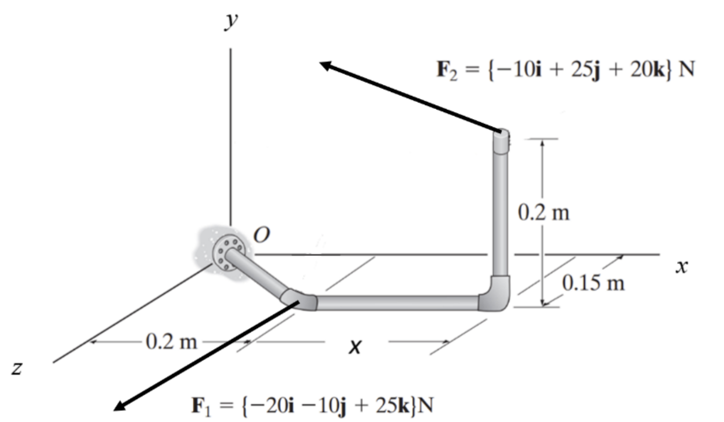
Name____

PROBLEM 1 (15 POINTS)

Determine the distance *x* such that the magnitude of the moment at *O* equals 15 N * m. x must be positive. (Figure not to scale)

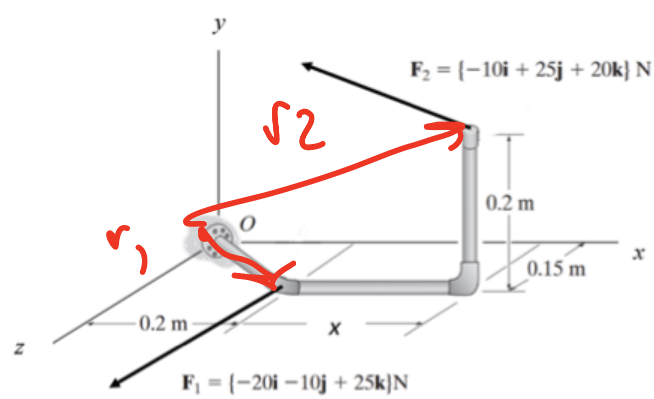


$$|\vec{M}_O| = 15\ \text{N.m}$$

$$\vec{M}_O = \bar{r}_1 \times \bar{F}_1 + \bar{r}_2 \times \vec{F}_2$$

1 (0.2; 0; 0.15)

2 (x+0.2; 0.2; 0.15)



$$\vec{M}_O = \underbrace{\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0.2 & 0 & 0.15 \\ -20 & -10 & 25 \end{vmatrix}}_{M_1} + \underbrace{\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x+0.2 & 0.2 & 0.15 \\ -10 & 25 & 20 \end{vmatrix}}_{M_2}$$

$$M_1 = \vec{i}\left(10(0.15)\right) - \bar{j}\left[0.2(25) - (-20)0.15\right] + 0.2(-10)\,\bar{k}$$

$$1.5\bar{i} - 8\bar{j} - 2\bar{k}$$

$$M_2 = \left[0.2(20) - 25(0.15)\right]\vec{i} = 0.25\vec{i}$$

$$-\vec{j}\left[20(x+0.2)-(-10)0.15\right] = -\left[20x+5.5\right]$$

$$\vec{k}\left[(25(x+0.2)-(-10)(0.2)\right] = (25x+7)\,\bar{k}$$

$$M\begin{cases}\vec{i}\;(1.5+0.25) = \bar{i}(1.75)\\ \bar{J}\;(-8-20x-5.5) = (-20x-13.5)\\ \bar{k}\;(-2+25x+7) = (25x+5)\end{cases}$$

$$\sqrt{1.75^2+(20x+13.5)^2+(25x+5)^2} = 15$$

$$\boxed{x = 0.0182\ m}$$

$$1.75^2+(20x)^2+40x(13.5)+13.5^2+(25x)^2+250x+25 = 225$$

$$(400+625)x^2+x(540+250)+3.0625+182.25+25-225$$

$$1025x^2+790x-14.687 = 0$$

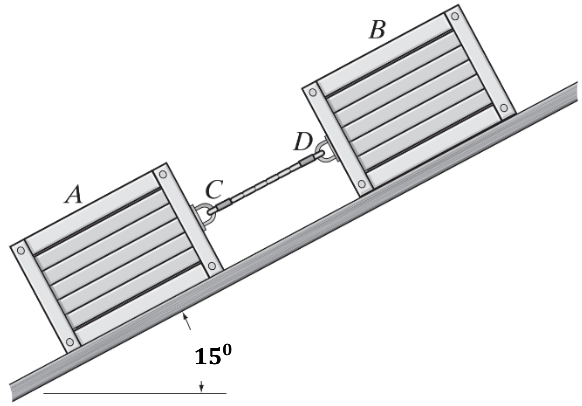
$$x = \frac{-790\pm\sqrt{790^2+4(1025)(14.687)}}{2(1025)} = \frac{-790\pm 827.23}{2(1025)}$$

