STAT-UB 103 Homework 5

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1 The area (p) under the standard normal probability distribution between the following pairs of z-scores

$$z_x = \frac{x - \mu_X}{\sigma_X}.$$

$$\varphi(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}.$$

$$\Phi(x) = \int_{-\infty}^{x} \varphi(t)\,\mathrm{d}t = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\,\mathrm{d}t.$$

$$P(z = x) = P(z \leq x) = \Phi(x).$$

$$\operatorname{probit}(p) = \Phi^{-1}(p) \text{ for } p \in (0, 1).$$

a.

$$z^* = 0.$$

$$z' = 2.$$

$$p = P(z < 2) - P(z \leq 0).$$

$$\Phi(2) \approx 0.9772\ldots$$

$$p = \Phi(2) - \frac{1}{2} \approx 0.4772\ldots$$

b.

$$z^* = 0.$$

$$z' = 3.$$

$$p = P(z < 3) - P(z \leq 0).$$

$$\Phi(3) \approx 0.9987\ldots$$

$$p = \Phi(3) - \frac{1}{2} \approx 0.4987\ldots$$

c.

$$z^* = 0.$$

$$z' = 1.5.$$

$$p = P(z < 1.5) - P(z \leq 0).$$

$$\Phi(1.5) \approx 0.9332\ldots$$

$$p = \Phi(1.5) - \frac{1}{2} \approx 0.4332\ldots$$

d.

$$z^* = 0.$$

$$z' = 0.8.$$

$$p = P(z < 0.8) - P(z \leq 0).$$

$$\Phi(0.8) \approx 0.7881\ldots$$

$$p = \Phi(0.8) - \frac{1}{2} \approx 0.2881\ldots$$

2 Find a value (z^*) of the standard normal variable z

a.

$$P(z \geq z^*) = 0.05.$$

$$1 - P(z < z^*) = 0.95.$$

$$z^* = \text{probit}(0.95) \approx 1.6449\ldots$$

b.

$$P(z \geq z^*) = 0.025.$$

$$1 - P(z < z^*) = 0.975.$$

$$z^* = \text{probit}(0.975) \approx 1.9600\ldots$$

c.

$$P(z \leq z^*) = 0.025.$$

$$z^* = \text{probit}(0.025) \approx -1.9600\ldots$$

d.

$$P(z \geq z^*) = 0.1.$$

$$1 - P(z < z^*) = 0.9.$$

$$z^* = \text{probit}(0.9) \approx 1.2816\ldots$$

e.

$$P(z > z^*) = 0.1.$$

$$1 - P(z \leq z^*) = 0.9.$$

$$z^* = \text{probit}(0.9) \approx 1.2816\ldots$$

3 Suppose X is a normally distributed random variable

$$\mu_X = 50.$$

$$\sigma_X = 3.$$

$$z_x = \frac{x - \mu_X}{\sigma_X}.$$

$$x = z_x \sigma_X + \mu_X.$$

$$x_0 = 3z^* + 50.$$

a.

$$P(x \leq x_0) = 0.8413.$$

$$P(z \leq z^*) = 0.8413.$$

$$z^* = \text{probit}(0.8413) \approx 0.9998\ldots$$

$$x_0 \approx 52.9994\ldots$$

b.

$$P(x > x_0) = 0.025.$$

$$P(x \leq x_0) = 0.975.$$

$$z^* = \text{probit}(0.975) \approx 1.9600\ldots$$

$$x_0 \approx 55.8800\ldots$$

c.

$$P(x > x_0) = 0.95.$$

$$P(x \leq x_0) = 0.05.$$

$$z^* = \text{probit}(0.95) \approx -1.6449\ldots$$

$$x_0 \approx 45.0654\ldots$$

d.

$$P(41 \le x < x_0) = P(x < x_0) - P(x \le 41) \approx 0.8630.$$

$$P(x \le 41) = P(z < -3) \approx 1.350 \times 10^{-3}.$$

$$P(x < x_0) = P(z < z^*) \approx 0.8630 + 1.350 \times 10^{-3} \approx 0.8643.$$

$$z^* \approx \text{probit}(0.8643) \approx 1.100.$$

$$x_0 \approx 53.30.$$

e. 10 percent of the values of x are less than x_0.

$$P(x \le x_0) = P(z \le z^*) = 0.1.$$

$$z^* = \text{probit}(0.1) \approx -1.2816\ldots$$

$$x_0 \approx 46.1553\ldots$$

f. 1 percent of the values of x are greater than x_0.

$$P(x < x_0) = P(z < z^*) = 0.99.$$

$$\text{probit}(0.99) \approx 2.3263\ldots$$

$$x_0 \approx 56.9790\ldots$$

4 Buy-side *vs.* sell-side analysts' earnings forecasts

Let a represent the sell-side ("ask") forecast error and b represent the buy-side ("buy" or "bid") forecast error.

$$\begin{array}{lclclcl} \mu_a & \approx & -0.05 & \mu_b & \approx & 0.85 \\ \sigma_a & \approx & 0.85 & \sigma_b & \approx & 1.93 \end{array}$$

$$z_x = \frac{x - \mu_X}{\sigma_X}.$$

a. The probability that a buy-side analyst has a positive forecast error of 2 or higher.

$$P(b \geq 2) = P(z \geq z_2).$$

$$z_2 \approx \frac{2 - 0.85}{1.93} \approx 0.596.$$

$$P(b \geq 2) \approx \Phi(0.596) \approx 0.276.$$

b. The probability that a sell-side analyst has a positive forecast error of 2 or higher.

$$P(a \geq 2) = P(z \geq z_2).$$

$$z_2 \approx \frac{2 + 0.05}{0.85} \approx 2.4.$$

$$P(a \geq 2) \approx \Phi(2.4) \approx 0.0079.$$

5 A Pepsi machine at a Burger King store

Let X represent the volume filled.

$$\sigma_X = 0.2 \text{ oz}.$$

$$P(X > 8 \text{ oz}) = P\left(z > z_{(8 \text{ oz})}\right) = 0.01.$$

$$P(X \leq 8 \text{ oz}) = P\left(Z \leq z_{(8 \text{ oz})}\right) = 0.99.$$

$$z_{(8 \text{ oz})} = \text{probit}(0.99) \approx 2.3263\ldots$$

$$z_x = \frac{X - \mu_X}{\sigma_X}.$$

$$\mu_X = X - z_x \sigma_X.$$

$$z_{(8 \text{ oz})} = \frac{8 \text{ oz} - \mu_X}{0.2 \text{ oz}} \approx 2.3263\ldots$$

$$\mu_X \approx 8 \text{ oz} - (0.2 \text{ oz})(2.3263\ldots) \approx 7.5346\ldots$$

6 Annual stock returns for a particular company

$$\mu_X = 16\%.$$

$$\sigma_X = 10\%.$$

a.
$$P(X > 30\%) = 1 - P(z \leq 1.4) \approx 0.0808\ldots$$

b.
$$P(X < 0) = 1 - P(z \leq -1.6) \approx 0.0548\ldots$$