Feb 19, 2024

P-1

(Quantum Teleportation)

Super dense Coding

We will teleport a quantum state from one location to another.

Alice $\alpha|0\rangle+\beta|1\rangle$ $\longrightarrow$ Bob $\alpha|0\rangle+\beta|1\rangle$.

Quantum Cloning Theorem :- We can't able to Copy any arbitrary quantum state.

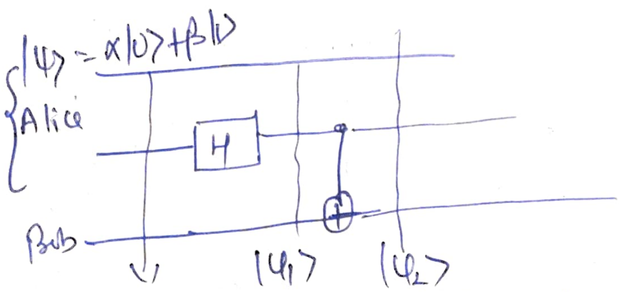
Copy is not possible but Move is possible
$\Downarrow$
teleportation

$|\psi\rangle = \alpha|0\rangle+\beta|1\rangle$

Alice

Bob

Step-1



$$|\psi_0\rangle = (\alpha|0\rangle+\beta|1\rangle) \otimes |0_A\rangle \otimes |0_B\rangle$$

$$|\psi_1\rangle = (\alpha|0\rangle+\beta|1\rangle) \otimes \left(\frac{|0\rangle_A+|1_A\rangle}{\sqrt{2}}\right) \otimes |0_B\rangle = (\alpha|0\rangle+\beta|1\rangle)\frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 0_B\rangle)$$

$$|\psi_2\rangle = (\alpha|0\rangle+\beta|1\rangle) \otimes \left(\frac{|0_A 0_B\rangle+|1_A 1_B\rangle}{\sqrt{2}}\right)$$

$$= \alpha|0\rangle\left(\frac{|0_A 0_B\rangle+|1_A 1_B\rangle}{\sqrt{2}}\right) + \beta|1\rangle\left(\frac{|0_A 0_B\rangle+|1_A 1_B\rangle}{\sqrt{2}}\right)$$

$$\frac{1}{2}\Big(\cancel{|00\rangle} \, \alpha|000\rangle + \beta|001\rangle + \alpha|011\rangle + \beta|010\rangle + \alpha|100\rangle - \beta|101\rangle + \alpha|111\rangle - \beta|110\rangle \Big)$$

Repeat

$$\frac{1}{2}\Big[\alpha|0\,0_A 0_B\rangle + \beta|0\,0_A 1_B\rangle + \alpha|0\,1_A 1_B\rangle + \beta|0\,1_A 0_B\rangle + \alpha|1\,0\,0_B\rangle - \beta|1\,0_A 1_B\rangle + \alpha|1\,1_A 1_B\rangle - \beta|1\,1_A 0_B\rangle \Big]$$

$$\frac{\frac{1}{2}\left(\alpha|0\rangle+\beta|1\rangle\right)}{\sqrt{\langle\psi|\psi\rangle}}$$

$$= \frac{1}{2}\Big[|00\rangle\left(\alpha|0_B\rangle+\beta|1_B\rangle\right) + |01\rangle\left[\alpha|1\rangle+\beta|0\rangle\right] + |10\rangle\left[\alpha|0\rangle-\beta|1_B\rangle\right] + |11\rangle\left[\alpha|1_B\rangle-\beta|0_B\rangle\right] \Big]$$

Bobs end

$\lvert 00\rangle$	$\alpha\lvert 0\rangle+\beta\lvert 1\rangle$	— I Apply	$\alpha\lvert 0\rangle+\beta\lvert 1\rangle$
$\lvert 01\rangle$	$\alpha\lvert 1\rangle+\beta\lvert 0\rangle$	$\rightarrow \begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\alpha\lvert 0\rangle+\beta\lvert 1\rangle$
$\lvert 10\rangle$	$\alpha\lvert 0\rangle-\beta\lvert 1\rangle$	$\xrightarrow{Z} \begin{bmatrix}1&0\\0&-1\end{bmatrix}$	$\alpha\lvert 0\rangle+\beta\lvert 1\rangle$
$\lvert 11\rangle$	$\alpha\lvert 1\rangle-\beta\lvert 0\rangle$	$\begin{bmatrix}0&1\\-1&0\end{bmatrix}$	$\alpha\lvert 0\rangle+\beta\lvert 1\rangle$

