

DIGITAL LOGIC CIRCUITS

Logic Gates

Boolean Algebra

Map Specification

Combinational Circuits

Flip-Flops

Sequential Circuits

Memory Components

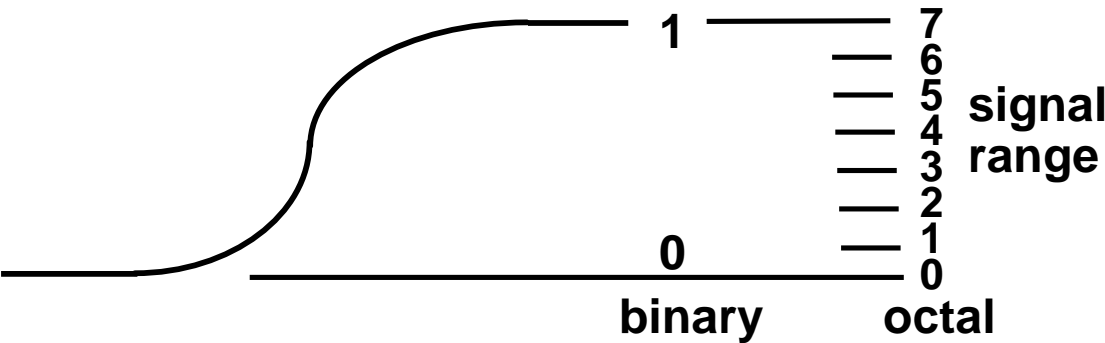
Integrated Circuits

LOGIC GATES

Digital Computers

- Imply that the computer deals with digital information, i.e., it deals with the information that is represented by binary digits
- Why *BINARY* ? instead of Decimal or other number system ?

* Consider electronic signal

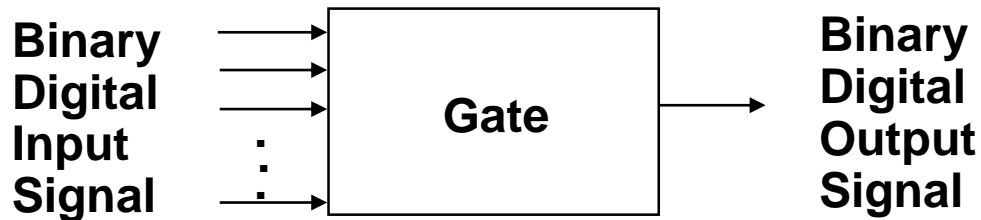


* Consider the calculation cost - Add

	0	1
0	0	1
1	1	10

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

BASIC LOGIC BLOCK - GATE -



Types of Basic Logic Blocks

- **Combinational Logic Block**

Logic Blocks whose output logic value depends only on the input logic values

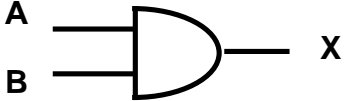


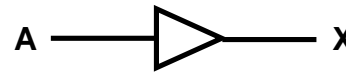
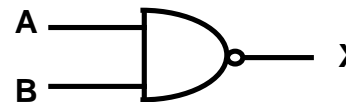
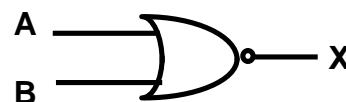
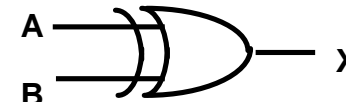
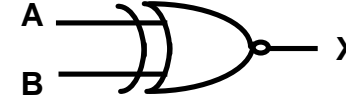
- **Sequential Logic Block**

Logic Blocks whose output logic value depends on the input values and the state (stored information) of the blocks

Functions of Gates can be described by

- Truth Table
- Boolean Function
- Karnaugh Map

COMBINATIONAL GATES

Name	Symbol	Function	Truth Table															
AND		$X = A \cdot B$ or $X = AB$	<table><tr><th>A</th><th>B</th><th>X</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	X	0	0	0	0	1	0	1	0	0	1	1	1
A	B	X																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$X = A + B$	<table><tr><th>A</th><th>B</th><th>X</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	X	0	0	0	0	1	1	1	0	1	1	1	1
A	B	X																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
I		$X = A$	<table><tr><th>A</th><th>X</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	A	X	0	1	1	0									
A	X																	
0	1																	
1	0																	
Buffer		$X = A$	<table><tr><th>A</th><th>X</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table>	A	X	0	0	1	1									
A	X																	
0	0																	
1	1																	
NAND		$X = (AB)'$	<table><tr><th>A</th><th>B</th><th>X</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	X	0	0	1	0	1	1	1	0	1	1	1	0
A	B	X																
0	0	1																
0	1	1																
1	0	1																
1	1	0																
NOR		$X = (A + B)'$	<table><tr><th>A</th><th>B</th><th>X</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	X	0	0	1	0	1	0	1	0	0	1	1	0
A	B	X																
0	0	1																
0	1	0																
1	0	0																
1	1	0																
XOR Exclusive OR		$X = A \oplus B$ or $X = A'B + AB'$	<table><tr><th>A</th><th>B</th><th>X</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	X	0	0	0	0	1	1	1	0	1	1	1	0
A	B	X																
0	0	0																
0	1	1																
1	0	1																
1	1	0																
XNOR Exclusive NOR or Equivalence		$X = (A \oplus B)'$ or $X = A'B' + AB$	<table><tr><th>A</th><th>B</th><th>X</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	X	0	0	1	0	1	0	1	0	0	1	1	1
A	B	X																
0	0	1																
0	1	0																
1	0	0																
1	1	1																

BOOLEAN ALGEBRA

Boolean Algebra

- * Algebra with Binary(Boolean) Variable and Logic Operations
- * Boolean Algebra is useful in Analysis and Synthesis of Digital Logic Circuits
 - Input and Output signals can be represented by Boolean Variables, and
 - Function of the Digital Logic Circuits can be represented by Logic Operations, i.e., Boolean Function(s)
 - From a Boolean function, a logic diagram can be constructed using AND, OR, and I

Truth Table

- * The most elementary specification of the function of a Digital Logic Circuit is the Truth Table
 - Table that describes the Output Values for all the combinations of the Input Values, called *MINTERMS*
 - n input variables $\rightarrow 2^n$ minterms

LOGIC CIRCUIT DESIGN

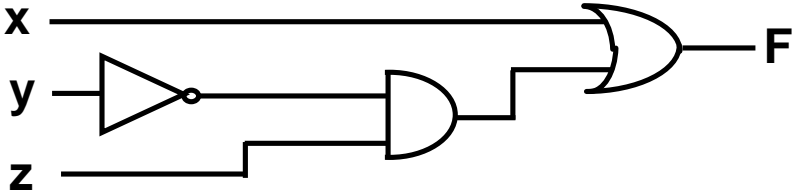
Truth Table

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Boolean Function

$F = x + y'z$

Logic Diagram



BASIC IDENTITIES OF BOOLEAN ALGEBRA

$$[1] \quad x + 0 = x$$

$$[3] \quad x + 1 = 1$$

$$[5] \quad x + x = x$$

$$[7] \quad x + x' = 1$$

$$[9] \quad x + y = y + x$$

$$[11] \quad x + (y + z) = (x + y) + z$$

$$[13] \quad x(y + z) = xy + xz$$

$$[15] \quad (x + y)' = x'y'$$

$$[17] \quad (x')' = x$$

$$[2] \quad x \cdot 0 = 0$$

$$[4] \quad x \cdot 1 = x$$

$$[6] \quad x \cdot x = x$$

$$[8] \quad x \cdot x' = 0$$

$$[10] \quad xy = yx$$

$$[12] \quad x(yz) = (xy)z$$

$$[14] \quad x + yz = (x + y)(x + z)$$

$$[16] \quad (xy)' = x' + y'$$

[15] and [16] : De Morgan's Theorem

Usefulness of this Table

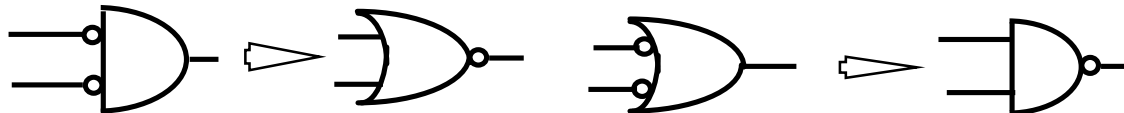
- Simplification of the Boolean function
 - Derivation of equivalent Boolean functions to obtain logic diagrams utilizing different logic gates
 - Ordinarily ANDs, ORs, and Inverters
 - But a certain different form of Boolean function may be convenient to obtain circuits with NANDs or NORs
- Applications of De Morgans Theorem

