ENGINEERING ANALYSIS II (EA2)

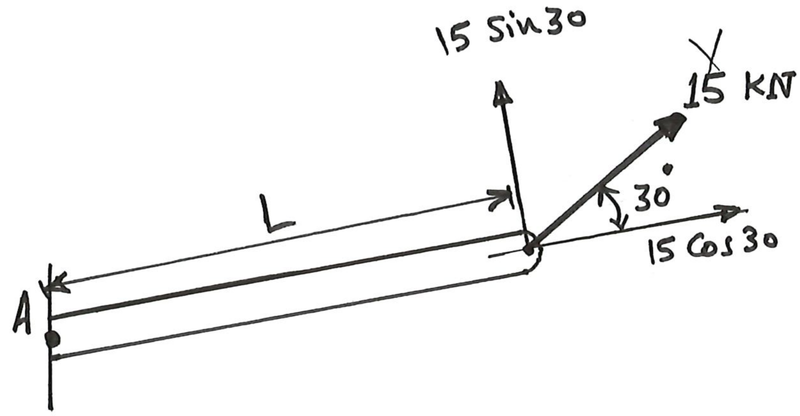
H.W#4 Solutions

Solution of H.W #4 [EA2]

Problem No 1 [4.8]:-

Req:- L_{max} of the beam.

1 FBD



$\therefore M_A = 15 \sin 30 * L$

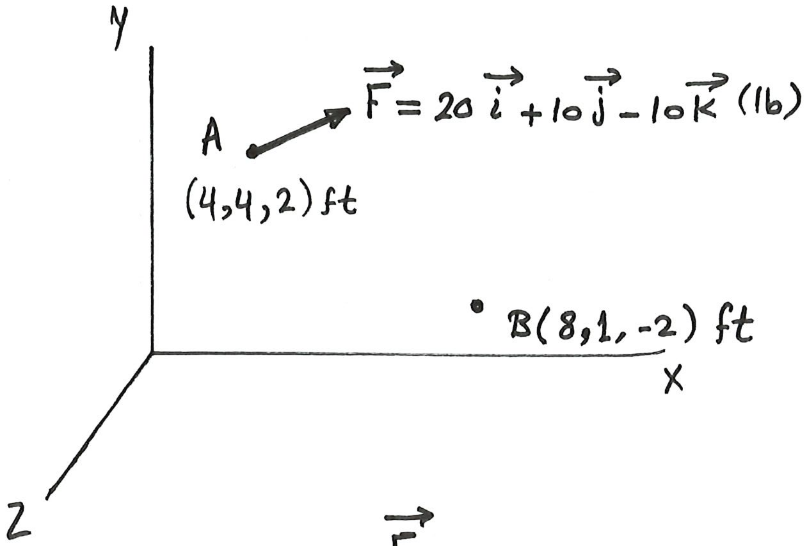
$= 7.5\,L \quad (KN.m)$

$\therefore M_A = M_{A max} = 18\ KN.m \quad (\text{Given})$

$\therefore 7.5\,L = 18 \Rightarrow \boxed{L = 2.4m} \Leftarrow \max L$

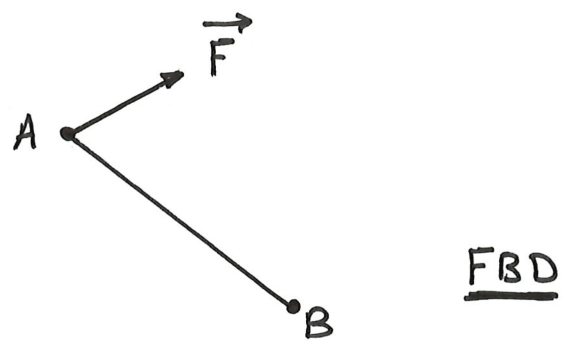
Shady Gomaa

Problem No (2) [4.56]



Req:- $|M_B|$

① FBD



[2]

$$\vec{M}_B = \vec{r}_{BA} \times \vec{F}$$

start with B

- $\vec{r}_{BA} = -4\vec{i} + 3\vec{j} + 4\vec{k}$ (ft)

