

FINC-UB 1 Homework 2

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September 6, 2023

1 The binomial distribution

1. The expected value is $\mu = \$2.00 \left(\frac{1}{2}\right) + (-\$1.00) \left(\frac{1}{2}\right) = \0.50 .
2. The expected value is $\mu = \$14.00(10\%) + (-\$2.00)(90\%) = -\$0.40$.

2 Historical stock returns

$$r_t^i = \frac{P_t^i - P_{t-1}^i}{P_{t-1}^i}.$$

1. See the daily returns of each equity security below.

Date	TSLA	AAL	PFE	DAL
—	—	—	—	—
6/1/2021	−0.21%	1.77%	−0.59%	0.04%
6/2/2021	−3.01%	4.66%	0.75%	0.15%
6/3/2021	−5.33%	−3.45%	0.46%	−3.43%
6/4/2021	4.58%	−2.53%	0.46%	−0.33%
6/7/2021	1.01%	−0.21%	−0.41%	0.22%
6/8/2021	−0.25%	−0.12%	−0.36%	2.08%
6/9/2021	−0.80%	−1.53%	2.47%	−1.11%
6/10/2021	1.89%	−1.64%	2.19%	−0.52%
6/11/2021	−0.04%	0.30%	−1.30%	0.48%
6/14/2021	1.28%	−2.29%	−1.30%	−1.29%
6/15/2021	−2.97%	−0.87%	−0.10%	−0.81%
6/16/2021	0.92%	0.18%	−0.73%	−0.02%
6/17/2021	1.94%	−2.63%	0.46%	−2.09%
6/18/2021	1.09%	0.27%	−1.70%	0.88%
6/21/2021	−0.40%	0.72%	1.57%	1.80%
6/22/2021	0.46%	−1.51%	0.48%	−0.87%
6/23/2021	5.27%	0.41%	−1.41%	−0.22%
6/24/2021	3.54%	0.68%	0.36%	−0.80%
6/25/2021	−1.17%	−0.58%	−0.54%	−1.07%
6/28/2021	2.51%	−3.74%	0.36%	−2.95%
6/29/2021	−1.16%	−1.45%	−0.05%	−0.58%
6/30/2021	−0.16%	0.62%	0.15%	0.91%

Date	TSLA	AAL	PFE	DAL
7/1/2021	−0.26%	1.41%	1.02%	2.24%
7/2/2021	0.14%	−0.14%	0.43%	−0.18%
7/6/2021	−2.85%	−2.19%	−1.11%	−2.06%
7/7/2021	−2.26%	−3.33%	0.15%	−1.62%
7/8/2021	1.27%	0.20%	−0.25%	−1.10%
7/9/2021	0.63%	2.65%	0.92%	2.02%
7/12/2021	4.38%	−0.24%	0.38%	−0.16%
7/13/2021	−2.50%	−3.93%	−0.28%	−3.55%
7/14/2021	−2.27%	3.00%	0.76%	−1.57%
7/15/2021	−0.43%	−0.78%	0.35%	1.65%
7/16/2021	−0.98%	−3.27%	0.65%	−3.12%
7/19/2021	0.31%	−4.14%	−0.50%	−3.74%
7/20/2021	2.21%	8.38%	2.24%	5.45%
7/21/2021	−0.79%	4.09%	−0.07%	2.34%
7/22/2021	−0.92%	−1.12%	1.10%	−1.32%
7/23/2021	−0.91%	0.19%	0.51%	−1.58%
7/26/2021	2.21%	4.15%	0.31%	3.12%
7/27/2021	−1.95%	−2.81%	0.69%	−1.70%
7/28/2021	0.34%	0.79%	3.21%	0.83%
7/29/2021	4.69%	−2.13%	−1.52%	−0.17%
7/30/2021	1.45%	−3.73%	0.05%	−3.23%

Date	TSLA	AAL	PFE	DAL
8/2/2021	3.27%	−1.57%	2.69%	−1.13%
8/3/2021	0.01%	−0.55%	3.91%	0.81%
8/4/2021	0.17%	−2.51%	−1.07%	−3.57%
8/5/2021	0.52%	7.51%	−0.29%	4.28%
8/6/2021	−2.17%	0.53%	0.02%	0.83%
8/9/2021	2.10%	−2.19%	2.02%	−2.55%
8/10/2021	−0.53%	1.85%	4.81%	2.98%
8/11/2021	−0.31%	1.38%	−3.90%	2.08%
8/12/2021	2.04%	−3.77%	2.01%	−1.86%
8/13/2021	−0.70%	−2.89%	2.62%	−1.55%
8/16/2021	−4.32%	−0.40%	0.89%	0.08%
8/17/2021	−2.98%	−2.13%	3.09%	−2.88%
8/18/2021	3.50%	−1.03%	−2.20%	−0.34%
8/19/2021	−2.25%	−2.14%	−1.03%	−1.86%
8/20/2021	1.01%	−1.23%	−0.16%	0.50%
8/23/2021	3.83%	3.30%	2.48%	2.86%
8/24/2021	0.31%	3.77%	−3.10%	3.37%
8/25/2021	0.38%	1.61%	−1.80%	1.90%
8/26/2021	−1.41%	−1.24%	−0.27%	−1.28%
8/27/2021	1.53%	1.61%	−1.65%	1.84%
8/30/2021	2.67%	−3.51%	0.34%	−3.88%

