

Engineering Analysis I, Fall 2017

Midterm 2

Name _____

Section number _____

Section number	Discussion time	Instructor
30	9:00 a.m.	Ilya Mikhelson
31	10:00 a.m.	Ilya Mikhelson
32	10:00 a.m.	Iman Hassani Nia
33	11:00 a.m.	Iman Hassani Nia
34	12:00 noon	Randy Berry

This exam is closed-book and closed-notes. Calculators, computers, phones, or other computing/communication devices are not allowed.

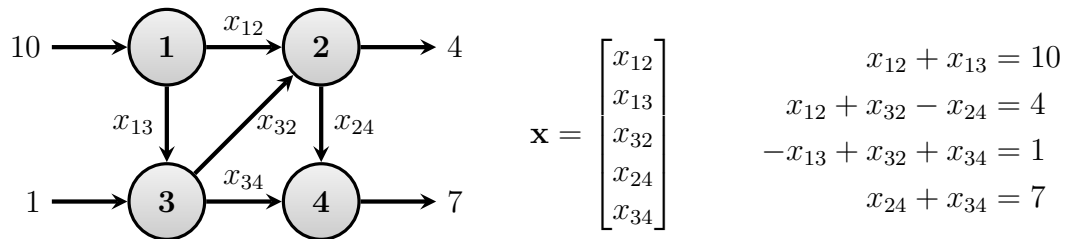
Students should skip this page—it is only for graders.

Question	Points	Score
1	12	
2	10	
3	13	
4	14	
5	10	
6	32	
7	9	
Total:	100	

Answer each question in the space provided. There are 7 questions for a total of 100 points.

Problem 1

Consider the following network flow diagram and flow balance equations:



- (a) [2 points] Write the flow balance equations in the form $A\mathbf{x} = \mathbf{b}$, with \mathbf{x} as defined above.
- (b) [3 points] Find the reduced row echelon form of the augmented matrix $[A \ \mathbf{b}]$.
- (c) [3 points] Write the solution set for your system of equations in parametric vector form.

- (d) [2 points] Find a parametric vector form solution in which the flows in the particular solution are all non-negative. There may be more than one right answer.

- (e) [2 points] Draw a solution to the network flow problem with $x_{12} = 0$ and $x_{24} = 0$. Make sure no flows are negative!

Problem 2

Let

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 & 1 & 4 \\ 2 & 1 & -2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 & 3 & -4 \\ 0 & 1 & 4 & 2 \\ 2 & -2 & 6 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find each of the following quantities or write “not defined” if the operation is not defined.

(a) [2 points] A^{100}

(b) [2 points] C^{-1}

(c) [2 points] $(C^T)^{-1}$

(d) [2 points] $(BB^T)C$

(e) [2 points] AB

Problem 3

- (a) Suppose T is a linear transformation from \mathbb{R}^2 to \mathbb{R}^2 . Write the standard matrix for each of the following transformations.

- i. [2 points] T scales vectors vertically by a factor of 3.

- ii. [2 points] T first reflects across the line at -45 degrees through the origin and then rotates clockwise by 90 degrees about the origin.

- iii. [2 points] T first projects onto the line at 45 degrees through the origin and then rotates clockwise by 90 degrees about the origin.

- (b) [2 points] Select all transformations from part (a) that are **onto**. Put a check mark (\checkmark) in the box next to **EACH** correct answer.

☐ i.☐ ii.☐ iii.

- (c) [2 points] Select all transformations from part (a) that are **one-to-one**. Put a check mark (✓) in the box next to **EACH** correct answer.

☐ i.

☐ ii.

☐ iii.

- (d) [3 points] Given a linear transformation $T : \mathbb{R}^7 \rightarrow \mathbb{R}^2$, suppose that $T(2\mathbf{x}) = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$ and $T(\mathbf{x} + \mathbf{y}) = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ for two vectors \mathbf{x} and \mathbf{y} in \mathbb{R}^7 . What is $T(\mathbf{y})$?

