Ans 1: $\rightarrow$

$$\boxed{k\cdot\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0}$$

(partial differential eq^n.)

$u(0,t) = 0$ $\quad$ $u(L,t) = 0$

$u(x,0) = 20$

Try separation

$$\boxed{u(x,t) = X_{(x)} \cdot T_{(t)}} \checkmark = X_{(x)} \cdot T_{(t)}$$

$$k\cdot X''\cdot T = XT'$$

$$\boxed{\frac{X''}{X} = \frac{T'}{kT} = -\lambda} \checkmark$$

$X'' + \lambda X = 0$ —— (i)

$T' + \lambda k T = 0$ —— (ii)

3 cases: $\rightarrow$

$\lambda = 0$ $\quad$ $X_{(x)} = c_1 + c_2 x$

$\lambda = -\alpha^2 < 0$ $\quad$ $X_{(x)} = c_1 \cdot e^{\alpha x} + c_2 e^{-\alpha x}$

$\lambda = \alpha^2 > 0$ $\quad$ $X(x) = c_1 \cos\alpha x + c_2 \sin\alpha x$

Now to ②, We get →

$u(0,t)=0$

$u(0,t) = x(0)\cdot T(t)$

$u(L,t) = X(L)\cdot T(t)$

We dont want $T(t)=0$
leading to trivial sol^n.

so, $x(0)=0$ and $x(L)=0$

Returning to 3 cases, we get →

not interested in case 1 ($\lambda=0$) :→ $X(0)=0$ $x(L)=0$

$X(x) = c_1 + c_2 x$

$X(0)=0$

$0 = X(x) = c_1 + c_2(0)$

$c_1=0$ $c_2=0$ ($c_2=0$)

$X(L) = c_1 + c_2(L)$

$0 = X(0) = c_1$

$= (c_1=0)$

$(0 = c_2)$

trivial case:

$\lambda < 0$ $X(x)=0$ $(u=0)$

$c_1=0$

$\lambda > 0$

λ positive

$X(x) = c_1\cdot\cos\alpha x + c_2\sin\alpha x$

$0 = X(0) = c_1\cdot\cos(0) + c_2\sin(0)$

$= c_1 + 0$

$0 = c_1$

$(c_1=0)$

$0 = X(L) = \cancel{c_1\cos\alpha L} + c_2\sin(\alpha L)$

$(c_2\cdot\sin(\alpha L)=0)$

c_2 not $\neq 0$

$\sin(\alpha L) \overset{\sin^{-1}}{=} (0) = n\pi$

$\alpha L = n\pi$

$\alpha = \frac{n\pi}{L}$ $(n=1, \ldots$

So, we have non-trivial solⁿ when

$$\lambda = \alpha^2 = \frac{n^2\pi^2}{L^2} \quad \text{for } n = 1, 2, 3.$$

Eigenvalue

$$\boxed{X(x) = c_2 \sin\left(\frac{n\pi}{L}\right)x}$$

Now, to second eqⁿ, we get

$$T' + k\cdot\lambda T = 0$$

$$\int \frac{T'}{T} = \int -k\lambda \cdot dt$$

$$\ln(T) = -k\lambda t + C$$

$$\boxed{T = c_3 \cdot e^{-\lambda k t}}$$

$$T = c_3 \cdot e^{-\left(\frac{n^2\pi^2}{L^2}\right)\cdot t}$$

$$U_n = X(x) \cdot T(t)$$

$$= A_n \cdot \sin\left(\frac{n\pi}{L}\right)x \cdot e^{-k\left(\frac{n^2\pi^2}{L^2}\right)\cdot t}$$

So at this point $u_n(x,t)$ is a sol^n of eq ① and satisfies the Boundary value conditions

Now to cond ② $\rightarrow$

$$u(x,0) = f(x) = 20$$

$$u(x,0) = A_n \cdot \sin\left(\frac{n\pi}{L}\right)x = f(x)$$

By superposition.