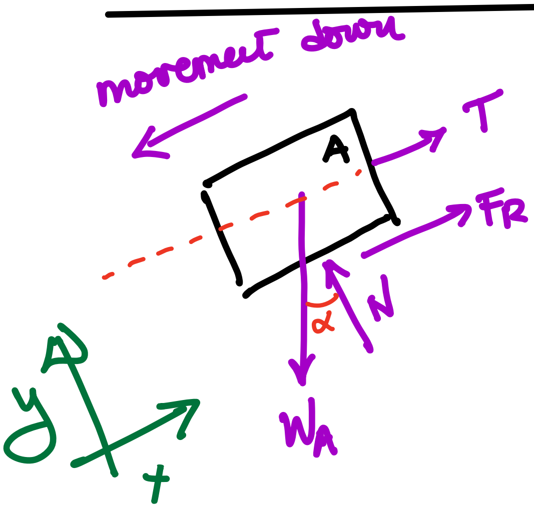
$$\boxed{x = 0.0182\ m}$$

Name____

PROBLEM 2 (20 POINTS)

Crate B weighs 150 lb. It is connected to crate A by a cable (CD on the figure) and placed on an inclined plane. Let w_A be the weight of crate A. What value of w_A will cause the system to be on the verge of sliding down? The coefficients of static friction between the crates and the plane are $\mu_A = 0.25$ and $\mu_B = 0.35$.





$$\begin{cases} T + F_R - W_A \sin\alpha = 0 & (1) \\ N - W_A \cos\alpha = 0 & (2) \\ F_R = \mu_A N & (3) \end{cases}$$

$$\underset{(3)}{F_R = \mu_A N} \underset{(2)}{=} \mu_A (W_A \cos\alpha)$$

into (1)

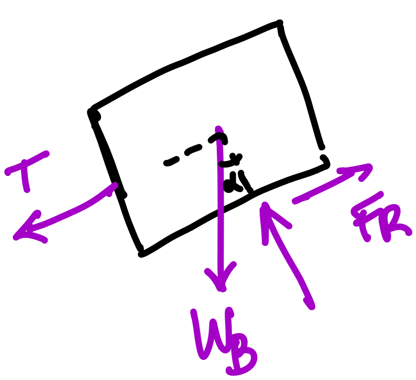
$$T + \mu_A W_A \cos\alpha - W_A \sin\alpha = 0$$

$$T = W_A (\sin\alpha - \mu_A \cos\alpha)$$

$$\left[W_A = \frac{T}{\sin\alpha - \mu_A \cos\alpha} \right] \quad (4)$$

we can get T from eq of block B

$$\left. \begin{array}{l} -T - W_B \sin\alpha + F_R = 0 \\ N - W_B \cos\alpha = 0 \\ F_R = \mu_B N \end{array} \right\}$$



$$T = F_R - W_B \sin\alpha$$

$$T = \mu_B W_B \cos\alpha - W_B \sin\alpha$$

$$\left.\begin{array}{l} \mu_B = 0.35 \\ W_B = 150 \text{ lb} \\ \alpha = 15^\circ \end{array}\right\} \rightarrow T = 11.88 \text{ lb}$$

$\downarrow$

into eq 4

$$\left.\begin{array}{l} T = 11.88 \\ \mu_A = 0.25 \\ \alpha = 15^\circ \end{array}\right\}$$

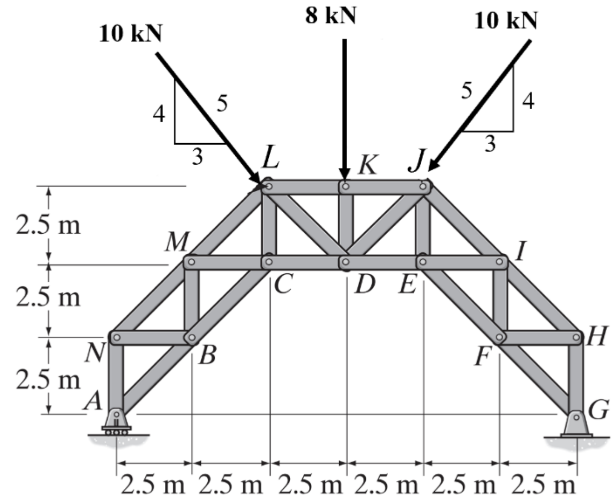
$$W_A = \frac{11.88}{\sin 15 - 0.25 \cos 15} = 685.69$$

