STAT-UB 103 Homework 6

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1 A recent survey

Let it be assumed each instance of a credit-card balance paid in full is completely independent of all other instances. Each person either pays or does not pay their credit card bill in full. Since there are a fixed number of independent binary trials (n), each with a known probability of success (p), the probability that the number of bills paid (X) is equal to a given value (k) can be modeled with a binomial random variable.

$$n = 400.$$

$$p \approx 0.30.$$

$$P(X = k) = p_k = \binom{n}{k} p^k (1-p)^{n-k}.$$

a.

$$P(X \geq k) = \sum_{i=k}^{n} p_i = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}.$$

$$P(X \geq 110) \approx \sum_{i=110}^{400} \binom{400}{i} (0.30)^i (0.70)^{400-i} \approx 0.87.$$

b.

$$P(X < k) = \sum_{i=0}^{k-1} \binom{n}{i} p^i (1-p)^{n-i}.$$

$$\begin{aligned}
P(125 \leq X < 140) &= P(X < 140) - P(X < 125) \\
&= P(X < 140) - P(X < 125) \\
&\approx \left[\sum_{i=0}^{139} \binom{400}{i} (0.30)^i (0.70)^{400-i}\right] - \left[\sum_{i=0}^{124} \binom{400}{i} (0.30)^i (0.70)^{400-i}\right] \\
&\approx 0.29.
\end{aligned}$$

2 The daily returns on a portfolio

Let X represent the daily returns on a portfolio, μ_X represent the mean daily return, and σ_X represent the standard deviation of daily returns.

$$\mu_X \approx 0.001.$$

$$\sigma_X \approx 0.002.$$

a. The probability that the number of positive-return days of the next 100 days (Y) greater than or to a given value (k) can be modeled with a binomial random variable.

$$P(Y \geq k) = \sum_{i=k}^{n} p_i = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}.$$

$$n = 100.$$

$$p \approx P(X > 0) \approx 1 - P(X \leq 0).$$

The probability of a positive portfolio return on a given day (p) is dependent on the return itself, which can be modeled with a Gaussian random variable.

$$z_x = \frac{x - \mu_X}{\sigma_X}.$$

$$z_0 \approx \frac{0 - 0.001}{0.002} \approx -0.5.$$

$$\varphi(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}.$$

$$\Phi(x) = \int_{-\infty}^{x} \varphi(t)\,\mathrm{d}t = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\,\mathrm{d}t.$$

$$P(z \leq x) = \Phi(x).$$

$$p \approx 1 - P(z \leq -0.5) \approx 1 - \Phi(-0.5) \approx 0.7.$$

$$P(Y \geq 60) \approx \sum_{i=60}^{100} \binom{100}{i} (0.7)^i (0.3)^{100-i} \approx 1.$$

b. The probability that the average return for the portfolio over the next 100 days ($\bar{X}$) exceeds 0.0015 follows from the Central Limit Theorem.

$$\mu_{\bar{X}} = \mu_X \approx 0.001.$$

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} \approx \frac{0.002}{\sqrt{100}} \approx 0.0002.$$

$$\begin{aligned}
P(\bar{X} > 0.0015) &= P\left(z > \frac{0.0015 - \mu_X}{\sigma_{\bar{X}}}\right) \\
&\approx P\left(z > \frac{0.0015 - 0.001}{0.0002}\right) \\
&\approx 1 - P(z \leq 2.5) \\
&\approx 1 - \Phi(2.5) \\
&\approx 0.006.
\end{aligned}$$

3 In May 1983

Let it be assumed each instance of a defective smoke detector is completely independent of all other instances. Each smoke detector is either defective or not effective. Since there are a fixed number of independent binary trials (n), each with a known probability of success (p), the probability that the number of defective smoke detectors discovered (X) is less than or equal to a given value (k) can be modeled with a binomial random variable.

$$P(X \leq k) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}.$$

The null hypothesis (H_0) is the Commission's suggestion that 40 percent of Honeywell smoke detectors are defective.

$$n = 2000.$$

$$H_0 : p = 0.4.$$

$$P(X \leq 4 \,|\, H_0) = \sum_{i=0}^{4} \binom{2000}{i} (0.4)^i (0.6)^{2000-i} \approx 2 \times 10^{-433}.$$

4 Game

Let it be assumed each coin flip is completely independent of all other coin flips. Each coin lands on either the obverse side or the reverse side. Since there are a fixed number of independent binary trials (n), each with a known probability of success (p), and because the number of obverse coins is material ($np \geq 10$) and the number of reverse coins is material ($(n)(1-p) \geq 10$), the number of obverse coins (X) can be modeled with a binomial random variable, and it is appropriate to use the normal approximation to the binomial distribution.

$$n = 100.$$

$$p = 0.5.$$

$$np = 50 \geq 10.$$

$$(n)(1-p) = 50 \geq 10.$$

Let c represent the compensation received for a given number of obverse coins (x).

$$c_x = \begin{cases} \$20, & x \geq 60 \\ -\$1, & x < 60. \end{cases}$$

$$\begin{aligned} P(X \geq 60) &= P\left(z \geq \frac{60 - \mu_X}{\sigma_X}\right) \\ &= P\left(z \geq \frac{60 - np}{\sqrt{(n)(p)(1-p)}}\right) \\ &\approx 1 - P\left(z < \frac{59.5 - np}{\sqrt{(n)(p)(1-p)}}\right) \\ &\approx 1 - P(z \leq 1.9) \\ &\approx 1 - \Phi(1.9) \\ &\approx 0.02871\ldots \end{aligned}$$

$$\begin{aligned} \mu_C &= \sum c p_c \\ &= \$20 \times P(X \geq 60) - \$1 \times P(X < 60) \\ &= \$20 \times P(X \geq 60) - \$1 \times (1 - P(X \geq 60)) \\ &= \$21 \times P(X \geq 60) - \$1 \\ &\approx -\$0.3970\ldots \\ &\approx -\$0.40. \end{aligned}$$

The expected loss is \$0.40. This is not a good game to play.

5 Data on body temperatures

The 95-percent confidence interval for μ is (98.1220, 98.3765). Yes, the results of the confidence interval are surprising, since the confidence interval does not contain the assumed population mean temperature of 98.6°F.

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
130	98.2492	0.7332	0.0643	(98.1220, 98.3765)

μ: population mean of Temp

Figure 1: Ninety-five-percent confidence interval for temperature.