Ans 1 :

$$S = s^{E} \cdot T + dE \quad \text{——— (1)}$$

$$D = dT + \epsilon^{T} \cdot E \quad \text{——— (2)}$$

(a) STRESS (T) vs CHARGE form: ⟶

Rewriting eq (1) and (2) in terms of independent variables

Now, we'll multiply eq (1) by c^{E}

We get: ⟶ $-c^{E} \cdot S = T + c^{E} \cdot dE$

$$\therefore \; T = c^{E} S - c^{E} d E$$

Substituting value of T in equation (1),

We get: ⟶ $$D = d' c^{E} \cdot S + (\epsilon^{T} - d' \cdot c^{E} d) E$$

$$\therefore \; T = c^{E} \cdot S - eE$$

$$D = e' S + \epsilon^{S} E$$

Here, (a) $e = c^{E} \cdot d$ (b) $\epsilon^{S} = \epsilon^{T} - d' c^{E} \cdot d$

(b) STRAIN VOLTAGE:

multiplying equation (2) by β^T, we get:→

$$E = -\beta^T \cdot d \cdot T + \beta^T \cdot D$$

substituting eq (6) in (1), we get →

$$S = (s^E - d\beta d')T + d\beta^T \cdot D$$

Here,

$$g = d\beta T$$
$$s^D = s^E - d\beta^T \cdot d'$$

(c). Equation (3): →

$$S = s^D T + gD$$

multiplying above equation with c^D, we get:→

$$T = c^D S - c^D \cdot g^D \quad \text{——— (a)}$$

Now, as we know

$$E = -g'T + \beta^T \cdot D$$

substituting the value of T from eq (a),

we get:→

$$E = -g'c^D \cdot S + (\beta^T + g'c^D g)D$$

$$T = c^D S - hD$$

$$\boxed{E = -h^{t}S + \beta^{S}D}$$

where, (i) $\boxed{h = c^{D}\cdot g}$

(ii) $\boxed{\beta^{S} = \beta^{T} + g^{t}c^{D}g}$

Sol^{n} 6.2: $\longrightarrow$

martensitic fraction $(\xi) = 0$

$T_{ambient} = 25^{\circ}C$

$\tau_s^{cr} = 105 \times 10^{6}\,Pa$

$\boxed{\tau_s^{cr} = 105\,MPa}$

$\tau_f^{cr} = 160 \times 10^{6}\,Pa$

$\boxed{\tau_f^{cr} = 160\,MPa}$

(A)

stress required: to lead ~~martensite~~ Austenite : $\xrightarrow{\text{transformation}}$ martensite

$$\boxed{M_f < \theta_0 < A_s}$$

$$\text{Stress}(\tau) = \tau_s^{cr} + C_m(\theta_0 - M_s)$$

$$\tau = 105 + 7(25-17) = 161\,MPa \qquad \boxed{\tau = 161\,MPa}$$

(B)

stress required to: martensite : $\xrightarrow{\text{transformation}}$ Austenite

$$\tau = \tau_f^{cr} + C_m\cdot(\theta_0 - M_s)$$

$$= 160 + 7(25-17)$$

$$\# \quad \boxed{\tau = 216\,MPa}$$

Ans 3: Problem 11.2: $\longrightarrow$

Given: $\rightarrow$

(a) $C = 1.2\,\mu f$

(b) $V = 100\,V$

(c) $f = 40\,Hz$

(a) peak power: $\longrightarrow$

$$= P_{peak} = \frac{1}{2} C V^2 \omega$$

$$= \frac{1}{2} \times (1.20 \times 10^{-6}) \times (100)^2 \times (80\pi)$$

$$\therefore \boxed{P_{peak} = 1.508\,W}$$

(b) Power dissipated: $\longrightarrow$

$$P_{diss} = \frac{2 V I_0}{\pi} \quad \text{——— (1)}$$

$$i(t) = \frac{C \cdot dv(t)}{dt}$$

$$V(t) = V \sin(\omega t)$$

$$\boxed{V(t) = 100 \sin(80\pi t)}$$

$$\therefore \; i(t) = (1.2 \times 10^{-6})(100)(80\pi \cos(80\pi t))\,A$$

$$\boxed{i(t) = 30.16\,mA}$$

$$P_{diss} = \frac{2 \times 100 \times (30.16 \times 10^{-3})}{\pi}$$

$$\boxed{P_{diss} = 1.92\,W}$$

(b) consider,

$V = 200V \qquad f = 40Hz$

Peak power $= \frac{1}{2} \times (1.2 \times 10^{-6}) \times (200)^2 \times (80\pi)$

