(Lecture-7)

No cloning Theorem :-

We Cannot able to Copy any unknown arbitrary quantum state.

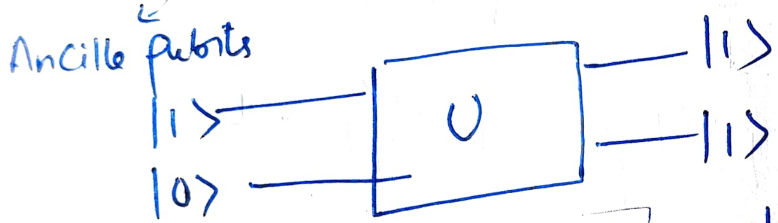
Proof :-

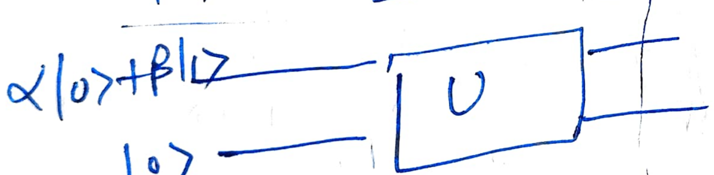
We prove it by Contradiction.

Let us Assume we have a cloning System.

T } pair. $T^\dagger$
S $S^\dagger$

$TT^\dagger = I$





$\alpha|0\rangle+\beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$

$\alpha|0\rangle+\beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$

$$U\big((\alpha|0\rangle+\beta|1\rangle)(|0\rangle)\big) = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}\otimes\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\alpha^2\\ \alpha\beta\\ \beta\alpha\\ \beta^2\end{pmatrix}$$

Unitary matrix they are Linear in operation ✓

$$f(x+y) = f(x)+f(y)$$

$$U[\alpha|0\rangle|0\rangle + \beta|1\rangle|0\rangle] = U[\alpha|0\rangle|0\rangle] + U[\beta|1\rangle|0\rangle]$$

$$= \alpha[U|0\rangle|0\rangle] + \beta[U|1\rangle|0\rangle]$$

$$= \alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle = \begin{pmatrix}\alpha\\ 0\\ 0\\ \beta\end{pmatrix}$$

$\alpha\beta = 0$

$\Downarrow$

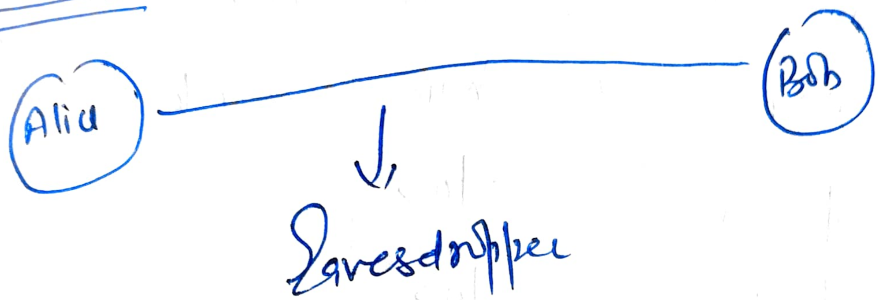
either $\alpha=0$ or $\beta=0$

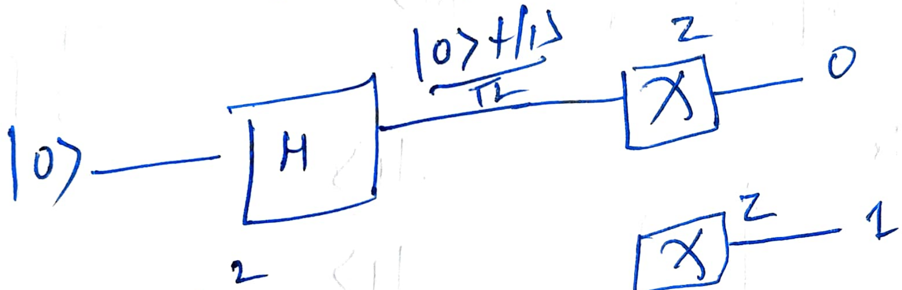
If $\alpha=0$, $\alpha|0\rangle+\beta|1\rangle = \beta|1\rangle$ so $\beta=1$

If $\beta=0$, $\alpha|0\rangle+\beta|1\rangle = \alpha|0\rangle$ so $\alpha=1$ } $\Rightarrow$ Something like classical bit

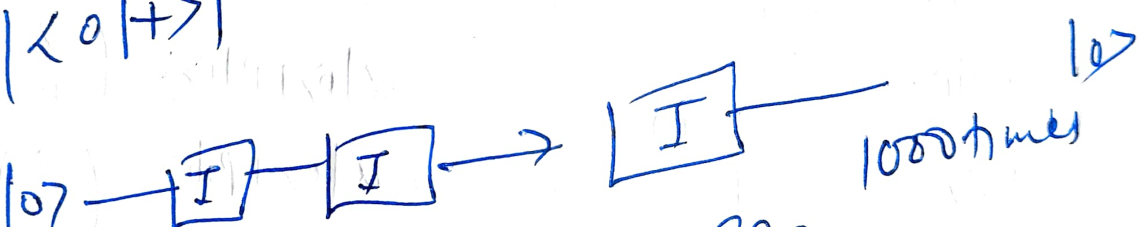
Hence we can say we cannot able to Copy any arbitrary unknown quantum state.

