P-1

$$\rho = \sum_i P_i |x_i\rangle\langle x_i|$$

$|+\rangle \longrightarrow |-\rangle \quad \frac{1}{4}$

$|+\rangle \longrightarrow |+\rangle \quad \frac{3}{4}$

(Mixed and Pure State) $\quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ Pure State

whenever we transmit a pure quantum state (qubits)

Alice $\longrightarrow$ Bob

qubits in pure state

$|+\rangle \longrightarrow |-\rangle$
phaseflip Error

Noise or Environment these pure qubits usually changes

Alice $\longrightarrow$ Bob

$|+\rangle \longrightarrow$ with probability P changed it is to $|-\rangle$

with " $1-P$ it is Remaining $|+\rangle$

$$\left.\begin{array}{l} |+\rangle \longrightarrow |-\rangle \quad \frac{1}{4} \\ |+\rangle \longrightarrow |+\rangle \quad \frac{3}{4} \end{array}\right]$$

they are represent by density matrix.

Pure state $\boxed{\rho = |\psi\rangle\langle\psi|}$ is defined by Outer product

$$\rho = \sum_i P_i |x_i\rangle\langle x_i|$$

$$= P_0 |-\rangle\langle -| + P_1 |+\rangle\langle +|$$

$$= \frac{1}{4}|-\rangle\langle -| + \frac{3}{4}|+\rangle\langle +|$$

$\alpha|0\rangle + \beta|1\rangle \Longrightarrow \alpha|1\rangle + \beta|0\rangle$ bit flip Error

$\Longrightarrow \alpha|0\rangle - \beta|1\rangle$ phase Flip Error

⋮

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$ $|\alpha|^2 + |\beta|^2 = 1$

Normalization Coefficient

$\rho = |\psi\rangle\langle\psi|$

$= \begin{bmatrix}\alpha\\ \beta\end{bmatrix}\begin{bmatrix}\alpha^{+} & \beta^{+}\end{bmatrix} = \begin{bmatrix}\alpha\alpha^{+} & \alpha\beta^{+}\\ \beta\alpha^{+} & \beta\beta^{+}\end{bmatrix}$

$= \begin{bmatrix}|\alpha|^2 & \alpha\beta^{+}\\ \beta\alpha^{+} & |\beta|^2\end{bmatrix}$

$\text{Trace}\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} = a_{11} + a_{22}$

$|\alpha|^2 + |\beta|^2$

$\text{trace}(\rho) = 1 \Longrightarrow \text{trace}(\rho^2) = 1$ Pure state

$\text{trace}(\rho^2) < 1$ For Mixed state

$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$ $\rho = |+\rangle\langle+|$

$= \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

$\rho^2 = \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} = \frac{1}{4}\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

$\text{trace}(\rho^2) = \frac{1}{2}(1+1) = 1$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\rho =$ ______

$\boxed{\rho^2 = \rho}$ for pure state

$\rho^2 =$ ______ classical probabilities

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$$\rho = \frac{1}{4}|-\rangle\langle -| + \frac{3}{4}|+\rangle\langle +|$$

$$= \frac{1}{4}\cdot\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\end{bmatrix} + \frac{3}{4}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} + \frac{3}{8}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$= \begin{bmatrix}\frac{4}{8} & -\frac{1}{8}+\frac{3}{8}\\-\frac{1}{8}+\frac{3}{8} & \frac{1}{8}+\frac{3}{8}\end{bmatrix} = \begin{bmatrix}\frac{1}{2} & \frac{1}{4}\\\frac{1}{4} & \frac{1}{2}\end{bmatrix}$$

$$\rho^2 = \begin{bmatrix}\frac{1}{2} & \frac{1}{4}\\\frac{1}{4} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}\frac{1}{2} & \frac{1}{4}\\\frac{1}{4} & \frac{1}{2}\end{bmatrix} = \begin{bmatrix}\frac{1}{4}+\frac{1}{16} & \frac{1}{8}+\frac{1}{8}\\\frac{1}{8}+\frac{1}{8} & \frac{1}{16}+\frac{1}{4}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{5}{16} & \frac{2}{8}\\\frac{2}{8} & \frac{5}{16}\end{bmatrix}$$

$$\text{trace}(\rho^2) = \frac{5}{16}+\frac{5}{16}$$

$$= \frac{10}{16} < 1$$

$|\psi\rangle$ and $\rho$ Normalization satisfied in both

Measurement $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$|\langle 0|\psi\rangle|^2 = |\alpha|^2$$