$\boxed{P_{peak} = 6.032 W}$

$V(t) = 200 \sin(80\pi t)$

$i(t) = (1.2 \times 10^{-6}) \times (200) \times (80\pi \cos(80\pi t))$ A

$\boxed{i(t) = 60.32 \text{ mA}}$

$P_{dissipated} = \frac{2 \times 200 \times 60.32 \times 10^{-3}}{\pi}$

$= 7.68 W$

(c) consider,

$\boxed{V = 100V}$ $\boxed{f = 80Hz}$

$$P_{peak} = \frac{1}{2} \times (1.2 \times 10^{-6}) \times (100)^2 \times (160\pi)$$

$$\boxed{P_{peak} = 3.016\ W}$$

$$v(t) = 100 \sin(160\pi)$$

$$i(t) = (1.2 \times 10^{-6}) \times (100) \times (160\pi \cos(160\pi t))$$

$$\boxed{i(t) = 60.32\ mA}$$

$$P_{dissipated} = \frac{2 \times 100 \times (60.32 \times 10^{-3})}{\pi}$$

$$\# \ \boxed{P_{diss} = 3.84\ W}$$

Ans 5: →

$$(K_s + K_p^D) r - \theta q = B_f \cdot f$$

known — known — known

$$-\theta' r + \varphi_p^{s-1} q = B_v \cdot v$$

known — known — known

$$① \quad \boxed{K_s = \int_{v_s} B_r(x)' \, \varphi_s \cdot B_r(x) \, dv_s}$$

$$② \quad K_p^D = \sum_{i=1}^{N_p} \int_{v_p^i} B_r(x)' R_s^{i\,\prime} C^D R_s^i B_r(x) \, dv_p^i$$

$$③ \quad \theta = \sum_{i=1}^{N_p} \int_{v_p^i} B_r(x)' R_s^{i\,\prime} h R_D^i \cdot B_q^i(x) \cdot dv_p^i$$

$$④ \quad \varphi_p^{s-1} = \sum_{i=1}^{N_p} \int_{v_p^i} B_q^i(x)' R_D^{i\,\prime} \beta^s \cdot R_D^i B_q^i(x) \cdot dv_p^i$$

To find: →

① free deflection: →

$$f = 0$$

② Block force: →

$$(L.H.S = 0)$$

$$u(H_0) = 0$$

$$H_d \cdot K^{-1} \theta \cdot \varphi_p^s B_v \cdot v + H_D K^{-1} B_f \cdot f.$$

$$K = K_s + K_p^D - \theta \varphi^s \theta'$$

5.2

$$\cancel{K_{(p)}^{D} \hat{\epsilon}^{D} \int_{-W/2}^{W/2} \int_{0}^{L} \int_{-t_{(p)}/2}^{t_{(p)}/2} \left\{ z^2 \left[4 \right. \right.}$$

$$\hat{C}^{D} = C_{11}^{D} - 2 v_{12} \cdot C_{12}^{D} - \cancel{2 v_{13} \cdot C_{13}^{D} + v_{12}^2 C_{22}^{D} + 2 v_{12} \cdot v_{13} C_{23}^{D} + v_{13}^2 \cdot C_{33}^{D}}$$

$$\hat{h} = h_{13} - \cancel{v_{12} h_{23} - v_{13} h_{33}} \quad \boxed{(\hat{h} = h_{13})}$$

$$(\hat{C}^{D} = C_{11}^{D})$$

$$B_2(x) = \begin{bmatrix} 0 & 0 & z\partial^2/\partial x^2 \\ 0 & 0 & -v_{12} \cdot z \cdot \partial^2/\partial x^2 \\ 0 & 0 & -v_{13} \cdot z \cdot \partial^2/\partial x^2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ x^2 - Lx & x^3 - L^2 x \end{bmatrix}$$

(a)1 $z \frac{\partial^2}{\partial x^2}(x^2 - Lx)$

$z \frac{\partial}{\partial x}(2x - L) = 2z.$

~~(b) $-v_{12} \cdot z \cdot \frac{\partial^2}{\partial x^2}$~~

(a)2 $z \frac{\partial^2}{\partial x^2}(x^3 - L^2 x)$

$= z \cdot \frac{\partial}{\partial x}(3x^2 - L^2)$

$= z \cdot$

$\cdot z)\, 6x.$

$$= \left[\begin{array}{l} \text{(b)} -v_{12} \cdot z \cdot \frac{\partial^2}{\partial x^2}(x^2 - Lx) \\ = -v_{12} \cdot z \cdot \frac{\partial}{\partial x}(2x - L) \\ - 2 \cdot z \cdot v_{12} \end{array} \right.$$

