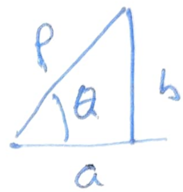
(Bloch Sphere)

$|\psi\rangle = c_0|0\rangle + c_1|1\rangle$

c_0 and c_1 are Complex Number s.t.

$|c_0|^2 + |c_1|^2 = 1$

$|\psi\rangle = r_1 e^{i\phi_1}|0\rangle + r_2 e^{i\phi_2}|1\rangle$



$\frac{b}{p} = \sin\theta$

Global phase have no impact on quantum state.

$\frac{a}{p} = \cos\theta$

$a + ib$

Polar form (p, θ)

$= e^{i\phi_1}\left[r_1|0\rangle + r_2 e^{i\phi_2} e^{-i\phi_1}|1\rangle \right]$

$a+ib$

$= p\cos\theta + i p\sin\theta$

Global phase of $e^{-i\phi_1}$

$e^{-i\phi_1}|\psi\rangle = e^{-i\phi_1} e^{i\phi_1}\left[r_1|0\rangle + r_2 e^{i(\phi_2-\phi_1)}|1\rangle\right]$

$= p(\cos\theta + i\sin\theta)$

$= pe^{i\theta}$

$= r_1|0\rangle + r_2 e^{i\phi}|1\rangle$ — (1)

$\phi = \phi_2 - \phi_1$

$|c_0|^2 + |c_1|^2 = 1$

$|\psi\rangle = c_0|0\rangle + c_1|1\rangle$

$= r_1 e^{i\phi_1}|0\rangle + r_2 e^{i\phi_2}|1\rangle$

$|r_1 e^{i\phi_1}|^2 + |r_2 e^{i\phi_2}|^2 = 1$

$|r_1|^2 |e^{i\phi_1}|^2 + |r_2|^2 |e^{i\phi_2}|^2 = 1$

$|e^{i\phi_1}|^2 = e^{i\phi_1} e^{-i\phi_1}$

$= e^{i\phi_1 - i\phi_1}$

$= e^0 = 1$

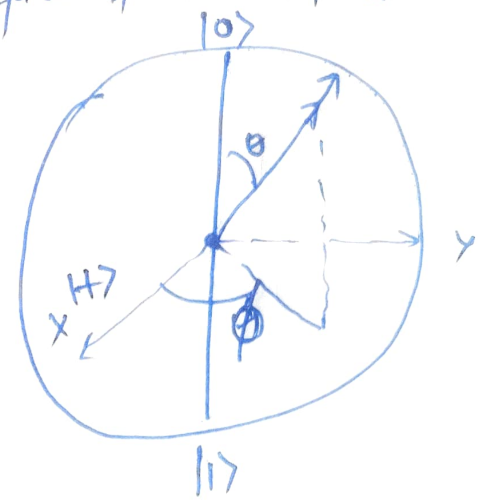
$\Rightarrow |r_1|^2 + |r_2|^2 = 1$

$\Rightarrow \boxed{r_1 = \cos\frac{\theta}{2} \qquad r_2 = \sin\frac{\theta}{2}}$

Put r_1 and r_2 in (1)

$e^{-i\phi_1}|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2} e^{i\phi}|1\rangle$

Flex: Bloch introduced Bloch Sphere. To Represent any Single Qubit on the Surface of Bloch Sphere.



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}\, e^{i\phi}|1\rangle$$

$|0\rangle, |1\rangle$ orthonormal

$\langle 0|1\rangle = 0$

$\langle 0|0\rangle = 1$

$\langle 1|1\rangle = 1$

$\theta = 0 \quad \phi = 0$

z-axis $|0\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\theta = \pi/2$

$e^{i\phi} = 1$

$\phi = 0$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\cos\frac{\pi}{4}|0\rangle + \sin\frac{\pi}{4} \cdot 1\, |1\rangle$$

$|1\rangle = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2} e^{i\phi}|1\rangle$$

$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \quad \theta = \pi/2 \quad \phi = \pi$

$e^{i\pi} = \cos\pi + i\sin\pi$

$= -1$

$$\cos\frac{\pi}{4}|0\rangle + \sin\frac{\pi}{4}(-1)|1\rangle$$

$= |-\rangle$

$X^2 = I \qquad Y^2 = I \qquad Z^2 = I$

Generalized Euler formula

A be any matrix such that $A^2 = I$

$$e^{i\theta A} = (\cos\theta) I + i(\sin\theta) A$$

$$\underbrace{e^{i\theta A}} \to 1 + (i\theta A) + \frac{(i\theta A)^2}{2!} + \frac{(i\theta A)^3}{3!} + \cdots$$

$$e^{-i\frac{\theta}{2}X} = \left(\cos\tfrac{\theta}{2}\right)I + i\sin\tfrac{\theta}{2}X$$

$$= \cos\left(-\frac{\theta}{2}\right)I + i\sin\left(-\frac{\theta}{2}\right)X$$

$\theta \Rightarrow$ anticlockwise

$\frac{\theta}{2} \Rightarrow$ $|0\rangle$, $|1\rangle$ orthogonal

$$= \cos\frac{\theta}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i\sin\frac{\theta}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\theta/2 \end{bmatrix}$$

Put $\theta = \pi$

$$\Downarrow$$

$$\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \underbrace{-i}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$e^{i\phi} = -i$$

$\hookrightarrow \cos\phi + i\sin\phi$

$\phi = \pi/2$

$$\Downarrow$$

$$\cos(-\pi/2) + i\sin(-\pi/2)$$

$$= 0 - i\sin\pi/2$$

$$= -i$$

$$R_z(\theta) = e^{-i\theta/2 Z}$$

$$= \left(\cos\frac{\theta}{2}\right)I + i\left(\sin\frac{\theta}{2}\right)Z$$

$$= \cos\frac{\theta}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - i\sin\frac{\theta}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\frac{\theta}{2} - i\sin\theta/2 & 0 \\ 0 & \cos\frac{\theta}{2} + i\sin\frac{\theta}{2} \end{bmatrix}$$

$$e^{i\theta/2} = \cos\theta/2 + i\sin\theta/2$$

$$e^{-i\theta/2} = \cos\theta/2 - i\sin\theta/2$$

$$R_z(\theta) = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

$\theta = \pi$

$$e^{i\theta/2} = i$$

$$e^{-i\theta/2} = -i$$

$$= \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} = -i\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$R_y(\theta) = e^{-i\theta/2 Y} = \left(\cos\frac{\theta}{2}\right)I - i\sin\frac{\theta}{2}(Y)$$