$$x'y' = (x + y)'$$

I, AND → NOR

$$x' + y' = (xy)'$$

I, OR → NAND



EQUIVALENT CIRCUITS

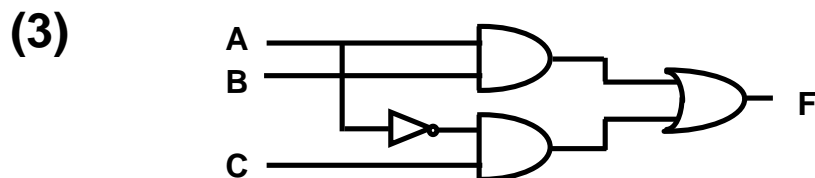
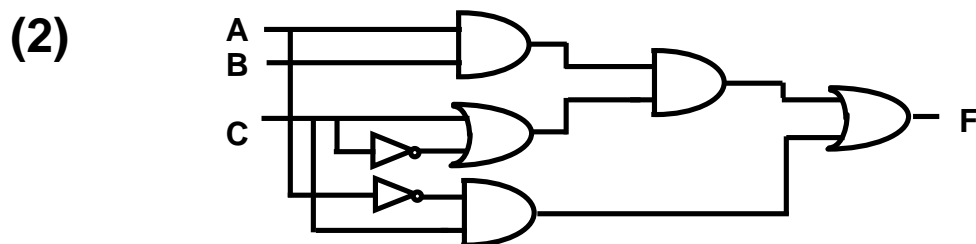
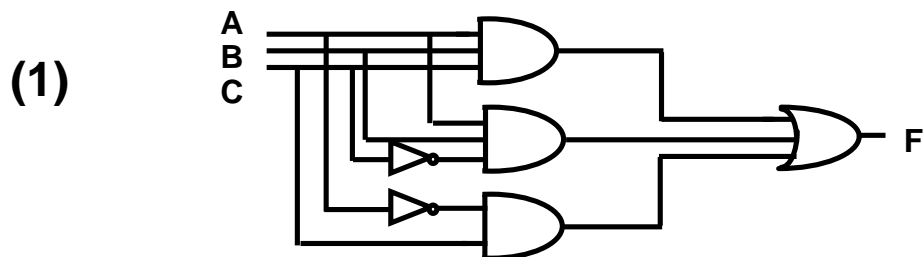
Many different logic diagrams are possible for a given Function

$$F = ABC + ABC' + A'C \quad \dots\dots\dots (1)$$

$$= AB(C + C') + A'C \quad [13] \dots\dots (2)$$

$$= AB \cdot 1 + A'C \quad [7]$$

$$= AB + A'C \quad [4] \dots\dots (3)$$



COMPLEMENT OF FUNCTIONS

A Boolean function of a digital logic circuit is represented by only using logical variables and AND, OR, and Invert operators.

→ Complement of a Boolean function

- Replace all the variables and subexpressions in the parentheses appearing in the function expression with their respective complements

$$A, B, \dots, Z, a, b, \dots, z \Rightarrow A', B', \dots, Z', a', b', \dots, z'$$
$$(p + q) \Rightarrow (p + q)'$$

- Replace all the operators with their respective complementary operators

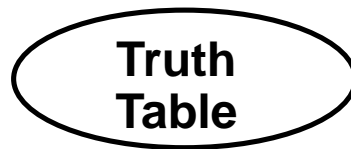
$$\text{AND} \Rightarrow \text{OR}$$
$$\text{OR} \Rightarrow \text{AND}$$

- Basically, extensive applications of the De Morgan's theorem

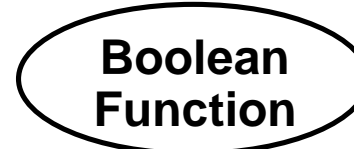
$$(x_1 + x_2 + \dots + x_n)' \Rightarrow x_1' x_2' \dots x_n'$$

$$(x_1 x_2 \dots x_n)' \Rightarrow x_1' + x_2' + \dots + x_n'$$

SIMPLIFICATION



Unique

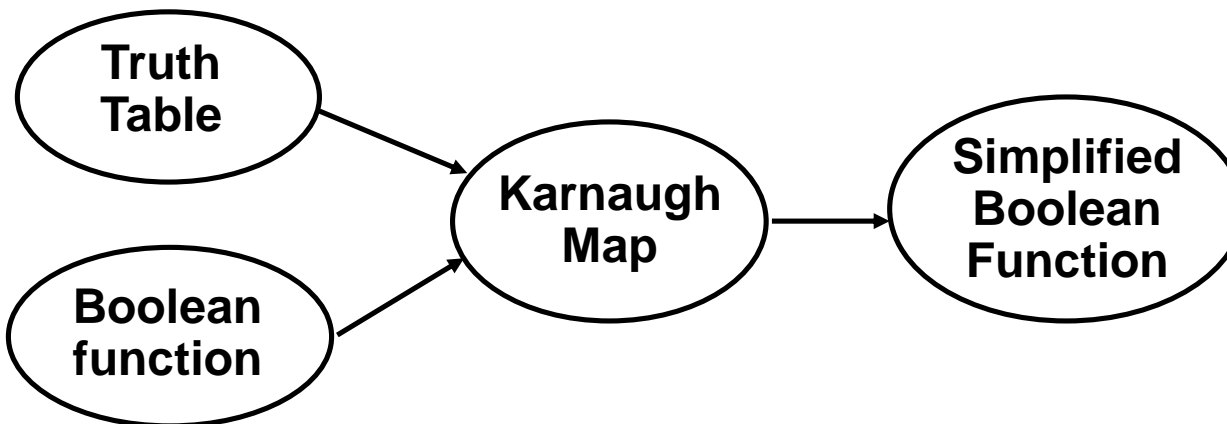


Many different expressions exist

Simplification from Boolean function

- Finding an equivalent expression that is least expensive to implement
- For a simple function, it is possible to obtain a simple expression for low cost implementation
- But, with complex functions, it is a very difficult task

Karnaugh Map (K-map) is a simple procedure for simplifying Boolean expressions.



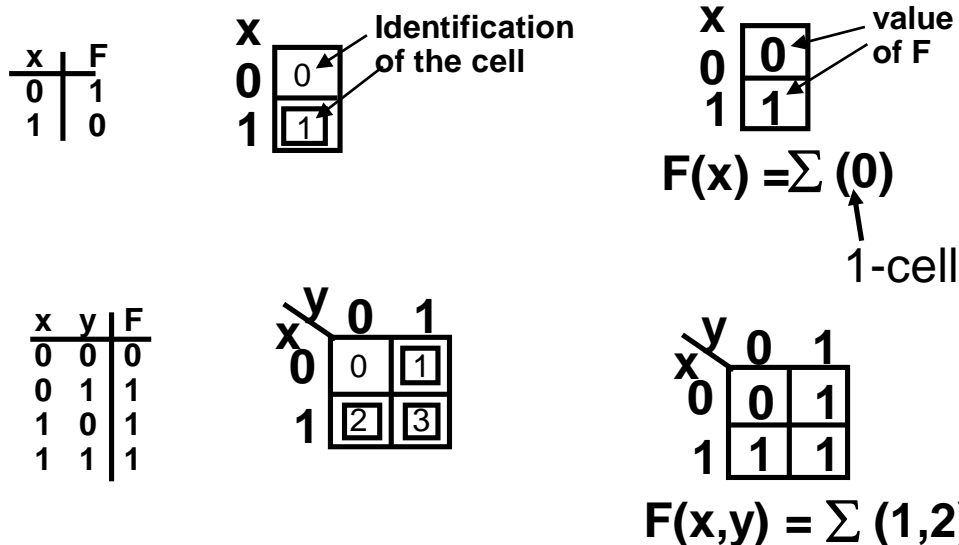
KARNAUGH MAP

Karnaugh Map for an n-input digital logic circuit (n-variable sum-of-products form of Boolean Function, or Truth Table) is

- Rectangle divided into 2^n cells
- Each cell is associated with a *Minterm*
- An output(function) value for each input value associated with a minterm is written in the cell representing the minterm
→ 1-cell, 0-cell

Each Minterm is identified by a decimal number whose binary representation is identical to the binary interpretation of the input values of the minterm.

