Northwestern University

Math 230-1 First Midterm Examination
Fall Quarter 2019
Tuesday 22 October

Last name: ____________________ Email address: ____________________

First name: ____________________ NetID: ____________________

Instructions

- This examination consists of 6 questions for a total of 60 points.
- Read all problems carefully before answering.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- **Show and justify all of your work**. Unsupported answers may not earn credit.
- **Terminology**: by "familiar named surface" we will mean a member of one of the following types of surfaces:

plane	cylinder	
ellipsoid	elliptic paraboloid	hyperbolic paraboloid
cone	hyperboloid of one sheet	hyperboloid of two sheets

1. (5 points) Compute the angle θ (in radians) between $\mathbf{v} = \langle \sqrt{3}, 3, 2\rangle$ and $\mathbf{w} = \langle -\sqrt{3}, -3, 2\rangle$.

   Your answer cannot be expressed in terms of inverse trigonometric functions; i.e., the answer is a familiar angle.

2. (5 points) Let $\mathcal{C}$ be the the conic in $\mathbb{R}^3$ defined by the following system of equations:

$$\begin{aligned} \frac{(x-1)^2}{9} + \frac{(z-2)^2}{25} &= 1 \\ y &= 3 \end{aligned}$$

   (a) Describe $\mathcal{C}$ qualitatively: include what type of conic it is, what its center is, and how it is situated in $\mathbb{R}^3$.

   (b) Give a vector parametrization $\mathbf{r}(t)$ for $\mathcal{C}$. Include explicit bounds $a \leq t \leq b$ ensuring that the entire curve is parametrized. **No justification required**.

3. (10 points) Let $\mathbf{v}$ and $\mathbf{w}$ be two nonzero vectors.

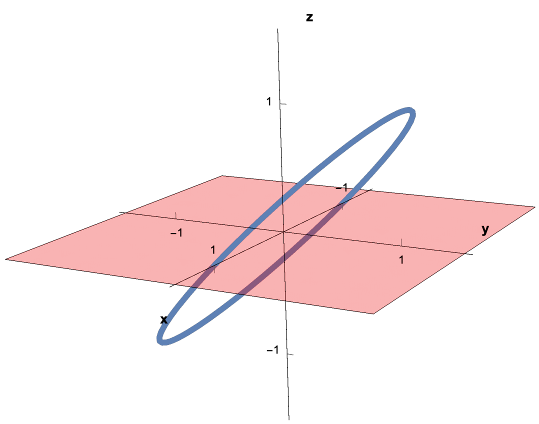
   (a) Give the dot product formula for $\operatorname{proj}_{\mathbf{w}} \mathbf{v}$. **No justification required**.

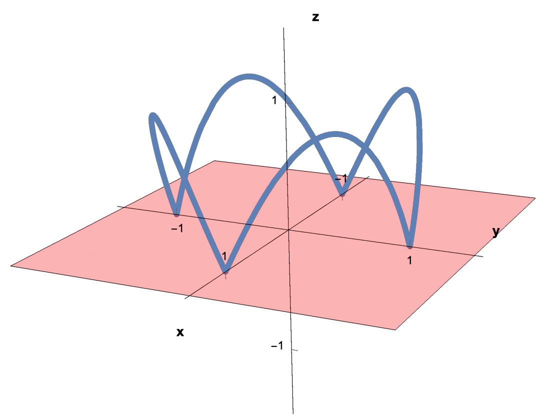
   (b) Now suppose $\mathbf{v}$ is parallel to $\mathbf{w}$. Show, using only the formula in (a), that $\operatorname{proj}_{\mathbf{w}} \mathbf{v} = \mathbf{v}$.
   You should begin by expressing with a vector equation what it means for $\mathbf{v}$ to be parallel to $\mathbf{w}$.

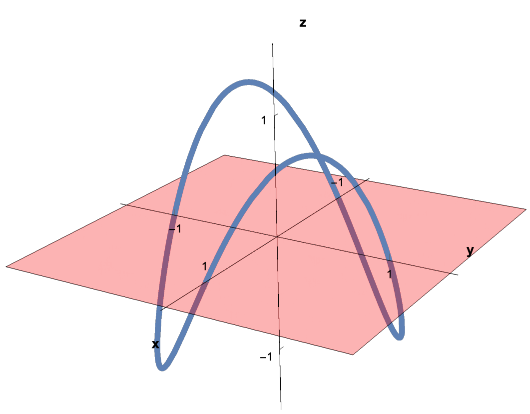
4. (15 points) Let $\mathcal{C}$ be the curve with parametrization $\mathbf{r}(t) = \langle \cos t, \sin t, \sin(2t) \rangle$.

