Practice Final Exam

Engineering Analysis 1

Name ______________________________ **Section** ____________

Clearly circle or box your solutions.

Check that your exam booklet has 11 pages

You may leave answers as fractions, where appropriate.

1. (16 points total)

   (a) The questions below are independent of each other and use the following matrices and vectors: $B = \begin{bmatrix} 2 & 4 \\ 0 & 8 \end{bmatrix}$ $\quad \mathbf{y} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\quad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ $\quad \mathbf{u} = \begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix}$

      i. (2 points) Calculate $\mathbf{y}^T B^2$.

      ii. (2 points) Calculate $\mathbf{u} \cdot \mathbf{v}$.

      iii. (2 points) Find the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

(b) (3 points) Put A into reduced row echelon form, and circle the pivot positions.

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 2 & 0 & 3 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

(c) (4 points) Let $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. What is the determinant of P?

(d) (3 points) Assume that you are given a MATLAB vector `b` with at least one element. Complete the loop below that sums all of the elements of `b` and put the result in `bSum`. Do **not** use the `sum()` function.

```
bSum = 0;

for  ____________________________

    ____________________________________________________;

end

disp(bSum);
```

2. (10 points, 1 point each) **Circle** True (T) or False (F). Statements are only true when they are true in every possible case, and otherwise they are false.

   (a) If $\|\mathbf{v}\| = 0$, then $\mathbf{v} = \mathbf{0}$.

   T F

   (b) There is no vector in R^n that is orthogonal to every other vector in R^n.

   T F

   (c) If $T(\mathbf{x}) = A\mathbf{x}$ is a one-to-one and onto transformation, then A must be a square matrix.

   T F

   (d) For any matrix A, it is true that $A^T A = I$.

   T F

   (e) If $\mathbf{y}$ is in $\operatorname{Col} A$, then there is a vector in the domain of the transformation $T(\mathbf{x}) = A\mathbf{x}$ that is mapped to $\mathbf{y}$.

   T F

   (f) If $T(\mathbf{x})$ is one-to-one, then the domain of the transformation is the same as the range.

   T F

   (g) If $\dim \operatorname{Nul} A = 1$, then $A\mathbf{x} = \mathbf{y}$ has an infinite number solutions for any $\mathbf{y}$.

   T F

   (h) If A is $m \times n$ and B is $p \times q$, then AB is only defined if $n = p$.

   T F

   (i) The line $x_2 = 3x_1 + 2$ is a subspace of R^2.

   T F

   (j) If $\mathbf{y}$ and $A\mathbf{x}$ are both in R^n, then $\|\mathbf{y} - A\mathbf{x}\|^2 = \|\mathbf{y}\|^2 + 2\mathbf{y} \cdot A\mathbf{x} - \|A\mathbf{x}\|^2$.

   T F

3. (12 points) Write a MATLAB function called `extrema` that is passed a single matrix A and returns four arguments

   1) `mx`, the maximum value of A
   2) `mn`, the minimum value of A
   3) `mxNum`, the number of times `mx` appears in A
   4) `mnNum`, the number of times `mn` appears in A

   Do not use built-in MATLAB functions `max`, `min`, `sort`, or `find`. Use instead `loops` that examine all the elements of A, as in the script below that you should complete.

```
function
mx=A(1,1);  mn=A(1,1);  mxNum=0;  mnNum=0;

for  ____________________

    for  ____________________









    end

end
```

4. (12 points total) Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$ be a basis for the subspace W.

(a) (2 points) Are the two vectors in the set $\mathcal{B}$ linearly independent?

(b) (1 point) What is the dimension of W?

(c) (3 points) Let $\mathbf{y} = \begin{bmatrix} 12 \\ 10 \\ 14 \end{bmatrix}$. What is $[\mathbf{y}]_{\mathcal{B}}$ (i.e. the coordinates of y with respect to the basis $\mathcal{B}$)?

(d) (1 point) Show that the two basis vectors are orthogonal.

(e) (3 points) What is the best approximation to $\mathbf{u} = \begin{bmatrix} 6 \\ 10 \\ 8 \end{bmatrix}$ in subspace W?

(f) (2 points) What is the distance from $\mathbf{u}$ to the nearest point in W?

5. (10 points total) Suppose you are given the matrix A and its reduced echelon form as follows:

$$A = \begin{bmatrix} 1 & -3 & 0 & -3 & 2 \\ 0 & 0 & 1 & -2 & 15 \\ 0 & 0 & 2 & -3 & 24 \\ 2 & -6 & 0 & -5 & -2 \\ -4 & 12 & -12 & 0 & 28 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 0 & -16 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (3 points) What is the dimension of the null space of A?

(b) (4 points) What is a basis for the null space of A?

(c) (3 points) Denote the columns of A as $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, $\mathbf{a}_4$, $\mathbf{a}_5$. Let $\mathcal{B}$ be the basis $\{\mathbf{a}_1, \mathbf{a}_3, \mathbf{a}_4\}$ for the column space of A. If the coordinates of $\mathbf{z}$ with respect to basis $\mathcal{B}$ are $[1 \;\; 0 \;\; 1]^T$, calculate $\mathbf{z}$.

6. (8 points) Let $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$.

   (a) (4 points) Find all eigenvalues of A.

   (b) (4 points) For each eigenvalue found in part (a), find a corresponding eigenvector.

7. (12 points) Let $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.

(a) (10 points) Find a least squares solution of the system $Ax = b$.

(b) (2 points) Is the solution from part (a) unique? Circle Yes (Y) or No (N):

Y N

8. (20 points) In this problem you will write two MATLAB functions to check various properties of an nxn (square) matrix A.

   (a) Write a function `isInvertible(A)` that returns 1 if the matrix is invertible and 0 if the matrix is not invertible.

   (b) Write a function `isOrthonormal(A)` that returns 1 if the columns of A are orthonormal and 0 if they are not.

   In your function you MAY use any of the MATLAB functions listed below, if you wish. (We will not explain during the exam what they do.)

`eye()`	`length()`	`min()`	`rank()`	`zeros()`
`inv()`	`max()`	`rref()`	`size()`	`det()`

   Do not perform any error checking.

```
function y = isInvertible(A)
```

```
function y = isOrthonormal(A)

[n,m]=size(A);  y=1;

% check that all columns have norm equal 1

for i=________

      if A(:,i)' * A(___________) ~=1

          y=0;

          break

      end

end

% check orthogonality

for k=1:n-1

       for j= ________

              if A(:,k)'  * A(________)  ~=0

                    y= __________;

                    break

              end

        end

end
```