$$\boxed{W_A = 686 \text{ lb}}$$

Name______________________________________

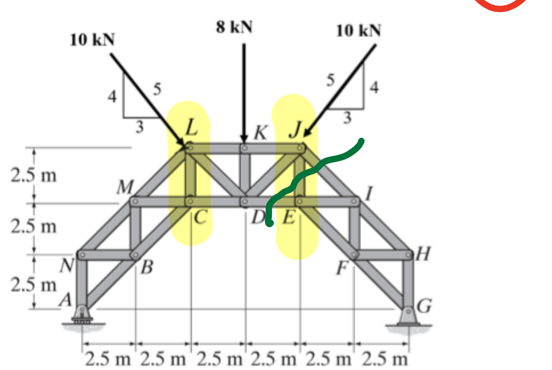
PROBLEM 3 (15 POINTS)

Determine the force in members CL and EJ of the truss, and state if the members are in tension or compression. Neglect the weight of the members. Write your results on the table.



BAR	MAGNITUDE (kN)	Tension or Compression
CL	12	T
EJ	12	T

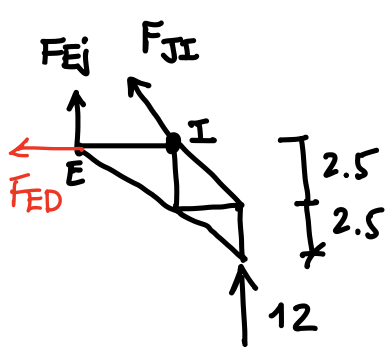
ok to say by symmetry here!



no need to calculate the reactions w/ eq if we see the symmetry! :)