Problem 4

(a) Consider the following matrix:

$$\begin{bmatrix} a & 0 & c & 0 \\ 0 & 1 & 1 & 0 \\ 0 & b & 0 & d \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here a , b , c , and d represent constant parameters. If this matrix is in **reduced** row echelon form, and there are *three pivots*, then what must be the values of these four parameters? For each parameter, either write a specific number or write “any number” if the parameter can have any value.

i. [2 points] What must be the value of a ?

i. _____

ii. [2 points] What must be the value of b ?

ii. _____

iii. [2 points] What must be the value of c ?

iii. _____

iv. [2 points] What must be the value of d ?

iv. _____

(b) [2 points] What is the reduced row echelon form of the following matrix?

$$\begin{bmatrix} 5 & 0 & 1 & 3 & 4 & 8 \\ 0 & 0 & 3 & 4 & 4 & 7 \\ 0 & 0 & 0 & 5 & 2 & 3 \\ 0 & 0 & 0 & 0 & 4 & -8 \\ 0 & 0 & 0 & 0 & 0 & -4 \end{bmatrix}$$

- (c) Consider a system of linear equations of the form $Ax = b$, where the matrix A is 4-by-5 and the column vector x consists of the five unknown variables x_1, x_2, \dots, x_5 . Suppose the reduced row echelon form of the augmented matrix is given by

$$[A \ b] \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- i. [2 points] Which of the unknown variables are free?

- ii. [1 point] How many solutions are there?

- iii. [1 point] Does the solution set pass through the origin in \mathbb{R}^5 (yes or no)?

Problem 5

Answer each question in the space provided. Consider the following matrices:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -5 & 3 \\ 0 & 4 & -2 \\ 0 & -2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- (a) [2 points] Is A invertible? If so, find its inverse.

- (b) [2 points] Is B invertible? If so, find its inverse.

- (c) [2 points] Is C invertible? If so, find its inverse.

- (d) [2 points] Is the product AB invertible? If so, find its inverse.

- (e) [2 points] Is the product AC invertible? If so, find its inverse.

Problem 6 32 points

Answer TRUE or FALSE for each of the following statements. You do not have to explain your answer. (2 points each)

- (a) If $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a linearly dependent set, then $\{\mathbf{x}, \mathbf{y}\}$ must be linearly independent.
(a) _____
- (b) If a matrix is in reduced row echelon form, then the entry in its bottom left-hand corner must be 0.
(b) _____
- (c) The solution set for a system of linear equations of the form $A\mathbf{x} = \mathbf{b}$ can contain exactly two distinct vectors.
(c) _____
- (d) If a matrix A has a row of all zeros, then the system of linear equations $A\mathbf{x} = \mathbf{b}$ must be inconsistent.
(d) _____
- (e) No two matrices can have the same reduced row echelon form.
(e) _____
- (f) If a linear transformation from \mathbb{R}^5 to \mathbb{R}^5 is one-to-one, then the span of the columns of its standard matrix must be the same as its codomain.
(f) _____
- (g) If a matrix product AB exists and is invertible, then both A and B must be invertible with $(AB)^{-1} = B^{-1}A^{-1}$.
(g) _____
- (h) If a matrix product AB exists and has all zero entries, then either A or B (or both) must have all zero entries.
(h) _____
- (i) If $A = B^T$, and the columns of B are linearly independent, then it must be the case that the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbb{R}^m .
(i) _____
- (j) If the span of the set $\{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\}$ in \mathbb{R}^m does not include all vectors in \mathbb{R}^m , then the linear transformation defined by the standard matrix $A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \cdots \ \mathbf{a}_n]$ is not onto.
(j) _____

- (k) A linear transformation defined by a matrix with more columns than rows cannot be onto.

(k) _____

- (l) A linear transformation defined by a matrix with more columns than rows must be onto.

(l) _____

- (m) A linear transformation defined by a matrix with more columns than rows cannot be one-to-one.

(m) _____

- (n) A linear transformation defined by a matrix with more columns than rows must be one-to-one.

(n) _____

- (o) If the system of linear equations $A\mathbf{x} = \mathbf{b}$ has a unique solution for every \mathbf{b} , then the linear transformation defined by A must be both onto and one-to-one.

(o) _____

- (p) If matrix A has more rows than columns, then $A\mathbf{x} = \mathbf{b}$ **cannot** have infinitely many solutions.

(p) _____

Problem 7 9 points

Write a function called `check_cons` that has 2 inputs and 1 output. The inputs are a matrix `A` and a vector `b`, and the output is a logical scalar `y`. The function should check if the system defined by $A\mathbf{x} = \mathbf{b}$ is consistent. If it is, then `y` should be `true`. Otherwise, it should be `false`. No error checking, help lines, etc. is necessary. Just provide the code. *Hint: use `rref`.*