# Engineering Analysis I, Fall 2018

# Midterm 2

Name ______________________________

Section number ____________

| Section number | Discussion time | Instructor |
|---|---|---|
| 30 | 9:00 a.m. | Randy Freeman |
| 31 | 10:00 a.m. | Michael Honig |
| 32 | 11:00 a.m. | Prem Kumar |
| 33 | 12:00 noon. | Prem Kumar |
| 35 | 11:00 a.m. | Michael Honig |

This exam is closed-book and closed-notes. Calculators, computers, phones, or other computing/communication devices are not allowed.

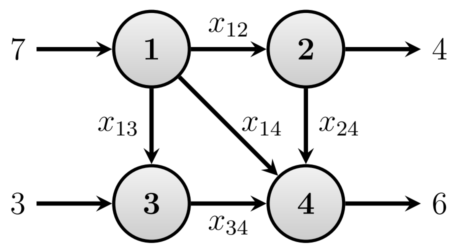
Students should skip this page—it is only for graders.

| Question | Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 10 | |
| 3 | 23 | |
| 4 | 9 | |
| 5 | 10 | |
| 6 | 27 | |
| 7 | 9 | |
| Total: | 100 | |

Answer each question in the space provided. There are 7 questions for a total of 100 points.

**Problem 1**

Consider the following network flow diagram and flow balance equations:




$$\mathbf{x} = \begin{bmatrix} x_{12} \\ x_{13} \\ x_{14} \\ x_{24} \\ x_{34} \end{bmatrix}$$

$$\begin{aligned} x_{12} + x_{13} + x_{14} &= 7 \\ x_{12} - x_{24} &= 4 \\ -x_{13} + x_{34} &= 3 \\ x_{14} + x_{24} + x_{34} &= 6 \end{aligned}$$

(a) [2 points] Write the flow balance equations in the form $A\mathbf{x} = \mathbf{b}$, with $\mathbf{x}$ as defined above.

(b) [3 points] Find the reduced row echelon form of the augmented matrix $[A \;\; \mathbf{b}]$.

(c) [3 points] Write the solution set for your system of equations in parametric vector form.

(d) [2 points] Find a parametric vector form solution in which the flows in the particular solution are all non-negative. There may be more than one right answer.

(e) [2 points] Draw a solution to the network flow problem which has $x_{14} = 0$. Make sure no flows are negative! There may be more than one right answer.

**Problem 2**

Let

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -3 & 1 & 4 \\ 2 & 1 & -2 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -2 & -2 \end{bmatrix}$$

Find each of the following quantities or write "not defined" if the operation is not defined.

(a) [2 points] $A^2$

(b) [2 points] $A^{-1}$

(c) [2 points] $B^2$

(d) [2 points] $BC$

(e) [2 points] $CB$

**Problem 3**

(a) i. [3 points] Suppose $T$ is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that maps the vector $\mathbf{x} = \begin{bmatrix}1\\0\end{bmatrix}$ to $T(\mathbf{x}) = \begin{bmatrix}2\\2\end{bmatrix}$ and the vector $\mathbf{x} = \begin{bmatrix}0\\-1\end{bmatrix}$ to $T(\mathbf{x}) = \begin{bmatrix}1\\-1\end{bmatrix}$. What is the image of $\mathbf{x} = \begin{bmatrix}1\\-1\end{bmatrix}$ under $T$?

ii. [3 points] Suppose $T$ is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ that maps the vector $\mathbf{x} = \begin{bmatrix}-1\\1.5\\2\end{bmatrix}$ to $T(\mathbf{x}) = \begin{bmatrix}2\\3\end{bmatrix}$. Write a vector $\mathbf{x}$ such that $T(\mathbf{x}) = \begin{bmatrix}1\\1.5\end{bmatrix}$. Is this $\mathbf{x}$ unique?

(b) Suppose $T$ is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ such that $T(\mathbf{x}) = A\mathbf{x}$. Write the standard matrix $A$ for each of the following transformations.

i. [2 points] $T$ reflects about the line $x_2 = -x_1$.

ii. [2 points] $T$ first rotates by 90 degrees counterclockwise, then projects onto the $x_2$ axis.

iii. [2 points] $T$ first scales horizontally by a factor of 2 then reflects about the $x_2$ axis.

(c) [1½ points] Select all transformations from part (b) that are **onto**. Put a check mark ✓ in the box next to **EACH** correct answer.

☐ i. ☐ ii. ☐ iii.

(d) [1½ points] Select all transformations from part (b) that are **one-to-one**. Put a check mark ✓ in the box next to **EACH** correct answer.

☐ i. ☐ ii. ☐ iii.

