NAME:____

INSTRUCTOR:____________________________

Question 1 (of 8)

(i) (1 pt) Find $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$.

(ii) (1 pt) Find a 2×2 matrix A that maps (0,0)→(0,0), (1,0)→(4,0), (0,1)→(4,2) and (1,1)→(8,2).

(iii) (1 pt) Find the $m \times n$ matrix A that yields the linear transformation

$$T(x_1, x_2, x_3, x_4) = \begin{bmatrix} 2x_1 - 3x_2 + 2x_4 \\ 5x_3 \\ 6x_1 - 2x_2 \end{bmatrix}.$$

POINTS:

(iv) (2 pts) For $A = \begin{bmatrix} -2 & 0 & 3 & 1 \\ 1 & 3 & 4 & 2 \\ 4 & 0 & -2 & 3 \end{bmatrix}$, determine if the associated transformation is one-to-one, onto, both or neither.

(v) (2 pts) Consider the vectors v_1, v_2 and v_3 below. For what values of h are v_1, v_2 and v_3 linearly independent? (If you can find no such values for h, state so.)

$$v_1 = \begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix},\quad v_2 = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix},\quad v_3 = \begin{pmatrix} 5 \\ 6 \\ h \end{pmatrix}.$$

(vi) (2 pts) Given $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$, find A^{-1}.

(vii) (1 pt) Simplify the expression $A(A^{-1}B)(B^{-1}C)(C^{-1}I)$ assuming that all matrix row-column dimensions allow the indicated operations and the inverse matrices exist.

POINTS:

Question 2 (of 8)

Indicate whether each statement below is TRUE or FALSE.

(i) (1 pt) The product AB of two matrices A and B is always defined when the number of rows of A is equal to the number of columns of B.

(ii) (1 pt) The matrix product $A^T A$ is defined for all matrices.

(iii) (1 pt) If the transformation $T(x) = Ax$ for the matrix A is both one-to-one and onto, then A must be invertible.

(iv) (1 pt) If the matrix product AB yields the zero matrix, then either A or B must be the zero matrix.

(v) (1 pt) If the matrix product AB yields the identity matrix, then B must equal the transpose of A.

(vi) (1 pt) If the matrix product $AB = BA$, then both matrices must be square.

(vii) (1 pt) If the matrix product AB is invertible, then both A and B must be invertible.

(viii) (1 pt) The inverse of the transpose of a matrix A must equal the transpose of the inverse of A.

POINTS:

Question 3 (of 8)

Suppose you are given the matrix A and its reduced echelon form as follows:

$$A = \begin{bmatrix} 2 & 1 & 4 & 2 & 6 \\ 1 & 0 & 1 & 3 & 4 \\ 3 & 1 & 5 & 2 & 7 \\ 1 & 1 & 3 & 7 & 10 \end{bmatrix}, \quad \texttt{rref}(\mathrm{A}) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(i) (1 pt) What is the dimension of the column space of A?

(ii) (1 pt) What is the dimension of the null space of A?

(iii) (2 pts) What is a basis for the column space of A?

(iv) (2 pts) What is a basis for the null space of A?

(v) (2 pts) Suppose $y = A(:,3) + A(:,4)$. What are the coordinates of y relative to the basis you gave in part (iii)?

(vi) (1 pt) Do the columns of A span $\mathbf{R}^3$?

POINTS:

Question 4 (of 8)

(i) (1 pt) Suppose A is a $p \times q$ matrix and the columns of A are linearly independent. What is the dimension of the null space of A?

(ii) (1 pt) Suppose A is a $p \times q$ matrix, which is already in row reduced echelon form. Suppose that every row has at least one non-zero entry. What is the dimension of the column space of A?

(iii) (1 pt) If A is a $p \times q$ matrix where $p>q$, what is the maximum number of vectors in a basis of the column space of A?

(iv) (1 pt) If the columns of A form a basis for the column space of A, what are the possible number of solutions for $Ax=b$? (Give all the possibilities.)

(v) (1 pt) If A is a $p \times q$ matrix and $p<q$, then the rank of A cannot be greater than p. (true/false)

(vi) (1 pt) If the coordinates of some vector x relative to a basis $\boldsymbol{B}$ are $c_1 = 1$, $c_2 = 0$ and $c_3 = 3$, and the coordinates of a different vector y relative to the same basis $\boldsymbol{B}$ are $d_1 = -1$, $d_2 = 2$ and $d_3 = 1$, what are the coordinates of $2x+y$ relative to $\boldsymbol{B}$?

POINTS:

Question 5 (of 8)

(i) (2 pts) Given the vector $x = \begin{bmatrix} 5 \\ 0 \\ 3 \\ 6 \end{bmatrix}$ and the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & -1 \\ 2 & -1 \\ -1 & 2 \end{bmatrix}$, find the orthogonal projection of x onto Col (A).