Vertical forces $8 + \frac{4}{5}10(2) = 24$

$A_y = G_y = \frac{24}{2} = 12$; $G_x = 0$ due to symmetry



$\Sigma M_I = 0 \quad -F_{EJ}(\cancel{2.5}) + 12(\cancel{2.5}) = 0$

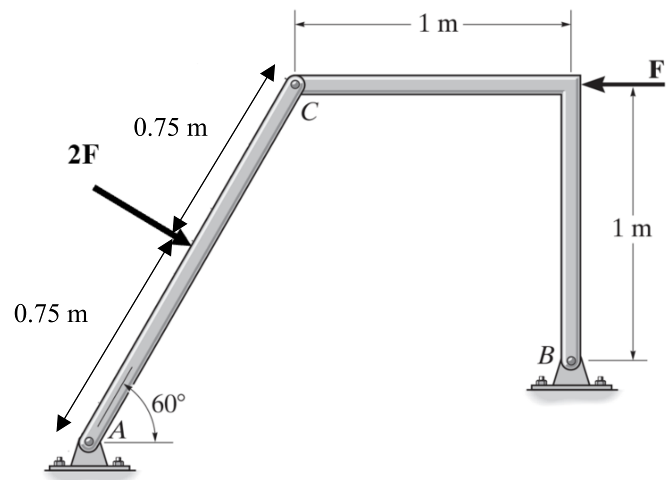
$\boxed{F_{EJ} = 12 \text{ kN}}$ T

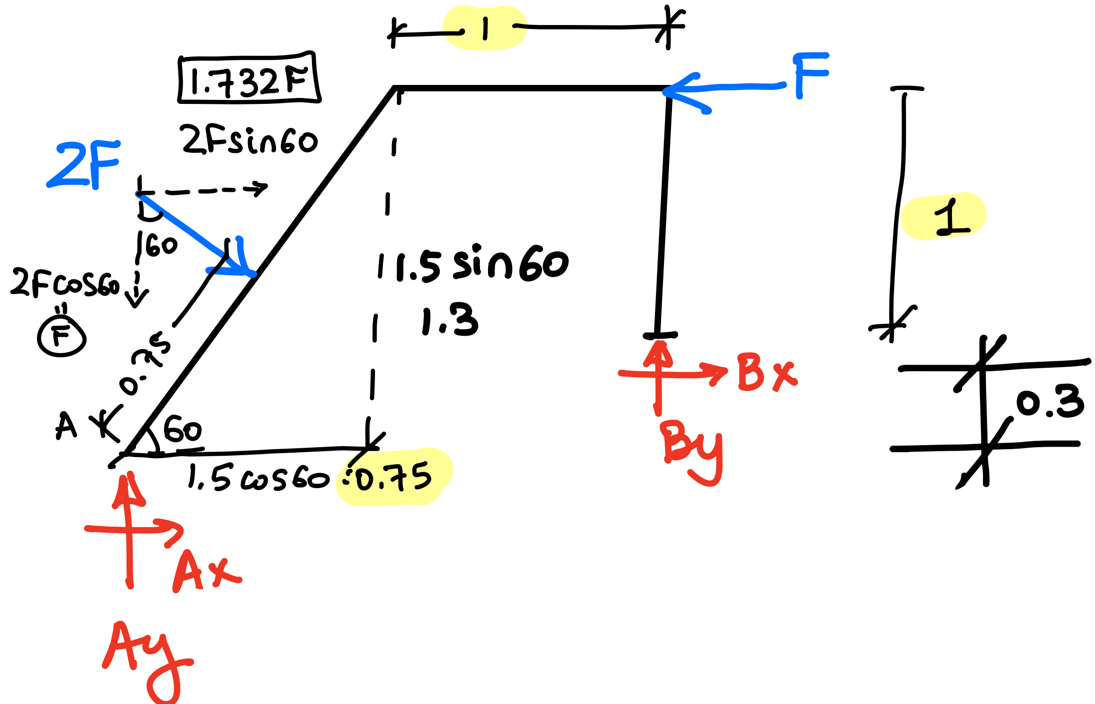
$\boxed{F_{CL} = 12 \text{ kN by symmetry}}$

Name____

PROBLEM 4 (20 POINTS)

Determine the horizontal and vertical components of reaction that the pins A and B exert on the two-member frame. Length of AC is 1.5 m. Set F = 200 lb. Draw a figure with your results and box it.

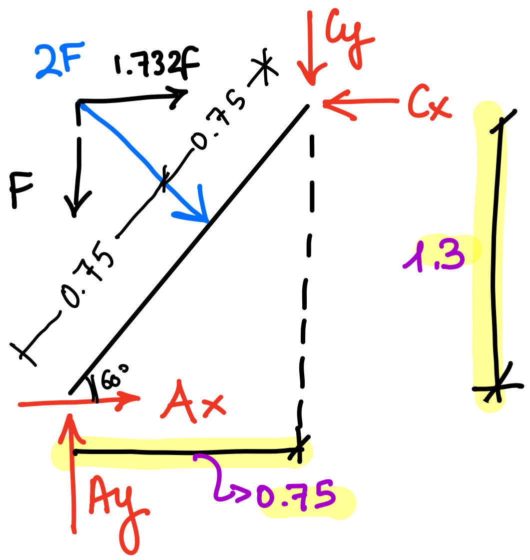


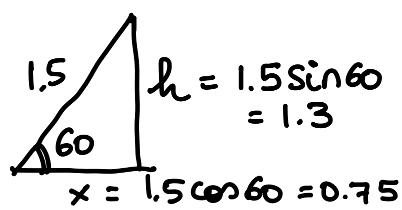


$\rightarrow \Sigma F_x = 0 \qquad A_x + B_x - F + 1.732F = 0 \qquad (*)$

$+\uparrow \Sigma F_y = 0 \qquad A_y + B_y - F = 0 \qquad (**)$

$$\overset{+}{\circlearrowleft} \Sigma M_A = 0 \quad -2F(0.75) + B_y(1.75) - B_x(0.3) + F(1.3) = 0 \quad (***)$$