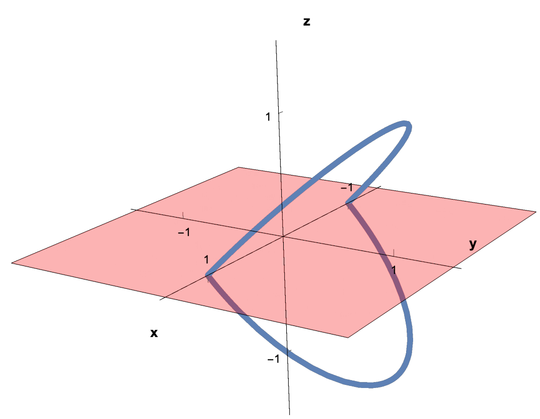
   (a) Exactly one of the figures below is a graph of $\mathbf{r}(t)$ for $0 \leq t \leq 2\pi$. Identify which is correct via a process of elimination: that is, indicate each incorrect graph with an 'X' and briefly explain why it cannot be a graph of $\mathbf{r}(t)$; then indicate the correct graph with a checkmark.

   Note: I've included a shaded portion of the xy-plane in each figure to help you visualize the curve.









4. contd. Let $\mathcal{C}$ be the curve with parametrization $\mathbf{r}(t) = \langle \cos t, \sin t, \sin(2t) \rangle$.

(b) Give the parametric equations for the tangent line to $\mathcal{C}$ at $P = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1)$.

(c) Show that the velocity vector of a particle moving along $\mathcal{C}$ according to $\mathbf{r}(t)$ never points in the vertical direction: i.e., is never parallel to the z-axis.

5. (15 points) Let M be the plane through the points $P = (0, 0, 0)$, $Q = (1, -1, 0)$, and $R = (1, 0, 1)$. Let N be the plane containing the point $S = (1, 0, -2)$ with normal vector $\mathbf{n} = \langle 2, 1, 1 \rangle$.

   (a) Find an equation for M.

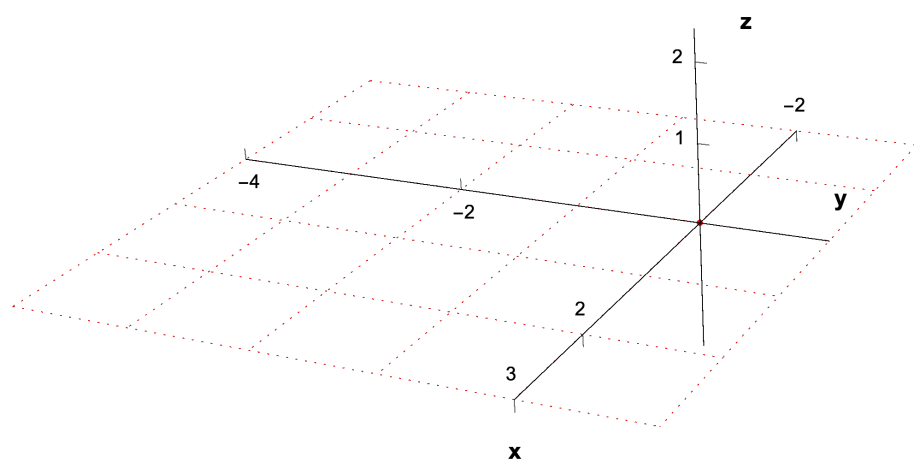
   (b) Determine whether the planes M and N intersect. If they do intersect, find the parametric equations for their line of intersection.

6. (10 points) Let $\mathcal{S}$ be the surface with equation $x^2 + y^2 + 4z^2 - 2x + 4y + 1 = 0$.

   (a) Identify $\mathcal{S}$ as one of our familiar named surfaces. You should first do some algebra to bring the equation into a more standard form.

   Justify your answer. You may reference your work in (b) if you like.

   (b) Find equations for the $(x = 1)$-, $(y = -2)-$ and $(z = 0)$-cross sections, and sketch these in the coordinate system below. Each cross section sketch must include at least 4 plotted points.



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DO NOT WRITE ON THIS PAGE.