EA 2 Design Project 2

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Contents

Problem 1

1a

Given: Robot arm with links of lengths L_1 and L_2, and at angles θ_1 and θ_2 to the x-axis and to the line extending from the first link, respectively. The setup is illustrated in Figure 1. Each link can rotate 360° independently without affecting the other link's rotation.

Find: The expression for the x and y coordinates of the end effector in terms of L_1, L_2, θ_1 and θ_2.

Solution: The required equations are:

$$\begin{aligned} x &= L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \\ y &= L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) \end{aligned} \tag{1}$$

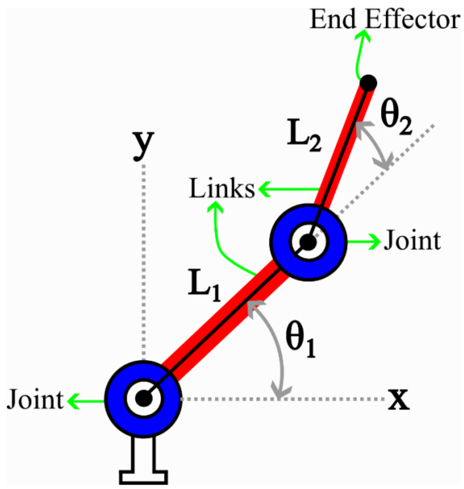

Figure 1: Robot arm with two links

Analysis: The x- and y-components are the sums of projections of the links on the x- and y-axes respectively.

1b

Given: L_1 and L_2 are 4 ft and 2 ft respectively. The robot arm is fully extended.

Plot: Possible positions $(x,\, y)$ of the end effector.

Solution: The coordinates of the end effector are:

$$\begin{aligned} x &= (6 \text{ ft}) \cos{(\theta_1)} \\ y &= (6 \text{ ft}) \sin{(\theta_1)} \end{aligned} \tag{2}$$

The possible positions $(x,\, y)$ of the end effector are plotted in blue in Figure 2.

Analysis: As we can see, the robot arm sweeps a circle of radius 6 ft about the center. This makes intuitive sense: when the two links are collinear, the robot arm will behave as if it has a single link at angle θ_1 to the x-axis, and sweep a circle of radius $L_1 + L_2$.

1c

Given: Only the smaller link L_2 moves. The lengths of the links are as described in Problem 1b.

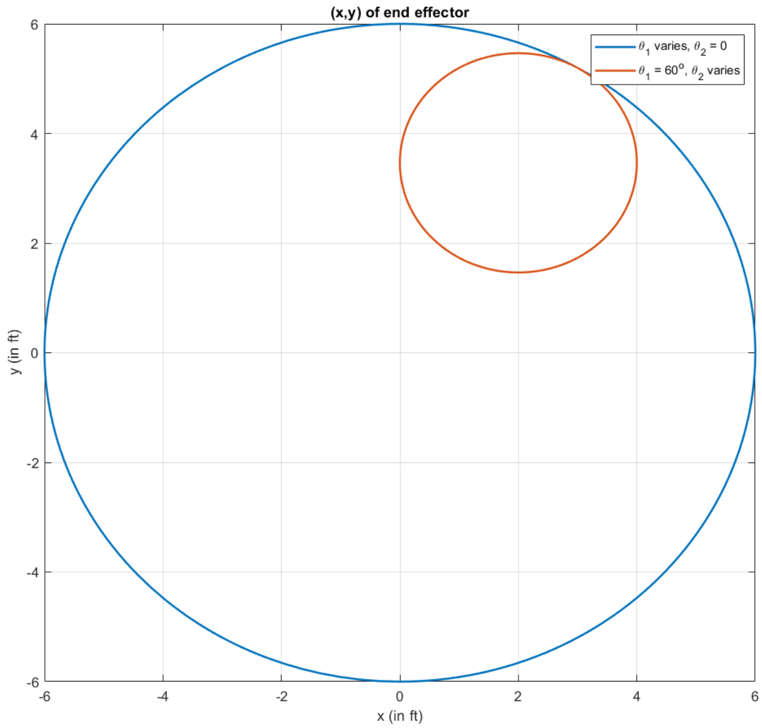

Figure 2: Plot of the possible positions of the end effector under conditions described in Problems 1b and 1c

Plot: Possible positions (x, y) of the end effector.

Solution: We pick $\theta_1 = 60°$. The coordinates of the end effector are:

$$\begin{aligned} x &= 2 \text{ ft} + (2 \text{ ft}) \cos(60° + \theta_2) \\ y &= 3.46 \text{ ft} + (2 \text{ ft}) \sin(60° + \theta_2) \end{aligned} \tag{3}$$

The possible positions (x, y) of the end effector are plotted in red in Figure 2.

Analysis: The robot arm sweeps a circle of radius 2 ft about the joint between the first link and the second link, located at roughly (2 ft, 3.46 ft). This makes intuitive sense as only the second link is moving about its joint to the first link. Since it has a fixed length of 2 ft and θ_2 varies from 0° to 360°, the locus of the points traversed by the end effector describes a circle.

