STAT-UB 103 Homework 3

Ishan Pranav

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1 The probability that a plane leaves on time (L) and arrives on time (A)

$$P(L) = 0.85.$$

$$P(L \cap A) = 0.60.$$

$$P(L \cap A) = P(L)P(A|L).$$

$$\frac{P(L \cap A)}{P(L)} = P(A|L).$$

$$P(A|L) = \frac{0.60}{0.85} = \frac{12}{17} \approx 0.7059\ldots$$

2 The chance of a "critical item failure" (F)

$$P(F) = \frac{1}{63}.$$

a.

$$P(\bar{0}) = 1 - \left(\frac{62}{63}\right)^8 \approx 0.1201\ldots$$

b.

$$P(\bar{0}) = 1 - \left(\frac{62}{63}\right)^{40} \approx 0.4727\ldots$$

3 In the packaged-cereals industry, 29% of vice presidents hold Master's degrees in business (M), 24% hold Bachelor's degrees in business (B), and 8% hold both

a.

$$P(M \cup B) = P(M) + P(B) - P(M \cap B).$$

$$P(M) = 0.29.$$

$$P(B) = 0.24.$$

$$P(M \cap B) = 0.08.$$

$$P(M \cup B) = 0.29 + 0.24 - 0.08 = 0.45.$$

b.

$$P(\overline{M \cup B}) = 1 - P(M \cup B) = 0.55.$$

c.

$$P(M \cap \bar{B}) = P(M) - P(M \cap B) = 0.29 = 0.08 = 0.21.$$

4 The probability that a child born in the US in 2004 will survive past a given age

$$P(80) = 0.2.$$

$$P(90) = P(80 \cap 90) = 0.1 = P(80)P(90|80).$$

$$P(90|80) = \frac{P(80 \cap 90)}{P(80)} = \frac{P(90)}{P(80)} = \frac{0.1}{0.2} = 0.5.$$

5 A survey of workers in the two plants of a manufacturing firm (one of which is A) and whether they responded with "poor" (B)

a.

$$P(A) = \frac{192}{192 + 248} = 0.4\overline{36}.$$

$$P(B) = \frac{48 + 80}{192 + 248} = 0.2\overline{90}.$$

$$P(A|B) = \frac{48}{48 + 80} = 0.375.$$

b. No, events A and B are not independent.

$$P(A) \neq P(A|B).$$

c. No, these probabilities are not equal.

$$P(B|A) = \frac{48}{192} = 0.25.$$

$$P(B|\bar{A}) = \frac{80}{248} = \frac{10}{31} \approx 0.3226\ldots$$

d.

$$P(\bar{B}) \neq P(\bar{B}|A).$$

$$P(\bar{B}) = 1 - \frac{48+80}{192+248} = 0.7\overline{09}.$$

$$P(\bar{B}|A) = \frac{192-48}{192} = 0.75.$$

6 Heads on a coin flip (H) and a red marble selected from a jar (R)

a.

$$P(R|H) = \frac{7}{7+3} = 0.7.$$

b.

$$P(R|\bar{H}) = \frac{15}{15+30} = 0.\overline{3}.$$

c.

$$P(R \cap H) = P(H)P(R|H) = (0.5)(0.7) = 0.35.$$

d.

$$P(R \cap \bar{H}) = P(\bar{H})P(R|\bar{H}) = (0.5)\left(\frac{1}{3}\right) = \frac{1}{6} = 0.1\overline{6}.$$

e.

$$P(R) = P(R \cap H) + P(R \cap \bar{H}) = 0.51\overline{6}.$$

f.

$$P(\bar{H}|\bar{R}) = \frac{P(\bar{R}|\bar{H})P(\bar{H})}{P(\bar{R})} = \frac{(0.5)(\frac{30}{30+15})}{1-P(R)} = \frac{\frac{1}{3}}{1-\frac{1}{6}-0.35} \approx 0.6897\ldots$$