UNIT - II

2 Electrostatics

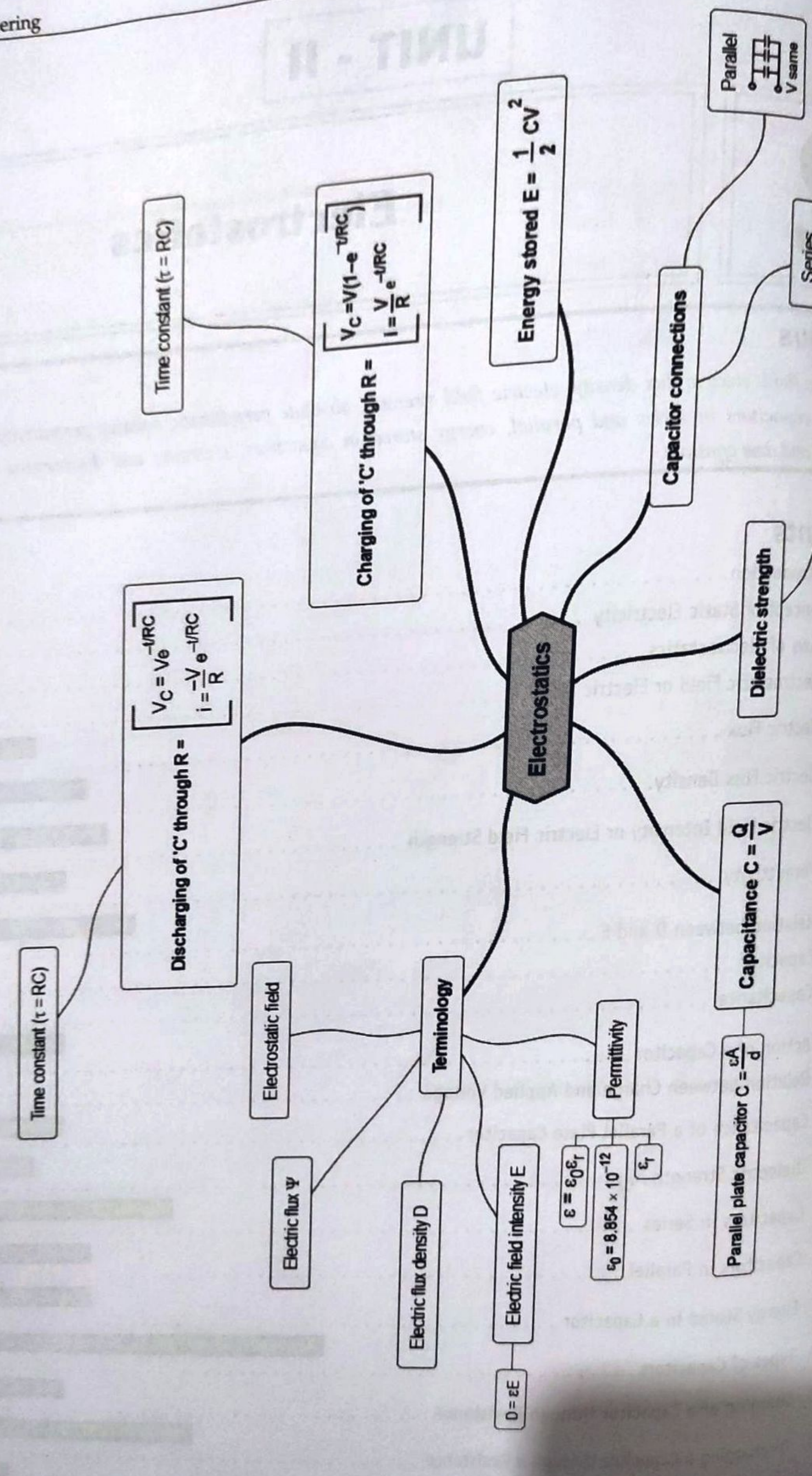
Syllabus

Electrostatic field, electric flux density, electric field strength, absolute permittivity, relative permittivity and capacitance. Capacitor, capacitors in series and parallel, energy stored in capacitors, charging and discharging of capacitors (no derivation) and time constant.

Contents

2.1 Introduction . . . . 2 - 3
2.2 Concept of Static Electricity . . . . 2 - 3
2.3 Laws of Electrostatics . . . . 2 - 3
2.4 Electrostatic Field or Electric Field . . . . 2 - 4
2.5 Electric Flux . . . . 2 - 5 (Dec.-03, 05, Marks 2)
2.6 Electric Flux Density . . . . 2 - 5 (May-03, Dec.-05,07, Marks 2)
2.7 Electric Field Intensity or Electric Field Strength . . . . 2 - 5 (Dec.-05,07,08, May-11, Marks 2)
2.8 Permittivity . . . . 2 - 6 (May-06, Dec.-07, Marks 2)
2.9 Relation between D and E . . . . 2 - 6 (Dec.-03,05, 07,08, May-11, Marks 2)
2.10 Capacitor . . . . 2 - 6
2.11 Capacitance . . . . 2 - 7 (Dec.-08, May-11, Marks 2)
2.12 Action of a Capacitor . . . . 2 - 7
2.13 Relation between Charge and Applied Voltage . . . . 2 - 8 (Dec.-08, May-11, Marks 2)
2.14 Capacitance of a Parallel Plate Capacitor . . . . 2 - 8 (Dec.-06, 11, Marks 8)
2.15 Dielectric Strength . . . . 2 - 10 (May-06, 08, 11, Dec.-06, 08, 09, Marks 5)
2.16 Capacitors in Series . . . . 2 - 10 (Dec.-05, 18, May-07, Marks 6)
2.17 Capacitors in Parallel . . . . 2 - 11 (May-12, Dec.-08, 10, Marks 6)
2.18 Energy Stored in a Capacitor . . . . 2 - 14 (May-06, 07, 08, 09, 10, 11, Dec.-05,08,10, 12, 14, 17, Marks 8)
2.19 Types of Capacitors . . . . 2 - 15 (Dec.-05, May-03, Marks 5)
2.20 Charging of a Capacitor through Resistance . . . . 2 - 15 (May-04,10,12,17, Dec.-04,07,09, 10, Marks 6)
2.21 Discharging a Capacitor through a Resistance . . . . 2 - 17 (May-06, Marks 6)

Mind Map - Electrostatics



2.1 : Introduction

- The branch of electrical engineering which deals with electricity at rest is called **electrostatics.**
- All the electric phenomena are produced due to the various types of charges. The moving charges produce current and magnetic effects. The accelerated charges produce radiation.
- Apart from moving and accelerated charges, there exists one more type of charge called **stationary charge** or **static charge.** Static charges are responsible for the generation of the forces on other charges which are called **electrostatic forces.** Electrostatics means the study of the static charges and the associated effects.

The static charges are classified as,

1. Point charge 2. Line charge
3. Surface charge 4. Volume charge

- The static charges may be situated at a point when they are called **point charges.** When the static charges are distributed along the telephone lines or power lines, they are called **line charges.** When distributed over the surfaces such as surfaces of plates of capacitor, they are called **surface charges.** Static charges may exist in the entire volume in the form of a charged cloud then they are called **volume charges.**

2.2 : Concept of Static Electricity

- It can be observed that when an ebonite rod is rubbed on a fur cloth, then the rod extracts electrons from fur cloth and behaves as negatively charged while fur cloth behaves as positively charged as it looses negatively charged electrons.
- This charged condition can not be sensed by eyes or by sense organs.
- But if such a charged ebonite rod is brought near light pieces of paper, it attracts these paper pieces as it is charged.
- This effect is due to static charge present on the rod. This is basic principle of static electricity.
- When two dissimilar metals are rubbed against each other then an electric charge gets developed on these two bodies. This charge is called static charge.

The magnitude and the nature of charge depends on,

1. Characteristics of two bodies.
2. Atmospheric conditions around the bodies.

- The phenomenon due to static charges is governed by the laws called laws of electrostatics.

2.3 : Laws of Electrostatics

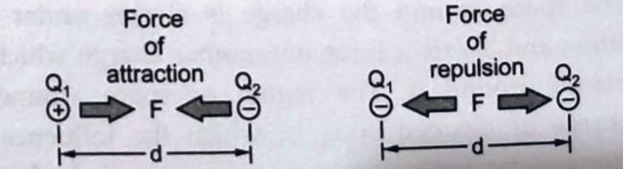
The two fundamental laws of electrostatics are as below :

1) Like charges repel each other and unlike charges attract each other.

- The law can be demonstrated by another simple experiment. The ebonite rod becomes negatively charged when rubbed against fur cloth. Now, if glass rod is rubbed against fur cloth, it gets positively charged. And if they are brought near each other, they try to attract each other. While two ebonite rods after rubbing against fur cloth, brought nearby, try to repel each other. This shows that like charges repel while unlike charges attract each other.

2) Coulomb's Inverse Square Law.

- The law states that the mechanical force, attraction or repulsion, between the two small charged bodies is
  - i) directly proportional to the product of the charges present on the bodies.
  - ii) inversely proportional to the square of the distance between the bodies and
  - iii) depends upon the nature of the medium surrounding the bodies.
- The Fig. 2.3.1 shows two point charges, separate by distance 'd' metres. The charges are Q_1 and C coulombs and K is the constant of proportionality.



(a) Unlike charges attract (b) Like charges repel

Fig. 2.3.1 Force between charges

- According to Coulomb's law, force between the charges can be mathematically expressed as,

$$F \propto \frac{Q_1 Q_2}{d^2}$$

So,
$$\boxed{F = \frac{K Q_1 Q_2}{d^2} \quad \text{Newtons}}$$

The constant of proportionality, K depends on the surrounding medium and is given by,

$$\boxed{K = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_r \varepsilon_0}}$$

where ε = Absolute permittivity of the medium = $\varepsilon_0 \varepsilon_r$

ε_0 = Permittivity of free space and ε_r = Relative permittivity of the medium

And
$$\boxed{\varepsilon_0 = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12} \quad \text{F/m}}$$

For air, $\varepsilon_r = 1$

The concept of permittivity is discussed later in this chapter.

If $Q_1 = Q_2 = 1\ \text{C}$ and $d = 1\ \text{m}$,

then,
$$F = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 9 \times 10^9 \ \text{N}$$

Key Point *Thus, one coulomb of charge may be defined as that charge, which, when placed in the air or vacuum at a distance of one metre away from an equal and similar charge, is repelled by a force of 9×10^9 N.*

Expected Question

1. *State and explain the laws of electrostatics.*

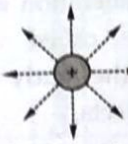
2.4 : Electrostatic Field or Electric Field

SPPU : Dec.-03, 05

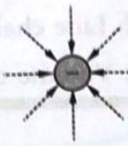
- The space around the charge is always under the stress and exerts a force on another charge which is placed around it. The region or space around a charge or charged body in which the influence of electrostatic force or stress exists is called **electric field** or **dielectric field** or **electrostatic field.**

2.4.1 Electric Lines of Force

- The electric field around a charge is imagined in terms of presence of lines of force around it. The imaginary lines, distributed around a charge, representing the stress of the charge around it are called **electric or electrostatic lines of force.**
- The pattern of lines of force around isolated positive charge is shown in Fig. 2.4.1 (a) while the pattern of lines of force around isolated negative charge is shown in Fig. 2.4.1 (b).



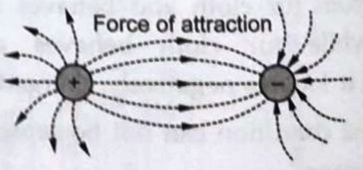
(a) Isolated positive charge



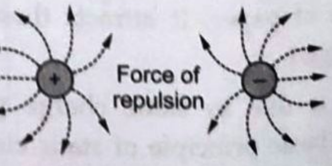
(b) Isolated negative charge

Fig. 2.4.1

- Such lines of force originate from the positive charge and terminate on the negative charge, when these charges are placed near each other.
- The positive and negative charges placed near each other exert the force of attraction on each other. This is shown in Fig. 2.4.1 (c).
- While when two like charges are near each other, such lines will be in opposite direction as shown in Fig. 2.4.1 (d). There exists a force of repulsion between them.



(c) Two equal unlike charges



(d) Two equal like charges

Fig. 2.4.1

- The properties of electric lines of force are,

1) The lines of force always originate from a positive charge and terminate at negative charge.
2) They always enter or leave a conducting surface, normally.
3) They are always parallel and never cross each other.
4) The lines travelling in the same direction repel each other, while traveling in the opposite directions attract one another.
5) They behave like a stretched rubber band and always try to contract.
6) They pass only through the insulating medium between the charges and do not enter the charged bodies.

Key Point *Hence, electric lines of force cannot form a closed loop as in case of the magnetic lines of force.*

Expected Questions

1. *Define electrostatic field.*
SPPU : Dec.-03, 05, Marks 2
2. *What are electric lines of force ? State its properties.*

2.5 : Electric Flux

SPPU : May-03, Dec.-05,07

- Theoretically, the lines of force emanating from a charge are infinite. Faraday suggested that the electric field should be assumed to be composed of very small bunches containing a fixed number of electric lines of force. Such a bunch or a closed area is called a **tube of flux.**

The total number of lines of force or tubes of flux in any particular electric field is called the ***electric flux.***

- This is represented by the symbol ψ. Similar to charge, unit of electric flux is also coulomb C.
- One coulomb o-f electric flux is defined as that flux which emanates from a positive charge of one Coulomb.

$\therefore$ Electric Flux, ψ = Q coulombs (numerically)

Expected Question

1. *Define electric flux.*
SPPU : May-03, Dec.-05,07, Marks 2

2.6 : Electric Flux Density

SPPU : Dec.-05,07,08, May-11

- Electric flux density is defined as the flux passing at right angles through unit area of surface. It is represented by symbol D and measured in Coulomb per square metre.
- If a flux of ψ Coulombs passes normally (at right angles) through an area of A m^2, then

$$\therefore \quad D = \frac{\psi}{A} = \frac{Q}{A} \ C/m^2 \quad \ldots \text{ as } \psi = Q$$

- The flux density is also called **displacement density.**

Expected Question

1. *Define electric flux density.*
SPPU : Dec.-05,07,08, May-11, Marks 2

2.7 : Electric Field Intensity or Electric Field Strength

SPPU : May-06, Dec.-07

- It is defined as the force experienced by a unit positive charge placed at any point in the electric field. It is represented by symbol E and measured in newton per coulomb.
- Suppose a charge of Q coulombs, placed at a point within an electric field, experiences a force of F newtons, then the intensity of the electric field at that point is given by,

$$\therefore \quad E = \frac{F}{Q} \quad N/C$$

- Higher the value of E, stronger is the electric field.
- It is also measured in volts per metre **(V/m)** and given by,

$$\therefore \quad E = \frac{V}{d} \quad \text{where } d = \text{distance of seperation and } V = \text{applied voltage}$$

Expected Question

1. *Define electric field intensity.*

SPPU : May-06, Dec.-07, Marks 2

2.8 : Permittivity SPPU : Dec.-03,05, 07,08, May-11

Permittivity can be defined as the ease with which a dielectric medium permits an electric flux to be established in it.

2.8.1 Absolute Permittivity

- The ratio of the electric flux density D to electric field intensity E at any point is defined as the **absolute permittivity.**
- It is denoted by ε and measured in units farads/metre, (F/m).

$$\therefore \quad \varepsilon = \frac{D}{E} \text{ F/m}$$

2.8.2 Permittivity of Free Space

The ratio of the electric flux density in a vacuum (or free space) to the corresponding electric field is defined as permittivity of the free space.

- It is denoted by ε_0 and measured in unit farads/m (F/m).

$$\therefore \quad \varepsilon_0 = \frac{D}{E} \text{ F/m in vacuum}$$

- The value of ε_0 is less than the value of permittivity of any medium.
- Experimentally, its value has been derived as,

$$\therefore \quad \varepsilon_0 = \frac{1}{36\pi\times 10^9} = 8.854\times 10^{-12} \text{ F/m}$$

2.8.3 Relative Permittivity

- To define the permittivity of the dielectric medium, the vacuum or free space is considered to be a reference medium. So, **relative permittivity** of vacuum with respect to itself is unity.
- The ratio of electric flux density in a dielectric medium to that produced in a vacuum by the same electric field intensity under identical conditions is called **relative permittivity.**
- It is denoted by ε_r and has no units.

$$\therefore \quad \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \quad \text{i.e.} \quad \varepsilon = \varepsilon_r \varepsilon_0$$

- It can also be defined as the ratio of the absolute permittivity of the dielectric medium to the permittivity of the free space.
- The relative permittivity of air is assumed to be one for all practical purposes.

Higher the value of ε_r, easier is the flow of electric flux through the materials.

*The **relative permittivity** is nothing but the **dielectric constant** of the material.*

Expected Questions

1. *What is permittivity ?*

SPPU : Dec.-05, 08, May-11, Marks 2

2. *Define absolute permittivity and state its unit.*

Dec.-03, 07, Marks 2

3. *Define permittivity of free space and state is value.*
4. *Define relative permittivity.* **Dec.-03, 07, Marks 2**

2.9 : Relation between D and E

- The electric flux density D and electric field intensity E are related to each other as,

$$D = \varepsilon E = \varepsilon_0 \varepsilon_r E \quad C/m^2$$

- The units of E are N/C but it is also measured in volts per meter (V/m).

$$E = \frac{V}{d} \text{ V/m where V = voltage across the distance d}$$

- The potential gradient is defined as the drop in potential per metre in the direction of electric field. Numerically its value is same as the electric filed intensity E.

Electric filed strength = Potential gradient (numerically) in V/m

2.10 : Capacitor

- A capacitor is nothing but the two conducting surfaces, separated by an insulating medium called **dielectric.** These conducting surfaces could be in the form of rectangular, circular, spherical or cylindrical in shape.

- The basic construction and symbol of a capacitor is shown in the Fig. 2.10.1 a and b.

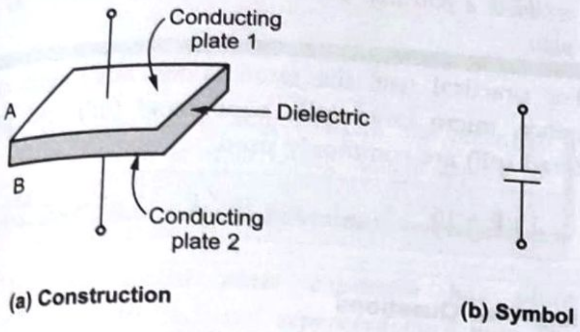


Fig. 2.10.1 Capacitor

- A capacitor is also called **condenser.** The commonly used dielectrics in capacitors are paper, mica, air etc.

Expected Question

1. *What is capacitor ? Which are the commonly used dielectrics in capacitor ?*

2.11 : Capacitance

SPPU : Dec.-08, May-11

- Capacitance is defined as the amount of charge required to create a unit potential difference between the plates.

The property of a capacitor to store an electric energy in the form of static charges is called its ***capacitance.*** *It is denoted as C.*

The capacitance is measured in farads (F).

Expected Question

1. *Define capacitance. State its unit.*

SPPU : Dec.-08, May-11, Marks 2

2.12 : Action of a Capacitor

- Consider a capacitor formed by two flat metal plates X and Y, facing each other and separated by an air gap or other insulating material used as a dielectric medium. There is no electrical contact or connection between them. Such a capacitor is called **parallel plate capacitor.**

- Consider a circuit in which such a capacitor across a battery with the help of a switch 'S' and a galvanometer 'G' in series. The arrangement is shown in the Fig. 2.12.1.

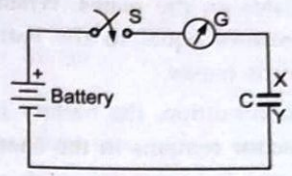


Fig. 2.12.1 A capacitor

- As soon as the switch 'S' is closed, the positive terminal of the battery attracts some of the free electrons from the plate 'X' of the capacitor.
- The electrons are then pumped from positive terminal of the battery to the negative terminal of the battery due to e.m.f. of the battery.
- Now, negative terminal and electrons are like charges and hence, electrons are repelled by the negative terminal to the plate 'Y' of the capacitor. The action is shown in Fig. 2.12.2.

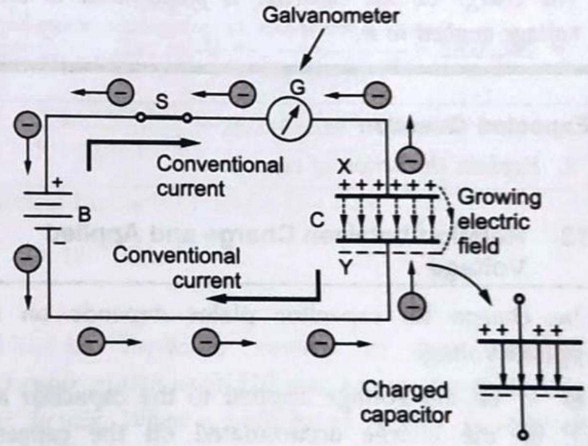


Fig. 2.12.2 Action of a capacitor

- So, plate 'X' becomes positively charged while plate 'Y' becomes negatively charged.
- The flow of electrons constitutes a current, in the direction opposite to the flow of electrons.
- This is the conventional current called **charging current** of the capacitor as shown in the Fig. 2.12.2. This can be experienced from the momentary deflection of the galvanometer 'G'. Because of this, there builds a potential difference across the plates 'X' and 'Y'. There builds an electric field between the two fields.

- But this potential difference across the plates, acts as a counter e.m.f. and starts opposing the movement of the electrons. The magnitude of this potential difference is proportional to the charge that accumulates on the plates. When this potential difference becomes equal to the battery e.m.f., the flow of electrons ceases.
- If under such condition, the battery is disconnected then the capacitor remains in the charged condition, for a long time. It stores an electrical energy and can be regarded as a reservoir of electricity.
- Now, if a conducting wire is connected across the two plates of capacitor, the electrons rush back to plate X from plate Y through the wire. So, there is a rush of current through the wire. This is called **discharging current of a capacitor.** Thus, the energy stored in the capacitor is released and is dissipated in the form of the heat energy in the resistance of the wire connected.
- The direction of the conventional current is always opposite to the flow of electrons.

The charge on the capacitor is proportional to the voltage applied to it.

Expected Question

1. *Explain the action of capacitor.*

2.13 Relation between Charge and Applied Voltage

- The charge on capacitor plates depends on the applied voltage.
- Let 'V' be the voltage applied to the capacitor and 'Q' be the charge accumulated on the capacitor plates, then mathematically, it can be written as,

$$Q \propto V \quad \text{i.e.} \quad Q = C\,V$$

- The constant of proportionality 'C' is called **capacitance** of the capacitor.

$$\therefore \quad C = \frac{Q}{V}$$

- From the above expression, the **capacitance** is defined as the ratio of charge acquired to attain the potential difference between the plates. It is the charge required per unit potential difference. It is measured in unit **farads.**

One farad capacitance is defined as the capacitance of a capacitor which requires a charge of one coulomb to establish a potential difference of one volt between its plates.

- For practical use, the farad is too large unit and hence, micro farad (μF), nano farad (nF) and pico farad (pF) are commonly used.

$$1\ \mu F = 10^{-6}\ F, \quad 1\ nF = 10^{-9}\ F, \quad 1\ pF = 10^{-12}\ F$$

Expected Questions

1. *State the relation between charge, capacitance and the applied voltage.*
2. *Define capacitance and its unit.* **SPPU : Dec.-08, May-11, Marks 2**

2.14 Capacitance of a Parallel Plate Capacitor

SPPU : Dec.-06, 11

- Consider a parallel plate capacitor, fully charged, as shown in the Fig. 2.14.1.

