1/8/2024

ENGINEERING ANALYSIS II (EA2)

Lecture # 03: Vectors

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Lecture Outlines:

1. Dot Product.
2. Example.

References:

1. Bedford, A., & Fowler, W. *Engineering Mechanics: Statics* (5th ed.).
2. Prof. Alarcon's lecture notes.

$\boxed{4}$ Magnitude of the total Force.

$$\vec{F} = \vec{F_{AB}} + \vec{F_{AC}}$$

$$\vec{F} = 20.2\,\vec{i} - 282.9\,\vec{j} + 198\,\vec{k}$$

$$\therefore |\vec{F}| = \sqrt{20.2^2 + (-282.9)^2 + 198^2} = 346 \text{ lb.}$$

End of Lecture 2

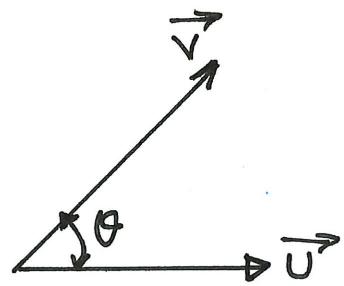
Lecture 3

Dot Product

⇒ It can be used in many applications including determining the angle betⁿ two lines in space.

⇒ It is a salar Product of 2 vectors.

$$\vec{U} \cdot \vec{V} = |\vec{U}||\vec{V}| \cos\theta$$



⇒ If the result is Zero, the two vectors are **Perpendicular**.

*** Properties:-**

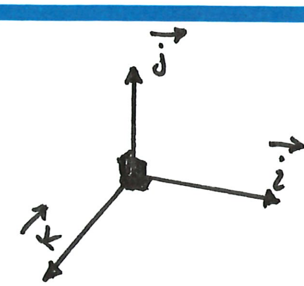
⇒ Commutative : $\vec{U} \cdot \vec{V} = \vec{V} \cdot \vec{U}$.

⇒ Distributive : $\vec{U} \cdot (\vec{V_1} + \vec{V_2}) = \vec{U} \cdot \vec{V_1} + \vec{U} \cdot \vec{V_2}$

Note:- $\vec{i} \cdot \vec{i} = |i||i| \cos 0 = 1$

$\vec{i} \cdot \vec{j} = |i||j| \cos 90 = 0$

$\therefore \vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$, otherwise $= 0$



* In Cartesian Components:-

$$\vec{U} \cdot \vec{V} = (U_x \vec{i} + U_y \vec{j} + U_z \vec{k}) \cdot (V_x \vec{i} + V_y \vec{j} + V_z \vec{k})$$

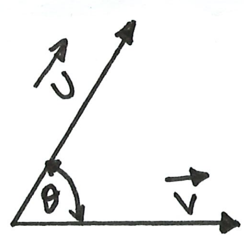
$$= U_x V_x (\vec{i} \cdot \vec{i}) + U_x \cdot V_y (\vec{i} \cdot \vec{j}) + (U_x \cdot U_z)(\vec{i} \cdot \vec{k}) + \cdots$$

$$\vec{U} \cdot \vec{V} = U_x V_x + U_y V_y + U_z V_z .$$

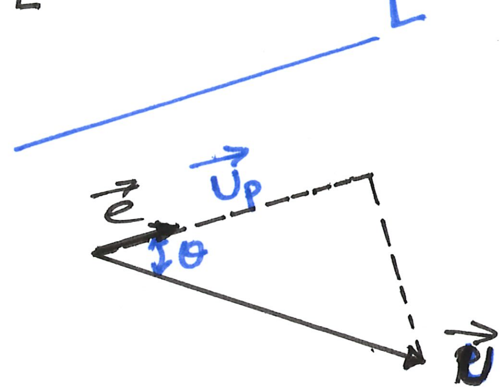
* Applications:-

$\boxed{1}$ ⇒ Angle betⁿ two vectors :-

$$\cos\theta = \frac{\vec{U} \cdot \vec{V}}{|\vec{U}||\vec{V}|} = \frac{U_x V_x + U_y V_y + U_z V_z}{|\vec{U}| \, |\vec{V}|}$$



$\boxed{2}$ ⇒ Projection of a $\underbrace{\text{vector}}_{\vec{U}}$ on a given $\underbrace{\text{line}}_{L}$



$$|\vec{U_p}| = |\vec{U}|\cos\theta$$

if $\vec{e}$ is a unit vector parallel to L

$$\vec{e}\cdot\vec{U} = \underbrace{|\vec{e}|}_{1.0}\,|\vec{U}|\cos\theta = |\vec{U}|\cos\theta$$

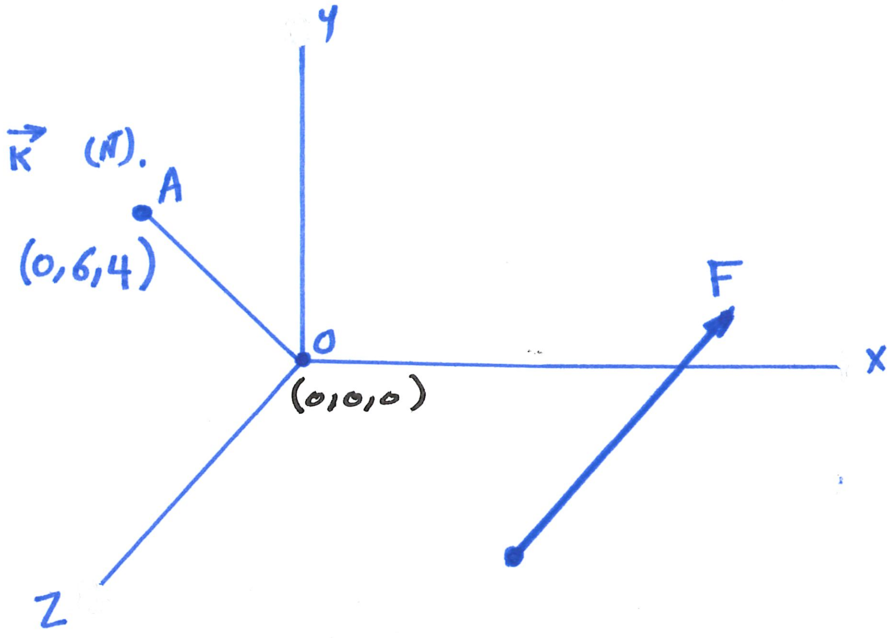
$$\therefore |\vec{U_p}| = \vec{e}\cdot\vec{U}$$

$$\therefore \vec{U_p} = \underbrace{(\vec{e}\cdot\vec{U})}_{\text{magnitude}}\,\underbrace{\vec{e}}_{\text{direction.}}$$

Example : Dot Product.

The force $\vec{F} = 10\vec{i} + 12\vec{j} - 6\vec{K}$ (N).

Determine the vector Components of $\vec{F}$ Parallel and Perpendicular to the line OA.



Sol.

$$\vec{F_p} = (\vec{e} \cdot \vec{F})\,\vec{e}$$ ⇐ General equation

Parallel Component.

Parallel to line (OA in our case)

$$\therefore \vec{F_{//}} = (\vec{e_{OA}} \cdot \vec{F})\,\vec{e_{OA}}$$

⇒ **Calculate $\vec{e_{OA}}$:-**

$$\underset{A-O}{\vec{r_{OA}}} = 6\vec{j} + 4\vec{K} \text{ (m)}, \quad |\vec{r_{OA}}| = \sqrt{6^2+4^2} = 7.21 \text{ (m)}$$

$$\vec{e_{OA}} = \frac{\vec{r_{OA}}}{|\vec{r_{OA}}|} = \frac{1}{7.21}(6\vec{j} + 4\vec{K}) = 0\vec{i} + 0.832\vec{j} + 0.555\vec{K}$$

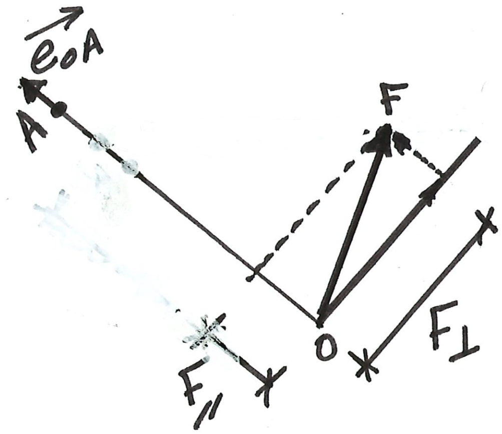
$$\vec{F}_{\parallel} = (\vec{e}_{OA} \cdot \vec{F})\, \vec{e}_{OA}$$

$\Rightarrow$ Calculate $\vec{F}_{\parallel}$ & $F_{\perp}$:-

$$\therefore \vec{F}_{\parallel} = [\underbrace{0 * 10 + 0.832 * 12 + (0.555 * -6)}_{6.65}]\,[0.832\,\vec{j} + 0.555\,\vec{k}]$$

✓✓ $\therefore \vec{F}_{\parallel} = 5.54\,\vec{j} + 3.69\,\vec{k}$ (N)

$$\vec{F} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$



$$\therefore \vec{F}_{\perp} = \vec{F} - \vec{F}_{\parallel}$$

$$= (10\,\vec{i} + 12\,\vec{j} - 6\,\vec{k}) - (0\,\vec{i} + 5.54\,\vec{j} + 3.69\,\vec{k})$$

✓✓ $\therefore F_{\perp} = 10\,\vec{i} + 6.46\,\vec{j} - 9.69\,\vec{k}$