(Lecture - 6)

Measurement $|\psi\rangle$ is any quantum state

and Measure $|\psi\rangle$ in Computational Basis

$\{|0\rangle, |1\rangle\}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \qquad |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|\langle 0|\psi\rangle|^2 = \left| \begin{bmatrix}1 & 0\end{bmatrix} \begin{bmatrix}\alpha\\ \beta\end{bmatrix} \right|^2 \qquad \langle 0| = (1 \quad 0)$$

$$= |\alpha|^2$$

braket (bra ket Notation)

Dirac Notation

$$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \qquad \langle 1| = (0 \quad 1)$$

$$|\langle 1|\psi\rangle|^2 = \left| \begin{bmatrix}0 & 1\end{bmatrix} \begin{bmatrix}\alpha\\ \beta\end{bmatrix} \right|^2 = |\beta|^2$$

Born Rule :-

For any given quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$P(|0\rangle) = |\alpha|^2$$
$$P(|1\rangle) = |\beta|^2$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

Relative phase of π

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + \underbrace{e^{i\pi}}_{\Downarrow}|1\rangle)$$

$$e^{i\pi} = \cos\pi + i\sin\pi = -1$$

Global phase

$|\psi\rangle$ Original State

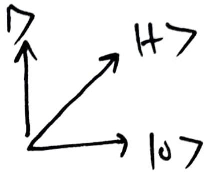
$e^{i\phi}|\psi\rangle$ & Complete state is multiplied with $e^{i\phi}$

$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

$\left.\begin{array}{l}|\langle 0|+\rangle|^2 = \frac{1}{2}\\ |\langle 1|+\rangle|^2 = \frac{1}{2}\end{array}\right\}$ Same way $\begin{array}{l}|\langle 0|-\rangle|^2 = \frac{1}{2}\\ |\langle 1|-\rangle|^2 = \frac{1}{2}\end{array}$

$|\langle +|+\rangle|^2 = \left|\frac{1}{\sqrt{2}}(1\ \ 1)\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\right|^2$



$= 1$

$|\langle -|+\rangle|^2 = 0$

$|\langle +|-\rangle|^2 = 0$

$|\langle -|-\rangle|^2 = 1$

Global phase

$\Downarrow$

No impact on quantum state

$|\psi_1\rangle = |0\rangle$

$|\psi_2\rangle = e^{i\phi}|0\rangle = \begin{bmatrix}e^{i\phi}\\0\end{bmatrix}$

$|\langle 0|0\rangle|^2 = 1$

$|\langle 0|e^{i\phi}0\rangle|^2 = \left|[1\ \ 0]\begin{bmatrix}e^{i\phi}\\0\end{bmatrix}\right|^2 = |e^{i\phi}|^2 \cancel{= |e^{i\phi}e^{-i\phi}| = |e^{i\phi-i\phi}|^2} \cancel{= |e^{0}|^2} = 1$

$e^{i\phi}e^{-i\phi}$

$= e^{i\phi-i\phi} = e^{0} = 1$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and $|\psi_2\rangle = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$

$= \begin{pmatrix}\alpha\\\beta\end{pmatrix}$ $\qquad = \begin{pmatrix}e^{i\phi}\alpha\\e^{i\phi}\beta\end{pmatrix}$

$|\langle 0|\psi_2\rangle|^2 = \left|[1\ \ 0]\begin{bmatrix}e^{i\phi}\alpha\\e^{i\phi}\beta\end{bmatrix}\right|^2 = |e^{i\phi}\alpha|^2 = |e^{i\phi}|^2|\alpha|^2$

$= e^{i\phi}e^{-i\phi}|\alpha|^2 = 1|\alpha|^2 = |\alpha|^2$

$|\langle 1|\psi_2\rangle|^2 = \left|[0\ \ 1]\begin{bmatrix}e^{i\phi}\alpha\\e^{i\phi}\beta\end{bmatrix}\right|^2 = |e^{i\phi}\beta|^2 = |\beta|^2$

$|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$ $\qquad |\psi_2\rangle = e^{i\phi}\left(\alpha|0\rangle + \beta|1\rangle\right)$ 6 2

