

Engineering Analysis 1 - Midterm 2 – Practice Problems

Problem 1

a)

Let

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 3 \end{bmatrix} \quad x = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Find each of the following quantities or write “not defined” if the operation is not defined.

i) A^2

ii) B^2

iii) AB

iv) $\mathbf{x}^T \mathbf{x}$

v) \mathbf{xx}^T

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b)

Let T be a linear transformation from \mathbf{R}^2 to \mathbf{R}^2 such that $T(\mathbf{x}) = A\mathbf{x}$. Write the standard matrix A for each of the following transformations

i) T reflects through the x_1 -axis

ii) T reflects through the line $x_2 = x_1$

iii) T expands in the x_1 direction by factor 3

iv) T projects onto the x_1 -axis

v) T first performs a horizontal shear that transforms \mathbf{e}_2 into $\mathbf{e}_2 + 4\mathbf{e}_1$ (leaving \mathbf{e}_1 unchanged) and then reflects points through the line $x_2 = x_1$

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Problem 2

a) Let A be an $m \times n$ matrix. Which of the following conditions imply that the linear system $A\mathbf{x} = \mathbf{b}$ has at least one solution for each choice of \mathbf{b} in \mathbf{R}^m ?
(Write all of the correct conditions in the box.)

- A. The columns of A span \mathbf{R}^m .
- B. A has more columns than rows.
- C. A has a pivot in every row.

b) Let A be an $m \times n$ matrix. If the number of rows is greater than the number of columns then which of the following are always true?
(Write all of the correct conditions in the box.)

- A. The columns of A cannot span \mathbf{R}^m .
- B. If $A\mathbf{x} = \mathbf{b}$ has a solution it will be unique.
- C. A has a pivot in every column.

c) Consider the vectors:

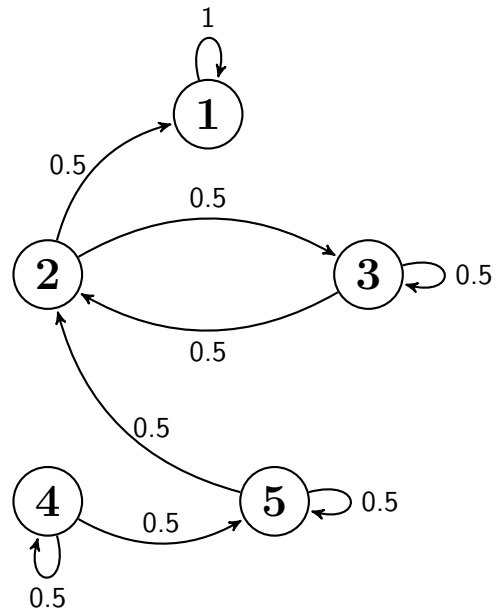
$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 6 \\ -4 \\ h \end{bmatrix}$$

Find all values of h for which \mathbf{w} is a linear combination of \mathbf{v}_1 and \mathbf{v}_2 .

d) For the vectors \mathbf{v}_1 and \mathbf{v}_2 in part (c), give a geometric description of $\text{Span}(\mathbf{v}_1, \mathbf{v}_2)$ (i.e. is it a single point, a line, a plane, or all of \mathbf{R}^3).

Problem 3

Consider the following Markov chain:



- (a) Write the transition matrix P

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- (b) If the state probabilities at $k = 3$ are $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, find the state probabilities at $k = 5$.

- (c) Find the steady-state state probabilities, π .

- (d) Does this Markov chain have an absorbing state?

- (e) Is the solution you found in part (c) unique? In other words, is this the only valid steady-state distribution?

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Problem 4

Write MATLAB statements that perform the following operations. Assume that a matrix A has been defined previously and that the instruction `[m n] = size(A)` has been executed.

- a) Given a pivot element of A , which may not be 1, use row operations to zero out the entries below the pivot. Let the row and column indices of the pivot be ii and jj , respectively.

- b) Given two row numbers, `nrow1` and `nrow2`, write MATLAB statements that will exchange rows `nrow1` and `nrow2` of A

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Problem 5

This question has 5 parts. *A part may have more than one correct choice. You need to write down all the correct choices to receive full credit.* Each part of this question refers to the four linear systems given by the following **augmented** matrices.

$$(A.) \begin{bmatrix} 1 & 2 & 5 & 3 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad (B.) \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (C.) \begin{bmatrix} 1 & 2 & 5 & 3 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (D.) \begin{bmatrix} 1 & 2 & 5 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

a) Which of the linear systems, if any, are in echelon form?

b) Which of the linear systems, if any, are in *reduced* echelon form?

c) Which of the linear systems are *not* consistent?

d) Which of the linear systems have a unique solution?

e) Suppose the linear system for each case is written in matrix form as $A\mathbf{x} = \mathbf{b}$. For which of the systems is \mathbf{b} in the span of the columns of A ?

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Problem 6

Answer true or false for each of the following. You do not have to explain your answer.

a) If $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent and \mathbf{z} is in $\text{Span}\{\mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent.

b) If T is a linear transformation, then $T(\mathbf{x})$ must be a vector that has the same number of elements as \mathbf{x} .

c) If T is a linear transformation, then $T(\mathbf{0}) = \mathbf{0}$.

d) A linear transformation T is one-to-one if and only if the columns of T 's standard matrix A are linearly independent.

e) The second column of AB equals A times the second column of B .

f) The transpose of AB , $(AB)^T$, always equals the transpose of A times the transpose of B , $A^T B^T$.

g) The inverse of AB , $(AB)^{-1}$, always equals the inverse of A times the inverse of B , $A^{-1} B^{-1}$.

h) If A is invertible, then the inverse of A^{-1} , $(A^{-1})^{-1}$, is A itself.

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i) If square matrices A and B satisfy $BA = I$, then $BA = AB$.

j) If the square matrix A is $n \times n$ and invertible, then the linear transformation $\mathbf{x} \rightarrow A\mathbf{x}$ maps \mathbf{R}^n onto \mathbf{R}^n .

k) If matrix A has m rows and 1 column, and matrix B has 1 row and m columns, then AB is an $m \times m$ matrix.

l) If matrix A has more columns than rows, then $A\mathbf{x} = \mathbf{b}$ cannot have exactly one solution.

m) If matrix A has more rows than columns, then $A\mathbf{x} = \mathbf{b}$ cannot have infinitely many solutions.
