Introduction to Linear Algebra and to Mathematics for Machine Learning

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Contents

1 Solving simultaneous equations

The simplest kind of simultaneous equation is one where we already know all but one of the variables:

$$\begin{aligned} 3x - y &= 2 \\ x &= 4 \end{aligned}$$

We can solve this by simply substituting the value of x into the first equation to find y.

Otherwise, a useful general principle is to *isolate one of the variables*. For example, we can take the second equation below away from the first to isolate x:

$$\begin{aligned} 3x - 2y &= 7 \\ 2x - 2y &= 2 \end{aligned}$$

But what if neither x nor y's coefficients in the two equations are the same?

One method is **elimination**. For example, to solve the below equations, multiply both sides of the first equation by 2 then take one equation away from the other:

$$\begin{aligned} 3x - 2y &= 4 \\ 6x + 3y &= 15 \end{aligned}$$

Another method is **substitution**, where we rearrange one of the equations to the form $x = ay + b$ or $y = cx + d$ and then substitute x or y into the other equation. Consider the following pair of equations:

$$\begin{aligned} -2x + 2y &= 20 \\ 5x + 3y &= 6 \end{aligned}$$

From the first equation, we can rewrite x as $y - 10$ and substitute this value of x into the second equation to find the value of y.

When systems of simultaneous equations have **more than two unknown variables**, first find one of the variables by elimination or substitution, which will lead to two equations and two unknown variables. Continue this process to find all the variables.

2 Vector operations

2.1 Vector addition

$$\boxed{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}}$$

For example, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ can be added to get $\begin{bmatrix} 2 + (-1) \\ 1 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2.2 Scalar multiplication

$$\boxed{k \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} ka_1 \\ ka_2 \\ \vdots \\ ka_n \end{bmatrix}}$$

For example, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and 4 can be multiplied to get $\begin{bmatrix} 2 \cdot 4 \\ 1 \cdot 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$