1/9/2024

ENGINEERING ANALYSIS II (EA2)

Lecture # 04: Vectors

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Lecture Outlines:

1. Cross Product.
2. Example.

References:

1. Bedford, A., & Fowler, W. *Engineering Mechanics: Statics* (5th ed.).
2. Prof. Alarcon's lecture notes.

Cross Product

⇒ It can be used for many applications including calculating moments of forces "very important application".

Definition:-

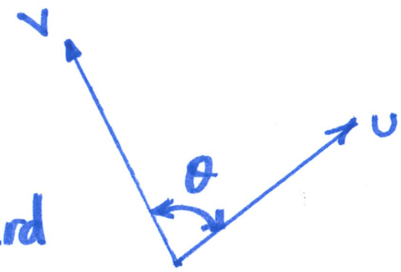
$\vec{U} \times \vec{V} = |\vec{U}|\,|\vec{V}| \sin\theta \, \vec{e}$

← therefore the cross product result is a vector, "Sometimes it is called "**Vector** Product""

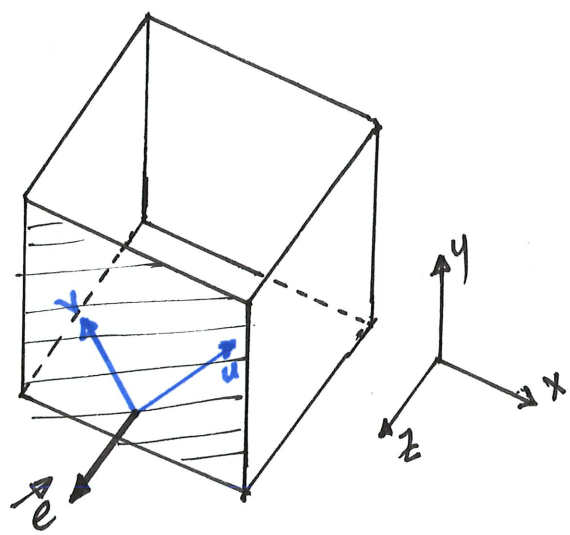
⇒ The product is Perpendicular to U, V Plan.

⇒ The direction $\vec{e}$ is based on "right-hand rule".

Put your right hand fingers in the direction of $\vec{U}$ (the first vector in the cross Product, and bend your hand toward the vector $\vec{V}$, the thumb points to $\vec{e}$.



Ex:- If vectors $\vec{U}$ & $\vec{V}$ are drawn in x-y Plan, $\vec{e}$ vector will Point to +ve z direction.



⇒ If the result of the cross Product is **Zero**, the two vectors are **Parallel**.

Properties:-

⇒ Non Commutative:- $\vec{U} \times \vec{V} \neq \vec{V} \times \vec{U}$ ← due to the right hand rule "will change the vector direction"

⇒ **Distributive:-**

$$\vec{U} \times (\vec{V_1} + \vec{V_2}) = \vec{U} \times \vec{V_1} + \vec{U} \times \vec{V_2}$$

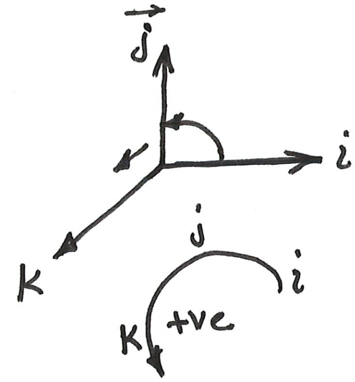
Note:

$$\vec{i} \times \vec{i} = |\vec{i}||\vec{i}| \underbrace{\sin\theta}_{0} \, \vec{e} = 0$$

$$\vec{i} \times \vec{j} = |\vec{i}||\vec{j}| \underbrace{\sin 90}_{1.0} \, \vec{e} = \vec{e}$$

where $\vec{e}$ is the unit vector Perpendicular to $\vec{i}$ and $\vec{j}$

May be $\vec{K}$ or $-\vec{K}$ "depending on the right hand rule"



$\vec{i} \times \vec{j} = \vec{K}$

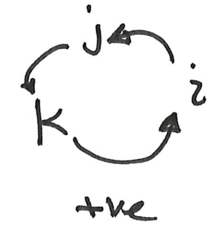
$\vec{j} \times \vec{K} = \vec{i}$

$K \times i = \vec{j}$

$\vec{j} \times \vec{i} = -\vec{K}$

$\vec{K} \times \vec{j} = -\vec{i}$

$\vec{i} \times \vec{K} = -\vec{j}$



* In Cartesian Components :-

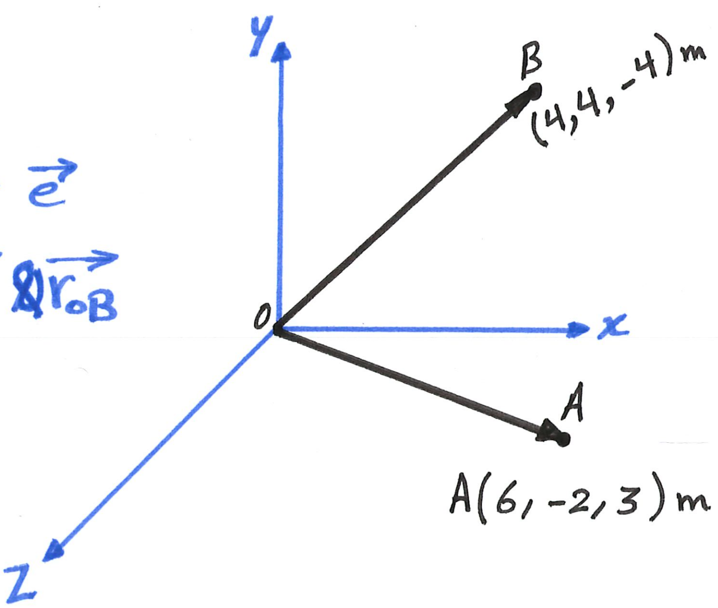
$$\vec{U} \times \vec{V} = \begin{array}{ccc} (+ve) & (-ve) & (+ve) \\ \begin{vmatrix} i & j & K \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix} \end{array}$$

