

# A Novel Model for Ship Trajectory Anomaly Detection Based on Gaussian Mixture Variational Autoencoder

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**Abstract**—The use of traditional methods in anomaly detection of multi-class ship trajectories showed some limitations in terms of robustness and learning ability of trajectory features. In view of this, an anomaly detection model for ship trajectory data based on Gaussian Mixture Variational Autoencoder (GMVAE) is proposed in this study using an unsupervised classification method. The proposed model modifies Variational Autoencoder (VAE) by changing the inferential distribution of prior distribution and approximate posterior to the Gaussian mixture model. A high-dimensional hidden space is constructed to learn the features of multi-class trajectory data, and the Dynamic Time Warping (DTW) method is applied to measure the error between the reconstructed trajectory and the original trajectory in order to judge whether the ship trajectory is abnormal. The Automatic Identification System (AIS) data from the US coastal areas are used to verify the proposed model, and the results are compared with other commonly used models in a manually labeled dataset. The research results indicate that the detection rate of the proposed model is 91.26%, and the false alarm rate is 0.68%, which performs the best. Using the Gaussian mixture model to describe the distribution of hidden space can improve the learning ability of multi-class trajectories of VAE, thus increasing the robustness of the model. This research can provide technical support for ship trajectory data analysis and risk management of maritime transportation.

**Index Terms**—Intelligent transportation, anomaly detection of ship trajectory, AIS, Gaussian Mixture Variational Autoencoder (GMVAE), unsupervised learning.

## I. INTRODUCTION

Safety and reliability have always been the key issues in the development of waterway transportation. With the continuous growth of the number of ships, the density of ship traffic flow in coastal ports, estuaries and other waters is keep increasing,

making the maritime traffic situation more and more complicated. Ship trajectory anomaly detection is necessary and important in order to improve the automation level of maritime supervision, optimize the efficiency of traffic flow, and ensure the safety of ship navigation.

A classification of methods defining abnormal ship trajectories is shown in Figure 1, in which anomaly detection methods are divided into rule-driven and data-driven methods. Rule-driven methods are based on the maritime rules formulated by the International Maritime Organization (IMO) and the actual navigational conditions as judgment criteria, such as whether the ship's driving complies with the General Regulations Abnormal shape of the trajectory (U-turn, multiple turns after avoidance), intrusion into the no-navigation zone, etc. Those that meet the rules are normal ship trajectories, otherwise, they are abnormal [1-2]. Based on the data-driven method, the characteristic of historical trajectory data is learned by establishing a mathematical model, in which either supervised learning methods or unsupervised learning methods can be used. In addition, the abnormal ship trajectory can also be determined based on the position of the ships and the patterns of motion. The abnormal position refers to the abnormal areas for a ship [3]. For example, when a ship travels to the water area out of the navigation rules, the trajectory will deviate from the customary route. The abnormal motion mainly refers to abnormal ship turning [4], U-turns [4], and reverse driving [5-6].

Data-driven anomaly detection methods mainly include nearest neighbor algorithms, the expert system, clustering algorithms, autoencoders, recurrent neural networks and other algorithm models. In a nearest neighbor method, La et al. [7] compared the trajectory data to be detected with all the data stored in the database, found the most similar ones, and then classified them. The expert system method formulates rules for abnormal trajectories through experienced experts, and classifies the trajectories according to these rules. Based on the communication principle of AIS, Wei et al. [8] proposed data error detection methods such as identification code verification of maritime mobile services, comprehensive detection of padding bits and verification codes, matching verification of dynamic information, static information, and abnormal location point detection. Based on an in-depth analysis of a large amount of AIS data, Wu et al. [9] summarized several types of abnormal

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AIS trajectories, and obtained corresponding detection rules for each type of feature, realizing automatic detection of trajectory abnormalities. The clustering algorithm aims to classify the data into multiple groups. The dense classes are regarded as normal trajectories, while the sparse classes are abnormal trajectories. Rong et al. [10] combined the trajectory compression algorithm with the clustering algorithm to identify the ship's navigation process. Relevant waypoints on routes where significant changes have occurred in. Zhen et al. [11] proposed an anomaly detection method combining ship trajectory clustering and the naive Bayesian classifier. The method first designed a similarity measure between ship trajectories according to the spatial and directional characteristics of ship trajectories, then hierarchical clustering is used to model and learn, and Naive Bayes is used to classify and detect anomalies in ship trajectories. Wei et al. [12] used Hausdorff's metric method to propose a ship motion pattern recognition method based on Density-Based Spatial Clustering of Applications with Noise (DBSCAN) [13-14] clustering algorithm, which provided theoretical support for trajectory anomaly detection.

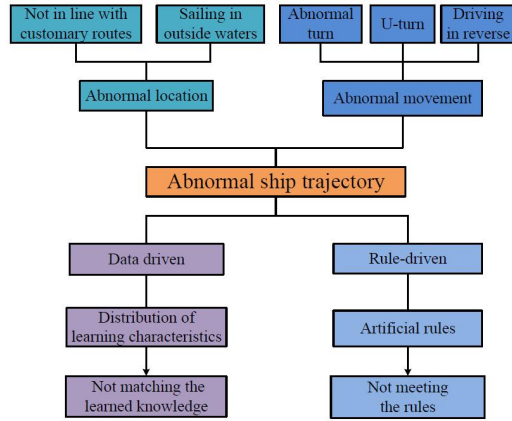


Fig.1. Definition of abnormal ship trajectory

As early as 2015, An et al. [15] discussed the superiorities of Variational Autoencoder (VAE) in data anomaly detection, and then VAE has been gradually applied and developed in this field. Roy et al. [16] proposed a road trajectory anomaly detection method based on Deep Autoencoder (DAE), and used historical trajectory data to train DAE to obtain the anomaly detection model. Qin et al. [17] proposed a hurricane trajectory anomaly detection method based on VAE. Used the trajectory segment obtained by scanning the fixed-length sliding window, the trajectory to be detected was taken as the input of the trajectory reconstruction model and output as the reconstructed trajectory. Finally, the reconstructed trajectory is compared with the input trajectory through the similarity measurement distance to find out the abnormal trajectory segment.

In the current research, the expert system needs to artificially stipulate some discrimination rules, which are prone to missed detection and false detection. At the same time, unbalanced positive and negative samples increase the difficulty of training, and the models such as autoencoders have weaker model robustness and lower complexity of latent space. There is still room for improvement in the ability to learn multiple categories of ship trajectory data.

The main contributions of this study are twofold.

