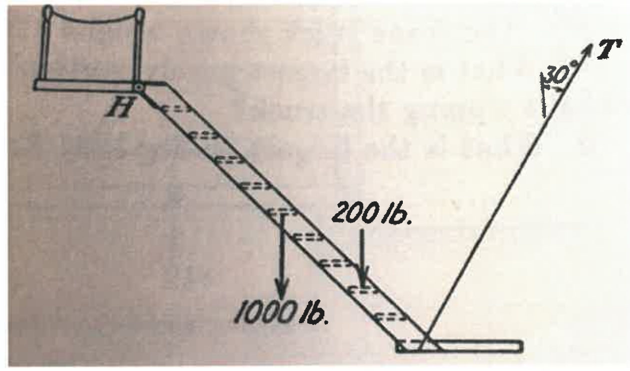
Problem 1 (10 points)

A ship's ladder is supported at the top by a hinge *H* (consisting of a horizontal and vertical reaction) and at the bottom by a rope with tension *T* pulling at 30 degree with respect to the vertical. The weight of the ladder is 1,000-lb and is considered to be concentrated at the center. A man weighting 200-lb stands at one-fifth distance from the bottom. The ladder itself is inclined at 45 degree.



a) Draw the Free Body Diagram of the problem (5 points)

b) Knowing that a hinge is a point where the sum of moments about it equals zero and that H is a hinge, determine the pull in the rope T. (5 points)

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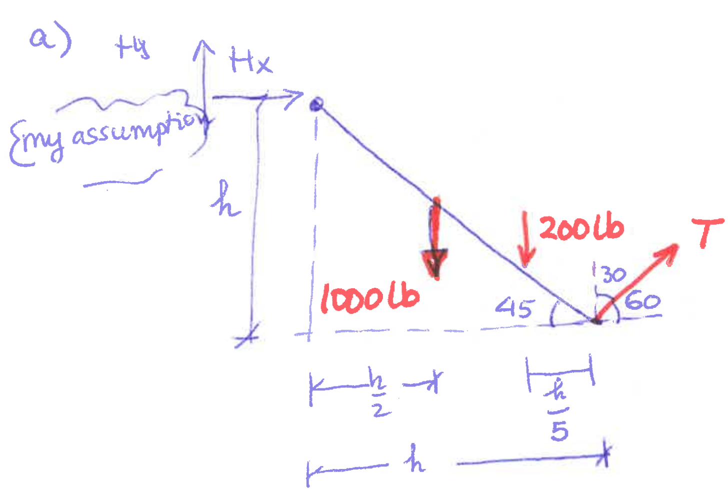
a)



T, 200, 100 each 1 point
angles 1 point
reactions 1 point

b) $\circlearrowleft + \sum M_H = 0$

$$-1000\frac{\cancel{h}}{2} - 200\left(\frac{4}{5}\cancel{h}\right) + \overset{T_x}{T\cos 60\,(\cancel{h})} + \overset{T_y}{T\sin 60\,\cancel{h}}$$

$$500\cancel{h} + 160\cancel{h} = T(\cos 60 + \sin 60)\cancel{h}$$

$$T = \frac{660}{0.5 + 0.866} = 483.15 \text{ lb}$$

$\boxed{T = 483 \text{ lb}}$

Additional Work Area:

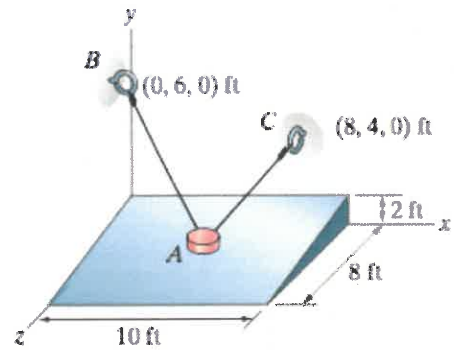
Problem 2 (10 points)

The 10 lb disk A is supported by the supported by the smooth inclined surface and the strings AB and AC. The disk is located at coordinates (5,1,4) ft.