$$u(x,t) = \sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty} A_n \cdot e^{-k\left(\frac{n^2\pi^2}{L^2}\right)\cdot t} \cdot \sin\left(\frac{n\pi}{L}\right)x$$

$$u(0,t) = f(x) = \left(\underbrace{\sum_{n=1}^{\infty} A_n \cdot \sin\left(\frac{n\pi}{L}\right)x}_{\text{Half Range expansion in Sine series}}\right)$$

Half-Range expansion

$$A_n = \frac{2}{L}\int_0^L f(x)\cdot \sin\frac{n\pi}{L}x\cdot dx$$

$$\left(\overline{\underline{f(x) = u(x,0) = 20}}\right)$$

heat generation Rate

$$= -2$$

Finally,

$$u(x,t) = A_n \cdot e^{-k\left(\frac{n^2\pi^2}{L^2}\right)\cdot t} \cdot \sin\left(\frac{n\pi}{L}\right)x$$

$$= \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\cdot \sin\left(\frac{n\pi}{L}\right)x\cdot dx\right) e^{-k(n^2\pi^2/L^2)t} \sin\left(\frac{n\pi}{L}\right)x$$

$$= \frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L (20\cdot \sin\left(\frac{n\pi}{L}\right)\cdot x\cdot dx\right) e^{-k\left(\frac{n^2\pi^2}{L^2}\right)\cdot t} \cdot \sin\left(\frac{n\pi}{L}\right)x\Big)$$

Ans2 : $\longrightarrow$

$$\frac{\partial u}{\partial t} = k \cdot \frac{\partial^2 u}{\partial x^2}$$

(i) $u(x,0) = f(x)$

(ii) $\frac{\partial u}{\partial x}(0,t) = 0$

(iii) $\frac{\partial u}{\partial x}(L,t) = 0$

$u(x,t) = \phi(x) \cdot G(t)$

\# Sol^n : $\longrightarrow$

$\frac{dG}{dt} = -k\lambda G$

$\frac{d^2\phi}{dx^2} = -\lambda\phi$

$\frac{d\phi}{dx}(0) = 0$

$\frac{d\phi}{dx}(L) = 0$

$\boxed{\frac{dG}{dt} - k\lambda G = 0}$ — ① or $r - k\lambda = 0$

\# $(r = k\lambda)$

$\boxed{\frac{d^2\phi}{dx^2} + \lambda\phi = 0}$ — ② or $r^2 + \lambda = 0$

\# $(r_{1,2} = \pm\sqrt{-\lambda})$

Case-1 : $\longrightarrow$ $(\lambda > 0)$

$(\phi(x) = c_1 \cos(\sqrt{\lambda} x) + c_2 \sin(\sqrt{\lambda} x))$

$$0 = \frac{d\phi}{dx}(0) = \sqrt{\lambda}\, c_2$$

$$\boxed{c_2 = 0}$$

$$0 = \frac{d\phi}{dx}(L) = -\sqrt{\lambda}\, c_1 \sin(L\sqrt{\lambda})$$

$\sin(L\sqrt{\lambda}) = 0$ $\quad L\sqrt{\lambda} = n\pi$ (for $n = 1, 2 \ldots$)

$$\lambda_{(n)} = \left(\frac{n\pi}{L}\right)^2 \qquad \phi_n(x) = \cos\left(\frac{n\pi x}{L}\right)$$

Case-2: $\longrightarrow$ $\boxed{\lambda = 0}$

$$\phi(x) = c_1 + c_2(x).$$

$$0 = \frac{d\phi}{dx}(0) = c_2$$

$$\boxed{\phi(x) = c_1}$$

$\phi(x) = 1$ (eigenfunction)

Case-3: $\longrightarrow$ $\phi(x) = c_1 \cosh(\sqrt{-\lambda}\, x) + c_2 \sinh(\sqrt{-\lambda}\, x)$

$$0 = \frac{d\phi}{dx}(0) = \sqrt{-\lambda}\, c_2 \qquad \boxed{c_2 = 0}$$

$$0 = \frac{d\phi}{dx}(L) = \sqrt{-\lambda}\, c_1 \sinh(L\sqrt{-\lambda})$$

$$\lambda_{(n)} = \left(\frac{n\pi}{L}\right)^2 \qquad \phi_n(x) = \cos\left(\frac{n\pi x}{L}\right)$$