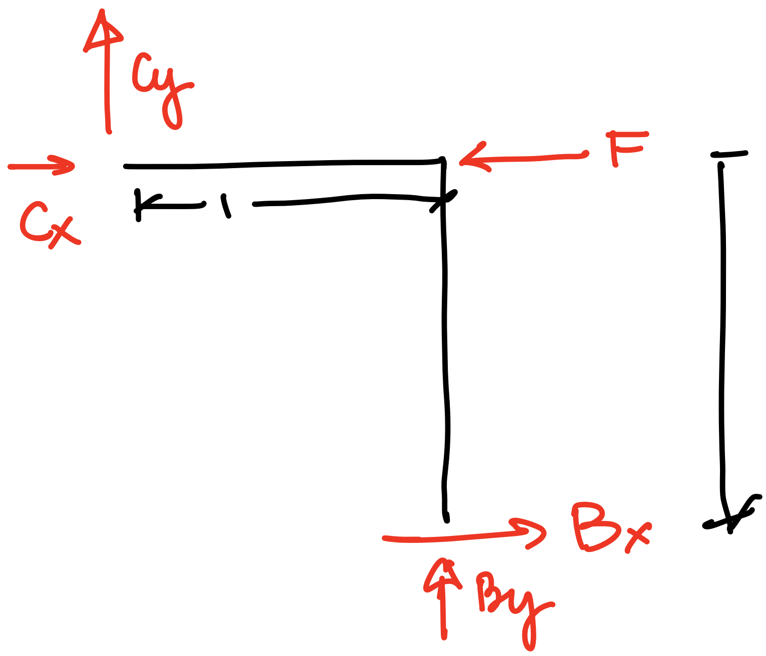




$\xrightarrow{+} \Sigma F_x = 0 \quad A_x + 1.732F - C_x = 0 \quad$ ①

$+\uparrow \Sigma F_y = 0 \quad A_y - F - C_y = 0 \quad$ ②

$\overset{+}{\circlearrowleft} \Sigma M_A = 0 \quad C_x(1.3) - C_y(0.75) - 2F(0.75) = 0 \quad$ ③



$\circlearrowleft^{+} \sum M_B = 0 \qquad -C_y(1) - C_x(1) + F = 0$

$+\uparrow \sum F_y = 0 \qquad C_y + B_y = 0 \rightarrow B_y = -C_y \quad (4) \quad (5)$

Eq (3) (2) (4) $\qquad \xrightarrow{+} \sum F_x = 0 \qquad B_x + C_x - F = 0 \quad (6)$

$$\begin{cases} C_x(1.3) - C_y(0.75) = 2F(0.75) \\ C_x + C_y = F \quad \times 0.75 \end{cases}$$

$C_x(1.3 + 0.75) = 2F(0.75) + 0.75F$

$C_x = \dfrac{3F(0.75)}{2.05} = 219.5$

$C_y = F - C_x = 200 - 219.5 = -19.5$

$\Downarrow \boxed{B_y = 19.5 \text{ lb}} \ (\uparrow)$

Into ② $A_y = C_y + F = -19.5 + 200$

$\boxed{A_y = 180.5 \text{ lb}}$

C_x into ①

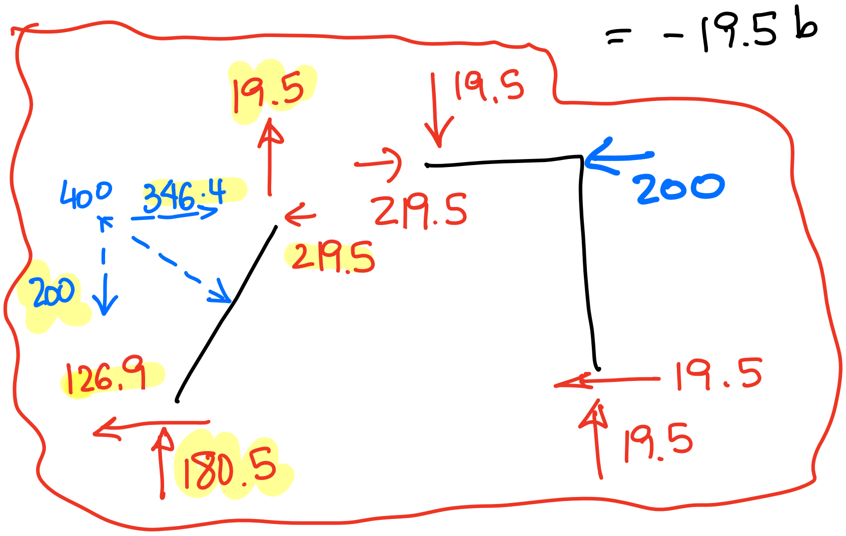
$A_x = C_x - 1.732F = 219.5 - 1.732(200)$

$\boxed{A_x = -126.9 \text{ lb}}$

B_x from ⑥

$B_x = F - C_x = 200 - 219.5$

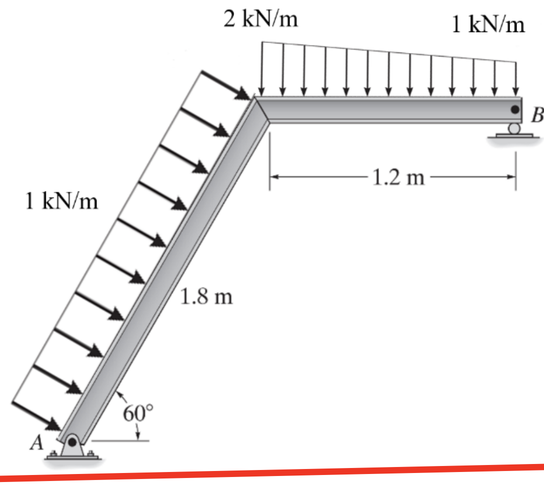
$= -19.5 \text{ b}$

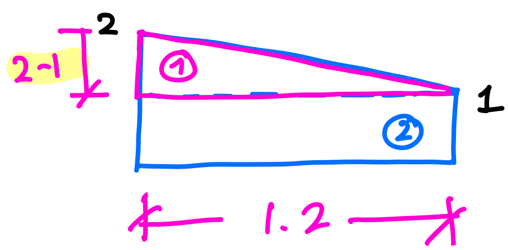


Name________________________________

PROBLEM 5 (15 POINTS)

