

# Homework Quiz 6

<b>Due</b> 3 Nov at 6:00	<b>Points</b> 210	<b>Questions</b> 22
<b>Available</b> 27 Oct at 6:00 - 3 Nov at 6:00	<b>Time limit</b> None	

This quiz was locked 3 Nov at 6:00.

## Attempt history

	Attempt	Time	Score
LATEST	<a href="#">Attempt 1</a>	362 minutes	203.33 out of 210

Score for this quiz: **203.33** out of 210  
Submitted 3 Nov at 3:25  
This attempt took 362 minutes.

Question 1

10 / 10 pts

The diagram shows a state transition system with 6 states, represented by blue circles labeled 1 through 6. The transitions and their probabilities are as follows:

- State 1: A self-loop with probability 0.95. A transition to State 2 with probability 0.05.
- State 2: A transition to State 3 with probability 0.6. A transition to State 5 with probability 0.2.
- State 3: A transition to State 4 with probability 0.5. A transition to State 6 with probability 0.5.
- State 4: A transition to State 3 with probability 0.7. A self-loop with probability 0.3.
- State 5: A transition to State 2 with probability 0.4. A self-loop with probability 0.8.
- State 6: A self-loop with probability 1.

In the Markov chain shown above, if  $\mathbf{s}(5) = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \\ 0.2 \\ 0.2 \\ 0 \end{bmatrix}$  what is the value of  $s_2(6)$ ?

Correct!

0.05

Correct Answers

0.05 (with margin: 0.001)

## Question 2

5 / 5 pts

State 1 from the previous problem is an absorbing state.

☐ True

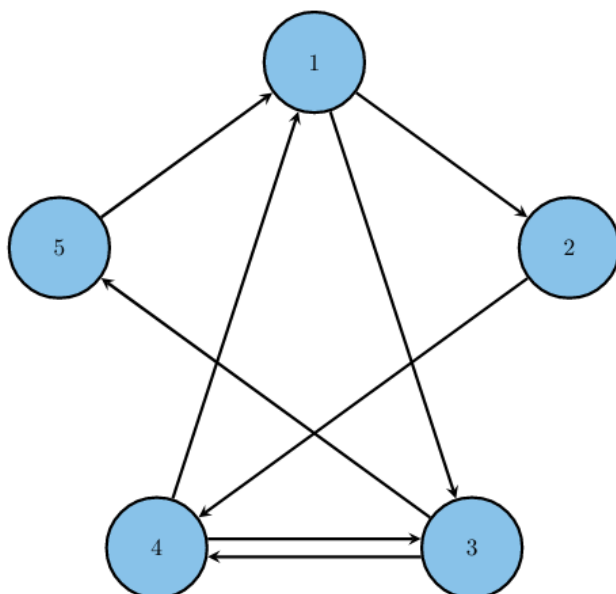
Correct!

☒ False

## Question 3

10 / 10 pts

Consider the PageRank network below with  $d = 0.5$ . Using Markov chain analysis, what is the value of the free variable such that  $(I - P^T)\mathbf{s}^* = \mathbf{0}$  and  $\sum_i s_i^* = 1$ ? Remember to first construct the Hyperlink matrix  $H$ , then use  $H$  to construct  $P$ . Enter your answer to 4 decimal places. (Hint: an example is done in section 2.2 of the Markov chain case study)



Correct!

0.1543

Correct Answers

0.1543 (with margin: 0.001)

#### Question 4

5 / 5 pts

Why can the Markov chain interpretation of Page Rank have self-loops even though the underlying network has no self-links?

☐

The Markov chain interpretation is independent of the underlying network.

☐

The Markov chain interpretation cannot have self-loops if the underlying network has no self-links.

Correct!

☒

When  $d < 1$ , there is a nonzero probability that we don't leave a page.

☐

Markov chains need self-loops to function.

## Question 5

10 / 10 pts

Which of the following collections of vectors from the exercises in section 1.7 in the Lay text are linearly independent? Check all that apply.

Correct!

☒ The columns of the matrix in Exercise 6:

$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

☐ The collection in Exercise 16:

$$\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$$

☐ The collection in Exercise 20:

$$\begin{bmatrix} 1 \\ 4 \\ -7 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

☐ The collection in Exercise 18:

$$\begin{bmatrix} 4 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

Correct!