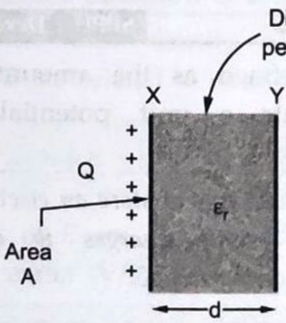


Fig. 2.14.1 Charged capacitor

- The surface area of each plate X and Y is say A m^2 and plates are separated by distance 'd'.
- The relative permittivity of the dielectric used in between is say ε_r.
- Let Q be the charge accumulated on plate X, then the flux passing through the medium is $\psi = Q$.
- The flux density,

$$D = \frac{\psi}{A} = \frac{Q}{A} \quad \text{... (1)}$$

- The electric field intensity,

$$E = \frac{V}{d} \quad \text{... (2)}$$

- We know that

$$D = \varepsilon E \quad \text{i.e.} \quad \frac{Q}{A} = \varepsilon \frac{V}{d}$$

$$\frac{Q}{V} = \frac{\varepsilon A}{d} \quad \text{But,} \quad \frac{Q}{V} = C$$

$$\therefore \quad \boxed{C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d} \quad F}$$

When the capacitor is fully charged, the potential difference across it is equal to the voltage applied to it.

Ex. 2.14.1 : *A parallel plate capacitor has plate surface area of 10 cm^2 and seperated by 4 mm. The dielectric used between the plates has relative permittivity of 5. Calculate*
i) Capacitance of parallel plate capacitor
ii) If the dielectric material is completely removed find the new value of capacitance

Sol. : Given : A = 10 cm^2 = 10×10^{-4} m^2,

d = 4 mm = 4×10^{-3} m, $\varepsilon_r = 5$ **To find : C**

i) $$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 10 \times 10^{-4} \times 5}{4 \times 10^{-3}}$$

$$= \mathbf{11.06 \ pF}$$

ii) When dielectric material is removed it will become air capacitor with $\varepsilon_r = 1$.

$$\therefore \quad C = \frac{8.854 \times 10^{-12} \times 1 \times 10 \times 10^{-4}}{4 \times 10^{-3}} = \mathbf{2.213 \ pF}$$

Ex. 2.14.2 : *The capacitance of parallel plate capacitor is 320 pF with certain dielectric material. The plate has cross section of 100 cm × 60 cm and seperated by 20 cm Calculate.*
i) relative permittivity of dielectric material.
ii) What should be distance between two plates in cm inorder to double the capacitance.

Sol. : Given : A = $100 \times 60 \times 10^{-4}$ m^2;

d = 20 cm = 0.2 m C = 320 pF,

To find : ε_r, d

i) $$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$\therefore \quad 320 \times 10^{-12} = \frac{8.854 \times 10^{-12} \times \varepsilon_r \times 100 \times 60 \times 10^{-4}}{0.20}$$

$$\varepsilon_r = \mathbf{12.047 \ pF}$$

ii) When capacitance is doubled

$$\therefore \quad 320 \times 2 \times 10^{-12} = \frac{8.854 \times 10^{-12} \times 12.047 \times 100 \times 60 \times 10^{-4}}{d}$$

$$d = \mathbf{10 \ cm}$$

OR $$C \propto \frac{1}{d}$$

Ex. 2.14.3 : *Two flat parallel plates measuring 1 m × 2 m and seperated by 10 cm are charged by transferring 10^{-6} coulombs from one plate to other. The permittivity of the oil between the plates is 2. Calculate, i) Capacitance of the parallel plates ii) Potential difference between the plates iii) Electric field intensity iv) Electric flux density between the plates.* **SPPU : Dec.-11, Marks 8**

Sol. : A = 1 × 2 = 2 m^2, d = 10 cm, ε_r = 2,

Q = 10^{-6} C

i) $$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 2 \times 2}{10 \times 10^{-2}} = \mathbf{354.16 \ pF}$$

ii) $$C = \frac{Q}{V} \quad \text{i.e.} \quad V = \frac{10^{-6}}{354.16 \times 10^{-12}} = \mathbf{2823.582 \ V}$$

iii) $$E = \frac{V}{d} = \frac{2823.582}{10 \times 10^{-2}} = \mathbf{28.235 \times 10^3 \ V/m}$$

iv) Electric flux ψ = Q hence $$D = \frac{\psi}{A} = \frac{10^{-6}}{2}$$

$$= \mathbf{5 \times 10^{-7} \ C/m^2}$$

Ex. 2.14.4 : *A capacitor consist of two parallel rectangular plates each 120 mm square separated by 1 mm in air. When a voltage of 1000 V is applied between the plates, calculate : i) The charge on the capacitor, ii) The electric flux density and iii) The electric field strength in the dielectric.* **SPPU : Dec.-06, Marks 6**

Sol. : A = 120 mm^2, d = 1 mm,

$\varepsilon = \varepsilon_0$ as air is dielectric, V = 1000 V.

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 120 \times 10^{-6}}{1 \times 10^{-3}} = \mathbf{1.0624 \times 10^{-12} \ F}$$

i) $Q = C \times V = 1.0624 \times 10^{-12} \times 1000 = \mathbf{1.0624 \times 10^{-9} \ C}$

ii) $D = \frac{Q}{A} = \frac{1.0624\times10^{-9}}{120\times10^{-6}} = \mathbf{8.854\times10^{-6}\ C/m^2}$

iii) $E = \frac{D}{\varepsilon_0} = \frac{8.854\times10^{-6}}{8.854\times10^{-12}} = 1\times10^{6}\ \text{V/m} = \mathbf{1\ MV/m}$

Expected Question

1. *Derive the expression for a cpacitance of a parallel plate capacitor.*

2.15 Dielectric Strength

SPPU : May-06, 08, 11, Dec.-06, 08, 09

- We know that, $E = \frac{V}{d}$ hence as the voltage on the capacitor is increased with a given thickness (d) or the thickness (d) is reduced with a given voltage (V), the electric intensity E increases.
- This intensity represents the force exerted on the charges on the molecules or the dielectric material.
- As E is increased, the centre of the positive charges is pushed in the direction of E and centre of the negative charges in the opposite direction.

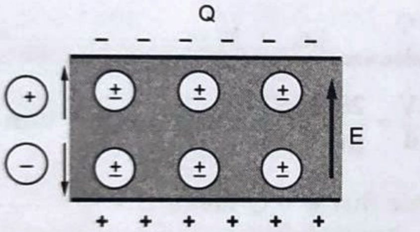


Fig. 2.15.1 Dielectric strength

- Every dielectric medium has its capacity to withstand the increasing E.
- If the applied voltage and hence E is increased beyond a certain limit, then forces on the molecules become sufficiently large. The electrons break away from the molecules causing ionization and free charges.
- The material then conducts due to ionization and the charge recombine, thereby vanish from the capacitor plates. The capacitor can no more hold the charge and is said to be breakdown. The dielectric medium is said to be punctured and becomes useless from using it as a dielectric.
- The ability of an insulating medium to resist its breakdown when a voltage is increased across it, is called its **dielectric strength.**
- This depends upon the temperature of the material and presence of air pockets and imperfections in the molecular arrangement of that material. It is generally expressed in **kV/cm of kV/mm.**

Key Point *The voltage at which the dielectric medium of the capacitor breaks down is known as* ***breakdown voltage*** *of the capacitor.*

- The factors affecting the dielectric strength are,
  1. Temperature
  2. Type of material
  3. Size, thickness and shape of the plates.
  4. Presence of air pockets in the material.
  5. Moisture content of the material.
  6. Molecular arrangement of the material.

Expected Questions

1. *Define dielectric strength.*

   SPPU : May-06, 11, Dec.-08, Marks 2

2. *What do you understand by dielectric strength and dielectric breakdown ?*

   SPPU : Dec.- 06, 09, May-08, Marks 5

2.16 Capacitors in Series

SPPU : Dec.-05, May-07

- Consider the three capacitors in series connected across the applied voltage V as shown in the Fig. 2.16.1.

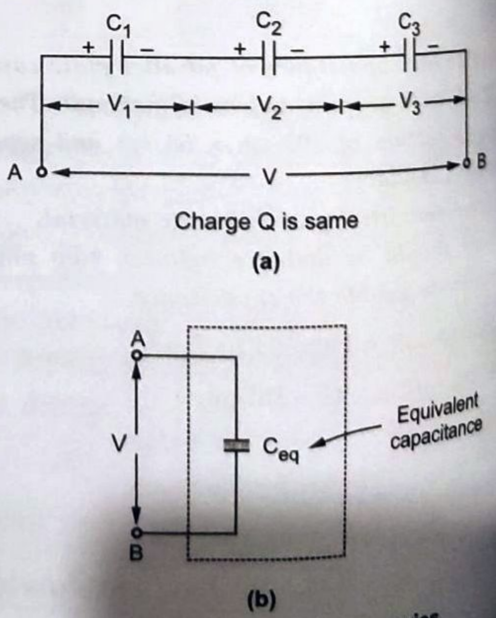


Fig. 2.16.1 Capacitors in series

- Suppose this pushes charge Q on C_1 then the opposite plate of C_1 must have the same charge. This charge which is negative must have been obtained from the connecting leads by the charge separation which means that the charge on the upper plate of C_2 is also Q. In short, **all the three capacitors have the same charge Q for the series capacitors.**

$$Q = C_1V_1 = C_2V_2 = C_3V_3$$

Giving, $$V_1 = \frac{Q}{C_1};\ V_2 = \frac{Q}{C_2};\ V_3 = \frac{Q}{C_3}$$

- If an equivalent capacitor also stores the same charge, when applied with the same voltage, then it is obvious that,

$$C_{eq} = \frac{Q}{V} \quad \text{or} \quad V = \frac{Q}{C_{eq}}$$

But, $$V = V_1 + V_2 + V_3$$

i.e. $$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \quad \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

It is easy to find V_1, V_2 and V_3 if Q is known.

For 'n' capacitors in **series,**

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

Key Points

- *For all the capacitors in series, the charge on all of them is always same, but the voltage across them is different.*
- *The smallest capacitor has the largest of the voltages across it and C_{eq} is lesser than any of the capacitors in the series string.*

For two capacitors in series,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Ex. 2.16.1 : ***Two capacitors of 8 μF and 2 μF are connected in series across 400 volt supply. Calculate (1) Resultant capacitance, (2) Charge on each capacitor and (3) p.d. across each capacitor.*** **SPPU : Dec.-05, May-07, Marks 6**

Sol. : $C_1 = 8\ \mu F$, $C_2 = 2\ \mu F$, $V = 400$ V

1) $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8\times10^{-6}\times2\times10^{-6}}{8\times10^{-6}+2\times10^{-6}} = \mathbf{1.6\ \mu F}$$

2) The charge on each capacitor is same.

$$Q = C_{eq} \times V = 1.6\times10^{-6}\times400 = \mathbf{6.4\times10^{-4}\ C}$$

3) $$Q = C_1 V_1 = C_2 V_2$$

$$\therefore \quad V_1 = \frac{Q}{C_1} = \frac{6.4\times10^{-4}}{8\times10^{-6}} = \mathbf{80\ V}$$

$$V_2 = \frac{Q}{C_2} = \frac{6.4\times10^{-4}}{2\times10^{-6}} = \mathbf{320\ V}$$

Key Point $V_1 + V_2 = V.$

Expected Question

1. *Derive the expression for an equivalent capacitance of a three capacitors connected in series.* **SPPU : Dec.-18, Marks 6**

2.17 Capacitors in Parallel

SPPU : May-12, Dec.-08, 10

- Consider three capacitors connected in parallel as shown in the Fig. 2.17.1.

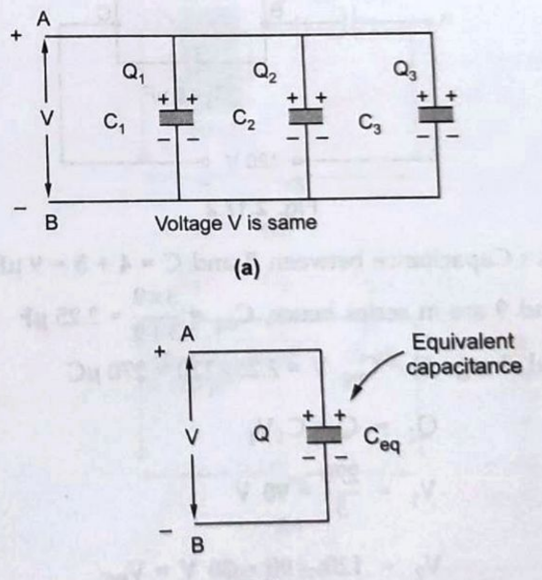


Fig. 2.17.1 Capacitors in parallel

When capacitors are in parallel, the same voltage exists across them, but charges are different.

$$\therefore \quad \boxed{Q_1 = C_1 V,\ Q_2 = C_2 V,\ Q_3 = C_3 V}$$

The total charge stored by the parallel bank of capacitors Q is given by,

$$Q = Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V$$
$$= (C_1 + C_2 + C_3) V \quad \text{... (2.17.1)}$$

- An equivalent capacitor which stores the same charge Q at the same voltage V, will have,

$$Q = C_{eq} V \quad \text{... (2.17.2)}$$

- Comparing (2.17.1) and (2.17.2),

$$C_{eq} = C_1 + C_2 + C_3$$

$$\therefore \quad \boxed{Q = C_1 V + C_2 V + C_3 V}$$

- It is easy to find Q_1, Q_2 and Q_3 if V is known.

> *For 'n' capacitors in parallel,*
> $$C_{eq} = C_1 + C_2 + \ldots + C_n$$

Ex. 2.17.1 : ***For the circuit shown below, find P.D. across and charge on each capacitor.***

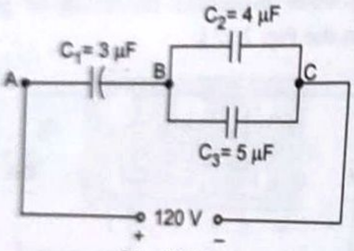


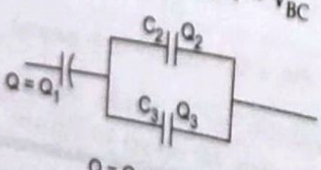
Fig. 2.17.2

Sol. : Capacitance between B and C = 4 + 5 = 9 μF

3 and 9 are in series hence, $C_{eq} = \dfrac{3 \times 9}{3+9} = 2.25\ \mu F$

Total charge $Q = C_{eq}\ V = 2.25 \times 120 = 270\ \mu C$

$$Q_1 = Q = C_1 V_1$$
$$V_1 = \frac{270}{3} = 90\ V$$
$$\therefore \quad V_2 = 120 - 90 = 30\ V = V_{BC}$$



$Q = Q_1 = Q_2 + Q_3$

Fig. 2.17.2 (a)

$\therefore$ Voltage across C_2 and C_3 is **30 V**

[As they are in parallel]

$$\therefore \text{ Charge} \quad Q_2 = C_2 V_2 = 4 \times 30 = \mathbf{120\ \mu C}$$
$$\text{Charge} \quad Q_3 = C_3 V_2 = 5 \times 30 = \mathbf{150\ \mu C}$$

$[Q_1 = Q = Q_2 + Q_3]$ i.e. $270\ \mu C = 120 + 150\ \mu C$

Ex. 2.17.2 : ***Two capacitors C_1 and C_2 connected in series are supplied at 120 V d.c. The p.d. across C_1 and C_2 is as shown. If capacitance of 2 μF is connected parallel [shunted] to C_2, it is found that p.d. across C_1 increases to 96 V. Determine values of C_1 and C_2.***

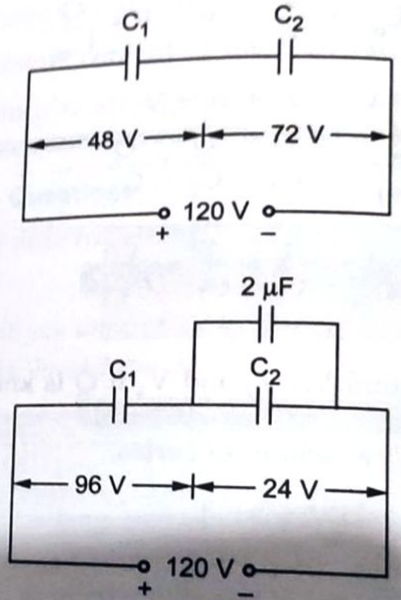


Fig. 2.17.3

Sol. : Let capacitors C_1 and C_2 are in μF

In series circuit $Q = C_1 V_1 = C_2 V_2$

$$\therefore \quad \frac{C_1}{C_2} = \frac{72}{48} = \frac{3}{2} \quad \therefore C_1 = \frac{3}{2} C_2 \quad \text{.... (1)}$$

When 2 μF is connected parallel to C_2, their equivalent is $(2 + C_2)$ μF for series circuit.

$$\therefore \quad (C_2 + 2) V_2 = C_1 V_1$$
$$(C_2 + 2) \times 24 = 96 \times \frac{3}{2} C_2 \quad \text{... From equation (1)}$$
$$\therefore \quad 24 C_2 + 48 = 144\ C_2$$
$$C_2 = 0.4\ \mu F$$
$$48 = 120\ C_2$$
$$C_1 = 1.5 \times 0.4 = 0.6\ \mu F$$

Ex. 2.17.3 : ***Two capacitors are connected in parallel having equivalent capacitance of 10 μF while same capacitors when connected in series have equivalent capacitance of 2 μF. Find the values of two capacitors.*** **SPPU : May-12, Marks 6**

Sol. : Case 1 : Capacitors in parallel

$$\therefore \quad \boxed{C_{eq} = C_1 + C_2 = 10\ \mu F} \quad \dots (1)$$

Case 2 : Capacitors in series

$$\therefore \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 2\ \mu F \quad \dots (2)$$

From (1),

$$C_2 = 10 \times 10^{-6} - C_1 \text{ and using in (2.17.4),}$$

$$\frac{C_1\left[10\times10^{-6} - C_1\right]}{10\times10^{-6}} = 2 \times 10^{-6} \quad \dots C_1 + C_2 = 10\ \mu F$$

$$\therefore \quad C_1^2 - 10\times10^{-6} C_1 + 20\times10^{-12} = 0$$

Solving, $C_1 = \mathbf{7.236\ \mu F}$, $C_2 = \mathbf{2.7639\ \mu F}$

or $C_1 = \mathbf{2.7639\ \mu F}$, $C_2 = \mathbf{7.236\ \mu F}$

Ex. 2.17.4 : ***Three capacitor A, B, C have capacitances 20, 50 and 25 μF respectively. Calculate (1) Charge on each when connected in parallel to a 250 V supply. (2) Total capacitance and (3) Potential difference across each when connected in series.*** **SPPU : Dec.-08, Marks 6**

Sol. : $C_A = 20\ \mu F$, $C_B = 50\ \mu F$, $C_C = 25\ \mu F$

1. Connected in parallel across V = 250 V. The voltage across each remains same as in parallel.

$$\therefore \quad V = \frac{Q_A}{C_A} = \frac{Q_B}{C_B} = \frac{Q_C}{C_C} = 250$$

$$\therefore \quad Q_A = 250\times C_A = 250\times20\times10^{-6} = \mathbf{5\ mC}$$

$$\therefore \quad Q_B = 250\times C_B = 250\times50\times10^{-6} = \mathbf{12.5\ mC}$$

$$\therefore \quad Q_C = 250\times C_C = 250\times25\times10^{-6} = \mathbf{6.25\ mC}$$

2. Total capacitance when connected in series is,

$$\frac{1}{C_{eq}} = \frac{1}{C_A} + \frac{1}{C_B} + \frac{1}{C_C}$$

$$= \frac{1}{20\times10^{-6}} + \frac{1}{50\times10^{-6}} + \frac{1}{25\times10^{-6}}$$

$$\therefore \quad C_{eq} = \mathbf{9.0909\ \mu F}$$

3. When connected in series, the charge Q remains same for all of them.

$$\therefore \quad Q = C_A V_A = C_B V_B = C_C V_C$$

$$\text{and} \quad Q = C_{eq} \times V = 9.099\times10^{-6}\times250 = 2.27272\ \text{mC}$$

$$\therefore \quad V_A = \frac{Q}{C_A} = 113.6363\ \text{V},$$

$$V_B = \frac{Q}{C_B} = 45.4544\ \text{V}, \quad V_C = \frac{Q}{C_C} = 90.909\ \text{V}$$

Key Point *Check that $V = V_A + V_B + V_C$*

Ex. 2.17.5 : ***Two capacitors of 50 μF each are connected in parallel with each other and this combination is connected in series with two capacitors of 80 μF and 40 μF each. Calculate equivalent capacitance of the circuit.*** **SPPU : Dec.-10, Marks 5**

Sol. : The arrangement is shown in the Fig. 2.17.4 (a).

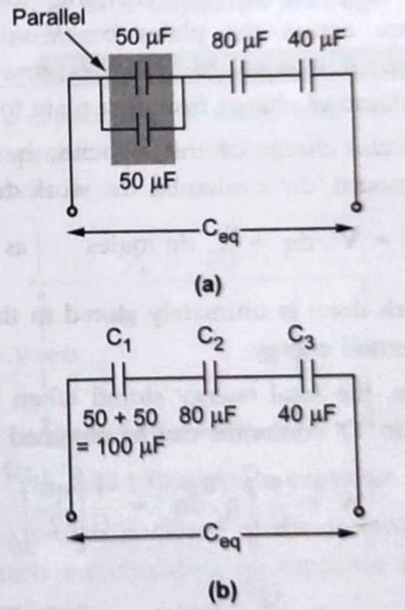


Fig. 2.17.4

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{100\times10^{-6}} + \frac{1}{80\times10^{-6}} + \frac{1}{40\times10^{-6}} = 47500$$

$$C_{eq} = \mathbf{21.0526\ \mu F}$$

Expected Question

1. *Derive the expression for an equivalent capacitance of a three capacitors connected in parallel.*

2.18 Energy Stored in a Capacitor

SPPU : May-06,07,08,09,10,11, Dec.-05,08,10,12,14,17

- When the capacitor is charged, energy is expended by the charging source. This is because charging the capacitor means the transfer of the charges from one plate to the another.
- This transfer is against the opposition due to potential difference across the plates. Due to this, there is expenditure of energy on the part of charging source. This energy is stored in the capacitor in terms of the electrostatic field set up in the dielectric medium.
- However, when the capacitor is discharged, this field collapses and energy stored in it is released.
- Let us determine the energy expended in charging a capacitor of capacitance C farads to a voltage V.
- Let at any instant of charging, the potential difference across the plates be 'V' volts. As per definition, it is equal to the work done in shifting one coulomb of charge from one plate to another.
- Now, if the charge of the capacitor is raised by a small amount 'dq' coulombs, the work done is,

$$dW = V \, . \, dq = \frac{q}{C} \, . \, dq \text{ joules} \qquad \left(\text{as } V = \frac{q}{C}\right)$$

- This work done is ultimately stored in the capacitor as a potential energy.
- Therefore, the total energy stored when it is finally charged to 'Q' coulombs can be obtained as,

$$W = \frac{1}{C}\int_0^Q q \, . \, dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$= \frac{Q^2}{2C} \text{ joules} \qquad \text{But } V = \frac{Q}{C}$$

$$\therefore \quad W = \frac{C^2 V^2}{2C}$$

$$\therefore \quad \boxed{\text{Energy stored, } W = \frac{1}{2} C V^2 \text{ joules}}$$

Ex. 2.18.1 : ***A parallel plate capacitor has plates, each of area of 100 cm², separated by a distance of 3 cm. The dielectric between the plates has relative permittivity of 2.2. The potential difference between the plates is 10 kV. Find 1) Capacitance of capacitor 2) The electric flux density, 3) The electric field strength, 4) Energy stored.***

SPPU : May-10, Marks

Sol. : $A = 100 \text{ cm}^2$, $d = 3$ cm, $\varepsilon_r = 2.2$, $V = 10$ kV

$$1) \; C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854\times10^{-12}\times2.2\times100\times10^{-4}}{3\times10^{-2}}$$

$$= \mathbf{6.493 \text{ pF}}$$

$$2) \; Q = CV = 6.493\times10^{-12}\times10\times10^{3} = 6.493\times10^{-8}$$

$$\therefore D = \frac{Q}{A} = \frac{6.493\times10^{-8}}{100\times10^{-4}} = \mathbf{6.493 \; \mu C/m^2} \quad \text{...Flux density}$$

$$3) \quad E = \frac{V}{d} = \frac{10\times10^{3}}{3\times10^{-2}} = \mathbf{333.333 \text{ kV/m}}$$

...Electric field strength

$$4) \text{ Energy} = \frac{1}{2}CV^2 = \frac{1}{2}\times6.493\times10^{-12}\times(10\times10^3)^2$$

$$= \mathbf{3.2465\times10^{-4} \text{ J}}$$

Ex. 2.18.2 : ***A capacitor is charged with 10 mC. If the energy stored in it is 1 joule, calculate the voltage across it and its capacitance.***

Sol. : The energy stored $= \frac{1}{2} C V^2$ **but Q = C V**

$$\therefore \quad W = \frac{1}{2} Q V \quad \text{i.e.} \quad 1 = \frac{1}{2}\times10\times10^{-3}\times V$$

$$\therefore \quad \mathbf{V = 200 \text{ V}}$$

$$\text{Now} \quad C = \frac{Q}{V} = \frac{10\times10^{-3}}{200} = \mathbf{50 \; \mu F}$$

Expected Question

1. *Derive an expression for the energy stored in a capacitor.* **SPPU : Dec.-05,08,10,12,14,17, May-06,07,08,09,11, Marks 5**

2.19 : Types of Capacitors

Dec.-05, May-03

- They are also classified based on the nature of the dielectric used as follows :-

i) **Air capacitors :** This type of capacitor consists of one set of fixed plates and another set of movable plates.

