MATH-UA 120 Section 6

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September 12, 2023

Palindrome

An integer n is a palindrome if it reads the same forward and backward when expressed in base-10.

1 Disprove: Given $a, b \in \mathbb{Z}$, if $a \mid b$, then $a \leq b$

Consider $a = 1 \in \mathbb{Z}, b = 0 \in \mathbb{Z}$. Let $c \in \mathbb{Z}$ such that $b = ac$. Note $0 = 1c$. Let $c = 0$. Note $0 = 1(0) = 0$. There exists $c \in \mathbb{Z}$ such that $b = ac$. Therefore, $a \mid b$. However, $1 > 0$, so $a > b$. We reject the claim that $a \mid b$ implies $a \leq b$.

2 Disprove: Given $a, b \in \mathbb{Z}$, if $a \geq 0$ and $a \mid b$, then $a \leq b$

Consider $a = 1 \in \mathbb{Z}, b = 0 \in \mathbb{Z}$. Note $1 \geq 0$, so $a \geq 0$. Let $c \in \mathbb{Z}$ such that $b = ac$. Note $0 = 1c$. Let $c = 0$. Note $0 = 1(0) = 0$. There exists $c \in \mathbb{Z}$ such that $b = ac$. Therefore, $a \mid b$. However, $1 > 0$, so $a > b$. We reject the claim that $a \mid b$ implies $a \leq b$.

3 Disprove: Given $a, b, c \in \mathbb{N}$, if $a \mid (bc)$, then $a \mid b$ or $a \mid c$

Consider $a = 4 \in \mathbb{N}, b = 2 \in \mathbb{N}, c = 6 \in \mathbb{N}$. Let $d \in \mathbb{Z}$ such that $bc = ad$. Note $12 = 4d$. Let $d = 3$. There exists $d \in \mathbb{Z}$ such that $bc = ad$. Therefore, $a \mid (bc)$. However, no such $x \in \mathbb{Z}$ exists such that $2 = 4x$ and no such $y \in \mathbb{Z}$ exists such that $6 = 4y$. Therefore, $a \nmid b$ and $a \nmid c$. We reject the claim that $a \mid (bc)$ implies $a \mid b$ or $a \mid c$.

4 Disprove: Given $a, b, c \in \mathbb{Z}$, if $a > 0$, $b > 0$, and $c > 0$, then $a^{(b^c)} = \left(a^b\right)^c$

Consider $a = 3 \in \mathbb{Z}, b = 3 \in \mathbb{Z}, c = 3 \in \mathbb{Z}$. Note $(a = b = c = 3) > 0$. However, $\left(3^{(3^3)} = 3^{27}\right) \neq \left((3^3)^3 = 27^3\right)$.

5 Disprove: Given $p, q \in \mathbb{Z}$, if p and q are prime, then $p + q$ is composite

Consider $p = 2 \in \mathbb{Z}, q = 3 \in \mathbb{Z}$. Note $(p = 2) > 1$. The only positive divisors of 2 are 1 and 2. Therefore, p is prime. Note $(q = 3) > 1$. The only positive divisors of 3 are 1 and 3. Therefore, q is prime. However, $p + q = 5$. Let $a \in \mathbb{Z}$; $p + q$ is called composite provided there exists $a \in \mathbb{Z}$ such that $1 < a < p + q$ and $a \mid (p + q)$. No such $a \in \mathbb{Z}$ exists, therefore $p + q$ is not composite. We reject the claim that p and q being primes implies that $p + q$ is composite.

6 Disprove: Given $p \in \mathbb{Z}$, if p is prime, then $2^p - 1$ is prime

Consider $p = 5 \in \mathbb{Z}$. Note $(p = 11) > 0$. The only positive divisors of 11 are 1 and 11. Therefore, p is prime. However, $2^p - 1 = 2^{11} - 1 = 2047$. 2047 is called prime provided the only positive divisors of 2047 are 1 and 2047. However, let $89(23) = 2047$. There exists $a \in \mathbb{Z}$ such that $2047 = 89a$. Thus, $89 \mid 2047$. Note $89 \neq 1$ and $89 \neq 2047$. Therefore $2^p - 1$ is not prime.

7 Disprove: Given $p \in \mathbb{Z}$, if p is a palindrome and p has more than 1 digit, then p is divisible by 11

Consider $p = 232$; p has more than 1 digit. However, there exists no $a \in \mathbb{Z}$ such that $232 = 11a$. Therefore, $11 \nmid 232$. We reject the claim that if p is a palindrome and p has more than 1 digit, then p is divisible by 11.