Northwestern University

MATH 230-1 Midterm 1
Spring Quarter 2022
April 26, 2022

Last name: _____ Email address: _____

First name: _____ NetID: _____

Instructions

- Mark your section.

Section	**Time**	**Instructor**	
41	10:00am	Hille	
51/53	11:00am	Hille	
61	12:00pm	Getzler	

- This examination consists of 11 pages, not including this cover page. Verify that your copy of this examination contains all 11 pages. If your examination is missing any pages, then obtain a new copy of the examination immediately.
- This examination consists of 6 questions for a total of 60 points.
- You have one hour to complete this examination.
- Do not use books, notes, calculators, computers, tablets, or phones.
- Write legibly and only inside of the boxed region on each page.
- Cross out any work that you do not wish to have scored.
- Show all of your work. Answers without sufficient justification may not earn full credit.

1. Mark each statement **True or False**. You do not need to justify your answers (no partial credit will be awarded).

   (a) (2 points) The lines $\mathbf{r}_1(t) = \langle t, t, t\rangle$ and $\mathbf{r}_2(t) = \langle 1+t, 1-t, 1\rangle$ intersect orthogonally at $(1, 1, 1)$.

   ☐ True ☐ False

   (b) (2 points) The volume of the parallelepiped spanned by $\mathbf{i}$, $\mathbf{i}+\mathbf{j}$ and $\mathbf{v} = \langle 1, 2, 3\rangle$ is 3.

   ☐ True ☐ False

   (c) (2 points) The planes $x - 2y + 3z - 10 = 0$ and $-2x + 4y - 6z = 4$ intersect along a line.

   ☐ True ☐ False

   (d) (2 points) Let $\mathbf{u}$ and $\mathbf{v}$ be three-dimensional vectors, with $\mathbf{v}$ nonzero. Then $\text{proj}_{2\,\mathbf{v}}(2\,\mathbf{u}) = 2\ \text{proj}_{\mathbf{v}}(\mathbf{u})$.

   ☐ True ☐ False

   (e) (2 points) The line $L : x = t, y = t, z = t$ and the plane $x + y - 2z = 1$ intersect in a point.

   ☐ True ☐ False

2. Consider the following two lines

$$L_1 : x = 1 + t, y = -6 + 2t, z = 4 + 2t$$
$$L_2 : x = -s, y = -2 + s, z = 14 + 4s.$$

(a) (6 points) The lines L_1 and L_2 intersect at a point P. Find the coordinates of the point P.

(b) (6 points) Determine the acute angle (in radians) between the lines at the point of intersection. Justify your answer.

3. (a) (6 points) Let

   - E_1 be the plane containing the points $(0, 0, 0), (1, 1, 1)$ and $(2, -2, 3)$, and
   - E_2 be the plane passing through the point $(1, 2, 3)$ with normal vector $\mathbf{n} = \langle 1, 2, 3 \rangle$.

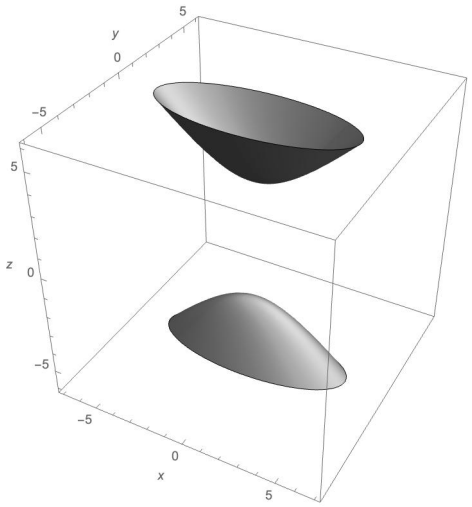
   Find equations for the planes E_1 and E_2.

(b) (6 points) Let

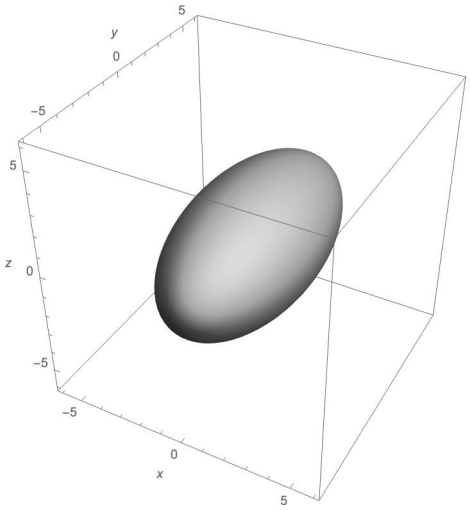
- E_3 be the plane with equation $x + y = 10$, and
- E_4 be the plane with equation $x + 2y + 3z = 20$.