- Its capacitance can be changed by changing the position of the movable plates.
- This type is mainly used for radio work where the capacitance is required to be varied. These are aslo used in laboratory circuits, oscillators and radar systems, tuning capacitors in recievers etc.

ii) **Paper capacitors :** This consists of metal foils interleaved with paper impregnated with wax or oil and it is rolled into a compact form.

- These are used in power supplies, timing circuits, as a power factor correcting device, for starting of single phase motors etc.

iii) **Mica capacitors :** It consists of alternate layers of mica and metal foil clamped together tightly. Use of mica makes its cost high.

- The lossed in these capacitors are very less.
- It is mainly used in high frequency circuits which requires greater accuracy, high voltages and less dielectric loss.
- These are used as bypass and blocking capacitors in electronic circuits, in filter circuits, for precise tuning circuits, as a coupling capactors etc.

iv) **Ceramic capacitors :** It has a metallic coatings on the opposite faces of a thin disc of ceramic material like barium titanate, hydrous silicate of magnesia, etc.

- It is used in high frequency radio and electronic circuits, filter circuits etc.

v) **Electrolytic capacitors :** These are most commonly used and consists of two aluminium foils, one with an oxide film and one without. The foils are interleaved with a material such as a paper saturated with a suitable electrolyte.

- The aluminium oxide film is formed on the one foil by passing it through an electrolytic bath. This oxide film acts as a dielectric.
- These are used where very large capacitance values are required so used in electronic and filter circuits. These are also used for photo flash, as a blocking capacitors, for starting of single phase motors, in electornic amplifiers etc.
- The main limitations of this type are the low insulation resistance and suitability only for those circuits where the voltage applied to the capacitor never reverses its direction.

vi) **Poly carbonate capacitors :** This is a recent development where a film of polycarbonate, metallised with aluminium is wound to form the capacitor elements.

- It has a relative permittivity of 2.8 and has a high resistivity with very low dielectric loss.

> **Expected Question**
>
> 1. *How capacitors are classified on the basis of nature of dielectric used.*
>
> SPPU : Dec.-05, May-03, Marks 5

2.20 : Charging of a Capacitor through Resistance

SPPU : May-04,10,12,17, Dec.-04,07,09,10

- Consider a capacitor C in series with the resistance R. The capacitor has initially no charge and no voltage across it.

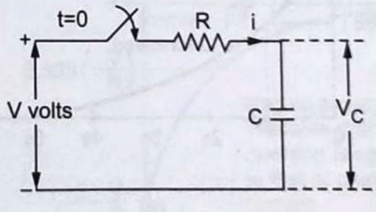


Fig. 2.20.1 Charging a capacitor

- When the switch is closed at the instant t = 0, the charge starts accumulating on capacitor and current starts flowing.
- The rate of rise of charge at start is high and later becomes slow and behaves in exponential manner till it reaches equal to the source voltage V.
- The current at the instant of closing the switch is high and as the voltage across capacitor V_C at start is zero this initial current can be expressed as,

$$i = \frac{V - V_C}{R} = \frac{V}{R}\ A \quad \text{...maximum charging current}$$

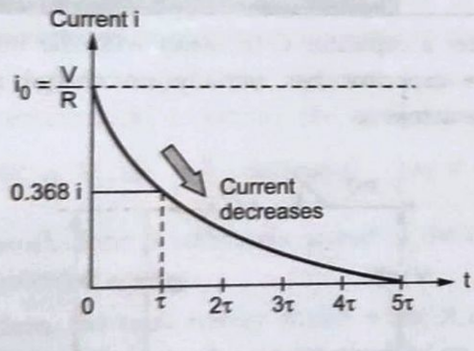
- As capacitor starts charging, the capacitor voltage V_C increases and finally after a certain period achieves a value equal to V.
- Then the charging current reduces to zero. Theoretically, the current becomes zero only after an infinite time. In actual practice the voltage across capacitor and current achieve their steady state values equal to V and zero respectively, in a relatively short time.
- The variation of charging current and capacitor voltage V_C against time is shown in Fig. 2.20.2 (a) and (b).
- The expressions for the charging current and the voltage across the capacitor are,

$$\text{Current } i = \frac{V}{R} e^{-t/CR} \text{ and}$$

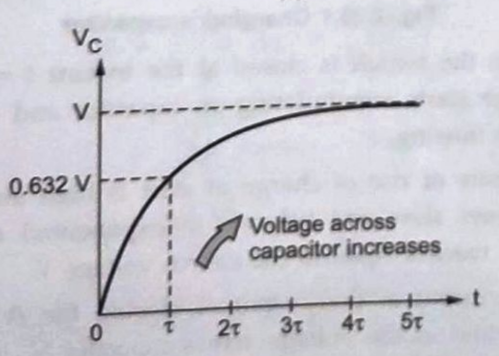
$$\therefore \quad \text{Voltage } V_C = V\left(1 - e^{-t/CR}\right)$$

- At t = 0, V_C = 0 V and the current is given by,

$$\boxed{i_0 = \frac{V}{R}\ A}$$



(a)



(b)

Fig. 2.20.2 Variation of charging current and voltage V_C

- **The capacitor acts as short circuit at start and acts as open circuit in steady state.**
- The term CR in the above equations is called the **Time Constant of the R-C charging circuit** and denoted by τ, measured in seconds.

$$\boxed{\tau = CR \text{ seconds} = \text{Time constant}}$$

- The time constant of the R-C series circuit is defined as **time required by the capacitor voltage to rise from zero to 0.632 of its final steady state value during charging.**
- **Time constant** is also defined as the time required for the charging current of the capacitor to fall to 0.368 of its initial maximum value, starting from its maximum value.

2.20.1 Initial Rate of Rise of Capacitor Voltage

- The initial rate of rise of capacitor voltage is high however when the capacitor charges this rate is reduced.
- The initial rate of rise of capacitor voltage can be obtained from $\frac{d V_C}{d t}$ at t = 0.
- Differentiating V_C with respect to t and equating to zero,

$$\boxed{\textit{Initial rate of rise of capacitor voltage} = \frac{V}{CR} = \frac{V}{\tau}\ \text{V/sec}}$$

Ex. 2.20.1 : ***A 12 μF capacitor in series with a 1.2 MΩ resistor is connected across a 100 volt d.c. supply. Determine : 1) The time constant of the circuit. 2) The initial value of charging current. 3) The initial rate of rise of voltage across the capacitor. 4) The voltage across the capacitor after 4 sec.*** **SPPU : Dec.-09, Marks 6**

Sol. : C = 12 μF, R = 1.2 MΩ, V = 100 V

1) τ = Time constant = RC = $1.2 \times 10^6 \times 12 \times 10^{-6}$

= **14.4 sec**

2) Initial charging current at t = 0 is

$$i = \frac{V}{R} = \frac{100}{1.2 \times 10^6} = 83.333\ \mu A$$

3) Initial rate of rise of capacitor voltage is,

$$\left.\frac{dV_C}{dt}\right|_{t=0} = \frac{V}{CR} = \frac{V}{\tau} = \frac{100}{14.4} = 6.944 \text{ V/sec}$$

4) The equation of capacitor voltage is,

$$V_C = V\,(1 - e^{-t/\tau}) = 100\,(1 - e^{-t/14.4})$$

At $t = 4$ sec, $V_C = 100\,(1 - e^{-4/14.4}) = \mathbf{24.2534\ V}$

Ex. 2.20.2 : *A 80 μF capacitor in series with a 1000 Ω resistor is connected suddenly across a 110 volt d.c. supply. Determine : 1) The time constant of the circuit. 2) The initial value of charging current 3) The equation of current 4) Value of current at t = 0.08 sec.*

SPPU : May-12, Marks 6

Sol. : $C = 80\ \mu F$, $R = 1000\ \Omega$, $V = 110\ V$

1) τ = Time constant $= RC = 1000 \times 80 \times 10^{-6}$

$= 0.08$ sec

2) Initial charging current at $t = 0$ is,

$$i = \frac{V}{R} = \frac{110}{1000} = \mathbf{0.11\ A}$$

3) The equation of current is,

$$i = \frac{V}{R}e^{-t/CR} = \frac{110}{1000}e^{-t/0.08} = \mathbf{0.11\ e^{-12.5\,t}\ A}$$

4) i at t = 0.08 sec is,

$$i = 0.11\,e^{-12.5\,t} = 0.11\,e^{-12.5 \times 0.08} = \mathbf{0.04046\ A}$$

Expected Question

1. *An initially uncharged capacitor in series with a resistor is connected to a d.c. supply at t = 0. Sketch the waveforms of current and the voltage across the capacitor. State their expressions. Define the time constant.*

May-04, 10, 17, Dec.-04, 07, 10 Marks 5

2.21 Discharging a Capacitor through a Resistance

SPPU : May-06

- Consider that a capacitor 'C' is being discharged through a resistor R by closing the switch at t = 0 as shown in the Fig. 2.21.1.

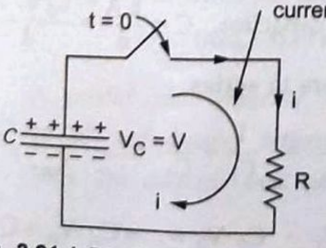
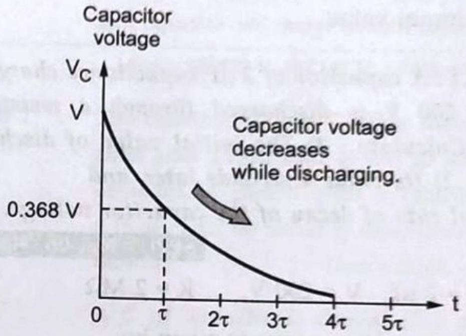
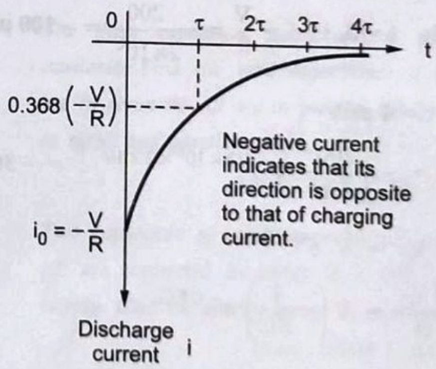


Fig. 2.21.1 Discharging of a capacitor

- At the time of closing the switch the capacitor 'C' is fully charged to V volts and it discharges through resistance 'R'.
- As time passes, charge and hence the capacitor voltage V_C decreases gradually and hence discharge current also gradually decreases exponentially from maximum to zero.
- The variation of capacitor voltage and discharging current as a function of time is shown in the Fig. 2.21.2.
- As direction of current is opposite to that of charging current, it is mathematically considered as negative. Hence graph of current against time is in fourth quadrant.



(a)



(b)

Fig. 2.21.2 Variation of discharge current and voltage

- The expressions for the discharging current and the voltage across the capacitor are,

$$\therefore \quad \boxed{V_C = V\,e^{-t/CR}\ \text{volts} \quad \text{and} \quad i = -\frac{V}{R}e^{-t/CR}}$$

Key Point *The negative sign indicates that the direction of the discharge current is the reverse to that of charging current.*

At the time of discharging also the term CR is called as **time constant** denoted by τ.

$\therefore \quad \boxed{\tau = CR \text{ seconds} = \text{Time constant}}$

- Discharging **time constant** can be defined as,

i) The time required for capacitor voltage to fall to 0.368 of its initial maximum value on discharge from its initial maximum value.

ii) The time required during which the capacitor discharge current falls to 0.368 of its initial maximum value.

Ex. 2.21.1 : ***A capacitor of 2 μF capacitance charged to p.d. of 200 V is discharged through a resistor of 2 MΩ. Calculate : 1) The initial value of discharged current 2) Its value 4 seconds later, and 3) Initial rate of decay of the capacitor voltage.***

SPPU : May-06, Marks 6

Sol. : C = 2 μF, V = 200 V, R = 2 MΩ

i) The discharging current is given by

$$i = -\frac{V}{R}e^{-t/RC}$$

Initially t = 0, $i = -\frac{V}{R} = -\frac{200}{2\times 10^6} = \mathbf{-100\ \mu A}$

ii) At t = 4 sec,

$$i = -\frac{200}{2\times 10^6}\, e^{-4/2\times 10^6\times 2\times 10^{-6}} = \mathbf{-36.7879\ \mu A}$$

iii) $\quad V_C = V\, e^{-t/RC}$

$$\therefore \quad \frac{dV_C}{dt} = V\left(-\frac{1}{RC}\right)e^{-t/RC}$$

Initial rate of decay is at t = 0,

$$\left.\frac{dV_C}{dt}\right|_{t=0} = -\frac{V}{RC} = -\frac{200}{2\times 10^6 \times 2\times 10^{-6}}$$

$$= \mathbf{-50\ V/sec}$$

Negative sign indicates decay of voltage and current.

Expected Question

1. *An initially charged capacitor is discharged through a resistor. Sketch the waveforms of current and the voltage across the capacitor. State their expressions. Define the time constant.*

Formulae at a Glance

Electric flux, ψ = Q coulombs (numerically)

- Electric flux density or displacement density,

$$D = \frac{\psi}{A} = \frac{Q}{A}\ C/m^2 \quad \dots \text{ as } \psi = Q = \text{Charge}$$

Electric field intensity, $\quad E = \frac{F}{Q}$ N/C

$E = \frac{V}{d}$ where d = Distance of seperation

and V = Applied voltage

- E is also measured in **V/m**.

Permittivity, $\quad \varepsilon = \frac{D}{E}$ F/m

Permittivity of free space,

$$\varepsilon_0 = \frac{1}{36\pi\times 10^9} = \mathbf{8.854 \times 10^{-12}}\ \text{F/m}$$

Relative permittivity, $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ i.e. $\varepsilon = \varepsilon_r \varepsilon_0$ no units

- **The relative permittivity** is nothing but the **dielectric constant** of the material.

Electric field strength = Potential gradient (numerically) in V/m

$$D = \varepsilon E = \varepsilon_0 \varepsilon_r E \ \ C/m^2$$

Coulomb's law, $F = \frac{K Q_1 Q_2}{d^2}$ Newtons where

$$K = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_r \varepsilon_0}$$

- Capacitance, Q = C V i.e. $C = \frac{Q}{V} = \frac{\text{Charge}}{\text{Voltage}}$
- Capacitance is measured in **farads**.

1 μF = 10^{-6} F, 1 nF = 10^{-9} F, 1 pF = 10^{-12} F

- Parallel plate capacitor, $C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d}$ F

For 'n' capacitors in **series**,

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

$$C_1 V_1 = C_2 V_2 = \dots = C_n V_n = Q$$

- For all the capacitors in series, the charge on all of them is always same, but the voltage across them is different.
- Capacitors in parallel,

$$Q_1 = C_1 V, Q_2 = C_2 V, Q_3 = C_3 V$$

For 'n' capacitors in parallel,

$$C_{eq} = C_1 + C_2 + \ldots + C_n$$

- For the capacitors in parallel,
- $Q = Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V$
- Energy stored, $W = \frac{1}{2} C V^2$ joules
- The expressions for the charging current and the voltage across the capacitor are,
- R-C charging, Current $i = \frac{V}{R} e^{-t/CR}$ and

Voltage $V_C = V(1 - e^{-t/CR})$

$$\tau = CR \text{ seconds} = \text{Time constant}$$

- The expressions for the discharging current and the voltage across the capacitor are,
- R-C discharging, $V_C = V e^{-t/CR}$ volts

and $i = -\frac{V}{R} e^{-t/CR}$.

Examples for Practice

Ex. 1 : *A parallel plate capacitor has an area of 10 cm^2 and distance between the plates is 2 mm. The dielectric used between the plates has relative permittivity of 3. Determine the capacitance of the parallel plate capacitor.*

[Ans. : 13.28 pF]

Ex. 2 : *The potential gradient between the plates in the above capacitor is 12 kV/cm, determine the voltage across the plates, charge, electric flux density and electricity flux between the plates..*

[Ans. : 31.87 nC, 31.87 μC/m^2]

Ex. 3 : *A parallel plate capacitor has an area of 10 cm^2 and distance between the plates is 2mm. The dielectric used between the plates has relative permittivity of 3. Determine the capacitance of the parallel plate capacitor.*

[Ans. : 13.28 pF]

Ex. 4 : *Three capacitors are connected in series across a 120 V supply, the voltage across them are 30, 40 and 50 and the charge on each is 4500 μC. What is the value of each capacitor and equivalent capacitor of the series combination ?*

[Ans. : 150 μF, 112.5 μF, 90 μF, 37.5 μF]

Ex. 5 : *Two capacitors of capacitance 15 μF and 20 μF are connected in series to a 600 V DC supply. Find the potential difference across each capacitor and charge on each capacitor.*

[Ans. : 342.857 V, 257.143 V, 5.1428 × 10^{-3} C]

Ex. 6 : *Two capacitors C_1 and C_2 when connected in series gives a capacitance of 0.03 μF and when connected in parallel gives a capacitance of 0.16 μF. Find the values of capacitance of each capacitor.* **[Ans. : 40 nF, 0.12 μF]**

Ex. 7 : *A 5 μF capacitor is charged to a potential difference of 100 V and then it is connected across an uncharged 3 μF capacitor. Calculate p.d. across the capacitors.* **[Ans. : 62.5 V]**

Ex. 8 : *The three capacitors 2, 3 and 6 μF are available. Find the total capacitance if i) All are in series ii) All are in parallel iii) 2 and 3 in series and parallel with 6.*

[Ans. : 1 μF, 11 μF, 7.2 μF]

Ex. 9 : *Two capacitors of capacitance 15 μF and 20 μF are connected in series to a 600 V DC supply. Find the energy stored in each case.*

[Ans. : 0.8816 J, 0.6612 J]

Ex. 10 : *Two capacitors 3 μF and 4 μF are connected in series across 100 V d.c. supply. Calculate :*
i) The voltage across each capacitor
ii) The energy stored across each capacitor and
iii) Equivalent capacitance of the combination.

[Ans. : i) 57.143 V, 42.857 V ii) 4.8979 mJ, 3.6734 mJ iii) 1.7143 μF]

Ex. 11 : *A capacitor having capacitance of 4 μF is connected in series with a resistance of 1MΩ across 200 volt d.c. supply. Calculate*
i) The time constant. ii) The initial charging current.
iii) The time taken by capacitor to raise upto 160 volt.

SPPU : Dec.-03

[Ans. : 4 sec, 200 μA, 6.4377 sec]

Ex. 12 : *An uncharged capacitance of 30 μF, connected in series with a resistance of 500-ohm, is suddenly connected across 100 V d.c. supply. Find (i) Time constant of the circuit (ii) Initial current (iii) Current after 0.05 second (iv) Final energy stored in the capacitor.*

SPPU : May-04

[Ans. : 0.015 sec, 0.2 A, 7.1347 mA, 0.15 J]

UNIT - II

3 A.C. Fundamentals

Syllabus

Sinusoidal voltages and currents, their mathematical and graphical representation, Concept of cycle, Period, frequency, instantaneous, peak(maximum), average and r.m.s. values, peak factor and form factor. Phase difference, lagging, leading and in phase quantities and phasor representation. Rectangular and polar representation of phasor.

Contents

3.1 Introduction . . . . 3 - 3

3.2 Generation of A.C. Voltage . . . . 3 - 4

3.3 Terminology Related to Alternating Quantity . . . . 3 - 5

May-06, 07, 09, Dec.-07, 10, Marks 6

3.4 Equation of an Alternating Quantity . . . . 3 - 7

May-98,99,02,07,09, Dec.-01,03,04,05,07,09, Marks 8

3.5 Effective Value or R.M.S. Value . . . . 3 - 10

Dec.-04,05,07,08,10,11,14, May-05,06,07,08,11,13,16,19, Marks 5

3.6 Average Value . . . . 3 - 12

May-05,06,09,11,12, Dec.-05,07,08,11,12,13,15,16, Marks 5

3.7 Form Factor (K_f) . . . . 3 - 13

May-05,07,11,12, Dec.-05,08, Marks 2

3.8 Crest or Peak Factor (K_p) . . . . 3 - 13

May-2000, 01, 05, 07, 09, 11,12,18, Dec.-05, 08,17, Marks 6

3.9 R.M.S. Value of Complex Waveform . . . . 3 - 15

Dec.-04, Marks 4

3.10 Phasor Representation of an Alternating Quantity . . . . 3 - 16

3.11 Concept of Phase and Phase Difference . . . . 3 - 17

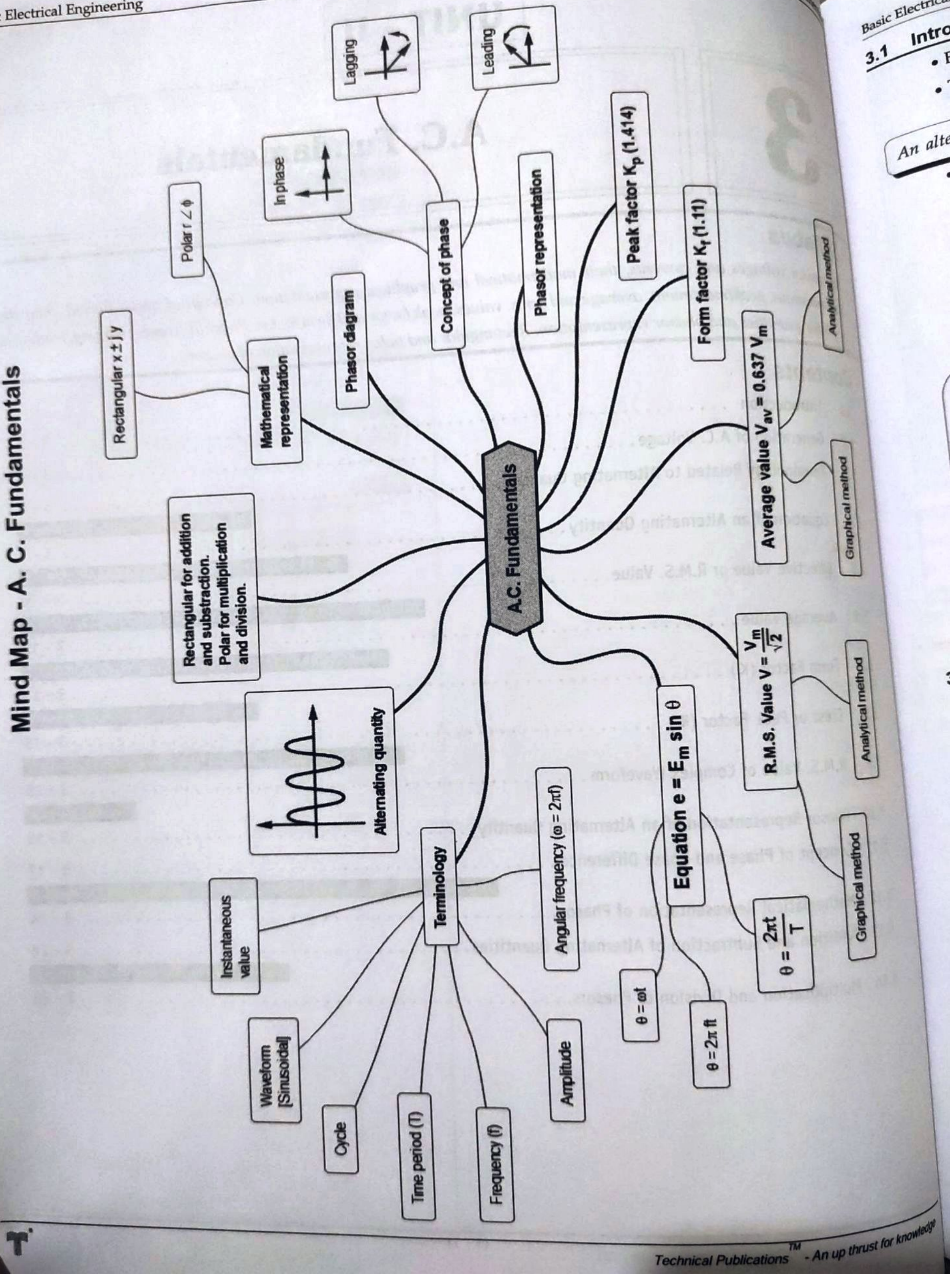
Dec.-99, 02, 05, 06, 08, 10, 15 May-01, 04, 05, 06, 08, 10,16,18, Marks 8

3.12 Mathematical Representation of Phasor . . . . 3 - 24

3.13 Addition and Subtraction of Alternating Quantities . . . . 3 - 26

Dec.-03, 08, 09, 11, May-01, 12, Marks 6

3.14 Multiplication and Division of Phasors . . . . 3 - 28


Mind Map - A. C. Fundamentals
A.C. Fundamentals
Rectangular for addition and substraction. Polar for multiplication and division.
Mathematical representation
Rectangular x ± j y
Polar r ∠ ϕ
Phasor diagram
Concept of phase
In phase
Lagging
Leading
Phasor representation
Peak factor K_p (1.414)
Form factor K_f (1.11)
Average value V_{av} = 0.637 V_m
Analytical method
Graphical method
R.M.S. Value $V = \frac{V_m}{\sqrt{2}}$
Analytical method
Graphical method
Equation $e = E_m \sin \theta$
$\theta = \frac{2\pi t}{T}$
$\theta = \omega t$
$\theta = 2\pi ft$
Alternating quantity
Terminology
Angular frequency ($\omega = 2\pi f$)
Instantaneous value
Waveform [Sinusoidal]
Cycle
Time period (T)
Frequency (f)
Amplitude

3.1 Introduction

- Electrical supply used for commercial and domestic purposes is alternating.
- The d.c. supply has constant magnitude with respect to time. The Fig. 3.1.1 (a) shows a graph of such current with respect to time.

