EECT INNOVATIVE PROJECT

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AIM

1) To use FINITE ELEMENT ANALYSIS (FEA) Method to Calculate the Inductance of a Gapped EI-Core Inductor.
2) To calculate the Result upto a more accurate level than regular Magnetic Circuit Theory Approximations by not neglecting Fringing and Flux leakage.

INTRODUCTION

A common task that FEMM might be applied to is the calculation of the inductance of a gapped inductor. Although the inductance can be estimated using simple magnetic circuit theory, the circuit approach typically ignores flux leakage and fringing effects.

To resolve these non-ideal effects in more detail a finite element analysis can be employed. The goal of the present example is to show how inductance is calculated in FEMM simulation and to compare that result to the approximation obtained through a magnetic circuit approach.

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THEORY

From Ampere's Circuital Law, we know that Magneto Motive Force (MMF) is equivalent to:

$$R\phi = ni$$

$$\phi = \frac{ni}{R}$$

$$n\phi = \frac{n^2 i}{R} = Li$$

$$L = \frac{n^2}{R}$$

This shows us that Inductance is equal to the Product of Square of number of Cores and Permeance. However, in the above example, factors such as Flux Leakage due to Fringing have been neglected. If we have to consider these conditions as well we can use the Finite Element Method to solve the problem.

SOFTWARE USED - Finite Element Method Magnetics (FEMM)

The finite element method (FEM) is the most widely used method for solving problems of engineering and mathematical models. Typical problem areas of interest include the traditional fields of structural analysis, heat transfer, fluid flow, mass transport, and electromagnetic potential. Studying or analyzing a phenomenon with FEM is often referred to as finite element analysis (FEA).

The FEM is a particular numerical method for solving partial differential equations in two or three space variables (i.e., some boundary value problems). To solve a problem, the FEM subdivides a large system into smaller, simpler parts that are called finite elements.

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Example Geometry

The example geometry is composed of a gapped, laminated EI core with the cross-section as pictured in Figure 1. An E core with a center pole 0.5" wide and outer poles 0.25" wide is separated from an I-shaped return path by an air gap 0.025" thick.
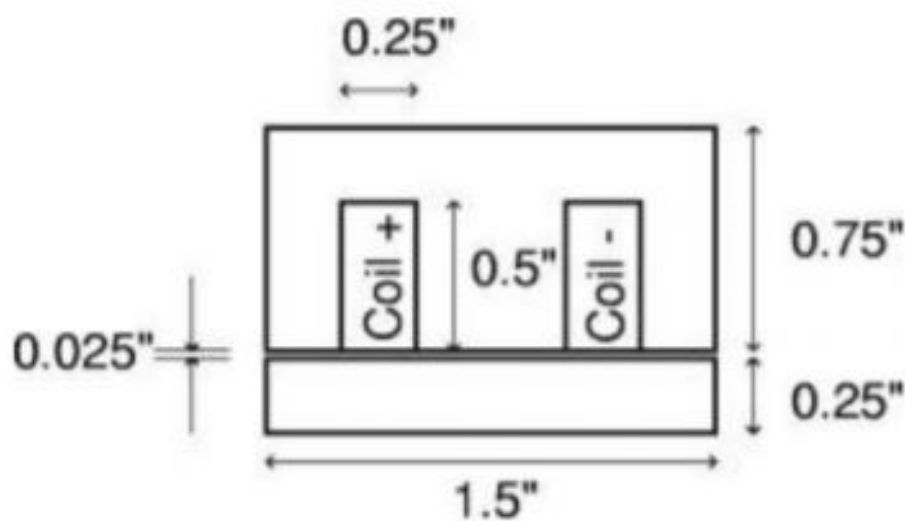

Figure 1: Example Geometry

The winding that lies in the slots of the E consists of 66 turns of 18 AWG wire heavy build insulation, for approximately a 66% copper fill fraction in the coil window. It is assumed that the core extends for 1 inch in the into-the-page direction. For the purposes of this model, the material is assumed to be a linear ferromagnetic material with a relative permeability of 2500.