- (1) A Gaussian mixture variational autoencoder is proposed to solve the problems that the trajectory anomaly detection model based on variational autoencoder is insensitive to multi-class trajectory data, and the inferred distribution of a single Gaussian prior and approximate posterior has poor variability.
- (2) The GMVAE model is used to realize the variational autoencoder is insensitive to multi-class trajectory data.

The novelties of this paper are threefold.

- (1) It develops a new method to effectively deal with ship trajectory anomaly detection.
- (2) It systemically realizes the GMVAE model to detect multi-class trajectory and the inferred distribution of a single Gaussian prior anomaly data
- (3) The study on abnormal ship track, provide targeted technical support for ship track data analysis and risk control methods for ship supervision departments, which is conducive to reducing the occurrence of maritime traffic accidents and ensuring the safety of people's lives and property.

The rest of this paper is organized as follows: Section II provides details of the methods. The information of the data set and the investigated water area is shown in Section III, and the experiments are presented in Section IV. This research is concluded in Section V.

## II. METHODS

The proposed anomaly detection framework is shown in Figure 2. The input is the original ship trajectory data  $T_0$ .  $T_1$  is the trajectory data after GMVAE reconstruction, and the output is the DTW distance between the original trajectory and the reconstructed trajectory. By calculating the original trajectory and the DTW distance between the reconstructed trajectory, and comparing the DTW distance with the set threshold  $Thd$ , if the distance is greater than a certain threshold, it is considered abnormal, otherwise, it is normal.

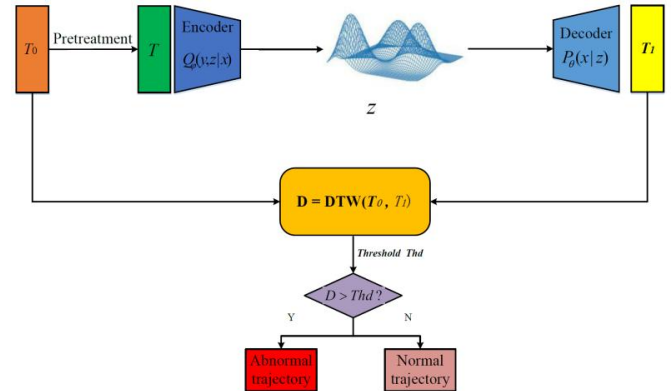


Fig.2. The process of the anomaly detection model for ship trajectory data

### A. Variational Autoencoder (VAE)

Generally, the theory of VAE can be explained by two viewpoints: variational inference and neural network. This section will first use variational inference to explain them, and then investigates the connections between them. Assuming that the input is  $x$ , and the hidden variable is  $z$ . The purpose of VAE is to find the posterior distribution  $P(z|x)$ . The posterior distribution can be expressed according to Bayes' theorem

using likelihood function  $P(x|z)$ , and the marginal probability density function  $P(x)$  of prior distributions  $P(z)$  and  $x$  for:

$$P(z|x) = \frac{P(x|z)P(z)}{P(x)} \quad (1)$$

$P(x)$  can be calculated by  $P(x) = \int dz P(x|z)P(z)$ , but this integral calculation for  $z$  is generally impossible to find. Therefore, it is necessary to use variational inference to find an easy-to-calculate distribution  $Q_\phi(z|x)$  (such as the Gaussian distribution) to approximate the posterior distribution  $P(z|x)$ , where  $\phi$  is the approximate posterior parameter. Next, minimize the KL divergence between  $Q_\phi(z|x)$  and  $P(z|x)$  to obtain the optimal approximate distribution. The KL divergence between  $Q_\phi(z|x)$  and  $P(z|x)$  can be expressed as:

$$\begin{aligned} D_{KL} D_{KL}[Q_\phi(z|x) \parallel P(z|x)] &= \sum_z Q_\phi(z|x) \log \frac{Q_\phi(z|x)}{P(z|x)} \\ &= E_{Q_\phi(z|x)} \left[ \log \frac{Q_\phi(z|x)}{P(z|x)} \right] \\ &= E_{Q_\phi(z|x)} [\log Q_\phi(z|x) - \log P(z|x)] \end{aligned} \quad (2)$$

Substitute formula (1) into:

$$\begin{aligned} D_{KL}[Q_\phi(z|x) \parallel P(z|x)] &= E_{Q_\phi(z|x)} \left[ \log Q_\phi(z|x) - \log \frac{P(x|z)P(z)}{P(x)} \right] \\ &= E_{Q_\phi(z|x)} [\log Q_\phi(z|x) - \log P(x|z) - \log P(z) + \log P(x)] \end{aligned} \quad (3)$$

Since the expectation is on  $z$ , the term  $\log P(x)$  is a constant, so move it to the left:

$$\begin{aligned} D_{KL}[Q_\phi(z|x) \parallel P(z|x)] - \log P(x) &= \\ - E_{Q_\phi(z|x)} [\log P(x, z) - \log Q_\phi(z|x)] &= \end{aligned} \quad (4)$$

$\text{ELBO}(\phi)$

Therefore, minimizing KL divergence is equivalent to maximizing the lower variational bound (ELBO) in formula (4). Decomposing ELBO and formula (4) can be further rewritten as:

$$\begin{aligned} D_{KL}[Q_\phi(z|x) \parallel P(z|x)] - \log P(x) &= \\ = D_{KL}[Q_\phi(z|x) \parallel P(z)] - E_{Q_\phi(z|x)} [\log P(x|z)] \end{aligned} \quad (5)$$

Figure 3 shows the neural network framework of VAE.  $Q_\phi(z|x)$  is an encoder, which is used to encode data into hidden variables  $z$ .  $P_\theta(x|z)$  is the decoder, which is responsible for decoding the hidden variable  $z$  to generate reconstructed data, where  $\theta$  and  $\phi$  are the decoder neural network parameters (weights and biases). The goal of the model is to minimize the right part of the formula (5), where the first term on the right of the formula (5) is the KL divergence of the encoder  $Q_\phi(z|x)$  and the prior distribution  $P(z)$ , and the second term is the reconstruction of the data and the original data error.

Even though VAE introduces Bayesian theory, there are still some shortcomings in actual use. For example, in the field of image research, the reconstructed image is sometimes very blurry. Because the hidden variable  $z$  of VAE is too simple, it cannot accurately describe the characteristic distribution law of the original data. Therefore, it is necessary to construct a more complex, flexible, and descriptive distribution in practical applications, namely the Gaussian mixture model introduced in the next section.

