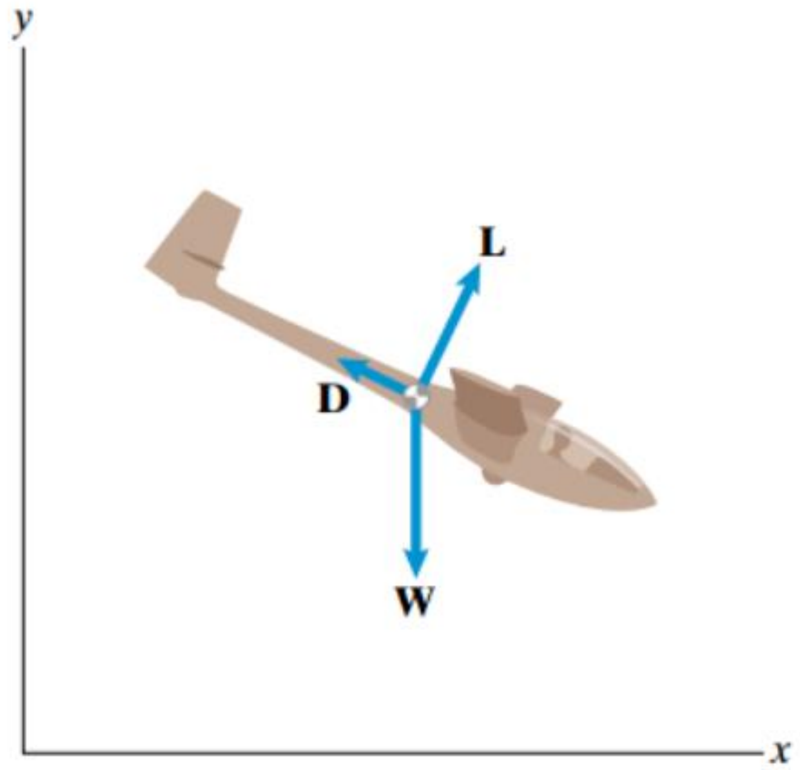
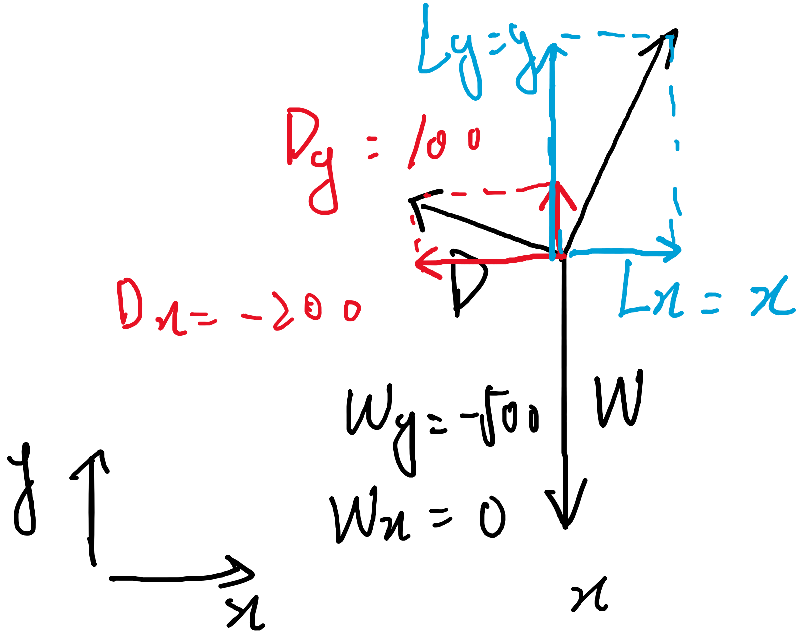
- Vectors

The forces acting on the sailplane are its weight $\mathbf{W} = -500\mathbf{j}$ (lb), the drag $\mathbf{D} = -200\mathbf{i} + 100\mathbf{j}$ (lb), and the lift $\mathbf{L}$. The sum of the forces $\mathbf{W} + \mathbf{L} + \mathbf{D} = \mathbf{0}$. Determine the components and the magnitude of $\mathbf{L}$.