($n = 1 \ldots$

$$\lambda_0 = 0 \qquad \boxed{\phi_0(x) = 1}$$

$$\lambda(n) = \left(\frac{n\pi}{L}\right)^2 \qquad \phi_n(x) = \cos\left(\frac{n\pi x}{L}\right) \qquad (n=0 \ldots\ldots$$

$$g(t) = c e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

$$u_n = A_n \cos\left(\frac{n\pi x}{L}\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

$$u(x,t) = A_n \cos\left(\frac{n\pi x}{L}\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \qquad n=0,1,2\ldots$$

$$u(x,t) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t}$$

$$\# \left(u(x,0) = f(x) = \sum_{n=0}^{\infty} A_n \cdot \cos\left(\frac{n\pi x}{L}\right) e^{-k\left(\frac{n\pi}{L}\right)^2 t} \right)$$

$$\# \left(u(x,0) = f(x) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) \right)$$

$$A(n) = \begin{cases} \frac{1}{L}\int_0^L f(x)\cdot dx & n=0 \\ \frac{2}{L}\int_0^L f(x)\cdot \cos\left(\frac{n\pi x}{L}\right) dx & n \neq 0 \end{cases}$$

Ans3: →

$$7x_1 + 2x_2 - 2x_3 - 4x_4 + 3x_5 = 8 \quad \text{—(i)}$$
$$-3x_1 - 3x_2 + 0x_3 + 2x_4 + x_5 = 1 \quad \text{—(ii)}$$
$$4x_1 - x_2 - 8x_3 + 0x_4 + 20x_5 = 1 \quad \text{—(iii)}$$

Observation: →

(i) x_1, x_2, x_3: BASIC VARIABLES (BV)

(ii) x_4, x_5 : FREE VARIABLES (FV)

(In non square matrix conditions)

(↑) - indicates pivot column

[x_1, x_2, x_3 can be represented in form of x_4 and x_5 : i.e the free variables in the equation]

$$\begin{array}{ccccc} (BV) & (BV) & (BV) & (FV) & (FV) \end{array}$$
$$\begin{bmatrix} 7 & 2 & -2 & 4 & 3 \\ -3 & -3 & 0 & 2 & 1 \\ 4 & -1 & 8 & 0 & 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$$

step 1: → $R_1 \longrightarrow R_1/7$

$$\left[\begin{array}{ccccc|c} 7/7 & 2/7 & -2/7 & 4/7 & 3/7 & 8/7 \\ -3 & -3 & 0 & 2 & 1 & 1 \\ 4 & -1 & 8 & 0 & 20 & 1 \\ (\uparrow) & & & & & \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & 0.28 & -0.28 & 0.57 & 0.42 & 1.142 \\ -3 & -3 & 0 & 2 & 1 & 1 \\ 4 & -1 & 8 & 0 & 20 & 1 \\ (\uparrow) & & & & & \end{array}\right]$$

step-2: → $R_2 \longrightarrow R_2 + 3(R_1)$

$$\left[\begin{array}{ccccc|c} 1 & 0.28 & -0.28 & 0.57 & 0.42 & 1.142 \\ 0 & -15/7 & -6/7 & 26/7 & 16/7 & 31/7 \\ 4 & -1 & 8 & 0 & 20 & 1 \\ (\uparrow) & & & & & \end{array}\right]$$

Step-3 : $\longrightarrow$

$$R_3 \longrightarrow R_3 - 4R_1$$

$$\left[\begin{array}{ccccc|c} 1 & 2/7 & -2/7 & 4/7 & 3/7 & 8/7 \\ 0 & -15/7 & -6/7 & 20/7 & 16/7 & 31/7 \\ 0 & -15/7 & -48/7 & 16/7 & 128/7 & -25/7 \end{array}\right]$$

Step : 4 $\longrightarrow$

$$R_2 \longrightarrow \left(\frac{7}{15} R_2\right)(-)$$

$$\left[\begin{array}{ccccc|c} 1 & 2/7 & -2/7 & 4/7 & 3/7 & 8/7 \\ 0 & 1 & 0.4 & -20/15 & -16/15 & -31/15 \\ 0 & -15/7 & -48/7 & 16/7 & 128/7 & -25/7 \end{array}\right]$$

