

NAME: _____

INSTRUCTOR: _____

Question 1 (of 8)

(i) (1 pt) Find $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$.

(ii) (1 pt) Find a 2×2 matrix A that maps $(0,0) \rightarrow (0,0)$, $(1,0) \rightarrow (4,0)$, $(0,1) \rightarrow (4,2)$ and $(1,1) \rightarrow (8,2)$.

(iii) (1 pt) Find the $m \times n$ matrix A that yields the linear transformation

$$T(x_1, x_2, x_3, x_4) = \begin{bmatrix} 2x_1 - 3x_2 + 2x_4 \\ 5x_3 \\ 6x_1 - 2x_2 \end{bmatrix}.$$

POINTS:

- (iv) (2 pts) For $A = \begin{bmatrix} -2 & 0 & 3 & 1 \\ 1 & 3 & 4 & 2 \\ 4 & 0 & -2 & 3 \end{bmatrix}$, determine if the associated transformation is one-to-one, onto, both or neither.

- (v) (2 pts) Consider the vectors v_1 , v_2 and v_3 below. For what values of h are v_1 , v_2 and v_3 linearly independent? (If you can find no such values for h , state so.)

$$v_1 = \begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 5 \\ 6 \\ h \end{pmatrix}.$$

(vi) (2 pts) Given $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$, find A^{-1} .



(vii) (1 pt) Simplify the expression $A(A^{-1}B)(B^{-1}C)(C^{-1}D)$ assuming that all matrix row-column dimensions allow the indicated operations and the inverse matrices exist.



Question 2 (of 8)

Indicate whether each statement below is TRUE or FALSE.

- (i) (1 pt) The product AB of two matrices A and B is always defined when the number of rows of A is equal to the number of columns of B .
- (ii) (1 pt) The matrix product $A^T A$ is defined for all matrices.
- (iii) (1 pt) If the transformation $T(x) = Ax$ for the matrix A is both one-to-one and onto, then A must be invertible.
- (iv) (1 pt) If the matrix product AB yields the zero matrix, then either A or B must be the zero matrix.
- (v) (1 pt) If the matrix product AB yields the identity matrix, then B must equal the transpose of A .
- (vi) (1 pt) If the matrix product $AB = BA$, then both matrices must be square.
- (vii) (1 pt) If the matrix product AB is invertible, then both A and B must be invertible.
- (viii) (1 pt) The inverse of the transpose of a matrix A must equal the transpose of the inverse of A .

POINTS:

Question 3 (of 8)

Suppose you are given the matrix A and its reduced echelon form as follows:

$$A = \begin{bmatrix} 2 & 1 & 4 & 2 & 6 \\ 1 & 0 & 1 & 3 & 4 \\ 3 & 1 & 5 & 2 & 7 \\ 1 & 1 & 3 & 7 & 10 \end{bmatrix}, \quad \text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

- (i)** (1 pt) What is the dimension of the column space of A ?

- (ii)** (1 pt) What is the dimension of the null space of A ?

- (iii)** (2 pts) What is a basis for the column space of A ?

- (iv)** (2 pts) What is a basis for the null space of A ?

- (v)** (2 pts) Suppose $y = A(:,3) + A(:,4)$. What are the coordinates of y relative to the basis you gave in part (iii)?

- (vi)** (1 pt) Do the columns of A span \mathbf{R}^3 ?

POINTS:

Question 4 (of 8)

- (i) (1 pt) Suppose A is a $p \times q$ matrix and the columns of A are linearly independent. What is the dimension of the null space of A ?

- (ii) (1 pt) Suppose A is a $p \times q$ matrix, which is already in row reduced echelon form. Suppose that every row has at least one non-zero entry. What is the dimension of the column space of A ?

- (iii) (1 pt) If A is a $p \times q$ matrix where $p > q$, what is the maximum number of vectors in a basis of the column space of A ?

- (iv) (1 pt) If the columns of A form a basis for the column space of A , what are the possible number of solutions for $Ax=b$? (Give all the possibilities.)

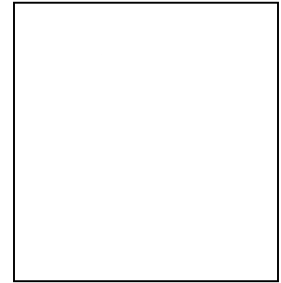
- (v) (1 pt) If A is a $p \times q$ matrix and $p < q$, then the rank of A cannot be greater than p . (true/false)

- (vi) (1 pt) If the coordinates of some vector x relative to a basis B are $c_1 = 1$, $c_2 = 0$ and $c_3 = 3$, and the coordinates of a different vector y relative to the same basis B are $d_1 = -1$, $d_2 = 2$ and $d_3 = 1$, what are the coordinates of $2x+y$ relative to B ?

POINTS:

Question 5 (of 8)

- (i)** (2 pts) Given the vector $x = \begin{bmatrix} 5 \\ 0 \\ 3 \\ 6 \end{bmatrix}$ and the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & -1 \\ 2 & -1 \\ -1 & 2 \end{bmatrix}$, find the orthogonal projection of x onto $\text{Col}(A)$.



