

# Span and Linear Independence

1. Write a function to determine if a vector  $\mathbf{b}$  is in the span of a set of vectors  $S = \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ . For example: is  $[4, 3, 1]'$  in the span of  $[1, 2, 1]', [3, -2, 6]', [5, 0, 0]'$ ?
  - (a) Your function should have 2 inputs and 1 output. The inputs should be a Matrix  $\mathbf{A}$  where the columns of  $\mathbf{A}$  are the vectors in  $S$  and a vector  $\mathbf{b}$ . The output should be the vector of constants  $\mathbf{c}$ .
  - (b) Remember that  $\mathbf{b}$  is in span  $S$  if there are constants  $c_1, c_2, \dots, c_n$  such that  $\mathbf{b} = c_1 \cdot \mathbf{s}_1 + c_2 \cdot \mathbf{s}_2 + \dots + c_n \cdot \mathbf{s}_n$ . You can create a matrix  $\mathbf{A}$  where the columns are the vectors in  $S = \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ . Set up an augmented matrix to determine the constants.
  - (c) If your program determines that  $\mathbf{b}$  is in span  $S$ , output a vector of the constants  $\mathbf{c} = [c_1, c_2, \dots, c_n]$ .
  - (d) Check to see if your program works by making  $\mathbf{A} = [[1, 2, 6]', [4, -3, 1]', [9, 0, 4]']$  and  $\mathbf{b} = [35, 13, 29]'$ . The constants should be  $c_1=2$ ,  $c_2=-3$ , and  $c_3=5$ . (Be careful creating the  $\mathbf{A}$  matrix and  $\mathbf{b}$  vector: Each vector inside  $\mathbf{A}$  should be a **column** and  $\mathbf{b}$  vector is also a column vector.)
2. Write a script to determine if the columns of a matrix are linearly independent. Your program should
  - (a) Ask for the user to input a matrix.
  - (b) Find the reduced echelon form of the matrix.
  - (c) Print using fprintf whether the columns of the matrix are linearly independent or dependent using the result of part b.