MATH 230-1: Written Homework 1
Northwestern University, Fall 2023

1. The collection of points (x, y, z) in $\mathbb{R}^3$ whose distance to $(1, 2, 3)$ is the same as their distance to $(-2, 1, 0)$ forms a plane with an equation of the form $ax + by + cz = d$, where a, b, c, and d are some constants. (It is not important at this stage to know why this equation describes a plane—we will see why in Week 3.)

(a) Find an equation for this plane. Hint: Setup an equation saying that the distance from (x, y, z) to $(1, 2, 3)$ is equal to the distance from (x, y, z) to $(-2, 1, 0)$, and simplify algebraically until your equation looks like $ax + by + cz = d$.

(b) Sketch the intersections of the plane you found with the xy-plane, with the yz-plane, and with the xz-plane.

(c) The portion of the plane in question that lies in the closed first octant (i.e., the region where $x \geq 0, y \geq 0$, and $z \geq 0$) has a triangular shape with a triangle as its boundary. Find the perimeter of this triangle.

2. The goal of this problem is to visualize the region in $\mathbb{R}^3$ consisting of points (x, y, z) satisfying all of the inequalities

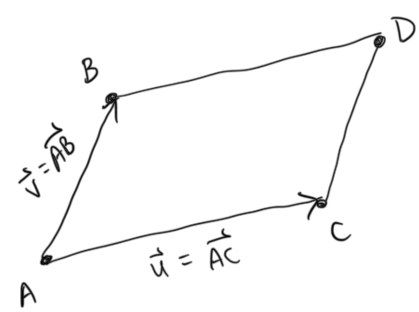
$$0 \leq y \leq 1,\ 0 \leq x \leq y^2, \text{ and } 0 \leq z \leq 1 - y.$$

(a) Sketch the surface with equation $x = y^2$, labeling clearly its intersection with the xy-plane and with the yz-plane. Then, in a separate drawing, sketch the surface with equation $z = 1 - y$, labeling clearly its intersection with the both the yz-plane and the xy-plane. Only sketch the portions of these surfaces that lie in the closed first octant.

(b) The points $(0, 0, 1)$ and $(1, 1, 0)$ both lie on the surfaces in (a) since their coordinates satisfy the equation of these two surfaces. Thus, these two points lie on the intersections of these two surfaces. Use this to give a rough sketch of this intersection, which is some kind of curve. You DO NOT have to try to describe this intersection algebraically in terms of equations, only give a rough sketch. Hint: The curve in question should lie above the intersection of $x = y^2$ and the xy-plane drawn in (a), and in front of the intersection of $z = 1 - y$ and the yz-plane.

(c) Sketch the region described by the inequalities at the start of the problem as best you can.

3. The goal of this problem is to use vector arithmetic to justify the fact that diagonals of parallelograms bisect each other, meaning cut each other in half. Consider the parallelogram



The diagonal of this parallelogram coming out of the point A is the sum $\mathbf{u} + \mathbf{v}$.

(a) What is the vector that begins at the point A and goes halfway up the diagonal $\mathbf{u} + \mathbf{v}$?

(b) Find, in terms of $\mathbf{u}$ and $\mathbf{v}$, the vector that describes the *other* diagonal of this parallelogram, namely the diagonal that goes between B and C. Then, find the vector that begins at the point A and ends halfway along this other diagonal.

(c) How do the results of (a) and (b) justify the fact about bisecting diagonals?