Step-5 : $\longrightarrow$ $R_3 \longrightarrow R_3 + \frac{7}{15}(R_2)\left(\frac{15}{7}\right)(R_2)$; $R_1 \longrightarrow R_1 - 2/7 R_2$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -0.4 & 8/15 & 11/15 & 26/15 \\ 0 & 1 & 0.4 & -20/15 & -16/15 & -31/15 \\ 0 & 0 & -6 & 2 & 16 & -8 \end{array}\right]$$

Step -6 $\quad R_3 \longrightarrow R_3/-6$

$$\left[\begin{array}{ccccc:c} 1 & 0 & -0.4 & -8/15 & 11/15 & 26/15 \\ 0 & 1 & 0.4 & -20/15 & -16/15 & -23/15 \\ 0 & 0 & 1 & -1/3 & -8/3 & 4/3 \end{array}\right]$$

For further simplification of the above system of Equations: $\rightarrow$

Step -7: $\longrightarrow$ $R_1 : \longrightarrow R_1 + 0.4R_3$

$R_2 : \longrightarrow R_2 - 0.4R_3$

$$\left[\begin{array}{ccccc:c} 1 & 0 & 0 & -2/3 & -1/3 & 34/15 \\ 0 & 1 & 0 & 0 \; \cancel{-32/15} & 0 & -23/15 \\ 0 & 0 & 1 & -1/3 & -8/3 & 4/3 \end{array}\right]$$

$x_1 - \frac{2}{3} x_4 - \frac{1}{3} x_5 = 34/15$ ——— (3)

$\cancel{x_2 - 32/15\, x_4 = -2.6}$ ——— (4)

$x_3 + x_4/3 - \frac{8}{3} x_5 = 4/3$ ——— (5)

$\cancel{x_2 = -2.6}$

\# $\boxed{x_1 = \frac{34}{15} - \frac{2}{3} x_4 + \frac{1}{3} x_5}$

\# $\boxed{\cancel{x_2 = -2.6 + \frac{32}{15} x_4}}$ $\quad (\cancel{x_2 = -2.6})$

$\boxed{x_2 = -23/15}$

\# $\boxed{x_3 = \frac{4}{3} - x_4/3 + \frac{8}{3} x_5}$

x_4, x_5 remain the free variables

Ans4

Eigen value Calculation

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} (4-\lambda) & 0 & 1 \\ -1 & (-\lambda-6) & -2 \\ 5 & 0 & -\lambda \end{bmatrix}$$

$$(4-\lambda)\left[(-\lambda)(-\lambda-6) - 0\right] - 0 + 1\left[0 - 5(-\lambda-6)\right]$$

$$= -\lambda^3 - 2\lambda^2 + 21\lambda + 30$$ (solving the cubic eqn, we get → THREE Eigen values)

$\boxed{\lambda_1 = 5}$

$\boxed{\lambda_2 = -1}$

$\boxed{\lambda_3 = -6}$

These are the Eigen values for the above matrix

(b) $A = \begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{bmatrix} (6-\lambda) & 3 & -8 \\ 0 & (-2-\lambda) & 0 \\ 1 & 0 & (-3-\lambda) \end{bmatrix} = 0$

$= (6-\lambda)\left[(-2-\lambda) \cdot (-3-\lambda) - 0\right] - 3\left[0\right] - 8\left[-1(-2-\lambda)\right] = 0$

$= -\lambda^3 + \lambda^2 + 16\lambda + 20$

<u>cubic eqn</u>:

Three eigen values: $\longrightarrow$

(i) $\lambda_1 = 5$

(ii) $\lambda_2 = -2$

(iii) $\lambda_3 = -2$

(c) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$|(A - \lambda I)| = 0$

$\begin{vmatrix} (0-\lambda) & 1 & 1 \\ 1 & (0-\lambda) & 1 \\ 1 & 1 & (0-\lambda) \end{vmatrix} = 0$

$= (-\lambda)[(-\lambda)(-\lambda) - (1)(1)] - 1[(-\lambda)(1) - (1)(1)] + 1[1 - (-\lambda)(1)]$