$-v_{12} \cdot z \cdot \frac{\partial^2}{\partial x^2}(x^2 - Lx)$

$= -v_{12} \cdot z \cdot \frac{\partial}{\partial x}(2x - L)$

$= -v_{12} \cdot z \cdot 2$

$-v_{12} \cdot z \cdot \frac{\partial^2}{\partial x^2}(x^3 - L^2 x)$

$= -v_{12} \cdot z \cdot \frac{\partial}{\partial x}(3x^2 - L^2)$

$= \underline{-v_{12} \cdot z \cdot 6x}$

$$B_x(x) = \begin{bmatrix} 2z & 6x\,z \\ -2z\cdot\nu_{12} & -(6x)(\nu_{12})\cdot z \\ -\nu_{13}\cdot 2z & -\nu_{13}\cdot 6z\cdot x \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_f = \begin{bmatrix} -L^2/4 \\ -3L^3/8 \end{bmatrix}$$

Input vector (in pinned condition)

$$B_q^1(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (1/wL) & 0 \end{bmatrix} \qquad B_q^2(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \left(\frac{1}{wL}\right) \end{bmatrix}$$

Textbook Expression 5.107 : (a) (equilibrium equations) (a) ✓

\# R.H.S : →

(a) Equilibrium Expression

R.H.S

$$= \begin{Bmatrix} -L^2/4 \\ -3L^3/8 \\ 0 \\ 0 \end{Bmatrix} f + \begin{Bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{Bmatrix} ✓$$

L.H.S : →

$$\begin{bmatrix} C_{11}^D \frac{bt^3L}{3} & C_{11}^D \frac{bt^3L^2}{2} & \frac{1}{4} h_{13} t^2 & -\frac{1}{4} h_{13} t^2 \\ C_{11}^D \frac{bt^3L^2}{2} & C_{11}^D \cdot bt^3L^2 & \frac{3L}{8} \times h_{13} t^2 & -\frac{3L}{8} h_{13} t^2 \\ \frac{1}{4} h_{13} t^2 & \frac{3L}{8} h_{13} t^2 & \frac{1}{2} \times \beta_{33}^S \frac{t}{bL} & 0 \\ -\frac{1}{4} h_{13} t^2 & -\frac{3L}{8} \times h_{13} t^2 & 0 & \frac{1}{2} \beta_{33}^S \cdot \frac{t}{bL} \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ q_M \\ q_e \end{bmatrix}$$

5.4

Expression stiffness matrix:

$$
\begin{bmatrix}
① \; \hat{C}_D \cdot w\, t_p^3 \cdot L/3 & ② \; \hat{C}^D w\, t_{p}^3 \cdot L^2/2 & ③ \; \hat{h}\, t_{p}^2/4 & ④ \; -\hat{h}\, t_{p}^2/4 \\
⑤ \; \hat{C}^D w\, t_p^3 L^2/2 & ⑥ \; \hat{C}^D w\, t_{p}^3 L^3 & ⑦ \; 3\hat{h}\, t_p^2 L/8 & ⑧ \; -3\hat{h}\, t_{p}^2 L/8 \\
⑨ \; \hat{h}\, t_{p}^2/4 & ⑩ \; 3h\, t_{p}^2 L/8 & ⑪ \; \beta_{33}^S\, t_{p}/2wL & ⑫ \; 0 \\
⑬ \; -\hat{h}\, t_{p}^2/4 & ⑭ \; -3h \cdot t_{p}^2 \cdot L/8 & ⑮ \; 0 & ⑯ \; \dfrac{\beta_{33}^S \cdot t_{p}}{2wL}
\end{bmatrix}
$$

$\hat{h} = h_{13} - \nu_{12} \cdot h_{23} - \nu_{13} \cdot h_{33}$

$\hat{h} = -5.58 \times 10^8$

C_{11}^D

①: $\hat{C}_D (w) \cdot t_{p}^3 \cdot L/3 = \dfrac{(76.6 \times 10^9) \times (10 \times 10^{-3}) \times (0.5 \times 10^{-3})^3 \times (60 \times 10^{-3})}{3} = \cancel{7.66 \times 10^{-3}} = 1.915 \times 10^{-3}$

②: $\dfrac{C_D (w) \cdot t_p^3 \cdot L^2}{2} = 1.72 \times 10^{-4}$: ⑤

③: $\dfrac{\hat{h}\, t_{p}^2}{4} = \dfrac{(\cancel{-58}\,-5.58 \times 10^8) \times (0.5 \times 10^{-3})^2}{4} = -3.487 \times 10^1 = ⑨$

$= ⑬$

④: $\dfrac{-\hat{h} \cdot t_{p}}{4} = (+)\, 3.487 \times 10^1$

⑤: $\dfrac{\hat{C}_D \cdot w \cdot t_{p}^3 \cdot L^2}{2} = \dfrac{(76.6 \times 10^9) \times (10 \times 10^{-3}) \times (0.5 \times 10^{-3})^3 \times (60 \times 10^{-3})^2}{2}$

