Problem 1

Homework 6

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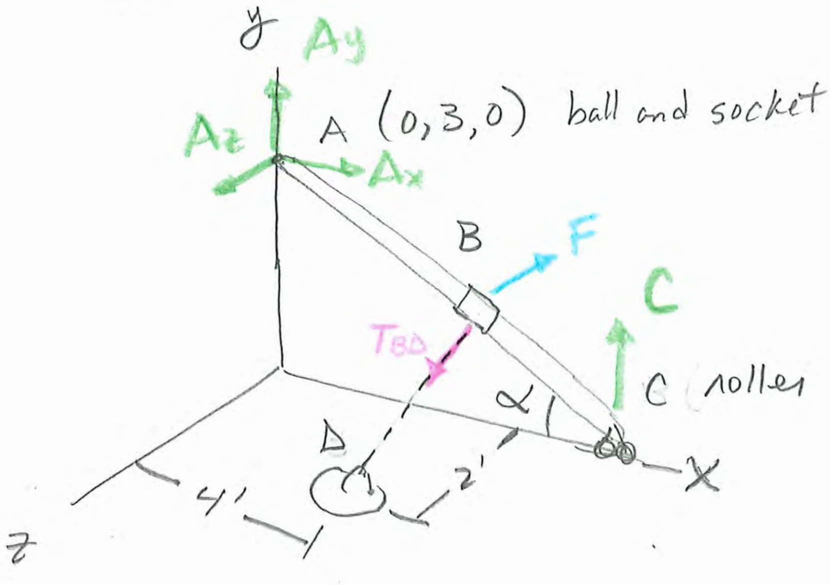
FBD



Given:

8ft bar

$\vec{F} = -50\,\vec{k}$ [lb]

B fixed at midpoint

Find:

$T_{CD} = ?$

$\vec{R}_A = ?$

$\vec{R}_C = ?$

5 unknowns 6 Eq. Eqs. Statically Determinate

A (0, 3, 0)

$\alpha = \sin^{-1}\left(\frac{3}{8}\right) = 22.02°$

$C_x = 8\cos(22.02) = 7.416$ ft

C (7.416, 0, 0)

$B_x = \frac{8\cos(22.02)}{2} = 3.708$ ft

B (3.708, 1.5, 0)

D (4, 0, 2)

$$\vec{e}_{BD} = \frac{\vec{r}_{BD}}{|\vec{r}_{BD}|} = \frac{(4-3.708)\vec{i} + -1.5\vec{j} + 2\vec{k}}{\sqrt{(4-3.708)^2+(-1.5)^2+(2)^2}}$$

$$\vec{e}_{BD} = \frac{0.292}{2.517}\vec{i} - \frac{1.5}{2.517}\vec{j} + \frac{2}{2.517}\vec{k}$$

$$\vec{e}_{BD} = 0.116\vec{i} - 0.596\vec{j} + 0.795\vec{k}$$

$$\vec{T}_{BD} = 0.116\,T_{BD}\,\vec{i} - 0.596\,T_{BD}\,\vec{j} + 0.795\,T_{BD}\,\vec{k}$$

$$\vec{F} = -50\vec{k}$$

$$\vec{C} = 0\vec{i} + C_y\vec{j} + 0\vec{k}$$

$$\Sigma M_A = 0$$

$$\vec{r}_{AB} \times \vec{T}_{BD} + \vec{r}_{AB} \times \vec{F} + \vec{r}_{AC} \times \vec{C} = 0$$

$$\begin{vmatrix} i & j & k \\ 3.708 & -1.5 & 0 \\ 0.116T_{BD} & -0.596T_{BD} & 0.795T_{BD} \end{vmatrix} + \begin{vmatrix} i & j & k \\ 3.708 & -1.5 & 0 \\ 0 & 0 & -50 \end{vmatrix}$$

$$+ \begin{vmatrix} i & j & k \\ 7.416 & -3 & 0 \\ 0 & C_y & 0 \end{vmatrix}$$

(i)

$$(-1.5)(0.795\,T_{BD}) + (-1.5)(-50) + 0 = 0 \quad (1)$$

$$-1.1925\,T_{BD} + 75 = 0$$

$$\boxed{T_{BD} = 62.9\ \text{lb}}$$

(j)

$$(-1)(\cancel{3.708})(0.795\,T_{BD}) + -\cancel{3.708}(-50) = 0$$

$$\boxed{T_{BD} = 62.9\ \text{lb}} \quad \text{Okay Check}$$

(k)

$$\left[(3.708)(-0.596\,T_{BD}) - (-1.5)(0.116\,T_{BD})\right] + (0) + 7.416\,C_y = 0$$

$$-2.03597\,T_{BD} + 7.416\,C_y = 0$$

$$\boxed{C_y = \frac{(2.03597)(62.9\ \text{lb})}{7.416} = 17.3\ \text{lb}}$$

$$\boxed{\vec{R}_c = 0\,\vec{i} + 17.3\,\vec{j} + 0\,\vec{k} \quad [\text{lb}]}$$

