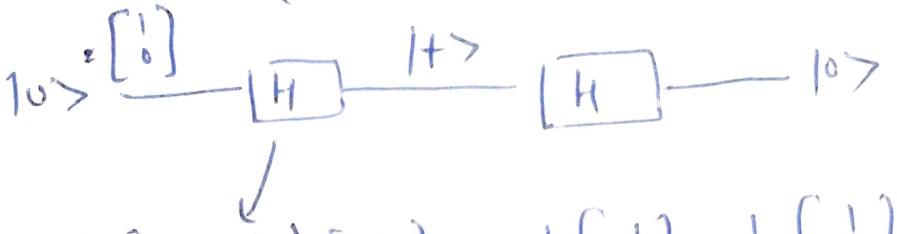
(Lecture - 3)

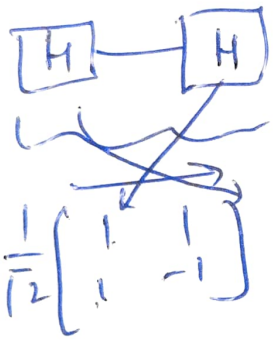


$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}$$

$$= |+\rangle$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\\1-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = |0\rangle$$

It indicate that Hadamard gate is Reversible in Nature.

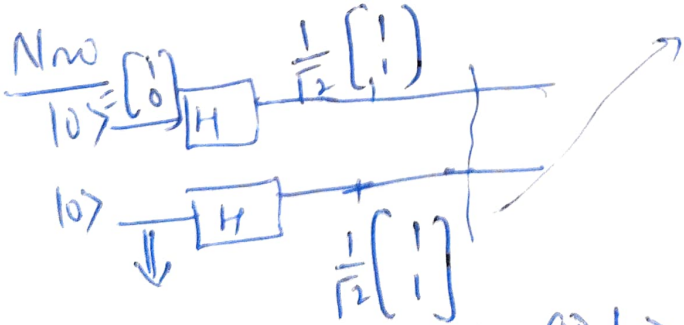


$$= I$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I$$

Now $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$

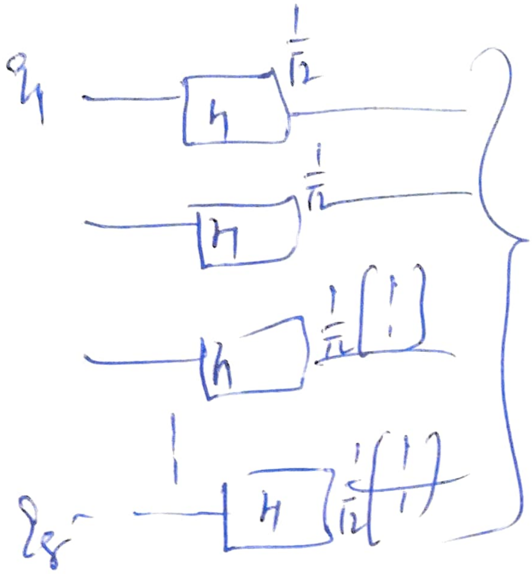


$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$|0\rangle|0\rangle = |00\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$



$|00000000\rangle$

$|$

$|11111111\rangle$

$$\left(\frac{1}{\sqrt{2}}\right)^8 \Rightarrow \left(\frac{1}{2}\right)^4 \Big(|00000000\rangle + |00000001\rangle + |00000010\rangle + |00000011\rangle + \_\_\_\_ \quad \_\_\_\_ + |11111111\rangle \Big)$$

RAND (3 , 0–10 , 100)