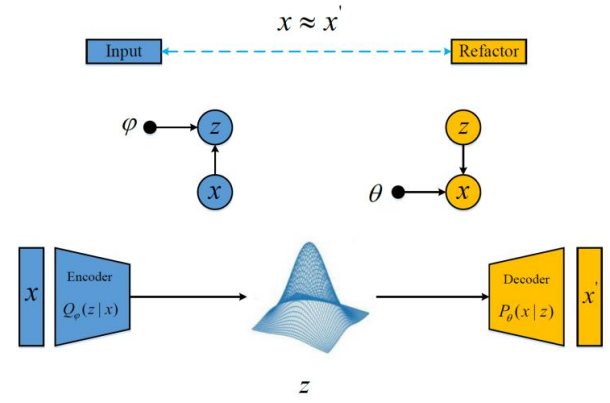


Fig.3. Schematic diagram of the structure of a variational autoencoder

### B. Gaussian Mixture Variational Automatic Variational Encoder (GMVAE)

The prior distribution of latent variable  $z$  and the approximate posterior inferred distribution are usually a single gaussian distribution, which will lead to the model's learning effect on data with higher complexity poor, resulting in unreal and incomplete reconstructed data. In theory, the Gaussian mixture distribution can describe the probability distribution of any shape. Therefore, according to the above properties, this paper sets the prior distribution  $P(z)$  of the hidden variable  $z$  and the approximate posterior inferred distribution  $Q_\phi(y, z|x)$  to the Gaussian mixture distribution to realize the model's learning and feature description of the original data more accurately. At the same time, variable  $y$  is introduced to represent the Gaussian distribution category corresponding to the data. According to the inference process of the variational autoencoder, the inference model of the GMVAE in this research can be expressed as:

$$Q_\phi(y, z|x) = Q_\phi(y|x)Q_\phi(z|x, y) \quad (6)$$

The latent variable space is a Gaussian mixture model composed of  $k$  Gaussian distribution. Where  $Q_\phi(z|x, y_i)$  is assumed to be a single Gaussian distribution, where  $i \in 0, 1, \dots, k-1$ . Therefore, the approximate posterior distribution becomes a Gaussian mixture. ELBO can be expressed as:

$$\text{ELBO}_m = E_{Q_\phi(y, z|x)} [\log P_\theta(x, y, z) - \log Q_\phi(y, z|x)] \quad (7)$$

Among them, the number of  $k$  in the Gaussian distribution is a hyperparameter, and the subscript  $m$  is ELBO distinguish the AE.  $P_\theta(x, y, z)$  can be expressed as  $P_\theta(x, y, z) = P_\theta(x|y, z)P_\theta(z|y)P(y)$ . Through the conditional independence hypothesis  $P_\theta(x|y, z) = P_\theta(z|y)$  (as shown in the probability graph model in Figure 4) can be obtained, and the joint probability can finally be expressed as:

$$P_\theta(x, y, z) = P_\theta(x|z)P_\theta(z|y)P(y) \quad (8)$$

Substituting formula (6) and formula (8) into formula (7),  $\text{ELBO}_m$  can be expressed as:

$$\begin{aligned} \text{ELBO}_m &= E_{Q_\phi(y, z|x)} [\log P(y)P_\theta(z|y)P_\theta(x|z) \\ &\quad - \log Q_\phi(y|x)Q_\phi(z|x, y)] \\ &= E_{Q_\phi(y, z|x)} \left[ \log P(y) - \log Q_\phi(y|x) + \log \frac{P_\theta(z|y)}{Q_\phi(z|x, y)} + \right. \\ &\quad \left. \log P_\theta(x|z) \right] \end{aligned} \quad (9)$$

Similar to VAE, the third and fourth terms in formula (9) respectively represent the contribution of the regularization

term and data reconstruction to the loss function. The prior distribution of  $y$  is the distribution.  $E_{Q(y,z|x)}[\log Q_\theta(y|x)]$  can be regarded as conditional entropy, reflecting the amount of information on  $x$  in  $y$ . In order to directly control the impact of clustering loss items on other loss items in the process of training the model, a weighting factor  $\alpha$  can be introduced to the mutual information between  $x$  and  $y$ :

$$\text{ELBO}_m = E_{Q(y,z|x)}[\log P(y) - \alpha \log Q_\theta(y|x) + \log \frac{P_\theta(z|y)}{Q_\theta(z|x,y)} + \log P_\theta(x|z)] \quad (10)$$

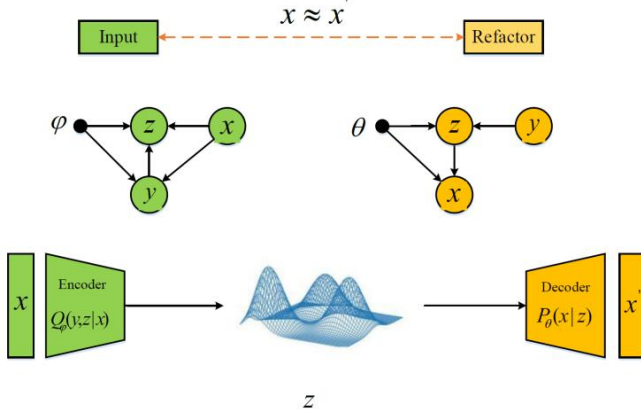


Fig.4. The Schematic diagram of the structure of GMVAE

Figure 4 is a detailed flow chart of the GMVAE algorithm. Table I summarizes the probability distribution used by the model.  $NN(*)$  in the diagram represents the neural network. First, the input data is clustered into  $k$  classes according to the probability( $NN(Q_y)$ ),  $Q(y|x)$  represents the probability that the data is allocated to these classes, and obeys the polynomial distribution. Since it is assumed that the hidden space of each cluster category obeys the Gaussian distribution, the neural network ( $NN(Q_z)$ ) in the encoder can be used to learn the characteristic law of each type of data, that is, the mean and variance of each Gaussian distribution ( $Q(z|x, y_i)$ ). Then, through the first sampling process and the mathematical calculation process of seeking expectations shown in formula (11), the hidden variable  $z$  of the data is obtained.

$$z = \sum_{i=0}^{k-1} P(y_i|x) z_i \quad (11)$$

In the decoding process,  $NN(P_z)$  generates the mean and variance from the corresponding category, and obtains the latent variable  $z$  through reparameterization sampling, where  $P(y)$  is uniformly distributed. Then, decode  $NN(P_x)$  to  $z$ , and obtain the corresponding reconstructed data  $x'$  through sampling and the expectation process shown in formula (12).

