ISEN 616 –Design and Analysis of Industrial Experiments

Design Optimization of Paper Helicopter to Increase Flight Time

PROJECT REPORT - GROUP 26

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Table of Contents

<u>Abstract</u>

The aim for the experiment is to demonstrate the orthogonal array technique improving the design of paper helicopters to make it fly the longest possible. The matrix is made using the Plackett-Burman Design $\mathrm{OA}\ (12{,}2^{11})$. The half-normal plot and the Hamada Wu analysis are two methods used to determine significant factors and their level settings. We compared results of both methods and validation experiment is performed utilizing the optimum level settings of the factors identified from the analysis.

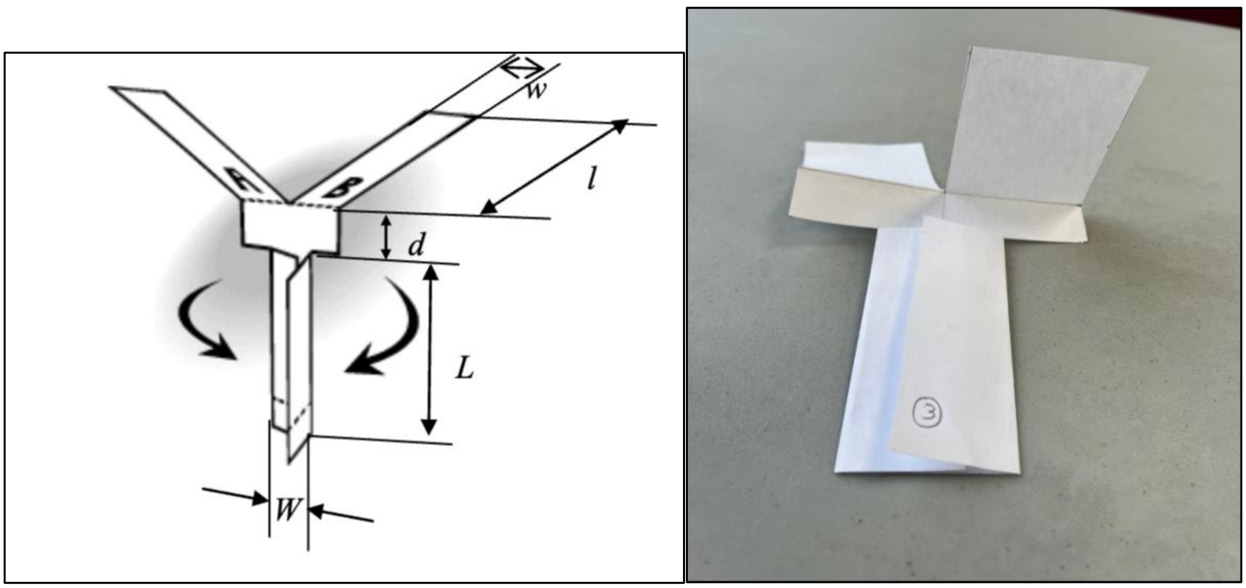

Figure 1: Construction of Paper Helicopter

1. Introduction

In this design, we build a matrix using Plackett-Burman Design $\mathrm{OA}\ (12,2^{11})$. To start the experiment, 12 paper helicopters are made by each team member with different dimensions and combinations to test for the best overall design.

Factors	Symbol	Dimension	
		- level	+ level
Wing length	l	3 inches	4.5 inches
Wing width	w	1.8 inches	2.4 inches
Body length	L	3 inches	4.5 inches
Body width	W	1.25 inches	2 inches
Middle body length	d	1 inch	1.5 inches
Fold at tip	F	no	yes

Table 1

The first step would be to use the graphical method called the half-normal plot. Using the half normal plot, we can identify the significant factors and effects.

Half normal plot consists of points:

$(\Phi^{-1}(0.5 + 0.5\ [i - 0.5]/\ I), |\hat{\theta}|_{(i)}$ for $i = 1\ldots., I$.

Φ = cumulative distribution function.
I = Effects and interaction considered.

The second approach is Hamada Wu method. Hamada Wu method focuses on two important principles: Effect sparsity principle and Effect heredity principle. According to the effect sparsity principles it states that the number of important factors in an experiment is very less. In this experiment we should figure out the factors (Wing length, Wing width etc.) for which the flight time is significant. The effects heredity principle states that for the interaction to be significant at least one of its parent main factors must be significant. In the above experiment, we will deem an interaction to be significant only if the main factors above are significant.

In often practical situations, there are many factors and many aliased effects that are difficult to interpret based on the designs with complex aliasing. Because of this difficulty these designs were used as screening factors to consider the main effects and not the interactions. Now this is possible if the interactions were negligible but that is not always the case. To overcome this, Hamada and Wu proposed a method that during a design if effect sparsity, effect heredity principles and correlation between partially aliased effects are small to moderate we can allow a certain modification that will reduce the significant effects and design will work even if the total number of effects are greater than the runs.