An alternating current (a.c.) is the current which changes periodically both in magnitude and direction.

- In alternating waveform there are two half cycles, one positive and other negative. These two half cycles make one cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly.
- The Fig. 3.1.1 (b) shows a graph of alternating current against time.
- In practice some waveforms are available in which magnitude changes but its direction remains same as positive or negative. This is shown in the Fig. 3.1.1 (c). Such waveform is called **pulsating d.c.** The waveform obtained as the output of full wave rectifier is an example of pulsating d.c.

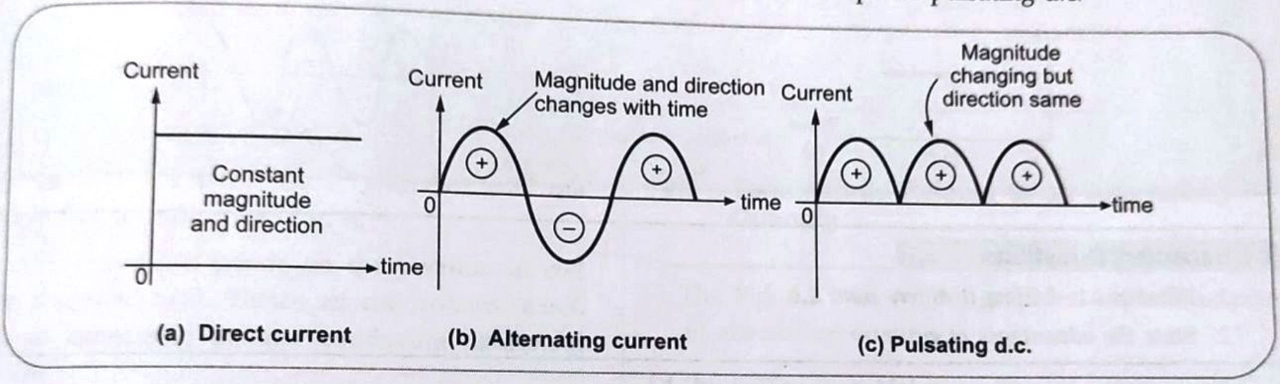


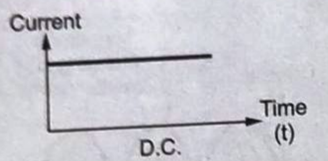
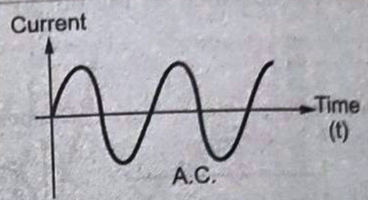
Fig. 3.1.1

3.1.1 Advantages of A.C.

1. The voltages in a.c. system can be raised or lowered with the help of a device called transformer. In d.c. system, raising and lowering of voltages is not so easy.
2. As the voltages can be raised, electrical transmission at high voltages is possible.
3. The a.c. transmission at high voltage is economical and efficient.
4. The construction and cost of alternators is low. High a.c. voltages of about 11 kV can be generated and can be raised up to 220 kV for transmission purpose.
5. A.C. electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.
6. Whenever it is necessary, a.c. supply can be easily converted to obtain d.c. supply.

- Due to these advantages, a.c. is used extensively in practice and hence, it is necessary to study a.c. principles.
- The standard waveform used for the a.c. purposes is **purely sinusoidal waveform.**

.1.2 Comparison between A.C. and D.C.

Sr. No.	D.C.	A.C.
1.	D.C. is direct current having constant magnitude and direction.	A.C. is alternating current whose direction and magnitude changes with time.
2.	The frequency of D.C. is zero.	There is finite frequency of A.C. It is 50 Hz in our nation.
3.	Raising and lowering of voltages is not easy.	Raising and lowering of voltages is very easy with the help of transformers.
4.	It is difficult to obtain A.C. from D.C.	It is easy to obtain D.C. from A.C. using rectifiers.
5.	The D.C. machines are expensive and require frequent maintenance.	The A.C. machines are cheaper and require less maintenance.
6.	The Figure shows the waveform of D.C. current. 	The Figure shows the waveform of A.C. current. 

Expected Questions

1. *What is a.c. ? How it differs from d.c. ?*
2. *State the advantages of a.c.*

3.2 Generation of A.C. Voltage

- The generators which generate purely sinusoidal alternating voltages are called alternators.
- The basic principle of an a.c. generation is the principle of electromagnetic induction. The sine wave is generated according to **Faraday's law of electromagnetic induction**.
- Whenever there is relative motion between the conductor and the magnetic field in which it is kept then an e.m.f. gets induced in that conductor.
- The velocity component which is responsible for cutting the magnetic flux and producing an e.m.f. is sine component i.e. $v \sin(\theta)$ where θ is the angle measured with respect to the plane of the flux.
- The magnitude of such an induced e.m.f. is given by,
  $e = Blv\sin(\theta)$ where B is the flux density, l is the active length of conductor and v is the velocity.
- As B, l and v are constants, the product Blv represents the maximum value of the induced e.m.f. denoted as E_m. Hence the induced e.m.f. is represented as $e = E_m \sin(\theta)$.
- A single turn alternator is shown in the Fig. 3.2.1.
- The two conductors ab and cd are connected together at one end to form a single turn coil.
- The other ends of the conductors are connected to the slip rings C_1 and C_2, mounted on the shaft.
- The coil is rotated in anticlockwise direction. The slip rings also rotate along with the coil.
- The brushes P and Q are resting on the slip rings and are stationary.
- The external resistance R is connected to the stationary brushes.

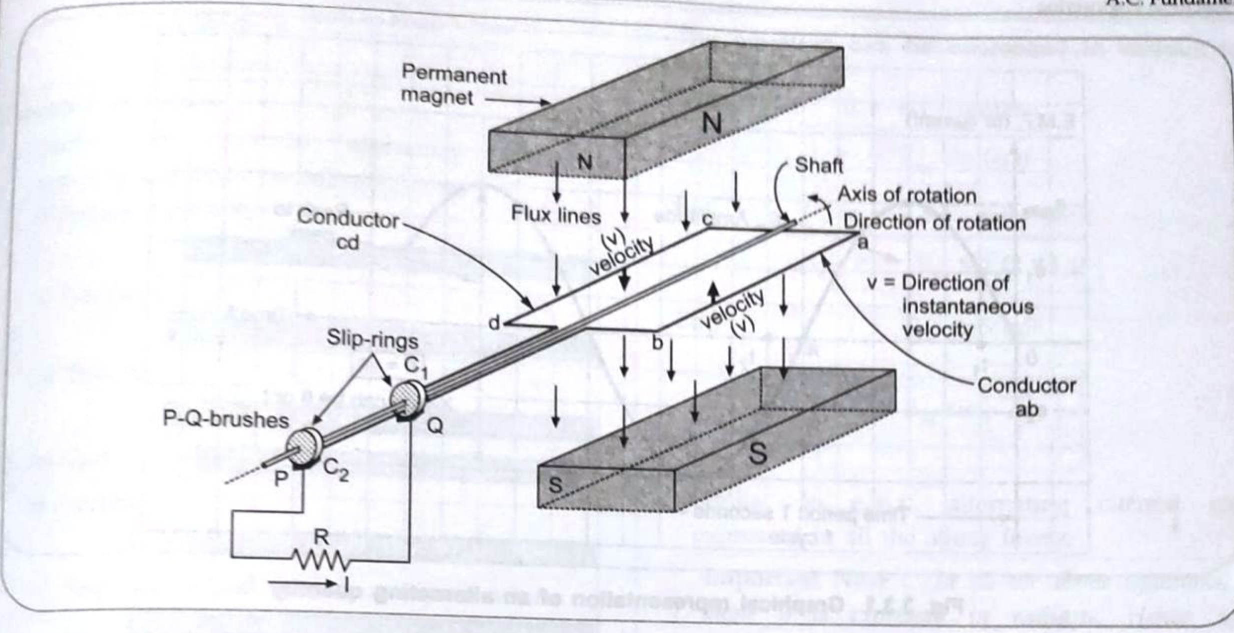


Fig. 3.2.1 Single turn alternator

- When the coil is rotated, each conductor cuts the magnetic flux to induce e.m.f in it.
- The e.m.f. induced depends on the position of coil in the magnetic field. Hence as coil rotates, e.m.f. varies in alternating manner producing sinusoidal behaviour.
- This e.m.f. drives current through resistance R.
- Thus if θ is the angle between velocity component with respect to the plane of the flux then as θ varies from 0° to 360°, the e.m.f. in a conductor varies in an alternating manner i.e. zero, increasing to achieve maximum in one direction, decreasing to zero, increasing to achieve maximum in other direction and again decreasing to zero. This set of variation repeats for every revolution as the conductors rotate in a circular motion with a certain speed.
- This variation of e.m.f. in a conductor is graphically represented by a purely sinusoidal waveform.
- When such an alternating e.m.f. is used to supply an electrical load then the resulting current waveform is also alternating i.e. purely sinusoidal in nature.

3.3 Terminology Related to an Alternating Quantity

SPPU : May-06, 07, 09, Dec.-07, 10

- The Fig. 3.3.1 shows the graphical representation of an alternating quantity.

1. Instantaneous Value : The value of an alternating quantity at a particular instant is known as its **instantaneous value.** e.g. e_1 and $-e_2$ are the instantaneous values of an alternating e.m.f. at the instants t_1 and t_2 respectively shown in the Fig. 3.3.1.

2. Waveform : The graph of instantaneous values of an alternating quantity plotted against time is called its **waveform.**

3. Cycle : Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a **cycle.**

- Such repetition occurs at regular interval of time. Such a waveform which exhibits variations that reoccur after a regular time interval is called **periodic waveform.**
- A cycle can also be defined as that interval of time during which a complete set of non-repeating events or waveform variations occur (containing positive as well as negative loops).

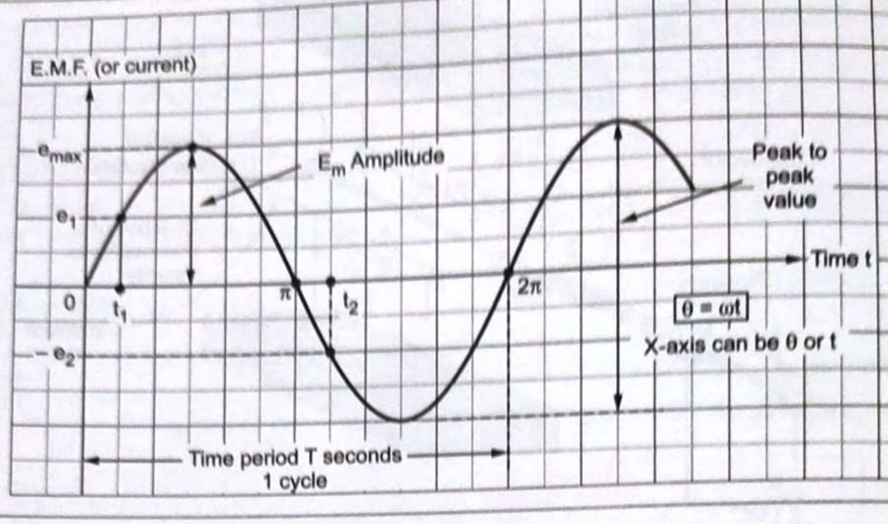


Fig. 3.3.1 Graphical representation of an alternating quantity

- One such cycle of the alternating quantity is shown in the Fig. 3.3.1.

One cycle corresponds to 2π radians or 360°.

4. Time Period (T) : The time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by T seconds.

- After every T seconds, the cycle of an alternating quantity repeats. This is shown in the Fig. 3.3.1.

5. Frequency (f) : The number of cycles completed by an alternating quantity per second is known as its **frequency.** It is denoted by f and it is measured in **cycles / second** which is known as **Hertz,** denoted as **Hz.**

- As time period T is time for one cycle i.e. seconds/cycle and frequency is cycles/second, we can say that frequency is reciprocal of the time period.

$$f = \frac{1}{T} \text{ Hz}$$

- As time period increases, frequency decreases while as time period decreases, frequency increases.
- **In our nation, standard frequency of alternating voltages and currents is 50 Hz.**

6. Amplitude : The maximum value attained by an alternating quantity during positive or negative half cycle is called its **amplitude.** It is denoted as E_m or I_m.

- Thus E_m is called peak value of the voltage while I_m is called peak value of the current.
- The amplitude is also called peak value or maximum value of an alternating quantity.

7. Angular Frequency (ω) : It is the frequency expressed in electrical radians per second.

- As one cycle of an corresponds to 2 π radians, the angular frequency can be expressed as (2π × cycles/sec.). It is denoted by 'ω' and its unit is radians/second. The relation between frequency 'f' and angular frequency 'ω' is,

$$\omega = 2\pi f \text{ radians/sec.} \quad \text{or} \quad \omega = \frac{2\pi}{T} \text{ radians/sec.}$$

Thus for 50 Hz, $\omega = 2\pi \times 50 = 314$ rad/sec

for 25 Hz, $\omega = 2\pi \times 25 = 157$ rad/sec

for 60 Hz, $\omega = 2\pi \times 60 = 377$ rad/sec

- The angle θ and the angular frequency ω are related to each other through time as,

$$\theta = \omega t \text{ radians} \quad \text{or} \quad \theta = 2\pi f t \text{ radians}$$

8. Peak to Peak Value : The value of an alternating quantity from its positive peak to negative peak is called its peak to peak value. It is denoted as I_{p-p} or V_{p-p}.

$$\text{Amplitude} = \frac{\text{Peak to Peak Value}}{2}$$

Expected Questions

1. *Sketch the sinusoidal alternating current waveform and define the following terms :*
   i) Instantaneous value
   SPPU : May-07, Dec.-07, 10, Mark 1
   ii) Waveform
   SPPU : May-06,09, Dec.-07,10, Mark 1
   iii) Time period
   SPPU : May-06,07,09, Dec.-10, Mark 1
   iv) Cycle **SPPU : May-09, Dec.-07,10, Mark 1**
   v) Frequency
   SPPU : May-06,07,09, Dec.-07,10, Mark 1
   vi) Amplitude or peak value or maximum value
   SPPU : May-06,07,09, Dec.-07,10, Mark 1

3.4 Equation of an Alternating Quantity

SPPU : May-98,99,02,07,09,Dec.-01,03,04,05,07,09

- As the standard waveform of an alternating quantity is purely sinusoidal, as stated earlier, the equation of an alternating voltage can be expressed as,

$$e = E_m \sin\theta \text{ volts}$$

where

E_m = Amplitude or maximum or peak value of the voltage

e = Instantaneous value of an alternating voltage

- Similarly equation of an alternating current can be expressed as,

$$i = I_m \sin\theta$$

where

I_m = Amplitude or maximum or peak value of the current.

i = Instantaneous value of an alternating current

Note If θ is expressed in radians in terms of π such as $\pi/3$, $\pi/4$ etc., then use $\pi = 180^\circ$ to express θ in degrees.

The equation can be expressed in various forms as :

Now, $\theta = \omega t$ radians

$$\therefore \quad e = E_m \sin(\omega t) \qquad \text{...(3.4.1)}$$

But, $\omega = 2\pi f$ rad / sec.

$$\therefore \quad e = E_m \sin(2\pi f t) \qquad \text{... (3.4.2)}$$

But, $f = \frac{1}{T}$ seconds

$$\therefore \quad e = E_m \sin\left(\frac{2\pi}{T} t\right) \qquad \text{... (3.4.3)}$$

- Similar to e.m.f., alternating current can be expressed in all the above forms.

Important Note : *In all the above equations, the angle θ is expressed in radians. Hence, while calculating the instantaneous value of the e.m.f. or current, it is necessary to calculate the sine of the angle expressed in radians.*

Ex. 3.4.1 : *The alternating voltage is given by $v(t) = 141.4 \sin(100\pi t)$. Find i) Maximum value ii) Frequency iii) periodic value.*

Sol. : Comparing with standard equation,

$$v(t) = V_m \sin(\omega t)$$

i) Maximum value = V_m = **141.4 V**

It is also known as peak value or amplitude

ii) $\omega = 2\pi f = 100\pi$

$$f = \frac{100}{2} = 50 \text{ Hz}$$

iii) Periodic time $T = \frac{1}{f} = \frac{1}{50} = 0.020$ sec.

OR $T = 20 \times 10^{-3}$ sec.

$$T = 20 \text{ ms}$$

Ex. 3.4.2 : *An alternating current of 50 Hz has maximum value of 14.14 A. i) Write down the expression for current. ii) Find the value of current at a) 1.666 ms b) 6.666 ms c) 13.333 ms d) At what time current would be 10 A measured from origin.*

Sol. : The current expression is given by,

$$i = 14.14 \sin \omega t$$

but $\omega = 2\pi f = 100\pi$

Note : *While solving problems, replace π by 180° to obtain angle in degrees. Otherwise convert calculator mode to radians to obtain sine of angle.*

$$i = \mathbf{14.14 \sin 100 \pi t}$$

i) When, $t = 1.666 \text{ ms} = 1.666 \times 10^{-3}$ sec

$$i(t) = 14.14 \sin(100 \times 180° \times 1.666 \times 10^{-3})$$

$$= 14.14 \sin(29.988°),\ \theta = 29.998° \cong 30°$$

$$i(t) = 14.14 \sin(30°)$$

$$i(t) = \mathbf{7.07\ A}$$

ii) When, $t = 6.666 \text{ ms} = 6.666 \times 10^{-3}$ sec

$$i(t) = 14.14 \sin(100 \times 180° \times 6.666 \times 10^{-3})$$

$$= 14.14 \sin(120°) = \mathbf{12.27\ A}$$

iii) When, $i(t) = 14.14 \sin(100 \times 180° \times 13.333 \times 10^{-3})$

$$= 14.14 \sin\theta$$

Where $\theta = 100 \times 180° \times 13.333 \times 10^{-3} = 240°$

$$i(t) = 14.14 \sin(240°) = \mathbf{-12.25\ A}$$

iv) At what time measured for origin, current would be 10 A

$$10 = 14.14 \sin(100 \times 180° \times t)$$

$$10 = 14.14 \sin\theta$$

$$\theta = 45° \quad \text{but } \theta = \omega t$$

Where $\theta = 100 \times 180° \times t$

$\therefore$ $45° = 100 \times 180° \times t$

$$\mathbf{t = 2.5 \times 10^{-3}\ sec}$$

Ex. 3.4.3 : ***An alternating current is given as : i(t) = 5 sin (314.15t) A. Find its amplitude, frequency and the time period.***

Sol. : Comparing the given equation with

$i(t) = I_m \sin(\omega t)$,

i) Amplitude $= I_m = 5$ **A**

ii) $\omega = 314.15$ rad/s

but $\omega = 2\pi f$

i.e. $f = \dfrac{\omega}{2\pi} = \dfrac{314.15}{2\pi} = \mathbf{50\ Hz}$

iii) Time period $T = \dfrac{1}{f} = \dfrac{1}{50} = \mathbf{0.02\ sec}$

Ex. 3.4.4 : ***An alternating current of frequency 60 Hz has a maximum value of 12 A :***
i) Write down the equation for instantaneous values
ii) Find the value of the current after 1/360 second
iii) Time taken to reach 9.6 A for the first time. In the above cases assume that time is reckoned as zero when current wave is passing through zero and increasing in the positive direction.

SPPU : May-98, 02, Dec.-09, Marks 8

Sol. : $f = 60$ Hz, $I_m = 12$ A,

$\omega = 2\pi f = 2\pi \times 60 = 377$ rad/sec

i) Equation of instantaneous value is,

$$i = I_m \sin \omega t = \mathbf{12 \sin 377\ t}$$

ii) $t = \dfrac{1}{360}$ sec

i.e. $i = 12 \sin 377 \dfrac{1}{360} = 12 \sin 1.0472 = \mathbf{10.3924\ A}$

Note sin of 1.0472 must be calculated in radian mode of calculator.

iii) $i = 9.6$ A

i.e. $9.6 = 12 \sin 377\ t$

i.e. $\sin 377\ t = 0.8$

$\therefore$ $377\ t = 0.9272$

i.e. $\mathbf{t = 2.459 \times 10^{-3}\ sec}$ ... $\sin^{-1}$ **in radian mode**

Ex. 3.4.5 : ***An alternating current varying sinusoidally with a frequency of 50 Hz has a peak value of current as $20\sqrt{2}$ Amp. At what time measured from negative maximum value instantaneous current will be $10\sqrt{2}$ Amp. ?***

SPPU : Dec.-05, Marks 8

Sol. : $I_m = 20\sqrt{2}$ A, $f = 50$ Hz

hence $i = I_m \sin(2\pi f t) = 20\sqrt{2} \sin(100\pi t)$ A

The waveform is shown in the Fig. 3.4.1.

To find time corresponding to point D after point C which is negative maximum.