$$x' = \sum_{i=0}^{k-1} P(y_i|x) x'_i \quad (12)$$

TABLE I  
THE PROBABILITY DISTRIBUTIONS OF GMVAE

Encoder	Decoder
$Q(z x, y) = N(\mu_z(x, y), \sigma_z^2(x, y))Q(z x, y)$	$P(y) = \text{Uniform}(\frac{1}{k})$
$Q(y x) = \text{Multinomial}(f(x))$	$P(z y) = N(\mu_z(y), \sigma_z^2(y))$
	$P(x z) = N(\mu_x(z), \sigma_x^2(z))$

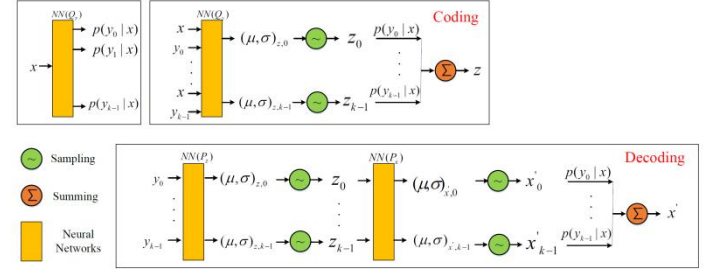


Fig.5. The process of GMVAE

### III. DATA SET AND WATER AREA INFORMATION ANALYSIS

#### A. Data Set

The data used in this research is the public AIS dataset from the U.S. Coast Guard (<https://marinecadastre.gov/ais/>). The main reason for selecting the AIS data in the coastal waters of the United States is that it belongs to one of the most commonly used public data sets in the research filed of maritime traffic modelling. See, for example, in some recent research, such as Zhang et al [24], (2021), Han et al [25], (2022), and Huang et al [26], (2022). This makes the validation of the proposed method and the repetition of experimental results possible. Since this is a general method, it can be used to other date source, and its applicability for the AIS date of inland rivers in China has been tested.

In the experiment, the data set from January to March 2019 was used for model training, and that in September 2019 was used for testing. The data set from January to March 2019 included 2229 different ships and 263149 trajectory points. After trajectory preprocessing, 3120 ship trajectories are obtained. At the same time, some data in the data set in September 2019 were manually marked with anomalies. Table II describes the establishment of the marked data set and the analysis process of abnormal ship tracks in detail. The basic information after labeling is shown in Table II.

TABLE II  
THE INFORMATION OF MANUALLY LABELED DATASET

Trajectory points	Trajectories	Abnormal trajectories	Abnormal proportion
42020	764	23	3.01%

#### B. Water Information Analysis

The analysis waters selected in this paper are located between 48°54'7.28" north latitude to 49°3'44.28", and 123°1'43.33" to 123°25'2.71" west longitude. Figure 6 shows the map and electronic chart of the waters under investigation.



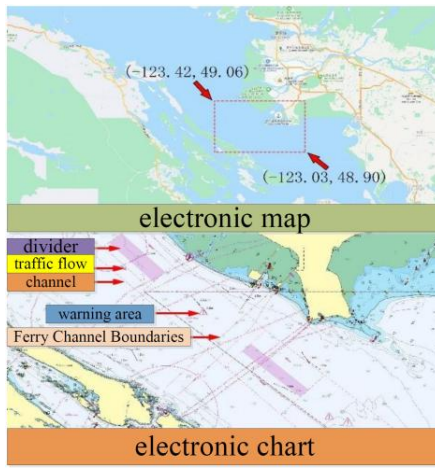


Fig.6. The map and ECDIS of the water

The chart analysis of the water area shows that it contains four traditional waterways. The traffic flow density map is drawn based on the real ship trajectory in January 2019. The visualization results of the real ship trajectory are basically the same as the traditional waterway results in the electronic chart, except that there is an additional route extending northward from the opposite bank of the wharf. Therefore, the ship trajectories in this water area are divided into five categories in this paper. Figure 7 shows the waterway and its traffic flow density in the analyzed waters.

TABLE III

THE PSEUDOCODE OF SHIP TRAJECTORY PREPROCESSING

The pseudocode for ship trajectory preprocessing	
Input: An AIS ship trajectory $T$ , $T$ contains $N$ trajectory points $S_i$ , the corresponding sampling time is $t_i$ , Where $i = 1, \dots, N$ . Velocity Threshold $Thd_v$ , Time Threshold $Thd_t$ and Trajectory Point Number Threshold $Thd_L$	
Step1 Traverse each trajectory point	
Step2 Calculate the distance $d_i$ between $S_i$ and $S_{i+1}$ , the time interval $t_{i+1} - t_i$ , and find the speed $v_i$	
Step3 If $t_{i+1} - t_i > Thd_t$ or $v_i < Thd_v$ , split the current trajectory point	
Step4 Continue to traverse the next trajectory point, $i = i + 1$ , if $i + 1 > N$ , end the traversal	
Step5 Traverse each sub-trajectory subT obtained by the segmentation, and calculate the number of sub-trajectory points $L$	
Step6 If $L < Thd_L$ , then cull the sub trajectory	
Step7 End the traversal of the sub trajectory	
Output: Get the trajectory data after eliminating invalid trajectories	

TABLE IV

THE THRESHOLD OF SHIP TRAJECTORY PREPROCESSING

Threshold type	default value
Time threshold	600 S
Speed threshold	1 Section
Trajectory point capacity threshold	20

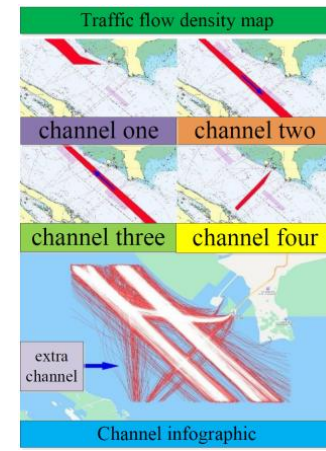


Fig.7. The traffic flow density and channel information of the water area

### C. Ship Trajectory Processing

The purpose of ship trajectory preprocessing is to eliminate invalid trajectories. The invalid trajectory is generally concentrated in a small area of water with little movement of the ship trajectory, or the sampling interval is too long to characterize the continuous motion characteristics of the ship trajectory. Eliminating invalid trajectories can eliminate the interference of ship anchoring points on the trajectory analysis of moving ships, reduce the complexity of ship trajectories, and help to mine high-value information of ship trajectories. Invalid trajectory elimination is usually based on the time intervals and ship speed changes. Table III shows the pseudo code of the processing steps, Table IV is the threshold parameter setting of the ship trajectory preprocessing, and Table V is the statistical results after the ship trajectory preprocessing.