2. Methodology

To start off, each member of the group created 12 paper helicopters having distinctive designs according to the randomization shown below. We used Plackett-Burman Design $\mathrm{OA}\ (12{,}2^{11})$ here. This design uses principles of replication and randomization having 11 factors and 12 runs.

Replication: This states that the experiment is repeated more than once for the same dimensions. Thus, each treatment being applied in many experimental units instead of one. This increases the statistical accuracy of an experiment. For the given experiment, 3 replicates (one each by a team member) were considered by dropping paper helicopter from a fixed height of 9 feet. The flight times were noted for each instance by a stopwatch. Thus, in total 36 readings were taken, and average time is calculated.

Randomization: This uses a chance mechanism assigning treatment to experimental units. The design has 11 factors, out of which the factors given in Table 1 are chosen at random.

2.1 Design Matrix

Run	A	B	C	D	E	F	7	8	9	10	11
1	+	+	−	+	+	+	−	−	−	+	−
2	+	−	+	+	+	−	−	−	+	−	+
3	−	+	+	+	−	−	−	+	−	+	+
4	+	+	+	−	−	−	+	−	+	+	−
5	+	+	−	−	−	+	−	+	+	−	+
6	+	−	−	−	+	−	+	+	−	+	+
7	−	−	−	+	−	+	+	−	+	+	+
8	−	−	+	−	+	+	−	+	+	+	−
9	−	+	−	+	+	−	+	+	+	−	−
10	+	−	+	+	−	+	+	+	−	−	−
11	−	+	+	−	+	+	+	−	−	−	+
12	−	−	−	−	−	−	−	−	−	−	−

Table 2

2.2 Paper helicopters

As shown below in figure 2, 36 paper helicopters were crafted, with dimensions in accordance with Table 1 and randomization in accordance with Design matrix in Table 2. From table 2, we get two factor levels "+" and "- ". Each run is decided by the factor levels and the values are taken from Table 1 for each level. Thus 12 different paper helicopters are crafted by each team member.

To conduct the experiment, each paper helicopter is dropped from a fixed height of 9 feet, and the flight time in which helicopter reaches the ground is noted by a stopwatch. For each run, we perform 3 readings and note down the readings to conduct further analysis.

