(April 15, 2024)

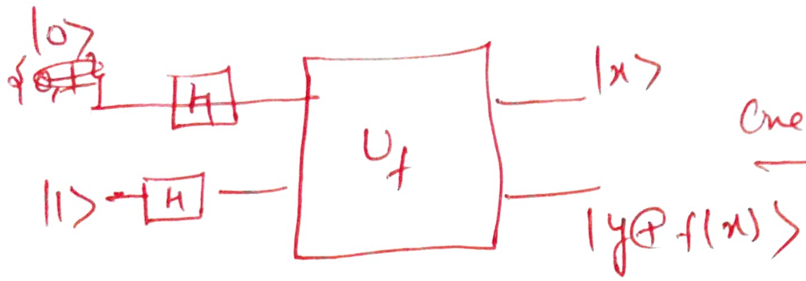
R1

Deutsch – Jozsa Algorithm

Deutsch Algorithm

$f: \{0,1\} \longrightarrow \{0,1\}$

Balanced or Constant



One evaluation

Deutsch-Jozsa algorithm

$f: \{0,1\}^n \longrightarrow \{0,1\}$

3-qubit

000
001
010
011
100
101
110
111

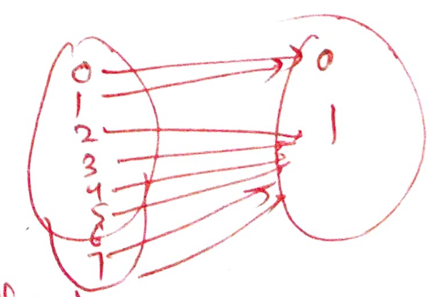
0

1

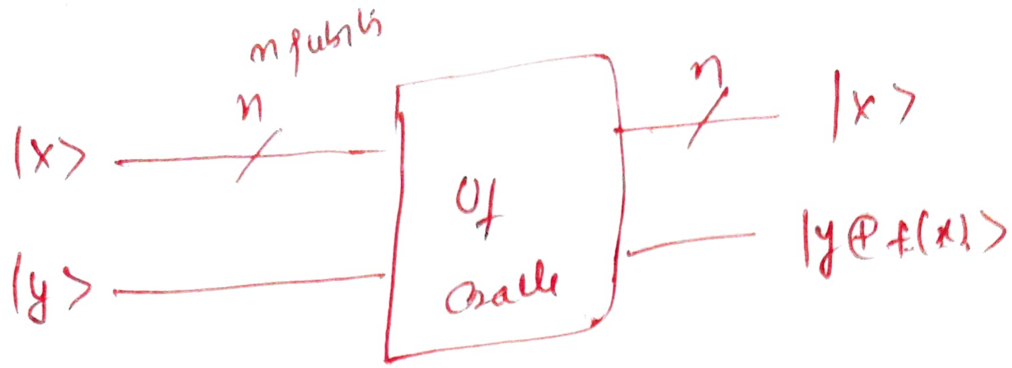
Assuming that the function is either Constant or Balanced.

$\boxed{\frac{2^n}{2}+1}$ Evaluation to decide that the function is Constant

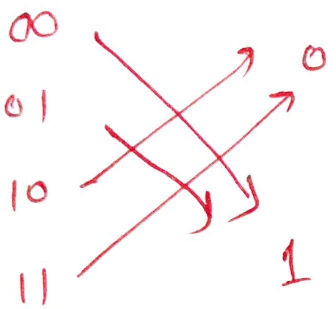
$n=3$ $\quad \frac{2^3}{2}+1 = 5$



Best Case Scenario for a function has Balanced, first two evaluation Results are different.



$\{0,1\}^2 \longrightarrow \{0,1\}$



$|\underbrace{00}_{x}\underbrace{0}_{y}\rangle \rightarrow |00\rangle|0\oplus1\rangle = |001\rangle$

$|\underbrace{00}_{x}\underbrace{1}_{y}\rangle \rightarrow |00\rangle|1\oplus1\rangle = |000\rangle$

$|\underbrace{01}\,0\rangle \rightarrow |01\rangle|0\oplus1\rangle = |011\rangle$

