

# Introduction to Linear Algebra and to Mathematics for Machine Learning

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## 1 Solving simultaneous equations

The simplest kind of simultaneous equation is one where we already know all but one of the variables:

$$\begin{aligned}3x - y &= 2 \\ x &= 4\end{aligned}$$

We can solve this by simply substituting the value of  $x$  into the first equation to find  $y$ .

Otherwise, a useful general principle is to *isolate one of the variables*. For example, we can take the second equation below away from the first to isolate  $x$ :

$$\begin{aligned}3x - 2y &= 7 \\ 2x - 2y &= 2\end{aligned}$$

But what if neither  $x$  nor  $y$ 's coefficients in the two equations are the same?

One method is **elimination**. For example, to solve the below equations, multiply both sides of the first equation by 2 then take one equation away from the other:

$$\begin{aligned}3x - 2y &= 4 \\ 6x + 3y &= 15\end{aligned}$$

Another method is **substitution**, where we rearrange one of the equations to the form  $x = ay + b$  or  $y = cx + d$  and then substitute  $x$  or  $y$  into the other equation. Consider the following pair of equations:

$$\begin{aligned} -2x + 2y &= 20 \\ 5x + 3y &= 6 \end{aligned}$$

From the first equation, we can rewrite  $x$  as  $y - 10$  and substitute this value of  $x$  into the second equation to find the value of  $y$ .

When systems of simultaneous equations have **more than two unknown variables**, first find one of the variables by elimination or substitution, which will lead to two equations and two unknown variables. Continue this process to find all the variables.

## 2 Vector operations

- **Vector addition:** 
$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}$$

- For example,  $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$  can be added to get  $\begin{bmatrix} 2 + (-1) \\ 1 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .

- **Scalar multiplication:** 
$$k \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} ka_1 \\ ka_2 \\ \vdots \\ ka_n \end{bmatrix}$$

- For example,  $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$  and 4 can be multiplied to get  $\begin{bmatrix} 2 \cdot 4 \\ 1 \cdot 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$