(e) Consider the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^4$ given by $T(\mathbf{x}) = \begin{bmatrix} x_1 + 2x_2 + 2x_3 \\ x_1 + 2x_2 + ax_3 \\ 2x_1 + 4x_2 + 4x_3 \\ x_2 \end{bmatrix}$

where $a$ is a constant.

i. [2 points] Write the standard matrix $A$ of this transformation.

ii. [2 points] For what values of $a$ (if any) is $T$ **onto**?

ii. ____________

iii. [2 points] For what values of $a$ (if any) is $T$ NOT **one-to-one**?

iii. ____________

iv. [2 points] For what values of $a$ (if any) is $T$ **invertible**?

iv. ____________

**Problem 4**

(a) Consider the following matrix:

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & a & b & 0 & 0 \\ 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 0 & d \end{bmatrix}$$

Here $a$, $b$, $c$, and $d$ represent constants. If this matrix is in *reduced* row echelon form, and there are *three pivots*, then what must be the values of these four parameters? For each parameter, either write a specific number or write "any number" if the parameter can have any value.

i. [1½ points] What must be the value of $a$?

i. ____________

ii. [1½ points] What must be the value of $b$?

ii. ____________

iii. [1½ points] What must be the value of $c$?

iii. ____________

iv. [1½ points] What must be the value of $d$?

iv. ____________

(b) [3 points] What is the reduced row echelon form of the following matrix?

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & 3 & 0 \\ 6 & 6 & 6 \end{bmatrix}$$

**Problem 5**

Answer each question in the space provided. Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -5 & 3 & 0 \\ 0 & 4 & -3 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 2 & 2 & 4 \\ 1 & 2 & 6 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) [2 points] Is $A$ invertible? If so, find its inverse.

(b) [2 points] Is $B$ invertible? If so, find its inverse.

(c) [2 points] Is $C$ invertible? If so, find its inverse.

(d) [2 points] Is the product $AB$ invertible? If so, find its inverse.

(e) [2 points] Is the product $AC$ invertible? If so, find its inverse.

**Problem 6** (27 points)
Answer TRUE or FALSE for each of the following statements. You do not have to explain your answer. (1.5 points each)

(a) In a network flows problem, you will always have more unknowns than equations.

(a) ____________

(b) If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a linearly independent set, then $\{\mathbf{x}, \mathbf{y}\}$ must be linearly independent.

(b) ____________

(c) Given the three row operations (exchange 2 rows, multiply a row by a non-zero scalar, and add a scalar multiple of a row to another row), any matrix can be put into echelon form without using the second of these operations.

(c) ____________

(d) Any two row equivalent matrices will have the same reduced echelon form.

(d) ____________

(e) If a matrix is in reduced row echelon form, then the entry in its bottom left-hand corner must be 0.

(e) ____________

(f) If a matrix $A$ has a row of all zeros, then the system of linear equations $A\mathbf{x} = \mathbf{b}$ must be inconsistent.

(f) ____________

(g) There is exactly one way to write the parametric vector form solution for a system of equations having multiple solutions.

(g) ____________

(h) The span of a set of vectors can contain exactly one vector.

(h) ____________

(i) If a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^4$ is one-to-one, then the span of the columns of its standard matrix must be $\mathbb{R}^4$.

(i) ____________

(j) If a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ is one-to-one, then the span of the columns of its standard matrix must be $\mathbb{R}^3$.

(j) ____________

(k) If a matrix product $AB$ exists and is invertible, then both $A$ and $B$ must be invertible with $(AB)^{-1} = B^{-1}A^{-1}$.

(k) ____________

(l) If a matrix product $AB$ exists and has all zero entries, then either $A$ or $B$ (or both) must have all zero entries.

(l) ____________

(m) If $A = B^T$, and the columns of $A$ are linearly independent, then it must be the case that the linear transformation defined by the standard matrix $B$ is onto.

(m) ____________

(n) A linear transformation defined by a matrix with more columns than rows cannot be onto.

(n) ____________

(o) A linear transformation defined by a matrix with more columns than rows must be onto.

(o) ____________

(p) A linear transformation defined by a matrix with more columns than rows cannot be one-to-one.

(p) ____________

(q) A linear transformation defined by a matrix with more columns than rows must be one-to-one.

(q) ____________

(r) If matrix $A$ has more rows than columns, then $A\mathbf{x} = \mathbf{b}$ **cannot** have infinitely many solutions.

(r) ____________

**Problem 7** (9 points)

Write a function called `in_span` that has 2 inputs and 1 output. The inputs are a matrix `A` and a column vector `b`, and the output is a logical scalar `y`. The columns of `A` form a set of vectors, i.e. `A` $= [\vec{a}_1\ \vec{a}_2\ \cdots\ \vec{a}_n]$. The function should check if `b` is in the span of the columns of `A`, i.e. `b` $\in \text{Span}\{\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n\}$ If it is, then `y` should be `true`. Otherwise, it should be `false`. No error checking, help lines, etc. is necessary. Just provide the code. *Hint: use* `rref`.