☒ The collection in Exercise 2:

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

Correct!

☒ The collection in Exercise 4:

$$\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -8 \end{bmatrix}$$

☐ The columns of the matrix in Exercise 8:

$$\begin{bmatrix} 1 & -3 & 3 & -2 \\ -3 & 7 & -1 & 2 \\ 0 & 1 & -4 & 3 \end{bmatrix}$$

## Question 6

10 / 10 pts

Two non-zero vectors in  $\mathbb{R}^3$  are linearly dependent if and only if they both lie on the same line through the origin.

Correct!

☒ True

☐ False

### Question 7

10 / 10 pts

If vectors  $x$ ,  $y$ , and  $z$  are such that  $z$  belongs to  $\text{span}\{x, y\}$ , then the collection

$\{x, y, z\}$  must be linearly dependent.

Correct!

☒ True

☐ False

### Question 8

10 / 10 pts

If the columns of a matrix are linearly dependent, then the matrix must have more columns than rows.

Correct!

☐ True

☒ False

### Question 9

10 / 10 pts

If a collection of vectors in  $\mathbb{R}^n$  contains the zero vector, then the collection must be linearly dependent.

Correct!

☒ True

☐ False

### Question 10

10 / 10 pts

If the third column of a 4x4 matrix is not a linear combination of the other three columns, then the columns of this matrix must be linearly independent.

☐ True

☒ False

Correct!

### Question 11

10 / 10 pts

If the set of vectors  $\{v_1, v_2, v_3, v_4, v_5\}$  is linearly independent, then so is  $\{v_1, v_2, v_4, v_5\}$ .

☒ True

☐ False

Correct!

### Question 12

10 / 10 pts

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix}$$

$$b = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$$

Suppose  $T()$  is defined by  $T(x) = Ax$ . Find a vector  $x$  whose image under  $T$  is  $b$ , and determine whether  $x$  is unique.

Correct!

- ☐  $x = [5; 3; -1]$  and the answer is unique.
- ☒  $x = [-5; -3; 1]$  and the answer is unique.
- ☐  $x = [-5; -3; 1]$  and the answer is not unique.
- ☐  $x = [5; 3; -1]$  and the answer is not unique.

### Question 13

10 / 10 pts

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -4 & 5 \\ 0 & 1 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 9 \\ 3 \end{bmatrix}$$

Suppose  $T()$  is defined by  $T(x) = Ax$ . Find a vector  $x$  whose image under  $T$  is  $b$ , and determine whether  $x$  is unique.

Correct!

- ☒  $x = [1; 1; 2]$  and the answer is not unique.
- ☐  $x = [7; 3; 0]$  and the answer is unique.
- ☐  $x = [-3; -1; 1]$  and the answer is not unique.
- ☐  $x = [1; 1; 2]$  and the answer is unique.

### Question 14

10 / 10 pts

Suppose  $T$  is a linear transformation from  $\mathbb{R}^3$  to  $\mathbb{R}^2$ , and suppose  $T(e_1) = [1; 2]$ ,  $T(e_2) = [4; -8]$ , and  $T(e_3) = [-6; 4]$ ,

where  $e_1$ ,  $e_2$ , and  $e_3$  denote the columns of the  $3 \times 3$  identity matrix from left to right. What is the matrix of this transformation  $T$ ?

- ☐ There is no matrix for this transformation.

Correct!

☐ [ 1 2 ; 4 -8 ; -6 4 ]

☒ [ 1 4 -6 ; 2 -8 4 ]

☐ [ -6 4 1 ; 4 -8 2 ]

### Question 15

10 / 10 pts

Suppose  $T$  is a linear horizontal shear transformation from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  that leaves  $e_1$  unchanged but maps  $e_2$  into  $3e_1 + e_2$  (here  $e_1$  and  $e_2$  denote the columns of the 2-by-2 identity matrix from left to right). What is the matrix of this transformation  $T$ ?

☐ There is no matrix for this transformation.

☐ [ 1 0 ; 1 3 ]

☐ [ 1 3 0 ; 0 1 0 ; 0 0 1 ]

☒ [ 1 3 ; 0 1 ]

Correct!