a) Draw the Free Body Diagram of the problem (2 points)

b) Write the equilibrium equation in vector form. (no need to calculate the tension vectors) (1 point)

c) Determine the unit vector in the normal direction to the surface. (7 points)



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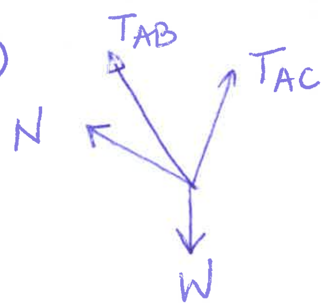
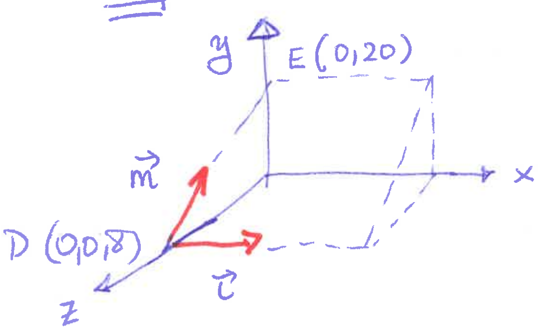
a)



b) $\sum \vec{F} = 0$

$\vec{T_{AB}} + \vec{T_{AC}} + \vec{N} + \vec{W} = 0$

c) The normal to a plane can be calculated knowing 2 any vectors on the plane. _ Easiest use



$\vec{m} = \vec{r}_{DE} = 2j - 8\vec{k}$

$\uparrow$ D-E

$\vec{i} = (1,0,0)$

$$\vec{v_n} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 2 & -8 \end{vmatrix} = 8\vec{j} + 2\vec{k}$$

$$|\vec{v_n}| = \sqrt{8^2+2^2} = 8.246$$

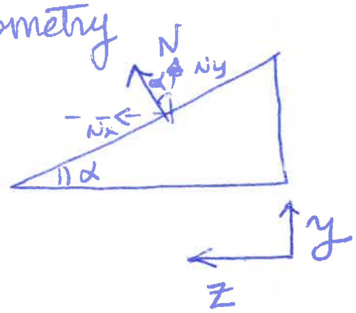
$$\vec{e_n} = \frac{\vec{v_n}}{|\vec{v_n}|} = \frac{8}{8.246}\vec{j} + \frac{2}{8.246}\vec{k} = 0.97\vec{j} + 0.243\vec{k}$$

$$\boxed{\vec{e_n} = 0.97\vec{j} + 0.243\vec{k}}$$

Additional Work Area:

* Taking other vectors the solution is the same for example $(10,0,0)$ $(0,1,-4)$.

* Using geometry



we can also define

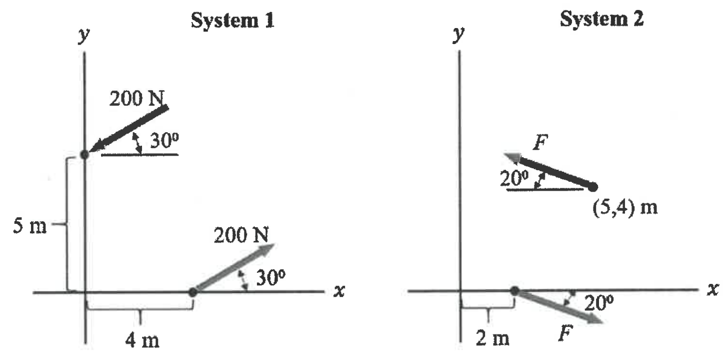
$\vec{N} = N\cos\alpha\,\vec{j} + N\sin\alpha\,\vec{k}$

$\alpha = \arctan \frac{2}{8} = 14 \rightarrow \alpha = 14°$ $\rightarrow \cos\alpha = 0.9701$, $\rightarrow \sin\alpha = 0.243$