$= 1.723 \times 10^{-4}$

6: $\hat{C}_D \times W \times t_p^3 \cdot L^3$

$$= (76.6\times10^9)\times(10\times10^{-3})\times(0.5\times10^{-3})^3\times(60\times10^{-3})^3$$

$$= 2.068\times10^{-5}.$$

7: $\dfrac{3\hat{h}\cdot t_p^2\cdot L}{8} = \dfrac{3\times \overset{(-5.58\times10^8)}{\cancel{(-6.5\times10^8)}}\times(0.5\times10^{-3})^2\times \overset{(60\times10^{-3})}{\cancel{100\times}}}{8}$

$$= \frac{3\times(-5.58\times10^8)\times(0.5\times10^{-3})^2\times(60\times10^{-3})}{8}$$

$$= -3.138 = 10$$

8: 3.138 :

$\boxed{9 : 4}$ $\boxed{10 : -3.138}$

11: $\dfrac{(2.8\times10^7)\times(0.5\times10^{-3})}{2\times(10\times10^{-3})\times(60\times10^{-3})} = 1.16\times10^7$

16: $1.16\times10^7.$

(b) stiffness matrix: $\longrightarrow$

$$K = \begin{bmatrix} 1.915\times10^{-3} & 1.723\times10^{-4} & -3.48\times10^{1} & 3.485\times10^{1} \\ 1.723\times10^{-4} & 2.06\times10^{-5} & -3.13\times10^{0} & 3.13\times10^{0} \\ -3.48\times10^{1} & -3.13\times10^{0} & 1.167\times10^{7} & 0 \\ 3.485\times10^{1} & 3.13\times10^{0} & 0 & 1.167\times10^{7} \end{bmatrix}$$

(c) FREE displacement: $\longrightarrow$ (At Zero Resistance force).

$$[k]\,r = b_{f}\cdot f + b_{r}\cdot v$$

Generalized coordinate calculation: $\longrightarrow$

$$r = [k^{-1}]\,[b_{f}\cdot f + b_{r}\cdot v]$$

$$[k^{-1}] = \begin{bmatrix} 0.72 & -1.787 & 0.677 & 0.744 \\ -0.80 & 1.829 & -0.757 & -0.697 \\ 0.02 & -0.135 & 0.101 & 0.11 \\ -0.0015 & 0.132 & -0.001 & -0.01 \end{bmatrix}$$

$$r = \begin{bmatrix} 3.50\times10^{-3} \\ 0 \\ 9.617\times10^{-8} \\ -9.617\times10^{-8} \end{bmatrix} = \begin{bmatrix} 3.50\times10^{-3} \\ 0 \\ 9.617\times10^{-8} \\ -9.617\times10^{-8} \end{bmatrix}$$

Cubic Expression: $\longrightarrow$

$$W(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$

(B) (C) free displacement (pin-pinned beam).

$$u_3 (L/2)_{f=0} = \frac{3}{4} \times \frac{\hat{h}}{c^D \times \beta^S_{33}} \times \frac{L^2}{t_p^2} \times \frac{1}{(1 - 3k^2/4)} \cdot V$$

BC's (pin-pinned condition)