Karnaugh Map



KARNAUGH MAP

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

		y			
		00	01	11	10
x \ yz	0	0	1	3	2
	1	4	5	7	6

		y			
		00	01	11	10
x \ yz	0	0	1	0	1
	1	1	0	0	0

$F(x,y,z) = \sum (1,2,4)$

u	v	w	x	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

		w			
		00	01	11	10
u \ vx	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10

		w			
		00	01	11	10
u \ vx	00	0	1	1	0
	01	0	0	0	1
	11	0	0	0	1
	10	1	1	1	0

$F(u,v,w,x) = \sum (1,3,6,8,9,11,14)$

MAP SIMPLIFICATION - 2 ADJACENT CELLS -

Rule: $xy' + xy = x(y + y') = x$

Adjacent cells

- binary identifications are different in one bit
→ minterms associated with the adjacent cells have one variable complemented each other

Cells (1,0) and (1,1) are adjacent

Minterms for (1,0) and (1,1) are

$$x \cdot y' \rightarrow x=1, y=0$$

$$x \cdot y \rightarrow x=1, y=1$$

$F = xy' + xy$ can be reduced to $F = x$
From the map

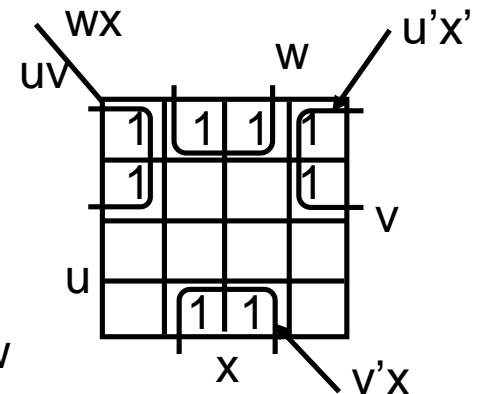
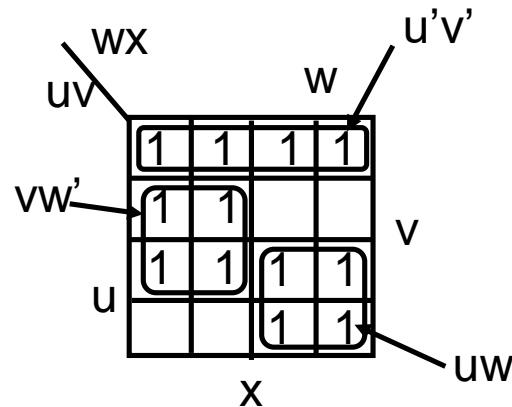
		y		0	1
		x		0	1
0				0	0
1				1	1

2 adjacent cells xy' and xy
→ merge them to a larger cell x

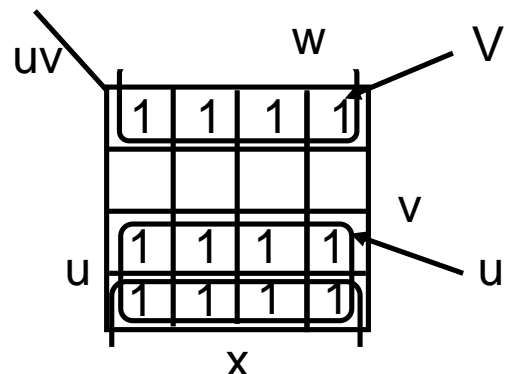
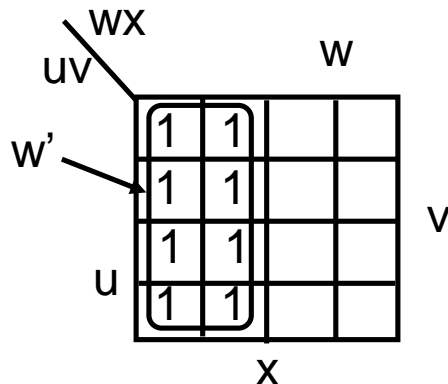
$$\begin{aligned}
 F(x,y) &= \sum (2,3) \\
 &= xy' + xy \\
 &= x
 \end{aligned}$$

MAP SIMPLIFICATION - MORE THAN 2 CELLS -

$$\begin{aligned}
 &u'v'w'x' + u'v'w'x + u'v'wx + u'v'wx' \\
 &= u'v'w'(x' + x) + u'v'w(x + x') \\
 &= u'v'w' + u'v'w \\
 &= u'v'(w' + w) \\
 &= u'v'
 \end{aligned}$$



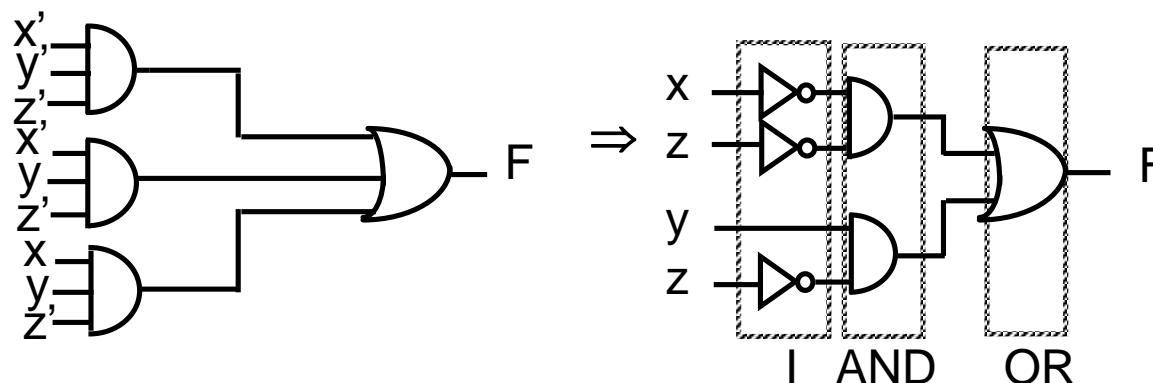
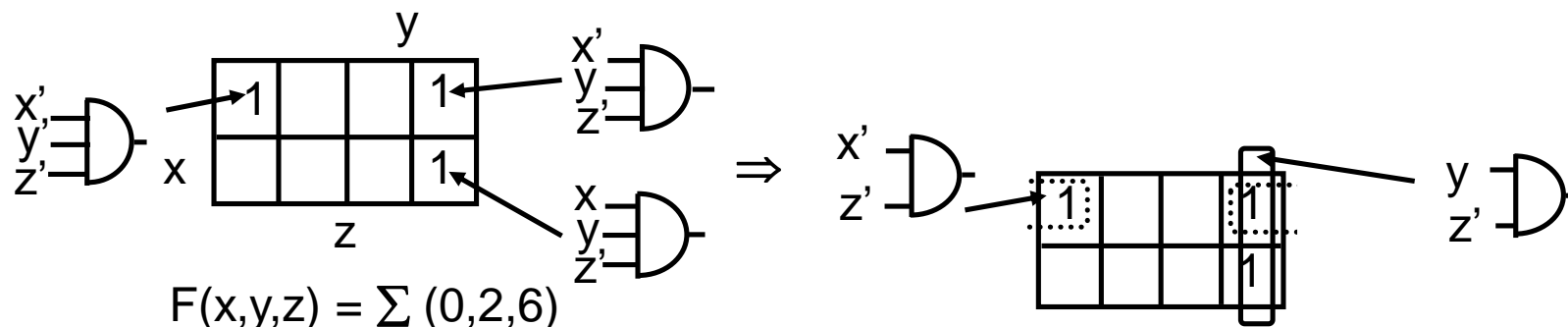
$$\begin{aligned}
 &u'v'w'x' + u'v'w'x + u'vw'x' + u'vw'x + uvw'x' + uvw'x + uv'w'x' + uv'w'x \\
 &= u'v'w'(x' + x) + u'vw'(x' + x) + uvw'(x' + x) + uv'w'(x' + x) \\
 &= u'(v' + v)w' + u(v' + v)w' \\
 &= (u' + u)w' = w'
 \end{aligned}$$



IMPLEMENTATION OF K-MAPS - Sum-of-Products Form -

Logic function represented by a Karnaugh map can be implemented in the form of I-AND-OR

A cell or a collection of the adjacent 1-cells can be realized by an AND gate, with some inversion of the input variables.