$|011\rangle \rightarrow |01\rangle|1\oplus1\rangle = |010\rangle$

$|100\rangle \rightarrow |10\rangle|0\rangle = |100\rangle$

$|101\rangle \rightarrow |10\rangle|1\rangle$

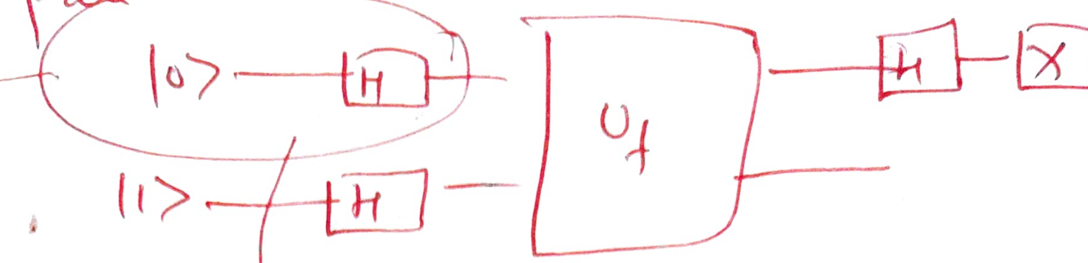
$|110\rangle \rightarrow |110\rangle$

$|111\rangle \rightarrow |111\rangle$

Input →

	000	001	010	011	100	101	110	111
000		1						
001	1							
010				1				
011			1					
100					1			
101						1		
110							1	
111								1

Hadamard Gate



qubit

Superposition H

$$|0\rangle \text{---} \boxed{H} \text{---} \frac{|0\rangle+|1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|0\rangle \text{---} \boxed{H} \text{---} \frac{|0\rangle+|1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} (-1)^{0\wedge 0} & (-1)^{0\wedge 1} \\ (-1)^{1\wedge 0} & (-1)^{1\wedge 1} \end{bmatrix}$$

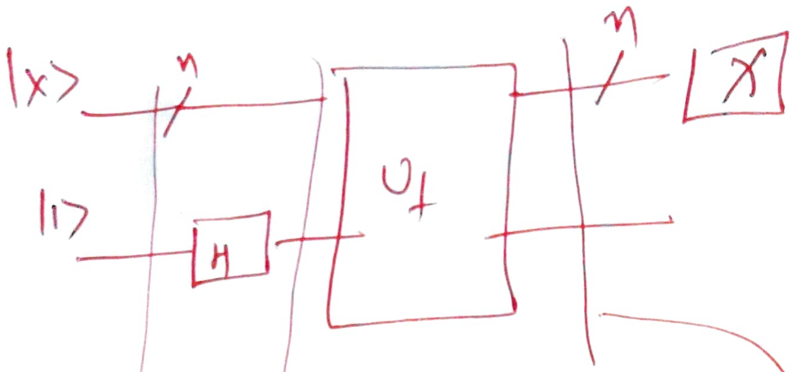
$$H^{\otimes 2} = \frac{1}{\sqrt{2}}\begin{bmatrix} (-1)^{0\wedge 0} & (-1)^{0\wedge 1} \\ (-1)^{1\wedge 0} & (-1)^{1\wedge 1} \end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix} (-1)^{0\wedge 0} & (-1)^{0\wedge 1} \\ (-1)^{1\wedge 0} & (-1)^{1\wedge 1} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} (-1)^{0\wedge 0} * (-1)^{0\wedge 0} & (-1)^{0\wedge 0} * (-1)^{0\wedge 1} & (-1)^{0\wedge 1} * (-1)^{0\wedge 0} & (-1)^{0\wedge 1} + (-1)^{0\wedge 1} \\ (-1)^{0\wedge 0} * (-1)^{1\wedge 0} & (-1)^{0\wedge 0} * (-1)^{1\wedge 1} & (-1)^{0\wedge 1} * (-1)^{1\wedge 0} & (-1)^{0\wedge 1} + (-1)^{1\wedge 1} \\ (-1)^{1\wedge 0} * (-1)^{0\wedge 0} & (-1)^{1\wedge 0} * (-1)^{0\wedge 1} & (-1)^{1\wedge 1} * (-1)^{0\wedge 0} & (-1)^{1\wedge 1} + (-1)^{0\wedge 1} \\ (-1)^{1\wedge 0} * (-1)^{1\wedge 0} & (-1)^{1\wedge 0} * (-1)^{1\wedge 1} & (-1)^{1\wedge 1} * (-1)^{1\wedge 0} & (-1)^{1\wedge 1} + (-1)^{1\wedge 1} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} (-1)^{0\wedge 0 \oplus 0\wedge 0} & (-1)^{0 \oplus 1} & & \\ (-1)^{0\wedge 0 \oplus 1\wedge 0} & & (-1)^{1\oplus 0} & \\ (-1)^{1\wedge 0 \oplus 0\wedge 0} & & (-1)^{1\oplus 0} & (-1)^{1\oplus 1} \\ (-1)^{1\wedge 0 \oplus 1\wedge 0} & & & \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

P-4



$|x,1\rangle \qquad |\cancel{x,} \, |0\rangle$

$$|x\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$

$$|x\rangle\left[\frac{|0\oplus f(x)\rangle-|1\oplus f(x)\rangle}{\sqrt{2}}\right]$$

$$= |x\rangle\left[\frac{|\cancel{0\oplus} f(x)\rangle-|\overline{f(x)}\rangle}{\sqrt{2}}\right]$$

$$(-1)^{f(x)}\,|x\rangle\left[\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right]$$