MATH 230-1: Discussion 2 Problems
Northwestern University, Fall 2023

1. Let $\mathbf{u} = \langle 3, -1, 2\rangle$ and $\mathbf{v} = \langle -1, 4, 1\rangle$. Find the vector of length 3 that points in the direction directly opposite that of the sum of the vector projection of $\mathbf{u}$ onto $\mathbf{v}$ and the vector projection of $\mathbf{v}$ onto $\mathbf{u}$.

2. A *parallelepiped* is a three-dimensional analogue of a parallelogram, which we can visualize as a "slanted" box. Find the volume of the parallelepiped with edges formed by the vectors $\mathbf{u} = \langle 5, 0, -2\rangle$, $\mathbf{v} = \langle 1, 1, 1\rangle$, and $\mathbf{u}\times\mathbf{v}$. The fact that $\mathbf{u}\times\mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$ is important to determining the correct way of computing the volume. Hint: Think of the parallelogram with edges $\mathbf{u}$ and $\mathbf{v}$ as being the *base* of this parallelepiped.

3. Consider the line with parametric equations

$$x = 3 + t,\ y = -1 + 4t,\ z = 2 - t$$

and the line with parametric equations

$$x = 1 + 4t,\ y = 1 + 2t,\ z = -3 + 4t.$$

Determine whether or not these lines intersect. If they do intersect, find the point of intersection, and if they do not intersect, determine if they are *skew*, meaning that they are not parallel and do not intersect. (In fact, skew lines lie in different parallel planes.)