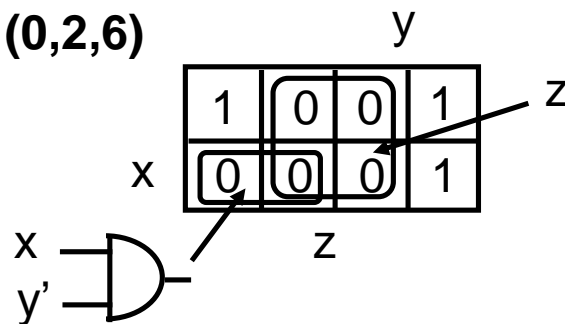


IMPLEMENTATION OF K-MAPS - Product-of-Sums Form -

Logic function represented by a Karnaugh map can be implemented in the form of I-OR-AND

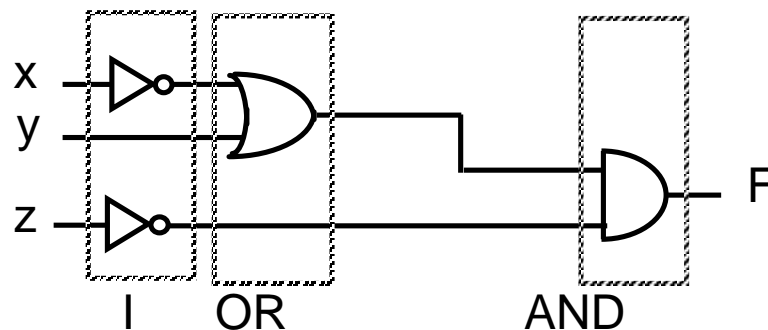
If we implement a Karnaugh map using 0-cells, the complement of F , i.e., F' , can be obtained. Thus, by complementing F' using DeMorgan's theorem F can be obtained

$$F(x,y,z) = (0,2,6)$$



$$F' = xy' + z$$

$$F = (xy')z' \\ = (x' + y)z'$$



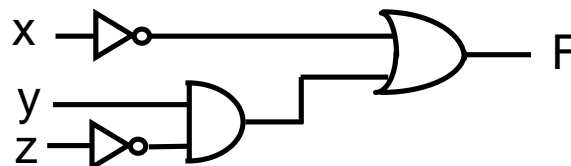
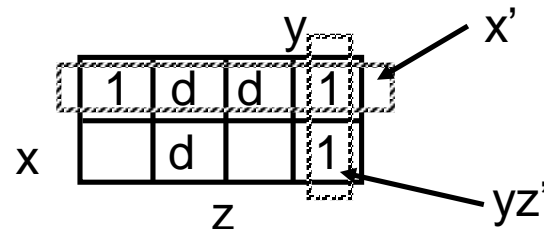
IMPLEMENTATION OF K-MAPS - Don't-Care Conditions -

In some logic circuits, the output responses for some input conditions are don't care whether they are 1 or 0.

In K-maps, don't-care conditions are represented by d's in the corresponding cells.

Don't-care conditions are useful in minimizing the logic functions using K-map.

- Can be considered either 1 or 0
- Thus increases the chances of merging cells into the larger cells
--> Reduce the number of variables in the product terms