TABLE V

THE RESULT OF SHIP TRAJECTORY PREPROCESSING

	Ship MMSI number	Trajectories	Total trajectory points
Before	871	/	263149
After	868	3120	144438

TABLE VI

MODEL STRUCTURE

Type	Parameter
Input layer size	100
Hidden space dimension	64
The number of nodes per layer of $NN(Q_y)$	512
The number of nodes per layer of $NN(Q_x)$	512
The number of nodes per layer of $NN(P_y)$	64
The number of nodes per layer of $NN(P_x)$	512

## IV. EXPERIMENT

### A. Model Building and Hyperparameter Setting

The experiments in this paper are carried out on the TensorFlow deep learning framework. Drawing on the method of Chen [18] and others, using cubic spline interpolation, each ship trajectory data is sampled into 50 points, the input format of the model training is  $[50, 2]$ , 50 represents the length of the trajectory after sampling, 2 indicates the longitude and latitude of the trajectory point, and the longitude and latitude are normalized to the maximum and minimum values; all neural

networks in the model are fully connected neural networks; the optimizer in the training process is the Adam optimization algorithm; The activation function of the classification layer of the neural network is Soft-max; in other neural networks, the activation function of the output layer that obtains the Gaussian variance is Soft-plus, and the activation functions of all the remaining hidden layers are Re-Lu; hyperparameter  $k$  is the number of classes obtained through the analysis of water information. When  $k$  is set to 1, the model is a VAE model. Table VI and Table VII introduce the model structure and hyperparameters in detail.

TABLE VII  
HYPERPARAMETERS

Type	Parameter
Gaussian mixture category $k$	5
Weighting factor $\alpha$	0.5
Batch size	64
Learning rate	0.001
Number of iterations	150
Number of training sets	3240
Number of test sets	500

The abnormal ship trajectories in this paper refer to (1) Violation of the "General Provisions of Ship Routing System"; (2) The number of identical trajectories in the set of ship trajectories is small; (3) Spatially deviates from other trajectories; (4) Most of the trajectories of neighboring ships have significantly different motion characteristics. On the contrary, normal ships should abide by the "General Regulations of Ship Routing System" [22]. When a ship runs in a specified channel according to the specified traffic operation route, the normal ship track usually has good continuity and smoothness [19].

### B. Analysis of Training Results

Figure 8 shows the change of the loss during the training process. The loss curve gradually converges. When the number of iterations reaches 60, the loss value tends to be stable. In addition, the classification accuracy of the neural network for the ship trajectory data test set was analyzed. Figure 9 shows the classification accuracy of the test set data. With the increase of the number of iterations, the neural network can accurately classify the 5 types of ship trajectories. The highest accuracy rate is 98.15%.

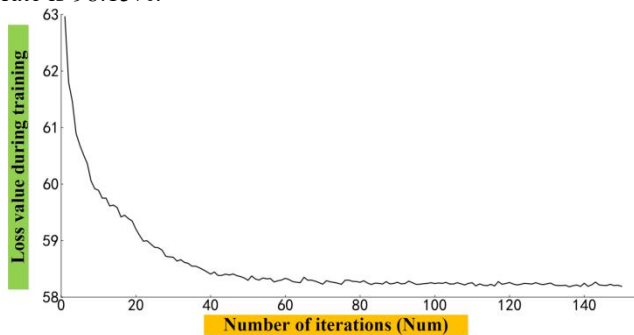


Fig.8. The curve of loss value

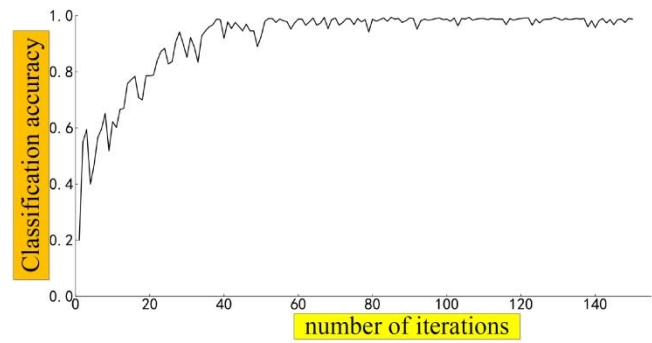


Fig.9. The accuracy of neural network

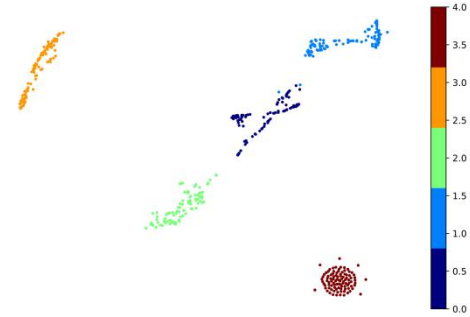


Fig.10. The visualization of GMVAE hidden space

The training results of GMVAE were evaluated by 2D visualization of high-dimensional latent variables in the latent space of GMVAE using the t-SNE data visualization tool. As shown in Figure 10, the latent space is divided into five categories, and the aggregation degree of each category of data is relatively high, indicating that the GMVAE model training has obtained an accurate Gaussian mixture model composed of five Gaussian distributions. Therefore, it can be seen from the above analysis results that the training process of the GMVAE model has achieved the expected effect.

### C. Experiment data water anomaly results

This paper first conducts anomaly detection of ship trajectories in the coastal waters of the United States from September 1st to September 10th, 2019. The data to be detected contains 353 ship trajectories, and the threshold is set to 0.07. The detection results are shown in Figure 11.

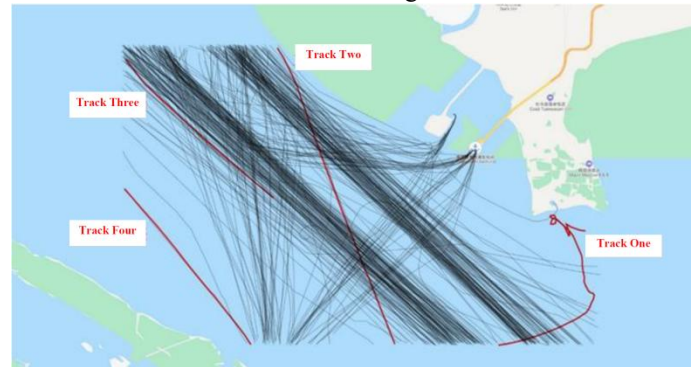


Fig.11. The anomaly detection result of the dataset

The red line segments in Figure 11 are the abnormal ship trajectories detected by the GMVAE model, and the number of detected abnormal ship trajectories is 4. No.1 ship's trajectory, the ship's navigation trajectory route is significantly different

from other ship trajectories, it does not travel in the waterway specified on the chart, and the range of continuous changes in the heading angle is too large, which does not conform to the general ship's driving rules; No. 2 ship's trajectory, according to the "General Regulations of Vessel Alignment System", the ship needs to travel in the specified direction of traffic operation, and the ship has the behavior of crossing the waterway in two directions; No. 3 ship's trajectory, the trajectory is incomplete. The trajectory segment of , the trajectory length is about half of the complete trajectory in the figure, and it is also judged as abnormal by the model; the No. 4 ship trajectory, from the results of the water area information analysis, it can be seen that the number of ship trajectories in this water area is very small. Because GMVAE is the learning of historical knowledge, for a small number of ship trajectories in historical data, GMVAE cannot perfectly learn the characteristic distribution of ship trajectories, and these trajectories are identified as abnormal trajectories. Table VIII describes in detail the MMSI, type and analysis of the abnormal ship trajectories in Figure 11.

