

# (Error Detection and Correction)

23/4/2024

Classical

Redundant Code

$$\begin{matrix} 0 \rightarrow 000 \\ 1 \rightarrow 111 \end{matrix}$$

Noise  $\begin{matrix} 100 \Rightarrow 0 \\ 101 \Rightarrow 1 \end{matrix}$

detect single Error & we can able to correct it

$$0 \rightarrow 000 \xrightarrow{\text{noise}} 101 = 1 \quad \text{Error}$$

Three bit Code & Majority checkup

detect only single Error.

$$\alpha|0\rangle + \beta|1\rangle \xrightarrow{\text{No-cloning theorem}} (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle)$$

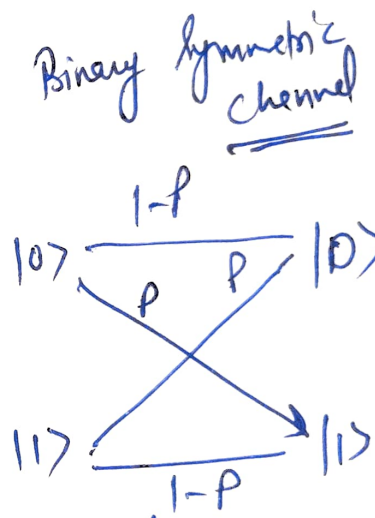
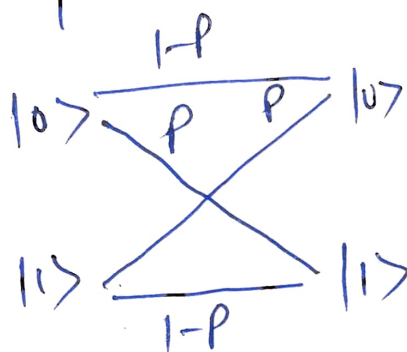
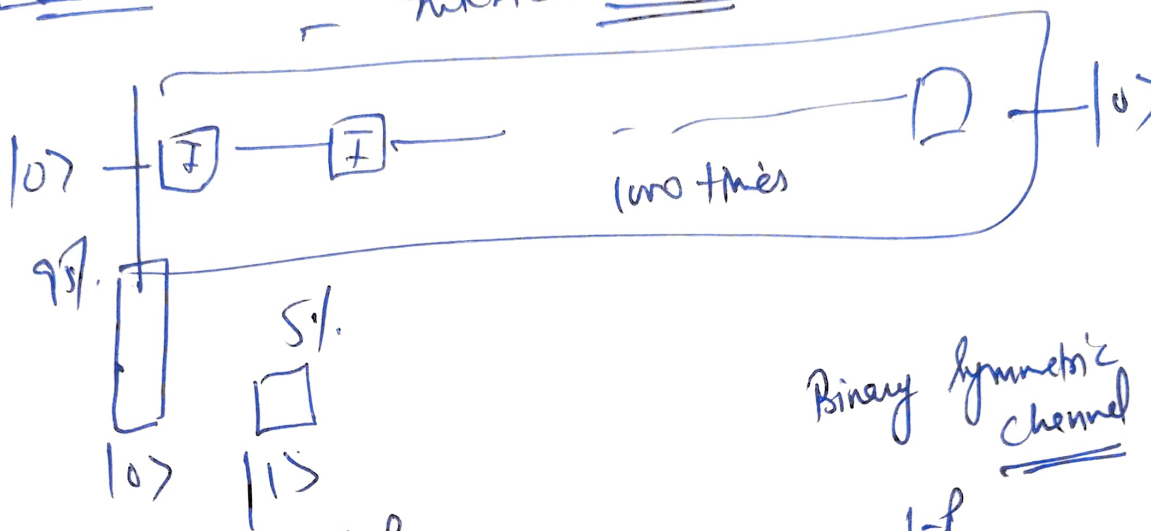
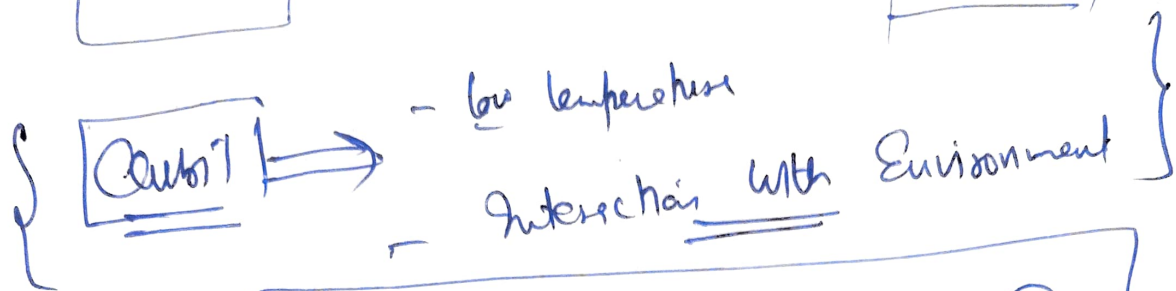
Quantum Computing

We can have following type of single bit Error.

