D(0,9m)-D(0,12m)}{D(0,12m)} = 1.834\% \quad \triangleleft$$

(b) The $3m$ discount factor is

$$D(0,3m) = \frac{1}{1+0.25\times 0.0165} = 0.995892$$

The $1y$ par swap rate with quarterly payment is

$$\begin{aligned} S &= \frac{1-D(0,12m)}{0.25\times(D(0,3m)+D(0,6m)+D(0,9m)+D(0,12m))} \\ &= 1.7877\% \quad \triangleleft \end{aligned}$$

(c) The relationship between a continuously compounded zero rate and the discount factor is

$$D(0,T) = e^{-R(0,T)\cdot T} \qquad \Rightarrow \qquad R(0,T) = -\frac{\log D(0,T)}{T}.$$

The zero rates are

$$\begin{aligned} R(0,3m) &= -\frac{\log D(0,3m)}{0.25} = 1.647\% \quad \triangleleft \\ R(0,6m) &= -\frac{\log D(0,6m)}{0.5} = 1.742\% \quad \triangleleft \\ R(0,12m) &= -\frac{\log D(0,12m)}{1} = 1.784\% \quad \triangleleft \end{aligned}$$

2. It is given that $FX_T = 1.39$, $FX_0 = 1.42$, $L_{\text{USD}}(0,6m) = 1.5\%$. Using interest rate parity relationship, we have

$$\begin{aligned} 1+0.5\cdot L_{\text{SGD}}(0,6m) &= \frac{FX_T}{FX_0}\Big(1+0.5\cdot L_{\text{USD}}(0,6m)\Big) \\ \Rightarrow \quad L_{\text{SGD}}(0,6m) &= \left[\frac{1.39}{1.42}\Big(1+0.5\cdot 0.015\Big)-1\right]\times\frac{1}{0.5} = -2.757\% \quad \triangleleft \end{aligned}$$

3. (a) Since the market is uncollateralized, the $6m$ discount factor is

$$D(0,6m) = \frac{1}{1+0.5\times 0.015} = 0.99256$$

Looking at the $1y$ IRS, we have

$$\begin{aligned} PV_{\text{fix}} &= PV_{\text{flt}} \\ 0.5\times 1.8\%\times (D(0,6m)+D(0,1y)) &= 1-D(0,1y) \\ \Rightarrow \quad D(0,1y) &= 0.98223 \end{aligned}$$

Looking at the $2y$ IRS, we have

$$0.5\times 2\%\times (D(0,6m)+D(0,1y)+D(0,1.5y)+D(0.2y)) = 1-D(0,2y)$$

The discount factor for $1.5y$ is linearly interpolated as

$$D(0,1.5y) = \frac{D(0,1y)+D(0,2y)}{2},$$

hence we have

$$\begin{aligned} 0.5\times 2\%\times (D(0,6m)+1.5D(0.1y)+1.5D(0,2y)) &= 1-D(0,2y) \\ \Rightarrow \quad D(0,2y) &= 0.96093, \end{aligned}$$

and

$$D(0,1.5y) = \frac{0.98223+0.96093}{2} = 0.97158$$

So the $1.5y$ par swap rate is given by

$$S_{1.5y} = \frac{1-D(0,1.5y)}{0.5\times (D(0,6m)+D(0,1y)+D(0,1.5y))} = 1.929\% \quad \triangleleft$$

(b) Looking at the $3y$ IRS, we have

$$\begin{aligned} 0.5\times 2.05\%\times \Big(&D(0,6m)+D(0,1y)+D(0,1.5y) \\ &+D(0,2y)+D(0,2.5y)+D(0,3y)\Big) = 1-D(0,3y) \end{aligned}$$

The discount factor for $2.5y$ is linearly interpolated as

$$D(0,2.5y) = \frac{D(0,2y)+D(0,3y)}{2}$$

hence we have

$$\begin{aligned} 0.5\times 2.05\%\times \Big(&D(0,6m)+D(0,1y)+D(0,1.5y) \\ &+1.5D(0,2y)+1.5D(0,3y)\Big) = 1-D(0,3y) \\ \Rightarrow \quad &D(0,3y) = 0.94056 \end{aligned}$$

and

$$D(0,2.5y) = \frac{0.96093+0.94056}{2} = 0.950745$$

So the forward starting swap has a par swap rate of

$$S = \frac{D(0,1y)-D(0,3y)}{0.5\times (D(0,1.5y)+D(0,2y)+D(0,2.5y)+D(0,3y))} = 2.179\% \quad \triangleleft$$