Northwestern University

Math 230-1 Final Exam
Fall Quarter 2021
Wednesday 8 December

Last name: ______________________ Email address: ______________________

First name: ______________________ NetID: ______________________

Instructions

- **Show and justify all of your work**. Unsupported answers may not earn credit.
- This examination consists of 9 questions for a total of 90 points.
- Read all problems carefully before answering.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- **Terminology**: by "familiar named surface" we will mean a member of one of the following types of surfaces:

plane	cylinder	
ellipsoid	elliptic paraboloid	hyperbolic paraboloid
cone	hyperboloid of one sheet	hyperboloid of two sheets

1. (10 points) Consider the points $A = (2, 1, 1), B = (3, 0, 1), C = (1, 6, -1)$, and $D = (5, 0, 0)$.

   (a) (3 points) Compute the area of ΔABC.

   (b) (3 points) Find the equation of the plane containing A, B, and C.

   (c) (4 points) Find the distance between D and the plane from part (b).

2. (10 points) Consider the set of points in $\mathbb{R}^3$ which are twice as far from the point $(1, 1, 1)$ as they are from $(0, 0, 0)$. In other words, these are the points (x, y, z) such that

$$2\sqrt{(x-1)^2 + (y-1)^2 + (z-1)^2} = \sqrt{x^2 + y^2 + z^2}.$$

The set of such points is a quadratic surface. Give the standard form of this quadratic surface, its center, and the familiar name of this surface. (Hint: Start by squaring both sides of the given equation.)

3. (10 points) Compute the following limits, or show that they do not exist.

   (a) (5 points) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$

   (b) (5 points) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$

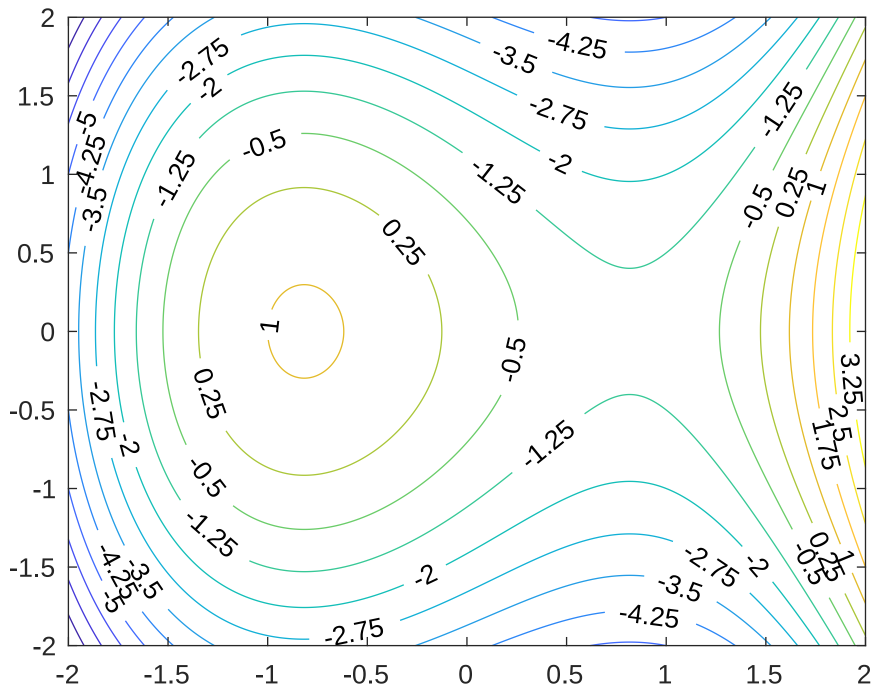
4. (10 points) Consider the smooth curve $\mathcal{C}$ parametrized by

$$\vec{r}(t) = \left\langle 2t + 3\cos(2t), 3t - 2\cos(2t), \sqrt{13}\sin(2t) \right\rangle$$

(a) (5 points) Find a parametrization of the tangent line to $\mathcal{C}$ at the point $P = (3, -2, 0)$.

(b) (5 points) Find the arc length of $\mathcal{C}$ from P to $Q = (2\pi + 3, 3\pi - 2, 0)$.

5. (10 points) Below is a level curve diagram for a function $f(x, y)$:



(a) (2 points) On the diagram above indicate roughly where the critical points are.

(b) (2 points) Based on the diagram, classify each of the critical points you found as local maxima, local minima, or saddle points.

(c) (2 points) Draw the gradient vector at the point $(1.5, 0)$.

(d) (2 points) Let $\mathbf{u} = \frac{\sqrt{2}}{2}\langle 1, 1\rangle$. Based on the diagram, is $D_{\mathbf{u}} f(0, 0)$ positive, negative, or zero? *Briefly* justify your answer.

(e) (2 points) Is $f_y(-1.5, 0)$ positive, negative, or zero? *Briefly* justify your answer.

6. (10 points) Consider the surface determined by the equation

$$ye^x + ze^{y^2} - z = 0,$$

and let $P = (0, 0, 1)$.

(a) (5 points) Find an equation for the tangent plane to the surface at P.

(b) (5 points) Find the normal line to the surface at P (your answer may be in either parametrized curve form or parametric equation form).

7. (10 points) Find the critical points of the function

$$f(x, y) = x^4 - xy^2 + 2xy$$

and classify them as local maxima, local minima, or saddle points.

8. (10 points) After moving into your new dorm, you realize there is no stove to cook with and you are forced to use an old hot plate from home. The hot plate is not functioning well, and its heating surface (a disk) has varying spots of hot and cold. Suppose we represent the surface of the hot plate as the unit disk in the plane, with temperature at a point (x, y) given by

$$T(x, y) = 30x^2 + 30y^2 - 10x + 200$$

in degrees Fahrenheit. Determine the hottest and coldest spots on the heating surface of the hot plate.

9. (10 points) Use the method of Lagrange multipliers to determine the points on the surface $\mathcal{S}$ given by

$$z^2 = xy + 4$$

which are closest to the origin.

(*Note: in this case the method of Lagrange multipliers does produce the points described, but in general you must be careful to check that the functions you are working with have certain 'nice' properties in order to guarantee such points exist. You are not required to do that here.*)

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