(1) bit flip Error :-  $\begin{matrix} |0\rangle \rightarrow |1\rangle \\ |1\rangle \rightarrow |0\rangle \\ \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle \end{matrix} \quad \left. \vphantom{\begin{matrix} |0\rangle \rightarrow |1\rangle \\ |1\rangle \rightarrow |0\rangle \\ \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle \end{matrix}} \right\} X$

(2) phase flip Error  $\begin{matrix} |0\rangle \rightarrow |0\rangle \\ |1\rangle \rightarrow -|1\rangle \\ \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle \end{matrix} \quad \left. \vphantom{\begin{matrix} |0\rangle \rightarrow |0\rangle \\ |1\rangle \rightarrow -|1\rangle \\ \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle \end{matrix}} \right\} Z \text{ Gate}$

(3) Combination of Both :- Bit flip as well as phase flip



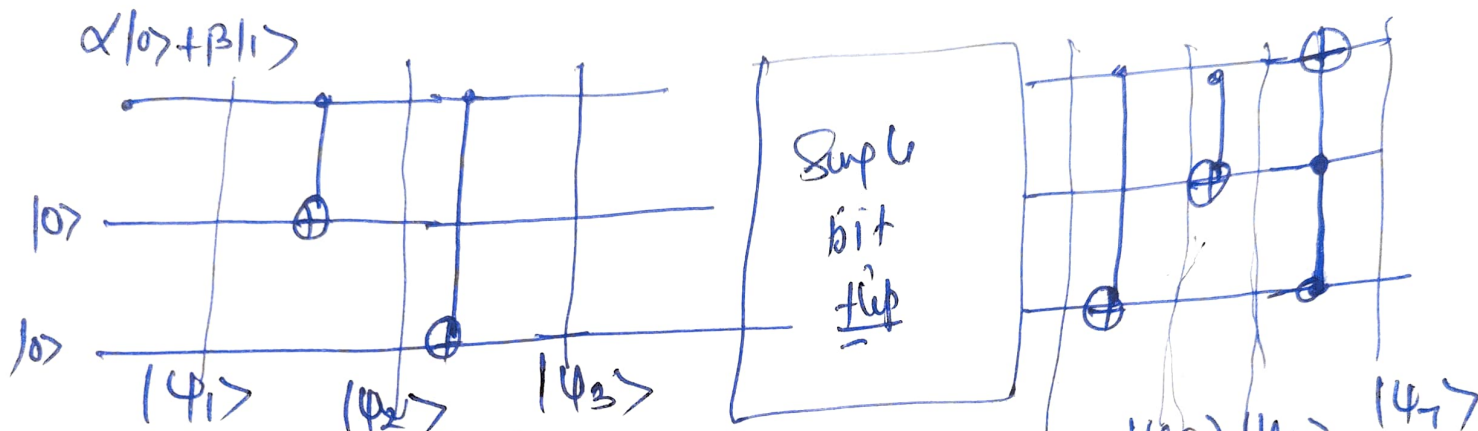
$P \Rightarrow$  Probability that qubit is change

- $\rightarrow$  No cloning
- $\rightarrow$  Measure the qubit } destroy the qubit
- $\rightarrow$  Error type } Bit flip
- $\rightarrow$  Phase flip

$\rightarrow$  Quantum Computation

(3) (3,1) repetition Code to correct single bit flip  
 " " " " " phase flip

Shor 9-bit Code to correct Bit flip as well phase flip



$$|\psi_1\rangle = (\alpha|0\rangle + \beta|1\rangle)|0\rangle|0\rangle$$

$$|\psi_2\rangle = (\alpha|00\rangle + \beta|11\rangle)|0\rangle =$$

$$|\psi_3\rangle = \underline{\alpha|000\rangle + \beta|111\rangle}$$

$$\underline{|\psi_4\rangle} = \alpha|000\rangle + \beta|111\rangle =$$

No Error

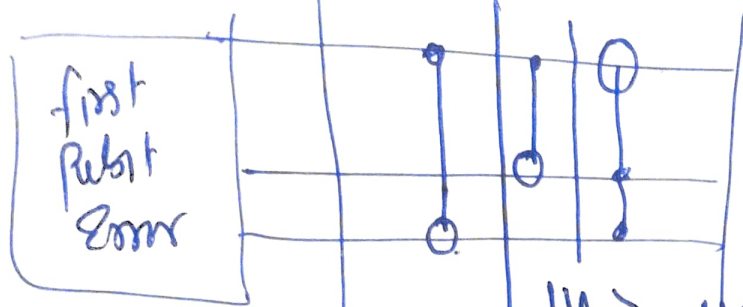
$$\alpha|000\rangle + \beta|110\rangle$$