Measure in $|+\rangle$ ~~State~~

$$|\langle + |\psi_1\rangle|^2 = \left|\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\right|^2 = \left|\frac{1}{\sqrt{2}}(\alpha+\beta)\right|^2$$

$$|\langle - |\psi_1\rangle|^2 = \left|\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\right|^2 = \left|\frac{1}{\sqrt{2}}(\alpha-\beta)\right|^2$$

$$|\langle + |\psi_2\rangle|^2 =$$

$$|\langle - |\psi_2\rangle|^2 =$$

(Y-gate)

$$y = iXZ = i\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \begin{bmatrix}\cancel{0} & \cancel{-1}\\ \cancel{1} & \cancel{1}\end{bmatrix}$$

$$= i\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$$

Pauli - X $\quad X$
Pauli - Y $\quad Y$
Pauli - Z $\quad Z$

$\Bigg\}$ Pauli family

$$e^{i\phi} = \cos\phi + i\sin\phi = i$$
$$\phi = \pi/2$$

$e^{i\phi}|y\rangle$ $\qquad$ $|y\rangle$ and $e^{i\phi}|y\rangle$ will be Same

$$\phi = \pi/2$$

$$= i\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix} = \begin{bmatrix}0 & -i^2\\ i^2 & 0\end{bmatrix} = \begin{bmatrix}0 & +1\\ -1 & 0\end{bmatrix}$$

Commutative

X and Y gate anti commutate in nature

$XY \neq YX$

$$XY = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

$$YX = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} = -1 \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

$$XY = -YX$$

$$XY + YX = 0$$

X and Z gate are anti commutate in nature

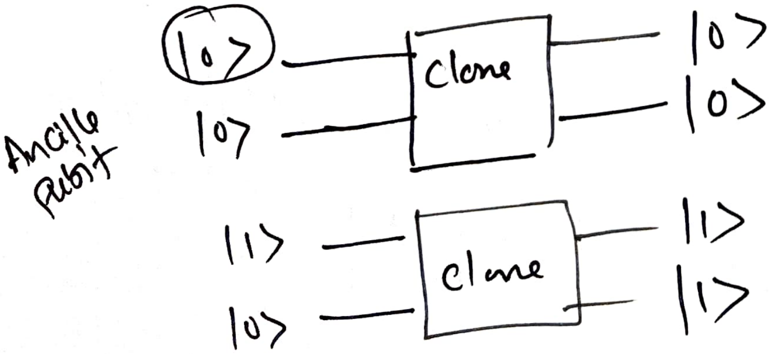
$$XZ = -ZX$$

$$\boxed{XZ + ZX = 0}$$

No cloning Theorem :-

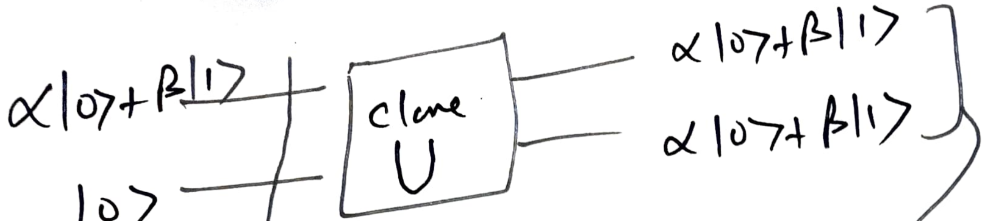
Any arbitrary unknown quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

We cannot able to clone any arbitrary quantum state.



$|0\rangle$ Blank copy of page.

$|1\rangle$



$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \otimes \begin{pmatrix}1\\ 0\end{pmatrix} \qquad \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \otimes \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\alpha^2\\ \alpha\beta\\ \beta\alpha\\ \beta^2\end{pmatrix}$$

Unitary operators are linear in nature.

$$f(x+y) = f(x)+f(y)$$

$$U\big((\alpha|0\rangle+\beta|1\rangle)(|0\rangle)\big) = U[\alpha|0\rangle|0\rangle+\beta|1\rangle|0\rangle]$$

$$= U[\alpha|0\rangle|0\rangle] + U[\beta|1\rangle|0\rangle]$$

$$= \alpha U[|0\rangle|0\rangle] + \beta U[|1\rangle|0\rangle]$$

$$= \alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle = \alpha|00\rangle+\beta|11\rangle = \begin{pmatrix}\alpha\\ 0\\ 0\\ \beta\end{pmatrix}$$

$\boxed{\alpha\beta = 0} \Rightarrow \alpha = 0$ or $\beta = 0$

$\alpha|0\rangle+\beta|1\rangle = \beta|1\rangle$ = classical 1 state

$\beta = 0$ $\quad \alpha|0\rangle$ = classical 0 state