$$|\langle 0|\psi\rangle|^2 = (\langle 0|\psi\rangle)(\langle 0|\psi\rangle)^\dagger$$

$$= (\langle 0|\underbrace{\psi\rangle\langle\psi}|0\rangle) = (\langle 0|\rho|0\rangle)$$

$$\langle 0|\rho|0\rangle = [1 \quad 0]\,\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}_{2\times 2}\begin{bmatrix}1\\0\end{bmatrix}_{2\times 1}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$|+\rangle\langle +| = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\frac{1}{\sqrt{2}}(1$$

$$= \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

$$= [1 \quad 0]\,\frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \frac{1}{2}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\boxed{\begin{aligned} P(|0\rangle) &= |\alpha|^2 \\ P_2(|1\rangle) &= |\beta|^2 \end{aligned}}$$

$$\rho = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}[\alpha^\dagger \quad \beta^\dagger]$$

$$\langle 0|\rho|0\rangle =$$

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Unitary operation

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \qquad Z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$|\psi_2\rangle = U|\psi\rangle = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\alpha\\ -\beta\end{pmatrix} = \alpha|0\rangle - \beta|1\rangle$$

$$\rho \text{ of } |\psi_2\rangle\langle\psi_2| = \begin{pmatrix}\alpha\\ -\beta\end{pmatrix}(\alpha^\dagger \quad -\beta^\dagger)$$

$$= \begin{pmatrix}\alpha\alpha^\dagger & -\alpha\beta^\dagger\\ -\beta\alpha^\dagger & \beta\beta^\dagger\end{pmatrix}$$

10:30 Grover

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$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

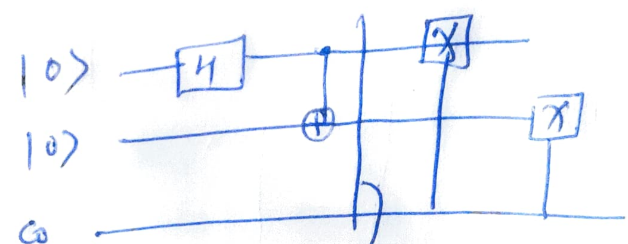
$$U\rho U^\dagger = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}\alpha\alpha^\dagger & \alpha\beta^\dagger\\ \beta\alpha^\dagger & \beta\beta^\dagger\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \qquad \rho = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}\begin{bmatrix}\alpha^\dagger & \beta^\dagger\end{bmatrix}$$

$$= \begin{bmatrix}\alpha\alpha^\dagger & \alpha\beta^\dagger\\ -\beta\alpha^\dagger & -\beta\beta^\dagger\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \begin{bmatrix}\alpha\alpha^\dagger & -\alpha\beta^\dagger\\ -\beta\alpha^\dagger & \beta\beta^\dagger\end{bmatrix}$$

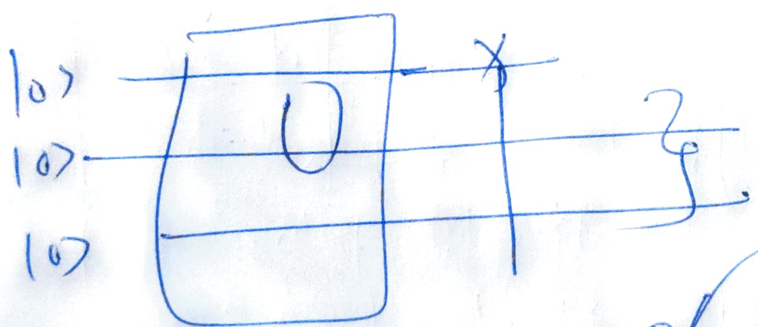
$|\psi\rangle \Rightarrow \quad U|\psi\rangle \longrightarrow \rho_2$

$\rho_1 \quad U\rho U^\dagger \longrightarrow \rho_2$

(Partial Measurement)




$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$




$\frac{1}{\sqrt{2}}|0\rangle \quad |0\rangle'$

$\cancel{\frac{1}{\sqrt{2}}}$

$|0\rangle \otimes \frac{1}{\sqrt{2}}|0\rangle$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

0

$|v\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 0\end{bmatrix}$

$\langle v| = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\end{bmatrix}$

v is a vector $\|v\| = \sqrt{\langle v|v\rangle}$

$$\|v\| = \sqrt{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 0\end{bmatrix}}$$

$$= \sqrt{\frac{1}{2}(1)} = \frac{1}{\sqrt{2}}$$

$$|\psi\rangle = \frac{1}{2}|00\rangle - \frac{i}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$$