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Circuit Theory Inductance

For a rough approximate of inductance, we can assume that there is no leakage and fringing, and that the contribution of the iron sections to the reluctance of the magnetic circuit is trivial in comparison to the contributions from the air. Then, the flux flowing in the magnetic circuit is obtained by solving the circuit equation:

$$R\phi = ni$$

for ϕ, the total flux linked by the coil. The R represents the reluctance of the magnetic circuit, which under our assumptions can be written as:

$$R = \frac{2g}{\mu_o a_{pole}} = 3.133(10^6)\frac{A}{Wb}$$

where $g = 0.025''$, $a_{pole} = 0.5$ in^2, and $\mu_o = 4\pi(10^{-7})$ H/m. Since the total flux linking the coil is then:

$$\phi = \frac{ni}{R}$$

and the total self-inductance is just that flux times the total number of turns:

$$n\phi = \frac{n^2 i}{R} = Li$$

so that the self-inductance is:

$$L = \frac{n^2}{R}$$

Since, in our case, there are 66 turns, the inductance works out to be:

$$L = 1.39 \text{ mH}$$

We would expect the inductance derived from finite element computations to be in the neighborhood of 1.39 mH.

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FINITE ELEMENT INDUCTANCE

For the purposes of creating a finite element solution, the inductor is placed at the center of a 2" by 2.5" box. On the boundaries of the domain, a boundary condition of A=0 is defined. For the purposes of performing the finite element analysis, a current of 1 A is somewhat arbitrarily applied to the coils. A fairly coarse mesh density with the mesh size constrained to be no larger than 0.05" is defined everywhere. The meshed problem domain is pictured in Figure 2.
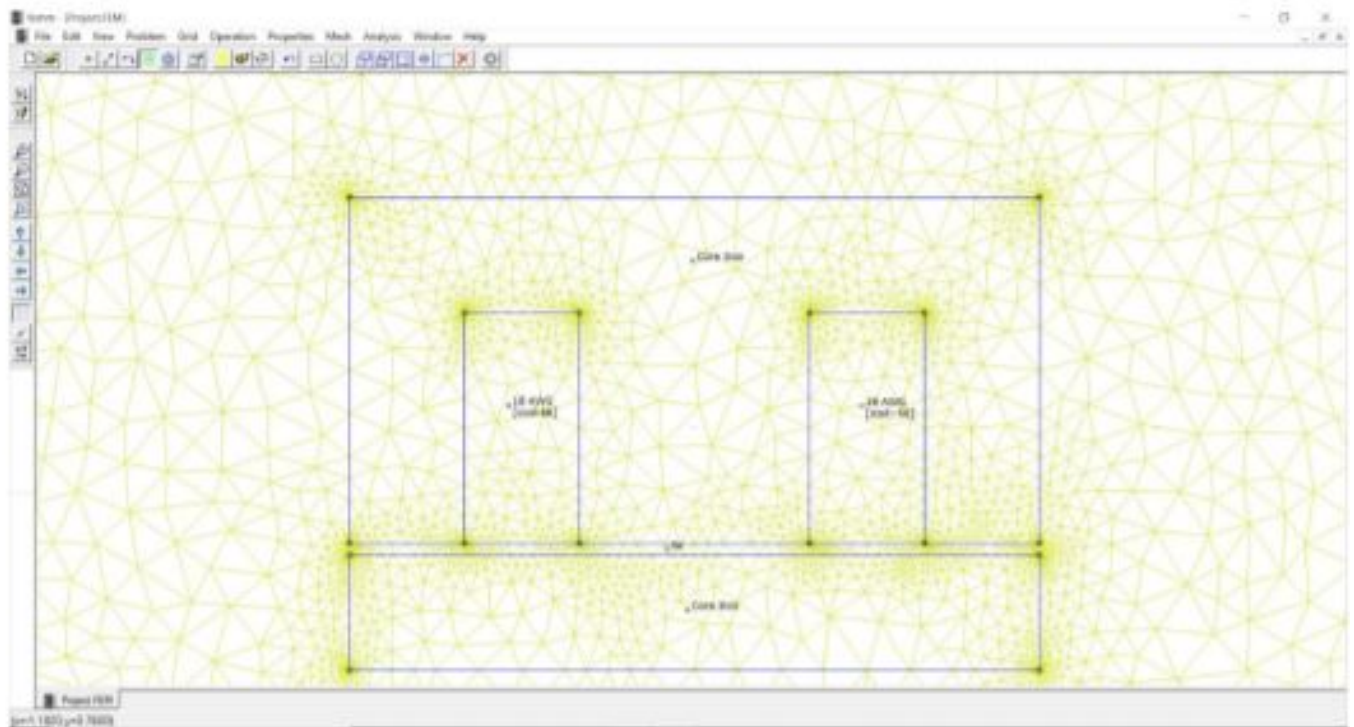
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Circuit Property ✕