1d

Given: The lengths of the links are as described in Problem 1b.

Plot: All possible positions (x, y) the end effector can reach.

Solution: The coordinates of the end effector are:

$$\begin{aligned} x &= (4\text{ ft})\cos(\theta_1) + (2\text{ ft})\cos(\theta_1+\theta_2) \\ y &= (4\text{ ft})\sin(\theta_1) + (2\text{ ft})\sin(\theta_1+\theta_2) \end{aligned} \tag{4}$$

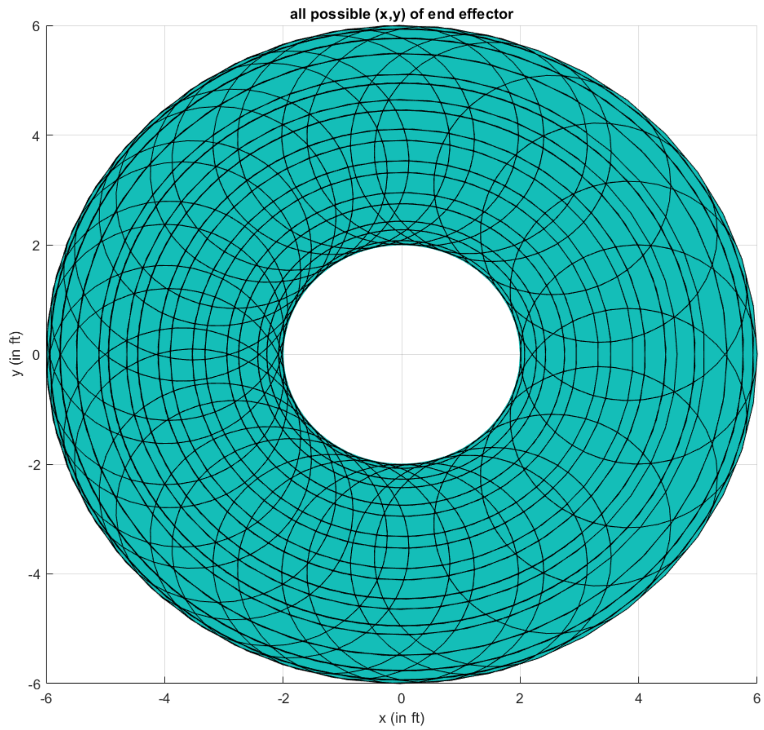

Figure 3: Plot of all points (x, y) reachable by the end effector

The collection of all points (x, y) possibly reachable by the end effector is plotted in Figure 3.

Analysis: The end effector can reach all points in an area between two circles – a large circle of radius 6 ft, and a smaller circle of radius 2 ft. The area's inner boundary is formed of points reached when $\theta_2 = 180°$ and the outer boundary is formed of points reached when $\theta_2 = 0°$.

Theory Manual

Problem 1a

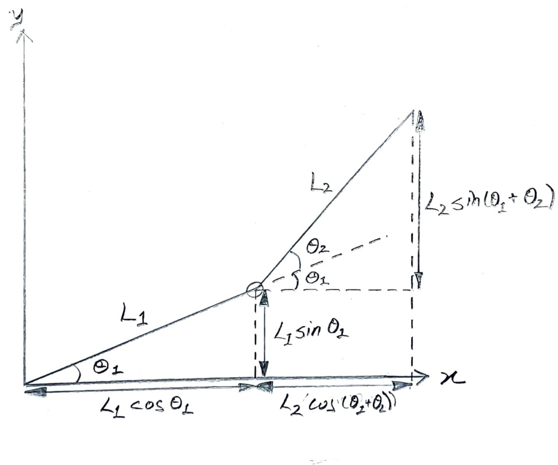

Figure 4: x- and y-coordinates of the end effector

As can be seen in Figure 4, the end effector is at a distance of $L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$ along the x-axis and $L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$ along the y-axis.

From this, we can infer the x- and y-coordinates of the end effector are:

$$x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2)$$
$$y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$$

Problem 1b

θ_2 is the angle made by the second link with the line obtained by extending the first link. So, when the two links are collinear, $\theta_2 = 0$. Referring to equations 1 derived in Section 1a, we can write that the coordinates of the end effector are:

$$
\begin{aligned}
x &= L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + 0) \\
&= L_1 \cos(\theta_1) + L_2 \cos(\theta_1) \\
&= (L_1 + L_2) \cos(\theta_1) \\
y &= L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + 0) \\
&= L_1 \sin(\theta_1) + L_2 \sin(\theta_1) \\
&= (L_1 + L_2) \sin(\theta_1)
\end{aligned}
$$

It is given that $L_1 = 4$ ft and $L_2 = 2$ ft. So, the coordinates of the end effector are:

$$
\begin{aligned}
x &= (6 \text{ ft}) \cos(\theta_1) \\
y &= (6 \text{ ft}) \sin(\theta_1)
\end{aligned}
$$

