MATH-UA 120 Discrete Mathematics

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PROPOSITION.

Claim. Let $a \in \mathbb{Z}$. $14 \mid a$ if and only if $7 \mid a$ and $2 \mid a$.

Proof. Let $a \in \mathbb{Z}$. We will demonstrate that $14 \mid a$ if and only if $7 \mid a$ and $2 \mid a$.

$\Rightarrow$) Suppose $14 \mid a$. Then there exists $n \in \mathbb{Z}$ such that $a = 14n$. Note $a = 7(2n)$. There exists $(2n) \in \mathbb{Z}$ such that $a = 7(2n)$. Thus, $7 \mid a$. Note also $a = 2(7n)$. There exists $(7n) \in \mathbb{Z}$ such that $a = 2(7n)$. Thus, $2 \mid a$. Therefore, $7 \mid a$ and $2 \mid a$.

$\Leftarrow$) Suppose $7 \mid a$ and $2 \mid a$. Then there exists $b \in \mathbb{Z}$ such that $a = 7b$. Since $2 \mid a$, there exists $c \in \mathbb{Z}$ such that $a = 2c$. Thus $a = 7b = 2c$. There exists $c \in \mathbb{Z}$ such that $7b = 2c$, so $2 \mid 7b$. Thus $7b$ is even, so either 7 is even or b is even; but 7 is not even, so b is even. Since b is even, $2 \mid b$ and there exists $d \in \mathbb{Z}$ such that $b = 2d$. Observe

$$\begin{aligned} a &= 7b \\ a &= 7(2d) \\ a &= 14d. \end{aligned}$$

There exists $d \in \mathbb{Z}$ such that $a = 14d$. Therefore, $14 \mid a$.

Hence, $14 \mid a$ if and only if $7 \mid a$ and $2 \mid a$. $\square$