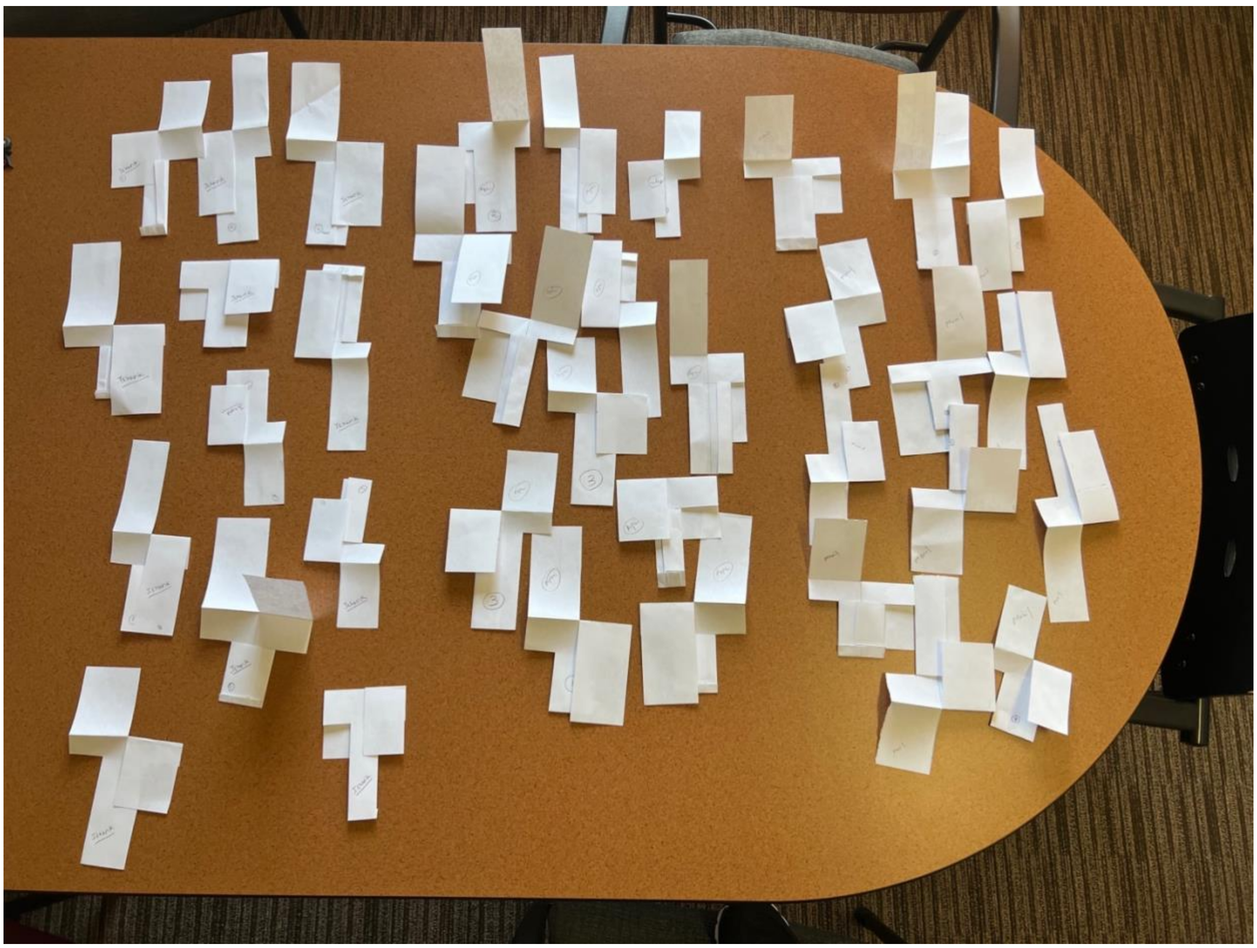

Figure 2: Paper Helicopter

2.3 Observations

The table below shows three different readings namely Y_1, Y_2, Y_3 noted for each run. Further in the analysis part of the experiment, we calculate the average and variances for each run.

Runs	l	w	L	W	d	F	f7	f8	f9	f10	f11	Y_1	Y_2	Y_3
1	1	1	-1	1	1	1	-1	-1	-1	1	-1	2.89	2.56	2.69
2	1	-1	1	1	1	-1	-1	-1	1	-1	1	2.43	2.64	2.7
3	-1	1	1	1	-1	-1	-1	1	-1	1	1	2.17	2.3	2.11
4	1	1	1	-1	-1	-1	1	-1	1	1	-1	2.31	2.24	2.24
5	1	1	-1	-1	-1	1	-1	1	1	-1	1	2.49	2.33	2.3
6	1	-1	-1	-1	1	-1	1	1	-1	1	1	2.76	2.5	2.63
7	-1	-1	-1	1	-1	1	1	-1	1	1	1	2.1	1.9	1.97
8	-1	-1	1	-1	1	1	-1	1	1	1	-1	1.84	1.9	1.98
9	-1	1	-1	1	1	-1	1	1	1	-1	-1	2.13	2.04	2.29
10	1	-1	1	1	-1	1	1	1	-1	-1	-1	2.88	2.96	2.83
11	-1	1	1	-1	1	1	1	-1	-1	-1	1	1.84	2.04	2.24
12	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	2.1	2.17	2.23

Table 3: Observation Table

2.4 Half-Normal Plot - Method

A half normal plot is a graphical method to compare relative magnitude of effects arranged In an order to identify which of the effects are significant.

Steps for half normal plot:
Step 1: Getting the main and interaction effect values
Step 2: These main and interaction effects are ordered in a pattern.
Step 3: A half normal plot is generated with half normal quantities on x-axis and effect values on y-axis.

Significant effects in Half-Normal plot are detected when they are away from the non-zero effects and their normal distribution is centered at a larger effect value. Similarly, insignificant effects have non-zero effect and their normal distribution is centered near the zero value. On plotting the graph, we need to segregate the off-line factors and declare them as significant.

2.5 Hamada Wu – Method

The steps to deploy the hamada Wu method is as follows:

Step 1: We use all the six factors from Table 1 and two factor interactions between them in stepwise regression to identify significant effects.
Step 2: To generate a model that has all the primary effects and interaction effects from step 1, we select all the significant factors from each model. We again apply stepwise regression to this model.
Step 3: In this step we use the effect heredity principle on significant factors in step 2 by selecting the interactions where at least one parent effect is significant.
Step 4: Iterate between step 2 and step 3, until the model stops changing.

3. Analysis

R- Code:
We are calculating y_bar and s^2 using the code snippet given below

```
# Calculating Y_bar
df$y_bar <- rowMeans(df[,c(13,14,15)])
df$y_bar

#calculating ln_s^2
df$s2 <- apply(df[,c(13,14,15)],1,sd,na.rm=T)
df$s2

df$ln_s2 <- log(df$s2)
```