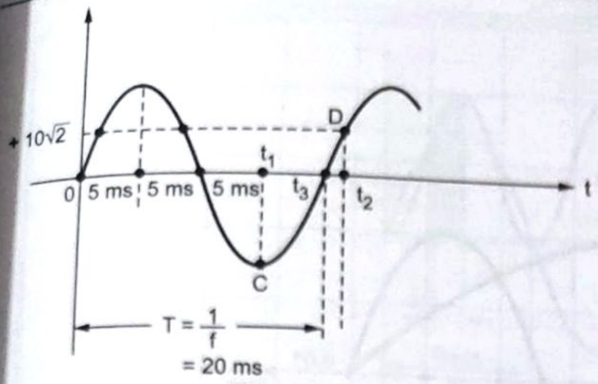


Fig. 3.4.1

$T = \frac{1}{f} = \frac{1}{50} = 0.02$ sec hence time corresponding to each quarter cycle is $\frac{T}{4} = 5$ ms

Thus at point C, $t_1 = 15$ ms.

For, $i = 10\sqrt{2}$

$$10\sqrt{2} = 20\sqrt{2}\,\sin(100\pi\, t)$$

$\therefore$ $t = 1.66$ ms

Thus from t_3 and t_2 time is 1.66 ms and t_1 to t_3 is quarter half cycle i.e. 5 ms.

$\therefore$ Total time for current to achieve $10\sqrt{2}$ A from negative maximum value is,

t_1 to t_2 $= 5 + 1.66 = $ **6.666 ms**

Ex. 3.4.6 : ***An alternating current of frequency 60 Hz has maximum value of 110 A. Calculate***

1) Its value $\frac{1}{600}$ sec. after the instant current is zero and its value decreasing thereafter.

2) Time required to reach 90 A after the instant current is zero and increasing positively.

SPPU : Dec.-09, Marks 6

Sol. : The instants are shown in the Fig. 3.4.2.

$$i = I_m \sin \omega t$$

$$= 110 \sin (2\pi f t) = 110 \sin (120\pi t) \text{ A}$$

1) t_1 is the time, $\frac{1}{600}$ sec after the instant current is zero and decreasing.

$\therefore$ $t_1 = 8.333 \times 10^{-3} + \frac{1}{600} = 0.01$ sec

$\therefore$ $i_1 = 110 \sin (120\pi \times 0.01)$

$= -$ **64.6563 A** ...Use sin in radian

2) t_2 is time at which $i = 90$ A after the instant current is zero and increasing.

$\therefore$ $90 = 110 \sin (120\pi t_1)$

i.e. $120\pi t_1 = \sin^{-1} \frac{90}{110}$ ... Radian mode

$\therefore$ $t_1 = \frac{0.9582}{120\pi} = $ **2.5418 msec**

Ex. 3.4.7 : ***For an A.C. circuit $e = 100 \sin \omega t$, calculate the value of e at $t = 0.005$ sec for i) 50 Hz and ii) 150 Hz. Sketch the waveform for e from $t = 0$ sec. to $t = 0.01$ sec for both cases on the same time axis.***

SPPU : Dec.-07, Marks 6

Sol. : i) $f = 50$ Hz, $\omega = 2\pi f = 100\pi$

i.e. $e = 100 \sin (100\pi t)$

At $t = 0.005$ sec, $e = 100 \sin (100\pi \times 0.005) = $ **100 V**

...(Use **radian mode** or use 180° instead of π)

ii) $f = 150$ Hz, $\omega = 2\pi f = 300\pi$

i.e. $e = 100 \sin (300\pi t)$

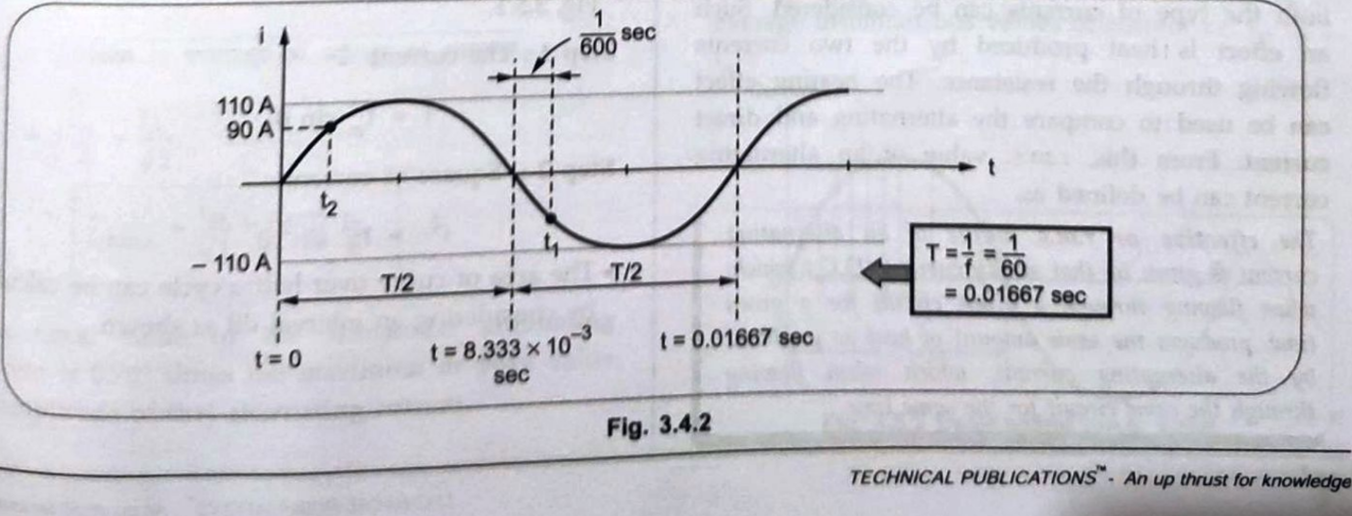


Fig. 3.4.2

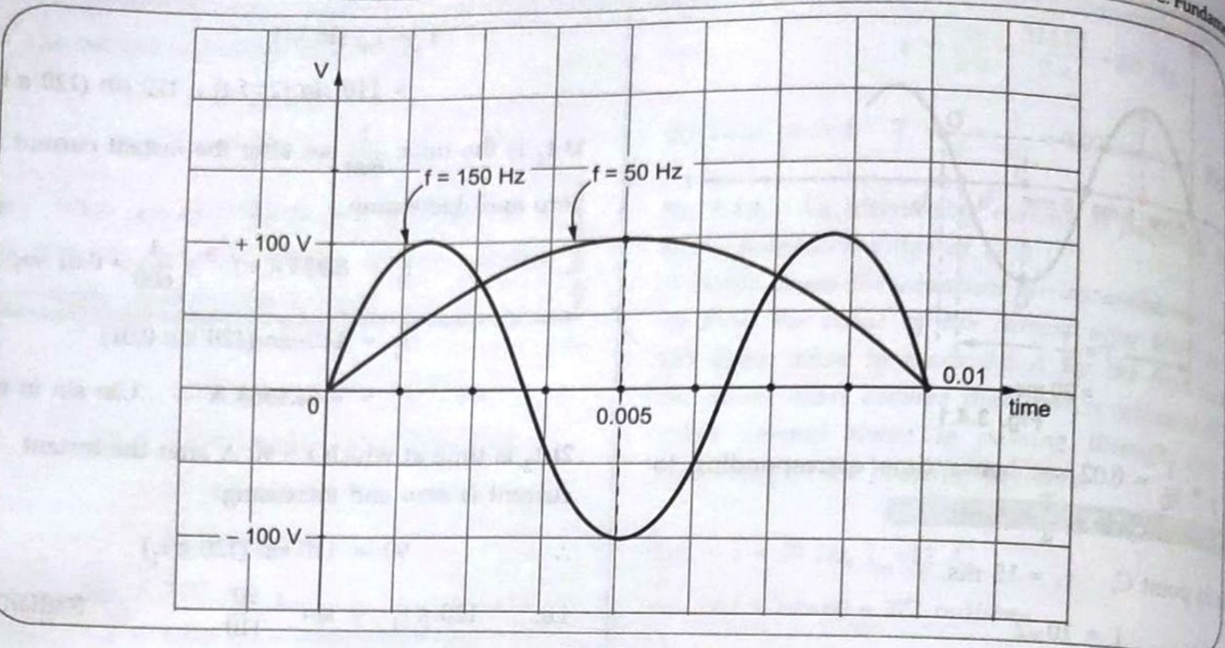


Fig. 3.4.3

At $t = 0.005$ sec, $e = 100 \sin (300\pi \times 0.005)$

$= -100$ V ...(Use **radian mode**)

The waveforms are shown in the Fig. 3.4.3.

> **Expected Question**
>
> 1. *State the equation of an alternating quantity. State its various forms.*

3.5 Effective Value or R.M.S. Value

SPPU : May-05, 06, 07, 08, 11, 13, 16, 19
Dec.-04, 05, 07, 08, 10, 11, 14

- An alternating current varies from instant to instant, while the direct current is constant, with respect to time.
- For the comparison of the two, a common effect to both the type of currents can be considered. Such an effect is heat produced by the two currents flowing through the resistance. The heating effect can be used to compare the alternating and direct current. From this, r.m.s. value of an alternating current can be defined as,

> *The **effective** or **r.m.s. value** of an alternating current is given by that steady current (D.C.) which, when flowing through a given circuit for a given time, produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.*

3.5.1 Analytical Method of Obtaining R.M.S. Value

> **Steps to find r.m.s. value of an a.c. quantity :**
>
> 1. Write the equation of an a.c. quantity. Observe its behaviour during various time intervals.
> 2. Find square of the a.c. quantity from its equation.
> 3. Find average value of square of an alternating quantity as,
>
> $$\text{Average} = \frac{\text{Area of curve over one cycle of squared waveform}}{\text{Length of the cycle}}$$
>
> 4. Find square root of average value which gives r.m.s. value.

- Consider sinusoidally varying alternating current and square of this current as shown in the Fig. 3.5.1.

Step 1 : The current

$$i = I_m \sin \theta$$

Step 2 : Square of current

$$i^2 = I_m^2 \sin^2 \theta$$

- The area of curve over half a cycle can be calculated by considering an interval $d\theta$ as shown.

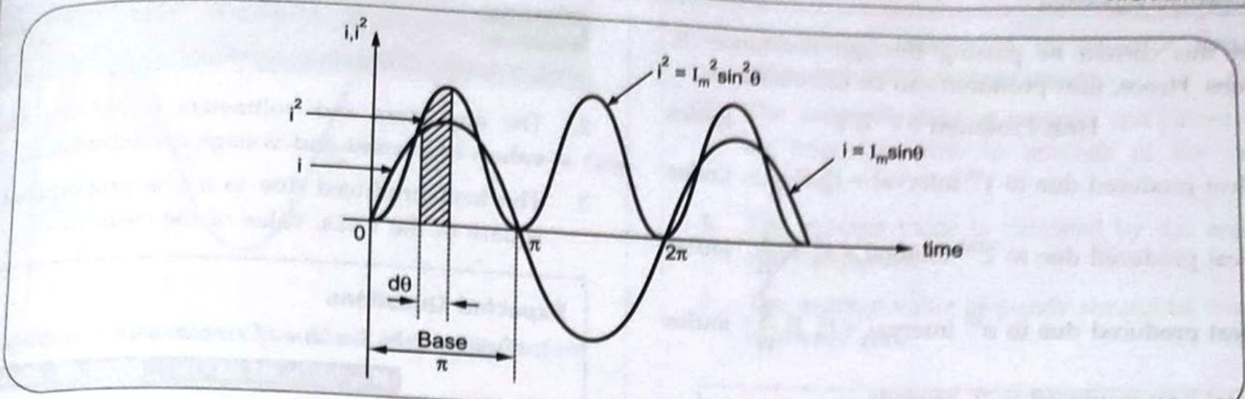


Fig. 3.5.1 Waveform of current and square of the current

Area of square curve over half cycle = $\int_0^{\pi} i^2 \, d\theta$ and length of the base is π.

Step 3 :

$\therefore$ Average value of square of the current over half cycle is,

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} i^2 \, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta = \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}[\pi] = \frac{I_m^2}{2}$$

Step 4 : Root mean square value i.e. r.m.s. value can be calculated as,

$$I_{r.m.s.} = \sqrt{\text{Mean or average of square of current}}$$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad \boxed{I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m}$$

- **The r.m.s. value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.**

> *The **instantaneous** values are denoted by **small** letters like i, e etc. while **r.m.s.** values are represented by **capital** letters like I, E etc.*

- The above result is also applicable to **sinusoidal** alternating voltages.

$$\therefore \quad \boxed{V_{r.m.s.} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m}$$

- The r.m.s. values are used for specifying alternating quantities. The given values such as 230 V, 110 V are r.m.s. values of alternating quantities unless and otherwise specified to be other than r.m.s.

3.5.2 Graphical Method

- Consider sinusoidally varying current. The r.m.s. value is to be obtained by comparing heat produced.
- Heat produced is proportional to square of current ($i^2 R$) so heat produced in both positive and negative half cycles will be the same. Hence, consider only positive half cycle, which is divided into 'n' intervals as shown in the Fig. 3.5.2.
- The width of each interval is 't/n' seconds and average height of each interval is assumed to be the average instantaneous values of current i.e. $i_1, i_2, \ldots, i_n$.

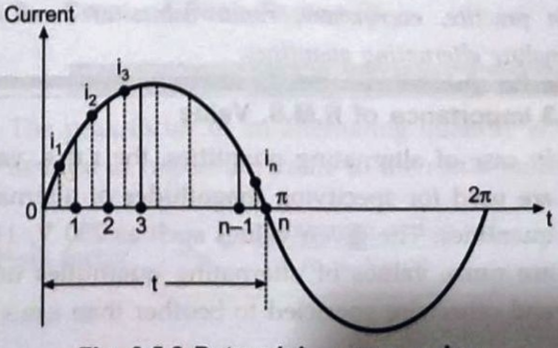


Fig. 3.5.2 Determining r.m.s. value

- Let this current be passing through resistance 'R' ohms. Hence, heat produced can be calculated as,

$$\text{Heat Produced} = i^2 R t \quad \text{joules}$$

$\therefore$ Heat produced due to 1st interval $= i_1^2 R \frac{t}{n}$ joules

$\therefore$ Heat produced due to 2nd interval $= i_2^2 R \frac{t}{n}$ joules

$\therefore$ Heat produced due to nth interval $= i_n^2 R \frac{t}{n}$ joules

$\therefore$ Total heat produced in 't' seconds

$$= R \times t \times \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$$

- Now, heat produced by direct current I amperes passing through same resistance 'R' for the same time 't' is $= I^2 R t$ joules
- For I to be the r.m.s. value of an alternating current, these two heats must be equal.

$$\therefore \quad I^2 R t = R \times t \times \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$$

$$\therefore \quad I^2 = \frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}$$

$$\therefore \quad \boxed{I = \sqrt{\frac{[i_1^2 + i_2^2 + ... + i_n^2]}{n}} = I_{r.m.s.}}$$

$I_{r.m.s.}$ = Square root of the mean of the squares of ordinates of the current.

This is called **Effective value** of an alternating current or **Virtual value** of an alternating current. This expression is equally applicable to sinusoidally varying alternating voltage as,

$$\boxed{V_{r.m.s.} = \sqrt{\frac{[V_1^2 + V_2^2 + ... + V_n^2]}{n}}}$$

In practice, everywhere, r.m.s. values are used to analyze alternating quantities.

3.5.3 Importance of R.M.S. Value

1. In case of alternating quantities, the r.m.s. values are used for specifying magnitudes of alternating quantities. The given values such as 230 V, 110 V are r.m.s. values of alternating quatntities unless and otherwise specicied to beother than r.m.s.

Key Point *In practice, everywhere, r.m.s. values are used to analyze alternating quantities.*

2. The ammeters and voltmeters record the r.m.s. values of current and voltage respectively.
3. The heat produced due to a.c. is proportional to square of the r.m.s. value of the current.

Expected Questions

1. *Define R.M.S. value of an alternating quantity.*

May-05,06,07,08, Dec.-05,07,08, Marks 2

2. *Derive the relation between r.m.s. value and the maximum value of an alternating quantity.*

Dec.-04,05,10,11,14, May-06,08,11,13,16,19, Marks 5

3.6 Average Value

May-05,06,09,11,12, Dec.-05,07,08,11,12,13,15,16

- The **average value** of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

For a symmetrical a.c., the average value over a complete cycle is zero as both positive and negative half cycles are exactly identical. Hence, the average value is defined for half cycle only.

- Average value can also be expressed by that steady current which transfers across any circuit, the same amount of charge as is transferred by that alternating current during the same time.

3.6.1 Analytical Method of Obtaining Average Value

- Consider sinusoidally varying current, $I = I_m \sin\theta$
- Consider the elementary interval of instant '$d\theta$' as shown in the Fig. 3.6.1. The average instantaneous value of current in this interval is say, 'i' as shown.
- The average value can be obtained by taking ratio of area under curve over **half cycle** to length of the base for half cycle.

$$\therefore \quad \boxed{I_{av} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}}$$

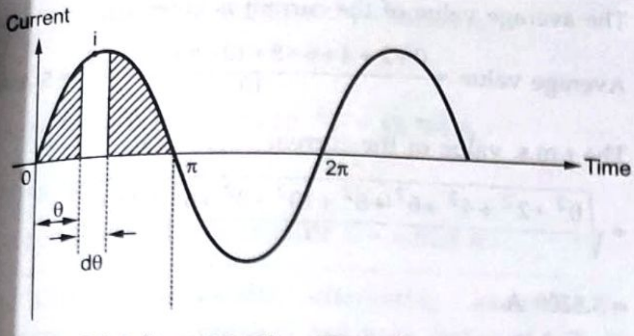


Fig. 3.6.1 Average value of an alternating current

$$I_{av} = \frac{\int_0^{\pi} i\, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} i\ d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta = \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[-\cos\pi + \cos 0] = \frac{I_m}{\pi}[2]$$

$$= \frac{2\, I_m}{\pi} = 0.637\ I_m$$

- For a purely sinusoidal waveform, the average value is expressed in terms of its maximum value as,

$$\therefore \quad \boxed{I_{av} = 0.637\ I_m \quad \text{and} \quad V_{av} = 0.637\ V_m}$$

3.6.2 Graphical Method

- Consider 'n' equal intervals of half cycle as shown in the Fig. 3.5.2. For r.m.s. value, we have calculated average value of the heat produced by the average currents during each of the 'n' intervals. In this case, it is necessary to determine the average value of current over half cycle.

Average value of current over half cycle

$$= \frac{i_1 + i_2 + \ldots\ldots + i_n}{n}$$

$$\therefore \quad I_{av} = \frac{i_1 + i_2 + \ldots\ldots + i_n}{n}$$

$$V_{av} = \frac{V_1 + V_2 + \ldots\ldots + V_n}{n}$$

3.6.3 Importance of Average Value

1. The average value is used for applications like battery charging.
2. The charge transferred in capacitor circuits is measured using average values.
3. The average values of voltages and currents play an important role in analysis of the rectifier circuits.
4. The average value is indicated by d.c. ammeters and voltmeters.
5. The average value of purely sinusoidal waveform is always zero.

Expected Questions

1. *Define average value of an alternating quantity.* **May-05,06, Dec.-05,08,16, Marks 2**
2. *Obtain the relation between average value and the maximum value of an alternating quantity.* **Dec.-05,07,11,12,13,15,16, May-06,09,11,12, Marks 5**

3.7 Form Factor (K_f) May-05,07,11,12, Dec.-05,08

- The form factor of an alternating quantity is defined as the ratio of r.m.s. value to the average value,

$$\text{Form factor,} \quad \boxed{K_f = \frac{\text{r.m.s. value}}{\text{average value}}}$$

- The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_f = \frac{0.707\ I_m}{0.637\ I_m} = 1.11$$

for sinusoidally varying quantity

Expected Question

1. *Define form factor.* **May-05,07,11,12, Dec.-05,08, Marks 2**

3.8 Crest or Peak Factor (K_p)

May-2000, 01, 05, 07, 09, 11,12,18 Dec.-05, 08,17

- The peak factor of an alternating quantity is defined as ratio of maximum value to the r.m.s. value.

$$\text{Peak factor} \quad \boxed{K_p = \frac{\text{maximum value}}{\text{r.m.s. value}}}$$

- The peak factor for sinusoidally varying alternating currents and voltages can be obtained as,

$$K_p = \frac{I_m}{0.707\, I_m} = 1.414 \text{ for sinusoidal waveform}$$

Ex. 3.8.1 : ***The equation of an alternating current is given by i = 42.42 sin 628t. Calculate its i) Maximum value ii) Frequency iii) RMS value iv) Average value v) Form factor.***

Sol. : Compare given equation with $i = I_m \sin(\omega t)$

i) $I_m = 42.42 \text{ A}$

ii) $f = \frac{\omega}{2\pi} = \frac{628}{2\pi} = 100 \text{ Hz}$

iii) $I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = 30 \text{ A}$

iv) $I_{av} = 0.637\, I_m = 27.0215 \text{ A}$

v) $K_f = \frac{\text{r.m.s.}}{\text{Average}} = \frac{30}{27.0215} = 1.11$

Ex. 3.8.2 : ***Calculate the r.m.s. value, average value, form factor, peak factor of a periodic current having following values for equal time intervals changing suddenly from one value to next as 0, 2, 4, 6, 8, 10, 8, 6, 4, 2, 0, –2, –4, –6, –8, –10, –8, ...*** **SPPU : May-01**

Sol. : The waveform can be represented as shown in the Fig. 3.8.1.

