MATH 230-1: Written Homework 4
Northwestern University, Fall 2023

1. Consider the curve which is the intersection of the surface with equation $y = e^x$ with the surface with equation $x^2 + z^2 = 4$.

(a) Find parametric equations for this curve.

(b) Find the points on this curve at which it intersects the surface with equation $\ln y = z$.

(c) Find parametric equations for the tangent line to this curve at one of the points (your choice!) found in part (b).

2. Suppose we launch a ball into the air from a height of 5 meters, at an angle of $\pi/4$ above the horizontal direction and at a speed of 100 meters per second. The only thing affecting the motion of the ball is the effect of gravity.

(a) Find the position vector $\mathbf{r}(t)$ of the ball at any time t.

(b) Determine the horizontal distance that the ball travels before it first hits the ground, and the maximal height attained along the way. Your answers should be actual numbers rounded to the nearest tenth, so use a calculator!

(c) Suppose now that a small rocket is attached to the ball, which gives it an additional (so gravity is still there) constant horizontal acceleration of 3 meters per second squared. The rest of the setup is the same as before. How far horizontally does the ball travel downfield now?

3. Consider the curve in the xy-plane with polar equation $r = \theta^2$ for $0 \leq \theta < 2\pi$.

(a) Find parametric equations for this curve, using t as the parameter.

(b) Find the arclength parameter function $s(t)$ for this curve, measured starting at the point with Cartesian coordinates $(\frac{\pi^2}{16} \cdot \frac{\sqrt{2}}{2}, \frac{\pi^2}{16} \cdot \frac{\sqrt{2}}{2})$

(c) Find the point on this curve that is at a distance of 1 away from the starting point in (b) as measured along the curve in the direction of increasing values of the parameter t. Give the answer in terms of numbers rounded to the nearest tenth, so again use a calculator.