Y_1	Y_2	Y_3	y_bar	s2	ln_s2
2.89	2.56	2.69	2.713333	0.16623277	-1.794366
2.43	2.64	2.70	2.590000	0.14177447	-1.953518
2.17	2.30	2.11	2.193333	0.09712535	-2.331753
2.31	2.24	2.24	2.263333	0.04041452	-3.208566
2.49	2.33	2.30	2.373333	0.10214369	-2.281375
2.76	2.50	2.63	2.630000	0.13000000	-2.040221
2.10	1.90	1.97	1.990000	0.10148892	-2.287806
1.84	1.90	1.98	1.906667	0.07023769	-2.655870
2.13	2.04	2.29	2.153333	0.12662280	-2.066543
2.88	2.96	2.83	2.890000	0.06557439	-2.724570
1.84	2.04	2.24	2.040000	0.20000000	-1.609438

Table 4: Analysis

3.1 Half-Normal Plot Analysis

A half normal plot is a graphical tool used in statistical analysis to assess whether a set of data follows a normal distribution. It is a type of probability plot created by plotting the absolute values of the data against the corresponding quantiles of the standard normal distribution.
Half-normal plot for location effect is calculated for all the factor as follows:
ME(A) = z(A+) - z(A-)
where z(A+) is the average of the Z_j values observed at A+ and z(A-) is the average of the Z_j values observed at A-

Factors	location_values
l	0.501666667
w	-0.072777778
L	-0.023888889
W	0.191666667
d	0.026111111
F	-0.013888889
f7	0.003888889
f8	0.063888889
f9	-0.226111111
f10	-0.086111111
f11	-0.046111111

Table 5: Location Values

Location Effect (Half Normal Analysis)

R-Code:

```
library(FrF2)
library(readxl)

df <- read_excel("data.csv.xlsx")

#Need to calculate Maineffects for location parameters
cols_to_sum <- c("l","w","L","W","d","F","f7","f8","f9","f10","f11")
location_val <- list()
for(col in cols_to_sum)
{
  s<- (sum(df$y_bar[df[[col]] == 1]) - sum(df$y_bar[df[[col]] == -1]))/6
  location_val<-append(location_val,s)
}
factors <- list("l","w","L","W","d","F","f7","f8","f9","f10","f11")
df_loc <- data.frame(unlist(factors),unlist(location_val))
names(df_loc) <- c("Factors","location_values")

#Plot Half Normal Plot for Locations
halfnorm(df_loc$location_values, nlab = 10, labs = as.character(df_loc$Factors), ylab = "Plot Data for Location",)
```

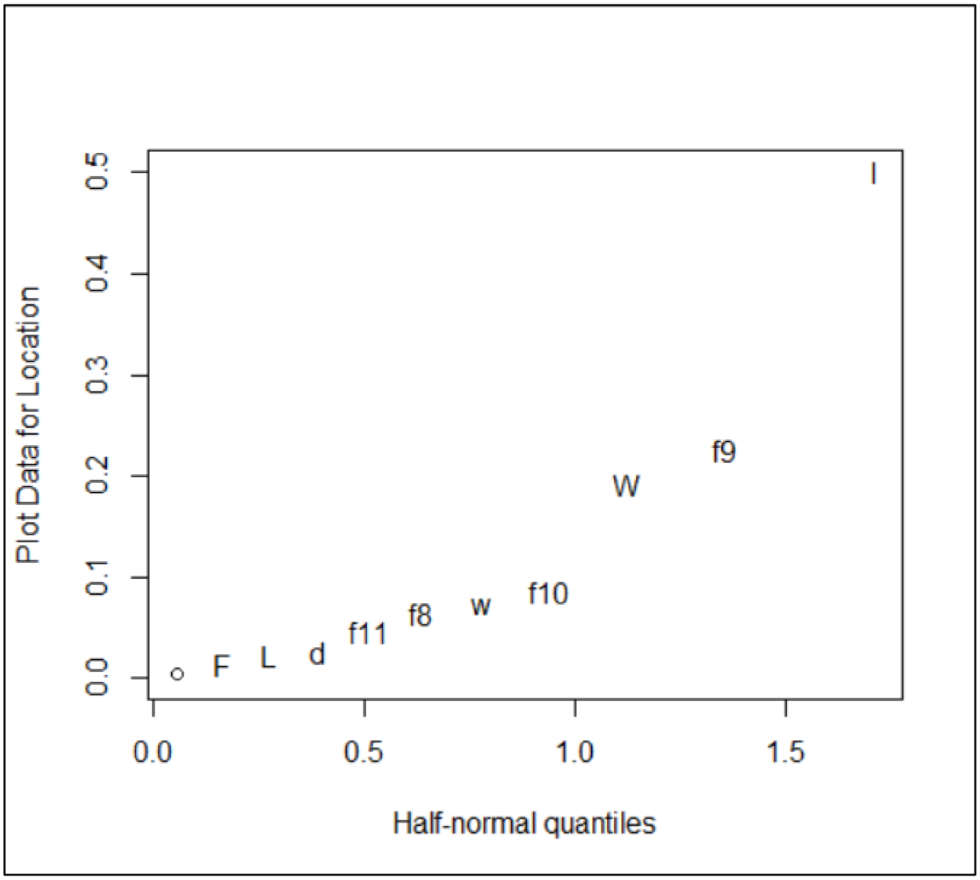

Figure 3: Half Normal Plot

The half normal plot for location is shown in the above plot. We neglect the effect of factors that fall on a line, technically we try to locate the outlier of the plot. This outlier has a significant effect. In this case, l is the outlier and has a significant effect on the location.