$= -\lambda^3 + 3\lambda + 2$

Eigen values: $\longrightarrow$ (i) $\boxed{\lambda_1 = 2}$ (ii) $\boxed{\lambda_2 = -1}$ (iii) $\boxed{\lambda_3 = -1}$

(d) $A = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

$\because \boxed{|(A - \lambda I)| = 0}$ ✓

$= \begin{bmatrix} (4-\lambda) & 0 & -1 \\ 0 & (3-\lambda) & 0 \\ 1 & 0 & (2-\lambda) \end{bmatrix} = 0$

$= (4-\lambda)[(3-\lambda)(2-\lambda) - 0] - 0 - 1[0 - 1(3-\lambda)]$

$= -\lambda^3 + 9\lambda^2 - 27\lambda + 27$

Eigen values

$(\lambda_1 = 3, 3, 3)$

$\begin{cases} \lambda_1 = 3 \\ \lambda_2 = 3 \\ \lambda_3 = 3 \end{cases}$ $\because$ Equal & Repeated Eigen values

Ans5: →

(a) $A = \begin{bmatrix} 3 & 2 \\ -9 & 5 \end{bmatrix}$

$= (3)(5) - (2)(-9)$

$= 15 + 18 = 33$

(b) $B = \begin{bmatrix} 3 & 5 & 4 \\ -2 & -1 & 8 \\ -11 & 1 & 7 \end{bmatrix}$

$3((-1)(7) - (8)(1)) - 5((7)(-2) - (8)(11)) + 4((-2)(1) - (11)(-1))$

$= 3(-7 - 8) - 5(\underbrace{-14 + 88}_{74}) + 4(-2 - 11)$

$\cancel{= 3(-15) - 5(\ldots) + 4(-13)}$

$\cancel{= -15 + 510 + 36}$

$\cancel{-15 - 370 + 36}$

$= 3(-15) - 5(74) + 4(-13)$

$= -45 - 370 - 52$

$= 467$

(c)

$$C = \begin{bmatrix} 2 & -6 & 2 \\ 2 & -8 & 3 \\ -3 & 1 & 1 \end{bmatrix}$$

$$= 2[(-8)(1) - (1)(3)] + 6[(2)(1) - (3)(-3)] + 2[(2)(1) - (-8)(-3)]$$

$$= 2[-8 - 3] + 6[2 + 9] + 2[2 - 24]$$

$$= 2[-11] + 6[11] + 2[-22]$$

$$= -22 + 66 - 44$$

$$= \cancel{-66} + \cancel{66} = 0$$

Ans 6 :

(i) Divergence $= \nabla \cdot F$ (Dot product)

(ii) curl $= \nabla \times F$ (cross product)

$\boxed{\vec{F} = x^2 y\,\hat{i} - (z^3 - 3x)\hat{j} + 4y^2 \cdot \hat{k}}$

$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$\nabla \cdot F = \left[\frac{\partial}{\partial x}(x^2 y)\right]\hat{i} + \left[\frac{\partial}{\partial y}(z^3 - 3x)\hat{j}\right] + \left[\frac{\partial}{\partial z}(4y^2)\cdot \hat{k}\right]$

$\boxed{\nabla \cdot F = 2x \cdot y\,\hat{i} + 0\hat{j} + 0\hat{k}}$

Now, curl

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & -(z^3 - 3x) & 4y^2 \end{bmatrix}$$

$= \cancel{\hat{i}\left[\frac{\partial}{\partial x}(-z^3+3x) - \frac{\partial}{}\right.}$

$\hat{i}\left[\frac{\partial}{\partial y}(4y^2) - \frac{\partial}{\partial z}(-z^3+3x)\right]$

$-\hat{j}\left[\frac{\partial}{\partial x}(4y^2) - \frac{\partial}{\partial z}(x^2 \cdot y)\right]$