$$\vec{T}_{BD} = 7.30\,\vec{i} - 37.5\,\vec{j} + 50.0\,\vec{k} \quad [lb]$$

$$\Sigma F_x = 0$$

$$A_x + 7.30 = 0$$

$$\boxed{A_x = -7.30 \text{ lb}}$$

$$\Sigma F_y = 0$$

$$A_y - 37.5 + 17.3 = 0$$

$$\boxed{A_y = 20.2 \text{ lb}}$$

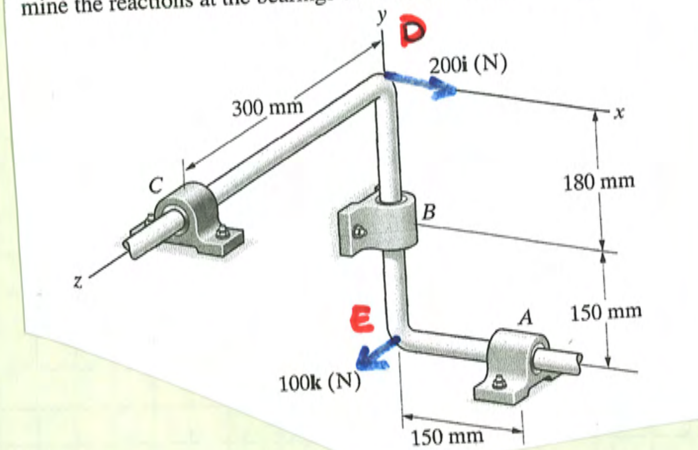
$$\Sigma F_z = 0$$

$$A_z + 50 - 50 = 0$$

$$\boxed{A_z = 0}$$

$$\boxed{\vec{R}_A = -7.30\,\vec{i} + 20.2\,\vec{j} + 0\,\vec{k} \quad [lb]}$$

5.117 The bearings at A, B, and C do not exert couples on the bar and do not exert forces in the direction of the axis of the bar. Determine the reactions at the bearings due to the two forces on the bar.

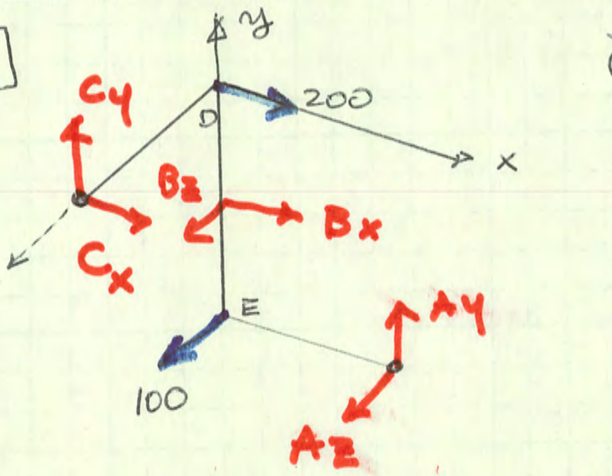


$\vec{B} = ?$

$\vec{C} = ?$

$\vec{A} = ?$

FBD



$\boxed{\text{EQ EQUATIONS}}$

$\sum \vec{F} = 0 \; ; \; \sum \vec{M}_A = 0$

$A(0.15 \, ; -0.33 \, ; 0)$

$D(0, 0, 0) \qquad C(0, 0, 0.3)$

$E(0, -0.33, 0)$

$B(0 \, -0.18, 0)$

$$\vec{M}_A = \vec{r}_{AE} \times 100\vec{k} + \vec{r}_{AD} \times (200\vec{i}) + \vec{r}_{AB} \times \vec{B} + \vec{r}_{AC} \times \vec{C}$$

$\vec{r}_{AE} = (-0.15 \, ; 0, 0) \quad ; \quad \vec{r}_{AB} = (-0.15 \, ; 0.15, 0)$

$\vec{r}_{AD} = (-0.15 \, ; 0.33, 0) \quad ; \quad \vec{r}_{AC} = (-0.15 ; 0.33 ; 0.3)$

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.15 & 0 & 0 \\ 0 & 0 & 100 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.15 & 0.33 & 0 \\ 200 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.15 & 0.15 & 0 \\ B_x & 0 & B_z \end{vmatrix} +$$

$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.15 & 0.33 & 0.3 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

(i) $0.15 B_z - 0.3 C_y = 0$ ①

(j) $0.15(100) + 0.15 B_z + 0.3 C_x = 0$ ②

(k) $-200(0.33) - 0.15 B_x - 0.15 C_y - 0.33 C_x = 0$ ③

4 unknowns B_x, B_z, C_x, C_z + 3 equations ⇒ need one more!

$\Sigma F_x = 0 \qquad B_x + C_x + 200 = 0$ ④

we use F_x to avoid introducing more unknowns you can also solve 6 eq + 6 unknowns.

solve 4 eq + 4 unknowns

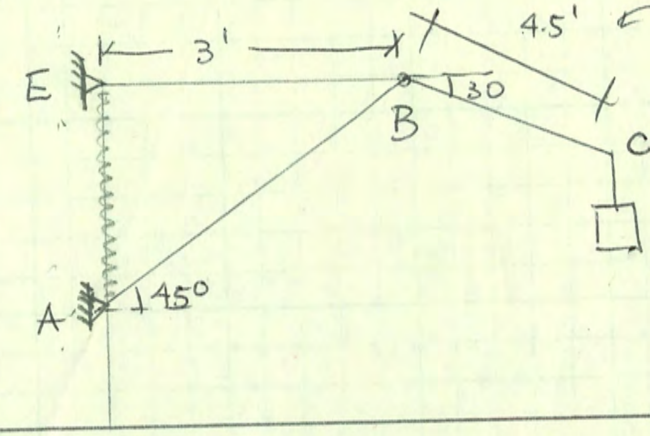
$$\left\{ \boxed{\begin{aligned} B_x &= 750\,N \\ B_z &= 1800\,N \\ C_x &= -950\,N \\ C_y &= 900\,N \end{aligned}} \right.$$

To get the other 2 reactions $\Sigma F_y = 0$, $\Sigma F_z = 0$

$\Sigma F_y = 0 \rightarrow A_y + C_y = 0 \rightarrow A_y = -C_y = \boxed{-900\,N = C_y}$

$\Sigma F_z = 0 \quad A_z + B_z + 100 = 0 \rightarrow A_z = \boxed{-1900\,N = A_z}$

5.127



← { not to scale }

suspended weight

600 lb weight

ABC 3 force member

$R_A = ?$

Force @ B = ?

