1 (Partial Measurement) (Feb 6, 2024) (1)

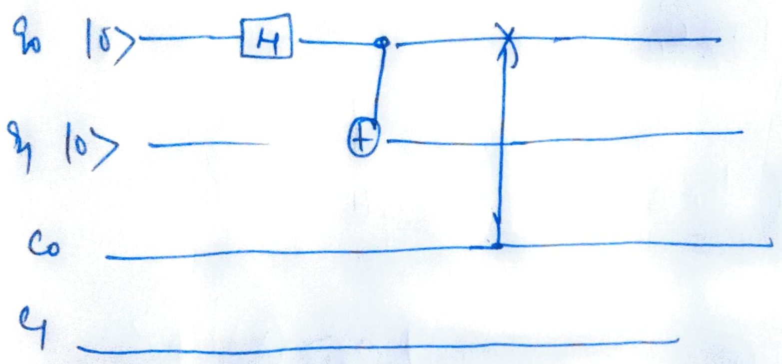
Quantum System with n qubits

$\downarrow$

If we try Measure n-1 qubits or n-2 qubits

---- or 1 qubit

$\rightarrow$ Partial Measurement



Example of Partial Measurement

$$|\psi\rangle = \frac{1}{2}\underbrace{|00\rangle}_{\text{first qubit}} - \frac{i}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle = \begin{array}{c|c} \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{i}{2} \\ \frac{1}{\sqrt{2}} \end{bmatrix} & \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} \end{array}$$

$$\| |\psi\rangle \| = \sqrt{\langle\psi|\psi\rangle} =$$

$$= \sqrt{\begin{bmatrix} \frac{1}{2} & 0 & \frac{i}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{i}{2} \\ \frac{1}{\sqrt{2}} \end{bmatrix}}$$

$$\langle\psi| = \begin{bmatrix} \frac{1}{2} & 0 & \frac{i}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 - \underbrace{\left(\frac{i}{2}\right)^2}_{\frac{i^2}{4}} + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{2}} = \sqrt{1}$$

$$= 1$$

②

We measure & output is 0

what will be the status of second qubit

$$\frac{\frac{1}{2}|00\rangle}{\sqrt{\langle v|v\rangle}}$$

$v = \frac{1}{2}|00\rangle = \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\sqrt{\langle v|v\rangle} = \sqrt{\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}}$

$= \sqrt{\frac{1}{4}} = \frac{1}{2}$

$$= \frac{\frac{1}{2}|00\rangle}{\frac{1}{2}} = |00\rangle$$

$$= |0\rangle \otimes |0\rangle$$

outcome of 1st qubit

2nd qubit will be in $|0\rangle$

$$|\psi\rangle = \frac{1}{2}|00\rangle - \frac{i}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$|\psi_1\rangle = -\frac{i}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$

$= \begin{bmatrix} 0 \\ 0 \\ -\frac{i}{2} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$\langle \psi_1| = \begin{bmatrix} 0 & 0 & \frac{i}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$$\frac{-\frac{i}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle}{\underbrace{\sqrt{\begin{bmatrix} 0 & 0 & \frac{i}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -\frac{i}{2} \\ \frac{1}{\sqrt{2}} \end{bmatrix}}}}$$

$$\sqrt{-\frac{i^2}{4} + \frac{1}{2}} = \sqrt{\frac{1}{4} + \frac{1}{2}} = \sqrt{\frac{1+2}{4}} = \frac{\sqrt{3}}{2}$$

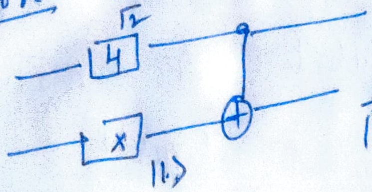
$$= \frac{-\frac{i}{2}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle}{\sqrt{3}/2} = -\frac{i}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}|10\rangle + \frac{1}{\sqrt{2}} \cdot \frac{2}{\sqrt{3}}|11\rangle$$

$$= -\frac{i}{\sqrt{3}}|10\rangle + \sqrt{\frac{2}{3}}|11\rangle$$

$$= |1\rangle \otimes \underbrace{\left(-\frac{i}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle\right)}$$

Exe. 3 $\frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle \right)$ Bell pair

Suppose $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$



Alice, Bob

$\frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle \right)$

Measure first qubit of Output is 0

$\frac{1}{\sqrt{2}}\left(\underbrace{|0_A 1_B\rangle} + |1_A 0_B\rangle \right)$

$\hookrightarrow \cancel{\frac{1}{\sqrt{2}}} \left(|0_A 1_B\rangle \right) = |0_A 1_B\rangle$

length of $\cancel{\frac{1}{\sqrt{2}}} \rightarrow$

$= |0_A\rangle \otimes |1_B\rangle$

$\frac{1}{5}$ $\frac{2}{5}$

Exe. 3

$|\psi\rangle = \frac{1}{\sqrt{5}} |0000\rangle - \sqrt{\frac{2}{5}} |0100\rangle + \sqrt{\frac{1}{5}} |0110\rangle + \sqrt{\frac{1}{5}} |1111\rangle$

$\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{4} |0000\rangle - \sqrt{\frac{2}{5}} |0100\rangle + \sqrt{\frac{1}{5}} |0110\rangle$

$\frac{4}{5} \longrightarrow$ length

$\Rightarrow \frac{\sqrt{5}}{4} |0000\rangle - \frac{\sqrt{5}}{2\sqrt{2}} |0100\rangle + \frac{1}{4} |0110\rangle$

$= |0\rangle \left(\frac{\sqrt{5}}{4} |00\rangle - \frac{\sqrt{5}}{2\sqrt{2}} |10\rangle + \frac{1}{4} |11\rangle \right) |0\rangle$

$\frac{5}{16} + \frac{5}{8} + \frac{1}{16} = \frac{5+10+1}{16} = 1$

(Entangled or Separable states)

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ $\quad\quad$ $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \_\_\_ \otimes |\psi_n\rangle$ Separable.

