Engineering Analysis I, Fall 2016

Midterm 1

Name _____

Section number _____

Section number	Discussion time	Instructor
30	9:00 a.m.	Ilya Mikhelson
31	10:00 a.m.	Ilya Mikhelson
32	11:00 a.m.	Michael Honig
33	12:00 noon	Michael Honig
34	12:00 noon	Randy Berry

This exam is closed-book and closed-notes. Calculators, computers, phones, or other computing/communication devices are not allowed.

Students should skip this page—it is only for graders.

Question	Points	Score
1	30	
2	20	
3	20	
4	18	
5	12	
Total:	100	

Answer each question in the space provided. There are 5 questions for a total of 100 points.

1. (a) [8 points] Which of the following MATLAB statements will *not* generate an error message?

☐
```
x = [1;2]'*[3;4]
```

☐
```
x = (5<6) == (6>7)
```

☐
```
x = [2 3]*[4;5;6]
```

☐
```
if 5 > 6
    x=2
end
```

☐
```
x = [4 5]<3
```

☐
```
x = sin(1:10)*cos(1:10)
```

☐
```
x = [3 2].^3
```

☐
```
while k=1:3
```

(b) [8 points] Which of the following eight blocks of code will square each element of an existing (and possibly non-square) matrix `A`?

☐
```
A = A.*A;
```

☐
```
A = A*A;
```

☐
```
A = A./(ones(size(A))./A);
```

☐
```
ii = 1;
x = size(A,1);
while ii<=x
     A(ii,:) = A(ii,:).^2;
     ii = ii + 1;
end
```

☐
```
A = A^2
```

☐
```
A = A.^2
```

☐
```
for ii = 1:size(A)
    A(ii,ii) = A(ii,ii).^2;
end
```

☐
```
for ii=1:size(A,1)
     for jj=1:size(A,2)
          x = A(ii,jj);
          x = x^2;
     end
end
```

(c) [4 points] Given the system of linear equations $2\mathbf{x}+\mathbf{A}\mathbf{x}=\mathbf{1}+\mathbf{x}$, where $\mathbf{1}$ is a 10×1 vector containing 1 for each entry, $\mathbf{x}$ is the vector of unknown variables, and $\mathbf{A}$ is a given matrix (not a scalar), what must be the dimensions of $\mathbf{x}$ and $\mathbf{A}$? Give your answers in the form $m \times n$ with specific values for m and n.

(d) [6 points] Given the MATLAB assignments

```
a = ones(1,10);
b = zeros(size(a));
b(a < 0.5) = a(a < 0.5) - 1;
b(a > 0.5) = a(a > 0.5);
```

indicate which of the following statements gives the same value for b. (Assume that a is given by the preceding assignment.)

- ☐ `b = ones(1,10);`
- ☐ `b = a - 1;`
- ☐ `b = zeros(1,10);`
- ☐ `b = (a.^2)/a;`
- ☐ `b = a;`
- ☐ `b = [zeros(1,5), ones(1,5)];`

(e) [4 points] Given the assignments

```
a = [4 3 8 2; 1 2 4 5; 4 7 2 3];
b = a([1 1], [2 2])
```

what does MATLAB return for b?

2. Suppose each section of code below is run in MATLAB. If MATLAB generates an error message for the given code section, write "error" on the associated line. Otherwise, write the value that the variable `x` will have after the code section is run.

(a) [4 points]

```
clear;
for ii = 1:100
    if ii > 5 && ii < 60
        x = x - ii;
    elseif ii >= 60 && ii <= 98
        x = x + ii;
    elseif ii > 98
        x = ii;
    else
        x = ii^2;
    end
end
```

(a) _____

(b) [4 points]

```
clear; x = 0;
for x <= 4
    x = x + x^2;
end
```

(b) _____

(c) [4 points]

```
clear; x = 4;
if x < 1 && y < 5
    x = 100;
end
```

(c) _____

(d) [4 points]

```
clear;
for k = 0.14:3:7
    if k == pi || k == 0.14
        x = 1;
    else
        x = x + 1;
    end
end
```

(d) _____

(e) [4 points]

```
clear; x = 0;
while x <= 20
    x = x + 4;
    y = y + 5;
end
```

(e) _____

3. [20 points] For this question, we will use a randomized experiment to estimate the value of π. In this experiment, we will generate random points in the square that extends from $(0, 0)$ to $(2, 2)$. For these points, we can test whether they lie in the circle having radius 1 centered at (1,1) by calculating their distance d to the center, which is

$$d(x, y) = \sqrt{(x-1)^2 + (y-1)^2}.$$

When we perform this random experiment many times, we expect that the fraction of (x, y) coordinates that fall inside the circle will approximate the ratio of the area of the circle to the area of the square. Since the area of the circle is π and the area of the square is 4, we can approximate π by multiplying the ratio of points that fall inside the unit circle by 4. Fill in the missing code below so that the script performs this calculation.

```
% Run 10,000 random trials
num_trials = 10000;

% Create vectors x and y each having num_trials entries randomly
% generated in the interval (0,2).  The rand() function generates
% random numbers in the interval (0,1).
x = _____
y = _____

% Compute a vector d of distances from the vectors x and y.  The
% sqrt() function will compute the square roots of the elements of
% its input.
d = _____

% Create a variable in_circle which counts how many entries in d
% lie inside the unit circle.  Use a loop or use logical indexing.









% Calculate the approximate value of pi based on our trials
approx_pi = _____
```

4. [18 points] Write a Matlab function called `solveLin` that uses left division to calculate a solution vector x to the system of equations

$$x = Mx + b$$

where M is a matrix and b is a vector. Note that you need to bring this into standard form before you can apply left division. The function should have two inputs `M` and `b`, and a single output `x`. Your function should start with an appropriate function declaration using the `function` keyword. Also, use `error()` to generate an appropriate error message if `M` is not square. You do not have to include any comments in your function or perform any other error checking.

5. Each problem below may have more than one correct choice. You must list all correct choices to receive full credit. Consider the following four matrices:

$$A = \begin{bmatrix} 1 & 4 & 8 & 2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 6 & 8 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 0 & 5 & 3 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & 0 & 6 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(a) [3 points] Which of the matrices, if any, are in row echelon form?

(b) [3 points] Which of the matrices, if any, are in reduced row echelon form?

(c) [3 points] Which of the matrices, if any, are not in reduced row echelon form but can be brought to reduced row echelon form with a *single* elementary row operation?

(d) [3 points] Which of the matrices, if any, are such that their reduced row echelon forms have exactly 3 pivots?