Ans 5: →

$$(K_s + K_p^D)r - \theta q = B_f \cdot f.$$

known

known

$$-\theta' r + C_p^{s-1} q = B_v \cdot v$$

known known known

① $$K_s = \int_{V_s} B_r(x)' \, C_s \cdot B_r(x) \, dv_s$$

② $$K_p^D = \sum_{i=1}^{N_p} \int_{V_p^i} B_r(x)' R_s^{i'} C^D R_s^i \, B_r(x) \cdot dV_p^i$$

③ $$\theta = \sum_{i=1}^{N_p} \int_{V_p^i} B_r(x)' R_s^i \, h R_D^i \cdot B_q^i(x) \cdot dV_p^i$$

④ $$C_p^{s-1} = \sum_{i=1}^{N_p} \int_{V_p^i} B_q^i(x)' R_D^{i'} \beta^s \cdot R_D^i B_q^i(x) \cdot dV_p^i$$

To find: →

① Free deflection: →

$$f = 0$$

② Block force: →

$$(L.H.S = 0)$$

$$u(H_0) = 0$$

$$H_d \cdot K^{-1} \theta \cdot C_p^s B_v \cdot v + H_D K^{-1} B_f \cdot f.$$

$$K = K_s + K_p^D - \theta C_p^s \theta'$$

$$\cancel{K_{(P)}^{D} \neq \hat{C}^{D} \int_{-W/2}^{W/2} \int_{0}^{L} \int_{-t_{(P)}/2}^{t_{(P)}/2} \left\{ Z^2 \left[4 \right.\right.}$$

$$\hat{C}^{D} = C_{11}^{D} - 2V_{12} \cdot C_{12}^{D} - \cancel{2V_{13} \cdot C_{13}^{D}} + \cancel{V_{12}^{2} C_{22}^{D}} + \cancel{2V_{12} \cdot V_{13} C_{23}^{D}} + \cancel{V_{13}^{2} \cdot C_{33}^{D}}$$

$$(\hat{C}^{D} = C_{11}^{D})$$

$$\hat{h} = h_{13} - \cancel{V_{12} \cdot h_{23}} - \cancel{V_{13} \cdot h_{33}} \qquad (\hat{h} = h_{13})$$

$$B_{2}(x) = \begin{bmatrix} 0 & 0 & Z\partial^2/\partial x^2 \\ 0 & 0 & -V_{12} \cdot Z \cdot \partial^2/\partial x^2 \\ 0 & 0 & -V_{13} \cdot Z \cdot \partial^2/\partial x^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & x^3 - L^2 x \\ x^2 - Lx & \end{bmatrix}$$

$$= \begin{bmatrix} \text{(b)} -V_{12} \cdot Z \cdot \frac{\partial^2}{\partial x^2}(x^2 - Lx) \\ = -V_{12} \cdot Z \cdot \frac{\partial}{\partial x}(2x - L) \\ -2 \cdot Z \cdot V_{12} \end{bmatrix}$$

(a)1 $Z \frac{\partial^2}{\partial x^2}(x^2 - Lx)$

$Z \frac{\partial}{\partial x}(2x - L) = 2Z.$

~~(b)~~ $\cancel{-V_{12} \cdot Z \cdot \frac{\partial^2}{\partial x^2}}$

(a)2 $Z \frac{\partial^2}{\partial x^2}(x^3 - L^2 x)$

$= Z \cdot \frac{\partial}{\partial x}(3x^2 - L^2)$

$= \cancel{Z} \cdot$

$\cdot Z) \, 6x.$

$-V_{12} \cdot Z \cdot \frac{\partial^2}{\partial x^2}(x^2 - Lx)$

$= -V_{12} \cdot Z \cdot \frac{\partial}{\partial x}(2x - L)$

$= -V_{12} \cdot Z \cdot 2$

$-V_{12} \cdot Z \cdot \frac{\partial^2}{\partial x^2}(x^3 - L^2 x)$

$= -V_{12} \cdot Z \cdot \frac{\partial}{\partial x}(3x^2 - L^2)$

$= \underline{-V_{12} \cdot Z \cdot 6x}$

$$B_x(x) = \begin{bmatrix} 2z & 6x\,z \\ -2z \cdot \nu_{12} & -(6x)(\nu_{12})\cdot z \quad \cancel{\nu_{12}\cdot 6} \\ -\nu_{13}\cdot 2z & -\nu_{13}\cdot 6z\cdot x \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_f = \begin{bmatrix} -L^2/4 \\ -3L^3/8 \end{bmatrix}$$

Input vector (pin pinned condition)

$$B_q^1(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (1/wL) & 0 \end{bmatrix} \qquad B_q^2(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \left(\frac{1}{wL}\right) \end{bmatrix}$$

Textbook Expression 5.107 : (a) (equilibrium equations) (a) ✓

R.H.S : →

(a) Equilibrium Expression

R.H.S

$$\begin{Bmatrix} -L^2/4 \\ -3L^3/8 \\ 0 \\ 0 \end{Bmatrix} f + \begin{Bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{Bmatrix} \checkmark$$

L.H.S : →

$$\begin{bmatrix} C_{11}^D \frac{bt^3L}{3} & C_{11}^D \frac{bt^3L^2}{2} & \frac{1}{4} h_{13} t^2 & -\frac{1}{4} h_{13} t^2 \\ C_{11}^D \frac{bt^3L^2}{2} & C_{11}^D \cdot bt^3L^2 & \frac{3L}{8} \times h_{13} t^2 & -\frac{3L}{8} h_{13} t^2 \\ \frac{1}{4} h_{13} t^2 & \frac{3L}{8} h_{13} t^2 & \frac{1}{2} \times \beta_{33}^S \frac{t}{bL} & 0 \\ -\frac{1}{4} h_{13} t^2 & -\frac{3L}{8} \times h_{13} t^2 & 0 & \frac{1}{2} \beta_{33}^S \cdot \frac{t}{bL} \end{bmatrix} \begin{bmatrix} q_2 \\ q_3 \\ q_4 \\ q_e \end{bmatrix}$$

5.4

Expression stiffness matrix.

$$\begin{bmatrix} ①\ \hat{C}_D \cdot w\, t_p^3 \cdot L/3 & ②\ \hat{C}^D w\, t_p^3 \cdot L^2/2 & ③\ \hat{h}\, t_p^2/4 & ④\ -\hat{h}\, t_p^2/4 \\ ⑤\ \hat{C}^D w\, t_p^3 L^2/2 & ⑥\ \hat{C}^D w\, t_p^3 L^3 & ⑦\ 3\hat{h}\, t_p^2 L/8 & ⑧\ -3\hat{h}\, t_p^2 L/8 \\ ⑨\ \hat{h}\, t_p^2/4 & ⑩\ 3h\, t_p^2 L/8 & ⑪\ \beta_{33}^S\, t_p/2wL & ⑫\ 0 \\ ⑬\ -\hat{h}\, t_p^2/4. & ⑭\ -3h \cdot t_p^2 \cdot L/8 & ⑮\ 0 & ⑯\ \dfrac{\beta_{33}^S \cdot t_p}{2wL} \end{bmatrix}$$

$\hat{h} = h_{13} - \nu_{12} \cdot h_{23} - \nu_{13} \cdot h_{33}$

$\hat{h} = \underline{-5.58 \times 10^8}$

C_{11}^D

①: $\hat{C}_D (w) \cdot t_p^3 \cdot L/3 = \dfrac{(76.6 \times 10^9) \times (10 \times 10^{-3}) \times (0.5 \times 10^{-3})^3 \times (60 \times 10^{-3})}{3} = \cancel{7.66 \times 10^{-3}} = \underline{\underline{1.915 \times 10^{-3}}}$

②: $\dfrac{C_D (w) \cdot t_p^3 \cdot L^2}{2} = \underline{\underline{1.72 \times 10^{-4}}}$: ⑤

③: $\dfrac{\hat{h}\, t_p^2}{4} = \dfrac{\cancel{-5.8}\,(-5.58 \times 10^8) \times (0.5 \times 10^{-3})^2}{4} = \underline{-3.487 \times 10^1} =$ ⑨

④: $\dfrac{-\hat{h} \cdot t_p}{4} = + 3.487 \times 10^1 =$ ⑬

⑤: $\dfrac{\hat{C}_D \cdot w \cdot t_p^3 \cdot L^2}{2} = \dfrac{(76.6 \times 10^9) \times (10 \times 10^{-3}) \times (0.5 \times 10^{-3})^3 \times (60 \times 10^{-3})^2}{2}$

$= \underline{1.723 \times 10^{-4}}$

⑥: $\hat{c}_D \times W \times t_{P}^{3} \cdot L^{3}$

$= (76.6 \times 10^{9}) \times (10 \times 10^{-3}) \times (0.5 \times 10^{-3})^{3} \times (60 \times 10^{-3})^{3}$

$= 2.068 \times 10^{-5}.$

⑦: $\dfrac{3\hat{h} \cdot t_{P}^{2} \cdot L}{8} = \dfrac{3 \times \overset{(-5.58 \times 10^{8})}{\cancel{(-6.5 \times 10^{8})}} \times (0.5 \times 10^{-3})^{2} \times \overset{(60 \times 10^{-3})}{\cancel{100 \times}}}{8}$

$= \dfrac{3 \times (-5.58 \times 10^{8}) \times (0.5 \times 10^{-3})^{2} \times (60 \times 10^{-3})}{8}$

$= -3.138 = $ ⑩

⑧: 3.138 :

$\boxed{⑨ : ④}$ $\boxed{⑩ : -3.138}$

⑪: $\dfrac{(2.8 \times 10^{7}) \times (0.5 \times 10^{-3})}{2 \times (10 \times 10^{-3}) \times (60 \times 10^{-3})} = 1.16 \times 10^{7}$

⑯: $1.16 \times 10^{7}.$

(b) stiffness matrix: $\longrightarrow$

$$K = \begin{bmatrix} 1.915\times10^{-3} & 1.723\times10^{-4} & -3.48\times10^{1} & 3.485\times10^{1} \\ 1.723\times10^{-4} & 2.06\times10^{-5} & -3.13\times10^{0} & 3.13\times10^{0} \\ -3.485\times10^{1} & -3.13\times10^{0} & 1.167\times10^{7} & 0 \\ 3.485\times10^{1} & 3.13\times10^{0} & 0 & 1.167\times10^{7} \end{bmatrix}$$

(c) FREE displacement: $\longrightarrow$ (At Zero Resistance force).

$$[k]\,r = b_{f}\cdot f + b_{v}\cdot v$$

\# Generalized coordinate calculation: $\longrightarrow$

$$r = [k^{-1}]\,[b_{f}\cdot f + b_{v}\cdot v]$$

$$[k^{-1}] = \cancel{\begin{bmatrix} 1.91e^{-3} \\ 1.723e^{-4} \\ -3.487e^{1} \end{bmatrix}} \begin{bmatrix} 0.72 & -1.787 & 0.677 & 0.744 \\ -0.80 & 1.829 & -0.757 & -0.697 \\ 0.02 & -0.135 & 0.101 & 0.11 \\ -0.0015 & 0.132 & -0.001 & -0.01 \end{bmatrix}$$

$$r = \begin{bmatrix} 3.50\times10^{-3} \\ 0 \\ 9.617\times10^{-8} \\ -9.617\times10^{-8} \end{bmatrix} = \begin{bmatrix} 3.50\times10^{-3} \\ 0 \\ 9.617\times10^{-8} \\ -9.617\times10^{-8} \end{bmatrix}$$

Cubic Expression: $\longrightarrow$

$$W(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$

(B) (C) free displacement (pin-pinned beam).

$$u_3(L/2)_{f=0} = \frac{3}{4} \times \frac{\hat{h}}{c^D \times \beta_{33}^S} \times \frac{L^2}{t_P^2} \times \frac{1}{(1 - 3k^2/4)} \cdot V$$