A−B

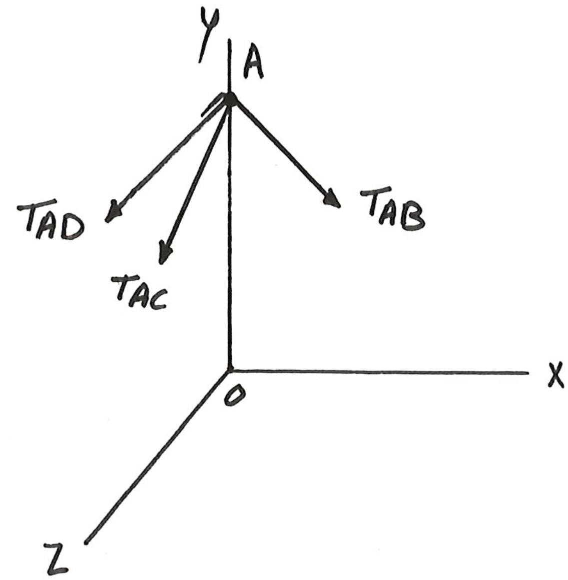
$$\therefore \vec{M}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 3 & 4 \\ 20 & 10 & -10 \end{vmatrix} = [(3*-10)-(4*10)]\vec{i} - [(-4*-10)-(4*20)]\vec{j} + [(-4*10)-(3*20)]\vec{k}$$

$$\therefore \vec{M}_B = -70\vec{i} + 40\vec{j} - 100\vec{k} \quad (lb.ft)$$

$$\therefore |M_B| = \sqrt{(-70)^2 + 40^2 + (-100)^2} = \boxed{128.5 \text{ lb.ft}}$$

Problem No 3 [4.69]:-

Req: $\vec{M_o}$



[1] FBD

[2] $\vec{M_o} = \vec{r_{oA}} \times \vec{T_A}$

where : $\vec{T_A} = \vec{T_{AB}} + \vec{T_{AC}} + \vec{T_{AD}}$

$\Rightarrow A(0,70,0),\ B(40,0,0),\ C(-40,0,40),\ D(-35,0,-35).$

$\Rightarrow \vec{T_{AB}} = |T_{AB}|\ \vec{e_{AB}}$

$$\vec{e_{AB}} = \frac{\vec{r_{AB}}}{|r_{AB}|} = \frac{40\vec{i} - 70\vec{j} + 0\vec{K}}{\sqrt{40^2 + (-70)^2 + 0^2}} = 0.496\vec{i} - 0.868\vec{j} + 0\vec{K}$$

$$\therefore \vec{T_{AB}} = 4\left[0.496\vec{i} - 0.868\vec{j} + 0\vec{K}\right] \ (KN) \quad (1)$$

$\Rightarrow \vec{T_{AC}} = |T_{AC}|\ \vec{e_{AC}}$

$$\vec{e_{AC}} = \frac{\vec{r_{AC}}}{|r_{AC}|} = \frac{-40\vec{i} - 70\vec{j} + 40\vec{K}}{\sqrt{(-40)^2 + (-70)^2 + 40^2}} = -0.444\vec{i} - 0.778\vec{j} + 0.444\vec{K}$$

$$\therefore \vec{T_{AC}} = 2\left[-0.444\vec{i} - 0.778\vec{j} + 0.444\vec{K}\right] \ (KN) \quad (2)$$

$$\Rightarrow \overrightarrow{T_{AD}} = |T_{AD}| \, \overrightarrow{e_{AD}}$$

$$\overrightarrow{e_{AD}} = \frac{\overrightarrow{r_{AD}}}{|r_{AD}|} = \frac{-35\vec{i} - 70\vec{j} - 35\vec{k}}{\sqrt{(-35)^2 + (-70)^2 + (-35)^2}} = -0.408\vec{i} - 0.816\vec{j} - 0.408\vec{k}$$

$$\therefore \overrightarrow{T_{AD}} = 2\left[-0.408\vec{i} - 0.816\vec{j} - 0.408\vec{k}\right]$$

$$\therefore \overrightarrow{T_A} = \overrightarrow{T_{AB}} + \overrightarrow{T_{AC}} + \overrightarrow{T_{AD}}$$

$$\overrightarrow{T_A} = 0.28\vec{i} - 6.66\vec{j} + 0.072\vec{k} \quad (KN)$$

$$\therefore \overrightarrow{M_o} = \overrightarrow{r_{oA}} \times \overrightarrow{T_A}, \quad \overrightarrow{r_{oA}} = 0\vec{i} + 70\vec{j} + 0\vec{k}$$

$$\therefore \overrightarrow{M_o} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 70 & 0 \\ 0.28 & -6.66 & 0.072 \end{vmatrix}$$