- (ii)** (2 pts) Suppose a vector y and its orthogonal projection onto a subspace W are given by

$$y = \begin{bmatrix} 2 \\ -3 \\ 3 \\ 1 \end{bmatrix}, \quad \text{proj}_W y = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 0 \end{bmatrix}.$$

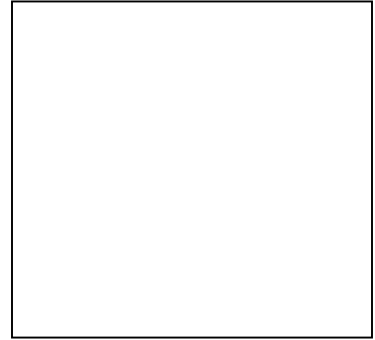
What is the distance from y to W ?



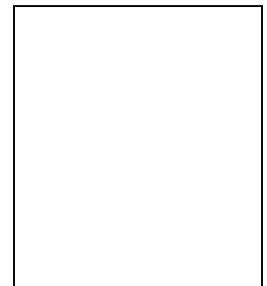
POINTS:

- (iii)** (2 pts) Apply Gram-Schmidt to transform the set $\{x_1, x_2\}$ into an orthogonal set of vectors, where

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}.$$



- (iv)** (2 pts) What is the least squares solution to $Ax = b$? $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ -3 & 3 \\ 1 & -1 \end{bmatrix}, b = \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}.$



Question 6 (of 8)

Parts (i) to (viii) (1 pt each) Let B be an $p \times q$ matrix with **linearly independent columns** and let x be a vector in the column space of B . The following segments of code can be used to calculate `coor`—the coordinates of x relative to B —if B has certain properties. You can assume that x , b , m (# rows of B) and n (# columns of B) are all previously defined.

For each of the segments of code, what is the necessary property of B for the code segment to correctly calculate `coor`? Each segment of code has exactly one correct answer and each answer is used at least once.

- a) All B
- b) B is a square matrix
- c) The columns of B are orthonormal.
- d) None of the Above. More information about B is needed.

EXAMPLE:

`coor = B \ x`

(You do not need to give an explanation – just the final answer). The answer is (a) —If x is in the column space of B , then $B * coor = x$ is consistent for any matrix B . The left division symbol will return the solution to $B * coor = x$ if the system is consistent, and this system is consistent for any B .

(i) `coor = zeros(n, 1);`
`for index = 1:n`
`coor(index) = dot (x, x)/dot(B(:, index), x);`
`end;`

(ii) `A = rref[B x];`
`coor = A(:, n+1);`

(iii) `coor = inv(B' * B) * B' * x;`

(iv) `coor = (eye(m,n) - B) \ x;`

(v) `coor = zeros(n, 1);`
`for index = 1:n`
`coor(index) = dot (B(:, index), x);`
`end;`

(vi) `coor = inv(B) * x;`

(vii) `coor = B' * x;`

(viii) `A = rref[B x];`
`coor = A(1:n, n+1);`

POINTS:

- (ix)** (2 pts) The following segment of code is used to identify the pivot columns of matrix A and store their position in a vector **pivot**. However, this segment contains one incorrect line. Find the incorrect line of code in this segment and replace it with the code necessary to correctly find the pivots in matrix A .

```
B = rref(A);

[m n] = size (A);

row = 1;
col = 1;

1. while (row <=m & col <= n)
2.     if (B(row, col))
3.         pivot(row) = col;
4.         col = col + 1;
5.         row = row + 1;
6.     else
7.         row = row+ 1;
8.     end;
9. end;
```

line:	new code:
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POINTS:

Question 7 (of 8)

- (i) (3 pts) Write a function called `projectit` whose input is a matrix A and whose output is a matrix Q . The matrix Q is obtained by normalizing each column of A , i.e., each column of Q is of norm 1. You can assume that A contains no zero columns.

- (ii) (3 pts) Write a function called `yonu` whose input is two vectors, y and u . The output is a vector w containing the projection of y onto the line containing u . You can assume that u is a non-zero vector.

POINTS:

Question 8 (of 8)

- (i) (2 pts) The purpose of the function `make_triang` given below is to set all elements of B below the diagonal to zero. What is the missing code?

```
function U= make_triang(B)
U=B;
[m n] = size(B);
for INSERT_CODE_1
    for INSERT_CODE_2
        U(irow,icol)= 0;
    end
end
```

- (ii) (2 points) The following Matlab script attempts to compute the least squares solution of $Ax = b$, for some predefined matrix A and vector b . Correct every line that is incorrect or write "OK" if the line is correct.

```
[m q]= size(A);
if rank(A) = q
    B= A*A;
    b= A'*b;
    xhat= B/b;
    disp(xhat);
else
    disp('dependent columns');
end
```

- (iii) (2 pts) The following Matlab script applies Gram-Schmidt to the n columns of a matrix X , which has been defined earlier. What is the missing code?

```
V(:,1)= X(:,1);
for iv = 2:n
    V(:,iv)= X(:,iv);
    for k = INSERT_CODE_1
        num = INSERT_CODE_2*V(:,k);
        den = V(:,k)'*V(:,k);
        proj = (num/den)*INSERT_CODE_3;
        V(:,iv) = V(:,iv)-proj;
    end
end
```

POINTS: