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ENGINEERING ANALYSIS II (EA2)

Lecture # 24: Ch6. Structures in Equilibrium

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Lecture Outlines:

1. Frames and Machines.
2. Determinacy and Stability of Frames.
3. Class Example 1.

References:

1. Bedford, A., & Fowler, W. *Engineering Mechanics: Statics* (5^{th} ed.).
2. Prof. Alarcon's lecture notes.

Frames and Machines

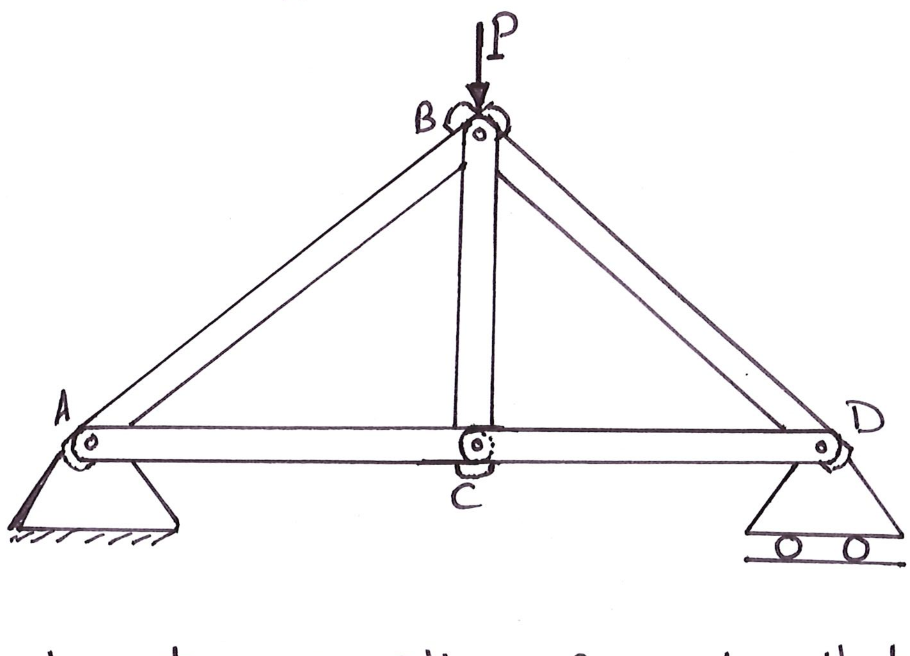
⇒ The main difference between frames & machines is that "frames" are designed to remain "stationary" and support loads. "Machines", on the other hand, are designed to move and apply loads. Yet, in terms of geometry, they are almost unique.

⇒ The structure is considered a "frame" if the assumptions of trusses are no longer available.

Truss Assumptions :-

1) Members are connected together ONLY at their ends.

2) The loads are applied ONLY at the joints.



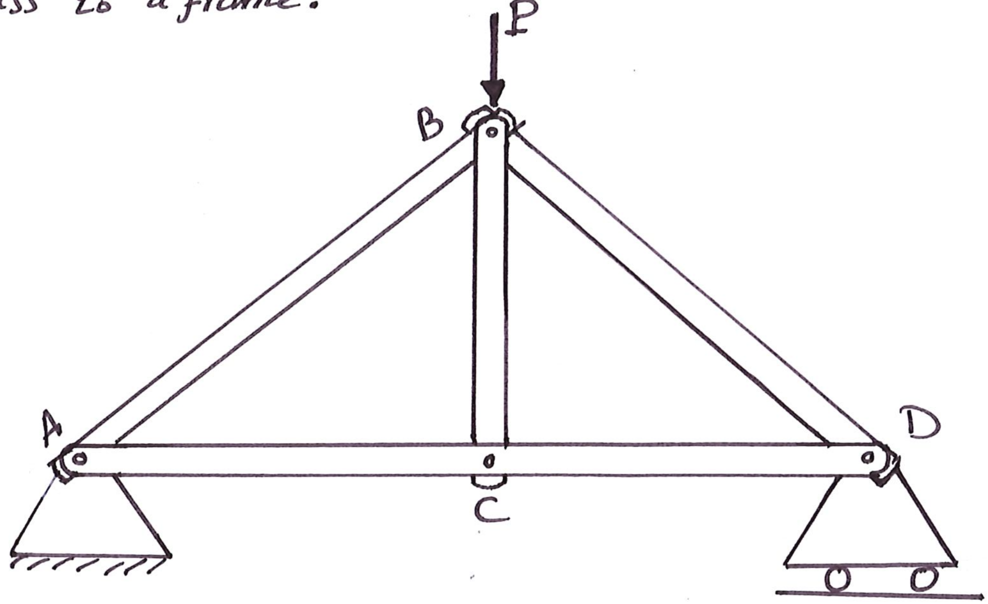
Both the above two assumptions guarantee that each member is a two force member. Thus, only an axial internal force will be experienced by each member "only one force".

