Sol^n 4.5

consider,

1D - poisson's Equation : $\longrightarrow$

$$-\frac{d^2u}{dx^2} = 2x \qquad 0 < x < 1$$

Boundary conditions $\rightarrow$ $\frac{du}{dx}(0) = 1. \qquad u(1) = 0$

① step-3 : $\rightarrow$ Derive the weak form : $\longrightarrow$

By weighted Residual method $\rightarrow$

$$\int_{\Omega} w \cdot \mathcal{L}u \cdot d\Omega = \int_a^b w\left[-\frac{d^2u}{dx^2} + q(x)\cdot u\right].dx$$

In our case, $a = 0, \quad b = 1.$

$$= \int_0^1 w \cdot \left[-\frac{d^2u}{dx^2}.dx\right] + \int_0^1 q(x)\cdot w\, u \cdot dx$$

$$= \int_0^1 \frac{du}{dx} \cdot \frac{dw}{dx} \cdot dx + \int_0^1 q(x) \cdot w \cdot u \cdot dx - w(b)\frac{du}{dx}(b) + w(a)\frac{du}{dx}(a)$$

$$= \int_0^1 \left[\frac{du}{dx} \cdot \frac{dw}{dx}\right] + q(x)\cdot w \cdot u\Big].dx - w(b)\bar{q}$$

$= B(w, u) + bt(w).$ $\longrightarrow$ Term non-zero, as the natural Boundary condition is non-homogeneous

where,

$$\int_0^1 W\cdot\left(-\frac{d^2u}{dx^2}\right)\cdot dx + \int_0^1 q(x)\cdot W\cdot u\cdot dx$$

Weight

$$\int W\left[\int_0^x\left[-\frac{d^2u}{dx^2}\right]\cdot dx + \int_0^1 u\cdot q(x)\cdot dx\right]$$

Integration By part, we get →

$$\left[W\cdot\frac{du}{dx}\right]_0^1 - \int_0^1\left[\frac{dW}{dx}\right]\left(-\frac{du}{dx}\right) - \left[\int_0^1 W\cdot \cancel{2x}(2x)\cdot dx\right]$$

$$= -\left[W\cdot\frac{du}{dx}\right]_0^1 + \int_0^1\left(\frac{dW}{dx}\right)\times\left(\frac{du}{dx}\right)\cdot dx$$

$(W = \delta u)$

$$= \left[\delta u\cdot\frac{du}{dx}\right]_0^1 + \int_0^1\left[\frac{d(\delta u)}{dx}\times\frac{du}{dx}\cdot dx\right] - \int_0^1 \delta u\,(2x)\cdot dx - \left[\left[\cancel{\delta(u)(1)\cdot\frac{du}{dx}(1)}\right] - \delta u(0)\cdot\frac{du}{dx}(0)\right]$$

$$= -\int_0^1 \frac{d(\delta u)}{dx}\times\frac{du}{dx}\cdot dx - \int_0^1(\delta u)\,(2x)\cdot dx$$

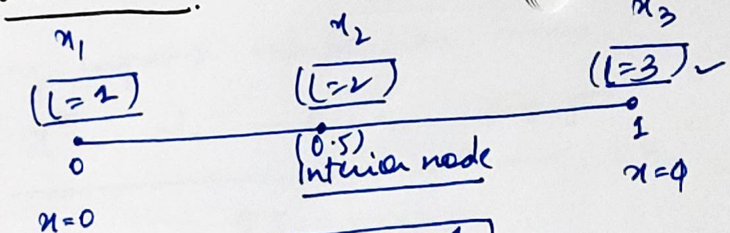
$$= -\int_0^1 \frac{d(\delta u)}{dx}\times\frac{du}{dx}\cdot dx - \int_0^1(\delta u)(2x)\cdot dx + \delta u(0)\cdot$$

$$= -\int_0^1 \frac{d(\delta u)}{dx}\times\frac{du}{dx}\cdot dx - \int_0^1(\delta u)(2x)\cdot dx + \delta u(0)\cdot$$

$$= -\int_0^1 \frac{d(\delta u)}{dx}\times\frac{du}{dx}\cdot dx - \int_0^1(\delta u)(2x)\cdot dx + \delta u(0)\cdot$$

Step-4 : — discretization

Now :