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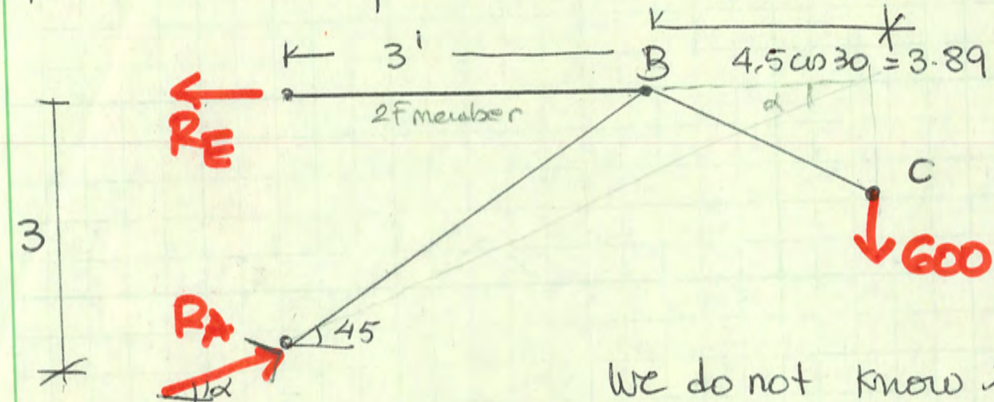
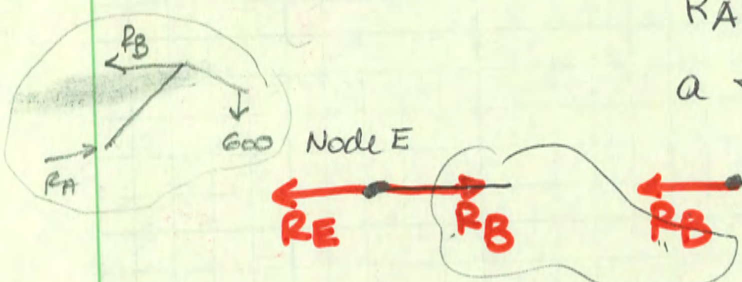
3 force member means their forces coincide @ 1 point since system is in equilibrium



$\overline{EC} = 6.8971$

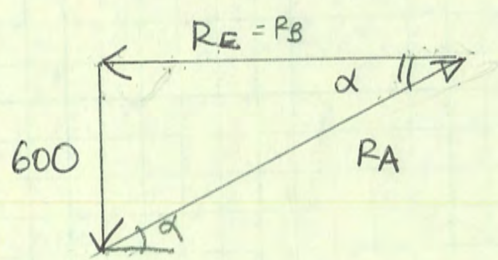
$\overline{EA} = 3$

We do not know the inclination of R_A. We do know R_E since it is a 2 force member so



$R_E = R_B$

Internal!



$\tan\alpha = \dfrac{\overline{EA}}{\overline{EC}} = \dfrac{3}{6.89} = 0.435$

$\alpha = 23.5°$

law of sin

$$\frac{R_A}{\sin 90} = \frac{600}{\sin\alpha} \rightarrow R_A = \frac{600}{\sin(23.5)} = 1504.7 \text{ lb}$$

$$\boxed{R_A = 1505 \text{ lb}}$$

$$\frac{R_E}{\sin(90-\alpha)} = \frac{R_A}{\sin 90^\circ} \rightarrow R_E = 1505 \sin(90-\alpha) = 1380 \text{ lb}$$

$$\boxed{R_E = 1380 \text{ lb}}$$

Since $R_E = R_B \Rightarrow R_B = 1380$ lb

$$\boxed{\begin{aligned} R_A &= 1505 \text{ lb } (\nearrow) \\ R_B &= 1380 \text{ lb } (\leftarrow) \end{aligned}}$$

Note: You can always also solve it using equations

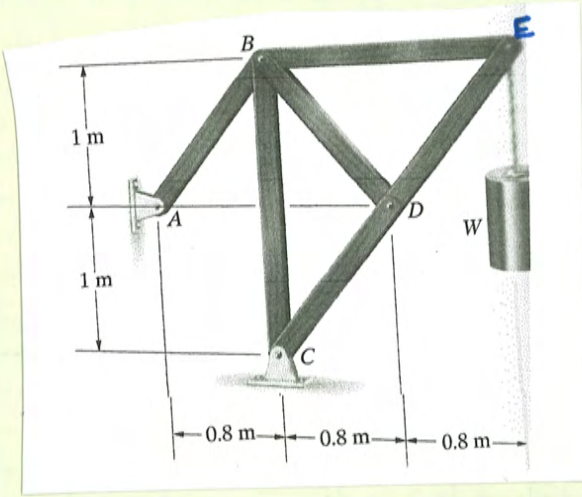
$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad R_A\cos\alpha - R_E = 0$ ①

