FINC-UB 1 Homework 2

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1 The binomial distribution

1. The expected value is $\mu = \$2.00\left(\frac{1}{2}\right) + (-\$1.00)\left(\frac{1}{2}\right) = \$0.50$.
2. The expected value is $\mu = \$14.00(10\%) + (-\$2.00)(90\%) = -\$0.40$.

2 Historical stock returns

$$r_t^i = \frac{P_t^i - P_{t-1}^i}{P_{t-1}^i}.$$

1. See the daily returns of each equity security below.

Date	TSLA	AAL	PFE	DAL
–	–	–	–	–
6/1/2021	−0.21%	1.77%	−0.59%	0.04%
6/2/2021	−3.01%	4.66%	0.75%	0.15%
6/3/2021	−5.33%	−3.45%	0.46%	−3.43%
6/4/2021	4.58%	−2.53%	0.46%	−0.33%
6/7/2021	1.01%	−0.21%	−0.41%	0.22%
6/8/2021	−0.25%	−0.12%	−0.36%	2.08%
6/9/2021	−0.80%	−1.53%	2.47%	−1.11%
6/10/2021	1.89%	−1.64%	2.19%	−0.52%
6/11/2021	−0.04%	0.30%	−1.30%	0.48%
6/14/2021	1.28%	−2.29%	−1.30%	−1.29%
6/15/2021	−2.97%	−0.87%	−0.10%	−0.81%
6/16/2021	0.92%	0.18%	−0.73%	−0.02%
6/17/2021	1.94%	−2.63%	0.46%	−2.09%
6/18/2021	1.09%	0.27%	−1.70%	0.88%
6/21/2021	−0.40%	0.72%	1.57%	1.80%
6/22/2021	0.46%	−1.51%	0.48%	−0.87%
6/23/2021	5.27%	0.41%	−1.41%	−0.22%
6/24/2021	3.54%	0.68%	0.36%	−0.80%
6/25/2021	−1.17%	−0.58%	−0.54%	−1.07%
6/28/2021	2.51%	−3.74%	0.36%	−2.95%
6/29/2021	−1.16%	−1.45%	−0.05%	−0.58%
6/30/2021	−0.16%	0.62%	0.15%	0.91%

Date	TSLA	AAL	PFE	DAL
7/1/2021	–0.26%	1.41%	1.02%	2.24%
7/2/2021	0.14%	–0.14%	0.43%	–0.18%
7/6/2021	–2.85%	–2.19%	–1.11%	–2.06%
7/7/2021	–2.26%	–3.33%	0.15%	–1.62%
7/8/2021	1.27%	0.20%	–0.25%	–1.10%
7/9/2021	0.63%	2.65%	0.92%	2.02%
7/12/2021	4.38%	–0.24%	0.38%	–0.16%
7/13/2021	–2.50%	–3.93%	–0.28%	–3.55%
7/14/2021	–2.27%	3.00%	0.76%	–1.57%
7/15/2021	–0.43%	–0.78%	0.35%	1.65%
7/16/2021	–0.98%	–3.27%	0.65%	–3.12%
7/19/2021	0.31%	–4.14%	–0.50%	–3.74%
7/20/2021	2.21%	8.38%	2.24%	5.45%
7/21/2021	–0.79%	4.09%	–0.07%	2.34%
7/22/2021	–0.92%	–1.12%	1.10%	–1.32%
7/23/2021	–0.91%	0.19%	0.51%	–1.58%
7/26/2021	2.21%	4.15%	0.31%	3.12%
7/27/2021	–1.95%	–2.81%	0.69%	–1.70%
7/28/2021	0.34%	0.79%	3.21%	0.83%
7/29/2021	4.69%	–2.13%	–1.52%	–0.17%
7/30/2021	1.45%	–3.73%	0.05%	–3.23%

Date	TSLA	AAL	PFE	DAL
8/2/2021	3.27%	–1.57%	2.69%	–1.13%
8/3/2021	0.01%	–0.55%	3.91%	0.81%
8/4/2021	0.17%	–2.51%	–1.07%	–3.57%
8/5/2021	0.52%	7.51%	–0.29%	4.28%
8/6/2021	–2.17%	0.53%	0.02%	0.83%
8/9/2021	2.10%	–2.19%	2.02%	–2.55%
8/10/2021	–0.53%	1.85%	4.81%	2.98%
8/11/2021	–0.31%	1.38%	–3.90%	2.08%
8/12/2021	2.04%	–3.77%	2.01%	–1.86%
8/13/2021	–0.70%	–2.89%	2.62%	–1.55%
8/16/2021	–4.32%	–0.40%	0.89%	0.08%
8/17/2021	–2.98%	–2.13%	3.09%	–2.88%
8/18/2021	3.50%	–1.03%	–2.20%	–0.34%
8/19/2021	–2.25%	–2.14%	–1.03%	–1.86%
8/20/2021	1.01%	–1.23%	–0.16%	0.50%
8/23/2021	3.83%	3.30%	2.48%	2.86%
8/24/2021	0.31%	3.77%	–3.10%	3.37%
8/25/2021	0.38%	1.61%	–1.80%	1.90%
8/26/2021	–1.41%	–1.24%	–0.27%	–1.28%
8/27/2021	1.53%	1.61%	–1.65%	1.84%
8/30/2021	2.67%	–3.51%	0.34%	–3.88%