Name | icoil

○ Parallel

Circuit Current, Amps

1

● Series

OK | Cancel

Boundary Property ✕

Name | BoundaryCondition | OK

Cancel

BC Type | Prescribed A ▼

Small skin depth parameters | **Prescribed A parameters**

μ, relative | 0 | A_0 | 0

c, MS/m | 0 | A_1 | 0

Mixed BC parameters

c_0 coefficient | 0 | A_2 | 0

c_1 coefficient | 0 | ϕ, deg | 0

Air Gap parameters

Inner Angle, Deg | 0

Outer Angle, Deg | 0

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Properties for selected block ✕

Block type	Core Iron ▾
Mesh size	0
☑ Let Triangle choose Mesh Size	
In Circuit	\<None> ▾
Number of Turns	1
Magnetization Direction	0
In Group	0

☐ Block label located in an external region

☐ Set as default block label

OK   Cancel

Properties for selected block ✕

Block type	18 AWG ▾
Mesh size	0
☑ Let Triangle choose Mesh Size	
In Circuit	icoil ▾
Number of Turns	66
Magnetization Direction	0
In Group	0

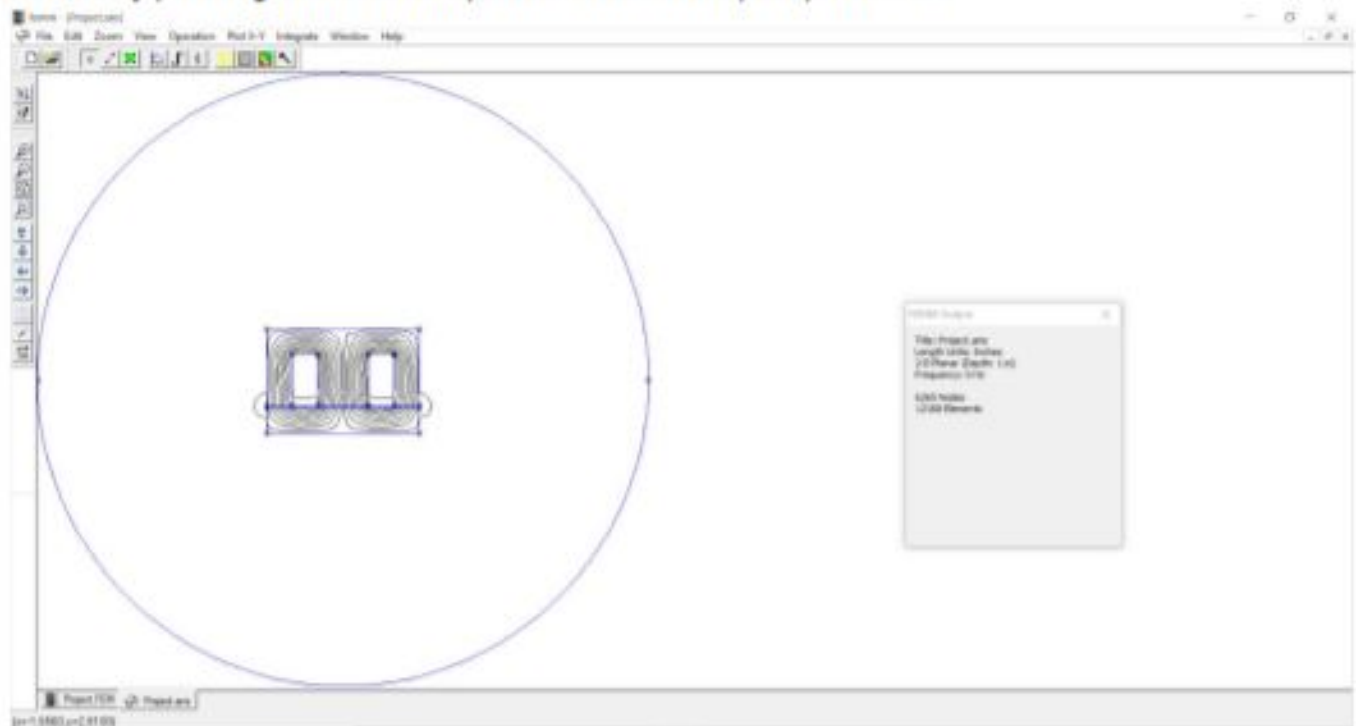☐ Block label located in an external region

