Northwestern University

Math 230-1 Second Midterm Examination
Fall Quarter 2019
Tuesday 19 November

Last name: _____ Email address: _____

First name: _____ NetID: _____

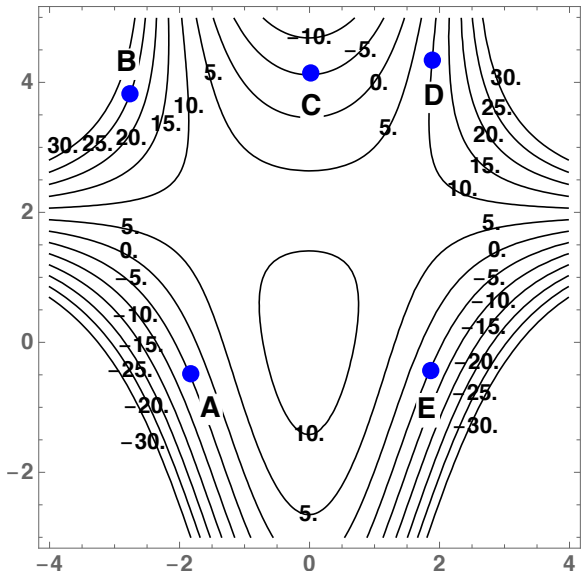
Instructions

- This examination consists of **??** questions for a total of **??** points.
- Read all problems carefully before answering.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- Exercise 2 has a blank page immediately following it in case you need more space.
- If you need more space for any other exercise, make use of one of the extra pages (pp. $9 - 10$) and indicate on the original exercise page where to find this additional work.
- **Show and justify all of your work**. Unsupported answers may not earn credit.

1. (10 points) Below you find a contour diagram of $z = f(x, y)$ with five points labelled A-E.

   For each condition below, write the letter of the labelled point on the contour diagram that satisfies it.

   No justification required!



(i) $f_x(P) > 0$ and $f_{xx}(P) > 0$

(i) $f_x(P) > 0$ and $f_{xx}(P) < 0$

(i) $f_x(P) < 0$ and $f_{xx}(P) > 0$

(i) $f_x(P) < 0$ and $f_{xx}(P) < 0$

(i) $f_x(P) = 0$

2. (15 points) Let $f(x, y) = x^2 - 2xy + y^2 - 3y$.
   (a) Find the directional derivative of f at $P = (1, 2)$ in the direction of the vector $\langle 1, 1 \rangle$.
   (b) Find the direction in which the directional derivative of f at $P = (1, 2)$ is maximized. Your answer must be a *unit* vector. Justify!
   (c) Find a direction in which the directional derivative of f at $P = (1, 2)$ is equal to 0. Your answer must be a *unit* vector. Justify!

ADDITIONAL PAGE FOR WORK ON EXERCISE 2 (IF NEEDED)
$f(x, y) = x^2 - 2xy + y^2 - 3y, P = (1, 2)$

3. (10 points) Let $\mathcal{S}$ be the surface defined by the equation $x \ln y + y \ln z - x = 0$.
   (a) Verify that $P = (1, 1, e)$ lies on $\mathcal{S}$.
   (b) Find an equation of the tangent plane to $\mathcal{S}$ at P.

4. (10 points) A particle moves in $\mathbb{R}^2$ along the curve $\mathcal{C}$ with parametrization

$$\mathbf{r}(t) = \left\langle t - \frac{t^3}{3},\ t^2 \right\rangle,\ -\infty < t < \infty$$

Find the distance it travels as it moves along $\mathcal{C}$ from point $P_1 = (6, 9)$ to point $P_2 = (0, 0)$.

Consolation: things factor nicely in the integral you need to compute.

5. (10 points) Decide whether the function f defined below is continuous at $P = (0, 0)$. Justify your answer, and indicate how you are using the definition of continuity at a point.

$$f(x, y) = \begin{cases} \frac{x^2}{2x^2+3y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

6. (15 points) Toxic green slime mold proliferates in dark and humid conditions. More precisely, we can model the concentration M of mold (in spores per m^2) with the function

$$M = f(L, h) = 100e^{-L^2}h^2,$$

where L is average ambient light (in lux) and h is percent humidity (in decimal form).

Suppose light and humidity vary in your room as

$$L = \sin(\pi x) - \cos(\pi y) + 2$$
$$h = e^{-x^2-y^2},$$

where x is your distance (in meters) east of the center of the room, and y is your distance (in meters) north of the center of the room.

(a) **Use the chain rule** to compute $\frac{\partial M}{\partial x}$ at $(x, y) = (0, 0)$.

6.contd. Recall, we have

$M = f(L,h) = 100e^{-L^2}h^2$
$L = \sin(\pi x) - \cos(\pi y) + 2$
$h = e^{-x^2-y^2}$

(b) Interpret your computation of $\frac{\partial M}{\partial x}\Big|_{(x,y)=(0,0)}$ in (a) as a statement about mold in your room. Your answer should be a full sentence, should be comprehensible to someone with no calculus background, and should include all numeric details of your result in (a), along with units.

(If you were not able to do part (a), just set $\frac{\partial M}{\partial x}\Big|_{(x,y)=(0,0)} = c$ for an undetermined constant c and express your answer in terms of c.)

YOU MUST SUBMIT THIS PAGE.

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

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DO NOT WRITE ON THIS PAGE.