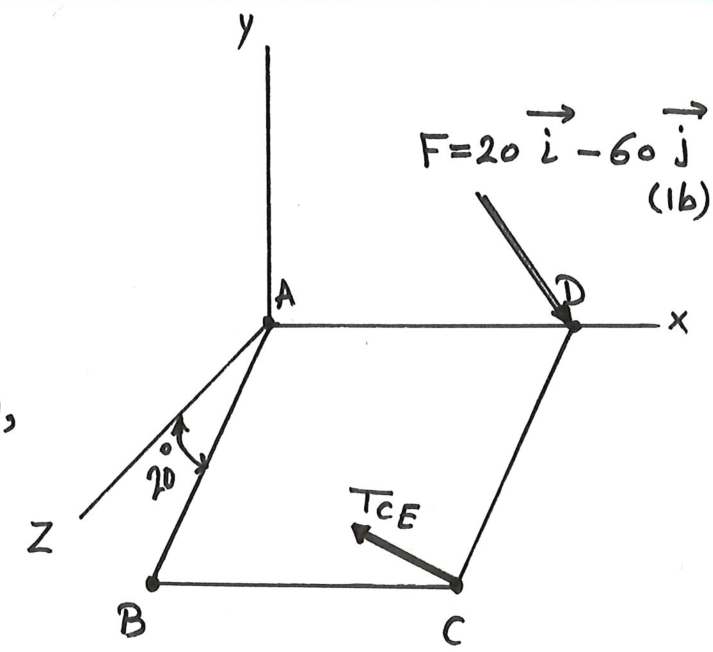
$$\therefore \overrightarrow{M_o} = \vec{i}\left[70 * 0.072\right] - \vec{j}\left[0\right] + \vec{k}\left[-(70 * 0.28)\right]$$

$$\boxed{\therefore \overrightarrow{M_o} = 5.04\vec{i} - 19.6\vec{k}}$$

Shady Gomaa

Problem No(4) [4.92]:-

1 FBD



2

A(0,0,0), B(0, −4 Sin 20, 4 Cos 20),

C(4, −4 Sin 20, 4 Cos 20),

D(4,0,0)

$$|M| = \vec{e}_{AB} \cdot (\vec{r}_{AD} \times \vec{F})$$

$$\Rightarrow \vec{r}_{AD} = 4\vec{i} + 0\vec{j} + 0\vec{K} \quad \text{(ft)}$$

$$\therefore \vec{r}_{AD} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ 4 & 0 & 0 \\ 20 & -60 & 0 \end{vmatrix} = 0\vec{i} + 0\vec{j} - 240\vec{K}$$

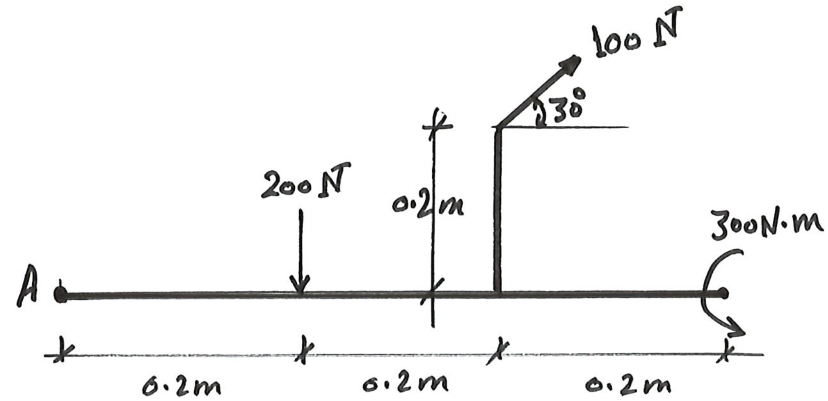
$$\Rightarrow \vec{e}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{0\vec{i} - (4\,\mathrm{Sin}\,20)\vec{j} + (4\,\mathrm{Cos}\,20)\vec{K}}{\sqrt{0^2 + (4\,\mathrm{Sin}\,20)^2 + (4\,\mathrm{Cos}\,20)^2}} = 0\vec{i} - 1.368\vec{j} + 0.94\vec{K}$$

