# MATH 230-1: Written Homework 7
## Northwestern University, Fall 2023

**1.** Consider the surface defined by the equation

$$zx + x^2y - y^3 = 0.$$

(a) Find the tangent plane to this surface at $(1, 1, 0)$. (Note there are two ways in which this could be done, either by using a gradient or by using the function $f(x, y)$ described in part (b).)

(b) Solve for $z$ in the given equation in order to express the surface near $(1, 1, 0)$ as the graph of a function $z = f(x, y)$, and use this to find the quadratic polynomial whose graph best approximates the given surface near $(1, 1, 0)$.

(c) Say you wanted to use the tangent plane in (a) to approximate the values of the function $f(x, y)$ in (b) for $(x, y)$ near $(1, 1)$ to within an accuracy of $0.001$. Find a value of $k$ such that the tangent plane approximation does give this desired accuracy on the rectangle $1 \leq x \leq k$, $1 \leq y \leq k$.

**2.** Let $f(x, y) = x^2 + y^3 - 3y + 10$.

(a) Find and classify the critical points of $f(x, y)$.

(b) Find the absolute maximum and absolute minimum of $f(x, y)$ among points in the triangle with vertices $(0, 0), (1, 0)$, and $(1, 1)$. Be sure to explain how you know $f(x, y)$ even has absolute extrema over this region in the first place.

(c) Find the absolute extrema of $f(x, y)$ among points in the circle $x^2+(y-1)^2 \leq 1$. (Hint: Treat the problem of finding absolute extrema on the boundary circle as one of optimizing a function subject to a single constraint equation.)

**3.** A certain company produces three products. The cost for producing the first product is $p_1$ dollars per unit, the cost of producing the second product is $p_2$ dollars per unit, and the cost of producing the third is $p_3$ dollars per unit. Assume the value derived from producing $x_1$ units of the first product, $x_2$ units of the second, and $x_3$ units of the third is given by the *utility function*

$$U(x_1, x_2, x_3) = x_1 x_2^2 x_3^3.$$

The larger the value of the utility function, the more worthwhile producing that many products is to the company. You can take it for granted that given some budget constraint as in (a) and (b) below, there are values of $x_1, x_2, x_3$ that maximize utility subject to that constraint.

(a) Assume the company has a budget of $100,000$ dollars to spend on producing its products. If $p_1 = 1$, $p_2 = 3$, and $p_3 = 5$, find the values of $x_1, x_2, x_3$ needed to maximize utility.

(b) More generally, with a budget of $B$ dollars and arbitrary prices $p_1, p_2, p_3$, determine how much of this budget the company should devote to producing the first product, to the second product, and to the third product in order to maximize utility.