$$W = -500\vec{j}$$

$$D = -200\vec{i} + 100\vec{j}$$

i is unit vector in x direction

j is unit vector in y direction

known: $W + D + L = 0$

set $L = x\vec{i} + y\vec{j}$

$$\Sigma F_x = 0 \Rightarrow L_x + D_x + W_x = 0$$

$$x + (-200) + 0 = 0 \Rightarrow x = 200$$

$$\Sigma F_y = 0 \Rightarrow L_y + D_y + W_y = 0$$

$$y + 100 - 500 = 0 \Rightarrow y = 400$$

$$L = 200\vec{i} + 400\vec{j} \text{ (lb)}$$

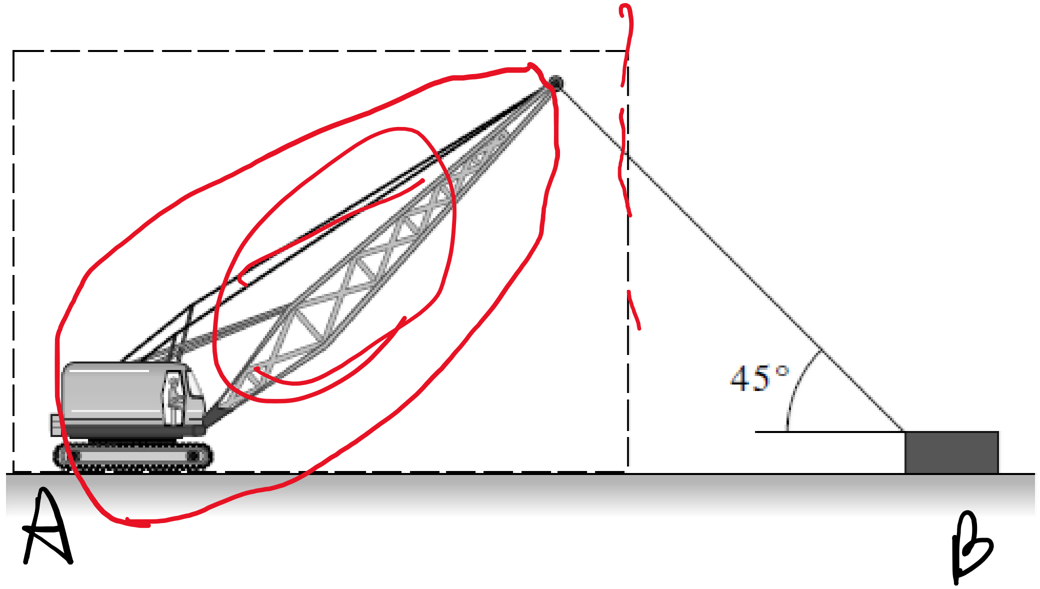
$$|L| = \sqrt{200^2 + 400^2}$$

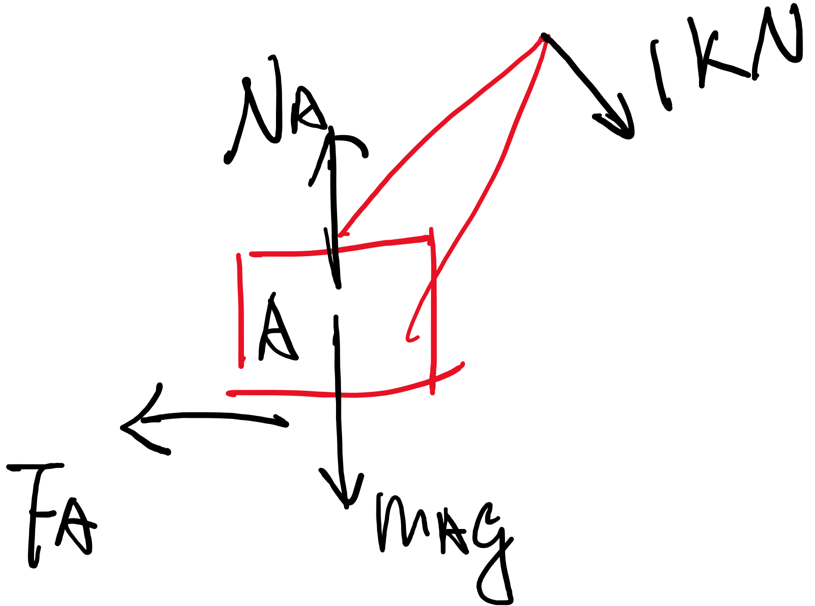
$$= 447 \text{ (lb)}$$

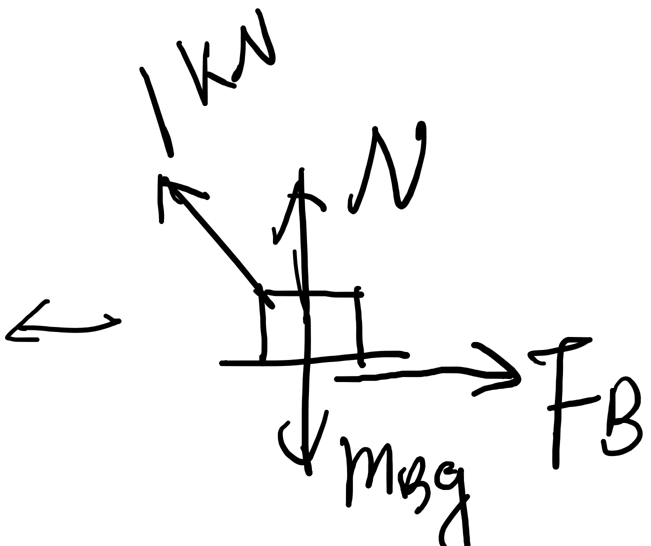
- # 2D Force

The mass of the crane is 20,000 kg. The crane's cable is attached to a caisson whose mass is 400 kg. The tension in the cable is 1 kN.

