Lecture - 2

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$

$\boxed{\text{Qubit}} \Rightarrow$ Quantum bit

$|0\rangle \Rightarrow$ Ket 0 $= \begin{pmatrix}1\\0\end{pmatrix}$

$|1\rangle \Rightarrow$ Ket 1 $= \begin{pmatrix}0\\1\end{pmatrix}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$$

Psi

$|\alpha|^2 + |\beta|^2 = 1$

α, β are Complex No.

$\alpha, \beta \in \mathbb{C}$

$\alpha = x_1 + iy_1$

$\beta = x_2 + iy_2$

$|\alpha|^2 =$ Probability that the system lies in $|0\rangle$ State

$|\beta|^2 =$ Probability that the System lies in $|1\rangle$ State.

$|\alpha|^2 = \alpha . \alpha^\dagger$ $\quad$ $\alpha^\dagger$ is Complex Conjugate of α

$\alpha = x_1 + iy_1$

$\alpha^\dagger = x_1 - iy_1$

$$|\alpha|^2 = (x_1 + iy_1)(x_1 - iy_1)$$

$$= x_1^2 + \cancel{iy_1x_1} - \cancel{ix_1y_1} - i^2y_1^2 \qquad i^2 = -1$$

$$= x_1^2 + y_1^2$$

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$$

$P(|0\rangle) = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = \frac{1}{3}$

$P(|1\rangle) = \sqrt{\frac{2}{3}} \cdot \sqrt{\frac{2}{3}} = \frac{2}{3}$

$P(|0\rangle) + P(|1\rangle) = 1$

$$|\phi\rangle = \left(\frac{1+i}{\sqrt{3}}\right)|0\rangle - \frac{i}{\sqrt{3}}|1\rangle$$

$$\alpha = \left(\frac{1+i}{\sqrt{3}}\right) \qquad \beta = -\frac{i}{\sqrt{3}}$$

$$|\alpha|^2 = \alpha . \alpha^\dagger = \left(\frac{1+i}{\sqrt{3}}\right)\left(\frac{1-i}{\sqrt{3}}\right)$$

$$= \frac{1}{3}\left(1-i^2\right) = 2/3$$

$$|\beta|^2 = \left(\frac{-i}{\sqrt{3}}\right)\left(\frac{i}{\sqrt{3}}\right) = -i^2/3 = 1/3$$

$$|\alpha|^2 + |\beta|^2 = 1$$

Valid Qubit

Classical System

$$00 \to \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad 01 \to \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad 10 \to \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad 11 \to \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Qubits in Quantum Systems

Tensor products

$$|\psi_1\rangle = |0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|\psi_2\rangle = |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|\psi\rangle = \underset{|01\rangle}{|\psi_1\psi_2\rangle} = |\psi_1\rangle \otimes |\psi_2\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix}$$

$$= \begin{pmatrix}1.0\\1.1\\0.0\\0.1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \begin{array}{l} \to |00\rangle \\ \to |01\rangle \\ \to |10\rangle \\ \to |11\rangle \end{array}$$

$$= 0\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + 1\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + 0\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + 0\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}\alpha_1\\\beta_1\end{pmatrix} \otimes \begin{pmatrix}\alpha_2\\\beta_2\end{pmatrix} = \begin{pmatrix}\alpha_1\alpha_2\\\alpha_1\beta_2\\\beta_1\alpha_2\\\beta_1\beta_2\end{pmatrix}$$

$$|\psi\rangle = 0|00\rangle + 1|01\rangle + 0|10\rangle + 0|11\rangle$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a.e & a.f & b.e & b.f \\ a.g & a.h & b.g & b.h \\ c.e & c.f & d.e & d.f \\ c.g & c.h & d.g & d.h \end{pmatrix}$$

$$|11\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = 0|00\rangle + 0|01\rangle + 0|10\rangle + 1|11\rangle$$

11

$|00\rangle = ?$

$|10\rangle = ??$

$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{array}$$

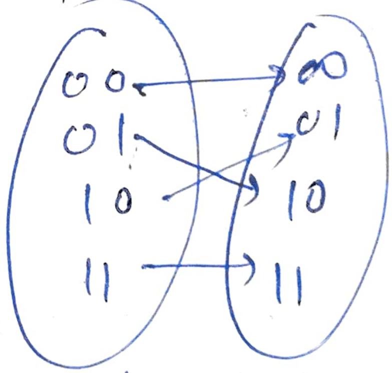
$$\begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} \otimes \begin{pmatrix} \alpha_2 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \beta_1\alpha_2 \\ \beta_1\beta_2 \end{pmatrix}$$

a	b	x
0	0	0
0	1	0
1	0	0
1	1	1

64 qubit system $\Longrightarrow 2^{64}$

(Hadamard gate) single qubit gate

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ — $\boxed{H}$ —



$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

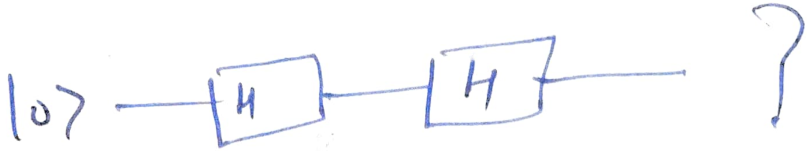
$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle = |-\rangle$$

a)



b)

c)