$(0 < x < 1)$

Step-5 :

Lagrange interpolation

$$u(x) = \sum_{i=1}^{n} N_i(x) \cdot u_i$$

At $i=1$ $\quad N_1(x) = \dfrac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} = \dfrac{(x-0.5)(x-1)}{(0-0.5)(0-1)}$

At $i=2$ $\quad N_2(x) = \dfrac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} = \dfrac{(x-0)(x-1)}{(0.5)(-0.5)}$

At $i=3$ $\quad N_3(x) = \dfrac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} = \dfrac{(x-0)(x-0.5)}{(1)(0.5)}$

$$N_1(x) = \frac{(x-0.5)(x-1)}{(0.5)} =$$

$$N_2(x) = \frac{x(x-1)}{-(0.5\times0.5)} =$$

$$N_3(x) = \frac{x(x-0.5)}{0.5} =$$

Continuing Lagrange interpolation →

$$N_1(x) = \frac{x^2 - x - 0.5x + 0.5}{0.5} = \frac{x^2 - 1.5x + 0.5}{(0.5)}$$

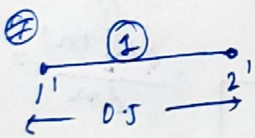
$$N_2(x) = \frac{x^2 - x}{0.25} = \frac{x^2 - x}{(0.25)}$$

$$N_3(x) = \frac{x^2 - 0.5x}{0.5} = \frac{(x^2 - 0.5x)}{(0.5)}$$

$$u(x) = N_1(x) \cdot u_1 + N_2(x) \cdot u_2 + N_3(x) \cdot u_3$$

$$= \left[\frac{x^2 - 1.5x + 0.5}{0.5}\right] u_1 + \left[\frac{x^2 - x}{0.25}\right] u_2 + \left[\frac{(x^2 - 0.5x)}{0.5}\right] u_3$$

Two node interpolation : →

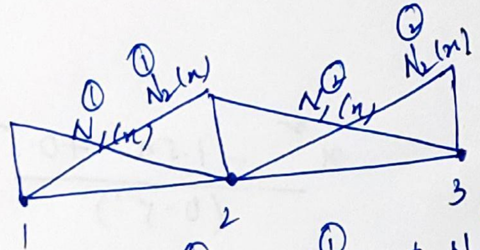


$$N_1^{(1)}(x) = \frac{(x - 0.5)}{0.5} = \frac{x - 0.5}{-0.5} = \frac{0.5 - x}{0.5}$$

$$N_2^{(1)}(x) = \frac{x - 0}{0.5}$$

$$N_2(x) = \frac{x}{0.5}$$

$$u(x) = N_1^{(1)}(x) \cdot u_1^{(1)} + N_2^{(1)}(x)\, u_2^{(1)}$$



$$N_1(x) = \frac{x - x_2}{(x_1 - x_2)}$$

$$N_2(x) = \frac{x - x_1}{(x_2 - x_1)}$$

1st element = $N_1^{(1)}(x) \cdot u_1^{(1)} + N_2^{(1)}(x)\, u_2^{(1)}$

2nd element = $N_1^{(2)}(x)\, u_1^{(2)} + N_2^{(2)}(x) \cdot u_2^{(2)}$

1st Element →

$$N_1^{(1)}(x) = \frac{0.5 - x}{0.5} \qquad N_2^{(1)}(x) = \frac{x}{0.5}$$

$$\frac{dN_1^{(1)}(x)}{dx} = \frac{1}{0.5}\left[0 - 1\right] \qquad \frac{dN_2^{(1)}(x)}{dx} = \frac{1}{0.5}\left[1\right]$$

$\boxed{\frac{dN_1^{(1)}(x)}{dx} = \frac{-1}{0.5}}$

$\boxed{\frac{dN_2^{(1)}(x)}{dx} = \frac{1}{0.5}}$

x_1 x_2

0.5   1.