$\boxed{W(0) = 0}$

$\boxed{W(L) = 0}$

$$k^2 = \frac{\hat{h}^2}{c^D \times \beta^S_{33}}$$

$W(0) = 0 \quad \boxed{a_0 = 0}$

$W(L) = a_0 + a_1 \cdot L + a_2 L^2 + a_3 L^3$

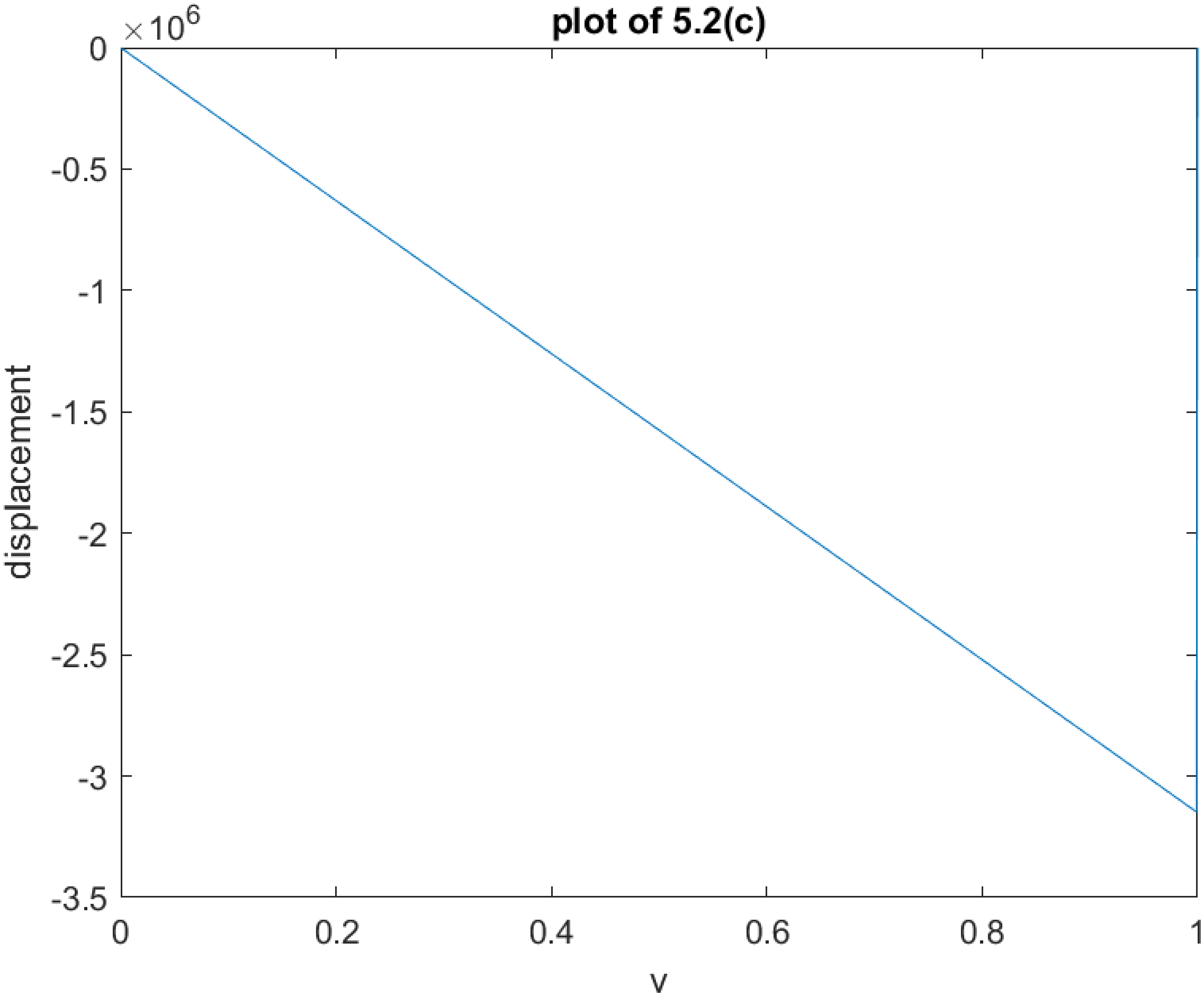
$0 = 0 + a_1 L + a_2 L^2 + a_3 L^3$

$$k^2 = \frac{(-5.58 \times 10^8)^2}{(76.6 \times 10^9) \times (2.8 \times 10^7)} = 1.457 \times 10^{-1}$$