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} = \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix} \otimes \begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix}$$

$$\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\0\\\frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\alpha_1\alpha_2\\ \alpha_1\beta_2\\ \beta_1\alpha_2\\ \beta_1\beta_2\end{pmatrix}$$

$\Rightarrow \underbrace{\alpha_1\beta_2 = 0}$ $\quad\quad$ $\beta_1\alpha_2 = 0$

$\alpha_1 = 0$ or $\beta_2 = 0$ $\quad\quad$ $\beta_1 = 0$ or $\alpha_2 = 0$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ } entangled state

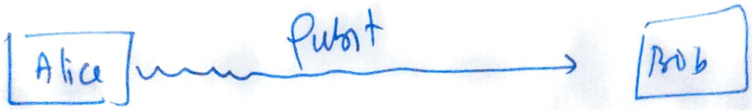
$$\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle$$
$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix} = \begin{pmatrix}\alpha_1\\ \beta_1\end{pmatrix} \otimes \begin{pmatrix}\alpha_2\\ \beta_2\end{pmatrix}$$

$\underbrace{\alpha_1\alpha_2 = 0}$. $\quad\quad$ $\beta_1\beta_2 = 0$ $\quad\quad$ $= \begin{pmatrix}\alpha_1\alpha_2\\ \alpha_1\beta_2\\ \beta_1\alpha_2\\ \beta_1\beta_2\end{pmatrix}$

$\alpha_1 = 0$ or $\alpha_2 = 0$ $\quad\quad$ $\beta_1 = 0$ $\beta_2 = 0$

(Super dense Coding)



Alice

$00 \longrightarrow I$

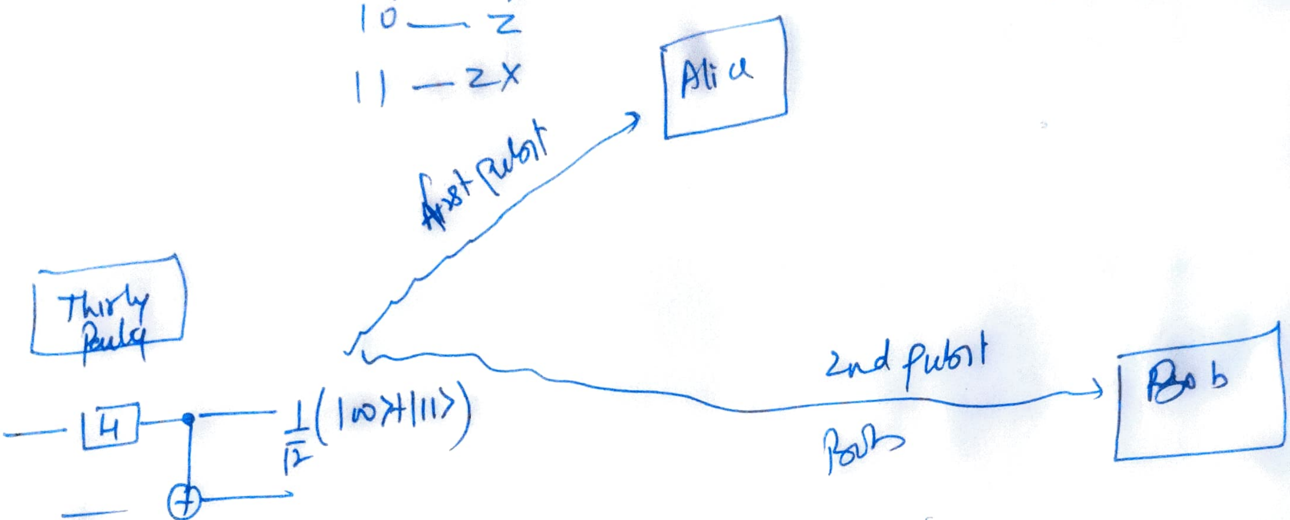
$01 \longrightarrow X$

$10 \longrightarrow Z$

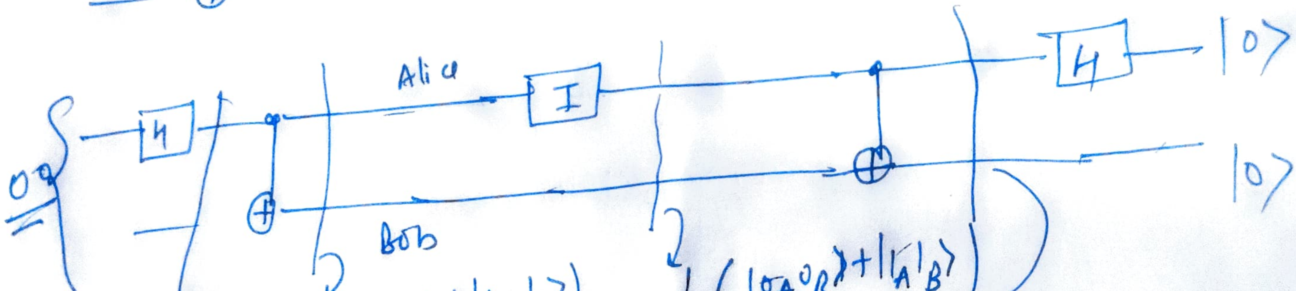
$11 \longrightarrow ZX$

Bob

Alice want to Send



$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$



$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

$\frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle)$

$\frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle)$

$\Downarrow$

$= \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 0_B\rangle)$