2. See the variance-covariance matrix below.

	TSLA	AAL	PFE	DAL
TSLA	0.00048	0.00004	-0.00002	0.00008
AAL	0.00004	0.00069	0.00001	0.00046
PFE	-0.00002	-0.00001	0.00025	-0.00000
DAL	0.00008	0.00046	-0.00000	0.00043

3. See the correlation matrix below.

	TSLA	AAL	PFE	DAL
TSLA	1.000	0.068	-0.058	0.177
AAL	0.068	1.000	-0.017	0.852
PFE	-0.058	-0.017	1.000	0.013
DAL	0.177	0.852	-0.013	1.000

4. See the autocorrelation figures below.

k	TSLA
1	0.006
2	-0.159
3	0.006

3 Firms and economic uncertainty: Part I

X	0	4	16	32
$P(X)$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{16}$

Let μ_X be the expected value of X, σ_X^2 be the variance of X, and σ_X be the standard deviation of X.

$$\begin{aligned}\mu_X &= \sum_{i=0}^{n-1} p_i x_i \\ &= \left(\frac{5}{16} \times 0\right) + \left(\frac{1}{4} \times 4\right) + \left(\frac{3}{8} \times 16\right) + \left(\frac{1}{16} \times 32\right) \\ &= 9.\end{aligned}$$

$$\begin{aligned}\sigma_X^2 &= \sum_{i=0}^{n-1} p_i (x_i - \mu_X)^2 \\ &= \left(\frac{5}{16}\right)(0-9)^2 + \left(\frac{1}{4}\right)(4-9)^2 + \left(\frac{3}{8}\right)(16-9)^2 + \left(\frac{1}{16}\right)(32-9)^2 \\ &= 83.\end{aligned}$$

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{83} \approx 9.1104\ldots$$

4 Firms and economic uncertainty: Part II

	IBM	DELL	Probability
Recession	0	-10%	10%
Normal	10%	10%	50%
Boom	20%	30%	40%

1. The expected return for IBM, μ_{IBM}, is 13%:

$$\mu_{\text{IBM}} = (0 \times 10\%) + (10\% \times 50\%) + (20\% \times 40\%) = 13\%.$$

The expected return for DELL, μ_{DELL}, is 16%:

$$\mu_{\text{DELL}} = (-10\% \times 10\%) + (10\% \times 50\%) + (30\% \times 40\%) = 16\%.$$

2. The variance for the returns of IBM, σ^2_{IBM}, is 41‱:

$$\sigma^2_{\text{IBM}} = (10\%)(0 - 13\%)^2 + (50\%)(10\% - 13\%)^2 + (40\%)(20\% - 13\%)^2 = 0.41‱.$$

The standard deviation for the returns of IBM, σ_{IBM}, is about 6.4%:

$$\sigma_{\text{IBM}} = \sqrt{\sigma^2_{\text{IBM}}} = \sqrt{0.41\%} \approx 6.4031\% \ldots$$

3. The variance for the returns of DELL, σ^2_{DELL}, is 164‱:

$$\sigma^2_{\text{DELL}} = (10\%)(-10\% - 16\%)^2 + (50\%)(10\% - 16\%)^2 + (40\%)(30\% - 16\%)^2 = 164‱.$$

The standard deviation for the returns of DELL, σ_{DELL}, is about 12.8%:

$$\sigma_{\text{DELL}} = \sqrt{\sigma^2_{\text{DELL}}} = \sqrt{1.64\%} \approx 12.8062\% \ldots$$

4. The covariance between the returns of IBM and DELL, $\sigma_{\text{IBM,DELL}}$, is 0.82%:

$$\sigma_{\text{IBM,DELL}} = \sum_{i=0}^{n-1} (\text{ibm}_i - \mu_{\text{IBM}})(\text{dell}_i - \mu_{\text{DELL}}) = 0.82\%.$$

5. The correlation between the returns of IBM and DELL, $\rho_{\text{IBM,DELL}}$, is:

$$\rho_{\text{IBM,DELL}} = \frac{\sigma_{\text{IBM,DELL}}}{\sigma_{\text{IBM}}\sigma_{\text{DELL}}} \approx 1.0005 \ldots$$

Since the correlation must be between -1 and 1, we conclude that the correlation is approximately 1.