R Code:

```
intercept <- mean(df$y_bar)
coeff_l   <- df_loc$location_values[df_loc$Factors=="l"]/2
print(paste0("y_ =", intercept,"+",coeff_l, "*l"))
```

We can easily get the location equation from the above table 5. The value of intercept is the avg value of the response(y). And the

$$y = 2.325 + 0.250 * x_l$$

<u>Dispersion (Half Normal plot Analysis)</u>

Half-normal plot for dispersion effect is calculated for all the factor as follows:

ME(A) = z(A+) - z(A-)

where z(A+) is the average of the Z_j values observed at A+ and z(A-) is the average of the Z_j values observed at A-

Factors	dispersion_values
l	-0.05313728
w	0.18372112
L	-0.21350367
W	0.22821621
d	0.57441613
F	0.16325970
f7	-0.03131314
f8	-0.08570915
f9	-0.20349107
f10	-0.15845934
f11	0.44636474

Table 6: Dispersion Values

R- Code:

```
> #need to calculate the Maineffects for the disperaion parameters
> cols_to_sum <- c("l","w","L","W","d","F","f7","f8","f9","f10","f11")
> dispersion_val <- list()
> for(col in cols_to_sum)
+ {
+   s<- (sum(df$ln_s2[df[[col]] == 1]) - sum(df$ln_s2[df[[col]] == -1]))/6
+   dispersion_val<-append(dispersion_val,s)
+ }
> df_dispersion <- data.frame(unlist(factors),unlist(dispersion_val))
> names(df_dispersion) <- c("Factors","dispersion_values")
> df_dispersion
   Factors dispersion_values
1        l       -0.05313728
2        w        0.18372112
3        L       -0.21350367
4        W        0.22821621
5        d        0.57441613
6        F        0.16325970
7       f7       -0.03131314
8       f8       -0.08570915
9       f9       -0.20349107
10     f10       -0.15845934
11     f11        0.44636474
```

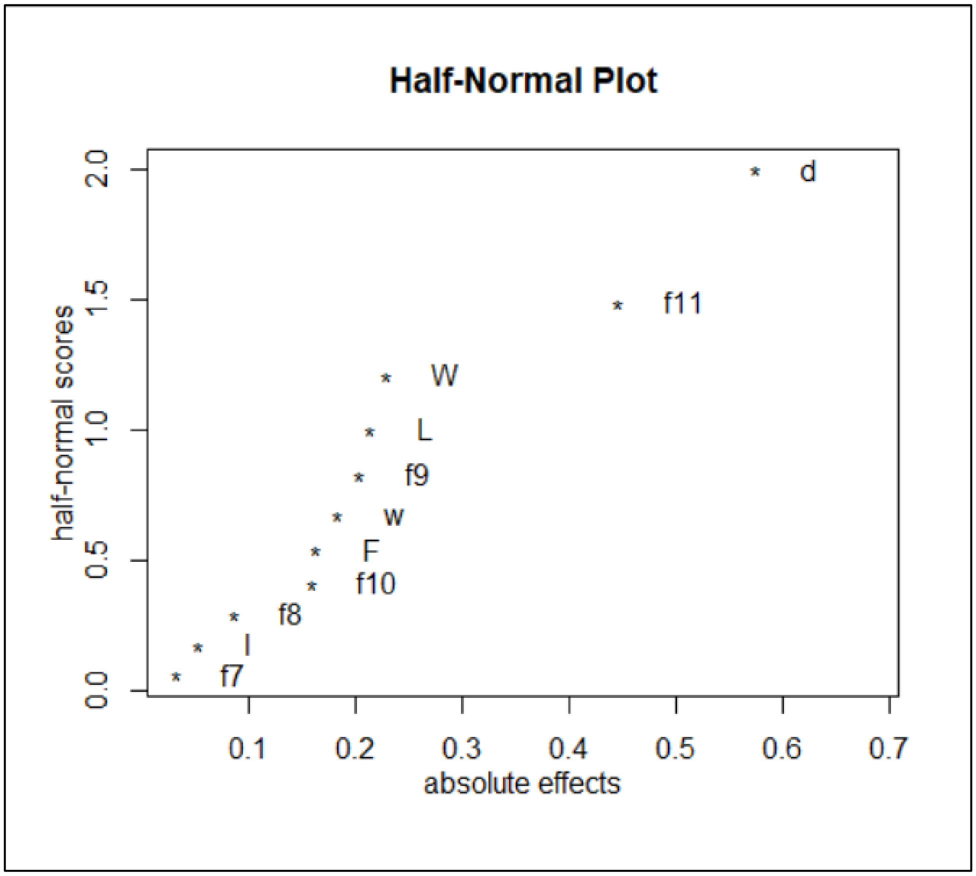

Figure 4: Half Normal Plot

From the above half normal plot for dispersion, we can say that only f11 and x_d are significant factors. We neglect the effect of factors that fall on a line, technically we try to locate the outlier of the plot.