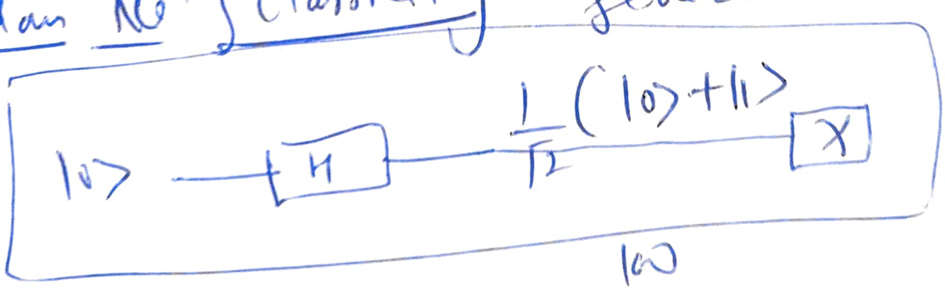
3 ⇓ initial seed

0–10 ⇓ Range

100 ⇓ How many No. u want to generate

Pseudo Random No. } classically

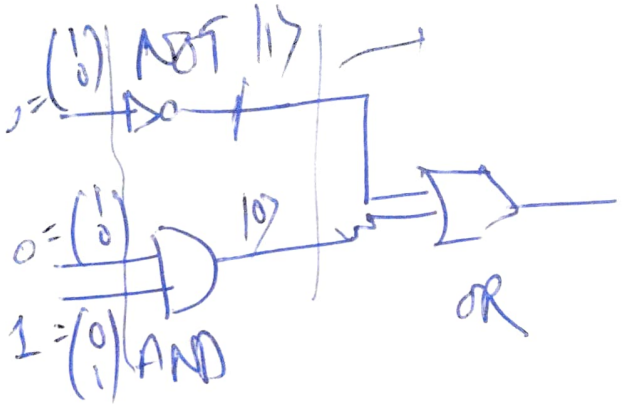
In Quantum

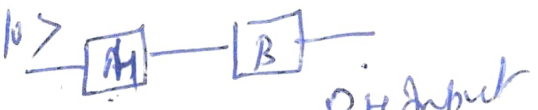


100

Random

0 1 0 0 1 0 0 1 ________





BA |0>
B H |0>
B |+>

OR * (NOT ⊗ AND) Input

$$\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)$$

NOT

|0>	|1>
|1>	|0>

	|0>	|1>
|0>	0	1
|1>	1	0

$$\begin{array}{c} \begin{matrix} 00 & 01 & 10 & 11 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \end{array} * \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2×4 · 4×8

00 → 0
01 → 0
10 → 0
11 → 1

$$\begin{array}{c} \begin{matrix} 00 & 01 & 10 & 11 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix}\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

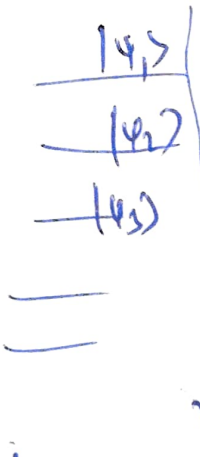
2×8 · 8×1 = 2×1

001 = $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ → 001 (8×1)

Tensor

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a\begin{pmatrix} e & f \\ g & h \end{pmatrix} & b\begin{pmatrix} e & f \\ g & h \end{pmatrix} \\ c\begin{pmatrix} e & f \\ g & h \end{pmatrix} & d\begin{pmatrix} e & f \\ g & h \end{pmatrix} \end{pmatrix}$$



$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle$$

CNOT gate
(Controlled NOT gate)

Controlled bit
Target bit

2 qubit gate

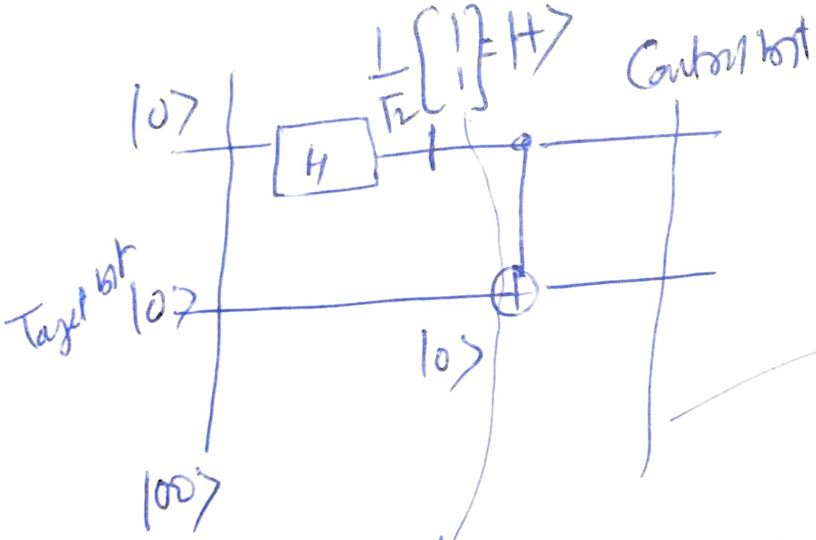
Control bit

Matrix Notation

	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
$\|00\rangle$	1	0	0	0
$\|01\rangle$	0	1	0	0
$\|10\rangle$	0	0	0	1
$\|11\rangle$	0	0	1	0

input		output	
Control bit	Target bit	Control bit	Target bit
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

$|00\rangle \longrightarrow |00\rangle$
$|01\rangle \longrightarrow |01\rangle$
$|10\rangle \longrightarrow |11\rangle$
$|11\rangle \longrightarrow |10\rangle$



$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix} \begin{matrix}00\\01\\10\\ \end{matrix}$$

$$= \frac{1}{\sqrt{2}}\left[|00\rangle + |10\rangle\right]$$

Control bit, Target bit

⇓ CNOT

$$\frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$$

$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$

$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

$$\begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \begin{matrix}00\\01\\10\\11\end{matrix}$$