

Northwestern University

MATH 230-1 Final Exam
Fall Quarter 2022
December 7, 2022

Last name: _____ Email address: _____

First name: _____ NetID: _____

Instructions

- Mark your section.

Section	Time	Instructor	
31	9:00	Lee	
41	10:00	Lee	
51	11:00	Cañez	
61	12:00	Schrader	
71	1:00	Tamarkin	
81	2:00	Tamarkin	

- This examination consists of 13 pages, not including this cover page. Verify that your copy of this examination contains all 13 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- This examination consists of 8 questions for a total of 100 points.
- You have two hours to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- Show all of your work. Unsupported answers may not earn credit.

1. (This problem has five parts and continues on the next page.) Determine whether each of the following statements is true or false. Justify your answer.

(a) (4 points) The area of the parallelogram in \mathbb{R}^3 with vertices $(0, 0, 0)$, $(1, 1, 1)$, $(1, -2, -1)$, and $(2, -1, 0)$ is smaller than 3.

(b) (4 points) There is a value of k such that the cross-section of the hyperbolic paraboloid $z = x^2 - y^2$ at $y = k$ is a hyperbola.

(c) (4 points) The limit $\lim_{(x,y) \rightarrow (1,2)} \frac{x^2 + 2y^2}{\sqrt{x^2 + y^2}}$ exists.

- (d) (4 points) There is a point $(a, b, f(a, b))$ on the graph of $f(x, y) = xy^2$ at which the tangent plane is parallel to the plane $z = 9x + 6y$.

- (e) (4 points) The function $f(x, y) = \cos(e^x + y)$ has an absolute maximum among points in the disk $(x - 2)^2 + (y + 1)^2 \leq 4$.

2. Consider the planes with equations

$$x + y + z = 1 \quad \text{and} \quad x - 2y - z = 2.$$

(a) (5 points) Find parametric equations for the line in which these planes intersect.

(b) (5 points) Determine if the line you found in part (a) intersects the plane $x + y + z = 0$.

3. Consider the surfaces with equations $y^2 + z^2 = 1$ and $x = z^2$
- (a) (5 points) Sketch or describe these surfaces. A valid description should indicate how the surface can be swept out by moving a specific curve in a specific plane in a specific direction.
- (b) (5 points) Find a vector function $\mathbf{r}(t)$ describing the curve along which these surfaces intersect and then find the acceleration vector for a point that moves along this curve.

4. Consider the function $f(x, y) = xe^{xy}$.

(a) (5 points) Find a vector giving a direction in which the rate of change of $f(x, y)$ at $(2, 1)$ is 0.

(b) (5 points) Compute all second-order partial derivatives of $f(x, y)$.

5. Consider the surfaces with equations $x^2 + y^2 - z^2 = 1$ and $xyz = 8$.

(a) (5 points) Find normal vectors to each surface at an arbitrary point (x, y, z) .

(b) (5 points) Justify the fact that there are no points at which these surfaces intersect at a right angle. (The angle between the surfaces at a point of intersection is just the angle between their normal vectors at that point.)

6. Consider the function $f(x, y) = e^{2x+3y}$.

(a) (5 points) Find an equation of the tangent plane to the graph of $z = f(x, y)$ at $(0, 0, 1)$.

(b) (5 points) Justify the fact that using the tangent plane found in part (a) to approximate the value of $e^{2(0.1)+3(0.2)}$ results in an error no larger than $\frac{9}{2}e(0.3)^2$. (Using the tangent plane to approximate the value is the same as using what the book calls the linearization to approximate the value.)

7. Consider the function $f(x, y) = x^2 + y^2 + xy^2$.

- (a) (10 points) Find all critical points of $f(x, y)$ and determine whether each is a local minimum, a local maximum, or a saddle point.

- (b) (10 points) Find the points at which $f(x, y)$ has an absolute maximum or an absolute minimum over the rectangle consisting of points (x, y) with $-3 \leq x \leq 2$ and $-1 \leq y \leq 1$.

8. (10 points) Determine the dimensions of the open rectangular box of surface area 100 that maximizes volume among all open rectangular boxes of surface area 100. (To be clear, saying that the box is “open” means that there is no top, only four sides and a bottom.)

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DO NOT WRITE ON THIS PAGE.