(a) Determine the magnitudes of the normal and friction forces exerted on the crane by the level ground.
(b) Determine the magnitudes of the normal and friction forces exerted on the caisson by the level ground.



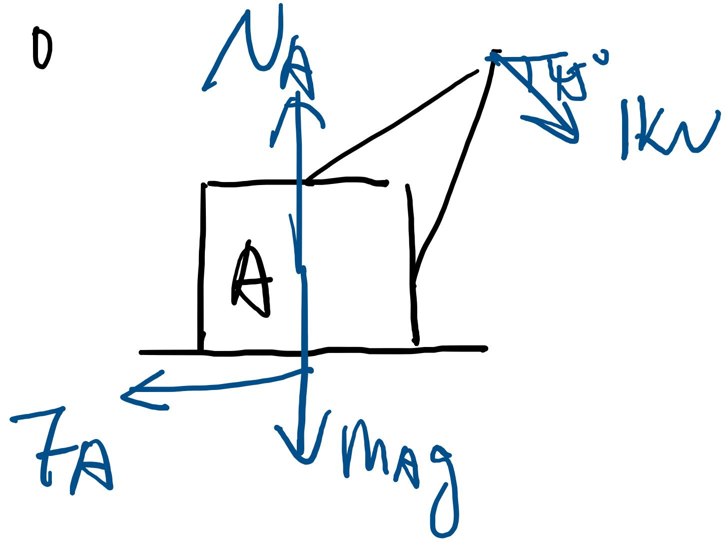




(a)

$$\sum F_{y_A}: \uparrow(+) \quad N_A - m_A g - 1\sin 45^\circ = 0$$

$$\sum F_{x_A}: \xrightarrow{(+)} \quad 1\cos 45^\circ - F_A = 0$$



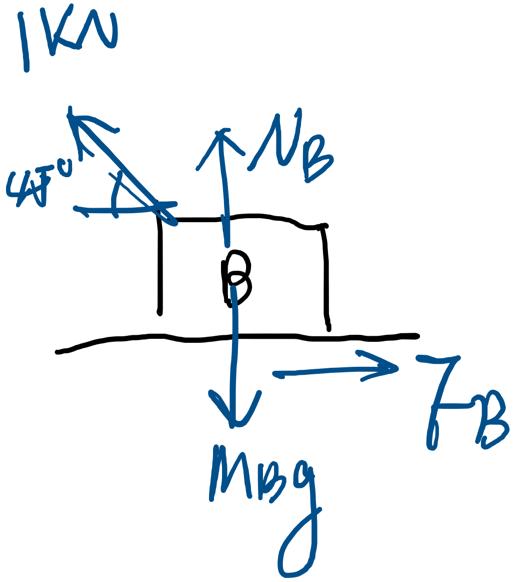
$$N_A = 186.7\,\mathrm{kN}$$

$$F_A = 0.707\,\mathrm{kN}$$

(b)

$$\sum F_{y_B} \uparrow(+): \quad N_B - m_B g + 1\sin 45^\circ = 0$$

$$\sum F_{x_B} \xrightarrow{(+)}: \quad F_B - 1\cos 45^\circ = 0$$



$$N_B = 3.22\,\mathrm{kN}$$

$$F_B = 0.707\,\mathrm{kN}$$

- Cross Product

  - Direction of the cross product

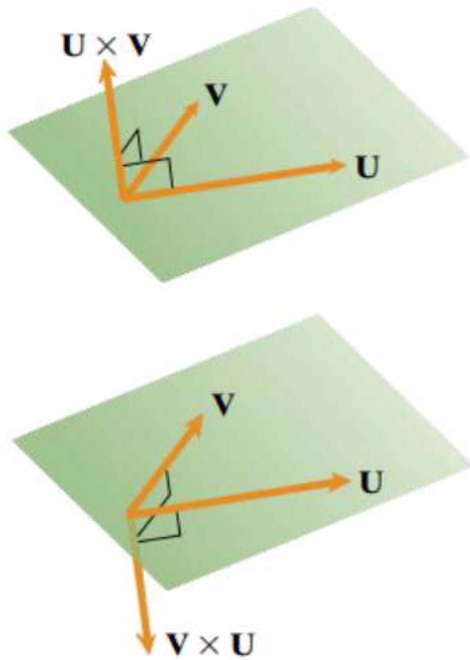


Figure 2.25
Directions of **U** × **V** and **V** × **U**.