2nd Element →

$$N_1^{(2)}(x) = \frac{x - 1}{(0.5 - 1)} = \frac{x-1}{-0.5}$$

$$\boxed{N_1^{(2)}(x) = \frac{1 - x}{0.5}}$$

$$N_2^{(2)}(x) = \frac{x - 0.5}{0.5}$$

$$\boxed{N_2^{(2)}(x) = \frac{x - 0.5}{0.5}}$$

$$\boxed{\frac{dN_1^{(2)}(x)}{dx} = \frac{-1}{0.5}}$$

$$\frac{dN_2^{(2)}(x)}{dx} = \frac{1}{0.5}\left[1 - 0\right) = \frac{1}{0.5}$$

$$\frac{dN_1^{(2)}(x)}{dx} = \frac{1}{0.5}(0 - 1) = \frac{-1}{0.5}$$

$$\boxed{\frac{dN_2^{(2)}(x)}{dx} = \frac{1}{0.5}}$$

Weak form
(L.H.S).

$$u^{(1)}(x) = \sum_{l=1}^{2} N_l(x) \cdot u_l = N_1^{(1)}(x) \cdot u_1^{(1)} + N_2^{(1)}(x)\, u_2^{(1)}$$

$$u^{(1)}(x) = \begin{bmatrix} N_1^{(1)}(x) & N_2^{(1)}(x) \end{bmatrix} \begin{bmatrix} u_1^{(1)} \\ u_2^{(1)} \end{bmatrix}$$

$$u^{(2)}(x) = \begin{bmatrix} N_1^{(2)}(x) & N_2^{(2)}(x) \end{bmatrix} \begin{bmatrix} u_1^{(2)} \\ u_2^{(2)} \end{bmatrix}$$

$$\frac{d(u^{(1)}(x))}{dx} = \begin{bmatrix} \frac{dN_1^{(1)}(x)}{dx} & \frac{dN_2^{(1)}(x)}{dx} \end{bmatrix} \begin{bmatrix} u_1^{(1)} \\ u_2^{(1)} \end{bmatrix}$$

$$\frac{d(u^{(2)}(x))}{dx} = \begin{bmatrix} \frac{dN_1^{(2)}(x)}{dx} & \frac{dN_2^{(2)}(x)}{dx} \end{bmatrix} \begin{bmatrix} u_1^{(2)} \\ u_2^{(2)} \end{bmatrix}$$

$$\delta u^{(1)}(x) = \begin{bmatrix} N_1^{(1)}(x) & N_2^{(1)}(x) \end{bmatrix} \begin{bmatrix} \delta u_1^{(1)} \\ \delta u_2^{(1)} \end{bmatrix}$$

$$\delta u^{(2)}(x) = \begin{bmatrix} N_1^{(2)}(x) & N_2^{(2)}(x) \end{bmatrix} \begin{bmatrix} \delta u_1^{(2)} \\ \delta u_2^{(2)} \end{bmatrix}$$

For 1st element

$$\int_0^{0.5} \begin{bmatrix} \delta u_1^{(1)} & \delta u_2^{(1)} \end{bmatrix} \begin{bmatrix} \frac{dN_1^{(1)}(x)}{dx} \\ \frac{dN_2^{(1)}(x)}{dx} \end{bmatrix} \begin{bmatrix} \frac{dN_1^{(1)}(x)}{dx} & \frac{dN_2^{(1)}(x)}{dx} \end{bmatrix} \begin{bmatrix} u_1^{(1)} \\ u_2^{(1)} \end{bmatrix} \cdot dx$$

(7)

$$= \begin{bmatrix} \delta u_1^{(1)} & \delta u_2^{(1)} \end{bmatrix} \begin{bmatrix} K_{11}^{(1)} & K_{12}^{(1)} \\ K_{21}^{(1)} & K_{22}^{(1)} \end{bmatrix} \begin{bmatrix} u_1^{(1)} \\ u_2^{(1)} \end{bmatrix} + \begin{bmatrix} \delta u_1^{(2)} & \delta u_2^{(2)} \end{bmatrix} \begin{bmatrix} K_{11}^{(2)} & K_{12}^{(2)} \\ K_{21}^{(2)} & K_{22}^{(2)} \end{bmatrix} \begin{bmatrix} u_1^{(2)} \\ u_2^{(2)} \end{bmatrix}$$