Advantage





$$|\langle 0|+\rangle|^2$$



$|0\rangle$ — 990

$|1\rangle$ — 10

Lecture - 2

$|\psi\rangle = C_0|0\rangle + C_1|1\rangle$

$\boxed{|C_0|^2 + |C_1|^2 = 1}$ & C_0, C_1 are Complex Numbers

$C_0 \to a_1 + ib_1$, $C_1 \to a_2 + ib_2$

4 Real No. required.

Felix Bloch

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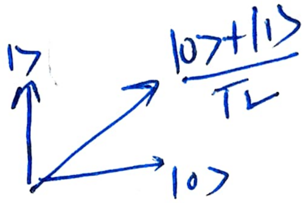
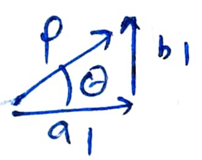
Bloch Sphere

Cartesian Notation



A Complex No $a_1 + ib_1$



$\rho = \sqrt{(a_1^2 + b_1^2)}$

Polar notation

$\cos\theta = \frac{a_1}{\rho}$ $\sin\theta = \frac{b_1}{\rho}$

$\rho\cos\theta + i\rho\sin\theta$

$\Rightarrow a_1 = \rho\cos\theta$

$b_1 = \rho\sin\theta$

$\Rightarrow \cancel{\rho(\cos}(\rho, \theta)$

$\rho(\cos\theta + i\sin\theta) = \rho e^{i\theta}$ Euler formula

$|C_0|^2 + |C_1|^2 = 1$

$|\psi\rangle = \boxed{C_0|0\rangle + C_1|1\rangle}$

$= r_0 e^{i\phi_0}|0\rangle + r_1 e^{i\phi_1}|1\rangle$

$e^{-i\phi_0}|\psi\rangle = e^{-i\phi_0}[r_0 e^{i\phi_0}|0\rangle + r_1 e^{i\phi_1}|1\rangle]$

$= r_0|0\rangle + r_1 e^{i(\phi_1 - \phi_0)}|1\rangle$

$|C_0|^2 + |C_1|^2 = 1$

$|r_0 e^{i\phi_0}|^2 + |r_1 e^{i\phi_1}|^2 = 1$

$|r_0 e^{i\phi_0}|^2 + |r_1|^2|e^{i\phi_1}|^2 = 1$

$\Rightarrow |r_0|^2|e^{i\phi_0}|^2 + |r_1|^2|e^{i\phi_1}|^2 = 1$

$\Rightarrow |r_0|^2 + |r_1|^2 = 1$

$|e^{i\phi_0}|^2 = e^{i\phi_0}e^{-i\phi_0}$

$= e^{i\phi_0 - i\phi_0}$

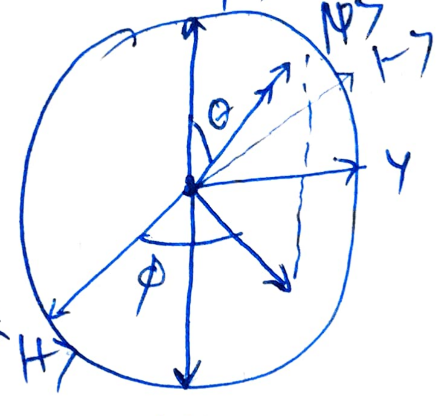
$= e^0 = 1$

$r_0 = \cos\theta/2$ $r_1 = \sin\theta/2$

$$e^{-i\phi_0}|\psi\rangle = r_0|0\rangle + r_1 e^{i(\phi_1-\phi_0)}|1\rangle$$

$$= \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}\, e^{i(\phi_1-\phi_0)}|1\rangle$$

$$\underbrace{e^{-i\phi_0}|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}}\, e^{i\phi}|1\rangle \qquad \phi = \phi_1 - \phi_0$$



θ = angle between qubit and z-axis

ϕ = Projection of $|\psi\rangle$ on xy plane and angle between the Projection of qubit with +x direction

$|0\rangle$ and $|1\rangle$ are orthonormal
↓
orthogonal + Normalization

$\theta = 0,\ \phi = 0$ z-axis $|0\rangle$

$\theta = \pi,\ \phi = 0$ $|1\rangle$ $\qquad \langle 0|1\rangle = 0$

$\theta = \pi/2 \quad \phi = 0$ x-axis

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

y axis

$$|i\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}$$

$$|-i\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -i\end{bmatrix}$$