Parametrize the intersection of the planes E_3 and E_4.

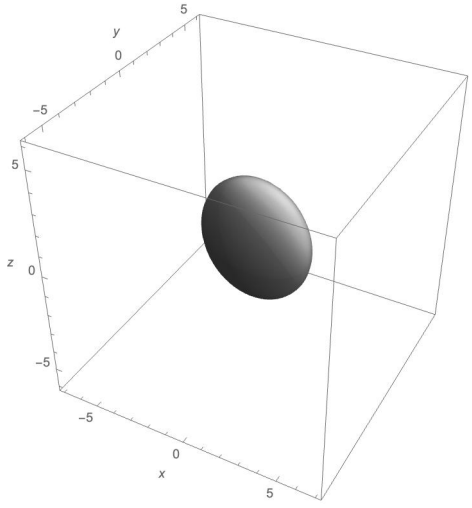
4. (6 points) Here are pictures of four quadric surfaces:



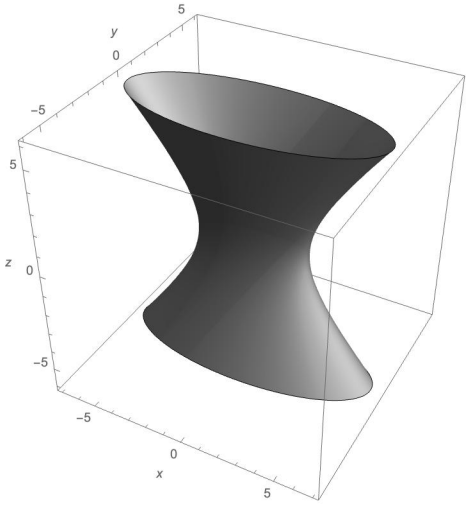
(A)



(B)



(C)



(D)

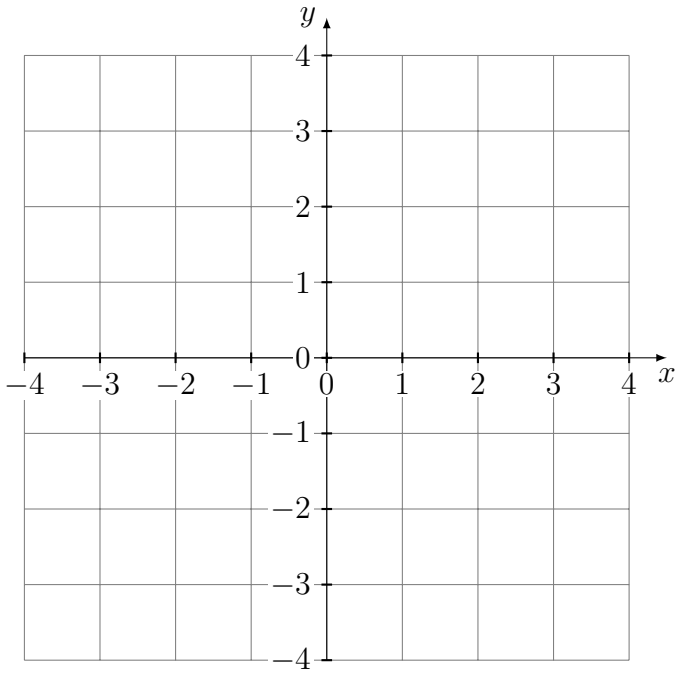
Identify which of these four surfaces corresponds to each of the following three equations:

☐ $\frac{x^2}{9} + y^2 - \frac{z^2}{9} = 1$ ☐ $\frac{x^2}{9} + y^2 + \frac{z^2}{9} = 1$ ☐ $\frac{x^2}{9} + y^2 - \frac{z^2}{9} = -1$

5. (a) (3 points) Parametrize the curve $\{2(x+1)^2 + y^2 = 4 \text{ and } y \geq 0\}$.

(b) (4 points) Find the tangent line at the point $(0, \sqrt{2})$.

(c) (3 points) Sketch this curve, together with the tangent line that you found in (c).



Coordinate System

6. (10 points) Consider the quadric surface S given by the equation

$$-x^2 + y^2 - z^2 = 1.$$

Intersecting S with the plane $az - y = 2$ (where a is a scalar), we obtain a conic section, which we can project to the xz-plane.

For which of the following values of a is the resulting conic in the xz-plane an ellipse:

☐ $a = -2$ ☐ $a = -1/2$ ☐ $a = 0$ ☐ $a = 1$.

The grading of this question will be based on the working you give justifying your answers.

THERE IS NO EXAMINATION MATERIAL ON THIS PAGE.

YOU MUST SUBMIT THIS PAGE.

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score.

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