COMBINATIONAL LOGIC CIRCUITS

Half Adder

x	y	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

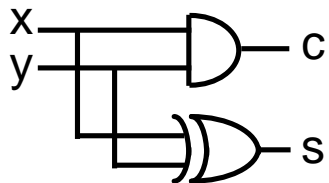
	y
x	0
	1

$$c = xy$$

	y
x	0
	1

$$s = xy' + x'y$$

$$= x \oplus y$$



Full Adder

x	y	c_{n-1}	c_n	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

	y
x	0
	1

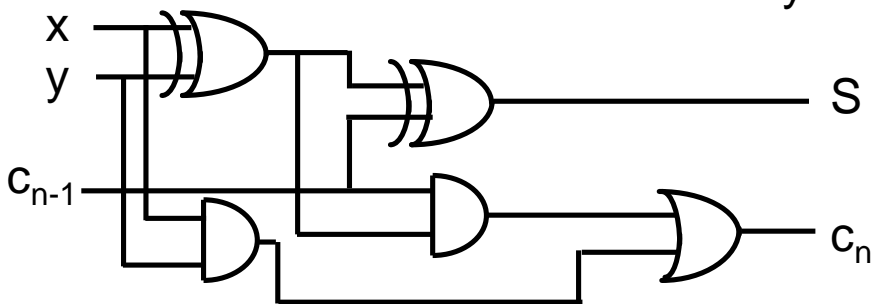
	y
x	0
	1

$$c_n = xy + xc_{n-1} + yc_{n-1}$$

$$= xy + (x \oplus y)c_{n-1}$$

$$s = x'y'c_{n-1} + x'yc'_{n-1} + xy'c'_{n-1} + xyc_{n-1}$$

$$= x \oplus y \oplus c_{n-1} = (x \oplus y) \oplus c_{n-1}$$



COMBINATIONAL LOGIC CIRCUITS

Other Combinational Circuits

Multiplexer

Encoder

Decoder

Parity Checker

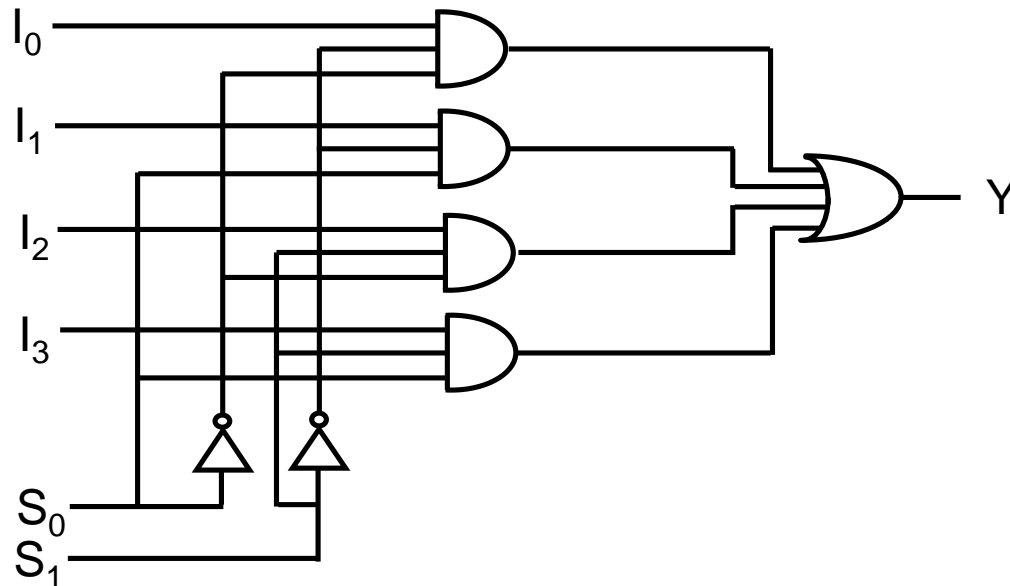
Parity Generator

etc

MULTIPLEXER

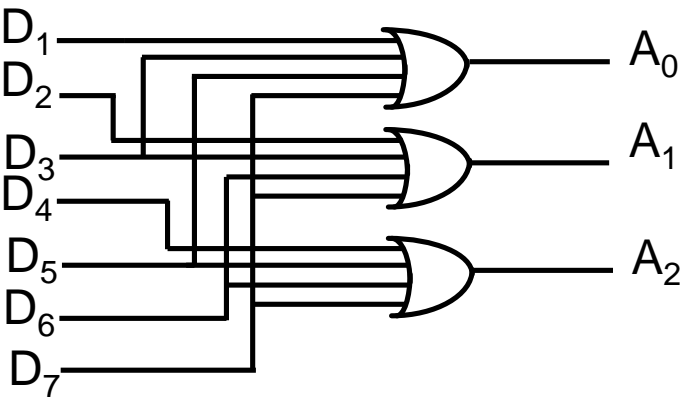
4-to-1 Multiplexer

Select		Output
S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3



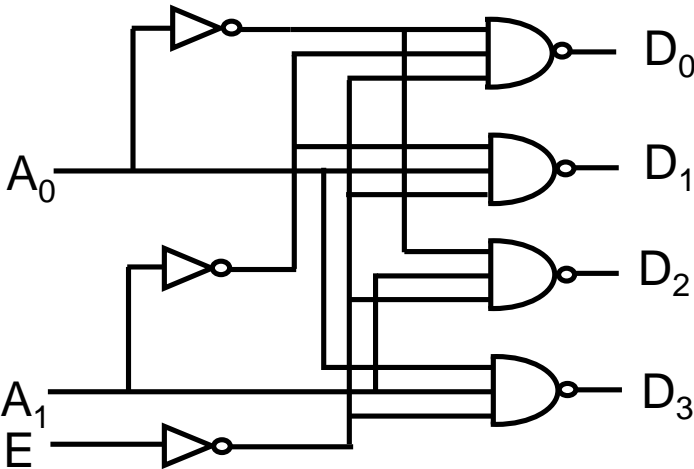
ENCODER/DECODER

Octal-to-Binary Encoder



2-to-4 Decoder

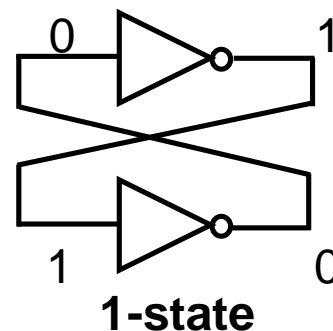
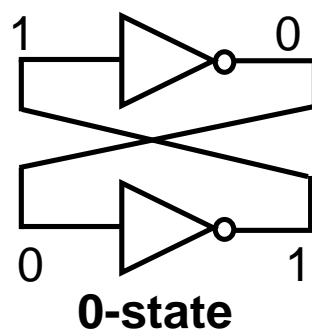
E	A ₁	A ₀	D ₀	D ₁	D ₂	D ₃
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0
1	d	d	1	1	1	1



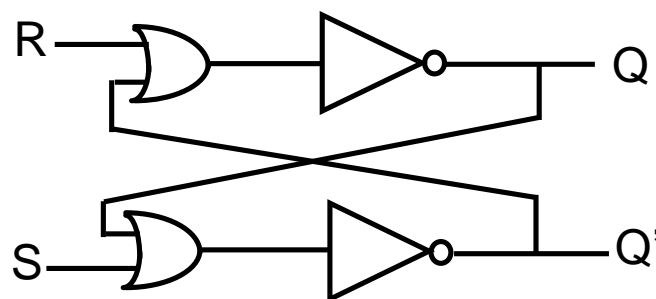
FLIP FLOPS

Characteristics

- 2 stable states
- Memory capability
- Operation is specified by a Characteristic Table



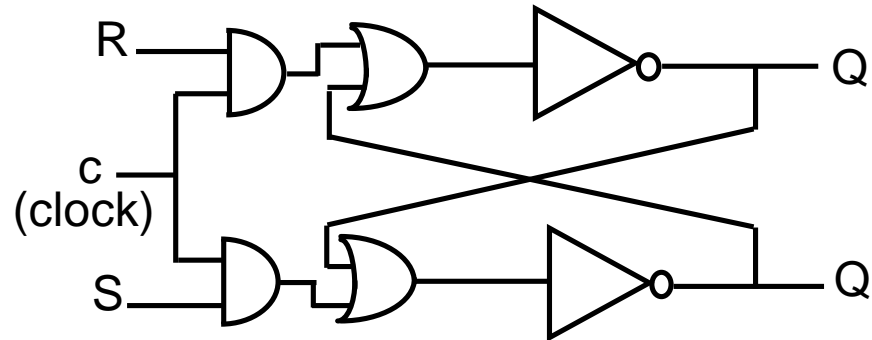
In order to be used in the computer circuits, state of the flip flop should have input terminals and output terminals so that it can be set to a certain state, and its state can be read externally.



S	R	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	indeterminate (forbidden)

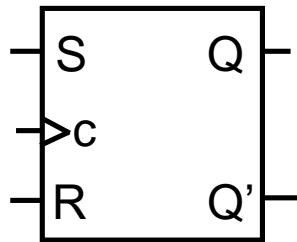
CLOCKED FLIP FLOPS

In a large digital system with many flip flops, operations of individual flip flops are required to be synchronized to a clock pulse. Otherwise, the operations of the system may be unpredictable.

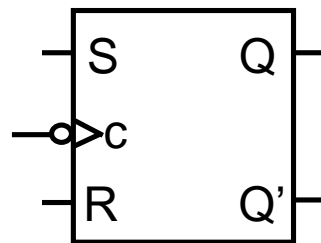


Clock pulse allows the flip flop to change state only when there is a clock pulse appearing at the c terminal.

We call above flip flop a Clocked RS Latch, and symbolically as

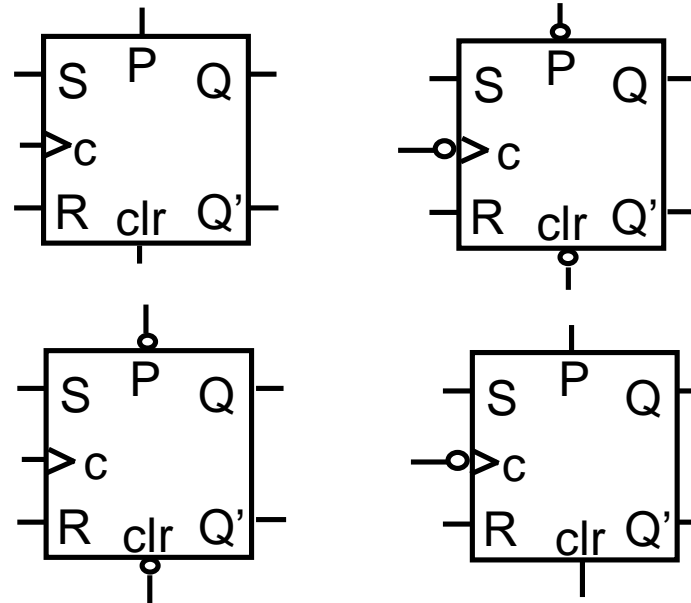
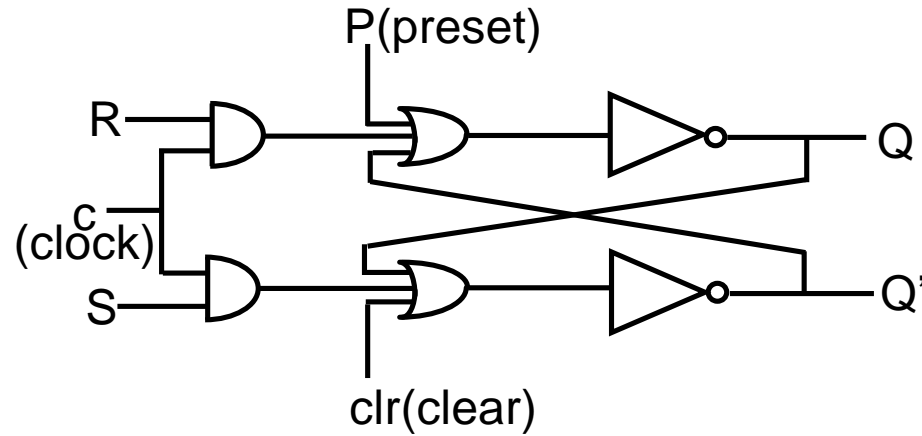