$+\hat{k}\left[\frac{\partial}{\partial x}(-z^3+3x) - \frac{\partial}{\partial y}(x^2 y)\right]$

$$= (8y + 3z^2)\hat{i} - \hat{j}(0+0) + \hat{k}(3 - x^2)$$

$$\boxed{\text{curl} f = (8y + 3z^2)\hat{i} + 0\hat{j} + (3 - x^2)\hat{k}}$$

Ans 7: $\rightarrow$ using divergence theorem: $\rightarrow$

Evaluate

$$\iint_S \vec{F} \cdot \vec{ds}$$

Divergence theorem: $\rightarrow$

As we know, $\boxed{\iint_S \vec{F} \cdot ds = \iiint_V div \vec{F} \cdot dV}$

Given: $\rightarrow$ (i) $\left(\vec{F} = yx^2 \cdot \hat{i} + (xy^2 - 3z^4)\hat{j} + (x^3+y^2)\hat{k}\right)$

(ii) Sphere: Radius 4 $\longrightarrow$ Origin (0,0) (Assumption)

Eq^n of circle: $\rightarrow$

Generic Eq^n of circle. $x^2 + y^2 = r^2$ (circle at Origin)

$$\boxed{x^2 + y^2 = 16}$$

Sol^n: $\longrightarrow$

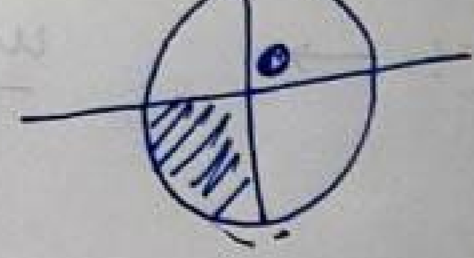
$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

(i) $\hat{i} \cdot \hat{i} = 1$
(ii) $\hat{j} \cdot \hat{j} = 1$
(iii) $\hat{k} \cdot \hat{k} = 1$

$$div\vec{F} = \nabla \cdot \vec{F}$$

$$= \frac{\partial}{\partial x}(yx^2)\hat{i} + \frac{\partial}{\partial y}(xy^2 - 3z^4) + \frac{\partial}{\partial z}(x^3+y^2)$$

$$= 2xy\hat{k} + 2xy + 0 \quad = \underline{4xy}$$



Now,

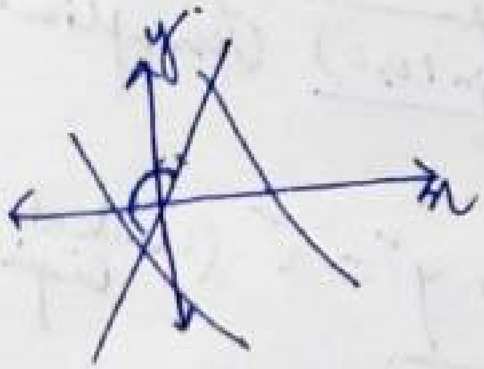
As per divergence theorem: →

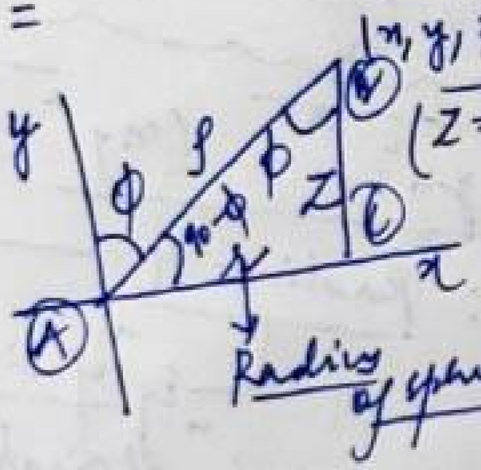
We get →

$$\iint_S F\cdot ds = \iiint \mathrm{div}\vec{F}\cdot dV$$

We'll be working in spherical coordinate (for ease of computation): →

=





$(Z = \rho\cos\phi)$

$r = \rho\cos(90-\phi)$

$(r = \rho\sin\phi)$

In ΔABC We get →

$(\rho^2 = Z^2 + r^2)$

$(x = r\cos\theta)$

$(y = r\sin\theta)$

$(x = (\rho\sin\phi)\cdot\cos\theta)$

$(y = (\rho\sin\phi)\cdot\sin\theta)$

$= \int\int\int$

$$= \int_{\pi}^{2\pi} \int_{\pi/2}^{\pi} \left(\tfrac{4}{5} \cdot \rho^5 \cdot \sin^3\phi \cdot \sin\theta \cdot \cos\theta \right) . d\phi . d\theta$$