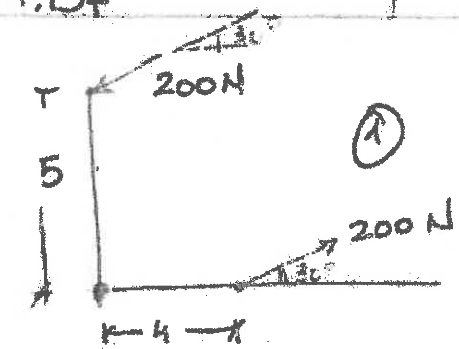
same as before! :)

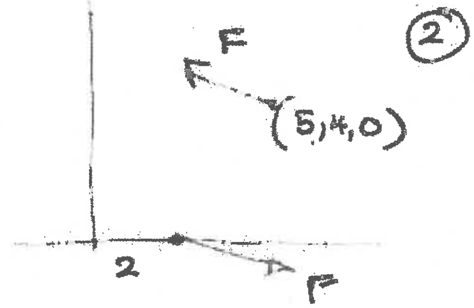
Problem 3 (10 points)

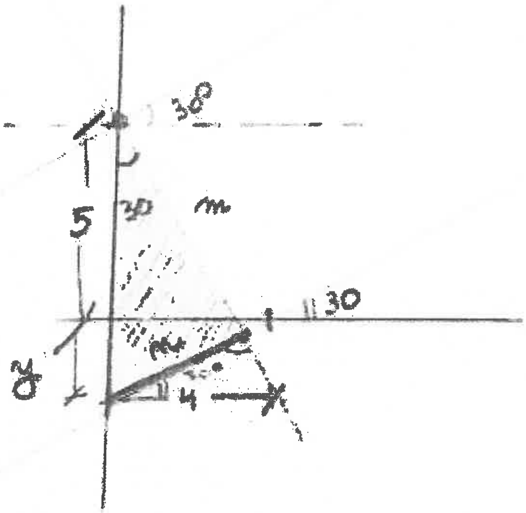
System 1 and 2 are equivalent. Determine force F.



Use components







$n = 4\cos 30 = 3.464$

$M^2 + m^2 = (5+y)^2$

$m = (5+y)\cos 30$

$n^2 + \left[(5+y)\cos 30\right]^2 = (5+y)^2$

$n^2 = (5+y)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 (5+y)^2$

$a^2\left[1 - \frac{3}{4}\right] = a^2\left[\frac{1}{4}\right] = \frac{a^2}{4}$

$n^2 = \frac{a^2}{4} \rightarrow n = \frac{a}{2} = \frac{5+y}{2}$

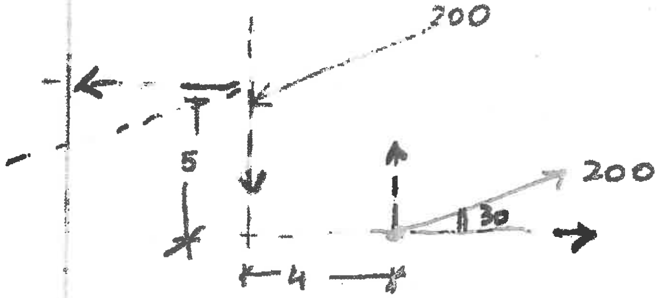
$\frac{5+y}{2} = 3.464 \rightarrow 5+y = 6.93 \rightarrow y = 1.93$

TOTAL = 5+1.93 = 6.93

$m = (5+y)\cos 30 = 6.93\,\frac{\sqrt{3}}{2} = 6.$

$M = F * d \sim 200 \times 6 = \underline{1200}\ N.m$

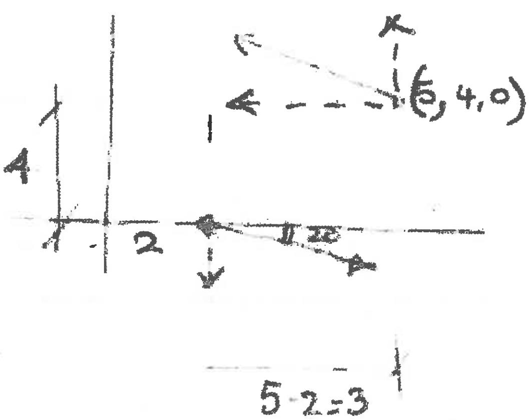
It is easier just to do it in components