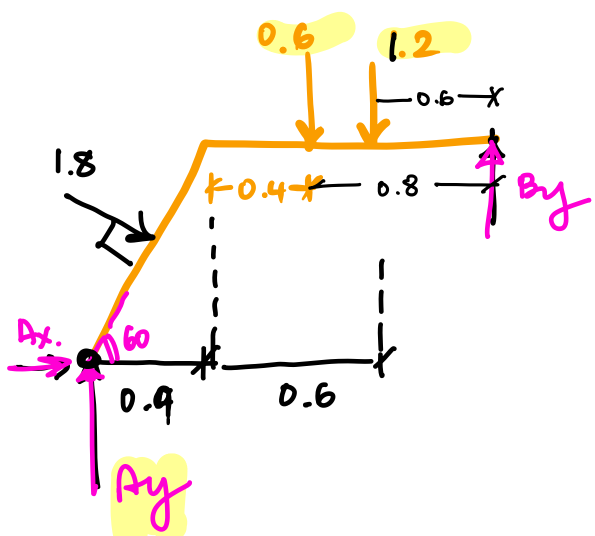
Determine the reactions at pin support A and roller support B. Draw a figure with your results and box it.

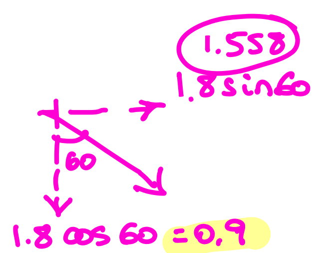




Area ① $\frac{1}{2}\,1.2 = 0.6$

Area ② $1.2 \times 1 = 1.2$





$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad A_x + 1.8\sin 60 = 0 \rightarrow \boxed{A_x = -1.56 \text{ kN}}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 0.6 - 1.2 + B_y - 0.9 = 0 \quad ②$

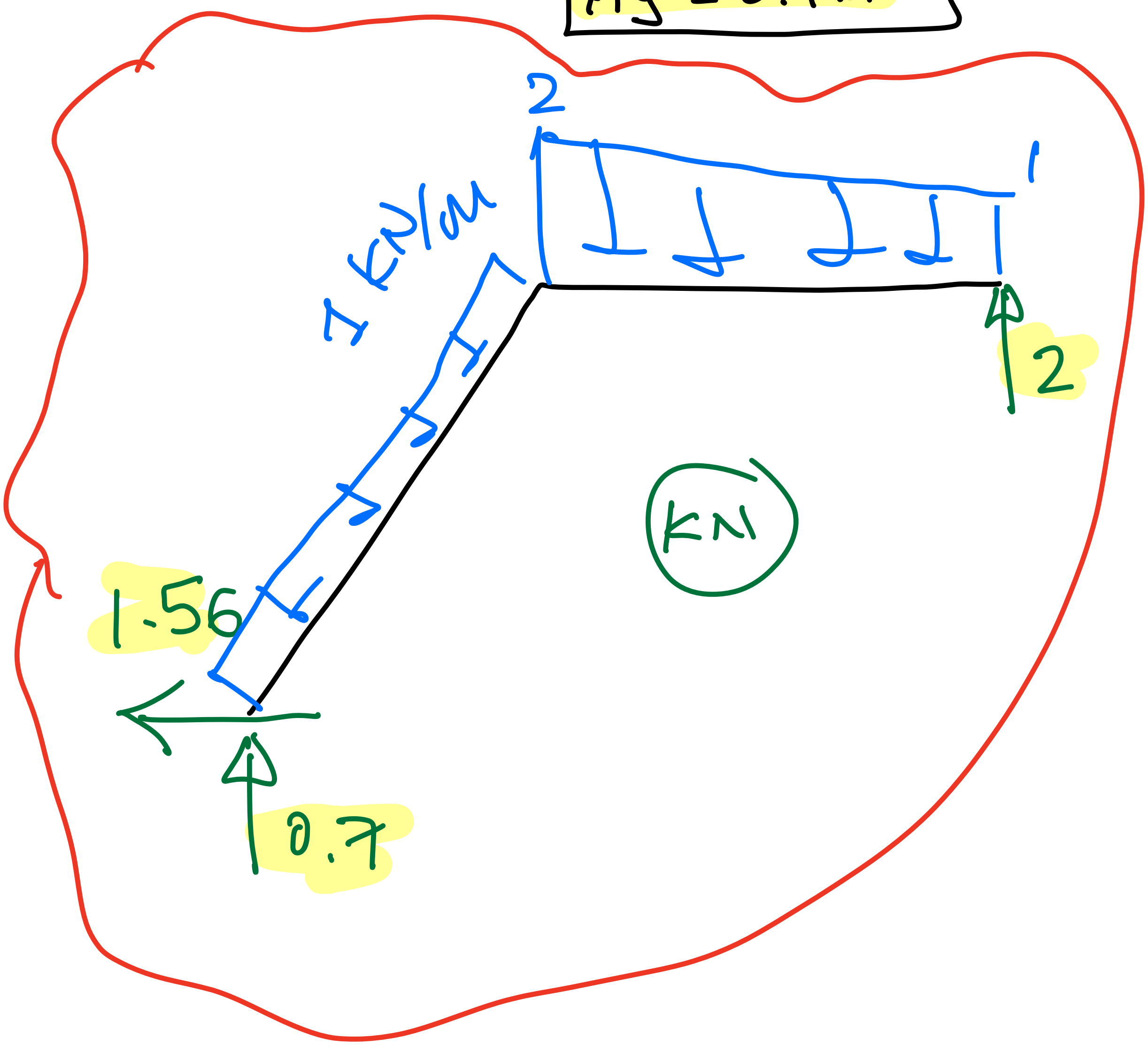
$\circlearrowleft \Sigma M_A = 0 \qquad -0.6(0.4+0.9) - 1.2(0.6+0.9) + B_y(0.9+1.2) - 1.8\left(\frac{1.8}{2}\right) = 0$

$$By = \frac{+0.78 \;\; + 1.8 + 1.62}{2.1} = 2$$

into 2

$$Ay = -By + \underbrace{2.7 = -2 + 2.7 = 0.7\text{ KN}}$$

$$\boxed{Ay = 0.7\text{KN}}$$



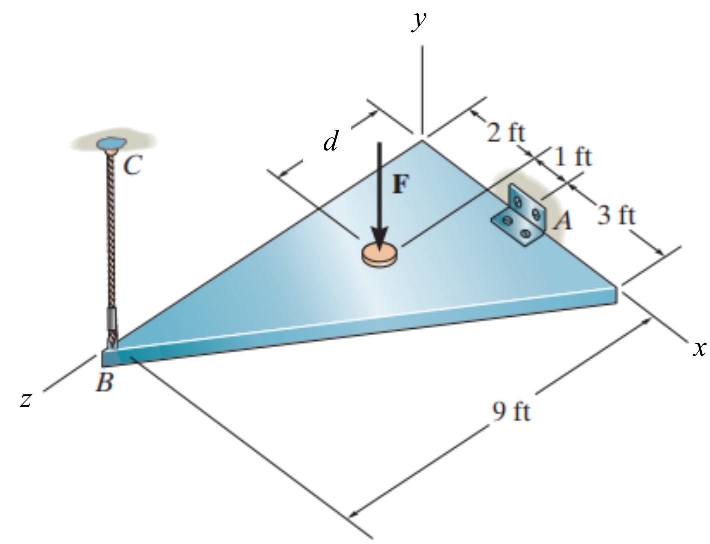
Name____

PROBLEM 6 (15 POINTS)

