MATH 230-1: Discussion 3 Problems
Northwestern University, Fall 2023

1. Justify the fact that there does not exist a plane containing the line with parametric equations

$$x = 4 - 2t,\ y = -9 + 2t,\ z = t \quad -\infty < t < \infty$$

which is perpendicular to the line with parametric equations

$$x = -7 + 4t,\ y = 3 + 2t,\ z = 6 - t \quad -\infty < t < \infty.$$

Then, find a way to modify the equation for z in this second line to make it so that there *is* such a plane, and find the distance from $(1, 1, 1)$ to this new plane.

2. Consider the quadric surface with equation

$$-3x^2 + 2y^2 + 5z^2 = 1.$$

(a) Sketch the cross-sections of this surface at $x = 0, \pm 1, \pm 2, \pm 3$.

(b) There are two values of k such that the cross-sections at $z = k$ are pairs of straight lines. Determine these values of k.

(c) Identify the surface and find the points on it that are closest to the origin.

3. Consider the quadric surface with equation

$$x = 1 - y^2 - 2z^2.$$

Sketch enough cross-sections that allow you to determine what this surface looks like, and then sketch the surface and identify it by name.