$+\uparrow \Sigma F_y = 0 \quad R_A\sin\alpha - 600 = 0$ ②

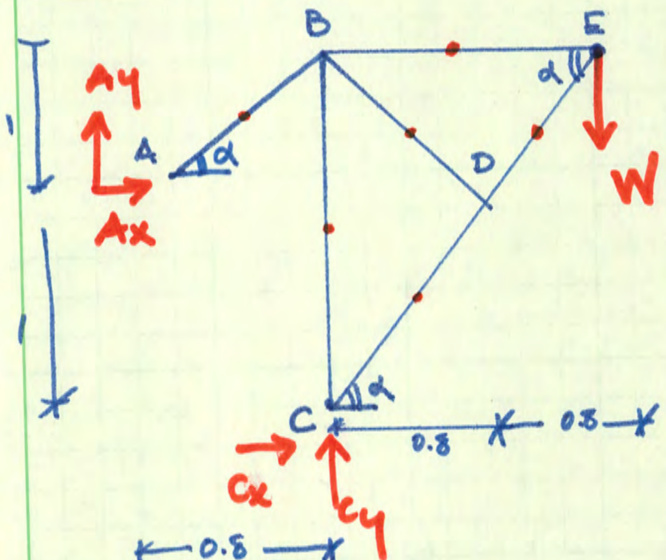
$+\circlearrowright \Sigma M_A = 0 \quad -600(\overline{BE}+\overline{BC}) + R_E(3) = 0 \rightarrow R_E = \frac{600(3.89)}{3}$ ③

$R_A \xleftarrow{\text{into ①}} R_B = R_E = 1380$ lb

or directly getting R_A from ② $\Rightarrow$ LOTS OF OPTIONS! :)



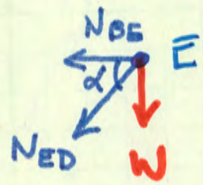
Axial forces $f(W) = ?$



$\tan\alpha = \dfrac{\cancel{2}(1)}{0.8\cancel{2}} \to \alpha = 51.34^\circ$

No need to calculate the reactions, we can start @ E.

Node E



$\overset{+}{\to} \Sigma F_x = 0 \qquad -N_{BE} - N_{DE}\cos\alpha = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0 \qquad -N_{ED}\sin\alpha - W = 0 \quad (2)$

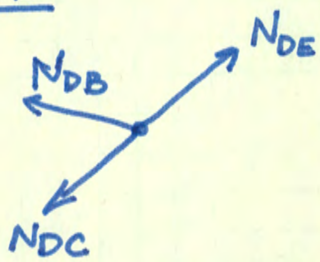
$\to N_{ED} = \dfrac{-W}{\sin\alpha}$

$\boxed{N_{ED} = -1.28\,W \; (C)}$

into (1) $\quad N_{BE} = -N_{DE}\cos\alpha = 1.28\,W\cos 51.34 = 0.8\,W \; (T)$

$\boxed{N_{BE} = 0.8\,W \, (T)}$

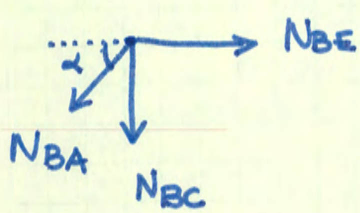
Node D



N_{DB} is a zero force member $\boxed{N_{DB}=0}$ because N_{DE} & N_{DC} are equal bc the node is in equilibrium

$$\left[N_{DE} = N_{DE} = -1.28\,W \;Ⓒ \right]$$

Node B



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad N_{BE} - N_{BA}\cos\alpha = 0$ ①

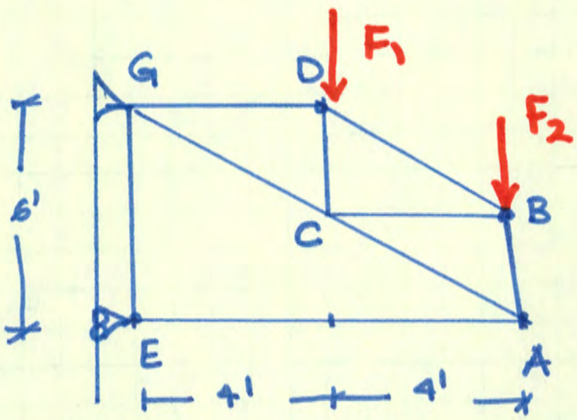
$+\uparrow \Sigma F_y = 0 \qquad -N_{BA}\sin\alpha - N_{BC} = 0$ ②