operates when
clock is high



operates when
clock is low

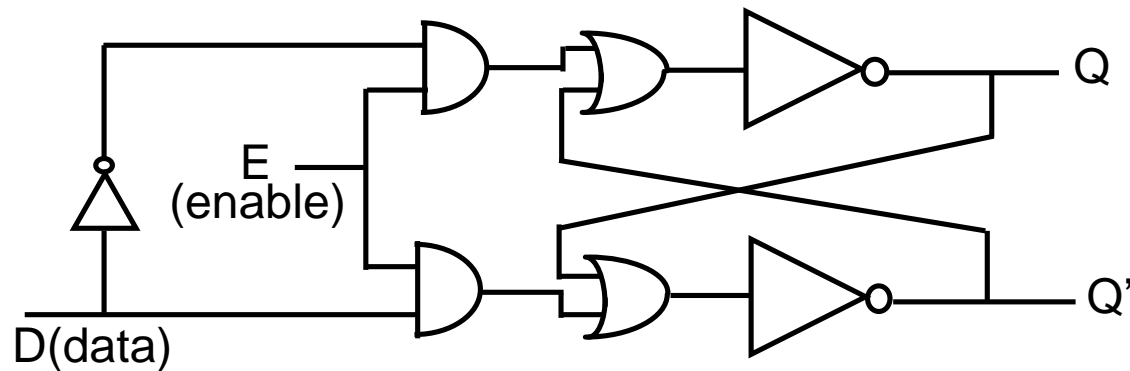
RS-LATCH WITH PRESET AND CLEAR INPUTS



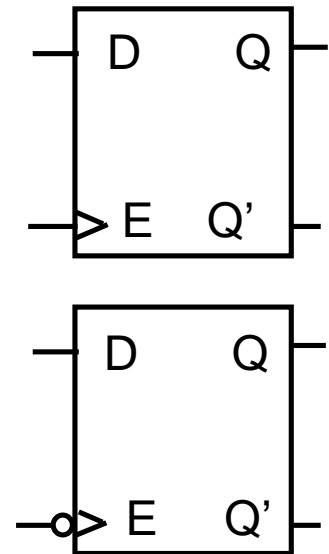
D-LATCH

D-Latch

Forbidden input values are forced not to occur by using an inverter between the inputs



D	Q(t+1)
0	0
1	1

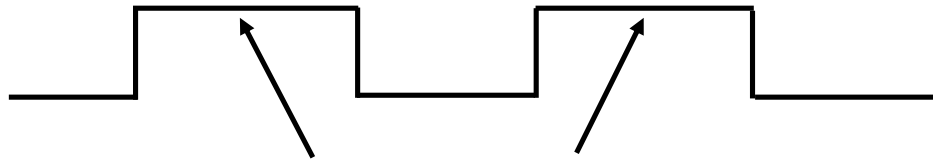


EDGE-TRIGGERED FLIP FLOPS

Characteristics

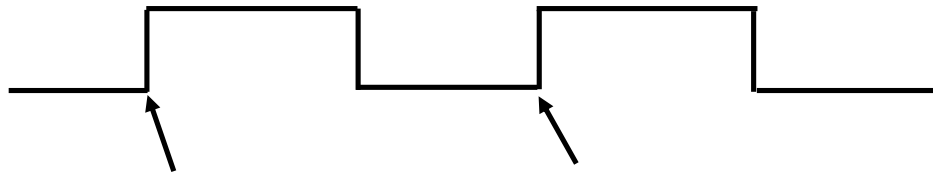
- State transition occurs at the rising edge or falling edge of the clock pulse

Latches



respond to the input only during these periods

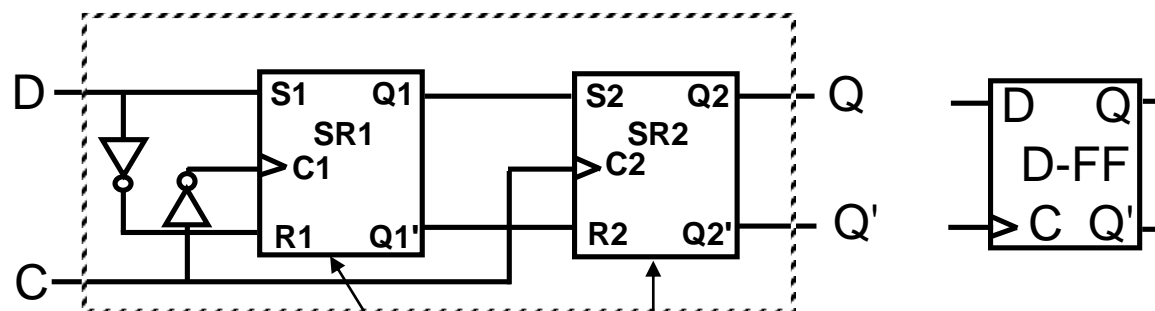
Edge-triggered Flip Flops (positive)



