April 9, 2024

P1

(Grover algorithm)

⇓

Time Complexity $\sqrt{N}$

If we are using n qubit $N = 2^n$

→ Time Complexity $\frac{\pi}{4}\sqrt{\frac{N}{M}}$

If there is one a single Solution so

$M = 1$ $\frac{\pi}{4}\sqrt{N}$

f(10) = 1 ✓
f(00) = 0 ✓
f(01) = 0
f(11) = 0

⇓

$\frac{\pi}{4} * \sqrt{4} = \frac{\pi}{4} * 2$

$= \frac{3.14}{2} = 1.57$

If there are two Solution
$M = 2$

Eg. N = 8 3 qubits

$\frac{\pi}{4}\sqrt{8} = \frac{\pi}{4} * 2\sqrt{2}$

$= \frac{3.14 * 2 * 1.44}{4} \simeq 2.21$ only 2 steps

$-V + 2a$

$-I + 2A$

53
38
17
23
19

A = 42

Inversion = 46
67
61

a = 42

31

53, 38, 17, 23, 79 $\quad A = 42$

$-V+2a = (-53+42)+42 = -11+42 = 31$

$-V+2a = -38+84 = 46$

$-V+2a = -17+84 = 67$

$-V+2a = -23+84 = 61$

$-V+2a = -79+84 = 5$

$$\begin{bmatrix} 53 \\ 38 \\ 17 \\ 23 \\ 79 \end{bmatrix} \longrightarrow \begin{bmatrix} 31 \\ 46 \\ 67 \\ 61 \\ 5 \end{bmatrix}$$

$-I+2A$

$$-I+2A = \begin{bmatrix} \frac{2}{5}-1 & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5}-1 & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5}-1 & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5}-1 & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5}-1 \end{bmatrix}_{5\times 5}$$

$$= \begin{bmatrix} -3/5 & 2/5 & 2/5 & 2/5 & 2/5 \\ 2/5 & -3/5 & 2/5 & 2/5 & 2/5 \\ 2/5 & 2/5 & -3/5 & 2/5 & 2/5 \\ 2/5 & 2/5 & 2/5 & -3/5 & 2/5 \\ 2/5 & 2/5 & 2/5 & 2/5 & -3/5 \end{bmatrix} \begin{bmatrix} 53 \\ 38 \\ 17 \\ 23 \\ 79 \end{bmatrix} = \begin{bmatrix} 31 \\ 46 \\ 67 \\ 61 \\ 5 \end{bmatrix}$$

$$-\frac{3}{5}*53 + \frac{2}{5}*38 + \frac{2}{5}*17 + \frac{2}{5}*23 + \frac{2}{5}*79 = 31$$

$$= \frac{1}{5}\left(-3*53 + 2*38 + 2*17 + 2*23 + 2*79\right)$$

$$= \frac{1}{5}\left(-5*53 + 2*38 + 2*53 + 2*17 + 2*23 + 2*79\right)$$

$$= \frac{1}{5}*-5*53 + \frac{1}{5}\left(2*38 + 2*53 + 2*17 + 2*23 + 2*79\right)$$