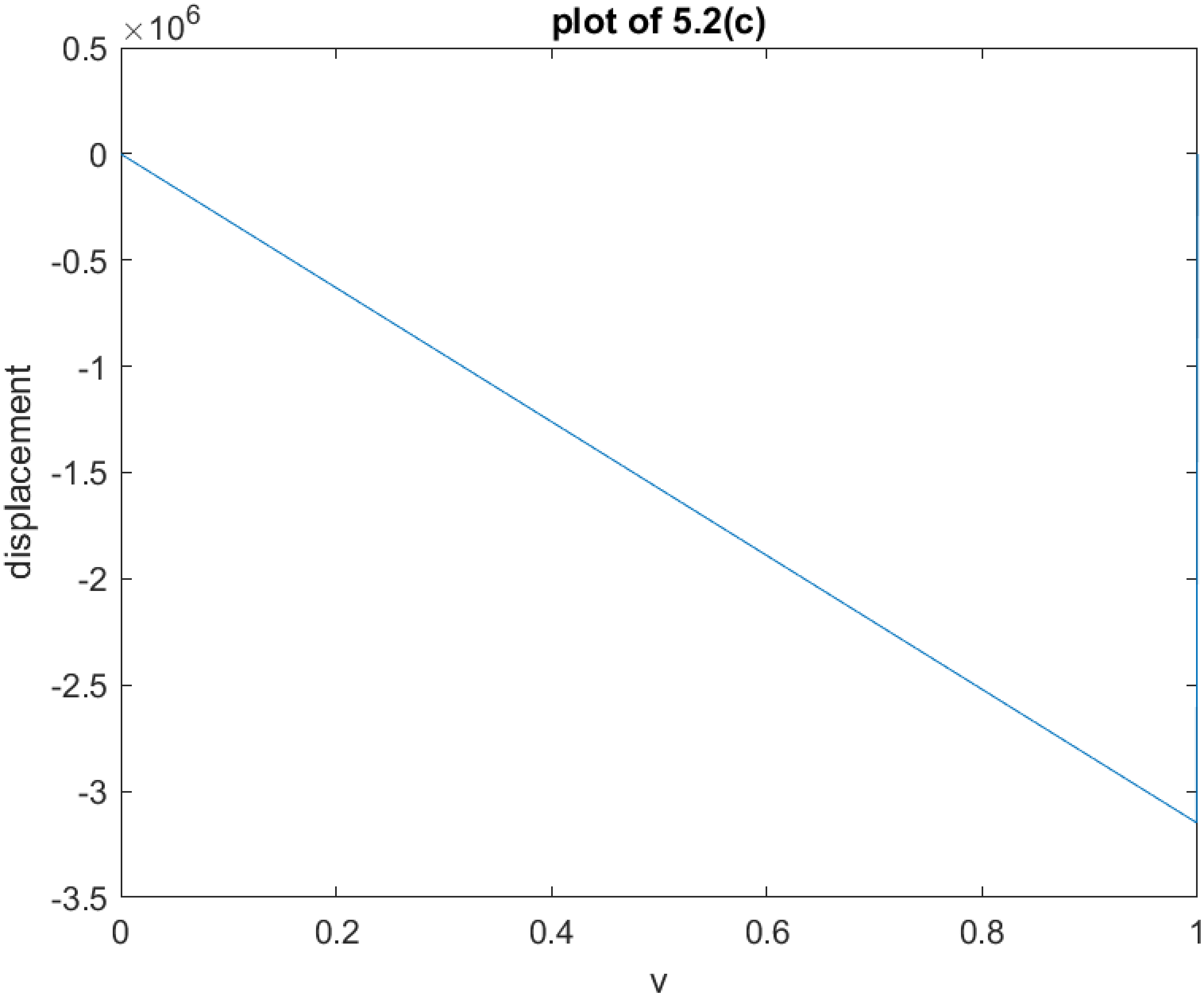
$$= \frac{3}{4} \times \frac{(-5.58 \times 10^8)}{(76.6 \times 10^9) \times (2.8 \times 10^7)} \times \frac{(60 \times 10^{-3})^2}{(0.5 \times 10^{-3})^2} \times \frac{1}{1 - \frac{3(1.45 \times 10^{-1})}{4}} \times V$$

$$= \frac{(-2.809 \times 10^{-6}) \times 1 \times 4}{(4 - 3(1.45 \times 10^{-1}))} = \underline{-3.151 \times 10^{-6}} \, V$$

$\boxed{\text{free displacement} = -3.151 \times 10^{-6} V}$



plot of 5.2(c)
$\times 10^6$
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
displacement
0
0.2
0.4
0.6
0.8
1
v


plot of 5.2(c)
×10^6
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
displacement
0
0.2
0.4
0.6
0.8
1
v

5.8 (Graph)

5-9

There have been two graphical plots.

One in with $\ominus V$ that means the voltage to be applied along the opp. dir^n of current flow

i.e polarity need to reversed.

(d) Displacement calculation : $\longrightarrow$

$$u_3(L/2) = \frac{3}{16\,\hat{c}^{D}} \times \frac{L^2}{W \cdot t_p^3} \times \frac{1 \times 4}{(4-3K^2)} \cdot f + \frac{3}{4}\frac{\hat{h}}{\hat{c}_D\,\beta_{33}^S} \times \frac{L^2}{t_p^2} \times \frac{1}{(1-\frac{3k^2}{4})} \cdot V$$