The average value of the current is given by,

$$\text{Average value} = \frac{0+2+4+6+8+10+8+6+4+2}{10} = 5 \text{ A}$$

The r.m.s. value of the current

$$= \sqrt{\frac{0^2+2^2+4^2+6^2+8^2+10^2+8^2+6^2+4^2+2^2}{10}}$$

$= 5.8309 \text{ A}$

$$\text{Form factor } K_f = \frac{\text{r.m.s.}}{\text{average}} = \frac{5.8309}{5} = 1.1661$$

$$\text{Peak factor } K_p = \frac{\text{maximum}}{\text{r.m.s.}} = \frac{10}{5.8309} = 1.715$$

Ex. 3.8.3 : ***A sinusoidal varying alternating current has r.m.s. value of 40 A and periodic time of 20 milliseconds. If the waveform of this current enters into positive half cycle at t = 0, find instantaneous value of the current at quarter cycle, t_1 = 7 ms and t_2 = 14 ms*** **SPPU : May-18, Marks 6**

Sol. : Given : I (RMS) = 40 A, T = 20 msec,

To find : i at t_1 and t_2

$$f = \frac{1}{T} = \frac{1}{20\times10^{-3}} = 50 \text{ Hz}, \quad I(\text{RMS}) = \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad I_m = \sqrt{2}\times 40 = 56.568 \text{ A}$$

$$\therefore i = I_m \sin(2\pi f t) = 56.568 \sin(314.159\, t) \text{ A}$$

At quarter cycle, $t = \frac{T}{4} = 5$ msec, Use radian mode

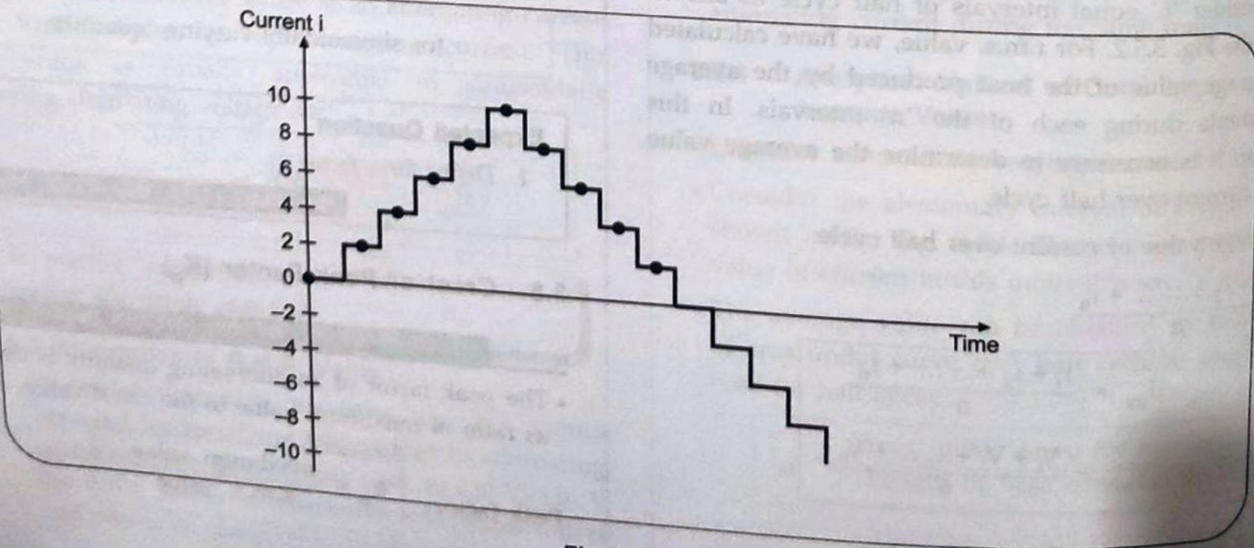


Fig. 3.8.1

$\therefore i = 56.568 \sin (314.159 \times 5 \times 10^{-3}) = \mathbf{56.568\ A}$

At $t_1 = 7$ msec,

$i = 56.568 \sin (314.159 \times 7 \times 10^{-3}) = \mathbf{45.764\ A}$

At $t_2 = 14$ msec,

$i = 56.568 \sin (314.159 \times 14 \times 10^{-3}) = \mathbf{-53.8\ A}$

Ex. 3.8.4 : *A sinusoidal alternating quantity is having form factor of 1.15 and peak factor of 1.57. If the maximum value of the voltage is 440 V calculate the average and RMS values of the voltage. Mention the relations of the factors.* **SPPU : Dec.-17, Marks 6**

Sol. : Given : $K_f = 1.15, K_p = 1.57, V_m = 440$ V

To find : Average and r.m.s. values

$$\therefore \quad K_f = \frac{\text{r.m.s.}}{\text{average}} \text{ and } K_p = \frac{\text{maximum}}{\text{r.m.s.}}$$

$$\therefore \quad K_f \times K_p = \frac{\text{maximum}}{\text{average}}$$

$$\therefore \quad 1.15 \times 1.57 = \frac{440}{\text{average}}$$

$$\therefore \quad \text{average} = \mathbf{243.7\ V}$$

$$\therefore \quad \text{r.m.s.} = K_f \times \text{average} = \mathbf{280.254\ V}$$

Expected Question

1. *Define peak factor.* **May-05,07,11,12, Dec.-05,08, Marks 2**

3.9 R.M.S. Value of Complex Waveform

SPPU : Dec.-04

- Consider a wire carrying simultaneously more than one alternating current of different magnitudes and frequencies alongwith certain d.c. current. It is required to calculate resultant **r.m.s. value** i.e. **effective value** of the current.
- Let the wire carries three different currents as shown in the Fig. 3.9.1.
- It is required to obtain resultant I_{rms} through the wire.
- Method is based on heating effect of various currents.
- Let

R = Resistance of wire

I_{rms} = Resultant r.m.s. value of current

and t = Time for which current is flowing

$\therefore H$ = Heat produced by resultant $= I_{rms}^2 \times R \times t$ ...(3.9.1)

- This heat produced is sum of the heats produced by the individual current components flowing for the same time t.

H_1 = Heat produced by d.c. component $= I_{dc}^2 \times R \times t$

H_2 = Heat produced by first a.c. component

$$= I_{rms2}^2 \times R \times t = \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 \times R \times t$$

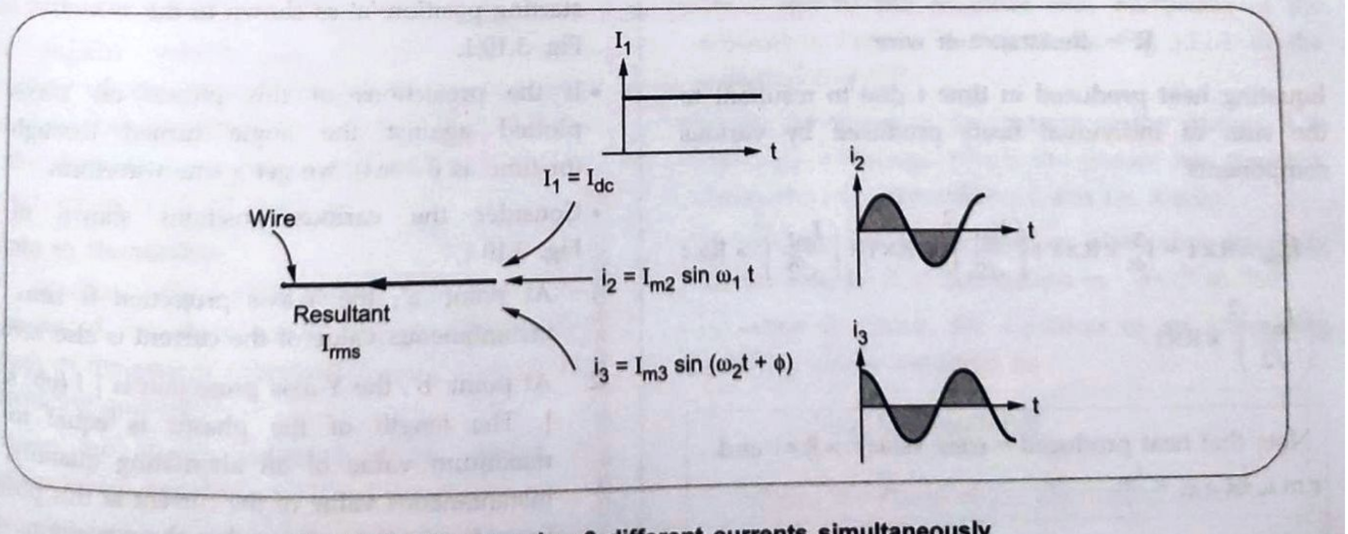


Fig. 3.9.1 Wire carrying 3 different currents simultaneously

H_3 = Heat produced by second a.c. component

$$= I^2_{rms3} \times R \times t = \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 \times R \times t$$

Note that for alternating currents $I_{rms} = I_m / \sqrt{2}$.

- Thus equating the total heat produced to sum of the individual heats produced,

$$H = H_1 + H_2 + H_3$$

i.e. $$I^2_{rms} Rt = I^2_{dc} Rt + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 Rt + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 Rt$$

$$\therefore \quad \boxed{I_{rms} = \sqrt{I^2_{dc} + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2}}$$

The result can be extended to n number of current components flowing through the wire.

Ex. 3.9.1 : ***Find the effective value of a resultant current in a wire which carries simultaneously a direct current of 10 A and alternating current given by,***

$$i = 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin(5\omega t + \pi/3).$$

SPPU : Dec.-04, Marks 4

Sol. : The **effective** value means **r.m.s.** value. It is based on the heating effect of the currents.

I_{dc} = 10 A, I_{m1} = 12 A, I_{m2} = 6 A, I_{m3} = 4 A,

Let, I_{rms} = Resultant r.m.s. value,

R = Resistance of wire.

Equating heat produced in time t due to resultant to the sum of individual heats produced by various components.

$$\therefore I^2_{rms} \times R \times t = I^2_{dc} \times R \times t + \left(\frac{I_{m1}}{\sqrt{2}}\right)^2 \times R \times t + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 \times R \times t + \left(\frac{I_{m3}}{\sqrt{2}}\right)^2 \times R \times t$$

Note that heat produced = (rms value)$^2 \times R \times t$ and r.m.s. of a.c. = $\frac{I_m}{\sqrt{2}}$

$$\therefore I^2_{rms} = 10^2 + \left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 = 198$$

$$\therefore I_{rms} = \sqrt{198} = \mathbf{14.0712\ A}$$

...Effective value of the resultant

Expected Question

1. *How to obtain r.m.s. value if number of alternating currents are passing through same wire ?*

3.10 Phasor Representation of an Alternating Quantity

- The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow in the phasor representation method.
- The length of the line represents the magnitude of the quantity and arrow indicates its direction. This is similar to a vector representation. Such a line is called a **phasor.**

The phasors are assumed to be rotated in anticlockwise direction with a constant speed ω rad/sec.

- One complete cycle of a sine wave is represented by one complete rotation of a phasor. The anticlockwise direction of rotation is purely a conventional direction which has been universally adopted.
- Consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a' as shown in the Fig. 3.10.1.
- If the projections of this phasor on Y-axis are plotted against the angle turned through 'θ', (or time as θ = ω t), we get a sine waveform.
- Consider the various positions shown in the Fig. 3.10.1.

1. At point 'a', the Y-axis projection is zero. The instantaneous value of the current is also zero.
2. At point 'b', the Y-axis projection is [l (ob) sin θ]. The length of the phasor is equal to the maximum value of an alternating quantity. So, instantaneous value of the current at this position is $i = I_m \sin \theta$, represented in the waveform.

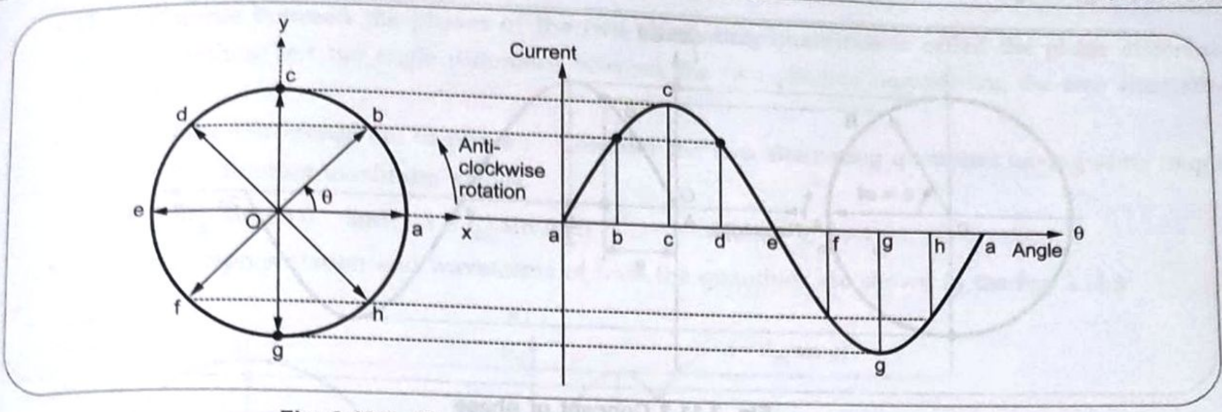


Fig. 3.10.1 Phasor representation of an alternating quantity

3. At point 'c', the Y-axis projection 'oc' represents entire length of the phasor i.e. instantaneous value equal to the maximum value of current I_m.
4. Similarly, at point d, the Y-axis projection becomes $I_m \sin \theta$ which is the instantaneous value of the current at that instant.
5. At point 'e', the Y-axis projection is zero and instantaneous value of the current is zero at this instant.
6. Similarly, at points f, g, h the Y-axis projections give us instantaneous values of the current at the respective instants and when plotted, give us negative half cycle of the alternating quantity.

- Thus, if the length of the phasor is taken equal to the maximum value of the alternating quantity, then its rotation in space at any instant is such that the length of its projection on the Y-axis gives the instantaneous value of the alternating quantity at that particular instant.
- The angular velocity 'ω' in an anticlockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle i.e. $\theta = \omega t$, where $\omega = 2 \pi f$ rad/sec.

Points to Remember :

- In practice, the alternating quantities are represented by their r.m.s. values. Hence, the length of the phasor represents r.m.s. value of the alternating quantity.
- Phasors are always assumed to be rotated in **anticlockwise** direction.
- Two alternating quantities of same frequencies can be represented on same phasor diagram.

Expected Question

1. *What is phasor ? How a rotating phasor represents an alternating quantity ?*

3.11 Concept of Phase and Phase Difference

SPPU : Dec.-99,02,06,08,10,15 May-01,04,05,06,10,16,18

- In the analysis of alternating quantities, it is necessary to know the position of the phasor representing that alternating quantity at a particular instant.
- It is represented in terms of angle θ in radians or degrees, measured from certain reference.

Phase : The phase of an alternating quantity at any instant is the angle φ (in radians or degrees) traveled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

- Let X-axis be the reference axis. So, phase of the alternating current shown in the Fig. 3.11.1 at the instant A is $\phi = 0°$.
- While the phase of the current at the instant B is the angle φ through which the phasor has traveled, measured from the reference axis i.e. X-axis.
- In general, the phase φ of an alternating quantity varies from $\phi = 0$ to 2π radians or $\phi = 0°$ to $360°$.
- In terms of phase, the equation of an alternating quantity can be modified as,

$$e = E_m \sin(\omega t \pm \phi)$$

where ϕ = Phase of the alternating quantity

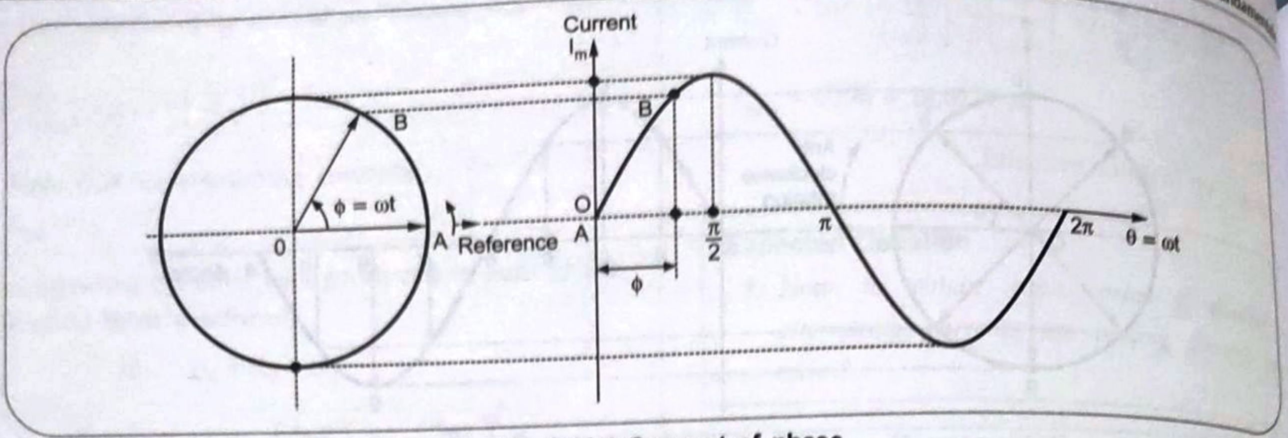


Fig. 3.11.1 Concept of phase

- Let us consider three cases;

Case 1 : $\phi = 0°$:

- When phase of an alternating quantity is zero, it is standard pure sinusoidal quantity having instantaneous value zero at t = 0. This is shown in the Fig. 3.11.2 (a).

Case 2 : Positive phase ϕ :

- When phase of an alternating quantity is positive it means that quantity has **some positive instantaneous value at t = 0.** This is shown in the Fig. 3.11.2 (b).

Case 3 : Negative phase ϕ :

When phase of an alternating quantity is negative it means that quantity has **some negative instantaneous value at t = 0.** This is shown in the Fig. 3.11.2 (c).

1. The **phase** is measured with respect to **reference direction** i.e. positive X-axis direction.
2. The **phase** measured in **anticlockwise** direction is **positive** while the **phase** measured in **clockwise** direction is **negative.**

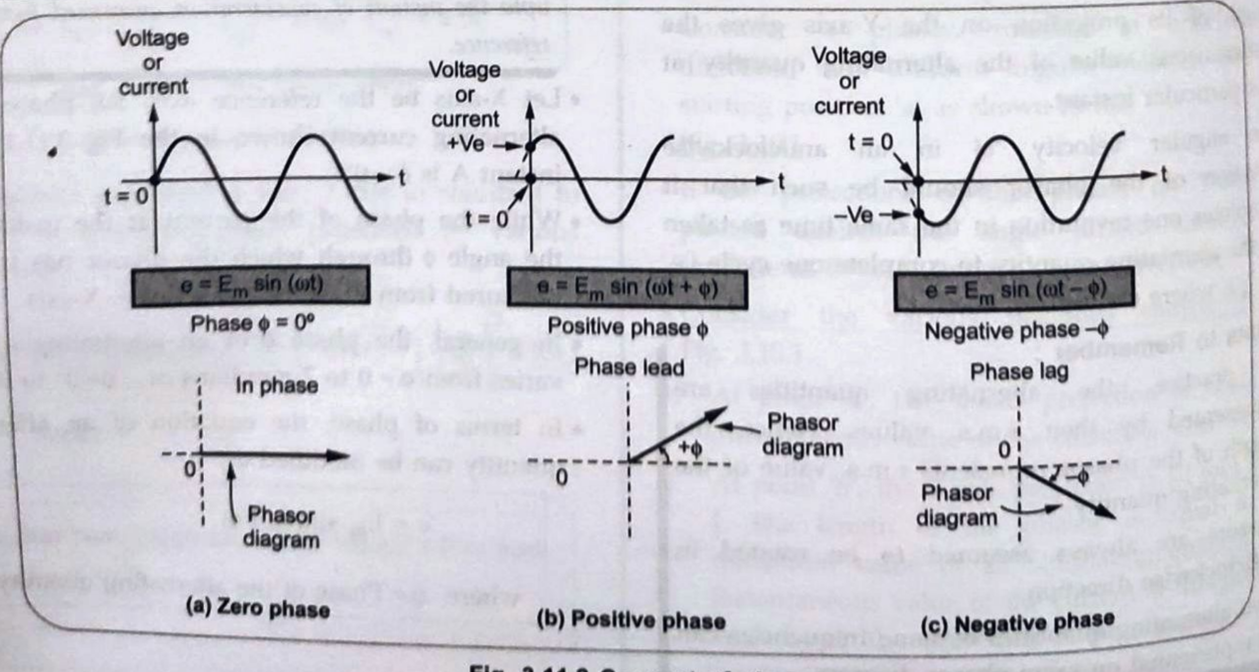


Fig. 3.11.2 Concept of phase

- **The difference between the phases of the two alternating quantities is called the phase difference** which is nothing but the angle difference between the two phasors representing the two alternating quantities.
- **1. Zero Phase Difference i.e. In phase :** Consider the two alternating quantities having same frequency f Hz having different maximum values.

$$e = E_m \sin(\omega t) \quad \text{and} \quad i = I_m \sin(\omega t) \qquad \text{where} \quad E_m > I_m$$

- The phasor representation and waveforms of both the quantities are shown in the Fig. 3.11.3.

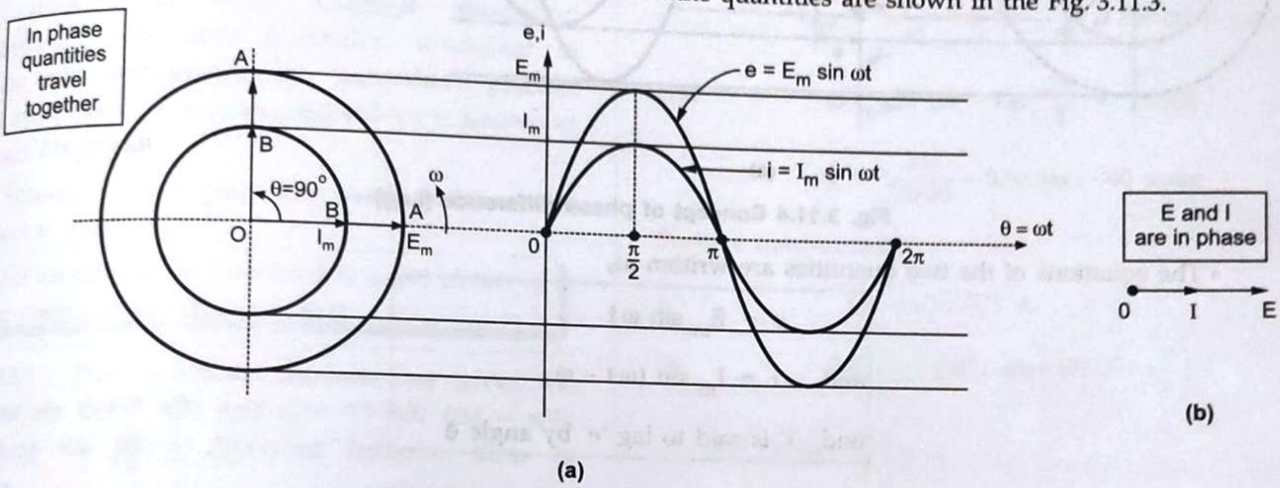


Fig. 3.11.3 In phase alternating quantities

- So, at any instant, we can say that the phase of voltage e will be same as phase of i. The difference between the phases of the two quantities is zero at any instant.

- When such ***phase difference*** *between the two alternating quantities is* ***zero,*** *the two quantities are said to be* ***in phase.***

- In the a.c. analysis, it is not necessary that all the alternating quantities must be always in phase. It is possible that if one is achieving its zero value and at the same instant the other is having some negative value or positive value then such two quantities are said to have **phase difference** between them.

2. Lagging Phase Difference : Consider an e.m.f. having maximum value E_m and current having maximum value I_m.

- Now, when e.m.f. 'e' is at its zero value, the current 'i' has some negative value as shown in the Fig. 3.11.4.
- Thus, there exists a phase difference ϕ between the two phasors.
- Now, as the two are rotating in anticlockwise direction, we can say that current is falling back with respect to voltage, at all the instants by angle ϕ. This is called **lagging phase difference.** The current i is said to lag the voltage e by angle ϕ.
- The current i achieves its maximum and zero values, ϕ angle later than the corresponding maximum and zero values of voltage.

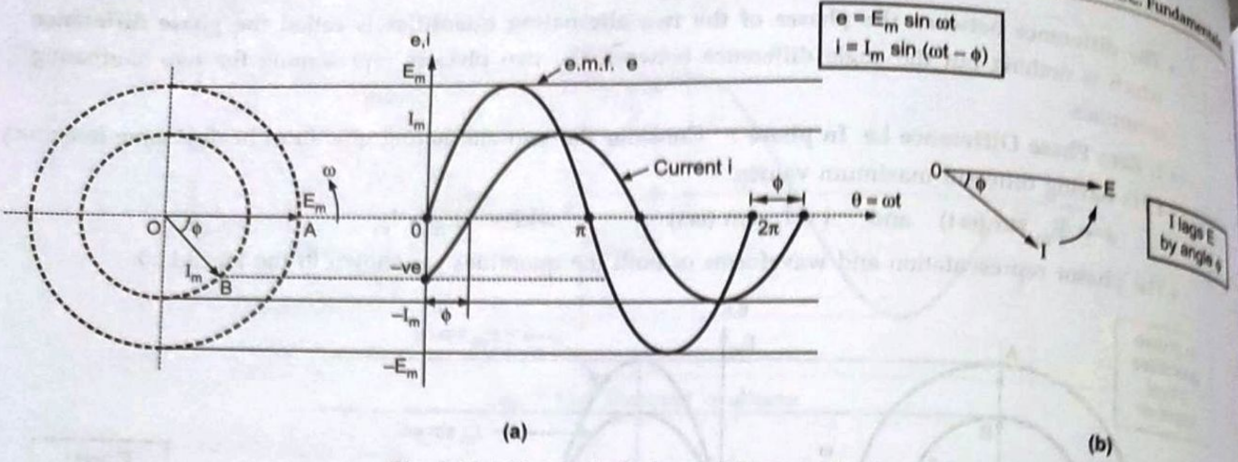


Fig. 3.11.4 Concept of phase difference (Lag)

- The equations of the two quantities are written as,

$$e = E_m \sin \omega t$$

and $\quad i = I_m \sin (\omega t - \phi)$

and 'i' is said to lag 'e' by angle ϕ

3. Leading Phase Difference :

- It is possible in practice that the current 'i' may have some positive value when voltage 'e' is zero. This is shown in the Fig. 3.11.5

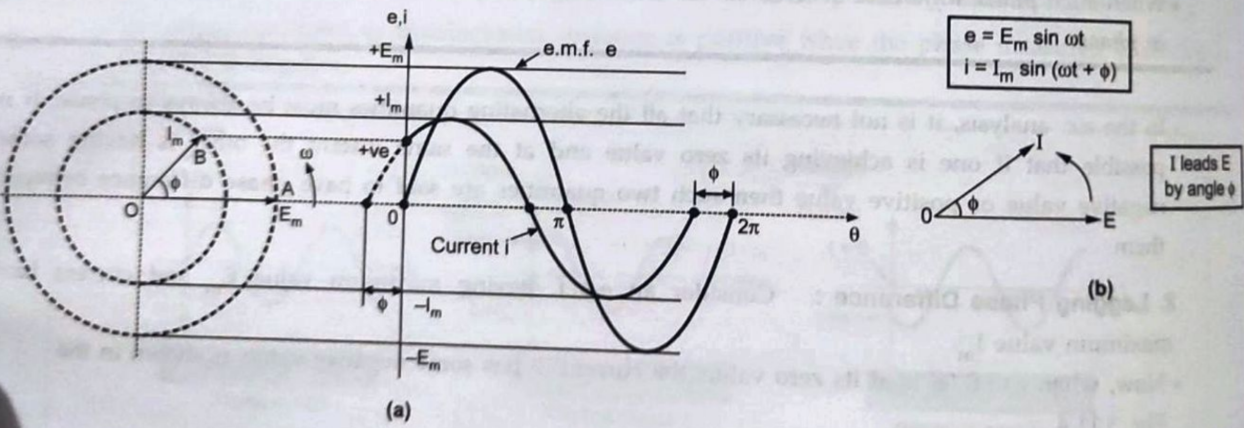


Fig. 3.11.5 Concept of phase difference (Lead)

- It can be seen that there exists a phase difference of ϕ angle between the two. But in this case, current 'i' is ahead of voltage 'e', as both are rotating in anticlockwise direction with same speed.
- Thus, current is said to be leading with respect to voltage and the phase difference is called leading **phase difference.**
- At all instants, current i is going to remain ahead of voltage 'e' by angle 'ϕ'.

