University of Warwick
Master of Science in Business Analytics

IB9190: Advanced Analytics: Models and Applications
2024-2025

Group Assignment Question and Guidance

Introduction

Given a dataset of N data points $\boldsymbol{x}_i \in \mathbb{R}^d$ which are labelled by $y_i \in \{-1,+1\}$, $i = 1, \ldots, N$, support vector machine (SVM) aims to find a separating hyperplane (H): $\boldsymbol{w}^T\boldsymbol{x} + b = 0$ with $\boldsymbol{w} \in \mathbb{R}^d$ and $b \in \mathbb{R}$, where $\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} = \sum_{j=1}^{d} w_j x_j$, that separates $\boldsymbol{x}_i$ into two classes defined by $y_i, i = 1, \ldots, N$. More specifically, if $y_i = +1$, $\boldsymbol{w}^T\boldsymbol{x}_i + b \geq 1$, and if $y_i = -1$, $\boldsymbol{w}^T\boldsymbol{x}_i + b \leq -1$ for all $i = 1, \ldots, N$. The best separating hyperplane (H) is the one with the maximum distance between two related parallel hyperplanes (H^+): $\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} + b = 1$ and (H^-): $\boldsymbol{w}^T\boldsymbol{x} + b = -1$.

Questions

1. (10%) Given that the distance between (H^+) and (H^-) is $\frac{2}{||\boldsymbol{w}||}$, where $\left||\boldsymbol{w}|\right| = \sqrt{\sum_{j=1}^{d} w_j^2}$, the Euclidean norm of $\boldsymbol{w}$, show that one can use the following optimisation problem to find the best SVM supporting hyperplane

$$\begin{aligned} \min_{\boldsymbol{w},b} \quad & \frac{1}{2}\left||\boldsymbol{w}|\right|^2 \\ \text{s.t.} \quad & y_i(\boldsymbol{w}^T\boldsymbol{x}_i + b) \geq 1, \qquad i = 1, \ldots, N. \end{aligned}$$

Explain in detail the decision variables, objective, and constraints.

2. (30%) This is a non-linear constrained optimisation problem and one can use the feasible direction method to solve it numerically.

The feasible direction method can be applied to the general constrained optimisation problem $\min_{z \in \mathcal{Z}} f(\boldsymbol{z})$, where $\boldsymbol{z} \in \mathbb{R}^n$ are decision variables and $\mathcal{Z} \subset \mathbb{R}^n$ is a bounded feasible set. It is an iterative algorithm, which is implemented in several iterations. Given a feasible solution $\boldsymbol{z}_k \in \mathcal{Z}$ at iteration k, the algorithm will update solution $\boldsymbol{z}_{k+1} \in \mathcal{Z}$ for iteration $k+1$ as $\boldsymbol{z}_{k+1} = \boldsymbol{z}_k + \tau_k \cdot \boldsymbol{d}_k$, where $\boldsymbol{d}_k$ is the direction and $\tau_k \in [0,1]$ is the step size. For feasible direction method, the direction can be computed as $\boldsymbol{d}_k = \boldsymbol{v}_k - \boldsymbol{z}_k$, where $\boldsymbol{v}_k$ is an optimal solution of the following optimisation problem $\min_{\boldsymbol{v} \in \mathcal{Z}} \nabla f(\boldsymbol{z}_k)^T \boldsymbol{v}$ with $\nabla f(\boldsymbol{z}) \in \mathbb{R}^n$ and $\nabla f(\boldsymbol{z})_i = \frac{\partial f(\boldsymbol{z})}{\partial z_i}$ is the partial derivative of $f(\boldsymbol{z})$ with respect to the variable z_i. The step size τ_k is an optimal solution of the following optimization problem $\min_{\tau \in [0,1]} f(\boldsymbol{z}_k + \tau \cdot \boldsymbol{d}_k)$.

In order to solve the problem $\min_{z \in \mathcal{Z}} f(\boldsymbol{z})$ with the feasible direction method, one would need to find an initial feasible solution $\boldsymbol{z}_0 \in \mathcal{Z}$ and continue to update the solution using the direction

and step size in each iteration. The algorithm can be stop when $||\boldsymbol{z}_k - \boldsymbol{z}_{k-1}|| \leq \epsilon$, where $\epsilon > 0$ is a tolerance parameter, which is normally very close to 0.
In order to guarantee the problem introduced in Q1 always has a bounded feasible set, one can impose the additional bound constraints $-M \leq w_j \leq M$ for $j = 1, \dots, d$, and $-M \leq b \leq M$, for a large enough parameter $M > 0$. Write down the steps of the feasible direction method for the optimization problem introduced in Q1 with these additional bound constraints including how to find the initial feasible solution, the optimization problem used to find the direction in each iteration, and the formulation to compute the step size in each iteration.