K_1

$$K_{11}^{(1)} = \int_0^{0.5} \left(\frac{dN_1^{(1)}(x)}{dx}\right)^2 . dx = \int_0^{0.5} \left(\frac{-1}{0.5}\right)^2 . dx = \int_0^{0.5} \left[\frac{1}{0.25}\right] . dx = [4x]_0^{0.5} = 4(0.5) - \cancel{4(0)}\,0 = 2$$

$$K_{12}^{(1)} = \int_0^{0.5} \left[\frac{dN_1^{(1)}(x)}{dx}\right] \times \left[\frac{dN_2^{(1)}(x)}{dx}\right] = K_{21} = \int_0^{0.5} \left[\frac{-1}{0.5}\right] \times \left[\frac{1}{0.5}\right] . dx = \int_0^{0.5} [-4 . dx] = [-4x]_0^{0.5} = -[4(0.5) - 4(0)] = -2$$

$$K_{21}^{(1)} = \int_0^{0.5} \left[\frac{dN_2^{(1)}(x)}{dx}\right]^2 = \int_0^{0.5} \left[\frac{1}{0.5}\right]^2 . dx = [4x]_0^{0.5} = 2$$

$$K_1 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

For second Element

$$K_{11}^{(2)} = \int_{0.5}^{1} \left[\frac{dN_1^{(2)}(x)}{dx}\right]^2 . dx = \int_{0.5}^{1} \left[\frac{-1}{0.5}\right]^2 = [4x]_{0.5}^{1} = 4(\cancel{0.5}\,1 - 0.5) = 4(0.5) = 2$$

$$K_{12}^{(2)} = \int_{0.5}^{1} \left[\frac{dN_1^{(2)}(x)}{dx}\right] \times \left[\frac{dN_2(x)}{dx}\right] = K_{21} = \frac{-1}{0.5} \times \frac{1}{0.5} = \int \frac{-1}{0.25} = -4[1 - 0.5] = -2$$

$$K_{21}^{(2)} = \int_0^{0.5} \left[\frac{dN_2^{(2)}(x)}{dx}\right]^2 = 4[\cancel{0}\,1 - 0.5] = 2$$

$$K_2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$\int_0^{0.5} \frac{d(\delta u)}{dx} \cdot \frac{du}{dx} \cdot dx = \begin{bmatrix} \delta u_1^{(1)} & \delta u_2^{(1)} \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_1^{(1)} \\ u_2^{(1)} \end{bmatrix}$$

$$\int_{0.5}^{1} \frac{d(\delta u)}{dx} \cdot \frac{du}{dx} \cdot dx = \begin{bmatrix} \delta u_1^{(2)} & \delta u_2^{(2)} \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_1^{(2)} \\ u_2^{(2)} \end{bmatrix}$$

Finished, discretization of weak form: →

Our weak form : →

$$\boxed{-\int_0^1 \frac{d(\delta u)}{dx} \cdot \frac{du}{dx} \cdot dx - \int_0^1 \delta u\,(2x) \cdot dx + \delta u(0)}$$

L.H.S →

$$\boxed{\int_0^1 \frac{d(\delta u)}{dx} \cdot \frac{du}{dx} \cdot dx = -\delta u(0) + \int_0^1 \delta u\,(2x) \cdot dx}$$

R.H.S → # $\boxed{-\delta u(0) + \int_0^1 \delta u\,(2x) \cdot dx}$ ✓

$$\left[\delta u^{①} \quad \delta u^{②}\right] \left[\int_0^{0.5} [N_1^{①}(x)]\right.$$

$$\delta u = \sum_{i=1}^{2} N_i(x) \cdot \delta u_i = N_1^{①}(x) \cdot (\delta u_1^{①}) + N_2^{①}(x)\,(\delta u_2^{①})$$

$$\delta u = \left[N_1^{①}(x) \quad N_2^{①}(x)\right] \begin{bmatrix} \delta u_1^{①} \\ \delta u_2^{①} \end{bmatrix}$$