$$= \int_{\pi}^{2\pi} \frac{8192}{15} \cos\theta \cdot \sin\theta \, . d\theta$$

$$= \int_{\pi}^{2\pi} \frac{(4096) \times 2 \times \cos\theta \times \sin\theta}{15} . d\theta$$

$$= \int_{\pi}^{2\pi} \frac{4096}{15} \times \sin 2\theta . d\theta$$

$$= \frac{4096}{15} \left[-\frac{\cos 2\theta}{2} \right]_{\pi}^{2\pi}$$

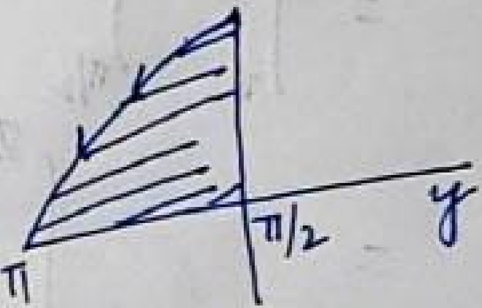
$$\frac{2048}{15} \times \left[-[\cos(4\pi) - \cos(2\pi)] \right]$$

$$\frac{2048}{15} \times \left[-[1 - 1] \right] = 0$$

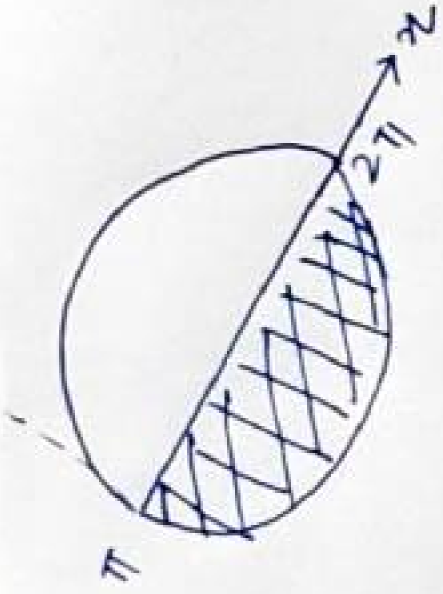
Substituting x, y by spherical coordinates

$$= \iiint_V 4(\rho\sin\phi\cdot\cos\theta)(\rho\sin\phi\cdot\sin\theta)\cdot dS$$

$\downarrow$



Shaded Region: Our Area of interest



dS to dv conversion,

calculation of Jacobian, we get: $\rightarrow$

$$J(\rho,\phi,\theta) = \begin{bmatrix} \partial x/\partial\rho & \partial x/\partial\phi & \partial x/\partial\theta \\ \partial y/\partial\rho & \partial y/\partial\phi & \partial y/\partial\theta \\ \partial z/\partial\rho & \partial z/\partial\phi & \partial z/\partial\theta \end{bmatrix}$$

$(z = \rho\cos\theta)$

$(y = \rho\sin\phi\cdot\sin\theta)$

$(x = \rho\sin\phi\cdot\cos\theta)$

$$J(\rho,\phi,\theta) = -\rho^2\cdot\sin\phi$$

$$= \iiint 4(\rho\sin\phi\cdot\cos\theta)(\rho\sin\phi\cdot\sin\theta)\cdot(-\rho^2\cdot\sin\phi)\cdot dv$$

$$= \int_{\pi}^{2\pi}\int_{\pi/2}^{\pi}\int_0^4 4(\rho\sin\phi\cdot\cos\theta)(\rho\sin\phi\cdot\sin\theta)\cdot d\rho\cdot d\phi\cdot d\theta$$

$$= \int_{\pi}^{2\pi}\int_{\pi/2}^{\pi}\int_0^4 -4\rho^4\sin^3\phi\cdot\sin\theta\cdot\cos\theta\cdot d\rho\cdot d\phi\cdot d\theta$$