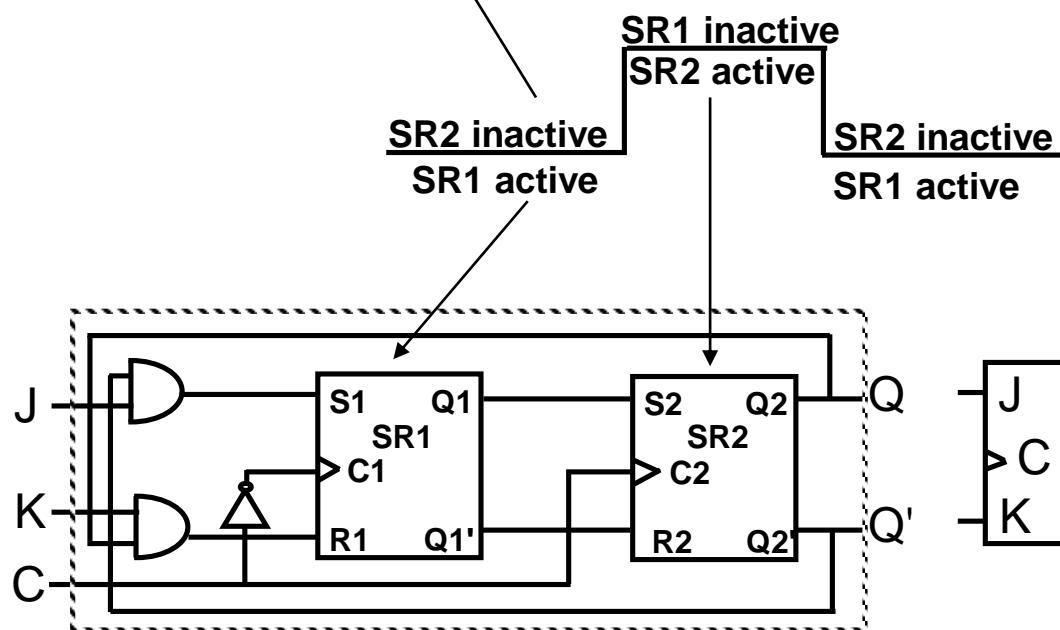
respond to the input only at this time

POSITIVE EDGE-TRIGGERED

D-Flip Flop



JK-Flip Flop

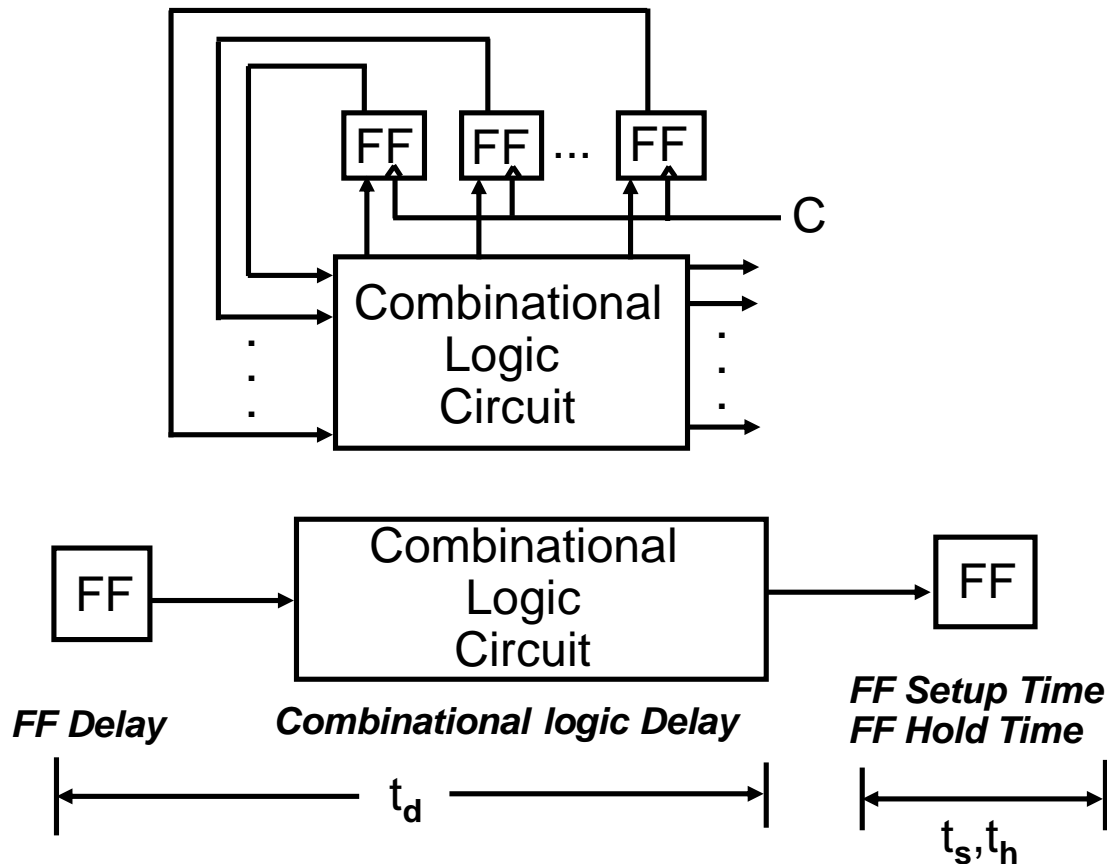


T-Flip Flop: JK-Flip Flop whose J and K inputs are tied together to make T input. Toggles whenever there is a pulse on T input.

CLOCK PERIOD

**Clock period determines how fast the digital circuit operates.
How can we determine the clock period ?**

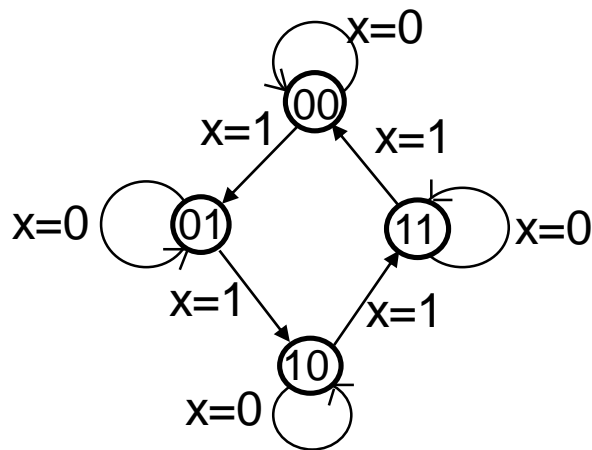
Usually, digital circuits are sequential circuits which has some flip flops



DESIGN EXAMPLE

Design Procedure:
Specification \Rightarrow State Diagram \Rightarrow State Table \Rightarrow
Excitation Table \Rightarrow Karnaugh Map \Rightarrow Circuit Diagram

Example: 2-bit Counter \rightarrow 2 FF's



current state		input	next state		FF inputs			
A	B		A	B	Ja	Ka	Jb	Kb
0	0	0	0	0	0	d	0	d
0	0	1	0	1	0	d	1	d
0	1	0	0	1	0	d	d	0
0	1	1	1	0	1	d	d	1
1	0	0	1	0	d	0	0	d
1	0	1	1	1	d	0	1	d
1	1	0	1	1	d	0	d	0
1	1	1	0	0	d	1	d	1