Hint: you would need to map the correct decision variables $\boldsymbol{z}$, the objective function $f(\boldsymbol{z})$ to be able to compute relevant partial derivatives to formulate the optimisation problem to find the direction, identify the feasible set $\mathcal{Z}$ to figure out to find a feasible solution, and finally, solve a univariate optimization problem to compute the step size.

3. (30%) Implement the feasible direction method in Python to solve the SVM problem presented in Q1 with additional bound constraints. Explain important implementation details based on results obtained from Q2 and all required inputs.
You are now given a dataset *Data.csv* of 3 classes of iris plants: *iris setosa*, *iris versicolor*, and *iris virginica* with 4 attributes: sepal length and width, and petal length and width (https://archive.ics.uci.edu/ml/datasets/iris). Use the implemented code to find a classifier which can identify whether a record belongs to the class of *iris setosa* or not. Explain in detail how you set the tolerance parameter and how it affects the running of the algorithm as well as the final solution for a given value of M. How do you set the value of M to make sure the solution obtained from the optimisation problem with additional bounded constraints in Q2 is indeed the solution of the original problem in Q1?

4. (30%) Instead of *iris setosa*, can you find a classifier which can exactly identify whether a record belongs to the class of *iris versicolor* using the given SVM optimisation problem? Show that in general the following modified optimisation problem can always be used to find an appropriate SVM separating hyperplane:

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\begin{aligned}
\min_{\boldsymbol{w},b,\boldsymbol{u}} \quad & \frac{1}{2}||\boldsymbol{w}||^2 + C \cdot \sum_{i=1}^{N} u_i \\
\text{s.t.} \quad & y_i(\boldsymbol{w}^T\boldsymbol{x}_i + b) \geq 1 - u_i, \qquad i = 1, \dots, N, \\
& u_i \geq 0, \qquad i = 1, \dots, N,
\end{aligned}
$$

where $C > 0$ is a parameter. Explain in detail the additional decision variables, the new component of the objective function, and the modified constraints. In general, how would you set the value for parameter C?

If you are given a new iris record, propose an algorithm using the modified optimisation problem to classify it into one of the three classes, *iris setosa*, *iris versicolor*, and *iris virginica*. Explain in detail the motivation behind the proposed algorithm.

Submission Format:	**The report needs to be typed with the maximum of 4 pages and a 2000-word limit. All formulations and notation need to be explained properly. The Python scripts (if needed) with proper comments need to be submitted together with the report on *my.wbs*.** **One member of each group should make the submission on behalf of the whole group.**
Assessment Weighting:	**20%**
Submission Deadline:	**Tuesday 25th February 2025 before 12:00:00 (UK time)**
Marks Released by:	**Tuesday 25th March 2025** (we will aim to release marks by this date, but in the event of an unavoidable delay a message will be posted on the module page)
Artificial Intelligence:	**PERMITTED**

<u>Use of Artificial Intelligence</u>

The University recognises an increasing number of technologies such as Artificial Intelligence and that they may be applicable in your completing this assessment. The assessment brief sets out specific requirements or restrictions, and your student handbook has further guidance and advice.

You are reminded that the inappropriate use of such a technology may constitute a breach of University policy, such as the Proofreading Policy or Regulation 11 (Academic Integrity). If you breach these policies, it may have significant consequences for your studies. Please make sure you read and understand the assessment brief and how AI may or may not be used.

When you submit the assessment, you will be required to explain the use of any AI. Failure to disclose at the point of submission may be prejudicial in any later investigations should they arise.

For this assessment, AI is:
<u>PERMITTED (neither prohibited or required)</u>
If you use a generative Artificial Intelligence (AI) in the process of completing this assessment you MUST set out clearly the following:

- WHY you used a generative AI
- WHAT it was used for
- WHICH AI was used; and
- If any generated content has been used directly in this submission, if so where.

Note that this declaration does NOT contribute towards the word count for the assessment.

You will also have to confirm in your declaration that the work remains yours and you have intellectual ownership of it. You may be called for viva or other interview to demonstrate such intellectual ownership. A failure to disclose the use of AI, or the use of a misleading description of its use may have significant consequences for your studies. As a result, keeping good records of your interactions is strongly advised.

Submitting your work

Before you submit your assessment, you should ensure you are familiar with the guidance and rules in the "Your Assessments" section of your Student Handbook, paying particular attention to:

- Academic Integrity (including Plagiarism)
- Referencing
- Word Count Policy and Formatting
- Confidentiality of your work
- Extensions and late submission of work
- Guidelines for Online Submission

Mitigating Circumstances

Mitigating circumstances MUST be submitted **within 20 working days following the submission deadline**. Mitigating circumstances not submitted by the relevant deadline are not required to be considered by the School/Department and may have to be considered by an Academic Appeals Committee as part of an academic appeal – for further information, please see: https://warwick.ac.uk/services/gov/calendar/section2/regulations/reg42academicappeals

Please see your Student Handbook for more guidance on mitigating circumstances

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