2. See the variance-covariance matrix below.

	TSLA	AAL	PFE	DAL
TSLA	0.00048	0.00004	-0.00002	0.00008
AAL	0.00004	0.00069	0.00001	0.00046
PFE	-0.00002	-0.00001	0.00025	-0.00000
DAL	0.00008	0.00046	-0.00000	0.00043

3. See the correlation matrix below.

	TSLA	AAL	PFE	DAL
TSLA	1.000	0.068	-0.058	0.177
AAL	0.068	1.000	-0.017	0.852
PFE	-0.058	-0.017	1.000	0.013
DAL	0.177	0.852	-0.013	1.000

4. See the autocorrelation figures below.

k	TSLA
1	0.006
2	-0.159
3	0.006

3 Firms and economic uncertainty: Part I

$$\begin{array}{c} X \\ P(X) \end{array} \parallel \begin{array}{c|c|c|c} 0 & 4 & 16 & 32 \\ \hline \frac{5}{16} & \frac{1}{4} & \frac{3}{8} & \frac{1}{16} \end{array}$$

Let μ_X be the expected value of X , σ_X^2 be the variance of X , and σ_X be the standard deviation of X .

$$\begin{aligned} \mu_X &= \sum_{i=0}^{n-1} p_i x_i \\ &= \left(\frac{5}{16} \times 0 \right) + \left(\frac{1}{4} \times 4 \right) + \left(\frac{3}{8} \times 16 \right) + \left(\frac{1}{16} \times 32 \right) \\ &= 9. \end{aligned}$$

$$\begin{aligned} \sigma_X^2 &= \sum_{i=0}^{n-1} p_i (x_i - \mu_X)^2 \\ &= \left(\frac{5}{16} \right) (0 - 9)^2 + \left(\frac{1}{4} \right) (4 - 9)^2 + \left(\frac{3}{8} \right) (16 - 9)^2 + \left(\frac{1}{16} \right) (32 - 9)^2 \\ &= 83. \end{aligned}$$

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{83} \approx 9.1104 \dots$$

4 Firms and economic uncertainty: Part II

	IBM	DELL	Probability
Recession	0	-10%	10%
Normal	10%	10%	50%
Boom	20%	30%	40%

1. The expected return for IBM, μ_{IBM} , is 13%:

$$\mu_{\text{IBM}} = (0 \times 10\%) + (10\% \times 50\%) + (20\% \times 40\%) = 13\%.$$

The expected return for DELL, μ_{DELL} , is 16%:

$$\mu_{\text{DELL}} = (-10\% \times 10\%) + (10\% \times 50\%) + (30\% \times 40\%) = 16\%.$$

2. The variance for the returns of IBM, σ_{IBM}^2 , is 0.41%:

$$\sigma_{\text{IBM}}^2 = (10\%)(0 - 13\%)^2 + (50\%)(10\% - 13\%)^2 + (40\%)(20\% - 13\%)^2 = 0.41\%.$$

The standard deviation for the returns of IBM, σ_{IBM} , is about 6.4%:

$$\sigma_{\text{IBM}} = \sqrt{\sigma_{\text{IBM}}^2} = \sqrt{0.41\%} \approx 6.4031\% \dots$$

3. The variance for the returns of DELL, σ_{DELL}^2 , is 1.64%:

$$\sigma_{\text{DELL}}^2 = (10\%)(-10\% - 16\%)^2 + (50\%)(10\% - 16\%)^2 + (40\%)(30\% - 16\%)^2 = 1.64\%.$$

The standard deviation for the returns of DELL, σ_{DELL} , is about 12.8%:

$$\sigma_{\text{DELL}} = \sqrt{\sigma_{\text{DELL}}^2} = \sqrt{1.64\%} \approx 12.8062\% \dots$$

4. The covariance between the returns of IBM and DELL, $\sigma_{\text{IBM,DELL}}$, is 0.82%:

$$\sigma_{\text{IBM,DELL}} = \sum_{i=0}^{n-1} (\text{ibm}_i - \mu_{\text{IBM}})(\text{dell}_i - \mu_{\text{DELL}}) = 0.82\%$$

5. The correlation between the returns of IBM and DELL, $\rho_{\text{IBM,DELL}}$, is:

$$\rho_{\text{IBM,DELL}} = \frac{\sigma_{\text{IBM,DELL}}}{\sigma_{\text{IBM}}\sigma_{\text{DELL}}} \approx 1.0005 \dots$$

Since the correlation must be between -1 and 1, we conclude that the correlation is approximately 1.