☐ Set as default block label

OK   Cancel

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APPROACH 1

Once the analysis is performed and the postprocessor is run, the inductance can be derived by pressing the Circuit Properties button in the post-processor:
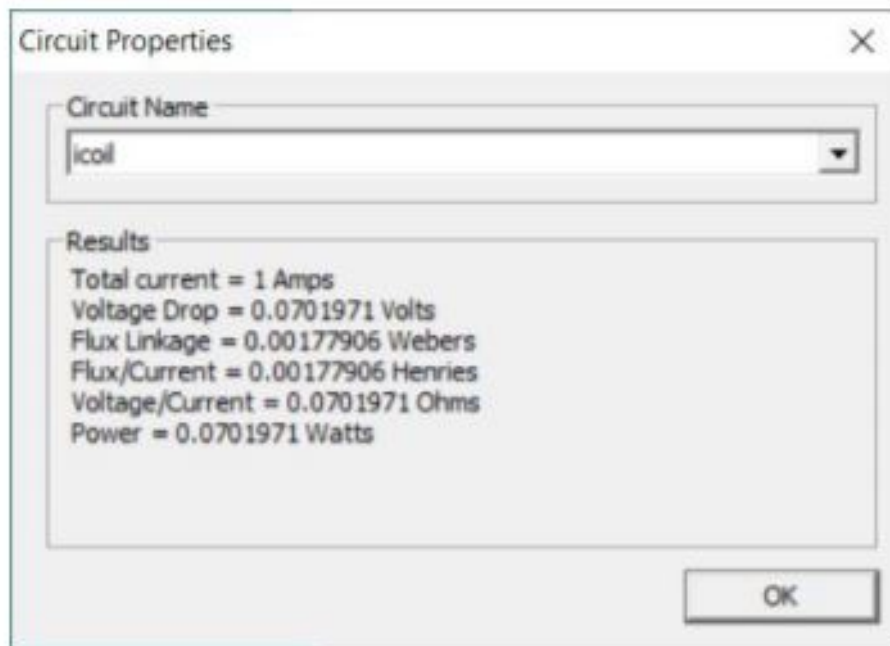
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FEMM Output ✕

Point: x=-0.01, y=-0.4
A   = 1.16716e-005 Wb/m
|B| = 0.0330701 T
Bx = 0.00386995 T
By = 0.0328429 T
|H| = 10.5266 A/m
Hx = 1.23184 A/m
Hy = 10.4542 A/m
mu_x= 2500 (rel)
mu_y = 2500 (rel)
E   = 0.174057 J/m^3
J   = 0 MA/m^2

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A dialog ⬛ appears with a number of properties of the winding as seen from the winding terminals. The Circuit Properties dialog for this example is shown below in the Figure below:-