Common Assumptions for frame analysis :-

1) Frame members are connected together by frictionless pins.

2) The weight of frame members may be neglected.

⇒ First, we need to know if the problem is a frame or a truss. Sometimes, it is confusing. For example, I will convert the above truss to a frame.



This is a frame since member ACD is one member. If they are seperated to AC & AD, then it will be a truss. The idea is if member ACD is continous, then

Point C is no longer considered a joint, and the force exerted by member BC on member ACD will be at point C. Thus, it will act as a force at point C in member ACD, which means that it is no longer a two force member → So it is a frame.

Determinacy and Stability of Frames

1) Statically determinate : unknowns = Equilibrium Equations.

2) Statically indeterminate : unknowns > Equilibrium Equations.

3) Unstable : unknowns < Equilibrium Equations

In frames: we will separate each member, and apply the equilibrium equations for each member. Thus, the number of equilibrium equations $= \underline{3 \times m}$

"m" is number of frame members

* The number of unknowns = unknowns at joints + unknowns at supports.

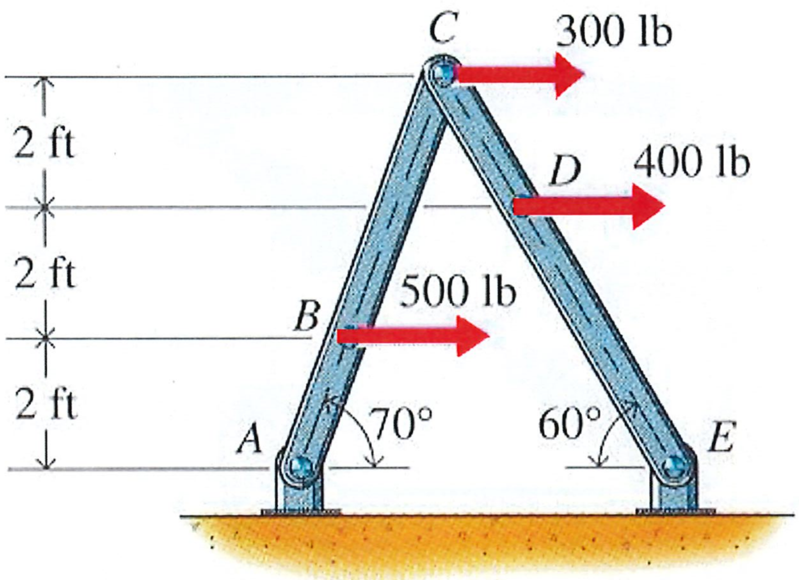
Since each joint has two unknows

$\therefore$ unknowns $= \underline{2j + \text{Support reactions}}$

"j" is number of joints exculding joints of supports.

Class Example 1

A two-bar frame is loaded and supported as shown below. Determine the reactions at supports *A* and *E* and the force exerted on member *ABC* by the pin at *C*. *Then, calculate the resultant force on member ABC at point C.*



Before starting our solution, let's check the determinacy

⇒ Unknowns $= 2j +$ Reactions

$= 2*1 + 4 = 6$ unknowns.

⇒ EQ EQ $= 3m = 3*2 = 6$ equations.

So, it is determinate

Note: All the problems, in this class, will be determinate; Indeterminate structures are not covered in this class.

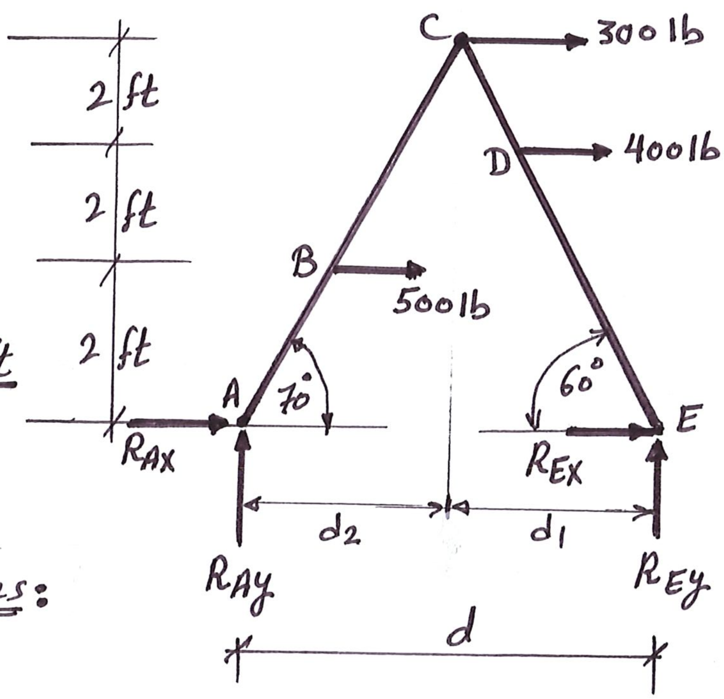
[1] FBD of the whole frame

From geometry,

$\tan 60 = \frac{6}{d_1}$, $\tan 70 = \frac{6}{d_2}$

$\therefore d = d_1 + d_2$

$d = \frac{6}{\tan 60} + \frac{6}{\tan 70} = 5.648\,ft$



[2] Calculate the reactions:

+↻ ⇒ $\Sigma M_A = 0$

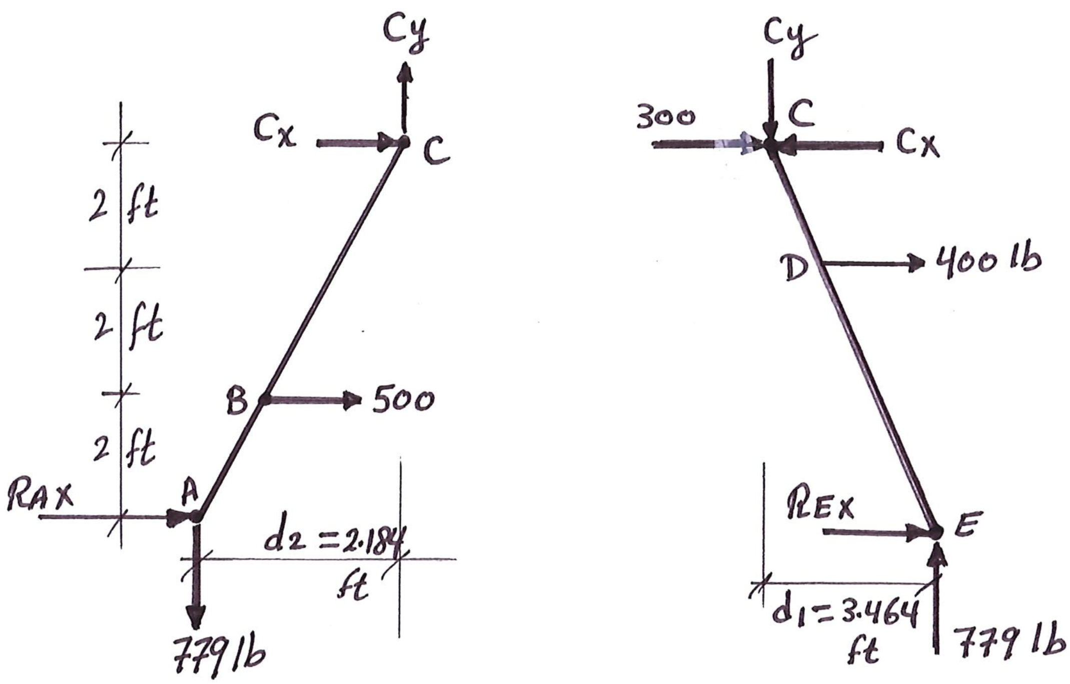
$-500*2 - 400*4 - 300*6 + R_{Ey}*5.648 = 0$

$\boxed{\therefore R_{Ey} = 779\ lb}$

+↑ ⇒ $\Sigma F_y = 0$ $\quad \therefore R_{Ay} + 779 = 0 \Rightarrow \boxed{R_{Ay} = -779\ lb} = 779\ lb \downarrow$

we still have the third equilibrium equation "$\Sigma F_x = 0$", but it will not help here because we have two unknown in x direction (R_{Ax} & R_{Ex}).

3 FBD of each member:-



If there is a force at any joint, we can assign this force in any member after separation. Here, I assigned the 300 lb force at joint C to member CDE.

⇒ From member ABC:-

$+\circlearrowleft \Sigma M_C = 0 \quad \therefore 500 * 4 + R_{Ax} * 6 + 779 * 2.184 = 0$

$$\boxed{\therefore R_{Ax} = -616.9 \text{ lb}}$$

$\rightarrow \Sigma F_x = 0 \quad \therefore R_{Ax} + 500 + C_x = 0$

$\therefore -616.9 + 500 + C_x = 0 \Rightarrow \boxed{C_x = 116.9 \text{ lb}}$

$\rightarrow \Sigma F_y = 0 \quad \therefore -779 + C_y = 0 \Rightarrow \boxed{C_y = 779 \text{ lb}}$

$\Rightarrow$ From member CDE

Here, we can use any equation to calculate R_{Ex}

$\Sigma F_x = 0 \quad \therefore 300 - C_x + 400 + R_{Ex} = 0$

$\therefore 300 - 116.9 + 400 + R_{Ex} = 0$

$\boxed{\therefore R_{Ex} = -583.1 \text{ lb}}$