$(b(x) = 2x)$

$$\delta u\,(b(x)) \cdot dx = \int_0^{0.5} \left[\delta u_1^{①} \quad \delta u_2^{①}\right] \begin{bmatrix} N_1^{①}(x) \\ N_2^{①}(x) \end{bmatrix} (2x) \cdot dx$$

$$= \left[\delta u_1^{①} \quad \delta u_2^{①}\right] \begin{bmatrix} \int_0^{0.5} N_1^{①}(x) \cdot b(x) \cdot dx \\ \int_0^{0.5} N_2^{①}(x) \cdot b(x) \cdot dx \end{bmatrix}$$

$$= \left[\delta u_1^{①} \quad \delta u_2^{①}\right] \begin{bmatrix} \int_0^{0.5} \left[\frac{0.5 - x}{0.5}\right] (2x) \cdot dx \\ \int_0^{0.5} \left[\frac{x}{0.5}\right] (2x) \cdot dx \end{bmatrix}$$

$$\begin{bmatrix} \delta u_1^{(1)} & \delta u_2^{(1)} \end{bmatrix} \begin{bmatrix} 0.083 \\ 0.166 \end{bmatrix} + \begin{bmatrix} \delta u_1^{(2)} & \delta u_2^{(2)} \end{bmatrix} \begin{bmatrix} \int_{0.5}^{1} N_1^{(2)}(x) \cdot b(x) \cdot dx \\ \int_{0.5}^{1} N_2^{(2)}(x) \cdot b(x) \cdot dx \end{bmatrix}$$

$$= \begin{bmatrix} \delta u_1^{(1)} & \delta u_2^{(1)} \end{bmatrix} \begin{bmatrix} 0.083 \\ 0.166 \end{bmatrix} + \begin{bmatrix} \delta u_1^{(2)} & \delta u_2^{(2)} \end{bmatrix} \begin{bmatrix} \int_{0.5}^{1} \frac{[1-x]}{0.5} (2x) \cdot dx \\ \int_{0.5}^{1} \left[\frac{x-0.5}{0.5}\right] (2x) \cdot dx \end{bmatrix}$$

$$= \begin{bmatrix} \delta u_1^{(1)} & \delta u_2^{(1)} \end{bmatrix} \begin{bmatrix} 0.083 \\ 0.166 \end{bmatrix} + \begin{bmatrix} \delta u_1^{(2)} & \delta u_2^{(2)} \end{bmatrix} \begin{bmatrix} 0.33 \\ 0.416 \end{bmatrix}$$

Step: 7 : $\rightarrow$

Assemble the global system equations $\rightarrow$

1st Element

$$\begin{bmatrix} \delta u_1 & \delta u_2 & \delta u_3 \end{bmatrix} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \begin{bmatrix} K_{11}^{(1)} & K_{12}^{(1)} & 0 \\ K_{21}^{(1)} & K_{22}^{(1)} & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

2nd Element : $\rightarrow$

$$\begin{bmatrix} \delta u_1 & \delta u_2 & \delta u_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{11}^{(2)} & K_{12}^{(2)} \\ 0 & K_{21}^{(2)} & K_{22}^{(2)} \end{bmatrix}$$

4✓

$$[\delta u_1 \quad \delta u_2 \quad \delta u_3] \begin{bmatrix} K_{11}^{(1)} & K_{12}^{(1)} & 0 \\ K_{21}^{(1)} & (K_{22}^{(1)} + K_{11}^{(2)}) & K_{12}^{(2)} \\ 0 & K_{21}^{(2)} & K_{22}^{(2)} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

L.H.S : →

$$= -[\delta u_1 \quad \delta u_2 \quad \delta u_3] \begin{bmatrix} 2 & -2 & 0 \\ -2 & (2+2) & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$= -[\delta u_1 \quad \delta u_2 \quad \delta u_3] \begin{bmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

R.H.S : →

$$[\delta u_1^{(1)} \quad \delta u_2^{(1)}] \begin{bmatrix} 0.083 \\ 0.166 \end{bmatrix} + [\delta u_1^{(2)} \quad \delta u_2^{(2)}] \begin{bmatrix} 0.33 \\ 0.416 \end{bmatrix} = [\delta u_1 \quad \delta u_2 \quad \delta u_3] \begin{bmatrix} f_1^{(1)} \\ f_2^{(1)} + f_1^{(2)} \\ f_2^{(2)} \end{bmatrix}$$

$$= [\delta u_1 \quad \delta u_2 \quad \delta u_3] \begin{bmatrix} 0.083 \\ 0.166 + 0.33 \\ 0.416 \end{bmatrix} =$$

R.H.S

$$[\delta u_1 \quad \delta u_2 \quad \delta u_3] \begin{bmatrix} 0.083 \\ 0.496 \\ 0.416 \end{bmatrix}$$

Now,

$$-[\delta u_1 \quad \cancel{\delta u_2} \quad \delta u_3] \begin{bmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = [\delta u_1 \quad \cancel{\delta u_2} \quad \delta u_3] \begin{bmatrix} 0.083 \\ 0.496 \\ 0.416 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0.083 \\ 0.496 \\ 0.416 \end{bmatrix}$$

step-8: Now, imposing Essential Boundary Conditions

(#) penalty method.