	B
	1
d	d
d	d

A

x

Ja

Ja = Bx

	B
d	d
d	d
	1

A

x

Ka

Ka = Bx

	B
	d
1	d
1	d
	d

A

x

Jb

Jb = x

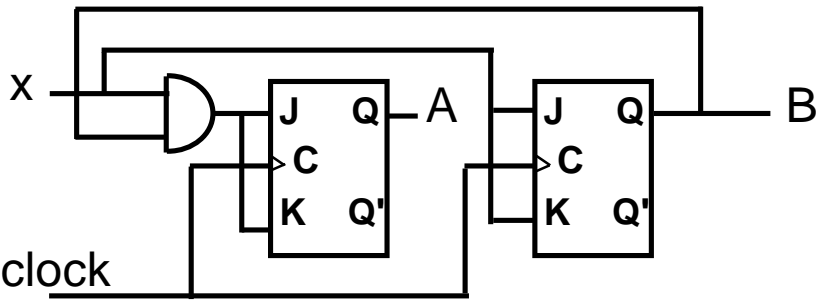
	B
d	
d	1
d	1
d	

A

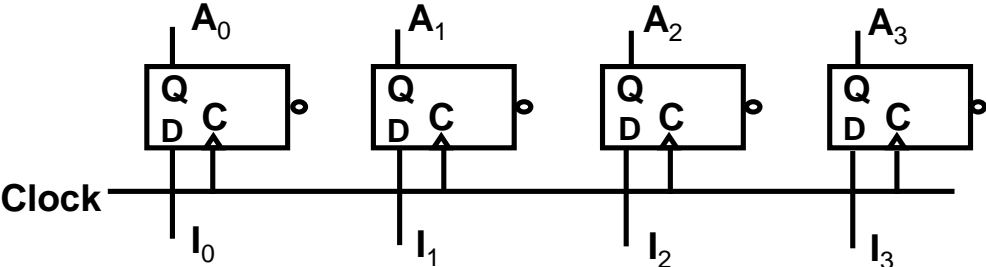
x

Kb

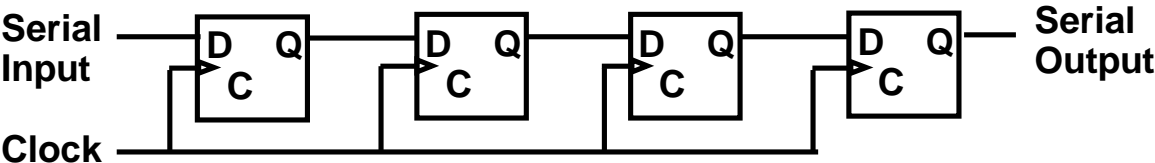
Kb = x



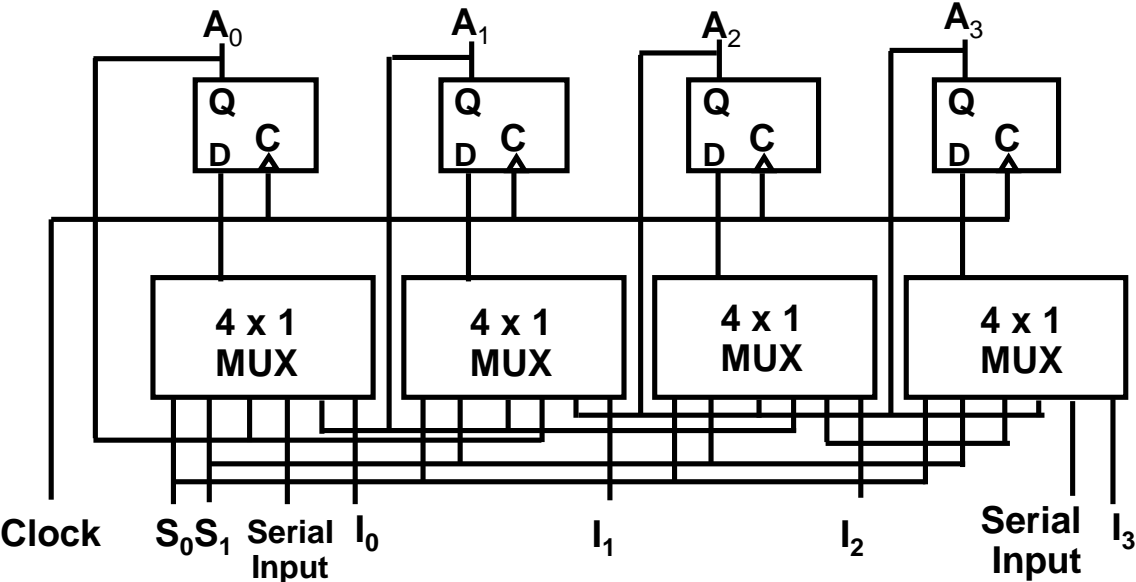
SEQUENTIAL CIRCUITS - Registers



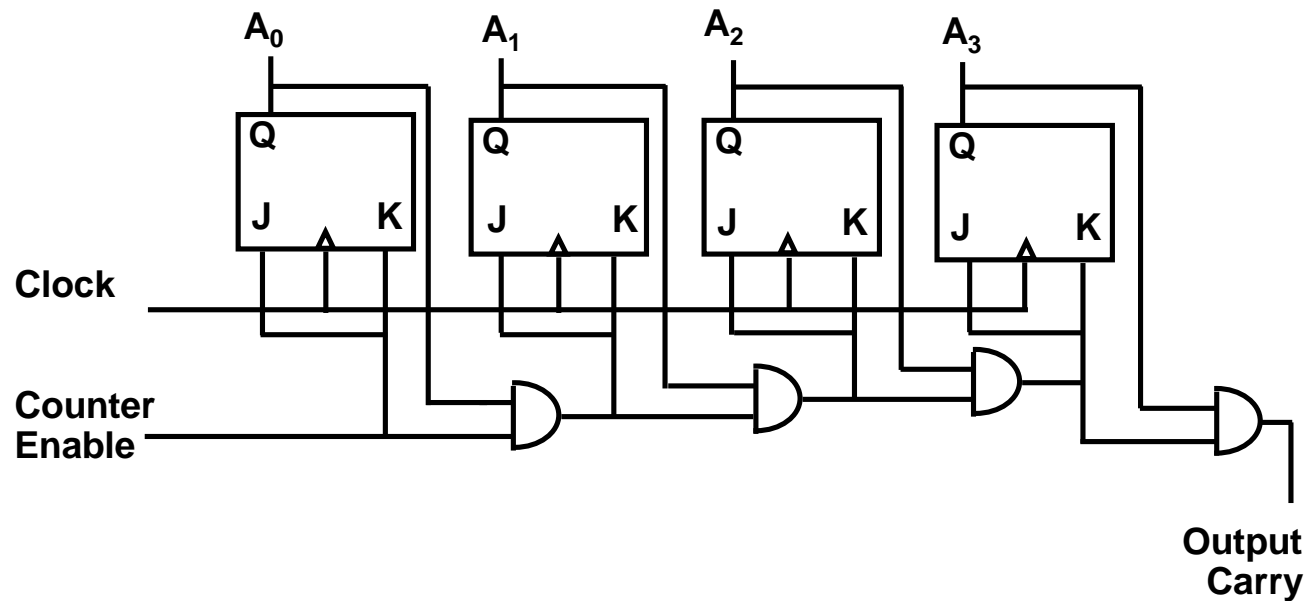
Shift Registers



Bidirectional Shift Register with Parallel Load



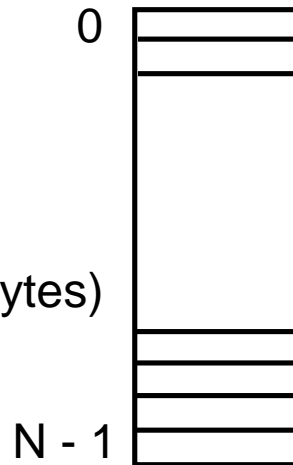
SEQUENTIAL CIRCUITS - Counters



MEMORY COMPONENTS

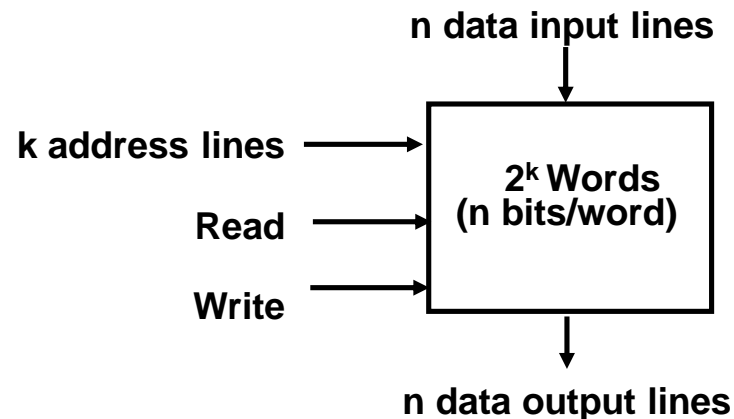
Logical Organization

words
(byte, or n bytes)



Random Access Memory

- Each word has a unique address
- Access to a word requires the same time independent of the location of the word
- Organization

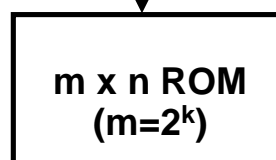


READ ONLY MEMORY(ROM)

Characteristics

- Perform read operation only, write operation is not possible
- Information stored in a ROM is made permanent during production, and cannot be changed
- Organization

k address input lines

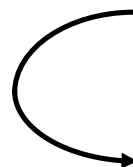


n data output lines

Information on the data output line depends only on the information on the address input lines.

--> Combinational Logic Circuit

$$\begin{aligned} X_0 &= A'B' + B'C \\ X_1 &= A'B'C + A'BC' \\ X_2 &= BC + AB'C' \\ X_3 &= A'BC' + AB' \\ X_4 &= AB \end{aligned}$$



$$\begin{aligned} X_0 &= A'B'C' + A'B'C + AB'C \\ X_1 &= A'B'C + A'BC' \\ X_2 &= A'BC + AB'C' + ABC \\ X_3 &= A'BC' + AB'C' + AB'C \\ X_4 &= ABC' + ABC \end{aligned}$$

Canonical minterms

address

Output

ABC	X ₀	X ₁	X ₂	X ₃	X ₄
000	1	0	0	0	0
001	1	1	0	0	0
010	0	1	0	1	0
011	0	0	1	0	0
100	0	0	1	1	0
101	1	0	0	1	0
110	0	0	0	0	1
111	0	0	1	0	1

TYPES OF ROM

ROM

- Store information (function) during production
- Mask is used in the production process
- Unalterable
- Low cost for large quantity production --> used in the final products

PROM (Programmable ROM)

- Store info electrically using PROM programmer at the user's site
- Unalterable
- Higher cost than ROM -> used in the system development phase
-> Can be used in small quantity system

EPROM (Erasable PROM)

- Store info electrically using PROM programmer at the user's site
- Stored info is erasable (alterable) using UV light (electrically in some devices) and rewriteable
- Higher cost than PROM but reusable --> used in the system development phase. Not used in the system production due to erasability

INTEGRATED CIRCUITS

Classification by the Circuit Density

- SSI -** several (less than 10) independent gates
- MSI -** 10 to 200 gates; Perform elementary digital functions;
Decoder, adder, register, parity checker, etc
- LSI -** 200 to few thousand gates; Digital subsystem
Processor, memory, etc
- VLSI -** Thousands of gates; Digital system
Microprocessor, memory module

Classification by Technology

- TTL -** Transistor-Transistor Logic
Bipolar transistors
NAND
- ECL -** Emitter-coupled Logic
Bipolar transistor
NOR
- MOS -** Metal-Oxide Semiconductor
Unipolar transistor
High density
- CMOS -** Complementary MOS
Low power consumption