Northwestern University

MATH 230-1 Midterm 2
Spring Quarter 2022
May 17, 2022

Last name: ________________ Email address: ________________

First name: ________________ NetID: ________________

Instructions

- Mark your section.

Section	**Time**	**Instructor**	
41	10:00am	Hille	
51/53	11:00am	Hille	
61	12:00pm	Getzler	

- This examination consists of 9 pages, not including this cover page. Verify that your copy of this examination contains all 9 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- This examination consists of 6 questions for a total of 60 points.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Show all of your work. Answers without sufficient justification may not earn full credit.
- The last two pages of the exam may be used to continue your answers if you need more room, or as scrap paper (cross all work out in your exam, including on these pages, that you don't want scored).

1. Mark each statement **True or False**. You do not need to justify your answers (no partial credit will be awarded).

   (a) (2 points) The arc length of the curve $\mathbf{r}(t) = \langle t+1, t+3, t+5 \rangle$ for $0 \leq t \leq 1$ is $\sqrt{3}$.

   ☐ True ☐ False

   (b) (2 points) Let $f(x, y)$ be a function of two variables defined on $\mathbb{R}^2$. Let $g(x) = f(x, 2)$, and suppose that $f(x, 2)$ is differentiable at $x = 1$ and $g'(1) = -1$, then $f_x(1, 2) = -1$.

   ☐ True ☐ False

   (c) (2 points) Every bounded function $f(x, y)$ on the square $0 \leq x, y \leq 1$ is continuous.

   ☐ True ☐ False

   (d) (2 points) The partial derivative of $f(x, y) = \sqrt{x - y^2}$ with respect to x exists and is continuous on the domain $x > y^2$.

   ☐ True ☐ False

   (e) (2 points) Let $f(x, y)$ be a function of two variables. The level curves $f(x, y) = 0$ and $f(x, y) = 1$ can intersect.

   ☐ True ☐ False

2. (9 points) The acceleration of a particle is constant and equal to $\mathbf{a}(t) = \langle -2, -2, -10 \rangle$. At time $t = 0$, the initial position and velocity of the particle are $\mathbf{r}_0 = \langle 0, 0, 10 \rangle$ and $\mathbf{v}_0 = \langle 1, 2, 5 \rangle$. Find the first positive time the particle crosses the xy-plane.

3. (9 points) Let

$$f(x,y) = \begin{cases} xy^2 \sin(x^{-1} + y^2), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Compute $\lim_{(x,y)\to(0,0)} f(x,y)$. Justify your answer.

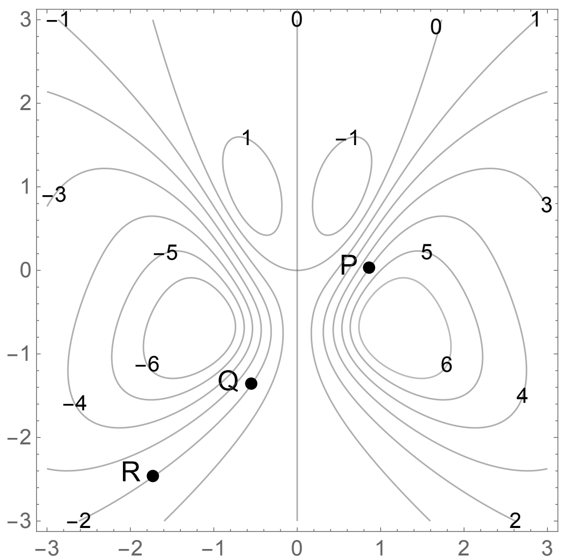
4. (9 points) Find all directions at the point $(1, 2, 2)$ in which the function

$$f(x, y, z) = 4x^2 + y^2 - 2y + z^2$$

is increasing most rapidly. Express directions as unit vectors. Justify your answer.

5. (9 points) Consider the level curve C of the function $g(x, y) = \ln(xy) + 2x^2 + 8(y - 1)^2$ given by the equation $g(x, y) = 10$. Find an equation for the tangent line to the point $(2, 1/2)$.

6. Here is the contour diagram of a function $f(x, y)$. Assume that the function, together with all of its first and second partial derivatives, are continuous. This contour diagram is used in both parts a) and b) of this question.



(a) (6 points) Using the Mean Value Theorem, show that there is a point S on the above contour diagram for which $f_{xx}(S) > 0$. (Continue your answer on the next page if you need more room.)

(b) (8 points) Three points P, Q and R are indicated in the contour diagram. Using only information from the contour diagram, order the four quantities 0, $f_x(P)$, $f_y(Q)$ and $f_x(R)$ in increasing order. Justify your answer.

THERE IS NO EXAMINATION MATERIAL ON THIS PAGE.

YOU MUST SUBMIT THIS PAGE. If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

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