

2) a)  $T(n) = T(n-1) + C$

$$T(n-1) = T(n-1-1) + C$$

$$T(n-1) = T(n-2) + C$$

$$T(n) = T(n-2) + 2C \quad - (2)$$

$$T(n-2) = T(n-2-1) + C$$

$$= T(n-3) + C$$

$$T(n) = T(n-3) + 3C \quad - (3)$$

$$T(n) = T(n-k) + kC$$

$$= T(n-(n-1)) + (n-1) \cdot C$$

$$= 1 + (n-1)C$$

$$O(n)$$

b)  $T(n) = 2T(n/2) + n \quad - (1)$

$$T(n/2) = 2T(n/4) + \frac{n}{2}$$

$$T(n/4) = 2T(n/8) + \frac{n}{4}$$

$$T(n) = 2 \left[ 2T\left(\frac{n}{4}\right) + \frac{n}{2} \right] + n$$

$$= 2^2 T\left(\frac{n}{4}\right) + \frac{2n}{2} + n$$

$$= 2^2 T\left(\frac{n}{4}\right) + 2n \quad \text{or} \quad 2^2 T\left(\frac{n}{2^2}\right) + 2n$$

Subst  $\frac{n}{4}$  in this

$$= 2^2 T\left(\frac{n}{4}\right)$$

$$= 2^2 \left[ 2T\left(\frac{n}{16}\right) + \frac{n}{4} \right] + 2n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

$$2^4 T\left(\frac{n}{2^4}\right) + 4n$$

$\vdots$  k times

$$2^k T\left(\frac{n}{2^k}\right) + kn$$

$$2^k T(1) + kn$$

$$n \cdot 1 + n \log n$$

$$O(n \log n)$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

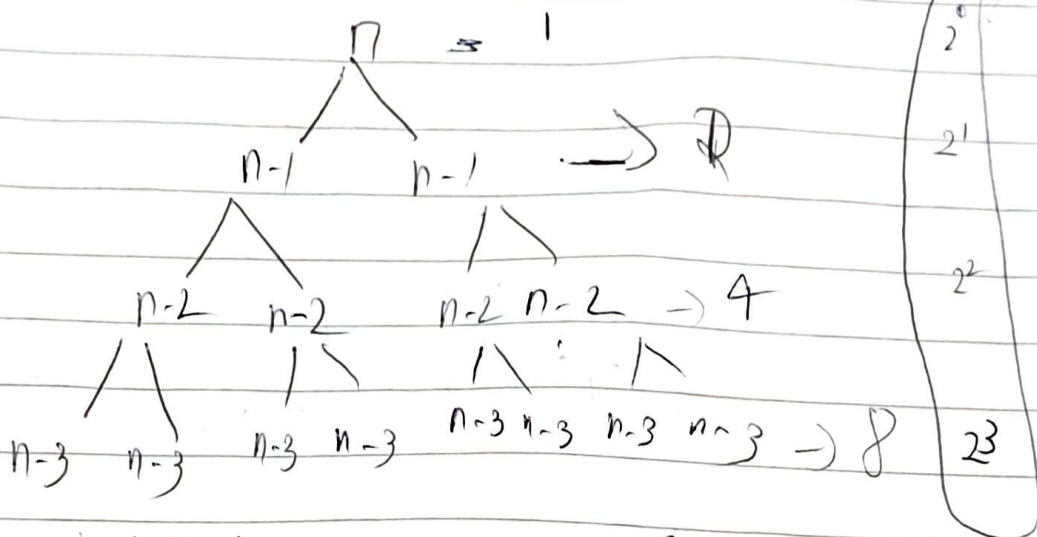
$$\log n = \log 2^k$$

$$\log n = R \times 1$$

$$R = \log n$$

$$3) T(n) = 2T(n-1) + 1$$

using Recursive tree approach



Left Side

$$n-k-1$$

$$k=n$$

Right

$$k=n$$

$$2^0 + 2^1 + 2^2 + \dots + 2^k$$

$$n=2$$

$$\text{Sum} = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1(2^n - 1)}{1}$$

$$= 2^n$$

$$O(2^n)$$

$$1) \quad T(2) = 3T(1) + 12 \times 2 - (1) \quad T(1) = 3T(0) + 12 \times 1 - (2)$$

$$T(0) = 5$$

Substitute this in (2)

$$T(1) = 3 \times 5 + 12 \\ = 27$$

Subst this in (1)

$$T(2) = 3 \times 27 + 24$$

$$\boxed{T(2) = 105}$$