- The equations of such two quantities are written as

$$e = E_m \sin \omega t$$

and $i = I_m \sin (\omega t + \phi)$

and 'i' is said to lead 'e' by angle ϕ.

3.11.1 Phasor Diagram

- The diagram in which different alternating quantities of the same frequency, sinusoidal in nature are represented by individual phasors indicating exact phase interrelationships is known as **phasor diagram.**
- All phasors have a particular fixed position with respect to each other .

Phasor diagram can be considered as a still picture of these phasors at a particular instant.

Ex. 3.11.1 : ***Two sinusoidal currents are given by, $i_1 = 10 \sin (\omega t + \pi/3)$ and $i_2 = 15 \sin (\omega t - \pi/4)$. Calculate the phase difference between them in degrees.***

Sol. : The phase of current i_1 is $\pi/3$ radians i.e. 60° while the phase of the current i_2 is $-\pi/4$ radians i.e. $-45°$. This is shown in the Fig. 3.11.6.

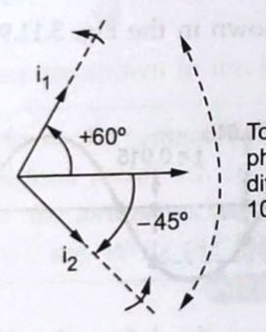


Fig. 3.11.6

Hence the phase difference between the two is,

$$\phi = \theta_1 - \theta_2 = 60° - (-45°) = \mathbf{105°}$$

And i_2 lags i_1.

Ex. 3.11.2 : ***The mathematical expression for the instantaneous value of an alternating current is $i = 7.071 \sin\left(157.08\, t - \frac{\pi}{4}\right)$ A. Find its effective value, periodic time and the instant at which it reaches its positive maximum value. Sketch the wave-form from $t = 0$ over one complete cycle.***

SPPU : Dec.-99, 06, Marks 6

Sol. : The given current is,

$$i = 7.071 \sin\left(157.08\, t - \frac{\pi}{4}\right) \text{ A}$$

Comparing this with,

$$i = I_m \sin(\omega t - \phi) \text{ A},$$

We get, $I_m = 7.071$ A

$\therefore \quad I_{rms} = \frac{7.071}{\sqrt{2}} = \mathbf{5\ A}$ ... Effective value

and $\omega = 157.08$ i.e. $\frac{2\pi}{T} = 157.08$

$\therefore \quad T = \frac{2\pi}{157.08} = 0.04 \text{ sec} = \mathbf{40\ msec}$

Positive maximum is,

$$I_m = +7.071 \text{ A}$$

i.e $\quad 7.071 = 7.071 \sin\left(157.08\, t - \frac{\pi}{t}\right)$

$\therefore \quad \sin\left(157.08\, t - \frac{\pi}{t}\right) = 1$

i.e. $\quad 157.08\, t - \frac{\pi}{t} = 1.5707$ Use radian mode

$\therefore \quad 157.08\, t = 2.3561$

i.e. $\quad t = 0.015 \text{ sec} = \mathbf{15\ msec}$

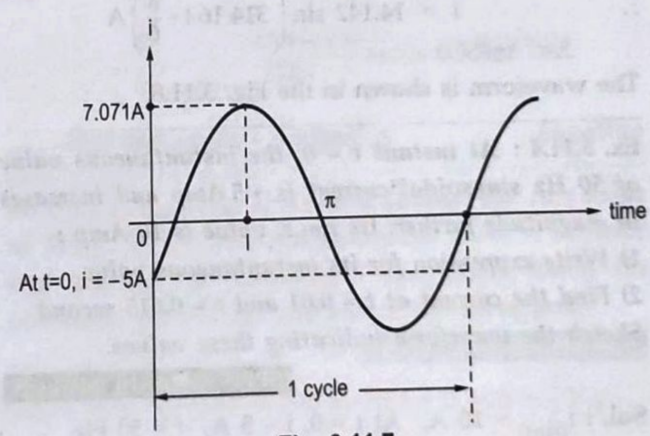


Fig. 3.11.7

Ex. 3.11.3 : ***A 50 Hz alternating current having rms value 10 A has instanteneous value of -7.07 A at $t = 0$. Write down the equation for current and sketch the waveform stating all currents and phase angle.***

SPPU : May-16, Marks 6

Sol. : $f = 50$ Hz, $I = 10$ A, $i = -7.07$ A at $t = 0$,

$I_m = \sqrt{2}\, I_{RMS} = \sqrt{2} \times I = \sqrt{2} \times 10 = 14.142$ A,

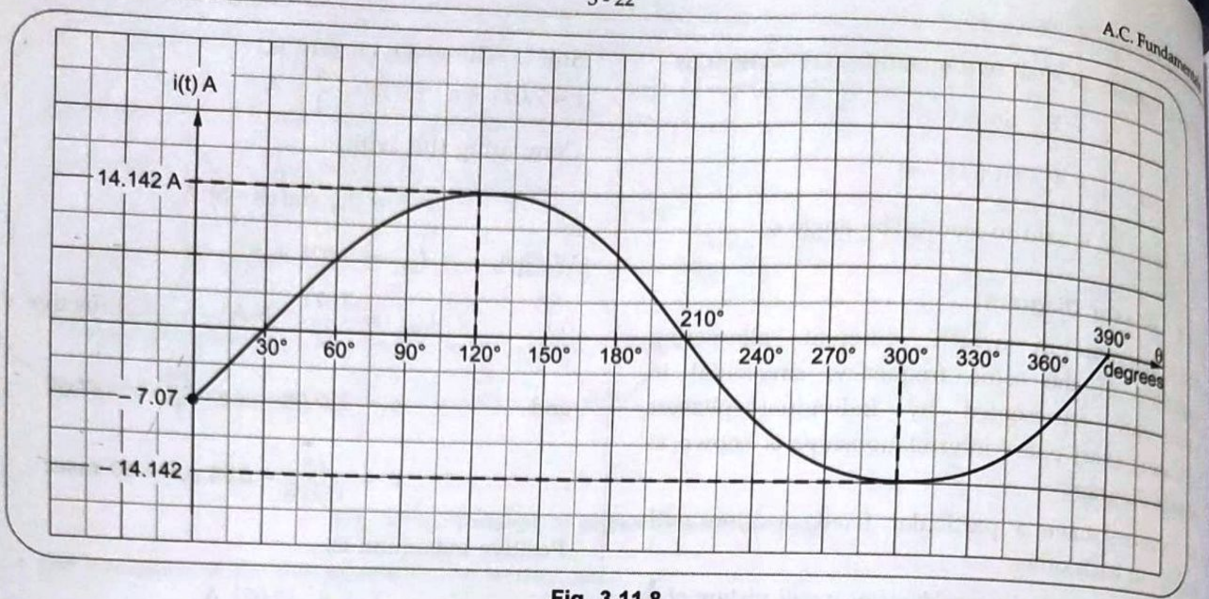


Fig. 3.11.8

$\omega = 2\pi f = 314.16$ rad/s

The general equation is,

$$i = I_m \sin(\omega t + \phi)$$

$$\therefore \quad i = 14.142 \sin(314.16\, t + \phi)$$

To find ϕ use $i = -7.07$ A at $t = 0$

$\therefore\ -7.07 = 14.142 \sin(\phi)$ i.e. $\phi = -30°$ or $\dfrac{\pi}{6}$ rad

$$\therefore \quad i = 14.142 \sin\left(314.16\, t - \frac{\pi}{6}\right) \text{A}$$

The waveform is shown in the Fig. 3.11.8.

Ex. 3.11.4 : ***At instant t = 0, the instantaneous value of 50 Hz sinusoidal current is +5 Amp and increases in magnitude further. Its r.m.s. value is 10 Amp :***
1) Write expression for its instantaneous value.
2) Find the current at t = 0.01 and t = 0.015 second Sketch the waveform indicating these values.

SPPU : May-10, Marks 8

Sol. : $I_{(rms)} = 10$ A, At $t = 0$, $i = 5$ A, $f = 50$ Hz

$$\therefore \quad I_m = \sqrt{2}\, I_{(rms)} = 14.1421 \text{ A}$$

As current has positive value at t = 0, the current has leading phase of ϕ

$i = I_m \sin(\omega t + \phi)$ ... $\omega = 2\pi f = 100\pi$ rad/sec

$\therefore\ 5 = 14.1421 \sin(0 + \phi)$ i.e. $\sin\phi = \dfrac{5}{14.1421}$

$$\therefore \quad \phi = 20.704° = 0.3613 \text{ rad}$$

1) $i = 14.1421 \sin(100\pi t + 0.3613)$ **A** ...Expression

2) At $t = 0.01$ sec,

$i = 14.1421 \sin(100\pi \times 0.01 + 0.3613) = -5$ **A**

At $t = 0.015$ sec,

$i = 14.1421 \sin(100\pi \times 0.015 + 0.3613) = -13.229$ **A**

The waveform is shown in the Fig. 3.11.9.

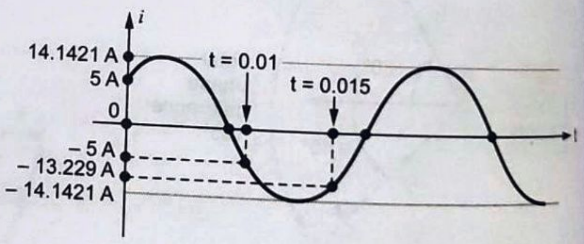


Fig. 3.11.9

Ex. 3.11.5 : *Sketch waveforms of currents and find its r.m.s. value and average value for the equation :*
a) $i_1 = 15 \sin(314.159\, t)$ *and*
b) $i_2 = 10 \sin(314.159\, t - \pi/2)$

Sol. :

For i_1, $I_{m1} = 15$ A,

$I_{RMS} = \dfrac{15}{\sqrt{2}}$ A, $I_{av} = 0.637 \times 15 = 9.555$ A

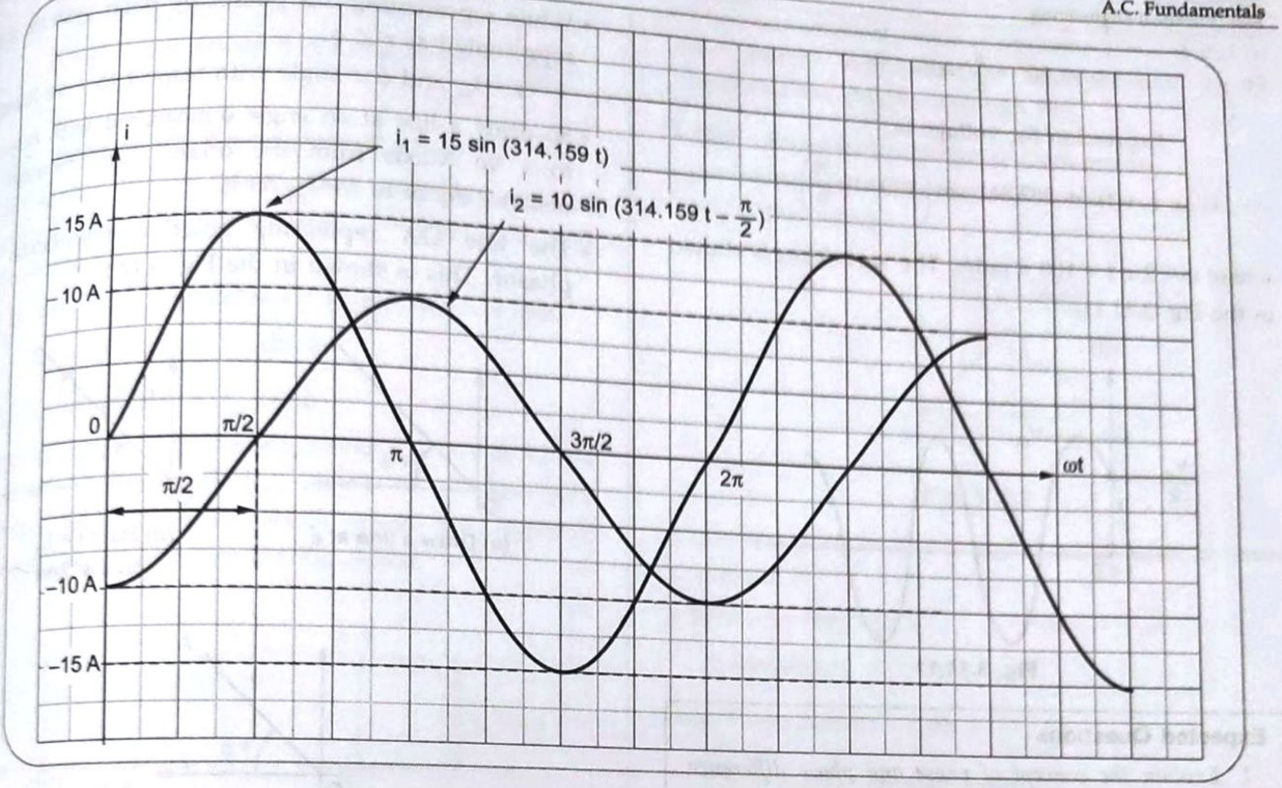


Fig. 3.11.10

For i_2, $I_{m2} = 10$ A,

$$I_{RMS} = \frac{10}{\sqrt{2}}\ \text{A},\ I_{av} = 0.637 \times 10 = \mathbf{6.37\ A}$$

The waveforms are shown in the Fig. 3.11.10.

Ex. 3.11.6 : ***A 50 Hz sinusoidal current has peak factor 1.4 and form factor 1.1. Its average value is 20 Amp. The instantaneous value of current is 15 Amp at t = 0 sec. Write the equation of current.***

SPPU : May-05, Marks 6

Sol. : $f = 50$ Hz, $K_p = 1.4$, $K_f = 1.1$, $I_{av} = 20$ A

$$K_p = \frac{I_m}{I_{rms}} \text{ while } K_f = \frac{I_{rms}}{I_{av}}$$

$$\therefore \quad K_f = \frac{(I_m / K_p)}{I_{av}} \quad \text{...from } K_p$$

$$\therefore \quad 1.1 = \frac{I_m / 1.4}{20}$$

i.e. $I_m = 20 \times 1.1 \times 1.4 = 30.8$ A

The current has i = 15 A at t = 0 hence its equation is,

$$i = I_m \sin(\omega t + \phi) = I_m \sin(2\pi f t + \phi)$$

At $t = 0$, $15 = 30.8 \sin (0 + \phi)$

$$\therefore \quad \phi = \sin^{-1}\left(\frac{15}{30.8}\right) = \mathbf{29.1444°}$$

$$= \frac{29.1444 \times \pi}{180} \text{ rad} = \mathbf{0.50866\ rad.}$$

$$\therefore i = 30.8 \sin(100\ \pi\ t + 0.50866) \text{ A} \quad \text{...Equation}$$

Ex. 3.11.7 : ***A 50 Hz sinusoidal voltage has rms value of 200 V. At t = 0 the instantaneous value is positive and half of its maximum value. Write down the expression for voltage and sketch the waveform.***

SPPU : Dec.-15, Marks 6

Sol. : $f = 50$ Hz, $V = 200$ V, At $t = 0$, $v(t) = \frac{1}{2} V_m$,

$$V_m = \sqrt{2} \times V = 200 \times \sqrt{2} = 282.842 \text{ V}$$

Let the equation of voltage is,

$$v(t) = V_m \sin (\omega t + \phi) \text{ and at } t = 0,\ v(t) = \frac{1}{2} V_m$$

$$\therefore \quad \frac{282.842}{2} = 282.842 \sin (\phi)$$

i.e. $\phi = 30^\circ = \frac{\pi}{6}$ rad

$\therefore$ Expression for voltage is,

$$v(t) = 282.842 \sin\left(100\pi t + \frac{\pi}{6}\right)$$

where $\omega = 2\pi f = 100\pi$ rad/s. The waveform is shown in the Fig. 3.11.11.

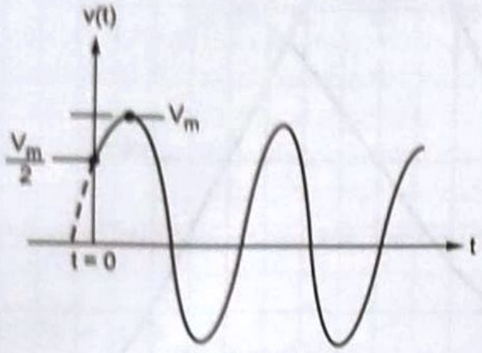


Fig. 3.11.11

Expected Questions

1. *Explain the concept of phase and phase difference in alternating quantities.*

   SPPU : Dec.-06,08, Marks 4

2. *What is phasor diagram ?*
3. *Explain the concept of lagging, leading and in phase phasors. Draw the respective waveform and phasor diagram for the same.*

   SPPU : May-18, Marks 6

3.12 Mathematical Representation of Phasor

- The algebraic operations such as addition, subtraction etc. with waveforms are much complicated and time consuming. Hence it is necessary to represent the phasors mathematically.
- Any phasor can be represented mathematically in two ways,

1) Polar co-ordinate system and

2) Rectangular co-ordinate system.

3.12.1 Polar Co-ordinate System

- Consider an alternating current given by, $i = I_m \sin(\omega t + \phi)$
- Thus its maximum value is I_m and phase is $+\phi$. The phase ϕ is always measured with respect to positive X-axis direction.
- While representing this phasor by polar system, it is represented as $r \angle \phi$.

  $r = I_m$ and ϕ is angle with respect to + ve X-axis.
- So draw a line at an angle ϕ measured with respect to + ve X-axis from the origin. And measure distance equal to $r = I_m$ on it.
- The line OA represents polar representation of phasor. This is shown in the Fig. 3.12.1.

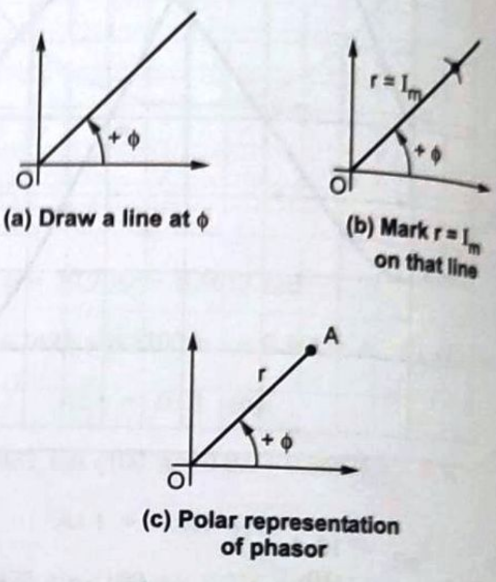


Fig. 3.12.1

- The angle ϕ can be positive or negative.
- The **positive** ϕ is measured in **anticlockwise** direction while the **negative** ϕ is measured in **clockwise** direction.
- The length r is always positive and called **magnitude** of the phasor. The angle ϕ is called **phase** of that phasor.
- Thus mathematically the polar representation of a phasor is,

$$\boxed{\text{Polar representation} = r \angle \pm \phi}$$

- **Practically instead of $r = I_m$, r.m.s. value is used as the magnitude r.**
- The polar form of an alternating quantity can be easily obtained from its instantaneous equation directly.

If $e = E_m \sin(\omega t \pm \phi)$ then polar form is,

$$E = E \angle \pm \phi \text{ where E is r.m.s. value} = \frac{E_m}{\sqrt{2}}.$$

3.12.2 Rectangular Co-ordinate System

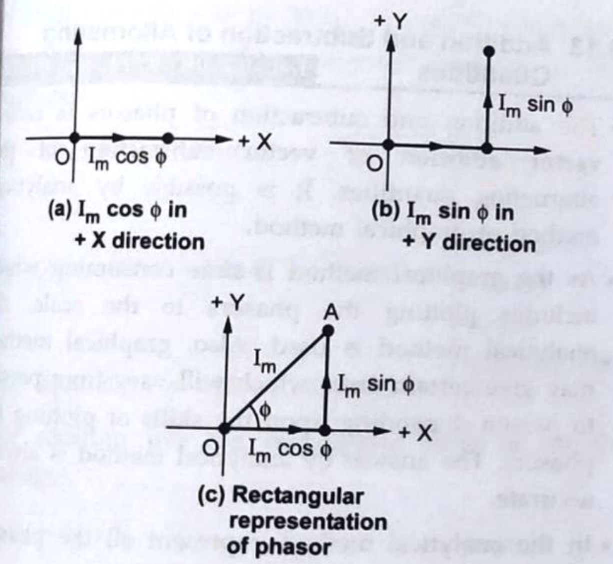
- Mathematically an alternating quantity can be divided into two components, X-component and Y-component.
- If an alternating current is $i = I_m \sin(\omega t + \phi)$ then,

$$\text{X-component} = I_m \cos \phi$$

and

$$\text{Y-component} = I_m \sin \phi$$

- Thus to represent the phasor, travel $(I_m \cos \phi)$ in +X direction then travel $(I_m \sin \phi)$ in +Y direction. Joining final point to origin gives the required phasor OA as shown in the Fig. 3.12.2.



(a) $I_m \cos \phi$ in + X direction

(b) $I_m \sin \phi$ in + Y direction

(c) Rectangular representation of phasor

Fig. 3.12.2

- The X and Y components can be positive or negative. To indicate that X and Y components are perpendicular to each other, the operator 'j' is used in mathematical representation of phasor in rectangular co-ordinate system.
- Thus mathematically the rectangular representation of a phasor is,

$$\boxed{\text{Rectangular representation} = \pm X \pm j\,Y}$$

- **The mathematical value of operator 'j' = $\sqrt{-1}$ but in the phasor representation multiplication by 'j' indicates the rotation through 90°.**

3.12.3 Polar to Rectangular Conversion

- Let a phasor is represented in polar form as shown in the Fig. 3.12.3.

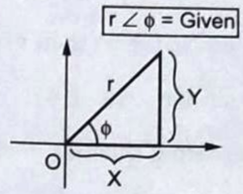


Fig. 3.12.3

- It is necessary to find X and Y components in terms of r and ϕ.
- From the Fig. 3.12.3,

$$\text{X component} = r \cos \phi$$

and

$$\text{Y component} = r \sin \phi$$

$$\therefore \quad \boxed{\text{Rectangular representation} = r \cos \phi + j\, r \sin \phi}$$

3.12.4 Rectangular to Polar Conversion

- Let a phasor is represented in rectangular form X + j Y, as shown in the Fig. 3.12.4.