$$\therefore |M| = (0\vec{i} - 1.368\vec{j} + 0.94\vec{K}) \cdot (0\vec{i} + 0\vec{j} - 240\vec{K})$$

$$\therefore |M| = 0.94 * -240 = \boxed{-225.5 \quad \text{(lb.ft)}}$$

Problem No 5. [4.117]

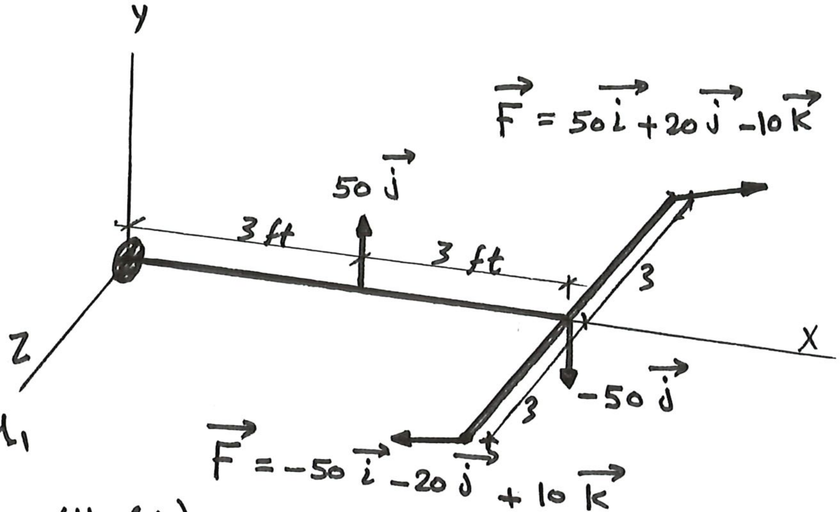
$$\Sigma M_A = 0.2 * (-200) - 0.2 * (100 \cos 30) + 0.4 * (100 \sin 30) + 300 = \boxed{262.7\ N \cdot m}$$



Problem No 6 [4.122]

$|M| = ?$

1 ⇒ FBD



2 For 50 lb → we can calculate M_1

$\therefore M_1 = -50 * 3 = -150\vec{K}$ (lb.ft)

For $\vec{F}$, the moment $M_2 = \vec{r} \times \vec{F}$

$$\therefore M_2 = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ 0 & 0 & 6 \\ 50 & 20 & -10 \end{vmatrix} = \vec{i}(-6 * 20) - \vec{j}(-6 * 50) + \vec{K}(0)$$

$\therefore M_2 = -120\vec{i} + 300\vec{j}$ (lb.ft)

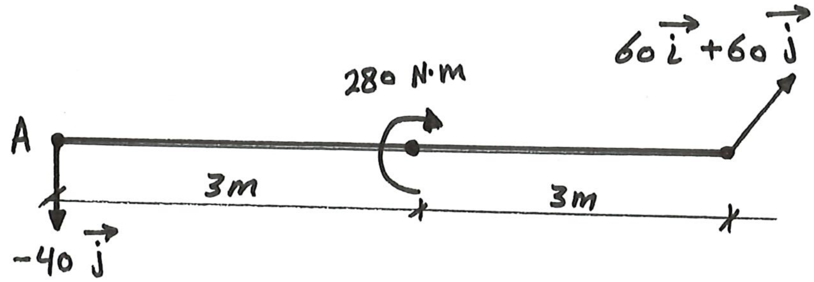
$\therefore \Sigma M = M_1 + M_2 = -120\vec{i} + 300\vec{j} - 150\vec{K}$

$\therefore |M| = \sqrt{(-120)^2 + (300)^2 + (-150)^2}$

$\therefore \boxed{|M| = 356.2 \text{ lb.ft}}$

Problem No (7) [4.139]

