MATH 230-1: Discussion 1 Problems
Northwestern University, Fall 2023

1. For the surface with equation $x^2 + z^2 = 4$ and for the surface with equation $z = 1 - y^2$, sketch or describe its intersection with the surfaces with each of the following equations:

$$(a)\ y = 0 \quad (b)\ y = 1 \quad (c)\ x = 1 \quad (d)\ x = 2$$

2. The equation $x^2 - 2x + y^2 + 8y + z^2 - 10z + 38 = 0$ describes a sphere. Find the point on this sphere closest to the xy-plane, the point closest to the xz-plane, and the point closest to the plane with equation $x = -5$. Also, sketch or describe the intersection of this sphere with the yz-plane.

3. Set $\mathbf{u} = \langle 2, 3\rangle$ and $\mathbf{v} = \langle -3, 6\rangle$, which we visualize as arrows drawn starting at the origin.

(a) Find the components of the vector of length 5 which points in the direction directly opposite that of the vector pointing from the endpoint of $\mathbf{v}$ to the endpoint of $\mathbf{u}$.

(b) Find a nonzero vector which is perpendicular to $2\mathbf{u} + \mathbf{v}$.

(c) If we write the vector $\langle -2, -1\rangle$ as $a\mathbf{u} + b\mathbf{v}$ for some numbers a and b, what will the signs of a and b be? Will b be smaller than or larger than 1? (Drawing a picture of $\mathbf{u}$, $\mathbf{v}$, and $\langle -2, -1\rangle$ may help!)