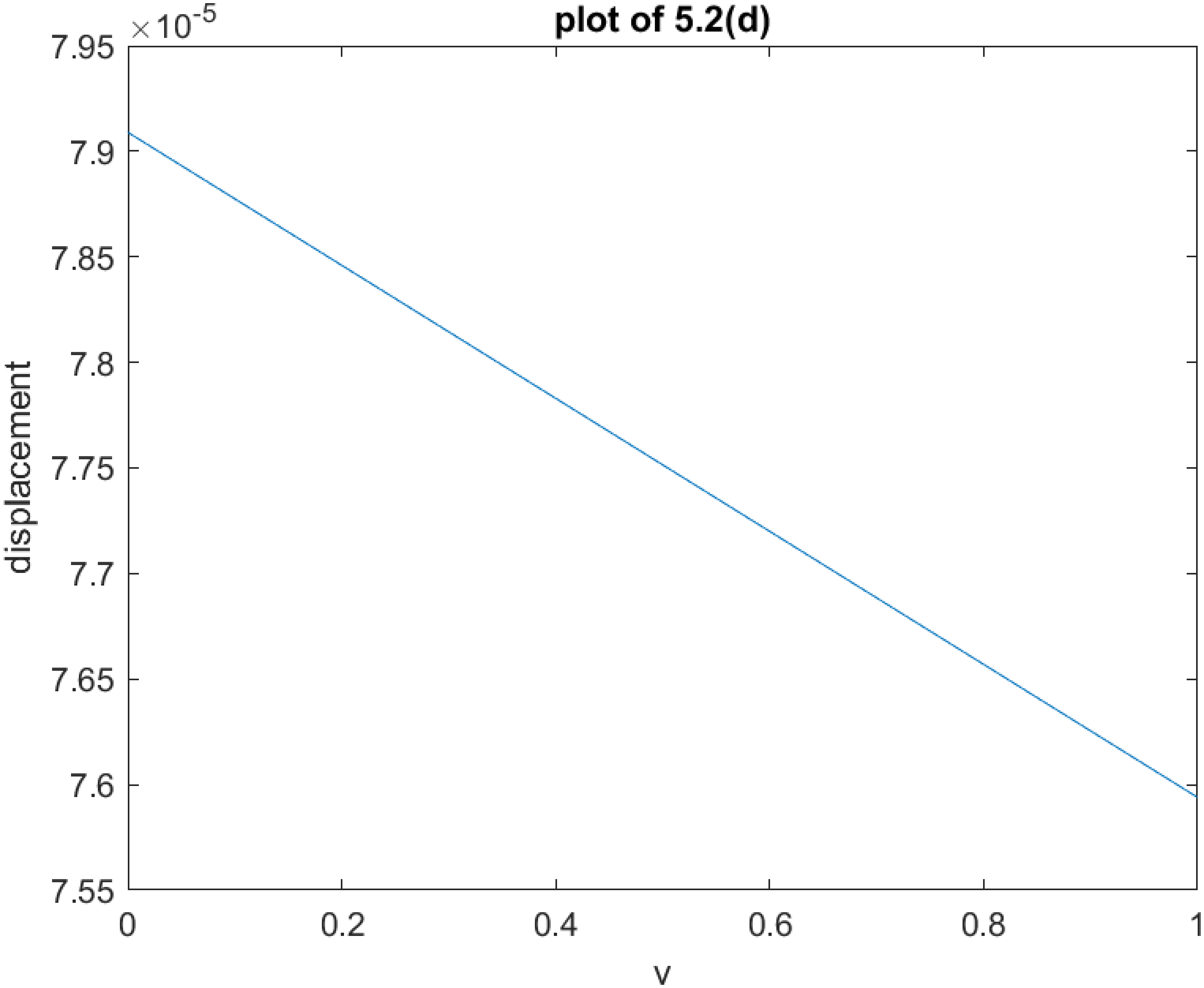
$(f = 10\,mN)$

$$= \left[\frac{3}{16 \times (76.6 \times 10^9)} \times \frac{(60 \times 10^{-3})^2}{(10 \times 10^{-3}) \times (0.5 \times 10^{-3})^3} \times \frac{4}{4 - 3(1.45 \times 10^{-1})} \times (10 \times 10^{-3})\right]$$

$$+ \left[\frac{3}{4} \times \frac{(-5.58 \times 10^8) \times (60 \times 10^{-3})^2}{(76.6 \times 10^9) \times (2.8 \times 10^7)} \times \frac{1}{(0.5 \times 10^{-3})^2 \times (4 - 3(1.45 \times 10^{-1}))} \times V\right]$$

$$= 7.909 \times 10^{-5} - 3.151 \times 10^{-6}\,V$$

plot on next page: 5.10


plot of 5.2(d)
×10^-5
7.95
7.9
7.85
7.8
7.75
7.7
7.65
7.6
7.55
displacement
0
0.2
0.4
0.6
0.8
1
v

(e). for N=8 condition:

$$f\Big|_{u_3(L/2)=0} = \frac{-3 \times \hat{h}}{\beta_{33}^{S}} \times \frac{w \times t_p}{L} \times V \qquad \text{Eq}^n\ (5.124)$$

