MATH-UA 120 Section 4

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Pythagorean theorem

If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then

$$a^2 + b^2 = c^2.$$

Conjecture

New theorems are the creations of mathematicians that begin as *conjectures*: statements about mathematics whose truth we have yet to establish. Conjectures are educated guesses.

1 Rewrite

a. The product of an odd integer and an even integer is even. Let x and y be integers. If x is odd and y is even, then xy is even.

b. The square of an odd integer is odd. If an integer x is odd, then x^2 is odd.

c. The square of a prime number is not prime. If an integer x is prime, then x^2 is not prime.

d. The product of two negative integers is negative. This, of course, is false. Let x and y be integers. If x and y are negative, then xy is negative.

e. The diagonals of a rhombus are perpendicular. If $\overline{AB}$ and $\overline{CD}$ are the diagonals of a rhombus, then $\overline{AB} \perp \overline{CD}$.

f. Congruent triangles have the same area. If $\triangle ABC \cong \triangle DEF$, then the area of $\triangle ABC$ is equal to the area of $\triangle DEF$.

g. The sum of three consecutive integers is divisible by three. If integers a, b, and c are consecutive (that is, $b = a + 1$ and $c = b + 1$), then $3|(a + b + c)$.

2 Indicate

a. If a polygon $PQRS$ is a square, then $PQRS$ is a rectangle.

b. If a polygon $PQRS$ is a rectangle, then $PQRS$ is a parallelogram.

c. If Joe is a grandfather, then Joe is male.

d. If Ellen resides in Los Angeles, then Ellen resides in California.

e. If this year is a leap year, then this year is divisible by 4.

f. None.

g. If $x > 0$, then $x^2 > 0$.

h. If $X < 0$, then $x^3 < 0$. If $x^3 < 0$, then $x < 0$. Thus, $x < 0$ if and only if $x^3 < 0$.

i. If $xy = 0$, then $x = 0$ or $y = 0$. If $x = 0$ or $y = 0$, then $xy = 0$. Thus, $xy = 0$ if and only if $x = 0$ or $y = 0$.

j. If $x = 0$ and $y = 0$, then $xy = 0$.

k. If $x = 0$ and $y = 0$, then $x + y = 0$.

3 A common mistake

Let condition A be "John lives in New York" and condition B be "John lives in the United States." Then, the statement "if A, then B" is true, but the statement "if B, then A" is false.

Let condition A be "A polygon P is a square" and condition B be "A polygon P is a rectangle." Then, the statement "if A, then B" is true, but the statement "if B, then A" is false.

4 Consider

The statement "if A, then B" is true unless A is true and B is false. An "or" statement is true unless both conditions are false. This makes the statement identical to the statement "If (not A) or B."

5 Consider

The statement "if A, then B" is identical to the statement "if (not B), then (not A)" because the positive and contrapositive are always identical.

6 Consider

The statement "if A iff B" is identical to the statement "(not A) iff (not B)."

7 An equilateral triangle

An equilateral triangle whose side lengths are $a = b = c = 1$ is not a violation of the Pythagorean theorem–even though $a^2 + b^2 \neq c^2$–because the Pythagorean Theorem only necessarily applies to right triangles. By definition, an equilateral triangle is not a right triangle: All angles of an equilateral triangle measure 30 degrees–none measure 90 degrees.

8 Nonsense

The statement "A *line* is the shortest distance between two points" is nonsense because the definition is incorrect and there is a category mismatch. A better definition is "A one-dimensional figure $\overline{AB}$ is called a *line segment* provided it connects two points A and B, and its length is the distance between A and B.

9 A rather grotesque claim

The claim "If you pick a guinea pig up by its tail, then its eyes will pop out" is mathematically true. This is a vacuous statement: Guinea pigs do not have tails.

10 The plural of lemma

The correct plural of *lemma* is *lemmata* (Latin third-declension noun, Greek origin).

11 Conjectures

a. Conjecture: The sums of consecutive odd numbers starting with 1, are perfect squares.

b. Conjecture: The sums of consecutive perfect cubes, starting with 1, are perfect squares.