From ① $\quad N_{BA} = \dfrac{N_{BE}}{\cos\alpha} = \dfrac{0.8W}{\cos 51.34}$

$$\left[N_{BA} = 1.28\,W \;Ⓣ \right]$$

into ② $\quad N_{BC} = -N_{BA}\sin\alpha$

$$\left[N_{BC} = -1.28W\,(0.78) = -W \;Ⓒ \right]$$



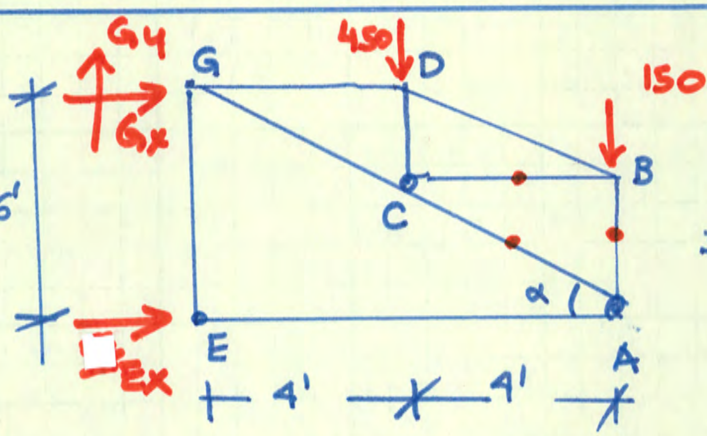
$F_1 = 450$ lb

$F_2 = 150$ lb

$N_{AB} = ?$

$N_{AC} = ?$

$N_{BC} = ?$



$\tan\alpha = \frac{6}{8}$

$\alpha = 36.87°$

Reactions : $\xrightarrow{+} \Sigma F_x = 0 \quad E_x + G_x = 0$ ①

$+\uparrow \Sigma F_y = 0 \qquad G_y = 450 + 150 = 600$ lb

$\circlearrowleft^{+} \Sigma M_G = 0 \qquad E_x(6) - 450(4) - 150(8) = 0$

$\hookrightarrow E_x = 500$ lb into ①

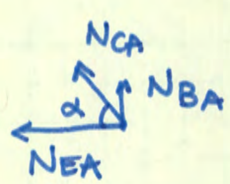
$[G_x = -E_x = -500 \text{ lb}]$

Node E $\quad \xrightarrow{E_x} \cdot \uparrow N_{EG} \rightarrow N_{EA}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad N_{EA} + E_x = 0$

$[N_{EA} = -E_x = -500 \text{ lb}]$ Ⓒ

Node A



$\left.\begin{array}{ll} \xrightarrow{+} \Sigma F_x = 0 & -N_{EA} - N_{CA}\cos\alpha = 0 \\ +\uparrow \Sigma F_y = 0 & N_{CA}\sin\alpha + N_{BA} = 0 \end{array}\right\}$ ① ②

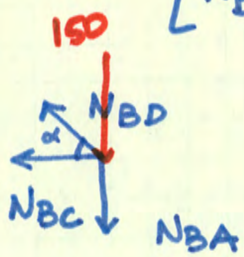
$$N_{CA}\cos\alpha = -N_{EA} \rightarrow N_{CA} = \frac{-N_{EA}}{\cos\alpha} = \frac{500}{\cos 36.87}$$

$$\left[N_{CA} = 625\ lb \ (T) \right]$$

into ② $N_{BA} = -N_{CA}\sin\alpha = -625 \sin 36.87$

$$\left[N_{BA} = -375\ lb \ (C) \right]$$

Node B



$$\begin{cases} \overset{+}{\rightarrow}\Sigma F_x = 0 & -N_{BC} - N_{BD}\cos\alpha = 0 \quad ① \\ \overset{+}{\uparrow}\Sigma F_y = 0 & -N_{BA} + N_{BD}\sin\alpha = 150 \quad ② \end{cases}$$

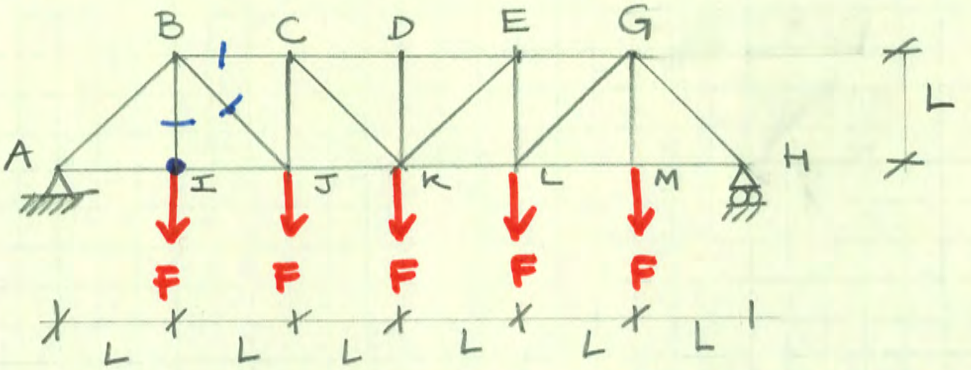
$\hookrightarrow N_{BD}\sin\alpha = N_{BA} + 150$

$$N_{BD} = \frac{N_{BA}+150}{\sin\alpha} = \frac{-375+150}{\sin 36.87} = -375\ lb$$

into ①

$$N_{BC} = -N_{BD}\cos\alpha = 375\cos 36.87 = 300\ lb \ (T)$$

$$\boxed{\begin{aligned} N_{AB} &= -375\ lb \ (C) \\ N_{AC} &= 625\ lb \ (T) \\ N_{BC} &= 300\ lb \ (T) \end{aligned}}$$



$F = 300\ kN$

$L = 8\ m$

$N_{BC} = ?$

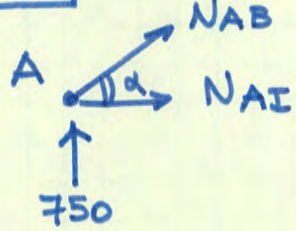
$N_{BI} = ?$

$N_{BJ} = ?$

This is ~~a~~ symmetric structure so we do not need to write the equilibrium equations, we know they are going to be the same $A_y = H_y = 2.5F = 2.5\,(300)$

$$\left[A_y = H_y = 750\ kN\right] (\uparrow)$$

Node A



$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad N_{AB} \cos\alpha + N_{AI} = 0$

