MATH 230-1: Discussion 7 Problems
Northwestern University, Fall 2023

1. (a) Find an equation for the tangent plane to the surface

$$z - x^2y + xy^3 = 2$$

at the point $(2, 1, 4)$. It might help to find a function $z = f(x, y)$ whose graph is this surface.

(b) Use the tangent plane found in (a) to approximate the value of $(2.1)^2(0.8) - (2.1)(0.8)^3 + 2$.

(c) Find a bound on the error in approximation used in (b).

2. (a) Find and classify the critical points of $f(x, y) = x^2 + y^2 + xy^2$.

(b) Find the absolute maximum and absolute minimum of $f(x, y) = x^2 + y^2 + xy^2$ among points (x, y) in the rectangle $-1 \leq x \leq 1, -1 \leq y \leq 1$. How do you know such absolute extrema exist before you find them?

3. An aquarium in the shape of an open rectangular box without a top is to hold 81 cubic feet of water and is to be built using slate for the rectangular base and glass for the sides. Slate costs \$12 per square foot and glass costs \$2 per square foot. Find the dimensions of the aquarium which minimize the cost.