$$|\psi_4\rangle$$

$$\alpha|000\rangle + \beta|100\rangle$$

$$\alpha|000\rangle + \beta|110\rangle$$

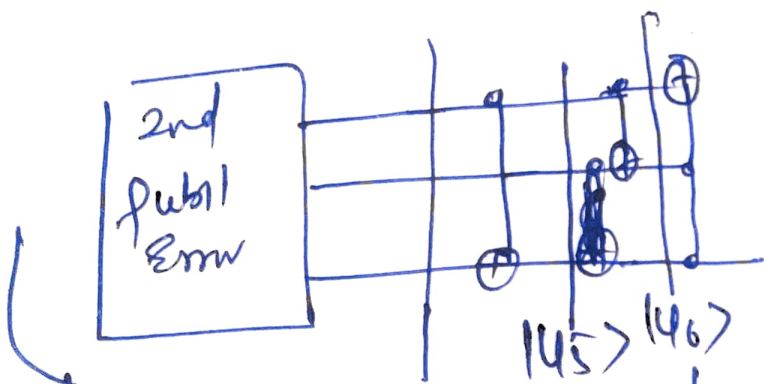
$$|\psi_7\rangle = \alpha|000\rangle + \beta|100\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |00\rangle$$



$$|000\rangle + \beta|111\rangle \quad |\psi_4\rangle \quad |\psi_5\rangle = \alpha|101\rangle + \beta|011\rangle$$

$$\alpha|1100\rangle + \beta|0111\rangle \quad |\psi_6\rangle = \alpha|1111\rangle + \beta|0111\rangle$$

$$|\psi_7\rangle = \alpha|0111\rangle + \beta|1111\rangle \\ = (\alpha|0\rangle + \beta|1\rangle) \otimes |111\rangle$$



$$\alpha|1000\rangle + \beta|1111\rangle \quad |\psi_4\rangle = \alpha|010\rangle + \beta|101\rangle$$

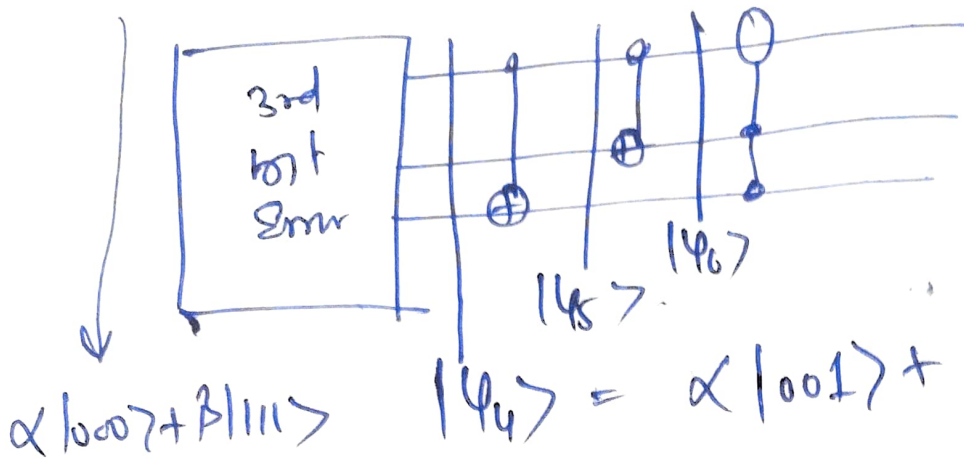
$$|\psi_5\rangle = \alpha|010\rangle + \beta|100\rangle$$

$$|\psi_6\rangle = \alpha|010\rangle + \beta|110\rangle$$

$$|\psi_7\rangle = \alpha|010\rangle + \beta|110\rangle = (\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

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### 3rd qubit Error



$$|\psi_4\rangle = \alpha|001\rangle + \beta|110\rangle$$

$$|\psi_5\rangle = \alpha|001\rangle + \beta|111\rangle$$

$$|\psi_6\rangle = \alpha|001\rangle + \beta|101\rangle = |\psi_7\rangle$$

$$= (\alpha|0\rangle + \beta|1\rangle)|01\rangle$$

### phase flip Error

$$HZH = X$$

$$HXH = Z$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