We get: $\boxed{u_{(x=L)} = 0}$

$\cancel{u_3 = 0}$

$$\underbrace{\begin{bmatrix} -2 & +2 & \cancel{0} \\ +2 & -4 & 0 \\ \cancel{0} & \cancel{2} & 10^{10} \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ \cancel{u_3}\,0 \end{bmatrix}}_{U} = \underbrace{\begin{bmatrix} 0.083 \\ 0.496 \\ \cancel{0.416}\ 10^{10} \end{bmatrix}}_{F}$$

$$0 - 2(u_2) + 10^{10}(u_3) = 10^{10}$$

$\boxed{u_3 = 0}$ ✓

Step-9 : →

We get

$$u = (K^{-1})(F)$$

$$= \begin{bmatrix} 2 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 10^{10} \end{bmatrix}^{-1} \begin{bmatrix} 0.083 \\ 0.496 \\ 10^{10} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -0.33 \\ -0.29 \\ 0 \end{bmatrix}}$$

$$\cancel{u^{(1)}(x) = N_1^{(1)}(x) \cdot u_1^{(1)} + N_2^{(1)}(x)\,(u_2)^{(1)}}$$

$$\cancel{=}$$

Verification: →

$$\int -\frac{d^2u}{dx^2} = \int 2x$$

Integrating both sides we get →

$$-\int \frac{du}{dx} = \int \frac{\cancel{2}x^2}{\cancel{2}} + C_1 \quad \text{—— ①}$$

$$\boxed{-u(x) = \frac{x^3}{3} + C_1 x + C_2} \quad \text{—— ②}$$

Applying BC's, we get →

$$\frac{du(0)}{dx} = 1$$

$u(1) = 0$ from eq ①

$$C_1 - \frac{du(0)}{dx} = (0)^2 + C_1$$

$$(-1 = C_1)$$

$u(1) = 0$ eq ②, we get →

$$u(1) = 0 = -\frac{x^3}{3} - C_1 x - C_2$$

$$(0) = -\frac{1}{3} - C_1(1) - C_2 \qquad \left(1 - \frac{1}{3}\right)$$

$$= -\frac{1}{3} + 1 - C_2$$

$$0 = \frac{2}{3} - C_2 \qquad \boxed{C_2 = 2/3}$$

Exact sol^n: $\longrightarrow$

$\boxed{u = -\frac{x^3}{3} + x - \frac{2}{3}}$

$u_1^{(1)} = u_1 =$

$u_2^{(1)} = u_2 =$ $u_1^{(2)} = u_2^{(1)} = u_2 =$

$u_2^{(2)} = u_3 =$

From weak form we get ~~$(N_1^{(1)} u_1^{(1)} + N_2^{(1)} u_2^{(1)})$~~

$= (N_1^{(1)} u_1^{(1)} + N_2^{(1)} u_2^{(1)})$

$u_1(x) = \left[\frac{0.5 - x}{0.5}\right][u_1] + \left[\frac{x}{0.5}\right][u_2]$ $u_2 = \left[\frac{1-x}{0.5}\right][u_2] + \left[\frac{x - 0.5}{0.5}\right][u_3]$

\# ~~$u(x) = \left[\frac{0.5 - x}{0.5}\right][0.33] + \left[\frac{x}{0.5}\right][0.28] + \left[\frac{1-x}{0.5}\right][0.28] + \left[\frac{x-0.5}{0.5}\right][0]$~~

\# plotted on the matlab: $\longrightarrow$

\# Through this problem was solved using three corner nodes.

The plot shows, sol^n is trying to be close at the Right Extreme Boundary.

A close approximation would be a sol^n with more nodes to imitate cubic behaviour of the curve.