Block Force (at $x = L/2$) ✓

$$= \frac{-3 \times (-5.58 \times 10^{8})}{(2.8 \times 10^{7})} \times \frac{(10 \times 10^{-3}) \times (0.5 \times 10^{-3}) \times V}{(60 \times 10^{-3})}$$

$$= 4.982\,mV$$

$$\boxed{f = 4.982\,mNV} \checkmark \qquad = 4.98 \times 10^{-3}\,V$$

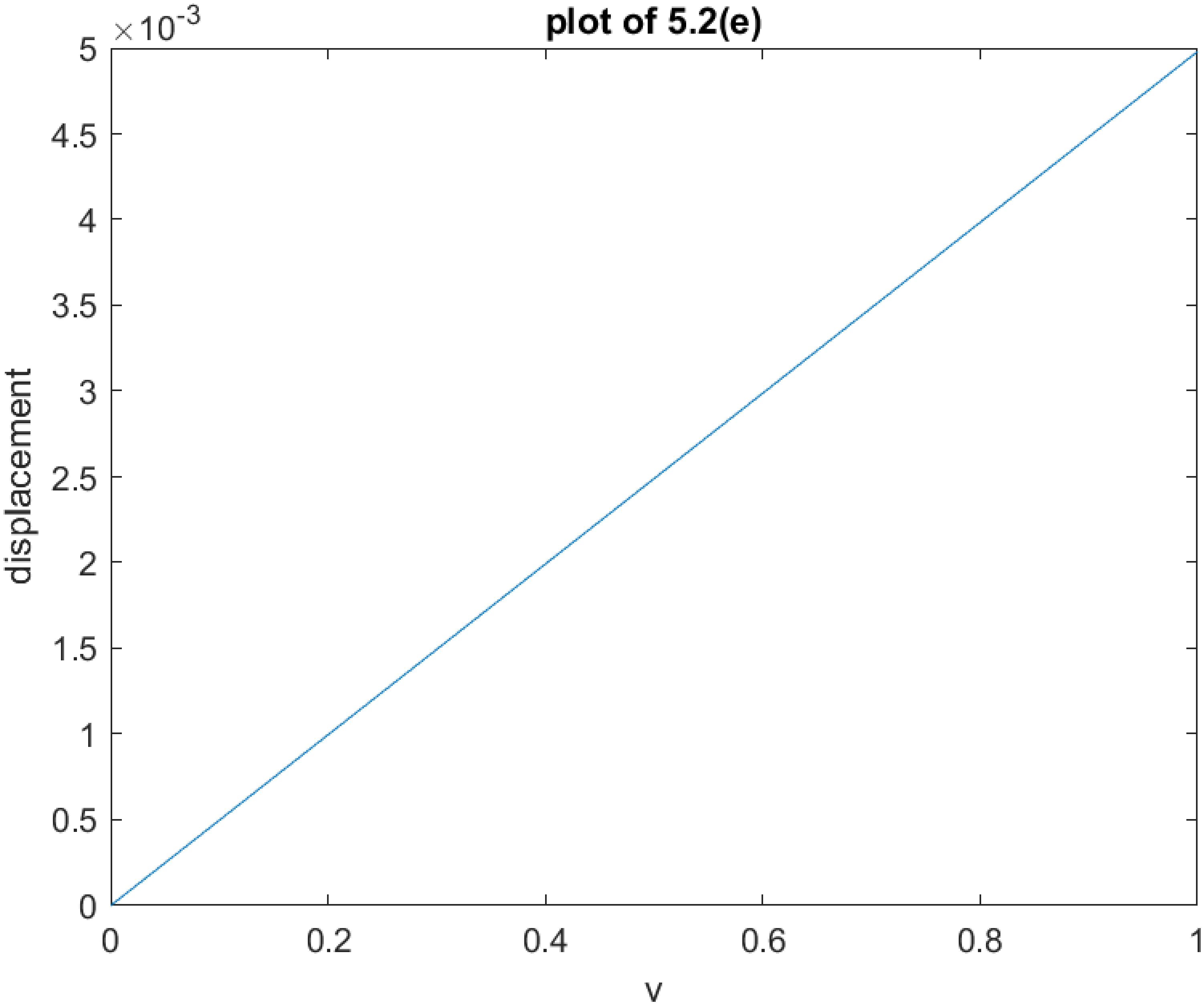
(f) Textbook Page: 231: →

Comparing the analysis of pin-pinned beam to that of cantilevered beam, we see that energy method is an Approximation method

Accuracy $\propto$ ~~st~~ No. of coefficients in shape function (or degree of shape function polynomial)

sol^n of pinned-pinned beam with sufficient number of terms allows us to compare the free deflection and blocked force geometry to that of cantilever beam.

LECTURE 11 Slide 30: → A plot of ~~deflection~~ deflection shape illustrates that beam is deflected less at tip. Expected as we are placing resistance force. ~~As usual~~ They are DIFFERENT as in cantilever there is NO RESISTANCE FORCE


plot of 5.2(e)
×10^-3
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
displacement
0
0.2
0.4
0.6
0.8
1
v