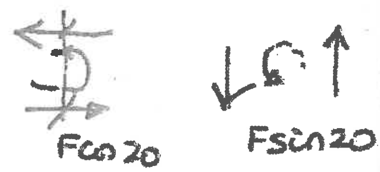


$M = 200 \cos 30 \times 5 + 200 \sin 30 \times 4$

$866 + 400 = \underline{\underline{1266}}$ N.m

System 2 :





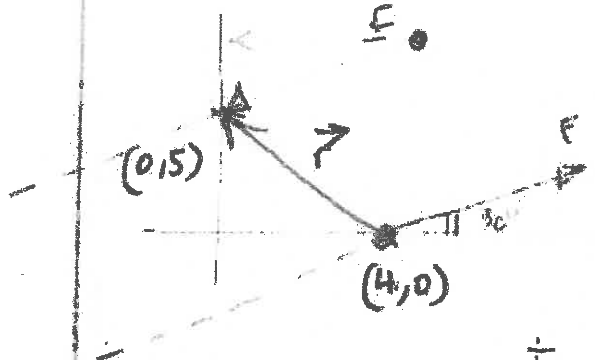
$F\cos 20 \; (4) + (F \sin 20)(3) = 1266$ ← from before

$F(3.758 + 1.026) = 1266$

$F = 264.58$

$\boxed{F \approx 265 \text{ N}}$

c) can also use the $\vec{r}$ between



$$\vec{M} = \vec{r} \times (-\vec{F}) = \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ \end{array} \begin{vmatrix} -4 & -5 & 0 \\ -200\cos & -200\sin & 0 \end{vmatrix} = -800\sin 30 - 1000\cos 30$$

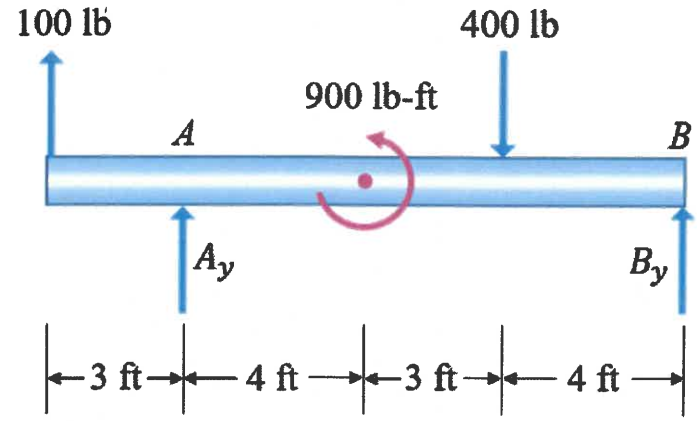
$$= 1266 \text{ N}\cdot\text{m}$$

as we saw before

Problem 4 (10 points)

Determine the vertical reactions A_y and B_y and clearly indicate their directions if:

(a) The sum of vertical forces equals 0;
(b) The sum of moments about A equals 0.



$\uparrow \Sigma F_y = 0$ $\qquad 100 + A_y - 400 + B_y = 0$ $\qquad$ (1)

$\circlearrowleft^{+} \Sigma M_A = 0$ $\qquad 900 - 400(4+3) + B_y(4+4+3) - 100\times 3 = 0$

(2)

$900 - 2800 + B_y\, 11 - 300 = 0$

$11\, B_y = 2200$

$\boxed{B_y = 200 \text{ lb } (\uparrow)}$

Into (1) $\qquad A_y = 400 - B_y - 100 = 300 - B_y = 300 - 200$

$\boxed{A_y = 100 \text{ lb } (\uparrow)}$