FINC-UB 1 Homework 6

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Fama–French three-factor model

Let t be a given month, and let μ_{i_t} represent the expected return of the hedge fund in month t. Let f represent the risk-free rate. Let λ_{M_t} represent the return of the market portfolio in month t. Let e_{i_t} represent the residual error with respect to the hedge fund's return in month t.

1 The baseline capital asset pricing model

Let $\alpha_{\text{CAPM}} \approx 0.5770\ldots$ be the constant coefficient from the baseline capital asset pricing model. Let $\beta_M \approx 1.4625\ldots$ be the regression coefficient for the return of the market portfolio. Then

$$\begin{aligned}\mu_{i_t} - f &= \alpha_{\text{CAPM}} + \beta_M \lambda_{M_t} + e_{i_t} \\ \mu_{i_t} - f &\approx 0.5770 + 1.4625\lambda_{M_t} + e_{i_t}.\end{aligned}$$

α_{CAPM}	$\approx$	$0.5770\ldots$	$t_{\alpha_{\text{CAPM}}}$	$\approx$	$2.8973\ldots$	$P_{(t\geq 2.8973)}$	$\approx$	$0.3846\ldots\%$
β_M	$\approx$	$1.4625\ldots$	t_{β_M}	$\approx$	$40.4768\ldots$	$P_{(t\geq 40.4768)}$	$\approx$	$0.0000\ldots\%$

2 Would you buy or sell this hedge fund?

If my only options are to buy the hedge fund or to sell it, I would choose to buy the hedge fund. According to the regression results, the hedge fund has a positive "alpha" with respect to the market at the 1% significance level $\left(P_{t<t_{\alpha_{\text{CAPM}}}} < 1\%\right)$. So, according to the capital asset pricing model, there is a good chance that the hedge fund has historically sustained "abnormal" excess returns above those predicted using the market excess returns. If the regression is to be trusted blindly, we expect that even in a month when the market provides a return of 0, the hedge fund will provide a return of $\alpha_{\text{CAPM}} \approx 0.58\%$. Of course, the regression only explains $r^2 \approx 62\%$ of the variation in the returns of the hedge fund. Still, given our model and data, the case for buying is stronger than the case for selling.

3 Fama–French three-factor model

Let $\alpha_{\text{FF}} \approx 0.0001\ldots$ be the constant coefficient from the Fama–French three-factor model. Let $\beta_M \approx 1.0120\ldots$ be the regression coefficient for the return of the market portfolio, $\beta_{\text{SMB}} \approx 1.3361\ldots$ be the regression coefficient for the market capitalization (size) factor, and $\beta_{\text{HML}} \approx 1.2340\ldots$ be the regression coefficient for the book-to-market (value) factor. Let λ_{SMB_i} be the size factor for the hedge fund, and λ_{HML_i} be the value factor for the hedge fund. Then

$$\begin{aligned}\mu_{i_t} - f &= \alpha_{\text{FF}} + \beta_M \lambda_{M_t} + \beta_{\text{SMB}} \lambda_{\text{SMB}_i} + \beta_{\text{HML}} \lambda_{\text{HML}_i} + e_{i_t} \\ \mu_{i_t} - f &= 0.0001 + 1.0120\lambda_{M_t} + 1.3361\lambda_{\text{SMB}_i} + 1.2340\lambda_{\text{HML}_i} + e_{i_t}.\end{aligned}$$

α_{FF}	$\approx$	$0.0001\ldots$	$t_{\alpha_{\text{FF}}}$	$\approx$	$0.2975\ldots$	$P_{(t\geq 0.2975)}$	$\approx$	$76.6115\ldots\%$
β_M	$\approx$	$1.0120\ldots$	t_{β_M}	$\approx$	$27757.3684\ldots$	$P_{(t\geq 27757.3684)}$	$\approx$	$0.0000\ldots\%$
β_{SMB}	$\approx$	$1.3361\ldots$	$t_{\beta_{\text{SMB}}}$	$\approx$	$25680.6429\ldots$	$P_{(t\geq 25680.6429)}$	$\approx$	$0.0000\ldots\%$
β_{HML}	$\approx$	$1.2340\ldots$	$t_{\beta_{\text{HML}}}$	$\approx$	$20669.4881\ldots$	$P_{(t\geq 20669.4881)}$	$\approx$	$0.0000\ldots\%$

According to the Fama–French three-factor model, the hedge fund is not performing much better than we expect. The "abnormal" return return provided by the hedge fund is $\alpha_{\text{FF}} \approx 0.00\%$. In fact, according to the Student t-statistic, the probability that $\alpha_{\text{FF}} = 0$ is $P \approx 76\%$. Thus the so-called "alpha" is not statistically significant and could be entirely fictious. The Fama–French model exposes the illusion of superior returns and reveals that the hedge fund is *not* beating expectations based on the market's excess return, the size of the hedge fund, and its book-to-market capitalization. We have no evidence that it is mispriced.

4 Why $\alpha_{\text{CAPM}} \neq \alpha_{\text{FF}}$

The simple and multivariate regressions produce different intercepts because the additional variables in the multivariate regression are able to "explain" some of the constant coefficient. In the multivariate regression, a portion of the original constant coefficient in the simple regression (α_{CAPM}) is accounted for by the introduction of the size and value factors (λ_{SMB_i} and λ_{HML_i}) multiplied by their respective coefficients (β_{SMB} and β_{HML}). Thus, $\alpha_{\text{FF}} < \alpha_{\text{CAPM}}$. Intuitively, the "abnormal returns" are in fact not abnormal. They can largely be explained by the introduction of the new factors.

5 Why CAPM's β_{M} is not equal to Fama–French's β_{M}

A variable is not guaranteed to have the same coefficient in a simple regression as in a multivariate regression. An ordinary least-squares regression is computed by minimizing the sum of the squares of the residuals. The simple regression is tasked with explaining as great a proportion of the residual errors as possible ($\max r^2$) using only λ_M, while the multivariate regression may use λ_M, λ_{SMB}, and λ_{HML} to accomplish the same. We know by comparing the values of r^2 that the simple regression is weaker than the multivariate regression. So the simple regression is "stretching" β_M to explain the data, when in fact there are multiple variables required to predict the excess return of a security. Thus β_M from the Fama–French model is probably more accurate than β_M from the capital asset pricing model. The assumption that β_M in one regression should equal β_M in another has no mathematical basis.