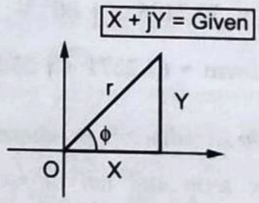


Fig. 3.12.4

- It is necessary to find r and ϕ in terms of X and Y.
- From the Fig. 3.12.4,

$$r = \sqrt{X^2 + Y^2}$$

and

$$\phi = \tan^{-1} \frac{Y}{X}$$

$$\therefore \quad \boxed{\begin{aligned} \text{Polar representation} &= r \angle \phi \\ &= \sqrt{X^2 + Y^2} \angle \tan^{-1} \frac{Y}{X} \end{aligned}}$$

The polar form always gives r.m.s. value of an alternating quantity.

Important Note : To obtain polar form from the instantaneous equation, express the given equation in sine form instead of cosine form.

If, $e = E_m \cos(\omega t \pm \phi)$ then express it as,

$$e = E_m \sin(\omega t + 90° \pm \phi)$$

$\therefore$ Phase of alternating quantity = $90° \pm \phi$.

Instead of using above relations, use the polar to rectangular (P → R) and rectangular to polar (R → P) functions available on calculator for the required conversions.

Ex. 3.12.1 : ***Write the polar form of the voltage given by, $V = 100 \sin(100\pi t + \pi/6)$ V. Obtain its rectangular form.***

Sol. : $V_m = 100$ V and $\phi = +\frac{\pi}{6}$ rad $= +30°$,

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 70.7106 \text{ V}$$

$\therefore$ In polar form = **$70.7106 \angle +30°$ V**

$\therefore$ Rectangular form = **$61.2371 + j\ 35.3553$ V**

Key Point *The r.m.s. value of an alternating quantity exists in its polar form and not in rectangular form. Thus to find r.m.s. value of an alternating quantity express it in polar form.*

Ex. 3.12.2 : ***Find r.m.s. value and phase of the current $I = 25 + j\ 40$ A.***

Sol. : The r.m.s. value is not 25 or 40 as it exists in polar form.

Converting it to polar form,

$$I = 47.1699\angle 57.99° \text{ A} = I_{rms} \angle \phi \text{ A}$$

$\therefore$ r.m.s. value of current = **47.1699 A**

Phase = **57.99°**

Ex. 3.12.3 : ***A voltage is defined as $-E_m \cos \omega t$. Express it in polar form.***

Sol. : To express a voltage in polar form express it in the form, $e = E_m \sin \omega t$

Now $e = -E_m \cos \omega t = -E_m \sin\left(\omega t + \frac{\pi}{2}\right)$

as $\sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t = E_m \sin\left(\omega t + \pi + \frac{\pi}{2}\right)$

$= E_m \sin\left(\omega t + \frac{3\pi}{2}\right)$ as $\sin(\pi + \theta) = -\sin\theta$

Now it can be expressed in polar form as,

$$e = E_m \angle +\frac{3\pi}{2} \text{ rad} = E_m \angle +270° \text{ V}$$

But +270° phase is nothing but −90°

hence $e = E_m \angle -90°$ **V**

3.13 Addition and Subtraction of Alternating Quantities

Dec.-03, 08, 09, 11, May-01, 12

- The addition and subtraction of phasors is called **vector addition** or **vector subtraction** of the alternating quantities. It is possible by analytical method or graphical method.
- As the graphical method is time consuming which includes plotting the phasors to the scale, the analytical method is used. Also, graphical method may give certain error which will vary from person to person depending upon the skills of plotting the phasors. The answer by analytical method is always accurate.
- In the analytical method, represent all the phasors in the **rectangular form.**
- Let $P = X_1 + j\ Y_1$ and $Q = X_2 + j\ Y_2$
- Then analytically while adding P and Q, their X components get added and corresponding Y components get added. Hence the resultant R is,

$$\boxed{R = P + Q = (X_1 + X_2) + j\ (Y_1 + Y_2)}$$

- While subtracting, their X components get subtracted and corresponding Y components get subtracted. Hence the resultant is,

$$\boxed{R = P - Q = (X_1 - X_2) + j\ (Y_1 - Y_2)}$$

Important : While performing addition and subtraction use rectangular form of representation of phasors.

- The result of the addition and subtraction, finally can be expressed in the polar form, but their individual polar forms cannot be used to perform addition and subtraction.

Ex. 3.13.1 : ***Find the resultant of three voltages given by $v_1 = 10 \sin \omega t$, $v_2 = 20 \sin(\omega t - \pi/4)$ and $v_3 = 30 \cos(\omega t + \pi/6)$*** **SPPU : Dec.-03, May-12, Marks 6**

Sol. : Express all the voltages in terms of $\sin(\omega t \pm \phi)$ first,

$v_1 = 10 \sin \omega t$

$v_2 = 20 \sin(\omega t - \pi/4) = 20 \sin(\omega t - 45°)$

$v_3 = 30 \cos(\omega t + \pi/6) = 30 \sin(90° + \omega t + \pi/6)$

$= 30 \sin(\omega t + 90° + 30°) = 30 \sin \quad ...\sin(90 + \theta) = \cos \theta$

The polar form and rectangular form of all the voltages,

$\bar{V}_1 = 10 \angle 0° \text{ V} = 10 + j0 \text{ V}$

$\bar{V}_2 = 20 \angle -45° \text{ V} = 14.142 - j\,14.1421 \text{ V}$

$\bar{V}_3 = 30 \angle 120° \text{ V} = -15 + j\,25.9807 \text{ V}$

} Using peak values of respective voltage hence resultant will

For addition use the rectangular form of all the voltages.

$\therefore \bar{V}_R = \bar{V}_1 + \bar{V}_2 + \bar{V}_3$

$= 10 + j0 + 14.142 - j\,14.1421 - 15 + j\,25.9807$

$= 9.142 + j\,11.8386 = \mathbf{14.9575 \angle 52.3239° \text{ V}}$

So resultant has peak value 14.9575 V and phase 52.3239°, so its equation is,

$$v_R = \mathbf{14.9575 \sin(\omega t + 52.3239°) \text{ V}}$$

Ex. 3.13.2 : ***In a prallale circuit the three branches, the instantaneous branch currents are represented by $i_1 = 10 \sin \omega t$, $i_2 = 20 \sin(\omega t + \pi/3)$, $i_3 = 12 \sin(\omega t + \pi/6)$ Write down the expression for the total instantaneous current in the form $i = i_m \sin(\omega t + \phi)$.*** **SPPU : Dec.-11, Marks 8**

Sol. : In parallel circuit, $\bar{I}_R = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

The polar forms from the given equations are,

$I_1 = \frac{10}{\sqrt{2}} \angle 0° \text{A}, \; I_2 = \frac{20}{\sqrt{2}} \angle +60° \text{ A}, \; I_3 = \frac{12}{\sqrt{2}} \angle -30° \text{ A}$

For addition convert the currents into rectangular form.

$\bar{I}_1 = 7.071 + j0 \text{ A},$

$\bar{I}_2 = 7.071 + j\,12.247 \text{ A}, \quad \bar{I}_3 = 7.3484 - j\,4.242 \text{ A}$

$\therefore \bar{I}_R = 21.4904 + j\,8.005 \text{ A} = 22.9328 \angle 20.43° \text{ A}$

$\therefore$ RMS value of total current = 22.9328 A,

Phase = 20.43°

$\therefore$ Maximum of total current = $\sqrt{2} \times 22.9328 = 32.4318$ A

$\therefore$ Expression for the total instantaneous current is,

$$i_R = \mathbf{32.4318 \sin(\omega t + 20.43°) \text{ A}}$$

Note In this example, in polar forms r.m.s. values of currents are used hence resultant gives r.m.s. value which is to be multiplied by $\sqrt{2}$ to obtain peak value.

Ex. 3.13.3 : ***A circuit consist of three parallel branches. The branch currents are given as $i_1 = 10 \sin \omega t$, $i_2 = 20 \sin(\omega t + 60°)$ and $i_3 = 7.5 \sin(\omega t - 30°)$. Find the resultant current and express it in the form $i = I_m \sin(\omega t \pm \phi)$. If the supply frequency is 50 Hz, calculate the resultant current when 1) t = 0 sec. and 2) t = 0.001 sec.*** **SPPU : Dec.-09, Marks 6**

Sol. : Comparing each current with $I = i_m \sin(\omega t + \phi)$

$I_{m1} = 10 \text{ A}, \; \phi_1 = 0°, \; I_{m2} = 20 \text{ A}, \; \phi_2 = 60°,$

$I_{m3} = 7.5 \text{ A}, \; \phi_3 = -30°$

The r.m.s. values of currents are $I = \frac{I_m}{\sqrt{2}}$

$$\therefore \quad \bar{I}_1 = \frac{10}{\sqrt{2}} \angle 0° \text{ A} = 7.071 + j\,0 \text{ A}$$

$$\therefore \quad \bar{I}_2 = \frac{20}{\sqrt{2}} \angle 60° \text{ A} = 7.071 + j\,12.2474 \text{ A}$$

$\therefore \quad \bar{I}_3 = \frac{7.5}{\sqrt{2}} \angle -30° \text{ A} = 4.5928 - j\,2.6516 \text{ A}$

$\bar{I}_T = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 18.7348 + j\,9.5958 \text{ A}$

$= 21.0492 \angle 27.121° \text{ A}$

$\therefore I_{Tm}$ = Maximum value = $\sqrt{2} \times 21.0492 = 29.768$ A

$\therefore i_T = I_{Tm} \sin(\omega t + \phi_T) = 29.768 \sin(\omega t + 27.121°)$ A

$\omega = 2\pi f = 100\pi$ rad/sec. ...f = 50 Hz

1) At t = 0 sec, $i_T = 29.768 \sin(27.121°) =$ **13.5703 A**

2) At t = 0.001 sec,

$i_T = 29.768 \sin(100\,\pi \times 0.001 + 27.121°)$

Now convert 27.121° to radians $= \frac{27.121° \times \pi}{180°}$

$= 0.47335$ rad

$\therefore i_T = 29.768 \sin(0.1\,\pi + 0.47335)$

= **21.0937 A** ...Calculate sin in radian mode

3.14 Multiplication and Division of Phasors

May-01

- The multiplication and division of phasors is performed using **polar form** of representation.
- Let P and Q be the two phasors such that,

$$P = X_1 + jY_1 \quad \text{and} \quad Q = X_2 + jY_2$$

- To obtain the multiplication P×Q both must be expressed in polar form

$\therefore \quad P = r_1 \angle \phi_1$ and $Q = r_2 \angle \phi_2$

Then $\boxed{P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2] = [r_1 \times r_2] \angle \phi_1 + \phi_2}$

Thus in multiplication of complex numbers in polar form, the magnitudes get multiplied while their angles get added.

- The result then can be expressed back to rectangular form, if required.
- Now consider the division of the phasors P and Q.

$$\boxed{\frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2}$$

Thus in division of complex numbers in polar form, the magnitudes get divided while their angles get subtracted.

Important : While performing multiplication and division use polar form of representation of phasors.

- The final result then can be expressed in rectangular form if required.

Remember :

- While **addition and subtraction,** use rectangular form.
- While **multiplication and division,** use polar form.

Ex. 3.14.1 : ***Perform the multiplication and division of the phasors, A = 10 + j 15 and B = 20 ∠ – 30°***

Sol. : Express both in polar form for performing multiplication and division.

$\therefore \quad A = 10 + j\,15 = 18.0277 \angle 56.31°,$

$B = 20 \angle -30°$

$\therefore \quad R = A \times B = (18.0277 \angle 56.31°)(20 \angle -30°)$

$= (18.0277 \times 20) \angle 56.31° - 30°$

$= \mathbf{360.554 \angle 26.31°}$

And $\quad R = \frac{A}{B} = \frac{18.0277 \angle 56.31°}{20 \angle -30°}$

$= \left(\frac{18.0277}{20}\right) \angle 56.31° - (-30°) = \mathbf{0.90138 \angle 86.31°}$

3.14.1 Another Way of Complex Number Representation

- In complex number analysis, the number may be expressed as $|r|e^{j\phi}$ where ϕ may be in degrees or radians. The mathematically $e^{j\phi} = \cos\phi + j\sin\phi$ while $e^{-j\phi} = \cos\phi - j\sin\phi$. Hence corresponding rectangular form can be obtained.
- For example if a particular current is given as $50\,e^{-j30°}$ then,

$50\,e^{-j30°} = 50\,[\cos 30° - j \sin 30°] = 43.3012 - j\,25$ A

- While a particular voltage given by $150\,e^{+j100°}$ is,

$$150\, e^{+j100^\circ} = 150\,[\cos 100^\circ + j \sin 100^\circ]$$
$$= -26.047 + j\, 147.721 \text{ V}$$

- If ϕ is given in radians, sin and cos must be calculated in radian mode.
- Infact $|r|e^{\pm j\phi}$ can be directly expressed in the polar form as $|r|\angle \pm\phi$ where ϕ may be in degrees or radians.

$\therefore \quad 50\, e^{-j30^\circ} = 50\angle -30^\circ \text{ A}$ while

$150\, e^{+j100^\circ} = 150\angle +100^\circ \text{ V}$

- This can be crosschecked by using rectangular to polar conversion.

Thus, $\boxed{|r|\, e^{\pm j\phi} = |r| \angle \pm\phi}$

Ex. 3.14.2 : ***Two currents $I_1 = 10\, e^{j50^\circ}$ and $I_2 = 5\, e^{-j100^\circ}$ flow in a 1-ph A.C. circuit. Estimate :- i) $I_1 + I_2$ ii) $I_1 - I_2$ and iii) I_1/I_2, in complex form.***

SPPU : May-01, Marks 6

Sol. : $I_1 = 10\, e^{j\,50^\circ}$ A and $I_2 = 5\, e^{-j\,100^\circ}$ A

Now $I_1 = 10\,[\cos 50^\circ + j \sin 50^\circ]$

$= 6.4278 + j\,7.66 \text{ A} = 10 \angle 50^\circ \text{ A}$

$I_2 = 5\,[\cos 100^\circ - j \sin 100^\circ]$

$= -0.8682 - j\,4.924 \text{ A} = 5 \angle -100^\circ \text{ A}$

i) $I_1 + I_2 = 5.5596 + j\,2.736 \text{ A} = \mathbf{6.196 \angle 26.2^\circ\ A}$

ii) $I_1 - I_2 = 7.296 + j\,12.584 \text{ A} = \mathbf{14.546 \angle 60^\circ\ A}$

iii) $I_1/I_2 = \dfrac{10\angle 50^\circ}{5\angle -100^\circ} = \mathbf{2 \angle +150^\circ\ A}$

Formulae At a Glance

- One cycle corresponds to 2π radians or 360°.
  - $f = \dfrac{1}{T}$ Hz
  - $\omega = 2\pi f$ radians/sec. or $\omega = \dfrac{2\pi}{T}$ radians/sec.
  - $\theta = \omega t$ radians or $\theta = 2\pi f t$ radians
- Equation of an alternating quantity,

$e = E_m \sin\theta$ volts

where E_m = Amplitude or maximum or peak value of the voltage.

e = Instantaneous value of an alternating voltage

$e = E_m \sin(\omega t)$

$= E_m \sin(2\pi f t)$

$= E_m \sin\left(\dfrac{2\pi}{T} t\right)$

- *In all the above equations, the angle θ is expressed in **radians**. Hence, while calculating the instantaneous value of the e.m.f. or current, it is necessary to calculate the sine of the angle expressed in radians or replace π by 180° to obtain angle in degrees.*

4. Find square root of average value which gives r.m.s. value.
   - For sinusoidal alternating quantities,

$I_{r.m.s.} = \dfrac{I_m}{\sqrt{2}} = 0.707\, I_m$ and $V_{r.m.s.} = \dfrac{V_m}{\sqrt{2}} = 0.707\, V_m$

   - For a purely sinusoidal waveform, the average value is,

$I_{av} = 0.637\, I_m$ and $V_{av} = 0.637\, V_m$

- Form factor, $K_f = \dfrac{\text{r.m.s. value}}{\text{average value}}$

$K_f = \dfrac{0.707\, I_m}{0.637\, I_m} = \mathbf{1.11}$ for sinusoidally varying quantity

- Peak factor, $K_p = \dfrac{\text{maximum value}}{\text{r.m.s. value}}$

$K_p = \dfrac{I_m}{0.707\, I_m} = \mathbf{1.414}$ for sinusoidal waveform

- $e = E_m \sin(\omega t \pm \phi)$ where

ϕ = Phase of the alternating quantity

  - $e = E_m \sin\omega t$ and $i = I_m \sin(\omega t + \phi)$ and 'i' is said to lead 'e' by angle ϕ
  - $e = E_m \sin\omega t$ and $i = I_m \sin(\omega t - \phi)$ and 'i' is said to lead 'e' by angle ϕ
- If $e = E_m \sin(\omega t \pm \phi)$ then polar form is, $E = E \angle \pm\phi$ where E is r.m.s. value $= \dfrac{E_m}{\sqrt{2}}$.
- Polar representation $= r \angle \pm\phi$ and Rectangular representation $= \pm X \pm jY$
- P→R conversion Rectangular representation $= r\cos\phi + j\, r\sin\phi$

- R→ P conversion,Polar representation = $r \angle \phi$
$= \sqrt{X^2 + Y^2} \angle \tan^{-1} \frac{Y}{X}$
- To obtain polar form from the instantaneous equation, express the given equation in sine form instead of cosine form.

If, $e = E_m \cos(\omega t \pm \phi)$ then express it as,
$e = E_m \sin(\omega t + 90° \pm \phi)$

$\therefore$ Phase of alternating quantity = $90° \pm \phi$.

Important : While performing addition and subtraction use rectangular form of representation of phasors and while performing multiplication and division use polar form of representation of phasors.

- To obtain the multiplication $P \times Q$, both must be expressed in polar form

$\therefore \quad P = r_1 \angle \phi_1$ and $Q = r_2 \angle \phi_2$
- Multiplication

$$P \times Q = [r_1 \angle \phi_1] \times [r_2 \angle \phi_2] = [r_1 \times r_2] \angle \phi_1 + \phi_2$$

- Division $\quad \frac{P}{Q} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \left|\frac{r_1}{r_2}\right| \angle \phi_1 - \phi_2$

Examples for Practice

Ex. 1 : *A sinusoidal voltage of 50 Hz has a maximum value of $200\sqrt{2}$ volts. At what time measured from a positive maximum value will the instantaneous voltage be equal to 141.4 volts ?* **Dec.-01**

[Ans. : 3.314×10^{-3} sec.]

Ex. 2 : *An alternating current varying sinusoidally at 50 Hz has an rms value of 10 amps. Write down equation for instantaneous value of current and find the value of current at,*

i) 0.0025 seconds after passing through positive maximum value.

ii) 0.0075 seconds after passing thorugh zero value and increasing negatively. **May-99**

[Ans. : 10 A, – 10 A]

Ex. 3 : *A sinusoidally varying alternating current has r.m.s. value of 20 A and periodic time of 20 milliseconds. If the waveform of this current enters into its positive half cycle at t = 0, find the instantaneous values of the current at t_1 = 6 milliseconds and t_2 = 12 milliseconds.* **May-02**

[Ans. : i = 26.899 A, i = – 16.625 A]

Ex. 4 : *A sinusoidal current of frequency 25 Hz has a maximum value of 100 A. How long will it take for the current to attain value of 20 A and 50 A, starting from zero. Sketch the waveform and show the times and currents.* **Dec.-04**

[Ans. : t_1 = 1.2818 ms, t_2 = 3.333 ms]

Ex. 5 : *An alternating current is given by i = 14.14 sin 377 t. Find (i) R.M.S. value of current, (ii) Frequency,*

(iii) Instantaneous value of current, when t = 3 ms and (iv) Time taken by current to reach 10 amp for 1st time after passing through zero. **May-07**

[Ans. : 10 A, 60 Hz, 12.794 A, 2.083 ms]

Ex. 6 : *Draw a neat sketch, in each case (not to scale), of the waveform and write the equation for instantaneous value, for the following :-*

i) Sinusoidal current of 10 A (r.m.s.), 50 Hz passing through its zero value at $\omega t = \pi/3$ radians and rising positively.

ii) Sinusoidal current of amplitude of 8 A, 50 Hz passing through its zero value at $\omega t = -\pi/6$ and rising positively. **May-01**

Ex. 7 : *The mathematical expression for the instantaneous value of an alternating current is $i = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$ – A. Find its effective value, periodic time and the instant at which it reaches its positive maximum value. Sketch the wave-form from t = 0 over one complete cycle.* **Dec.-99, 06**

[Ans. : 40 msec, 15 msec]

Ex. 8 : *Two sinusoidal sources of e.m.f. have r.m.s. values E_1 and E_2 and a phase difference α. When connected in series, the resultant voltage is 41.1 V.*

When one of the source is reversed, the resultant e.m.f. is 17.52 V. When phase displacement is made zero, the resultant e.m.f. is 42.5 V. Calculate E_1, E_2 and α.

[Ans. : E_1 = 28.14 V and E_2 = 14.35 V, E_1 = 14.35 V and E_2 = 28.14 V, 31.11°]

Ex. 9 : *An alternating current is represented by the expression,*

$i = 10 \sin\left(2\pi \times 60 \times t - \frac{\pi}{6}\right)$ *ampere. Find its periodic time. Also find i) Its instantaneous value at t = 0, ii) Time 't' at which it first reaches zero value after t = 0, and iii) Time at which it first reaches its negative maximum value after t = 0. Draw a neat sketch of its wave form for one cycle from time t = 0, and indicate in it the coordinates of the above three points.* **Dec.-02**

[Ans. : – 5 A, 1.388 msec, 0.01388 sec]

Ex. 10 : *At t = 0, the instantaneous value of a 60-Hz sinusoidal current is + 5 ampere and increases in magnitude further. Its r.m.s. value is 10-A.*

i) Write the expression for its instantaneous value.

ii) Find the current at t = 0.01 and t = 0.015 second.

iii) Sketch the wave form indicating these values. **May-04**

[Ans. : 14.1421 sin (120 p t + 0.3613) A, – 11.8202 A, – 3.7214 A]

Ex. 11 : *Find the resultant of the three voltages e_1, e_2 and e_3 where, $e_1 = 20 \sin(\omega t)$, $e_2 = 30 \sin\left(\omega t - \frac{\pi}{4}\right)$ and*

$e_3 = \cos\left(\omega t + \frac{\pi}{6}\right)$

[Ans. : 25.1058 sin (ω t + 32.33°) V]

Ex. 12 : *Four wires p, q, r, s are connected to a common point. The currents in lines p, q and r are $6 \sin\left(\omega t + \frac{\pi}{6}\right)$;*

$5 \cos\left(\omega t + \frac{\pi}{3}\right)$ *and* $\cos\left(\omega t + \frac{\pi}{3}\right)$ *respectively.*

Find the current in wire 's'.

[Ans. : 7.211 sin (ω t – 76.102°) A]

Ex. 13 : *Three voltages are connected as shown in the Fig. 3.1.*

If $V_a = 17.32 + j\,10$ V, $V_b = 30\angle 80°$ V, $V_c = 15\angle -100°$ V. Find. i) V_{12} ii) V_{23} iii) V_{34}.

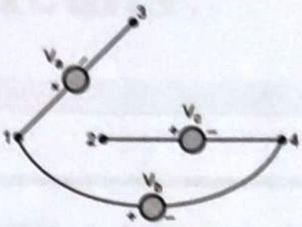


Fig. 3.1

[Ans. : i) 45∠80° V, ii) 35.6085∠– 74.51° V iii) 23∠121.78° V]

Ex. 14 : *If A = 4 + j 7; B = 8 + j 9 and C = 5 – j 6 then calculate, i)* $\frac{A+B}{C}$ *ii)* $\frac{A \times B}{C}$ *iii)* $\frac{A+B}{B+C}$ *iv)* $\frac{B-C}{A}$

[Ans. : i) 2.5607 ∠ 103.324° ii) 12.4291 ∠ 158.815° iii) 1.499 ∠ 40.14° iv) 1.8974 ∠ 18.435°]