#### D. Analysis of Threshold's Influence on Anomaly Detection Results

In order to verify the influence of the change of the threshold on the detection results of the GMVAE model, in this experiment, the selection of threshold in this paper is carried out according to the method in reference [24]. The outlier is defined as  $\omega_1$ , the sampling value at  $t$  is set to  $x_t$ , and the predicted value at time  $t$  is calculated as  $x'_t$ :

$$x'_t = x_{t-1} + (x_{t-1} - x_{t-2}) \quad (13)$$

$$\omega_1 = k_1 \times SOG_{m-1} \quad (14)$$

If  $|SOG_m - SOG'_m| > \omega_1$ , the  $SOG_m$  are abnormal data, the value  $SOG'_m$  can be obtained from formula (13), and the abnormal threshold  $\omega_1$  can be obtained from formula (14).

The threshold value of this experiment can be obtained by the above formulas, and the distance threshold was set to 0.060, 0.065, 0.069, and 0.080, respectively. Figures 12, 13, 14, and 15 show the detection results of abnormal ship trajectories in U.S. coastal waters from September 1st to September 10th, 2019 under different thresholds, and the red line segments in the figure are abnormal trajectories. When the threshold is 0.080, 0.069, 0.065, 0.060, the number of detected abnormal trajectories are 1, 5, 14, and 20, respectively. As the threshold increases, the number of detected anomalous trajectories decreases. Although the selection of the threshold will have a certain impact on the detection of abnormal ship trajectories, if the threshold is set in an appropriate range, the ship trajectories with obvious possibility of abnormality can be detected.

TABLE VIII

THE INFORMATION OF ANOMALOUS TRAJECTORIES

Number in Figure	MMSI	Ship type	Analysis of the abnormal cause
One	338140949	Sailing	The range of continuous change of heading angle is too large
Two	316006786	Fishing	Violation of the "General Provisions on

			Ship Routing System" Incomplete trajectory there were abnormalities in the navigation area
Three	316005616	Tugboat	
Four	366993790	Fishing	

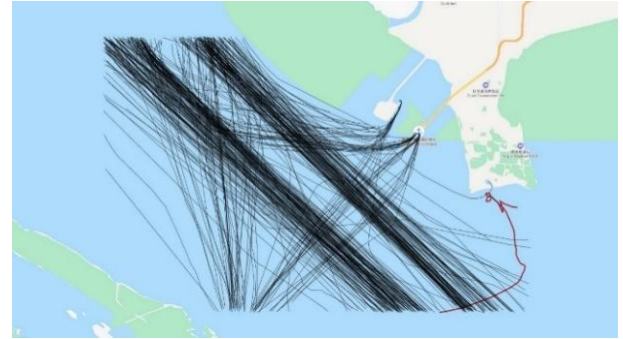


Fig.12. The detection result (Thd=0.080)



Fig.13. The detection result (Thd=0.069)

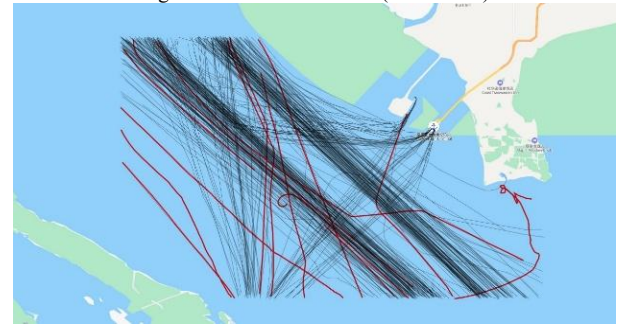


Fig.14. The detection result (Thd=0.065)

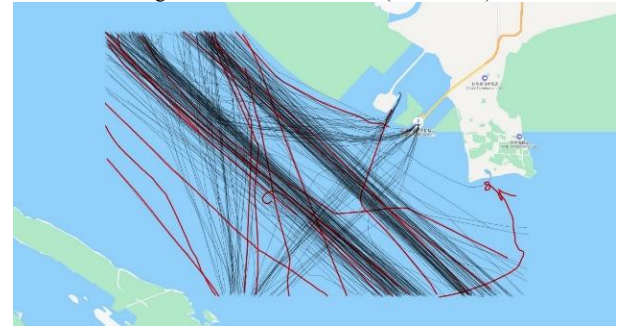


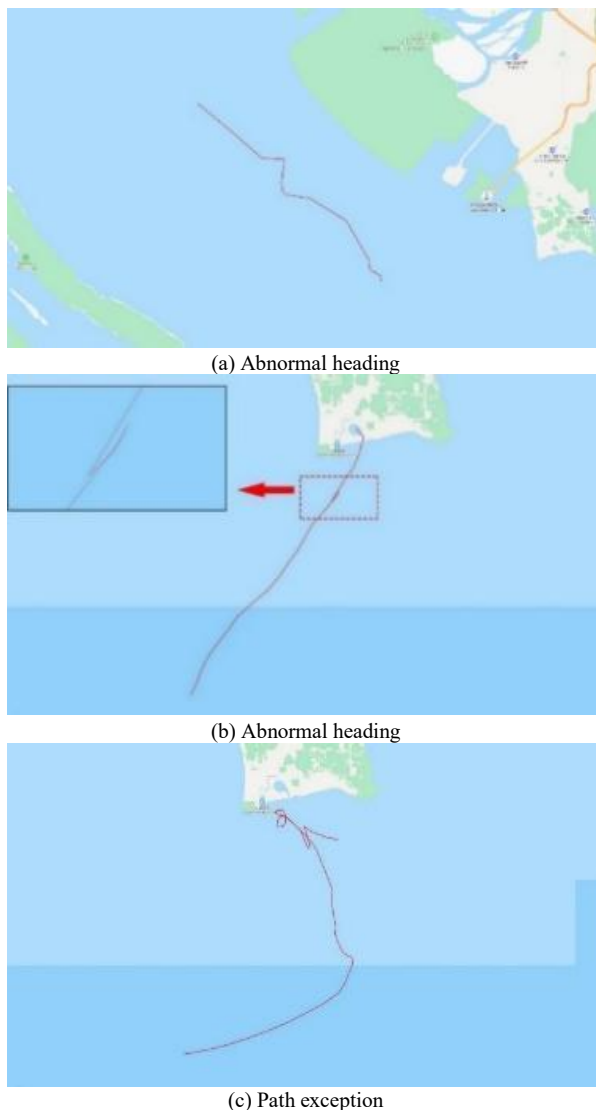
Fig.15. The detection result (Thd=0.060)

