MATH 230-1: Written Homework 2
Northwestern University, Fall 2023

1. The goal of this problem is to find the point on the line with parametric equations

$$x = 2 - 3t,\ y = -4 + t,\ z = 1 + t$$

that is closest to the point $(-3, 2, 1)$ in multiple ways. The point is to illustrate how the same problem can be approached by different methods.

(a) Find the function $f(t)$, depending on t, that gives the distance between $(-3, 2, 1)$ and a point $(2 - 3t, -4 + t, 1 + t)$ on the given line. Then, using single-variable calculus, find the value of t that minimizes this distance, and hence find the desired closest point.

(b) Take any two specific points P and Q (your choice!) on this line. Set $R = (-3, 2, 1)$, and compute the vector projection of $\overrightarrow{PR}$ onto $\overrightarrow{PQ}$. Explain (draw a picture!) how to use the sum of $\overrightarrow{OP}$ and this projection to find the desired closest point, and hence find its coordinates.

(c) Use the method described in the book, using a cross product, to find the distance from $(-3, 2, 1)$ to the given line. Then, find the value of t so that the function $f(t)$ from part (a) gives the distance you found, and hence find the closest point.

2. The goal of this problem is to illustrate the use of the dot product in justifying some facts about some standard geometric shapes.

(a) A *rhombus* is a parallelogram whose sides all have the same length. Use vectors to justify the fact that the diagonals of a parallelogram intersect each other at right angles exactly when that parallelogram is actually a rhombus.

(b) Use vectors to justify the fact that the diagonals of a parallelogram are equal in length exactly when that parallelogram is actually a rectangle.

3. Suppose we are given lines with parametric equations

$$x = 3 + t,\ y = 2 - t,\ z = -1 + 2t$$

and

$$x = -2 + 2t,\ y = 5 - t,\ z = 5 - 4t.$$

The goal of this problem is to find parametric equations for the line that is perpendicular to both of these lines and passes through their point of intersection.

(a) Find the point at which the lines above intersect.

(b) Find a vector that is perpendicular to both of these lines.

(c) Find parametric equations for the desired perpendicular line.