

ABSTRACT ALGEBRA

SUMMER OF SCIENCE

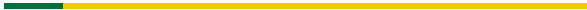
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INTRODUCTION



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The way I see it, algebraic structures are basically sets along with one or more operations that follow some basic properties. Some examples include groups, rings, and fields. Abstract algebra is then the study of these objects: how they behave on their own, and how they interact with each other.

MOTIVATION



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Out of the plethora of math topics offered, abstract algebra was one that fascinated me the most. I was particularly keen on knowing how a simple set of mathematical objects gives rise to a field as rich and as beautiful as algebra. I was also interested in how these abstract objects connect to the real world. Taking up abstract algebra as my topic study seemed like the perfect way to uncover these mysteries.

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1. **Group Theory:** The study of groups, their properties, some examples, maps between groups, and classification of groups.
2. **Ring Theory:** The study of rings, special classes of rings (such as domains and fields), maps between rings.
3. **Field Theory/Galois Theory:** The study of fields, extensions of fields, and some special classes of these extensions.

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And voíla! We have a group! The beautiful thing about this exercise is that the axioms of a group abstract out very intuitive and natural properties of symmetries.

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Just as before, we have a group!

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