Recitation 1

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- Units

Table.1: Systems of Units

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	newton* N $\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)$
U.S. Customary FPS	foot ft	second s	slug* $\left(\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)$	pound lb

$$1\ \text{lb} = (1\ \text{slug})(1\ \text{ft/s}^2).$$

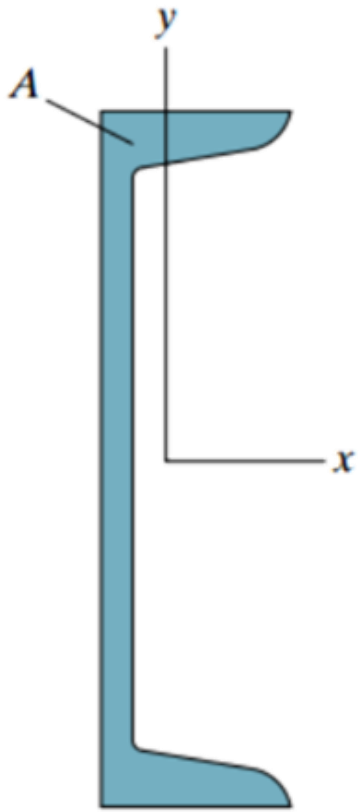
- Units

Table.2: Units Conversion

Time	1 minute	=	60 seconds
	1 hour	=	60 minutes
	1 day	=	24 hours
Length	1 foot	=	12 inches
	1 mile	=	5280 feet
	1 inch	=	25.4 millimeters
	1 foot	=	0.3048 meters
Angle	2π radians	=	360 degrees
Mass	1 slug	=	14.59 kilograms
Force	1 pound	=	4.448 newtons

- Units
  - Problem 1

The cross-sectional area of the C12×30 American Standard Channel steel beam is $A = 8.81\ in^2$. What is its cross-sectional area in mm^2?



$A = 8.81\ in^2$

According to last slide of units conversion

US		SI
$1\ in$	$=$	$25.4\ mm$
$1^2\ in^2$	$=$	$(25.4)^2\ mm^2$
$8.81\ in^2$	$=$	$8.81 \times (25.4)^2\ mm^2$
	$\approx$	$5683.86\ mm^2$

- Units
  - Problem 2

Suppose that in Einstein's equation

$$E = mc^2,$$

the mass m is in kilograms and the velocity of light c is in meters per second.

(a) What are the SI units of E?

(b) If the value of E in SI units is 20, what is its value in U.S. Customary base units?

Solution:

(a) $E = m c^2$

$m \rightarrow (kg)$

$c^2 \rightarrow (m/s)^2$

$$E = (kg) \cdot (m/s)^2$$

$$= kg \cdot m^2/s^2 \quad \text{or} \quad \frac{kg \cdot m^2}{s^2} \quad \text{or} \quad kg \cdot m^2 \cdot s^{-2}$$

b) $E = kg \cdot m^2/s^2$

According to table.2 : units conversion

US		SI
1 slug	=	14.59 kg
$\frac{1}{14.59}$ slug = 0.0685 slug	≈	1 kg
1S	=	1S

US		SI
1 ft	=	0.3048 m
$\frac{1}{0.3048}$ ft = 3.28 ft	=	1 m
$(3.28)^2$ ft^2 = 10.76 ft^2	=	1 m^2

So 1 kg · m^2/s^2 = $\frac{0.0685 \text{ slug} \cdot 10.76 \text{ ft}^2}{1 \text{ s}^2}$ = 0.737 slug ft^2/s^2

E = 20 kg · m^2/s^2 = 20 × 0.737 slug ft^2/s^2 = 14.74 slug ft^2/s^2

- Newtonian Gravitation
  - Problem 3

The acceleration due to gravity at sea level is $g = 9.81\ \text{m/s}^2$. The radius of the earth is 6370 km. The universal gravitational constant $G = 6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$. Use this information to determine the mass of the earth.

Solution: According to $F = \frac{G m_1 m_2}{r^2}$

at the sea level: $F = mg = m\frac{G m_E}{r_E^2}$

NOTE: be careful about the unit consistency.

$$m_E = \frac{g r_E^2}{G} = \frac{9.81\ \text{m/s}^2 \cdot (6370 \times 10^3)^2\ \text{m}^2}{6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2} = 5.97 \times 10^{24}\ \text{kg}$$