### Question 16

10 / 10 pts

Suppose  $T$  is the linear transformation from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  that first reflects points about the vertical axis and then rotates points counterclockwise by 90 degrees. What is the matrix of this transformation  $T$ ?

☒ [ 0 -1 ; -1 0 ]

☐ There is no matrix for this transformation.

☐ [ 0 1 ; 1 0 ]

Correct!



☐  $[-1 \ 0; 0 \ -1]$

### Question 17

10 / 10 pts

$A = [0 \ 0 \ 0 \ 0; 1 \ 1 \ 0 \ 0; 0 \ 1 \ 1 \ 0; 0 \ 0 \ 1 \ 1]$

The transformation  $T(x)=Ax$  is:

- ☐ both one-to-one and onto
- ☐ onto but not one-to-one
- ☐ one-to-one but not onto
- ☒ neither one-to-one nor onto

Correct!

### Question 18

10 / 10 pts

$A = [1 \ -5 \ 4; 0 \ 1 \ -6]$

The transformation  $T(x)=Ax$  is:

- ☒ onto but not one-to-one
- ☐ neither one-to-one nor onto
- ☐ both one-to-one and onto
- ☐ one-to-one but not onto

Correct!

### Question 19

10 / 10 pts

Suppose  $A$  is a 4-by-7 matrix. Then the transformation  $T(x)=Ax$  is:

- ☐ onto but not one-to-one
- ☐ both one-to-one and onto
- ☐ one-to-one but not onto
- ☐ neither one-to-one nor onto
- ☒ not enough information is provided

Correct!

### Question 20

10 / 10 pts

Suppose  $A$  is a matrix such that  $\text{rref}(A)$  is the identity matrix. Then the transformation  $T(x)=Ax$  is:

- ☐ not enough information is provided
- ☐ neither one-to-one nor onto
- ☐ onto but not one-to-one
- ☒ both one-to-one and onto
- ☐ one-to-one but not onto

Correct!

### Question 21

10 / 10 pts

If a matrix  $A$  is  $3 \times 6$  and the product  $AB$  is  $3 \times 2$ , then what is the size of  $B$ ?

- ☐  $2 \times 6$

Correct!

☐ 6x3

☒ 6x2

☐ 3x3

☐ 3x2

☐ 3x6

## Question 22

3.33 / 10 pts

Assume you are given the linear transformations T1, T2, T3, T4, T5 with corresponding matrices:

$$A1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

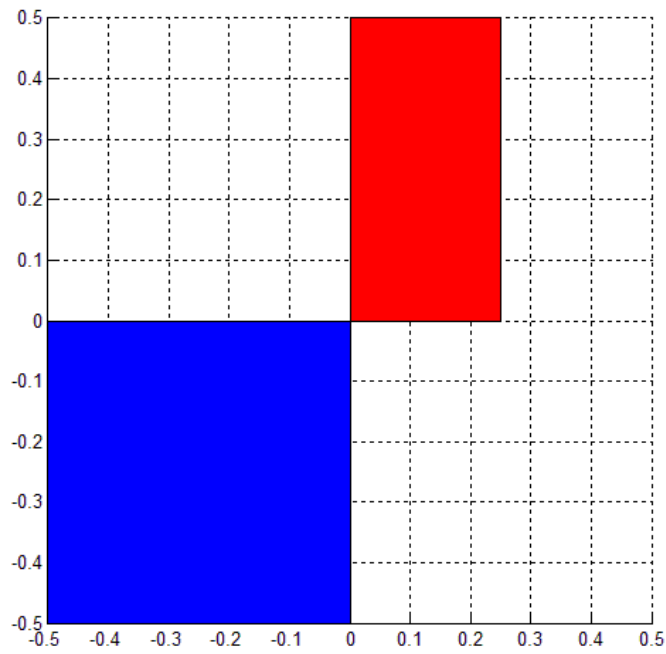
$$A2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A3 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$A4 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A5 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Select all possible sequences of transformations that would transform the red rectangle into the blue rectangle, in the provided image below. Note: transformations are applied in the order they are presented.



Correct!

☒ T4 -> T2 -> T1

☐ T5 -> T1 -> T2

Correct!

☒ T4 -> T1 -> T2

ou Answered

☒ T3 -> T4

orrect answer

☐ T3 -> T5

☐ T4 -> T5 -> T3

Quiz score: **203.33** out of 210