#### E. Comparison with VAE Trajectory Anomaly Detection Model

Under the guidance of experts in the maritime domain, according to Hawkins' definition of abnormality in the



introduction, Sidibé Abdoulaye et al. [20] defined the abnormal ship trajectory and "General Provisions for Ship Routing". At the same time, referring to the method of abnormal trajectory annotation of Han Zhaorong [21], statistical analysis and data annotation were carried out on the ship trajectory data in the coastal waters of the United States in September 2019, and a data set of ship trajectory annotation in the waters where the experimental data was located and established. The data set includes 764 pieces of normal ship trajectory data and 23 pieces of abnormal ship trajectory data. Table VIII introduces the specific information of manual annotation data set information. Part of the abnormal ship trajectory data is shown in Figure 16, in which the abnormal ship trajectory is mainly divided into abnormal heading and abnormal navigation path.



(d) Path exception

Fig. 16. The anomaly trajectories of the dataset

It can be seen in (a) that the variation range of the ship's heading angle with an abnormal trajectory is significantly higher than that of the historical ship trajectory in the channel; while in (b), the steering range of the ship is too large, which does not conform to the law of ship motion; the ship generally It is stipulated to drive in a fixed waterway, and the ship's trajectory path in (c) is far from the main waterway displayed by the chart and clustering results, so there is an abnormality; The ship's trajectory path in (d) crosses the waterway of two traffic running directions, so there is an abnormality.

In the training process, the category hyperparameter  $k$  of the GMVAE model is set to 1, and a VAE trajectory anomaly detection model with the same parameters as the GMVAE trajectory anomaly detection model is obtained [17]. This paper compares the detection results of the GMVAE trajectory anomaly detection model with the detection results of the VAE trajectory anomaly detection model on the custom data set, and takes the detection rate (DR) and false alarm rate (FAR) as the anomaly detection evaluation index is used to evaluate the anomaly detection results and verify the superiority of the method in this paper. The selection of the threshold needs to be determined by the actual situation. Through the analysis of the influence of the threshold on the abnormal detection results, it can be seen that if a higher detection rate is required, the lower position in the ranking can be selected for truncation; if a lower false alarm rate is required, you can choose to truncate at the first position in the sorting.

In this paper, the thresholds are set to 0.06, 0.065, 0.069, and the anomaly detection results of the two algorithms are compared. The results are shown in Figure 17. An excellent detection algorithm should have a high DR value and a low FAR value. Under different thresholds, the detection rate of GMVAE is always higher than that of VAE; except when the threshold is 0.069, the false alarm of GMVAE The rate is slightly higher than that of VAE, and it is lower than the false alarm rate of VAE at other thresholds. After analyzing the detection rate and false alarm rate of the two detection algorithms, 0.069 and 0.065 were selected as the best thresholds for GMVAE and VAE, respectively, and compared. The results are shown in Table IX. Comparing with other models, the GMVAE performs the best with respect to both detection rate (91.26%) and false alarm rate (0.68%). The analysis results of proved the superiorities of the proposed GMVAE algorithm.

TABLE IX



THE RESULT OF VAE AND GMVAE WITH THE BEST THRESHOLD

Model Method	Threshold	DR	FAR
GMVAE	0.069	91.26%	0.68%
VAE <sup>[8]</sup>	0.065	65.21%	2.01%
MA-GAN <sup>[23]</sup>	0.067	62.41%	2.76%
DTW <sup>[14]</sup>	0.063	54.36%	3.06%
BN <sup>[16]</sup>	0.061	42.76%	5.13%
Bi-LSTM <sup>[18]</sup>	0.060	40.58%	6.24%

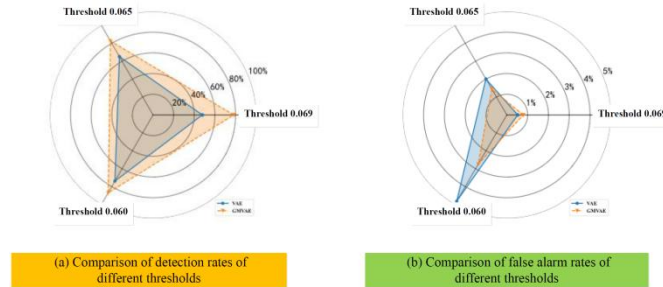


Fig.17. The result of VAE and GMVAE with different thresholds

## V. CONCLUSION

This paper proposes a method for anomaly detection of ship trajectories based on Gaussian mixture variational autoencoder. The prior distribution and approximate posterior inference distribution of the variational autoencoder are changed to a mixture Gaussian model to learn multi-class ship trajectories. On this basis, a GMVAE ship trajectory anomaly detection model is established. Anomaly detection experiments are carried out using the US coastal AIS ship trajectory data set, and the detection results are reasonably explained according to the definition of anomalous ship trajectories. Moreover, the influence of different thresholds on the detection effect of ship trajectory is analyzed. The experiment reveals that GMVAE with Gaussian mixture latent space has a stronger ability to describe the distribution of multi-class ship trajectory features than the original VAE, and the anomaly detection method using unsupervised classification can ensure the accuracy of detection while simplifying data annotation before model training.

In future research, the following two aspects can be further explored. First, the AIS data collected in the field can be analyzed, and ship trajectory data can be extracted for further exploration. Second, deep learning algorithms can be used to classify ship trajectories to distinguish the navigation characteristics of different types of ships in different waterways. Especially with the rapid development of graph neural networks, it can be considered in the research of ship trajectory classification, which may provide a new path for water traffic data mining.