5 Firms and economic uncertainty: Part III

1. Let μ_X be the expected return of small firms, μ_Y be the expected return of large firms, σ_X^2 be the variance of the return of small firms, and σ_Y^2 be the variance of the return of large firms.

$$\mu_X = \left(0 \times \frac{1}{10}\right) + \left(1 \times \frac{1}{5}\right) + \left(2 \times \frac{2}{5}\right) + \left(3 \times \frac{1}{5}\right) + \left(4 \times \frac{1}{10}\right) = 2.$$

$$\mu_Y = \left(-4 \times \frac{1}{10}\right) + \left(-2 \times \frac{1}{5}\right) + \left(4 \times \frac{2}{5}\right) + \left(7 \times \frac{1}{5}\right) + \left(8 \times \frac{1}{10}\right) = 3.$$

$$\sigma_X^2 = (0-2)^2\left(\frac{1}{10}\right) + (1-2)^2\left(\frac{1}{5}\right) + (2-2)^2\left(\frac{2}{5}\right) + (3-2)^2\left(\frac{1}{5}\right) + (4-2)^2\left(\frac{1}{10}\right) = 1.2.$$

$$\sigma_Y^2 = (-4-3)^2\left(\frac{1}{10}\right) + (-2-3)^2\left(\frac{1}{5}\right) + (4-3)^2\left(\frac{2}{5}\right) + (7-3)^2\left(\frac{1}{5}\right) + (8-3)^2\left(\frac{1}{10}\right) = 16.$$

2. Let $\sigma_{X,Y}$ be the covariance between the returns of small and large firms, and $\rho_{X,Y}$ be the correlation between those returns.

$$\begin{aligned}
\sigma_{X,Y} &= (0-2)(-4-3)\left(\frac{1}{10}\right) \\
&+ (1-2)(-2-3)\left(\frac{1}{5}\right) \\
&+ (2-2)(4-3)\left(\frac{2}{5}\right) \\
&+ (3-2)(7-3)\left(\frac{1}{5}\right) \\
&+ (4-2)(8-3)\left(\frac{1}{10}\right) \\
&= 4.2.
\end{aligned}$$

Let σ_X be the standard deviation of the returns of small firms and σ_Y be the standard deviation of the returns of large firms.

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y} = \frac{4.2}{\sqrt{1.2}\sqrt{16}} \approx 0.9585\ldots$$

3.

$$\mu_{3X-5Y} = 3\mu_X - 5\mu_Y = 3(2) - 5(3) = -9.$$

$$\begin{aligned}
\sigma_{3X-5Y}^2 &= 3^2\sigma_X^2 + 5^2\sigma_Y^2 - 2(3)(5)(\sigma_{X,Y}) \\
&= 9(1.2) + 25(16) - 30(4.2) = 284.8.
\end{aligned}$$

4.

$$\sigma_{3X,5Y} = (3)(5)(\sigma_{X,Y}) = 15(4.2) = 63.$$

$$\rho_{3X,5Y} = \frac{\sigma_{3X,5Y}}{\sigma_{3X}\sigma_{5Y}} = \frac{63}{(3)(\sqrt{1.2})(5)(\sqrt{16})} = \frac{4.2}{\sqrt{1.2}\sqrt{16}} = \rho_{X,Y} \approx 0.9585\ldots$$

5. Let Z be a random variable with mean $\mu_Z = 3$ and variance $\sigma_Z^2 = 5$. The covariance between X and Z is $\sigma_{X,Z} = 2$, and the covariance between Y and Z is $\sigma_{Y,Z} = 4$.

$$\begin{aligned}
\sigma_{3X,2Z-5Y} &= \sigma_{3X,2Z} - \sigma_{3X,5Y}; && \textit{bilinearity of covariance} \\
&= (3)(2)(\sigma_{X,Z}) - (3)(5)(\sigma_{X,Y}); && \textit{factor constant coefficient} \\
&= 6(2) - 15(4.2) && \textit{substitution} \\
&= -51.
\end{aligned}$$

$$\begin{aligned}
\rho_{3X,2Z-5Y} &= \frac{\sigma_{3X,2Z-5Y}}{\sigma_{3X}\sigma_{2Z-5Y}} && \textit{definition of correlation} \\
&= \frac{\sigma_{3X,2Z-5Y}}{3\sigma_X\sqrt{\sigma^2_{2Z-5Y}}} && \textit{definition of standard deviation} \\
&= \frac{\sigma_{3X,2Z-5Y}}{3\sigma_X\sqrt{2^2\sigma_Z^2 + 5^2\sigma_Y^2 - 2(2)(5)\sigma_{Z,Y}}} && \textit{definition of variance} \\
&= \frac{\sigma_{3X,2Z-5Y}}{3\sigma_X\sqrt{2^2\sigma_Z^2 + 5^2\sigma_Y^2 - 2(2)(5)\sigma_{Y,Z}}} && \textit{symmetry of covariance} \\
&= \frac{-51}{3\sqrt{1.2}\sqrt{4(5) + 25(16) - 2(2)(5)(4)}} && \textit{substitution} \\
&\approx -0.8416\ldots
\end{aligned}$$