$+\uparrow \Sigma F_y = 0 \quad N_{AB} \sin\alpha = -750$

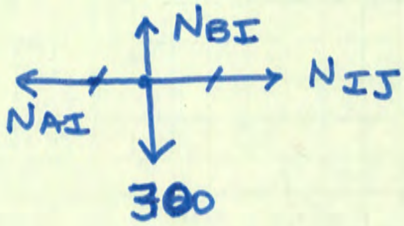
$\downarrow$

$N_{AB} = \dfrac{-750}{\sqrt{2}/2} = -1061\ kN$

into (1) $\quad N_{AI} = -N_{AB} \cos\alpha$

$\left[N_{AI} = +1061 \cos 45 = 750\ kN\right]$ (T)

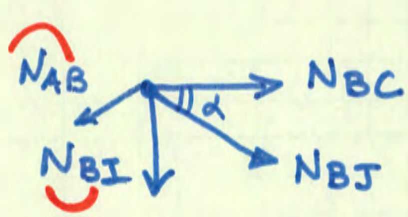
Node I



$\boxed{N_{BI} = 300\ kN \quad (T)}$

Node B

N_{AB}, N_{BI} known.



$\overset{+}{\rightarrow} \Sigma F_x = 0;\ N_{BC} + N_{BJ} \cos\alpha = N_{AB} \cos 45$

$+\uparrow \Sigma F_y = 0 \quad -N_{BI} - N_{BJ} \sin\alpha - N_{AB} \sin$

$\swarrow$

$N_{AB} \sin\alpha + N_{BJ} \sin\alpha = -N_{BI}$

$$N_{BJ}\sin\alpha = -N_{BI} - N_{AB}\sin\alpha$$

$$N_{BJ} = -\frac{N_{BI}}{\sin\alpha} - N_{AB} = \frac{-300}{\sin 45} + 1061 = 636\text{ kN}$$

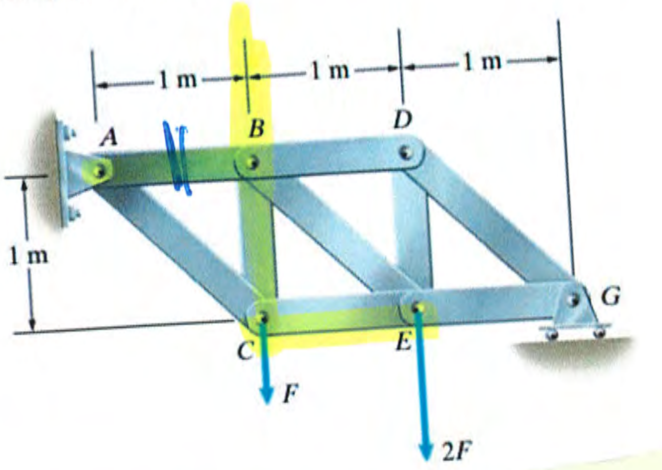
$$\boxed{N_{BJ} = 636\text{ kN (T)}}$$

into (1) $N_{BC} = N_{AB}\cos\alpha - N_{BJ}\cos 45$

$$N_{BC} = -1061\cos 45 - 636\cos 45 = -1200\text{ kN}$$

$$\boxed{\begin{aligned} N_{BC} &= -1200\text{ kN (C)} \\ N_{BI} &= 300\text{ kN (T)} \\ N_{BJ} &= 636\text{ kN (T)} \end{aligned}}$$

6.36 Use the method of sections to determine the axial forces in members *AB*, *BC*, and *CE*.



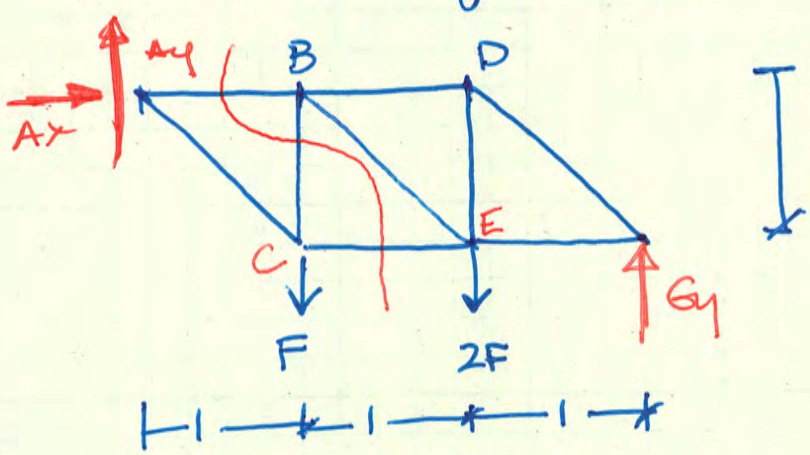
$F_{AB} = ?$

$F_{BC} = ?$

$F_{CE} = ?$

No matter what section we choose we cannot avoid calculating the reactions @ supports

1- Global equilibrium



$\xrightarrow{+} \Sigma F_x = 0 \qquad A_x = 0.$

$+\uparrow \Sigma F_y = 0 \qquad G_y + A_y - F - 2F = 0 \quad (1)$

$\circlearrowleft + \Sigma M_A = 0 \qquad -F(1) - 2F(2) + G_y(3) = 0 \quad (2)$

$3G_y = 4F + F = 5F$

$\boxed{G_y = \frac{5}{3}F} \; (\uparrow)$

G_y into ① to obtain A_y

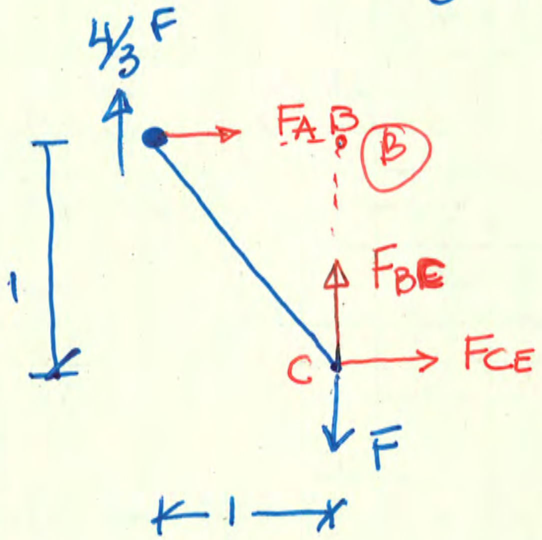
$$A_y = 3F - G_y = \left(3 - \frac{5}{3}\right)F = \frac{4}{3}F$$