(e) Exact sol^n: $\longrightarrow$ plot of the differential Eq^n

i.e strong form

$\downarrow$ integrated

Final Eq^n of the Exact sol^n for $x \in [0,1]$

$$\frac{x^3}{3} + x - \frac{2}{3}$$

(f) The above equation is plotted ✓

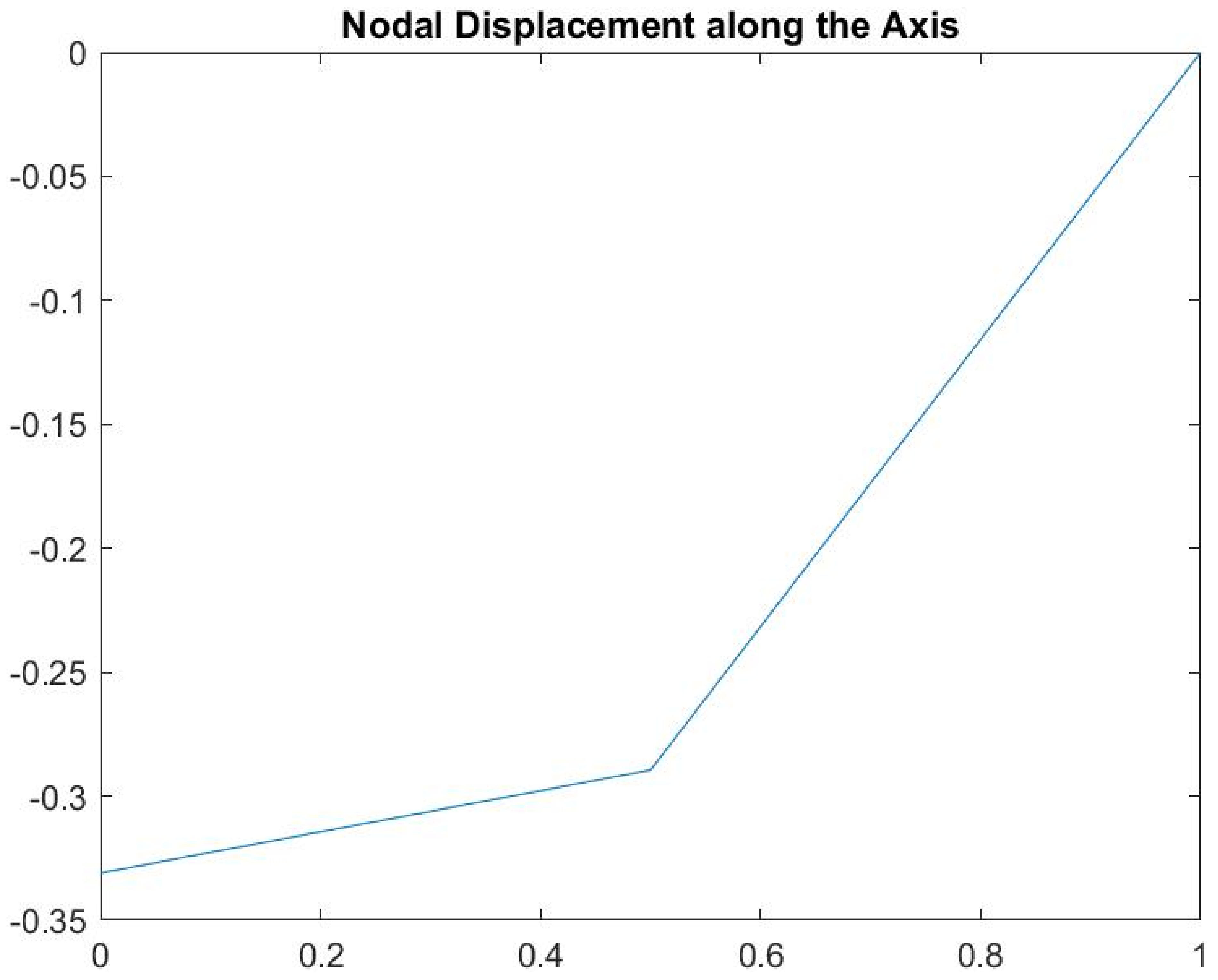
Now, ~~ago~~ Along with # ($u_1 = N_1^{(1)}(x) \cdot u_1 + N_2^{(1)}(x) \cdot u_2$) ✓

