

# Northwestern University

MATH 230-1 Midterm 2  
Fall Quarter 2022  
November 15, 2022

Last name: \_\_\_\_\_ Email address: \_\_\_\_\_

First name: \_\_\_\_\_ NetID: \_\_\_\_\_

## Instructions

- Mark your section.

Section	Time	Instructor	
31	9:00	Lee	
41	10:00	Lee	
51	11:00	Cañez	
61	12:00	Schrader	
71	1:00	Tamarkin	
81	2:00	Tamarkin	

- This examination consists of 11 pages, not including this cover page. Verify that your copy of this examination contains all 11 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- This examination consists of 5 questions for a total of 100 points.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- Show all of your work. Unsupported answers may not earn credit.

1. (This problem has four parts and continues on the next page.) Determine whether each of the following statements is true or false. Justify your answer.

(a) (5 points) If a ball is thrown from the ground with acceleration  $\mathbf{a}(t) = -g\mathbf{j}$  and initial velocity  $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$ , it will follow the path of the parabola  $y = -\frac{1}{2}gx^2 + x$ .

(b) (5 points) The level curves of  $f(x, y) = e^{x+y}$  are either empty or straight lines.

(c) (5 points) The limit of  $\frac{x^2y}{x^3-3xy^2+y^3}$  as  $(x, y)$  approaches  $(0, 0)$  exists.

(d) (5 points) There is a direction in which the directional derivative of  $f(x, y) = x + xy - 2y$  at  $(1, 1)$  is  $-10$ .

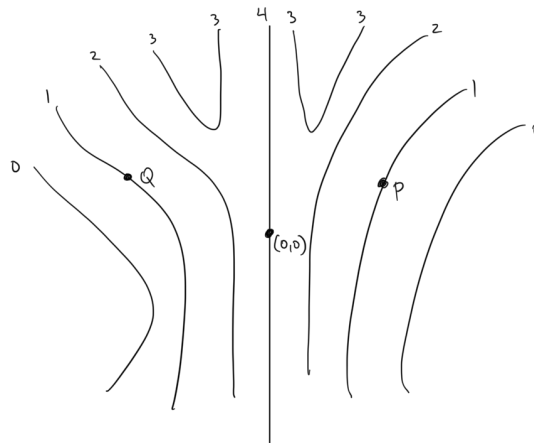
2. Consider the curve described by the vector function

$$\mathbf{r}(t) = \langle \sin t, 2t^{3/2}, \cos t \rangle, \quad t > 0.$$

(a) (10 points) Find parametric equations for the tangent line to this curve at the point where it intersects the plane  $y = \pi^{3/2}/4$ .

(b) (10 points) Find the arclength of the portion of the curve that begins at the point  $(1, 2(\pi/2)^{3/2}, 0)$  and ends at the point  $(0, 2\pi^{3/2}, -1)$ . The expression for your final answer should not involve an integral but can be left unsimplified in terms of square roots.

3. (This problem has two parts and continues on the next page.) Suppose  $z = f(x, y)$  is a continuous function with level curves



The labels on the level curves are the values of  $z$  at which they occur. Assume that other level curves occur at values of  $z$  strictly between those that are drawn. The  $x$ -axis (not drawn) passes through  $(0, 0)$  horizontally, and the  $y$ -axis is the level curve drawn at  $z = 4$ .

- (a) (10 points) Assuming that  $f_x$  and  $f_y$  exist at all points, determine whether each of the following is positive, negative, or zero.

$$f_x(Q), f_y(Q), f_x(0, 0), f_y(0, 0), f_x(P), f_y(P)$$

Justify your answer for  $f_x(0, 0)$  and  $f_y(0, 0)$  only.

(b) (15 points) Determine, with appropriate justification, the value of the following limit.

$$\lim_{(x,y) \rightarrow (0,0)} \left( f(x,y) + \frac{2x^2 + 3xy}{\sqrt{x^2 + y^2}} \right).$$

4. (10 points) Suppose the temperature at a point in  $\mathbb{R}^3$  is given by a function  $T(x, y, z)$  satisfying

$$\frac{\partial T}{\partial x}(0, 0, 0) = 1 = \frac{\partial T}{\partial y}(0, 0, 0), \quad \frac{\partial T}{\partial x}(-1, 0, 0) = 2 = \frac{\partial T}{\partial y}(-1, 0, 0), \quad \frac{\partial T}{\partial x}(-1, 0, 1) = 3 = \frac{\partial T}{\partial y}(-1, 0, 1)$$

and  $\frac{\partial T}{\partial z}(x, y, z) = 4$  at all points. (Note that some of these values are irrelevant in this problem.) If  $x, y, z$  are given in terms of two other variables  $u$  and  $v$  according to

$$x = \sin(u) \cos(v), \quad y = \sin(u) \sin(v), \quad z = \cos(u),$$

find  $\frac{\partial T}{\partial u}$  and  $\frac{\partial T}{\partial v}$  at  $(u, v) = (\frac{\pi}{2}, \pi)$ .

5. (This problem has three parts and continues on the next page.) Let  $f(x, y) = \frac{3x}{2x+y}$ .

(a) (10 points) Find the rate at which  $f$  is changing at  $(1, 1)$  in the direction of  $2\mathbf{i} + 3\mathbf{j}$ .

(b) (10 points) Find the direction in which the rate of change of  $f$  at  $(1, 1)$  is as large as possible and the rate of change in this direction.



- (c) (5 points) Find parametric equations for the line passing through  $(1, 1)$  which is perpendicular to the curve  $\frac{3x}{2x+y} = 1$ .

**THERE IS NO EXAMINATION MATERIAL ON THIS PAGE.**

**YOU MUST SUBMIT THIS PAGE.**

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

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