MATH 230-1: Written Homework 3
Northwestern University, Fall 2023

1. Consider the line L_1 with parametric equations

$$x = 4 - 6t,\ y = -3 - t,\ z = -2 + 3t \quad -\infty < t < \infty$$

and the line L_2 with parametric equations

$$x = t,\ y = 7 - 2t,\ z = 3 + 4t \quad -\infty < t < \infty.$$

(a) Justify the fact that these lines do not intersect, and are not parallel.

(b) The fact that L_1 and L_2 do not intersect and are not parallel implies they are *skew*, which means that they lie in different parallel planes. Find equations for these two planes. Hint: Two planes are parallel if they can be described using the same normal vector.

(c) Pick any point on L_1 and compute its distance to the plane containing L_2. Then pick any point on L_2 and compute its distance to the plane containing L_1. (You should get the same value, which is what we interpret as the distance between the skew lines, or between the corresponding parallel planes.)

2. Consider the quadric surface with equation

$$-2x^2 + 3y^2 - z^2 = 1.$$

(a) Sketch the cross-sections of this surface at $y = \pm 1/\sqrt{3}, \pm 1, \pm 2$ in a 2-dimensional xz-plane picture. For which values of k is there an empty cross-section at $y = k$? (In other words, for which k does the given surface not intersect the plane at $y = k$?)

(b) Sketch the given surface in 3-dimensional space, and in the same picture sketch the intersections of this surface with the planes at $z = 0, \pm 1, \pm 2$.

(c) Determine the point(s) on this surface that are closest to the origin.

3. Consider the quadric surface with equation

$$y = 2(x - 4)^2 - z^2.$$

(a) Sketch the cross-sections of this surface at $y = 0, \pm 1, \pm 2$ in a 2-dimensional xz-plane picture.

(b) What is the standard name we give to this type of surface?

(c) Determine the point(s) at which the line L_1 from Problem 1 intersects this surface.