Circuit Properties	✕
Circuit Name	
icoil	▼
Results	
Total current = 1 Amps Voltage Drop = 0.0701971 Volts Flux Linkage = 0.00177906 Webers Flux/Current = 0.00177906 Henries Voltage/Current = 0.0701971 Ohms Power = 0.0701971 Watts	
	OK

In the absence of permanent magnets or other coils, "Flux/Current" result can be interpreted directly as self inductance:

$$L = 1.77906 \text{ mH}$$

This result is higher than the 1.39 mH predicted by the simple circuit theory approach because the leakage and fringing effects neglected in the circuit approach tend to raise the inductance slightly.
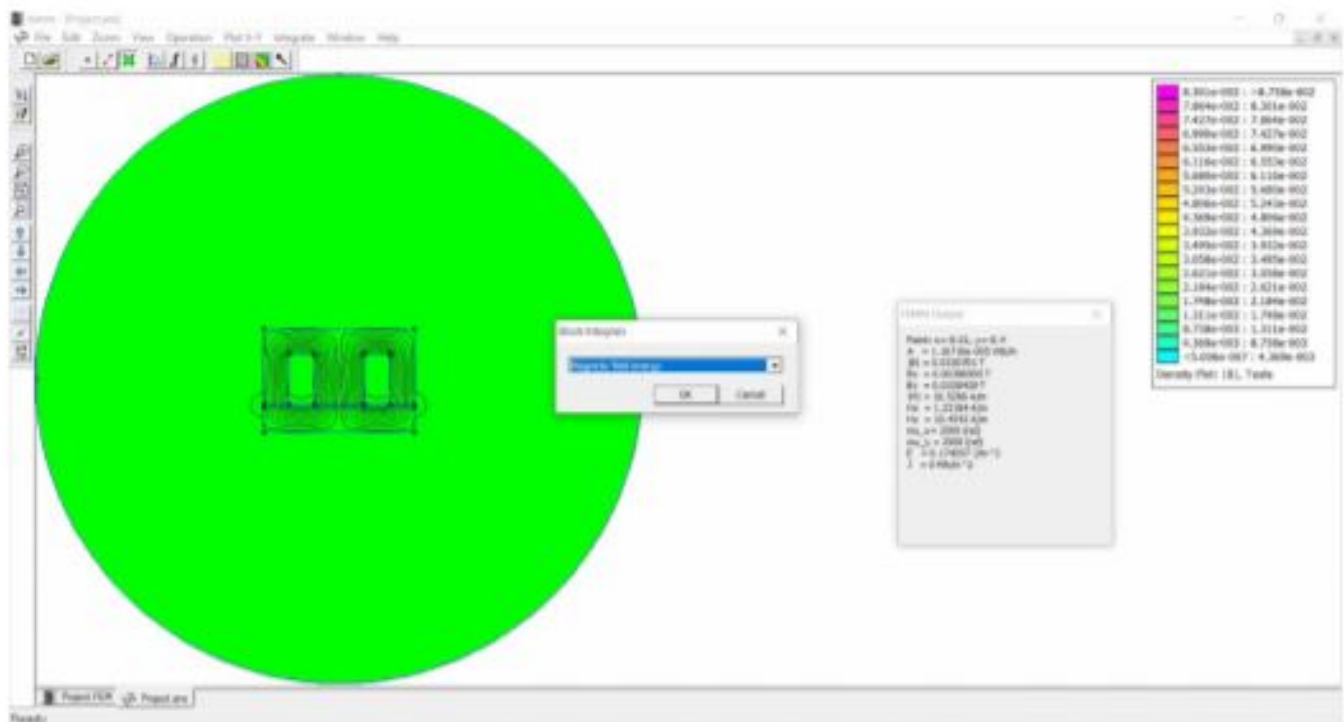
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APPROACH 2

An alternative approach is to obtain energy via the "Magnetic Field Energy" integral. For this approach, energy is obtained via:

$$W = \frac{1}{2} \int B \cdot H \; dV$$

where this integral is taken over the *entire* problem domain instead of just over the coils. To perform this integration in the postprocessor, switch to block integral mode by pushing the button on the taskbar.