BC's (pin-pinned condition)

$$\boxed{W(0)=0}$$
$$\boxed{W(L)=0}$$

$$k^2 = \frac{\hat{h}^2}{c^D \times \beta_{33}^S}$$

$W(0) = 0$ $\boxed{a_0 = 0}$

$$W(L) = a_0 + a_1 \cdot L + a_2 L^2 + a_3 L^3$$
$$0 = 0 + a_1 L + a_2 L^2 + a_3 L^3$$

$$k^2 = \frac{(-5.58 \times 10^8)^2}{(76.6 \times 10^9) \times (2.8 \times 10^7)} = 1.457 \times 10^{-1}$$

$$= \frac{3}{4} \times \frac{(-5.58 \times 10^8)}{(76.6 \times 10^9) \times (2.8 \times 10^7)} \times \frac{(60 \times 10^{-3})^2}{(0.5 \times 10^{-3})^2} \times \frac{1}{1 - \frac{3(1.45 \times 10^{-1})}{4}} \times V$$

$$= \frac{(-2.809 \times 10^{-6}) \times 1 \times 4}{(4 - 3(1.45 \times 10^{-1}))} = \underline{-3.151 \times 10^{-6}} \, V$$

$$\boxed{\text{free displacement} = -3.151 \times 10^{-6} V}$$

5.8 (Graph)

5.9

There have been two graphical plots.

One in with ⊖ V that means the voltage to be applied along the opp. dir.ⁿ of current flow

i.e polarity need to reversed.

ⓓ Displacement calculations : ⟶

$$u_3(L/2) = \frac{3}{16\,\hat{c}^D} \times \frac{L^2}{W \cdot t_p^3} \times \frac{1\times 4}{(4-3K^2)} \cdot f + \frac{3}{4}\,\frac{\hat{h}}{\hat{c}_3\,\beta_{33}^S} \times \frac{L^2}{t_p^2} \times \frac{1}{(1-\frac{3k^2}{4})} \cdot V$$

$(f = 10\,mN)$

$$= \left[\frac{3}{16\times(76.6\times10^9)} \times \frac{(60\times10^{-3})^2}{(10\times10^{-3})\times(0.5\times10^{-3})^3} \times \frac{4}{4-3(1.45\times10^{-1})} \times (10\times10^{-3})\right]$$

$$+ \left[\frac{3}{4} \times \frac{(-5.58\times10^8)\times(60\times10^{-3})^2}{(76.6\times10^9)\times(2.8\times10^7)} \times \frac{1}{(0.5\times10^{-3})^2\times(4-3(1.45\times10^{-1}))} \times V\right]$$

$$= 7.909\times10^{-5} \quad - \quad 3.151\times10^{-6}\,V$$

plot on next page: 5.10

for N=8 condition:

ⓔ. $$f\Big|_{u_3(L/2)=0} = \frac{-3\times\hat{h}}{\beta_{33}^{S}} \times \frac{w\times t_p}{L} \times V \qquad \text{Eq}^n\ (5.124)$$

BLOCK FORCE (at $x=L/2$) ✓ $$= \frac{-3\times(-5.58\times10^{8})}{(2.8\times10^{7})} \times \frac{(10\times10^{-3})\times(0.5\times10^{-3})\times V}{(60\times10^{-3})}$$

$$= 4.982\,mV$$

$$\boxed{f = 4.982\,mN\,V} \checkmark \quad = 4.98\times10^{-3}\,V$$

ⓕ Textbook Page: 231: ⟶
Comparing ~~the~~ analysis of pin-pinned beam to that of cantilevered beam, We see that Energy method is an Approximation method

Accuracy ∝ ~~s~~ No. of Coefficients in shape function (or degree of shape function polynomial)

sol^n of pinned-pinned beam with sufficient number of terms allows us to compare the free deflection and blocked force geometry to that of cantilever beam.

LECTURE 11 slide 30: ⟶
A plot of deflection shape illustrates that beam is deflected less at tip. Enforced as we are placing resistance force. ~~As we can~~ They are DIFFERENT as in cantilever there is NO RESISTANCE FORCE