1. F.B.D

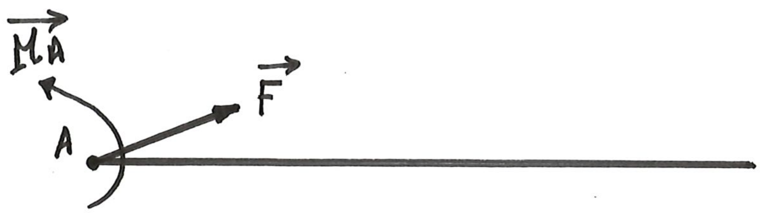


2. Convert the system of forces and couples to a single force & couple at A.

$$\vec{F} = -40\vec{j} + 60\vec{i} + 60\vec{j} = 60\vec{i} + 20\vec{j}\ (N)$$

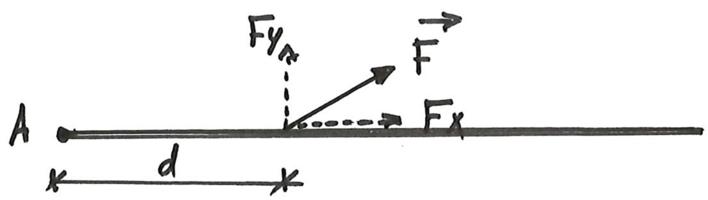
+ve

$$M_A = -280 + (6)*60 = 80\ N\cdot m$$



3. Convert the "M_A & $\vec{F}$" to $\vec{F}$ at a distance d from A.

x component of $\vec{F}$ doesn't affect the moment at A because its line of action passes through A.



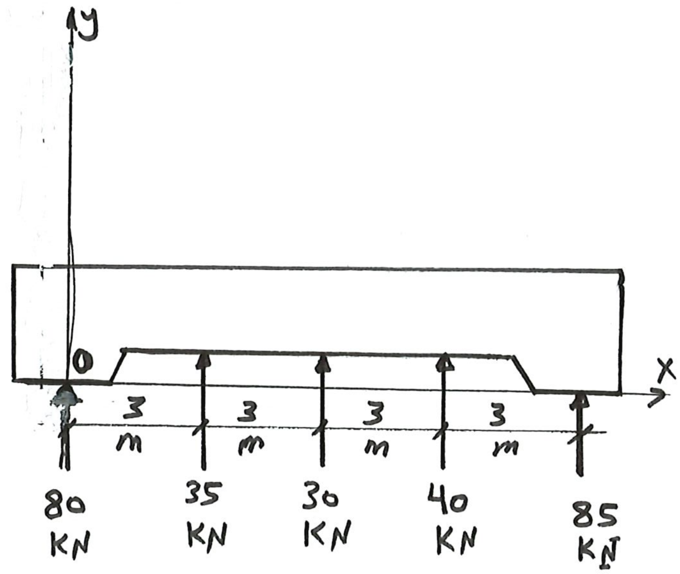
$\therefore F_y * d = M_A$

$\therefore 20 * d = 80$

$\therefore d = \frac{80}{20} = 4\,m$

Problem No (8) [4.143]

The equivalent system must have the Same Moment and total forces exerted by given system of forces.



"Given system"

$\Rightarrow F = 80 + 35 + 30 + 40 + 85$

$\boxed{F = 270 \text{ KN}}$

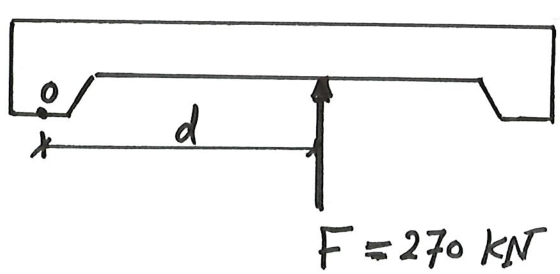
To get its location $\rightarrow \Sigma M_{\text{at any Point}}$ must be equal for the given system and the equivalent one.

$+ve \circlearrowleft \ \Sigma M_o = 3*35 + 6*30 + 9*40 + 12*85 = 1665 \text{ KN.m}$ ⇐ From the given system

$+ve \circlearrowleft \ M_o = d*F = 270\, d$ ⇐ From the equivalent system

$\therefore 1665 = 270\, d$

$\therefore \boxed{d = 6.167 \, m}$



"Equivalent system"