## VI. ACKNOWLEDGMENT

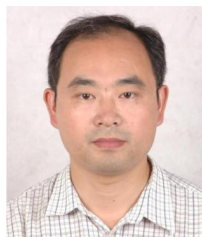
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## REFERENCES

- [1] D. Gong et al., "Memorizing normality to detect anomaly: Memory augmented deep autoencoder for unsupervised anomaly detection," in Proc. IEEE/CVF Int. Conf. Comput. Vis. (ICCV), Oct. 2019, pp. 1705–1714.
- [2] Y. Lyu, Z. Han, J. Zhong, C. Li, and Z. Liu, "A generic anomaly detection of catenary support components based on generative adversarial networks," IEEE Trans. Instrum. Meas., vol. 69, no. 5, pp. 2439–2448, May 2020.
- [3] H. Lv, C. Zhou, Z. Cui, C. Xu, Y. Li, and J. Yang, "Localizing anomalies from weakly-labeled videos," IEEE Trans. Image Process., vol. 30, pp. 4505–4515, 2021.
- [4] P. Wu and J. Liu, "Learning causal temporal relation and feature discrimination for anomaly detection," IEEE Trans. Image Process., vol. 30, pp. 3513–3527, 2021.
- [5] J.-C. Feng, F.-T. Hong, and W.-S. Zheng, "MIST: Multiple instance self-training framework for video anomaly detection," in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit. (CVPR), Jun. 2021, pp. 14009–14018.
- [6] W. Sultani, C. Chen, and M. Shah, "Real-world anomaly detection in surveillance videos," in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit., Jun. 2018, pp. 6479–6488.
- [7] C. Huang et al., "Online learning-based multi-stage complexity control for live video coding," IEEE Trans. Image Process., vol. 30, pp. 641–656, 2021.
- [8] K. K. Santhosh, D. P. Dogra, P. P. Roy, and B. B. Chaudhuri, "Trajectory-based scene understanding using Dirichlet process mixture model," IEEE Trans. Cybern., vol. 51, no. 8, pp. 4148–4161, Aug. 2021.
- [9] R. Morais, V. Le, T. Tran, B. Saha, M. Mansour, and S. Venkatesh, "Learning regularity in skeleton trajectories for anomaly detection in videos," in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit. (CVPR), Jun. 2019, pp. 11988–11996.
- [10] W. Liu, W. Luo, D. Lian, and S. Gao, "Future frame prediction for anomaly detection—A new baseline," in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit., Jun. 2018, pp. 6536–6545.
- [11] W. Luo, W. Liu, and S. Gao, "Remembering history with convolutional LSTM for anomaly detection," in Proc. IEEE Int. Conf. Multimedia Expo (ICME), Jul. 2017, pp. 439–444.
- [12] S. Lee, H. G. Kim, and Y. M. Ro, "BMAN: Bidirectional multi-scale aggregation networks for abnormal event detection," IEEE Trans. Image Process., vol. 29, pp. 2395–2408, 2020.
- [13] N. Li, F. Chang, and C. Liu, "Spatial-temporal cascade autoencoder for video anomaly detection in crowded scenes," IEEE Trans. Multimedia, vol. 23, pp. 203–215, 2021.
- [14] Y. Zhao, B. Deng, C. Shen, Y. Liu, H. Lu, and X.-S. Hua, "Spatiotemporal AutoEncoder for video anomaly detection," in Proc. 25th ACM Int. Conf. Multimedia, Oct. 2017, pp. 1933–1941.
- [15] W. Luo, W. Liu, and S. Gao, "A revisit of sparse coding based anomaly detection in stacked RNN framework," in Proc. IEEE Int. Conf. Comput. Vis. (ICCV), Oct. 2017, pp. 341–349.
- [16] M. Sabokrou, M. Khalooci, M. Fathy, and E. Adeli, "Adversarially learned one-class classifier for novelty detection," in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit., Jun. 2018, pp. 3379–3388.
- [17] P. Wu, J. Liu, and F. Shen, "A deep one-class neural network for anomalous event detection in complex scenes," IEEE Trans. Neural Netw. Learn. Syst., vol. 31, no. 7, pp. 2609–2622, Jul. 2020.
- [18] H. Park, J. Noh, and B. Ham, "Learning memory-guided normality for anomaly detection," in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit. (CVPR), Jun. 2020, pp. 14372–14381.

- [19] J. Wang and A. Cherian, "GODS: Generalized one-class discriminative subspaces for anomaly detection," in Proc. IEEE/CVF Int. Conf. Comput. Vis. (ICCV), Oct. 2019, pp. 8200–8210.
- [20] H. Song, C. Sun, X. Wu, M. Chen, and Y. Jia, "Learning normal patterns via adversarial attention-based autoencoder for abnormal event detection in videos," IEEE Trans. Multimedia, vol. 22, no. 8, pp. 2138–2148, Aug. 2020.
- [21] Z. Fang, J. Liang, J. T. Zhou, Y. Xiao, and F. Yang, "Anomaly detection with bidirectional consistency in videos," IEEE Trans. Neural Netw. Learn. Syst., vol. 33, no. 3, pp. 1079–1092, Mar. 2022.
- [22] C. Huang, J. Wen, Y. Xu, et al., "Self-supervised attentive generative adversarial networks for video anomaly detection," IEEE Trans. Image Process., vol. 20, pp. 621–626, 2021.
- [23] R. Liu, W. Liu, H. Li, et al., "Metro anomaly detection based on light strip inductive key frame extraction and MAGAN network," IEEE Transactions on Instrumentation and Measurement, vol. 71, pp. 1–14, 2022.
- [24] J. Hou, Y. Zhang, Q. Zhong, et al., "Divide-and-Assemble: Learning Block-wise Memory for Unsupervised Anomaly Detection," in Proc. IEEE/CVF Int. Conf. Comput. Vis. (ICCV), Oct. 2021, pp.5631–5641.
- [25] Wang, Y., Liu, J., Liu, R. W., Liu, Y., & Yuan, Z., "Data-driven methods for detection of abnormal ship behavior: Progress and trends," Ocean Engineering, pp.271-281, 2023.
- [26] Liu, R. W., Liang, M., Nie, J., Yuan, Y., Xiong, Z., Yu, H., & Guizani, N., "STMGCN: mobile edge computing-empowered vessel trajectory prediction using spatio-temporal multigraph convolutional network," IEEE Transactions on Industrial Informatics, vol.18 (11), pp.7977-7987, 2022.



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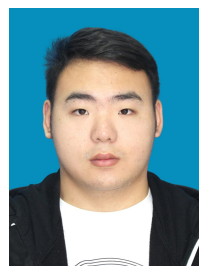


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