  - Component of the cross product

$$|\vec{U} \times \vec{V}| = |\vec{U}|\,|\vec{V}| \sin\theta$$

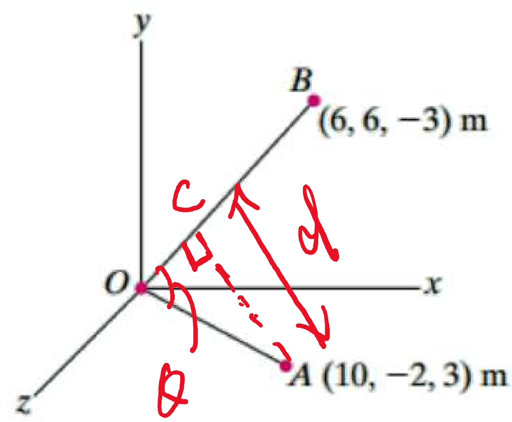
$$\rightarrow \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix} = \mathbf{i}\begin{vmatrix} U_y & U_z \\ V_y & V_z \end{vmatrix} - \mathbf{j}\begin{vmatrix} U_x & U_z \\ V_x & V_z \end{vmatrix} + \mathbf{k}\begin{vmatrix} U_x & U_y \\ V_x & V_y \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ U_x & U_y & U_z \\ V_x & V_y & V_z \end{vmatrix} = (U_yV_z - U_zV_y)\mathbf{i} - (U_xV_z - U_zV_x)\mathbf{j} + (U_xV_y - U_yV_x)\mathbf{k}$$

$$\sin\theta = \frac{|\vec{u} \times \vec{V}|}{|\vec{u}|\,|\vec{V}|}$$

- Cross Product
  - Problem 1

Consider the straight lines *OA* and *OB*.
(a) Determine the components of a unit vector that is perpendicular to both *OA* and *OB*.
(b) What is the minimum distance from point *A* to the line *OB*?



$$\vec{r}_{OA} = 10\vec{i} - 2\vec{j} + 3\vec{k} \quad (m)$$

$$\vec{r}_{OB} = 6\vec{i} + 6\vec{j} - 3\vec{k} \quad (m)$$

$$d = |\vec{r}_{OA}| \sin\theta$$

$$\vec{r}_{OA} = 10\vec{i} - 2\vec{j} + 3\vec{k}$$

$$\vec{r}_{OB} = 6\vec{i} + 6\vec{j} - 3\vec{k}$$

$$\vec{r}_{OA} \times \vec{r}_{OB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & -2 & 3 \\ 6 & 6 & -3 \end{vmatrix} = \vec{i}\begin{vmatrix} -2 & 3 \\ 6 & -3 \end{vmatrix} - \vec{j}\begin{vmatrix} 10 & 3 \\ 6 & -3 \end{vmatrix} + \vec{k}\begin{vmatrix} 10 & -2 \\ 6 & 6 \end{vmatrix}$$

$$= \vec{i}\left[(-2)\times(-3) - 6\times 3\right] - \vec{j}\left[10\times(-3) - 6\times 3\right] + \vec{k}\left[10\times 6 - (-2)\times 6\right]$$

$$= -12\vec{i} + 48\vec{j} + 72\vec{k}$$

$$\vec{e} = \frac{\vec{r}_{OA} \times \vec{r}_{OB}}{|\vec{r}_{OA} \times \vec{r}_{OB}|} = \frac{-12\vec{i} + 48\vec{j} + 72\vec{k}}{\sqrt{(-12)^2 + 48^2 + 72^2}}$$

$$= -0.137\vec{i} + 0.549\vec{j} + 0.824\vec{k}$$

(b) $d = |\vec{r}_{OA}| \, \mathrm{cm}\,\theta$

$$\sin\theta = \frac{|\vec{r}_{OA} \times \vec{r}_{OB}|}{|\vec{r}_{OA}| \, |\vec{r}_{OB}|}$$

$$d = \cancel{|\vec{r}_{OA}|} \cdot \frac{|\vec{r}_{OA} \times \vec{r}_{OB}|}{\cancel{|\vec{r}_{OA}|} \, |\vec{r}_{OB}|} = \frac{|\vec{r}_{OA} \times \vec{r}_{OB}|}{|\vec{r}_{OB}|} = \frac{\sqrt{(-12)^2 + 48^2 + 72^2}}{\sqrt{6^2 + 6^2 + (-3)^2}}$$

$= 9.71\,m$