$$\boxed{A_y = \frac{4}{3}F}$$ ($\uparrow$) same as assumed

Step 2 Isolate one cut $\Rightarrow$ write eq

$\Downarrow$ eq

Any cut will be in equilibrium



$\circlearrowleft \sum M_B = 0$

$$F_{CE}(1) - \frac{4}{3}F(1) = 0$$

$$\boxed{F_{CE} = \frac{4}{3}F}$$ (T)

$\overset{+}{\rightarrow} \sum F_x = 0$ $\quad F_{AB} + F_{CE} = 0 \rightarrow F_{AB} = -F_{CE}$

$$\boxed{F_{AB} = -\frac{4}{3}F}$$ (C)

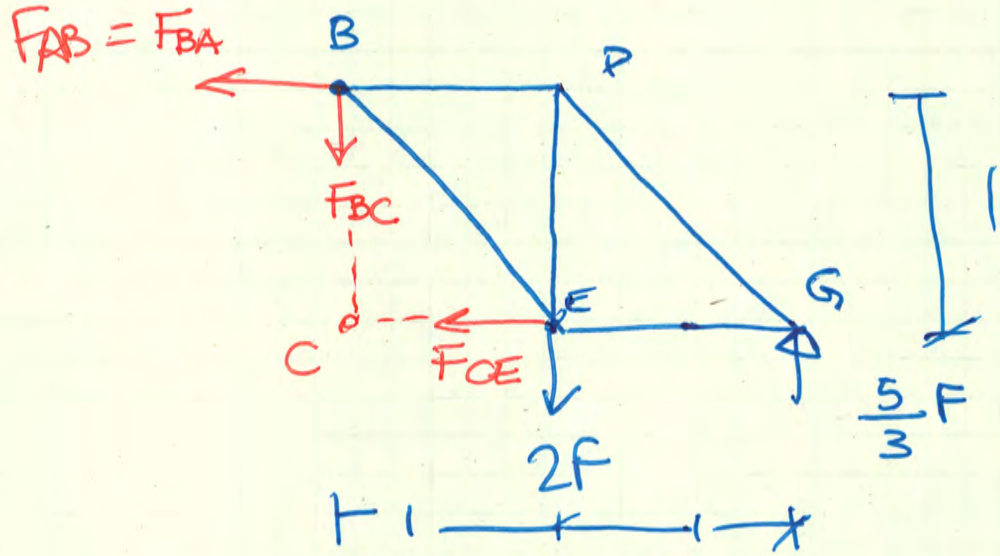
$+\uparrow \sum F_y = 0$ $\quad F_{BC} + \frac{4}{3}F - F = 0$

$$F_{BC} = F\left(1 - \frac{4}{3}\right) = -\frac{1}{3}F$$ (C)

$$\boxed{F_{BC} = -\frac{1}{3}F \text{ (C)}}$$

BAR	MAGNITUDE	C or T
AB	$\frac{4}{3}F$	C
BC	$F/3$	C
CE	$4/3\,F$	T

Note: you could have also used the other cut → Try it! the results are the same



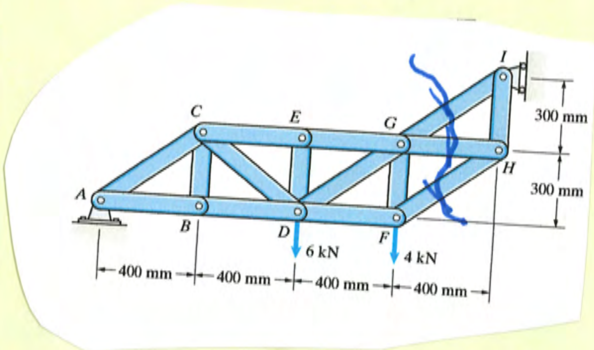
$\circlearrowleft^{+} \Sigma M_C = 0$ $\qquad F_{AB}(1) - 2F(1) + \frac{5}{3}F(2) = 0$

$$F_{AB} = \left(-\frac{10}{3} + 2\right)F = \frac{-10+6}{3}F$$

$$F_{AB} = -\frac{4}{3}F \quad \text{(C)}$$

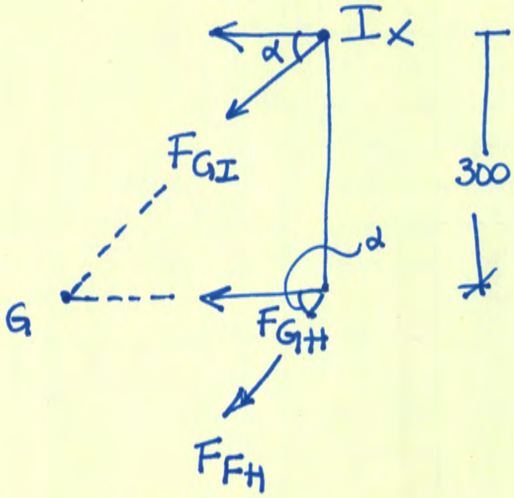
Same as before! :)

