MATH 230-1: Discussion 6 Problems
Northwestern University, Fall 2023

1. Let $f(x, y) = 2x^2 - y$.

    (a) Draw the level curves of f at $z = -2, -1, 0, 1, 2$.

    (b) Based on the level curves, explain why it makes sense that $f_x(0,0)$ is zero, $f_y(0,0)$ is negative, and $f_{yy}(0,0)$ is zero.

    (c) In which directions at $(3, -2)$ should we face so that the graph of f is as steep (either upward or downward) as possible?

2. (a) The water temperature, in polar coordinates, at a point (r, θ) in a lake is given by

$$T(r, \theta) = r^3 \sin(\theta/2)$$

in degrees celsius. Suppose a swimmer moves along some path in the lake, and that at the point $(r, \theta) = (2, \frac{\pi}{2})$ experiences a change in water temperature with respect to time of 10 degrees celsius per second. If at this instant the value of θ is changing at a rate of $\frac{\pi}{2}$ radians per second, find the rate at which the value of r is changing with respect to time. Your answer can be left unsimplified.

    (b) In which Cartesian direction is T increasing most rapidly at the Cartesian point with polar coordinates $(r, \theta) = (2, \frac{\pi}{2})$?

3. Let $f(x, y, z) = xy^2 z^3 + xy - 3ye^{yz}$.

    (a) Compute $\nabla f(x, y, z)$.

    (b) Give two directions in which the rate of change of f at the point $(1, 2, 0)$ is zero.

    (c) Find an equation for the tangent plane to the surface

$$xy^2 z^3 + xy - 3ye^{yz} = -4$$

at the point $(1, 2, 0)$.