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Then, select each region in the problem via left mouse button clicks. When the entire problem region is selected, push the Integral button and select the "Magnetic Field Energy" integral from the drop list of volume integrals.

Integral Result

0.000889528 Joules

OK
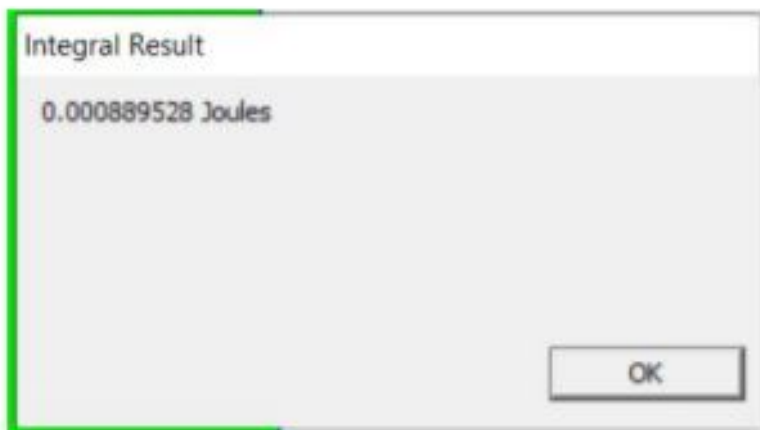
If *W* represents the integral result, the resulting inductance is:

$$L = \frac{2W}{i^2}$$

For the example problem, the result from the energy integration is:

W = 0.000889528 Joules

Which, when substituted into the formula and i = 1 A, yields a similar result as method (I):

L = 1.779056 mH

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CALCULATIONS

The experimental value of Inductance will be the Average of the Values calculated using the 2 Finite Element Methods above:

L(experimental) = [L(Approach I) + L(Approach 2)] /2

$$= (1.77906 + 1.779056) /2$$

$$= 1.779058 \text{ mH}$$

The theoretical value of Inductance is the one which has been calculated using Magnetic Circuit Theory:

L(theoretical) = 1.39 mH

Let us now find the value of Percentage Error in the Theoretical Inductance (E)

E = 100* [L(experimental) - L(theoretical)] / L(theoretical)

$$= 100* (1.779058 - 1.39) /1.39$$

$$= 28\%$$

So, as we can see the neglecting of Fringing Effect and Flux leakage may cause a significant error in our calculations of Inductance depending upon the dimensions of the Air Gap.

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Conclusions

A simple example has been presented that demonstrates how FEMM can be used to obtain inductance. A 2D planar problem was considered, and inductance results from two methods were compared with one another and with a "sanity check" inductance estimate from magnetic circuit theory.

Although the results from the two finite element approaches to deriving inductance are identical in this case, it is usually better to use method (I). The reason is that some boundary conditions (i.e. the asymptotic boundary condition used for approximating an "open boundary" problem) imply that some energy is stored outside the modeled problem domain. Internally, FEMM calculates the flux linkage using a method that accounts for this additional energy, whereas integrating B×H over all elements does not.

Strictly speaking, a single inductance coefficient implies a linear relationship between applied current and resulting flux. For this reason, an example problem with only linear materials was considered.

If the problem has nonlinear materials and there is significant saturation, there is no longer a linear relationship between current and flux. However, there are many situations in which a sinusoidal current is applied, and one would like to know the amplitude of the fundamental of the flux corresponding to the applied current. In this case, a harmonic analysis can be run in FEMM (i.e. when a frequency other than zero is specified in the Problem Definition). FEMM implements a nonlinear time harmonic formulation that computes the amplitude and phase of the fundamental portion of the magnetic field for time harmonic problems with nonlinear materials problems. For nonlinear time harmonic problems, the Circuit Properties dialog can again be used to harvest a coil's various terminal properties.

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