The value of intercept for the dispersion is equal to the avg value of response. The coefficient value of each factor variable is half of that factor variable's as seen in Table 6.

We get the following equation for dispersion:

$$\ln s^2 = -2.307 + 0.287\, x_d + 0.22318\, f11$$

Evaluation of the equations:

To increase the flight time of the airplanes, we need to follow the two-step approach. And we should ensure that both the objectives should be met simultaneously.

(1) Minimize the variance ($\ln s^2$)

(2) Maximize the flight time (y)

So, according to the two-step procedure we will minimize the variance first. We have the variance equation as follows

$$\ln s^2 = -2.307 + 0.287\, x_d + 0.22318\, f11$$

To minimize the variance, we need x_d to be negative factor and $f11$ to be also a negative factor Substituting $x_d = -1\ and\ f11 = -1$ we get,

$$\ln s^2 = -4.712$$

Now, going forward to step (2) to maximize the flight time (response variable). We have the equation of flight time as follows:

$$y = 2.325 + 0.250 * x_l$$

To maximize the above equation, we need x_l to be a positive factor. Substituting x_l = +1 we get,

$$y = 2.575$$

Thus, maximum flight time as per theoretical half normal plot approach is 2.575 sec. We can see from the above two equations that our adjustment factor is x_l as it affects the location value but does not account for changes in the dispersion equation.

3.2 Hamada Wu Approach

Location Effect Analysis

Step 1:

For each factor X, consider X and all 2fi's XY involving X. Use stepwise regression to identify significant effects. Repeat this for each X and keep the best model.

For factor l

```
> m11=lm(y_bar~l+l:w+l:L+l:W+l:d+l:F,data=df)
> ols_step_both_p(m11)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	l	addition	0.699	0.669	23.7500	-3.2616	0.1802
2	l:w	addition	0.790	0.743	16.1880	-5.5619	0.1588
3	l:F	addition	0.878	0.833	8.8590	-10.1076	0.1282

Significant factors identified: $l, l{:}w, l{:}F$

For factor w

```
> m12=lm(y_bar~w+w:l+w:L+w:W+w:d+w:F,data=df)
> ols_step_both_p(m12)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	w:L	addition	0.139	0.053	-1.3570	9.3565	0.3049

Significant factors: $w{:}L$

For factor L

```
> m13=lm(y_bar~L+L:l+L:w+L:W+L:d+L:F,data=df)
> ols_step_both_p(m13)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	L:d	addition	0.244	0.168	2.5000	7.8057	0.2858
2	L:W	addition	0.487	0.373	1.1180	5.1409	0.2480
3	L	addition	0.489	0.297	3.0960	7.1037	0.2627
4	L:d	removal	0.245	0.077	4.4780	9.7805	0.3009

Significant factors: $L: d, L: w, L$

For factor W

```
> m14=lm(y_bar~W+W:l+W:w+W:L+W:d+W:F,data=df)
> ols_step_both_p(m14)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	W:L	addition	0.244	0.168	0.8880	7.8057	0.2858

Significant factors: $W: L$

For factor d

```
> m15=lm(y_bar~d+d:l+d:w+d:L+d:W+d:F,data=df)
> ols_step_both_p(m15)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	d:L	addition	0.244	0.168	-1.1640	7.8057	0.2858

Significant factors : $d: L$

For factor F

```
> m16=lm(y_bar~F+F:l+F:w+F:L+F:W+F:d,data=df)
> ols_step_both_p(m16)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	F:W	addition	0.151	0.066	0.2260	9.1982	0.3028

Significant factors : $F: W$

Step 2:

We consider all the significant effects coming from step (1) which are lw, lF, wL, LW, dL, FW along with all the main effects

```
> m21 <- lm(y_bar~l+w+L+W+d+F+l:w+l:F+w:L+L:W+d:L+F:W,df)
> ols_step_both_p(m21)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	l	addition	0.699	0.669	NaN	-3.2616	0.1802
2	w	addition	0.801	0.757	NaN	-6.2366	0.1544

Significant factors: l, w

Step 3:

For this step we consider all the main effects and interaction effects whose at least 1 factor is significant as obtained from step (2). Using effect heredity, consider (1) effects identified in 2 and (2) 2fi's with at

least one parent factor appearing in the main effects in (1). Use stepwise regression to identify significant effects.