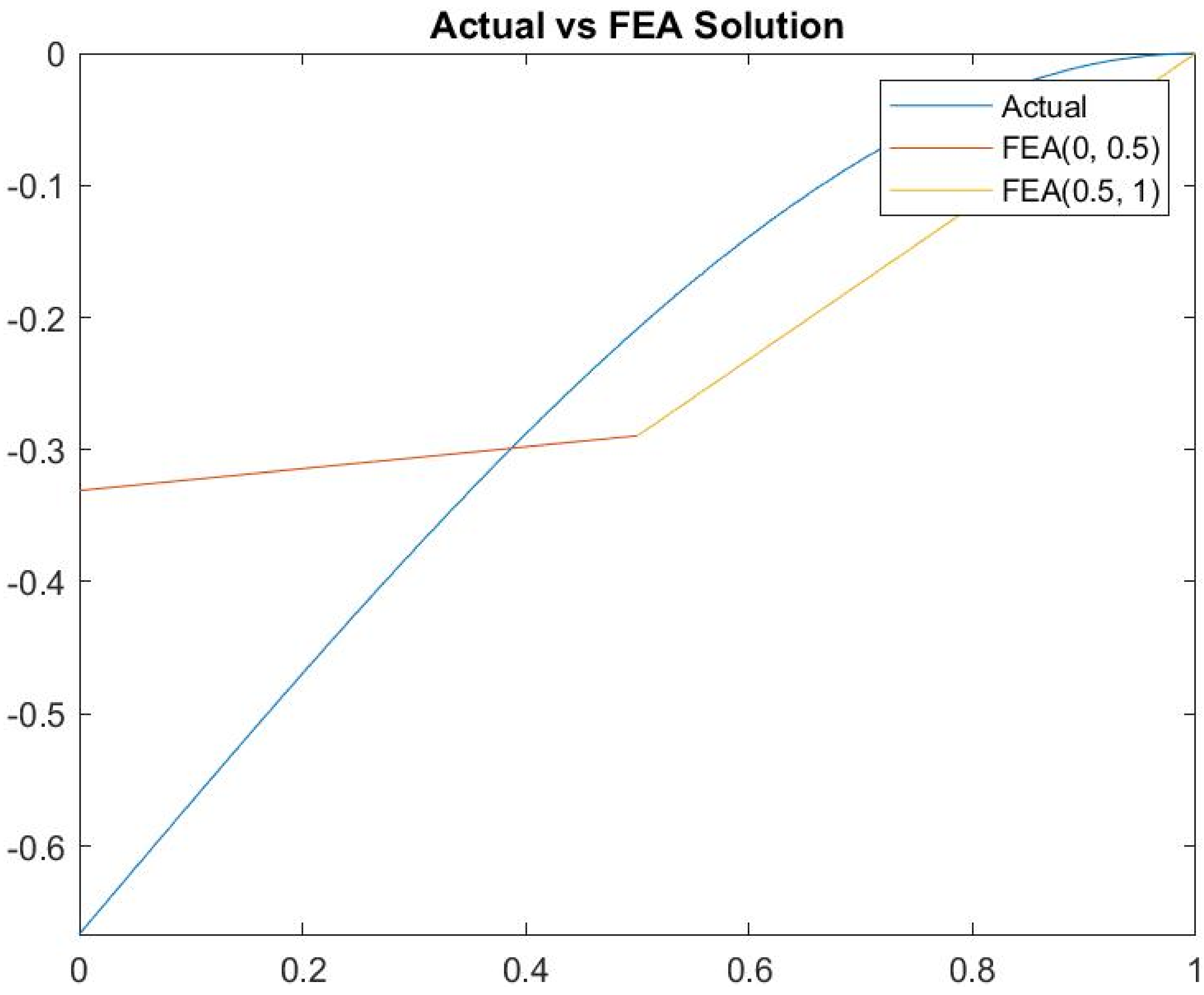
($u_2 = N_1^{(2)}(x)\, u_2 + N_2^{(2)}(x) \cdot u_3$)

A close approximation of the sol^n would be.

by increasing the number of nodes

and further discretizing in the entire domain


Nodal Displacement along the Axis
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
0
0.2
0.4
0.6
0.8
1


Actual vs FEA Solution
Actual
FEA(0, 0.5)
FEA(0.5, 1)
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
0.2
0.4
0.6
0.8
1