Abstract Algebra

Summer of Science

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Introduction

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The way I see it, algebraic structures are basically sets along with one or more operations that follow some basic properties. Some examples include groups, rings, and fields. Abstract algebra is then the study of these objects: how they behave on their own, and how they interact with each other.

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2. **Ring Theory**: The study of rings, special classes of rings (such as domains and fields), maps between rings.
3. **Field Theory/Galois Theory**: The study of fields, extensions of fields, and some special classes of these extensions.

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Group Theory

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And voíla! We have a group! The beautiful thing about this exercise is that the axioms of a group abstract out very intuitive and natural properties of symmetries.

Group Theory - Rotations

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Just as before, we have a group!

Group Theory - Conclusion

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