$$= -53 + 2\left(\frac{38+53+17+23+79}{5}\right)$$

$$= -V + 2a$$

$-I+2A$

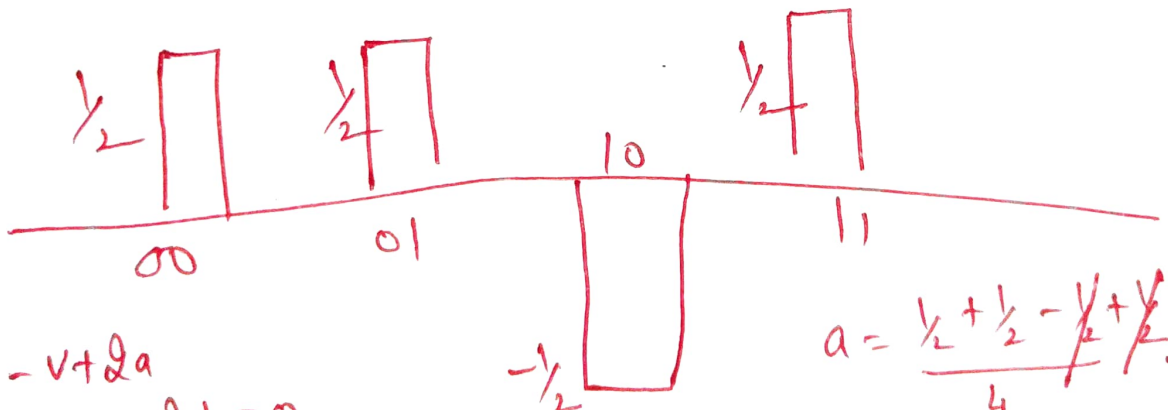
$$A = \begin{bmatrix} \frac{1}{2^n} & \frac{1}{2^n} & \frac{1}{2^n} & \cdots & \frac{1}{2^n} \\ | & & & & \\ \frac{1}{2^n} & & & & \frac{1}{2^n} \end{bmatrix} \quad N = 2^n$$

$n=2$ $|0\rangle$ — H — $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$|0\rangle$ — H —

$|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle \quad =$

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$

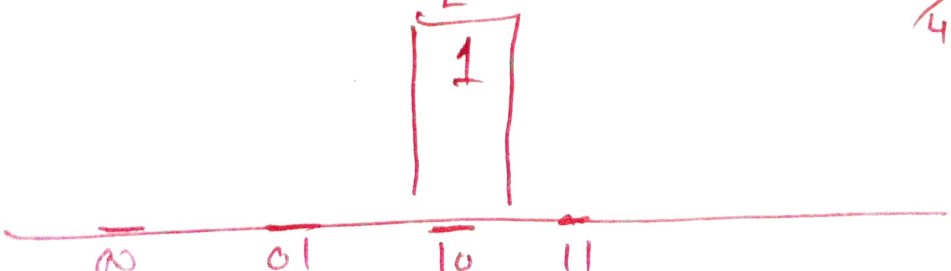


$-V+2a = -\frac{1}{2} + 2\cdot\frac{1}{4} = 0$

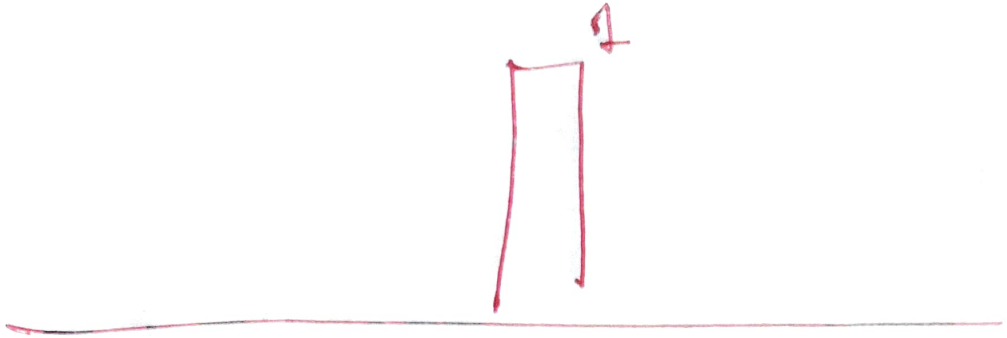
$a = \frac{\frac{1}{2}+\frac{1}{2}-\frac{1}{2}+\frac{1}{2}}{4} = \frac{1}{4}$

$-V+2a = \frac{1}{2} + 2\cdot\frac{1}{4} = 1$

$V = -\frac{1}{2}$



$\frac{\pi}{4}\sqrt{N} = (1)$



$a = 1/4$

$-V + 2a = -0 + 2 \cdot 1/4 = 1/2$

$|10\rangle \quad -V + 2a = -1 + 2 \cdot 1/4 =$