Problem 1c

We have picked a constant $\theta_1 = 60°$. Using equations 1 from Section 1a, we can write that the coordinates of the end effector are:

$$
\begin{aligned}
x &= L_1 \cos(60°) + L_2 \cos(60° + \theta_2) \\
&= \frac{1}{2} L_1 + L_2 \cos(60° + \theta_2) \\
y &= L_1 \sin(60°) + L_2 \sin(60° + \theta_2) \\
&= \frac{\sqrt{3}}{2} L_1 + L_2 \sin(60° + \theta_2)
\end{aligned}
$$

It is given that $L_1 = 4$ ft and $L_2 = 2$ ft. So, the coordinates of the end effector are:

$$
\begin{aligned}
x &= 2 \text{ ft} + (2 \text{ ft}) \cos(60° + \theta_2) \\
y &= 3.46 \text{ ft} + (2 \text{ ft}) \sin(60° + \theta_2)
\end{aligned}
$$

Problem 1d

It is given that $L_1 = 4$ ft and $L_2 = 2$ ft. Using equations 1 from Section 1a, we can write that the coordinates of the end effector are:

$$
\begin{aligned}
x &= (4 \text{ ft}) \cos(\theta_1) + (2 \text{ ft}) \cos(\theta_1 + \theta_2) \\
y &= (4 \text{ ft}) \sin(\theta_1) + (2 \text{ ft}) \sin(\theta_1 + \theta_2)
\end{aligned}
$$

Programmer Manual

Problems 1b & 1c

The variables used in the program are tabulated below:

Variable	Description
`theta`	Parameter used as angle θ_1 in Problem 1b and as angle θ_2 in Problem 1c. Its domain is specified in the `fplot()` function as $[0°, 360°]$.
`x1b`	The parametric function $x(\theta_1) = (6\text{ ft})\cos(\theta_1)$ used in Problem 1b.
`y1b`	The parametric function $y(\theta_1) = (6\text{ ft})\sin(\theta_1)$ used in Problem 1b.
`x1c`	The parametric function $x(\theta_2) = (4\text{ ft})\cos(60°) + (2\text{ ft})\cos(60° + \theta_2)$ used in Problem 1c.
`y1c`	The parametric function $y(\theta_2) = (4\text{ ft})\sin(60°) + (2\text{ ft})\sin(60° + \theta_2)$ used in Problem 1c.

Problem 1d

The variables used in the program are tabulated below:

Variable	Description
`theta1`	The parameter θ_1.
`theta2`	The parameter θ_2.
`x`	The parametric function $x(\theta_1, \theta_2) = (4\text{ ft})\cos(\theta_1) + (2\text{ ft})\cos(\theta_1 + \theta_2)$, representing the x-coordinate of the end effector.
`y`	The parametric function $y(\theta_1, \theta_2) = (4\text{ ft})\sin(\theta_1) + (2\text{ ft})\sin(\theta_1 + \theta_2)$, representing the y-coordinate of the end effector.

Appendices

Code for Problems 1b & 1c

```
%% Problems 1b and 1c

% Setting up parameter
syms theta;

% Initializing figure
figure('Position',[12 12 900 900]);

%% Problem 1b: full extension
% Setting up parametric functions x(theta) and y(theta)
% Here, they are x(theta1) and y(theta1)
% theta2 is a constant: 0 deg
x1b = 6*cos(theta);
y1b = 6*sin(theta);

% Creating plot
fplot(x1b, y1b, [0,2*pi], 'LineWidth', 1.5);
grid on;

% Titling and labeling axes
title('(x,y) of end effector');
xlabel('x (in ft)');
ylabel('y (in ft)');

%% Problem 1c: only smaller link moves
% Plotting on the same figure used in 1b
hold on;

% Setting up parametric functions x(theta) and y(theta)
% Here, they are x(theta2) and y(theta2)
% theta1 is a constant: 60 deg
x1c = 4*cos(pi/3) + 2*cos(theta);
y1c = 4*sin(pi/3) + 2*sin(theta);

% Creating plot
fplot(x1c, y1c, [0,2*pi], 'LineWidth', 1.5);

% Legend to distinguish 1b and 1c
legend('\theta_{1} varies, \theta_{2} = 0', ...
    '\theta_{1} = 60^{o}, \theta_{2} varies');
hold off;
```

Code for Problem 1d

```
%% Problem 1d

% Setting up parameters
syms theta1;
syms theta2;

% Setting up parametric functions x(theta1, theta2) and y(theta1,
    theta2)
x = 4*cos(theta1) + 2*cos(theta1 + theta2);
y = 4*sin(theta1) + 2*sin(theta1 + theta2);

% Initializing figure
figure('Position',[12 12 900 900]);
grid on;

% Creating plot
fsurf(x, y, 0);

% Viewing in 2D
view(2);

% Titling and labeling axes
title('all possible (x,y) of end effector');
xlabel('x (in ft)');
ylabel('y (in ft)');
```