```
> m41 <- lm(y_bar~l+w+L+F+d+W+l:w+l:F+w:d,df)
> ols_step_both_p(m41)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	l	addition	0.699	0.669	12.9990	-3.2616	0.1802
2	w	addition	0.801	0.757	7.8720	-6.2366	0.1544

Significant factors: l, w

Step 4:

Iterate between 2 and 3 until model stops changing.

```
> m41 <- lm(y_bar~l+w+L+F+d+W+l:w+l:F+w:d,df)
> ols_step_both_p(m41)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	l	addition	0.699	0.669	12.9990	-3.2616	0.1802
2	w	addition	0.801	0.757	7.8720	-6.2366	0.1544

```
> m42<-lm(y_bar~l+w+l:w+l:L+l:W+l:d+l:F+w:L+w:W+w:d+w:F,df)
> ols_step_both_p(m42)

                              Stepwise Selection Summary
---------------------------------------------------------------------------------
                      Added/                   Adj.
Step    Variable     Removed     R-Square    R-Square     C(p)       AIC        RMSE
---------------------------------------------------------------------------------
   1       l         addition       0.699       0.669      NaN     -3.2616     0.1802
   2      l:w        addition       0.790       0.743      NaN     -5.5619     0.1588
   3      l:F        addition       0.878       0.833      NaN    -10.1076     0.1282
   4       w         addition       0.933       0.894      NaN    -15.2520     0.1018
   5      w:d        addition       0.963       0.932      NaN    -20.3278     0.0819
---------------------------------------------------------------------------------
```

We can stop the iteration here as the model does not change anymore compared to step (3). The $Adjusted\ R^2$ value of the final model is 0.932.

Final Model:

```
> ols_step_both_p(m42)$model

Call:
lm(formula = paste(response, "~", paste(preds, collapse = " + ")),
    data = l)

Coefficients:
(Intercept)            l            w          l:w          l:F          w:d
    2.32583      0.27062     -0.06695     -0.09028      0.09170      0.05936
```

From the above model built we can get the location equation as follows:

$$y = 2.3258 + 0.2706\ x_l - 0.0669\ x_w - 0.0902\ x_{l:w} + 0.0917\ x_{l:F} + 0.0593\ x_{w:d}$$

<u>Dispersion Effect Analysis:</u>

<u>Step 1:</u>

```
> md11=lm(ln_s2~l+l:w+l:L+l:W+l:d+l:F,data=df)
> ols_step_both_p(md11)

                              Stepwise Selection Summary
------------------------------------------------------------------------------------
                      Added/                   Adj.
Step    Variable     Removed     R-Square    R-Square      C(p)        AIC        RMSE
------------------------------------------------------------------------------------
   1      l:L        addition       0.186       0.104     -0.4070     17.7408     0.4323
------------------------------------------------------------------------------------
```

Significant factors : $l: L$

```
> md12=lm(ln_s2~w+w:l+w:L+w:W+w:d+w:F,data=df)
> ols_step_both_p(md12)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	w:F	addition	0.298	0.228	0.9400	15.9606	0.4014
2	w	addition	0.342	0.196	2.3780	17.1818	0.4096
3	w:F	removal	0.044	-0.051	4.1690	19.6619	0.4684

Significant factors : $w{:}F, w$

```
> md13=lm(ln_s2~L+L:l+L:w+L:W+L:d+L:F,data=df)
> ols_step_both_p(md13)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	L:l	addition	0.186	0.104	-2.2890	17.7408	0.4323

Significant factors : $L{:}l$

```
> md14=lm(ln_s2~W+W:l+W:w+W:L+W:d+W:F,data=df)
> ols_step_both_p(md14)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	W:F	addition	0.130	0.043	-2.2920	18.5373	0.4469

Significant factors: $w{:}F$

```
> md15=lm(ln_s2~d+d:l+d:w+d:L+d:W+d:F,data=df)
> ols_step_both_p(md15)
```

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	d	addition	0.431	0.374	-0.5990	13.4296	0.3612

Significant factors: d

```
> md16=lm(ln_s2~F+F:l+F:w+F:L+F:W+F:d,data=df)
> ols_step_both_p(md16)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                   Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1      F:w       addition       0.298       0.228    -1.0030    15.9606    0.4014
   2       F        addition       0.333       0.184     0.6500    17.3498    0.4125
   3      F:w       removal        0.035      -0.062     1.6170    19.7779    0.4706
-------------------------------------------------------------------------------------
```

Significant factors: F

Step 2:

We consider all the significant effects coming from step (1) which are lL, WF along with all the main effects

```
> md21=lm(ln_s2~l+w+L+W+d+F+l:L+W:F,data=df)
> ols_step_both_p(md21)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                   Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       d        addition       0.431       0.374    -2.1180    13.4296    0.3612
-------------------------------------------------------------------------------------
```

Step 3:

For this step we consider all the main effects and interaction effects whose at least 1 factor is significant as obtained from step (2). Using effect heredity, consider (1) effects identified in 2 and (2) 2fi's with at

least one parent factor appearing in the main effects in (1). Use stepwise regression to identify significant effects.

```
> md31=lm(ln_s2~d+d:l+d:w+d:L+d:W+d:F,data=df)
> ols_step_both_p(md31)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                   Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       d        addition       0.431       0.374    -0.5990    13.4296    0.3612
-------------------------------------------------------------------------------------
```

Step 4:

```
> md41=lm(ln_s2~l+w+L+W+d+F,data=df)
> ols_step_both_p(md41)

                              Stepwise Selection Summary
-------------------------------------------------------------------------------------
                     Added/                   Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)        AIC        RMSE
-------------------------------------------------------------------------------------
   1       d        addition       0.431       0.374    -0.0650    13.4296    0.3612
-------------------------------------------------------------------------------------
```

We can stop the iteration here as the model does not change anymore compared to step (3). The $Adjusted\ R^2$ value of the final model is 0.431.

3.3 Final Model

```
> ols_step_both_p(md41)$model

Call:
lm(formula = paste(response, "~", paste(preds, collapse = " + ")),
    data = l)

Coefficients:
(Intercept)            d
    -2.3072       0.2872
```

$$\ln s^2 = -2.307 + 0.2872\, x_d$$

To increase the flight time of the airplanes, we need to follow the two-step approach. And we should ensure that both the objectives should be met simultaneously.

(1) Minimize the variance ($\ln s^2$)
(2) Maximize the flight time (y)

So, according to the two-step procedure we will minimize the variance first. We have the variance equation as follows

$$\ln s^2 = -2.307 + 0.2872\, x_d$$

From the above equation, we can clearly say that x_d should be a negative factor to reduce the variance.

We have the location equation as follows,

$$y = 2.3258 + 0.2706\, x_l - 0.0669\, x_w - 0.0902\, x_{l:w} + 0.0917\, x_{l:F} + 0.0593\, x_{w:d}$$

To maximize the flight time, we must maximize the above equation,
We set the factor $x_l = +1,\ x_w = -1,\ x_F = +1$
By substituting the value in the above equation, we get,

$$y = 2.9045\ sec$$

4. Validation

As concluded from the Hamada Wu experiment, we designed a paper helicopter, using the optimal levels of factors determined from the analysis. In our case for 2 factors L, W we were not able to determine the optimal levels from analysis, thus keeping the other factors level fixed (as achieved from the analysis), we designed 4 more helicopters with values of factors as : (L+, W+), (L-, W+), (L-, W-), (L+,W-).

After conducting the experiments, we identified the parameters values will result in maximizing the flight time as given in table below:

Factor	Symbol	Levels	Dimension
Wing Length	*l*	+	4.5 inches
Wing width	*w*	-	1. 8 inches
Body Length	*L*	+	4.5 inches
Body width	*W*	-	1.25 inches
Middle body length	*d*	-	1 inch
Fold at tip	*F*	+	Yes

Table 7

Using the above dimensions, we get an average flight time of 2.95 which is close to our theoretical calculation obtained 2.9045. Thus, this setting can be deemed as ideal for getting the maximum flight time.

Figure 5: Final Model

5. Conclusion

The Plackett-Burman design OA $(12,2^{11})$ was utilized in the Paper Helicopter Experiment to prolong the flight duration of a paper helicopter. We followed the procedure mentioned in the document to create several paper planes to test on. The data was collected for each paper plane. We tried our best to remove the influence of external factors like wind, scale error, temperature, paper thickness etc.

We used two methods to evaluate the flight times

(1) Half Normal Plots
(2) Hamada Wu Approach

In half normal plot, significant factors were identified using the plots and two equations were created each for location and dispersion. By solving for the two equations, we got the maximum flight time to be 2.575 seconds.
From this, we identified the significant factor which affected the flight time as l (wing length). The experiments suggest us to keep wing length as + factor which means that wing length (l) should be equal to 4.5 inch

In Hamada Wu approach, significant factors were identified using the step wise regression approach and we got a maximum flight time of 2.9045 seconds. The flight time depends on $x_l, x_w, x_{d,} x_F$ which translates to wing length, wing width, middle body length and fold at tip. Optimal levels of these factors are identified as follows: $x_l = +1,\ x_w = -1,\ x_F = +1, x_d = -1$.
Wing length: 4.5 inch
Wing width: 1.8 inch
Middle body length: 1 inch
Fold at tip: yes

Based on this we can observe that there is almost 12.79% increase in the flight time in the case of design from Hamada Wu compared to design achieved from Half-normal plot method.
Also, a validation experiment was conducted using the optimal levels identified from Hamada Wu approach and we got flight time of 2.95 which is almost same as the flight time of 2.9045 as calculated from the theoretical analysis.