MATH 230-1: Discussion 5 Problems
Northwestern University, Fall 2023

1. Consider the function $f(x, y) = xy$.

(a) Draw the level curves of f at $z = -3, -2, -1, 0, 1, 2, 3$.

(b) Based on the level curves, explain why $(0, 0)$ is sitting at a minimum of the single variable function $f(x, x)$, and at a maximum of the single variable function $f(x, -x)$. (Note, $f(x, x)$ gives the values of f along the line $y = x$, and $f(x, -x)$ gives the values of f along the line $y = -x$.)

(c) Based on the level curves, Find a point at which the graph of f slopes downward when facing in the direction of the vector $\mathbf{i}$ but upward when facing in the direction of the vector $\mathbf{j}$, and find a point at which the graph slopes upward in the direction of $\mathbf{i}$ but downward in the direction of $\mathbf{j}$.

2. Set $f(x, y) = \frac{x^4 - y^4}{x^2 + y^2}$.

(a) Find $\lim_{(x,y)\to(0,0)} f(x, y)$ by algebraically simplifying the expression for $f(x, y)$.

(b) Find $\lim_{(x,y)\to(0,0)} f(x, y)$ by converting to polar coordinates.

(c) Find $\lim_{(x,y)\to(0,0)} \cos(f(x, y) + 4)$. Be sure to justify your answer by appropriately applying the notion of continuity.

3. Consider the limit

$$\lim_{(x,y)\to(0,0)} \frac{2x^2 - 3xy - 4y^2}{x^2 + y^2}.$$

(a) Show that this limit does not exist by finding three lines passing through $(0, 0)$ along which the limit gives three different values.

(b) Determine the value of the limit when approaching $(0, 0)$ only along the curve $y = x^2$.

(c) Show that this limit does not exist by converting to polar coordinates.