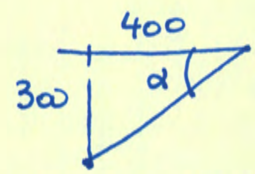
6.45 page 1 of 2



Method of sections

FH GH GI





$\tan\alpha = \frac{300}{400} \rightarrow \alpha = 36.87°$

1st calculate reaction $\circlearrowleft + \sum M_A = 0$

$$-6(400+400) - 4(400 * 3) + I_x(300+300) = 0$$

$$I_x = 16\ kN$$

$\circlearrowleft + \sum M_G = 0 \qquad I_x(300) - F_{FH}\sin\alpha\,(400) = 0$

$$F_{FH}\ \frac{16\ (300)}{\sin(36.87)\,400} = 20\ kN \quad \text{(T)}$$

$\circlearrowleft^{+} \ \Sigma M_H = 0 \qquad I_x(\cancel{300}) + F_{GI}\cos\alpha\,(\cancel{300}) = 0$

$$F_{GI} = -\frac{I_x}{\cos} = \frac{-16}{\cos 36.87}$$

$$F_{GI} = -20\ \text{kN} \ (C)$$

$\xrightarrow{+} \Sigma F_x = 0 \qquad -I_x - F_{GI}\cos\alpha - F_{GH} \cancel{=0}\ F_{FH}\cos\alpha = 0$

$$F_{GH} = -16 + 20\cos 36.87 - \cancel{20}\cos 36 = 0$$

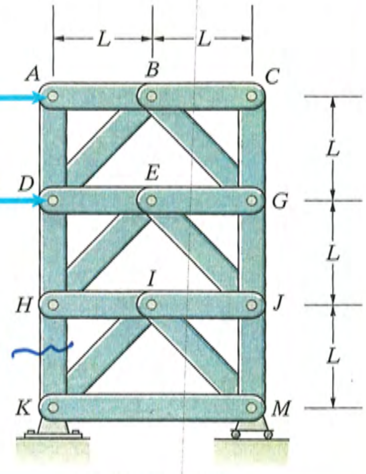
$$\boxed{F_{GH} = -16\ \text{kN} \ (C)}$$

$$\boxed{\begin{aligned} F_{FH} &= 20\ \text{kN} \ (T) \\ F_{GI} &= -20\ \text{kN} \ (C) \\ F_{GH} &= -16\ \text{kN} \ (C) \end{aligned}}$$

6.51. A·ALARCON

6.51 The load $F = 20$ kN and the dimension $L = 2$ m. Use the method of sections to determine the axial force in member HK.

Strategy: Obtain a section by cutting members HK, HI, IJ, and JM. You can determine the axial forces in members HK and JM even though the resulting free-body diagram is statically indeterminate.



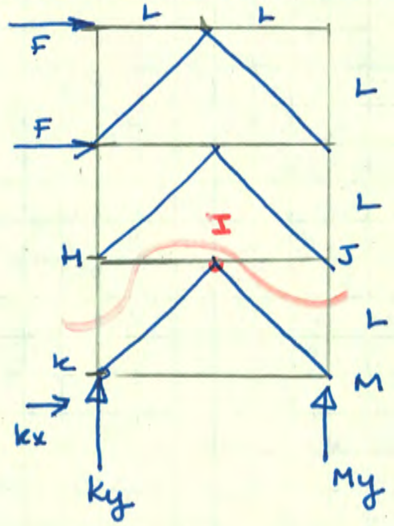
Problem 6.51

$F = 20$ kN

$L = 2$

$F_{HK} = ?$

GFBD



$\circlearrowleft^{+} \sum M_K = 0$

$M_y(2\cancel{L}) - F(2\cancel{L}) - F(3\cancel{L}) = 0$

$2M_y = 5F \rightarrow \boxed{M_y = \frac{5}{2}F} = \underline{\underline{50\text{ kN}}}$

$+\uparrow \sum F_y = 0$

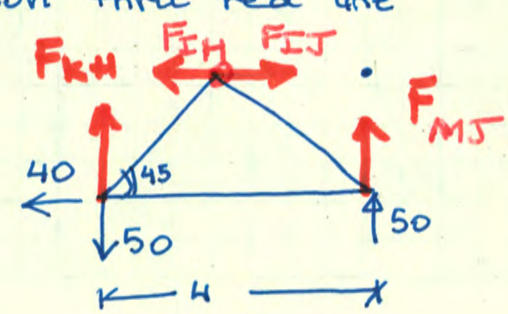
$K_y + M_y = 0 \rightarrow K_y = -M_y$

$\boxed{K_y = -5/2\, F} = -\frac{5}{2}(20) = -50$ kN

$\xrightarrow{+} \sum F_x = 0$ $\quad K_x + 2F = 0 \rightarrow K_x = -2F = -2(20) = -40$ kN

$\boxed{K_x = -40\text{ kN}}$

Section thru red line



$\circlearrowleft^{+} \sum M_J = 0$

$-F_{KH}(4) + 50(4) - 40(2) = 0$

$4F_{KH} = 50(4) - 40(2)$

$F_{KH} = 50 - \frac{40}{4}2 = \underline{\underline{30\text{ kN}}}$ (T)

$\boxed{F_{KH} = 30\text{ kN (T)}}$