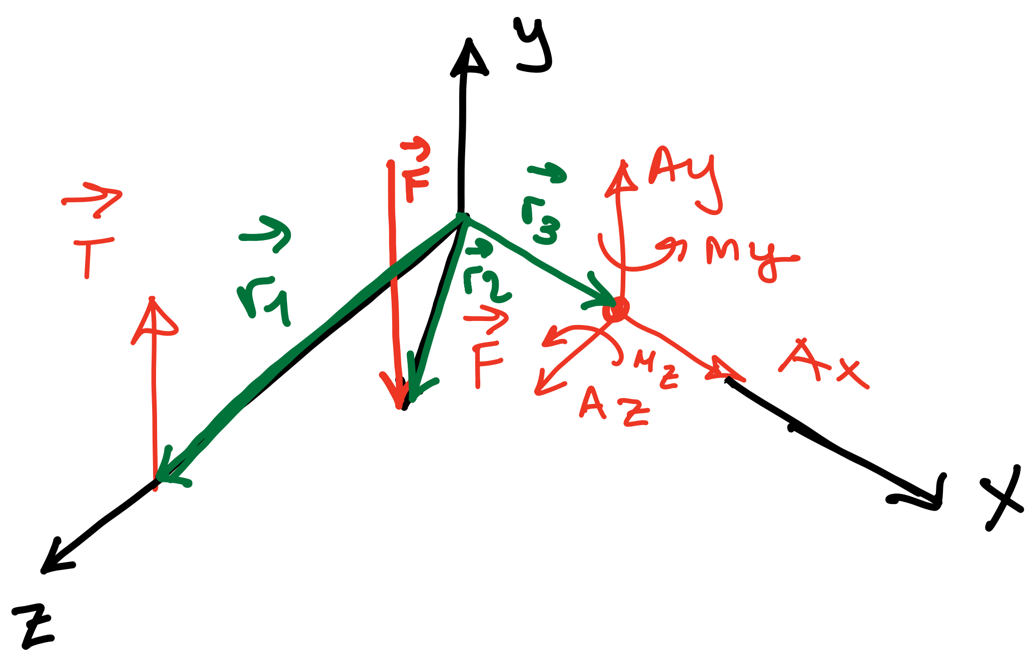
If cable BC is subjected to a tension of 300 lb and the force shown in the figure F = 900 lb

a) Determine distance *d*

b) Compute the components of the reaction at hinge A.



FBD



EQ EQ

$$\vec{T} + \vec{F} + \vec{A} = 0$$

$$\vec{T} = 300\vec{j}$$

$$\vec{F} = -900\vec{j}$$

$$\vec{A} = A_x\bar{i} + A_y\bar{j} + A_z\bar{k}$$

$$\sum\vec{F} = 0$$

$\sum F_x = 0$ $\quad \boxed{A_x = 0}$

$\sum F_y = 0$ $\quad 300 - 900 + A_y = 0$

$\boxed{A_y = 600 \text{ lb}}$

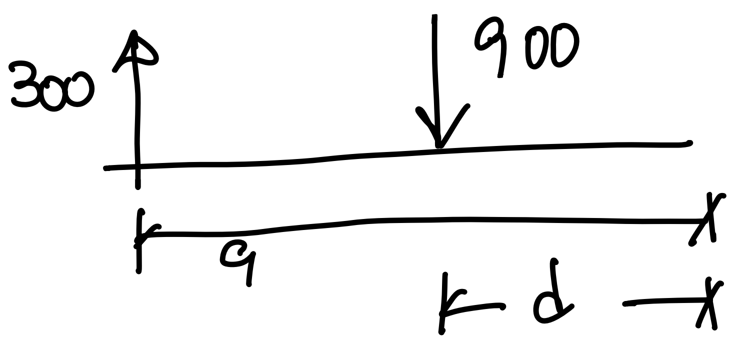
$\sum F_z = 0$ $\quad \boxed{A_z = 0}$

$\sum \vec{M}_0 = 0$

for simplicity! :)

$$\left\{ \begin{array}{l} \vec{r}_1 = 9\vec{k} \\ \vec{r}_2 = 2\vec{i} + d\vec{k} \\ \vec{r}_3 = 3\vec{i} \end{array} \right.$$

we can obtain d knowing that in a hinge about x → $M_x = 0$ – we can see in 2D



$$-3\cancel{00}(9) + 9\cancel{00}\, d = 0$$

$$\boxed{d = 3 ft}$$

$$\sum \vec{M}_O = \vec{r}_1 \times \vec{T} + \vec{r}_2 \times \vec{F} + \vec{r}_3 \times \vec{A} + M_z \bar{k} + M_y \bar{j}$$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 0 & 9 \\ 0 & 300 & 0 \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 0 & 3 \\ 0 & -900 & 0 \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 0 & 0 \\ 0 & 600 & 0 \end{vmatrix} + M_z \bar{k} + M_y \bar{j} = 0$$

$$\cancel{-2700\, i} - \left(\cancel{-2700\, \vec{i}}\right) \cancel{- 1800 k} + \cancel{1800 k} + M_z \bar{k} + M_y \bar{j} = 0$$

$$\boxed{\begin{aligned} M_z &= 0 \\ M_y &= 0 \end{aligned}}$$

$$\boxed{\left\{\begin{aligned} A_x &= 0 \\ A_y &= 600\ \text{lb} \\ A_z &= 0 \end{aligned}\right. \qquad \left\{\begin{aligned} M_x &= 0 \\ M_y &= 0 \\ M_z &= 0 \end{aligned}\right.}$$