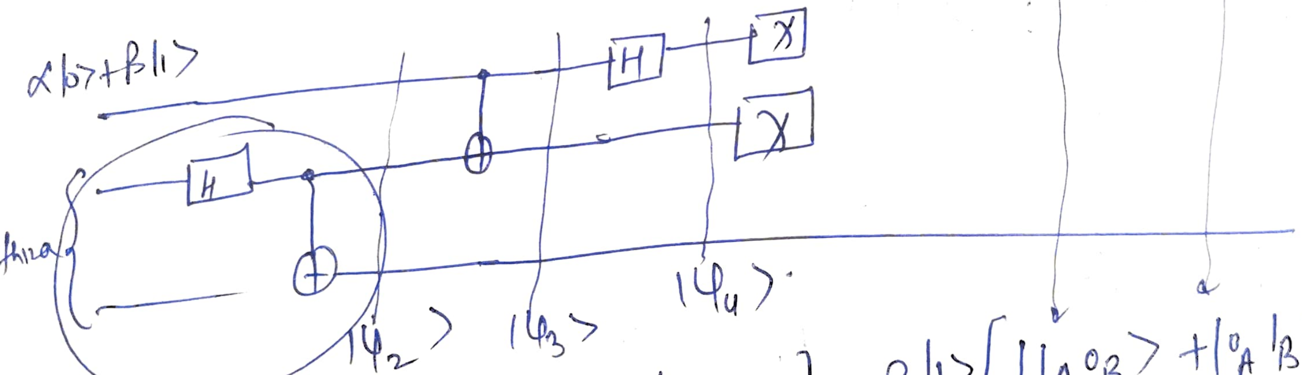
$\cancel{\alpha|0\rangle+\beta|1\rangle}$

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \otimes \begin{pmatrix}\frac{1}{\sqrt2}\\ 0\\ 0\\ \frac{1}{\sqrt2}\end{pmatrix} = \begin{array}{ll} \alpha\cdot\frac{1}{\sqrt2} & 000\\ 0 & 001\\ 0 & 010\\ \alpha\frac{1}{\sqrt2} & 011\\ \beta\frac{1}{\sqrt2} \rightarrow & 100\\ \vdots & \\ \beta\frac{1}{\sqrt2} \rightarrow & 111 \end{array}$$

$$= \alpha\frac{1}{\sqrt2}|000\rangle + \alpha\frac{1}{\sqrt2}|011\rangle + \beta\cdot\frac{1}{\sqrt2}|100\rangle + \beta\frac{1}{\sqrt2}|111\rangle$$

$$\alpha|0\rangle\frac{|00\rangle+|11\rangle}{\sqrt2} + \beta|1\rangle\left(\frac{|00\rangle+|11\rangle}{\sqrt2}\right)$$

$$|\psi_2\rangle = \alpha|0\rangle\left(\frac{|0_A0_B\rangle+|1_A1_B\rangle}{\sqrt2}\right) + \beta|1\rangle\left(\frac{|0_A0_B\rangle+|1_A1_B\rangle}{\sqrt2}\right)$$



$$|\psi_3\rangle = \underset{\frac{|0\rangle+|1\rangle}{\sqrt2}}{\alpha|0\rangle}\left[\frac{|0_A0_B\rangle+|1_A1_B\rangle}{\sqrt2}\right] + \underset{\frac{|0\rangle-|1\rangle}{\sqrt2}}{\beta|1\rangle}\left[\frac{|1_A0_B\rangle+|0_A1_B\rangle}{\sqrt2}\right]$$

$$|\psi_4\rangle = \frac{1}{2}\left[\alpha\left(\frac{|0\rangle+|1\rangle}{\cancel{\sqrt2}}\right)\left[|0_A0_B\rangle+|1_A1_B\rangle\right] + \beta\left(|0\rangle-|1\rangle\right)\left(|1_A0_B\rangle+|0_A1_B\rangle\right)\right]$$

$$\Rightarrow \frac{1}{2}\left[\alpha\left(|00_A0_B\rangle+|01_A1_B\rangle+|100_{AB}\rangle+|111_{AB}\rangle\right)+\right.$$
$$\left.\beta\left(|01_A0_B\rangle+|00_A1_B\rangle-|11_A0_B\rangle-|10_A1_B\rangle\right)\right]$$

Quantum Computing for Computer Scientist

Yanofsky

Bob

Alice Measure

Step-2 $|\varphi\rangle$ A entangled

Step-1 $|\psi\rangle$

A, B entangled

Partial Measurement $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$

first is $|0\rangle$ $\frac{1}{\sqrt{2}}|0\rangle$