5 Firms and economic uncertainty: Part III

1. Let μ_X be the expected return of small firms, μ_Y be the expected return of large firms, σ_X^2 be the variance of the return of small firms, and σ_Y^2 be the variance of the return of large firms.

$$\mu_X = \left(0 \times \frac{1}{10}\right) + \left(1 \times \frac{1}{5}\right) + \left(2 \times \frac{2}{5}\right) + \left(3 \times \frac{1}{5}\right) + \left(4 \times \frac{1}{10}\right) = 2.$$

$$\mu_Y = \left(-4 \times \frac{1}{10}\right) + \left(-2 \times \frac{1}{5}\right) + \left(4 \times \frac{2}{5}\right) + \left(7 \times \frac{1}{5}\right) + \left(8 \times \frac{1}{10}\right) = 3.$$

$$\sigma_X^2 = (0-2)^2 \left(\frac{1}{10}\right) + (1-2)^2 \left(\frac{1}{5}\right) + (2-2)^2 \left(\frac{2}{5}\right) + (3-2)^2 \left(\frac{1}{5}\right) + (4-2)^2 \left(\frac{1}{10}\right) = 1.2.$$

$$\sigma_Y^2 = (-4-3)^2 \left(\frac{1}{10}\right) + (-2-3)^2 \left(\frac{1}{5}\right) + (4-3)^2 \left(\frac{2}{5}\right) + (7-3)^2 \left(\frac{1}{5}\right) + (8-3)^2 \left(\frac{1}{10}\right) = 16.$$

2. Let $\sigma_{X,Y}$ be the covariance between the returns of small and large firms, and $\rho_{X,Y}$ be the correlation between those returns.

$$\begin{aligned} \sigma_{X,Y} &= (0-2)(-4-3) \left(\frac{1}{10}\right) \\ &\quad + (1-2)(-2-3) \left(\frac{1}{5}\right) \\ &\quad + (2-2)(4-3) \left(\frac{2}{5}\right) \\ &\quad + (3-2)(7-3) \left(\frac{1}{5}\right) \\ &\quad + (4-2)(8-3) \left(\frac{1}{10}\right) \\ &= 4.2. \end{aligned}$$

Let σ_X be the standard deviation of the returns of small firms and σ_Y be the standard deviation of the returns of large firms.

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y} = \frac{4.2}{\sqrt{1.2} \sqrt{16}} \approx 0.9585 \dots$$

- 3.

$$\mu_{3X-5Y} = 3\mu_X - 5\mu_Y = 3(2) - 5(3) = -9.$$

$$\begin{aligned} \sigma_{3X-5Y}^2 &= 3^2 \sigma_X^2 + 5^2 \sigma_Y^2 - 2(3)(5)(\sigma_{X,Y}) \\ &= 9(1.2) + 25(16) - 30(4.2) = 284.8. \end{aligned}$$

4.

$$\sigma_{3X,5Y} = (3)(5)(\sigma_{X,Y}) = 15(4.2) = 63.$$

$$\rho_{3X,5Y} = \frac{\sigma_{3X,5Y}}{\sigma_{3X}\sigma_{5Y}} = \frac{63}{(3)(\sqrt{1.2})(5)(\sqrt{16})} = \frac{4.2}{\sqrt{1.2}\sqrt{16}} = \rho_{X,Y} \approx 0.9585 \dots$$

5. Let Z be a random variable with mean $\mu_Z = 3$ and variance $\sigma_Z^2 = 5$. The covariance between X and Z is $\sigma_{X,Z} = 2$, and the covariance between Y and Z is $\sigma_{Y,Z} = 4$.

$$\begin{aligned} \sigma_{3X,2Z-5Y} &= \sigma_{3X,2Z} - \sigma_{3X,5Y}; && \text{bilinearity of covariance} \\ &= (3)(2)(\sigma_{X,Z}) - (3)(5)(\sigma_{X,Y}); && \text{factor constant coefficient} \\ &= 6(2) - 15(4.2) && \text{substitution} \\ &= -51. \end{aligned}$$

$$\rho_{3X,2Z-5Y} = \frac{\sigma_{3X,2Z-5Y}}{\sigma_{3X}\sigma_{2Z-5Y}} \quad \text{definition of correlation}$$

$$= \frac{\sigma_{3X,2Z-5Y}}{3\sigma_X\sqrt{\sigma_{2Z-5Y}^2}} \quad \text{definition of standard deviation}$$

$$= \frac{\sigma_{3X,2Z-5Y}}{3\sigma_X\sqrt{2^2\sigma_Z^2 + 5^2\sigma_Y^2 - 2(2)(5)\sigma_{Z,Y}}} \quad \text{definition of variance}$$

$$= \frac{\sigma_{3X,2Z-5Y}}{3\sigma_X\sqrt{2^2\sigma_Z^2 + 5^2\sigma_Y^2 - 2(2)(5)\sigma_{Y,Z}}} \quad \text{symmetry of covariance}$$

$$= \frac{-51}{3\sqrt{1.2}\sqrt{4(5) + 25(16) - 2(2)(5)(4)}} \quad \text{substitution}$$

$$\approx -0.8416 \dots$$