This is the result of the cross Product written as determinant.
"easy to use it in solving the proplems"

Example :-

a) Determine $\vec{r}_{oA} \times \vec{r}_{oB}$

b) Determine the unit vector $\vec{e}$ that is Perpendicular to $\vec{r}_{oA}$ & $\vec{r}_{oB}$



a) Sol.

$$\vec{r}_{oA} \times \vec{r}_{oB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ 6 & -2 & 3 \\ 4 & 4 & -4 \end{vmatrix}$$

$$= \vec{i}\left[(-2)(-4) - (3)(4)\right] - \vec{j}\left[(6)(-4) - (3)(4)\right] + \vec{K}\left[(6)(4) - (-2)(4)\right]$$

$\therefore \vec{r}_{oA} \times \vec{r}_{oB} = -4\vec{i} + 36\vec{j} + 32\vec{K}$ (m^2) $\Leftarrow$ Perpendicular to $\vec{r}_{oA}$ & $\vec{r}_{oB}$

b) Let's name $\vec{r_{OA}} \times \vec{r_{OB}} = \vec{R}$

$\therefore \vec{R} = |R|\, e_{\perp}$ because any vector can be written as a multiplication of its magnitude times the unit vector.

because $\vec{R}$ is $\perp$ to $\vec{r_{OA}}$ & $\vec{r_{OB}}$

$$|R| = \sqrt{(-4)^2 + 36^2 + 32^2} = \underline{48.3} \quad m^2$$

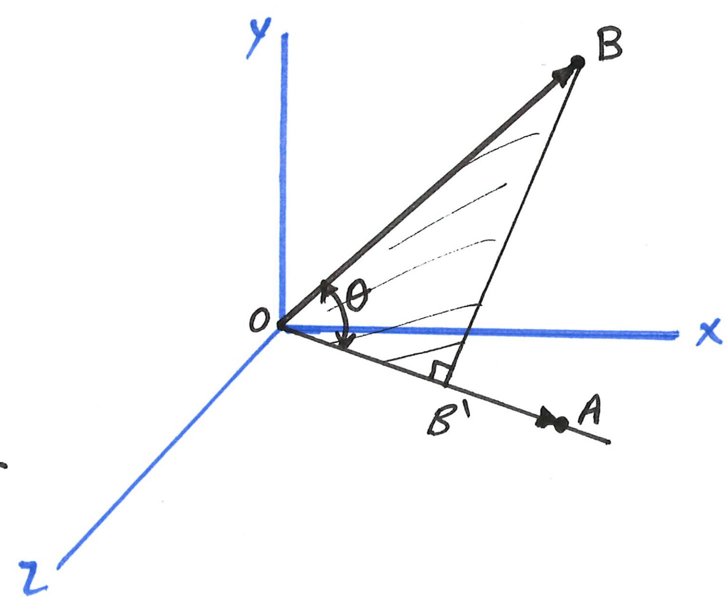
$$\therefore e_{\perp} = \frac{1}{48.3}\left(-4\vec{i} + 36\vec{j} + 32\vec{k}\right)$$

$$\therefore e_{\perp} = -0.083\vec{i} + 0.75\vec{j} + 0.66\vec{k}$$

Example :-

Referring to the same figure in the Previous example: use the cross product to determine the shortest straight line from B to OA.

Sol.



⇒ The shortest line from B to line OA can be calculated when it is ⊥ to line OA.

⇒ First Calculate angle θ using Cross Product definition.

⇒ then, use triangle OBB' to Calculate the length BB'

$$\sin\theta = \frac{|\vec{r_{oA}} \times \vec{r_{oB}}|}{|\vec{r_{oA}}|\,|\vec{r_{oB}}|} \quad (1)$$

we used here magnitude to get rid of $\vec{e}$.

$$\boxed{\vec{U} \times \vec{V} = |\vec{U}|\,|\vec{V}| \sin\theta \, \vec{e}}$$

This is the original form

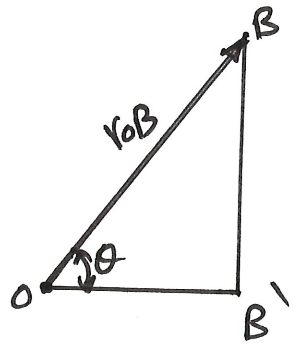
$$\therefore |\vec{r_{oA}}| = \sqrt{6^2 + (-2)^2 + 3^2} = 7 \text{ m}$$

$$|\vec{r_{oB}}| = \sqrt{4^2 + 4^2 + (-4)^2} = 6.93 \text{ m}$$

Sub in (1),

$$\therefore \sin\theta = \frac{48.3}{7 * 6.93} = 0.996$$

$$\sin\theta = \frac{BB'}{r_{oB}} = 0.996 = \frac{BB'}{6.93}$$



$$\therefore \underline{BB' = 6.9\text{m}}$$