(ii) (2 pts) Suppose a vector y and its orthogonal projection onto a subspace W are given by

$$y = \begin{bmatrix} 2 \\ -3 \\ 3 \\ 1 \end{bmatrix}, \quad proj_W\, y = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 0 \end{bmatrix}.$$

What is the distance from y to W?

POINTS:

(iii) (2 pts) Apply Gram-Schmidt to transform the set $\{x_1, x_2\}$ into an orthogonal set of vectors, where

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}.$$

(iv) (2 pts) What is the least squares solution to $Ax = b$? $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ -3 & 3 \\ 1 & -1 \end{bmatrix}, b = \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}.$

POINTS:

Question 6 (of 8)

Parts (i) to (viii) (1 pt each) Let B be an $p \times q$ matrix with **linearly independent columns** and **let x be a vector in the column space of B**. The following segments of code can be used to calculate `coor`—the coordinates of x relative to B— if B has certain properties. You can assume that x, b, m (# rows of B) and n (# columns of B) are all previously defined.

For each of the segments of code, what is the necessary property of B for the code segment to correctly calculate `coor`? Each segment of code has exactly one correct answer and each answer is used at least once.

a) All B
b) B is a square matrix
c) The columns of B are orthonormal.
d) None of the Above. More information about B is needed.

```
EXAMPLE:
coor = B\x
```

(*You do not need to give an explanation – just the final answer*). The answer is (a) —If x is in the column space of B, then B*$coor$ = x is consistent for any matrix B. The left division symbol will return the solution to B*$coor$ = x if the system is consistent, and this system is consistent for any B.

(i)
```
coor = zeros(n, 1);
for index = 1:n
    coor(index) = dot (x, x)/dot(B(:, index), x);
end;
```

(ii)
```
A = rref[B x];
coor = A(:, n+1);
```

(iii)
```
coor = inv(B' * B) * B' * x;
```

(iv)
```
coor = (eye(m,n) - B)\x;
```

(v)
```
coor = zeros(n, 1);
for index = 1:n
    coor(index) = dot (B(:, index), x);
end;
```

(vi)
```
coor = inv(B)* x;
```

(vii)
```
coor = B' * x;
```

(viii)
```
A = rref[B x];
coor = A(1:n, n+1);
```

POINTS:

(ix) (2 pts) The following segment of code is used to identify the pivot columns of matrix A and store their position in a vector **pivot**. However, this segment contains one incorrect line. Find the incorrect line of code in this segment and replace it with the code necessary to correctly find the pivots in matrix A.

```
B = rref(A);

[m n] = size (A);

row = 1;
col = 1;

1.   while (row <=m & col <= n)
2.        if (B(row, col))
3.             pivot(row) = col;
4.             col = col + 1;
5.             row = row + 1;
6.        else
7.             row = row+ 1;
8.        end;
9.   end;
```

line:	**new code:**

Question 7 (of 8)

(i) (3 pts) Write a function called `projectit` whose input is a matrix A and whose output is a matrix Q. The matrix Q is obtained by normalizing each column of A, i.e., each column of Q is of norm 1. You can assume that A contains no zero columns.

(ii) (3 pts) Write a function called `yonu` whose input is two vectors, y and u. The output is a vector w containing the projection of y onto the line containing u. You can assume that u is a non-zero vector.

POINTS:

Question 8 (of 8)

(i) (2 pts) The purpose of the function `make_triang` given below is to set all elements of B below the diagonal to zero. What is the missing code?

```
function  U= make_triang(B)
U=B;
[m n] = size(B);
for INSERT_CODE_1
     for INSERT_CODE_2
          U(irow,icol)= 0;
     end
end
```

(ii) (2 points) The following Matlab script attempts to compute the least squares solution of $Ax = b$, for some predefined matrix A and vector b. Correct every line that is incorrect or write "OK" if the line is correct.

```
[m q]= size(A);
if rank(A) = q

  B= A*A;

  b= A'*b;

  xhat= B/b;

  disp(xhat);
else
  disp('dependent columns');
end
```

(iii) (2 pts) The following Matlab script applies Gram-Schmidt to the n columns of a matrix X, which has been defined earlier. What is the missing code?

```
V(:,1)= X(:,1);
for iv = 2:n
  V(:,iv)= X(:,iv);
  for k = INSERT_CODE_1
    num = INSERT_CODE_2*V(:,k);
    den = V(:,k)'*V(:,k);
    proj = (num/den)*INSERT_CODE_3;
    V(:,iv) = V(:,iv)-proj;
  end
end
```

POINTS: