UNIT - III

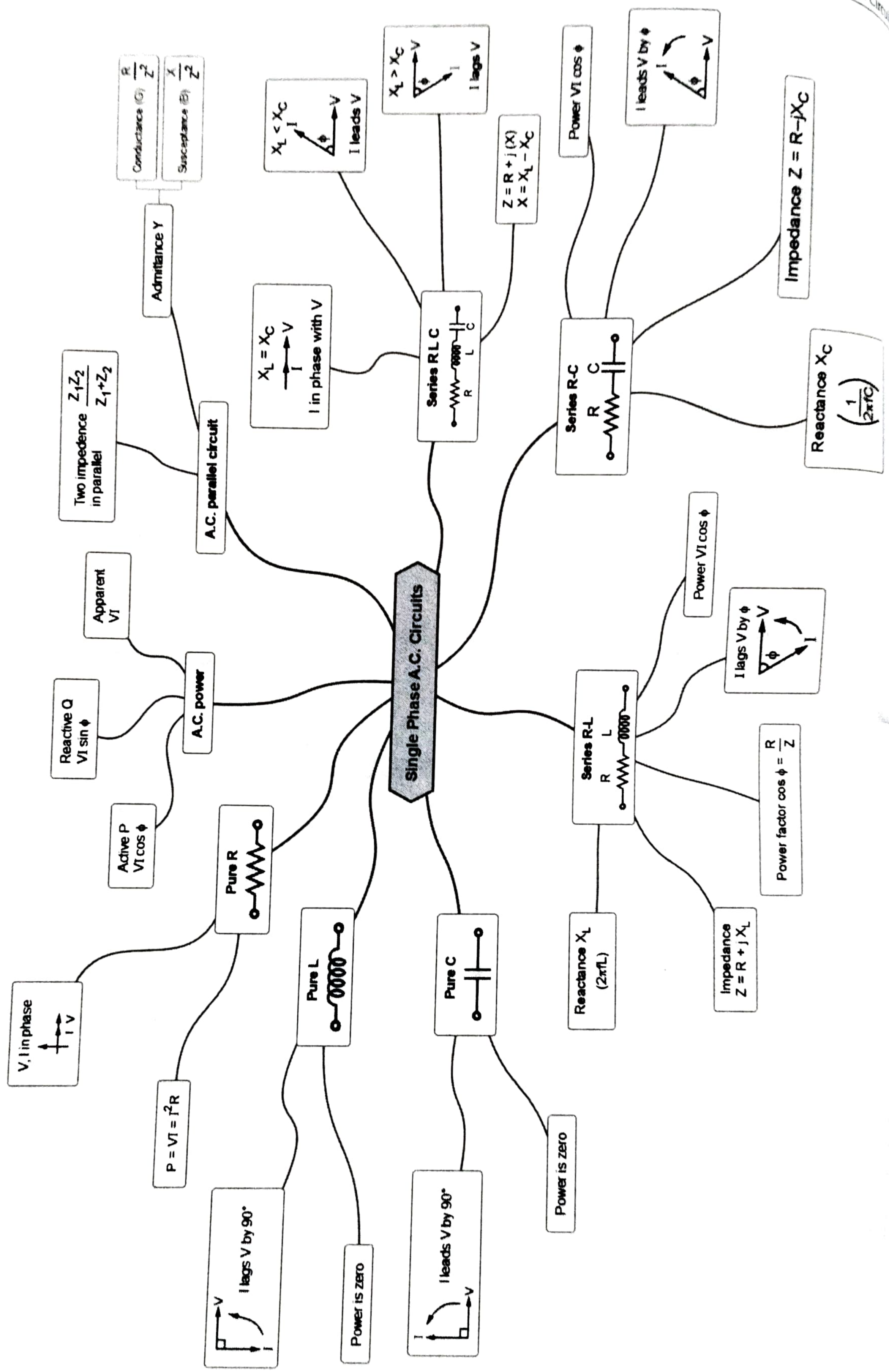
4 Single Phase A.C. Circuits

Syllabus

Study of AC circuits consisting of pure resistance, pure inductance, pure capacitance, series R-L, R-C and R-L-C circuits, phasor diagrams, voltage, current and power waveforms, resonance in series RLC circuits, concept of impedance, concept of active, reactive, apparent, complex power and power factor, Parallel AC circuits (No numericals), concept of admittance.

Contents

Mind Map - Single Phase A. C. Circuits



4.1 : A.C. through Pure Resistance

SPPU : Dec.-05, May-18,19

- Consider a simple circuit consisting of a pure resistance 'R' ohms connected across a voltage $v = V_m \sin \omega t$.

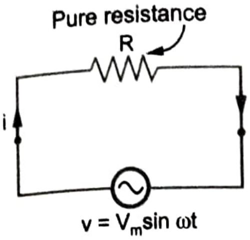


Fig. 4.1.1 Pure resistive circuit

- The circuit is shown in the Fig. 4.1.1.
- According to Ohm's law, we can find the equation for the current i as,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

i.e.

$$i = \left(\frac{V_m}{R}\right) \sin(\omega t)$$

- This is the equation giving instantaneous value of the current.
- Comparing this with standard equation, $i = I_m \sin(\omega t + \phi)$

$$\boxed{I_m = \frac{V_m}{R} \quad \text{and} \quad \phi = 0°}$$

So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while as $\phi = 0$, it indicates that it is in phase with the voltage applied.

In purely resistive circuit, the current and the voltage applied are in phase with each other.

- The waveforms of voltage and current and the corresponding phasor diagram is shown in the Fig. 4.1.2 (a) and (b).
- In the phasor diagram, the phasors are drawn in phase and there is no phase difference in between them. Phasors represent the **r.m.s. values** of alternating quantities.

4.1.1 Power

- The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin \omega t$$

$$= V_m I_m \sin^2(\omega t)$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$\therefore \quad \boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos(2\omega t)}$$

- From the above equation, it is clear that the instantaneous power consists of two components,

 1) Constant power component $\left(\frac{V_m I_m}{2}\right)$

 2) Fluctuating component $\left[\frac{V_m I_m}{2} \cos(2\omega t)\right]$ having frequency, double the frequency of the applied voltage.

- **The average value of the fluctuating cosine component of double frequency is zero, over one complete cycle.**
- So, average power consumption over one cycle is equal to the constant power component i.e. $\frac{V_m I_m}{2}$ which is half of the peak power $V_m I_m$.

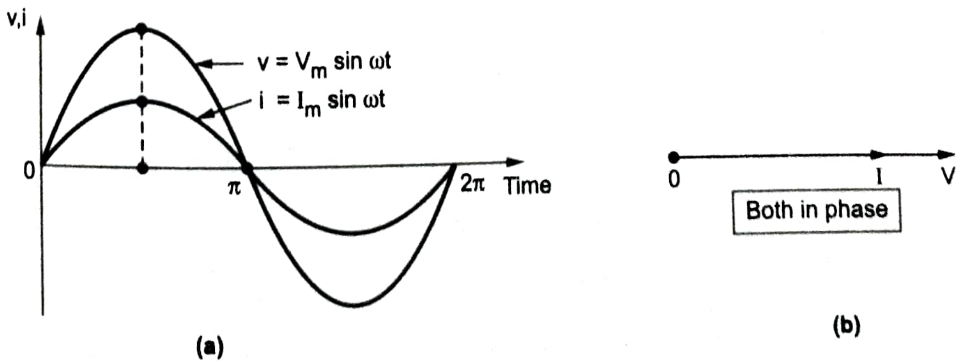


Fig. 4.1.2 A. C. through purely resistive circuit

$$\therefore \quad P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\therefore \quad \boxed{P_{av} = V_{rms} \times I_{rms} \text{ watts}}$$

- Generally, r.m.s. values are indicated by capital letters.

$$\therefore \quad \boxed{P_{av} = V \times I \quad \text{watts} = I^2 R \quad \text{watts}}$$

- The Fig. 4.1.3 shows the waveforms of voltage, current and power.

Ex. 4.1.1 : ***The alternating current expression is given by i = 14.14 sin (100 πt) Amp. Determine : [i] Maximum value of current [ii] RMS value of current [iii] Average value of current [iv] Form factor [v] Peak factor [vi] Power consumed when if flow through resistance of 10 Ω.*** **SPPU : May-19, Marks 7**

Sol. : Given : $i(t) = 14.14 \sin(100\pi t)$, $R = 10\ \Omega$

To find : I_m, I_{rms}, I_{av}, K_f, K_p, P

Comparing i(t) with $i(t) = I_m \sin(\omega t)$

i) $I_m = \mathbf{14.14\ A}$ ... Maximum

ii) $I_{rms} = \frac{I_m}{\sqrt{2}} = \mathbf{10\ A}$ ... RMS

iii) $I_{av} = 0.637\ I_m = \mathbf{9\ A}$ ... Average

iv) $K_f = \frac{\text{rms}}{\text{average}} = \frac{10}{9} = \mathbf{1.11}$ ... Form factor

v) $K_p = \frac{\text{maximum}}{\text{rms}} = \mathbf{1.414}$ ... Peak factor

vi) $P = V_{rms} \times I_{rms}$ but $I_{rms} = \frac{V_{rms}}{R}$

$$\therefore \quad P = I_{rms}^2 \times R = 10^2 \times 10 = 1\ \text{kW}$$

Expected Questions

1. *Prove that the voltage and current in purely resistive circuit are in phase.* **SPPU : Dec.-05, May-18, Marks**
2. *Derive an expression for the instantaneous power in a pure resistor energised by sinusoidal voltage.*

4.2 A.C. through Pure Inductance

SPPU : May-08,11,14,15,17, Dec.-08

- Consider a simple circuit consisting of a pure inductance of L henries, connected across a voltage given by the equation,
  $v = V_m \sin \omega t$.
- The circuit is shown in the Fig. 4.2.1.
- Pure inductance has zero ohmic resistance. Its internal resistance is zero. The coil has pure inductance of L henries (H).

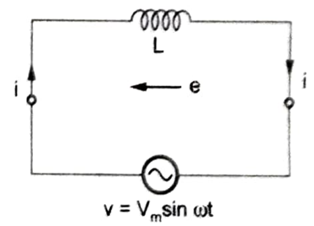


Fig. 4.2.1 Purely inductive circuit

- When alternating current 'i' flows through inductance 'L', it sets up an alternating magnetic field around the inductance.

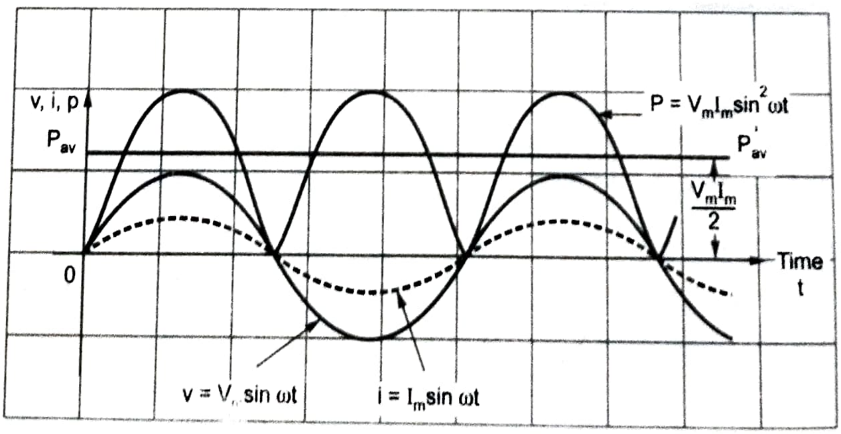


Fig. 4.1.3 v, i and p for purely resistive circuit

- This changing flux links the coil and due to self inductance, e.m.f. gets induced in the coil. This e.m.f. opposes the applied voltage.
- The self induced e.m.f. in the coil is given by,

$$e = -L\frac{di}{dt}$$

- At all instants, applied voltage, v is equal and opposite to the self induced e.m.f., e

$$v = -e = -\left(-L\frac{di}{dt}\right)$$

$$v = L\frac{di}{dt}$$

i.e. $$V_m \sin \omega t = L\frac{di}{dt} \quad \text{i.e.} \quad di = \frac{V_m}{L}\sin \omega t \, dt$$

$$i(t) = \int di = \int \frac{V_m}{L}\sin \omega t \, dt$$

$$= \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$= -\frac{V_m}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right)$$

$$\ldots \cos \omega t = \sin\left(\frac{\pi}{2}-\omega t\right)$$

$$i(t) = \frac{V_m}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right)$$

$$\ldots \sin\left(\frac{\pi}{2}-\omega t\right) = -\sin\left(\omega t-\frac{\pi}{2}\right)$$

$$\therefore \quad i(t) = I_m \sin\left(\omega t-\frac{\pi}{2}\right)$$

where $$I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$$

and $$X_L = \omega L = 2\pi f L \ \Omega \text{ as } \omega = 2\pi f$$

- The above equation clearly shows that the current is purely sinusoidal and having phase angle of $-\frac{\pi}{2}$ radians i.e. $-90°$. This means that **the current lags voltage applied by 90°.**
- The negative sign indicates lagging nature of the current.

Waveforms and Phasor Diagram

- The Fig. 4.2.2 shows the waveforms and the corresponding phasor diagram.

In purely inductive circuit, current lags voltage by 90°.

4.2.1 Concept of Inductive Reactance

- It is shown that,

$$X_L = \omega L = 2\pi f L \ \Omega$$

- The term, X_L, is called **Inductive Reactance** and is measured in **ohms.**
- The **inductive reactance** is defined as the opposition offered by the inductance of a circuit to the flow of an alternating sinusoidal current.

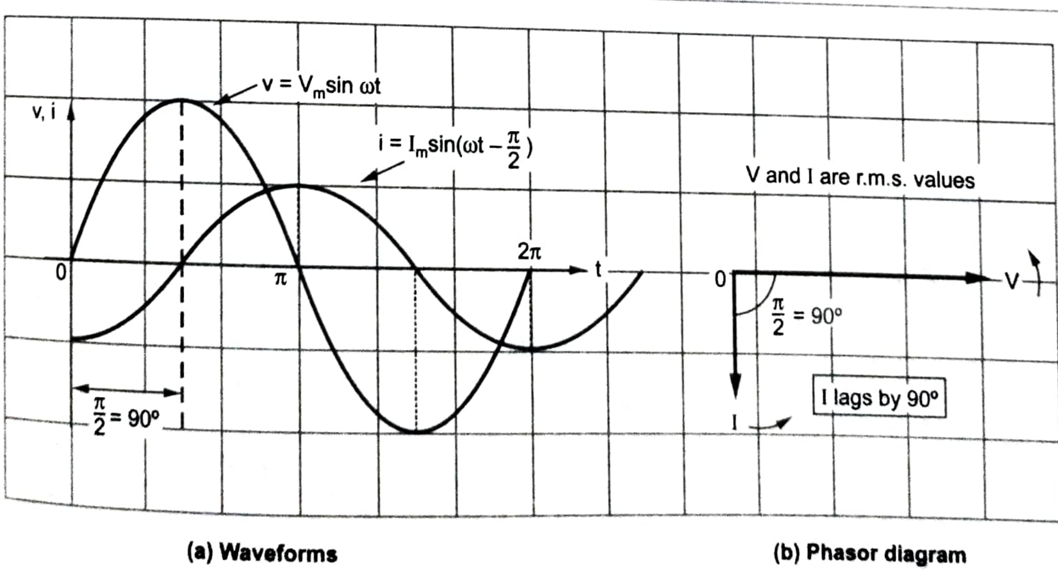


Fig. 4.2.2 A.C. through purely inductive circuit

- It is measured in ohms and it depends on the frequency of the applied voltage.
- The inductive reactance is directly proportional to the frequency for constant L.

$$X_L \propto f, \quad \text{for constant L}$$

- So, graph of X_L Vs f is a straight line passing through the origin as shown in the Fig. 4.2.3.

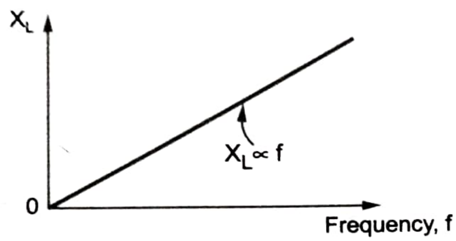


Fig. 4.2.3 X_L Vs f

If frequency is zero, which is so for d.c. voltage, the inductive reactance is zero. Therefore, it is said that the inductance offers zero reactance for the d.c. or steady current.

4.2.2 Power

- The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$\therefore \quad p = v \times i$$

$$= V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= - V_m I_m \sin (\omega t) \cos (\omega t) \text{ as } \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= - \cos \omega t$$

$$\therefore \quad \boxed{p = - \frac{V_m I_m}{2} \sin (2 \omega t)}$$

as $2 \sin \omega t \cos \omega t = \sin 2 \omega t$

This power curve is a sine curve of frequency double than that of applied voltage.

- The average value of sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} - \frac{V_m I_m}{2} \sin (2 \omega t) \, d (\omega t) = 0$$

- The Fig. 4.2.4 shows voltage, current and power waveforms.
  It can be observed from it that when power curve is positive, energy gets stored in the magnetic field established due to the increasing current while during negative power curve, this power is returned back to the supply. The areas of positive loop and negative loop are exactly same and hence, **average power consumption is zero.**

Pure inductance never consumes power.

- The average energy stored in an inductor is given by $E = \frac{1}{2} L I^2$ joules.

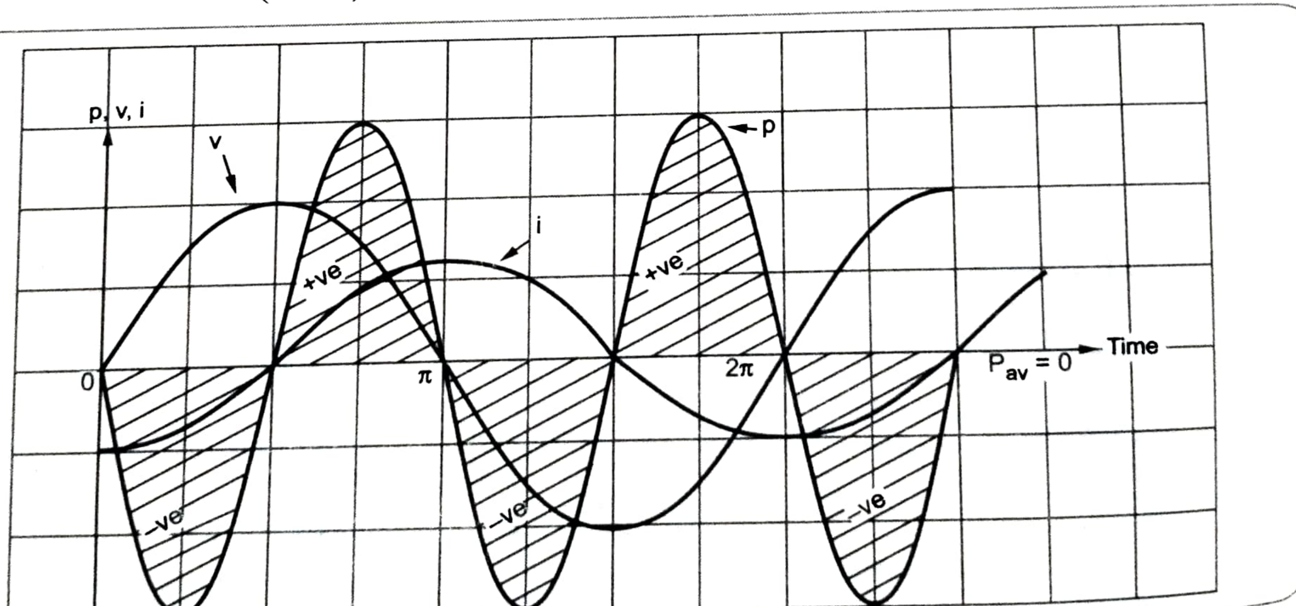


Fig. 4.2.4 Waveforms of voltage, current and power

Ex. 4.2.1 : *Find the expression for current which will flow when a pure inductor of 0.2 H is connected across 230 V, 50 Hz, AC supply. Draw the phasor diagram.*

SPPU : Dec.-16, Marks 6

Sol. : Given : L = 0.2 H, V = 230 V, f = 50 Hz

To find : i(t)

$$X_L = 2\pi f L = 62.831\ \Omega$$

$$I = \frac{V}{X_L} = \frac{230}{62.831} = 3.66\ A$$

This is rms current

$$I_m = \sqrt{2}\, I = \sqrt{2} \times 3.66$$

$$= 5.177\ A,\quad \omega = 2\pi f = 314.159\ rad/s$$

Expression for the current is,

$$i(t) = I_m \sin(\omega t - \phi),\ \phi = 90^\circ$$

$$i(t) = \mathbf{5.177 \sin(314.159\, t - 90^\circ)\ A}$$

The phasor diagram is shown in the Fig. 4.2.5.

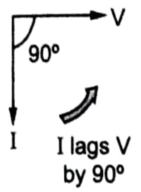


Fig. 4.2.5

Expected Questions

1. *Derive the expressions for current and power for purely inductive circuit when the voltage applied to it is $v(t) = V_m \sin \omega t$. Draw the corresponding waveforms.*

SPPU : May-08,14,15,17, Dec.-08, Marks 8

2. *Prove that in a purely inductive circuit the current lags voltage by 90°.*

SPPU : May-11, Marks 4

3. *Explain the concept of inductive reactance. How it depends on the frequency ?*
4. *Show that the average power consumed by pure inductor is zero.*

4.3 : A.C. through Pure Capacitance

SPPU : Dec.-05,06,17 May-06,08,09,11,13,18

- Consider a simple circuit consisting of a pure capacitor of C-farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$. The circuit is shown in the Fig. 4.3.1.

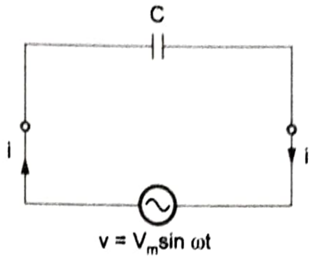


Fig. 4.3.1 Purely capacitive circuit

- The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by,

$$q = C\,v = C\,V_m \sin \omega t$$

- Current is rate of flow of charge.

$$i(t) = \frac{dq}{dt} = \frac{d}{dt}(C\,V_m \sin \omega t)$$

$$= C\,V_m \frac{d}{dt}(\sin \omega t) = C\,V_m\, \omega \cos \omega t$$

$$\therefore \quad i(t) = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \quad \boxed{\begin{aligned} & i(t) = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \\ \text{where} \quad & I_m = \frac{V_m}{X_C} \\ \text{and} \quad & X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega \end{aligned}}$$

- The above equation clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. + 90°.
- This means **current leads voltage applied by 90°.** The positive sign indicates leading nature of the current.

Waveforms and Phasor Diagram :

- The Fig. 4.3.2 shows waveforms of voltage and current and the corresponding phasor diagram.
- The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.

In purely capacitive circuit, current leads voltage by 90°.

4.3.1 Concept of Capacitive Reactance

- It is shown that, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \ \Omega$
- The term X_C is called **Capacitive Reactance** and is measured in ohms.
- The **capacitive reactance** is defined as the opposition offered by the capacitance of a circuit to the flow of an alternating sinusoidal current.
- X_C is measured in ohms and it depends on the frequency of the applied voltage.
- The capacitive reactance is inversely proportional to the frequency for constant capacitor C.

$$X_C \propto \frac{1}{f} \quad \text{for constant C}$$

The graph of X_C Vs f is a rectangular hyperbola as shown in Fig. 4.3.3.

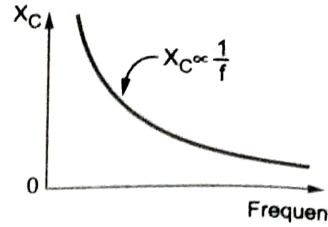


Fig. 4.3.3 X_C Vs f

If the frequency is zero, which is so for d.c. voltage, the capacitive reactance is infinite. Therefore, it is said that the capacitance offers open circuit to the d.c. or it blocks d.c.

4.3.2 Power

- The expression for the instantaneous power can be obtained by taking the product of instantaneous voltage and current.

$$p = v \times i$$

$$= V_m \sin(\omega t) \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin(\omega t) \cos(\omega t)$$

$$\text{as } \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\therefore \quad \boxed{p = \frac{V_m I_m}{2} \sin(2\omega t)}$$

as $2 \sin \omega t \cos \omega t = \sin 2 \omega t$

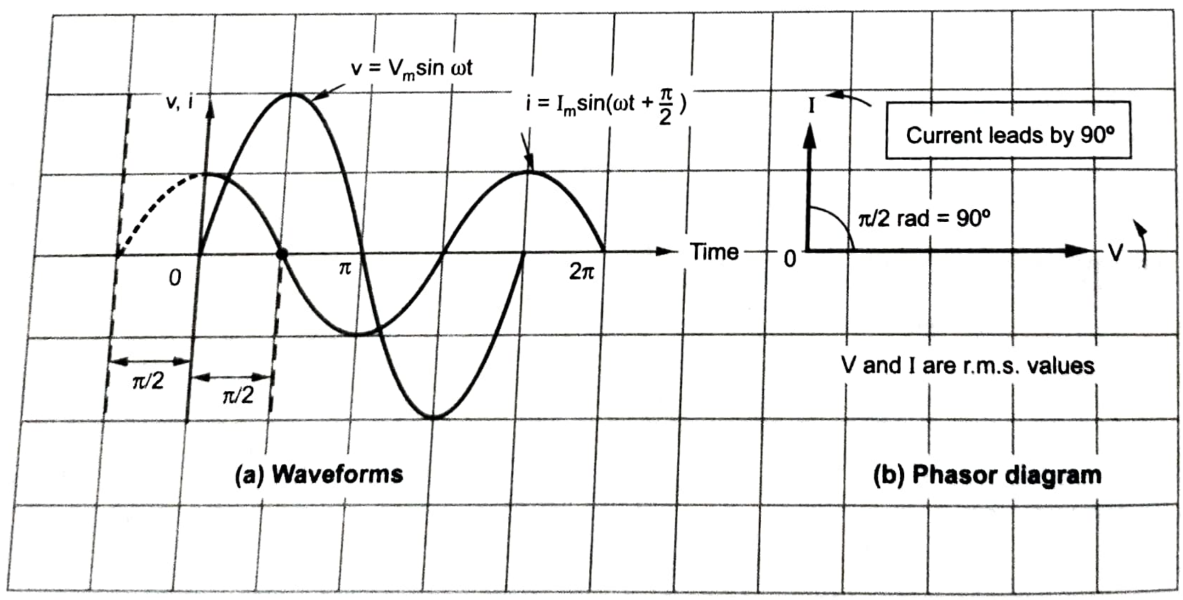


Fig. 4.3.2 A.C. through purely capacitive circuit

- Thus power curve is a sine wave of frequency double that of applied voltage.
- **The average value of sine curve over a complete cycle is always zero.**

$$P_{av} = \int_0^{2\pi} \frac{V_m I_m}{2} \sin (2\,\omega t)\, d(\omega t) = 0$$

- The Fig. 4.3.4 shows waveforms of current, voltage and power.
- It can be observed from the figure that when power curve is positive, in practice, an electrostatic energy gets stored in the capacitor during its charging while the negative power curve represents that the energy stored is returned back to the supply during its discharging.
- The areas of positive and negative loops are exactly the same and hence, **average power consumption is zero.**

Pure capacitance never consumes power.

- The average energy stored in a capacitor is given by,

$$E = \frac{1}{2} C V^2 \text{ joules.}$$

Ex. 4.3.1 : ***A 50 μF capacitor is connected across a single phase 230 V, 50 Hz supply. Calculate 1) The reactance offered by the capacitor 2) The maximum current and 3) The r.m.s. value of the current drawn by the capacitor.*** **SPPU : May-13, Marks 6**

Sol. : $C = 50\ \mu F,\ V = 230\ V,\ f = 50\ Hz$

1) $$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = \mathbf{63.662\ \Omega}$$

2) $$I_m = \frac{V_m}{X_C} = \frac{\sqrt{2}\,V}{X_C} = \frac{\sqrt{2} \times 230}{63.662}$$

$$= \mathbf{5.1093\ A} \text{ (Magnitude)}$$

3) $$I(\text{R.M.S.}) = \frac{I_m}{\sqrt{2}} = \mathbf{3.6128\ A} \text{ (Magnitude)}$$

Expected Questions

1. *Show that the current through purely capacitive circuit leads the applied voltage by 90°.* **SPPU : Dec.-05,17, May-09,11, Marks 5**
2. *Derive the expressions for current and power for purely capacitive circuit when the voltage applied to it is $v(t) = V_m \sin \omega t$. Draw the corresponding waveforms.* **SPPU : May-06,08,18, Dec.-06, Marks 8**
3. *Explain the concept of inductive reactance. How it depends on the frequency ?*
4. *Show that the average power consumed by pure capacitor is zero.*

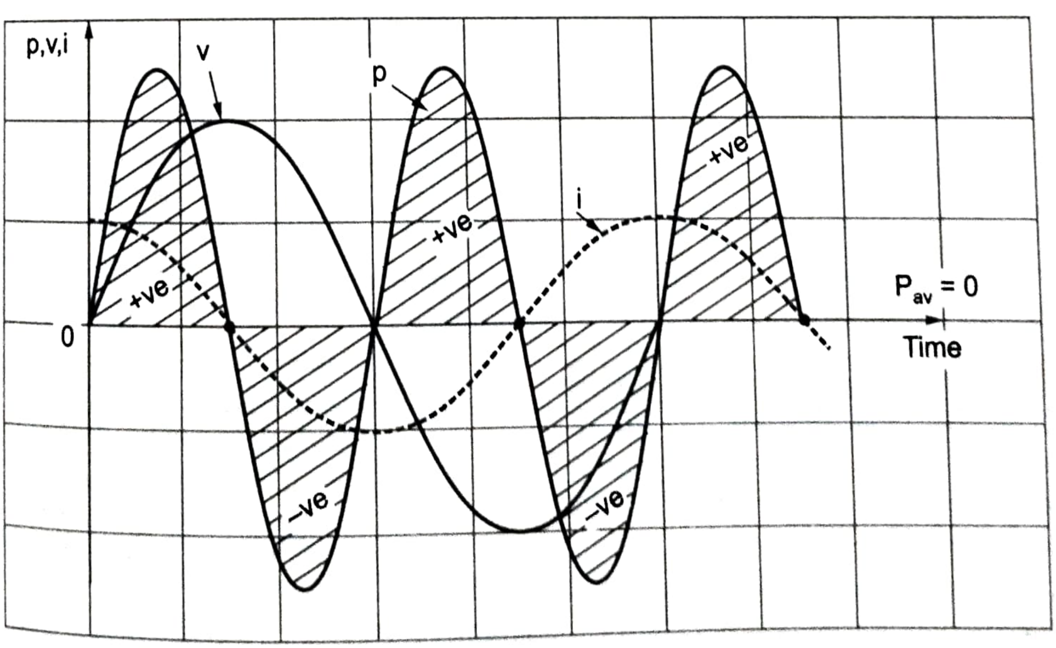


Fig. 4.3.4 Waveforms of voltage, current and power

4.4 : Impedance

SPPU : May-03, Dec.-07

- The opposition offered by an electric circuit to the flow of an alternating current is called an impedance. It is denoted by Z. It is the ratio of an alternating voltage to an alternating current through the circuit.
- Impedance is complex and is expressed in polar or rectangular form.
- For pure resistance voltage and current are in phase hence impedance does not introduce any phase angle. So impedance of a pure resistance can be expressed in polar and rectangular form as,

$$Z = R + j0 = R \angle 0° \text{ ohms.}$$

- For a pure inductance, the current lags voltage by 90° hence the inductive reactance X_L produces a phase lag of 90°.
- For a pure inductance, if voltage is $V\angle 0°$ then current is $I\angle -90°$ hence its impedance in polar and rectangular form is given by,

$$Z = \frac{V\angle 0°}{I\angle -90°} = \frac{V}{I}\angle 90° = X_L\angle 90° = 0 + j\,X_L \text{ ohms}$$

- For a pure capacitance, the current leads voltage by 90° hence the capacitive reactance X_C produces a phase lead of 90°.
- For a pure capacitance, if voltage is $V\angle 0°$ then current is $I\angle +90°$ hence its impedance is given by,

$$Z = \frac{V\angle 0°}{I\angle +90°} = \frac{V}{I}\angle -90° = X_C\angle -90° = 0 - j\,X_C \text{ ohms}$$

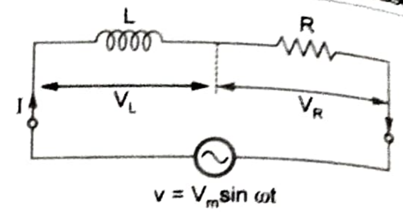
Expected Question

1. *Define impedance.*

4.5 : A.C. through Series R-L Circuit

SPPU : Dec.-98,03,04,06,07,08,09,2000,10,11,14,16 May-99,2000,01,04,05,07,08,10,12,16,17,19

- Consider a circuit consisting of pure resistance R ohms connected in series with a pure inductance of L Henries as shown in the Fig. 4.5.1 (a).
- The series combination is connected across a.c. supply given by $v = V_m \sin \omega t$.



(a) Series R-L circuit

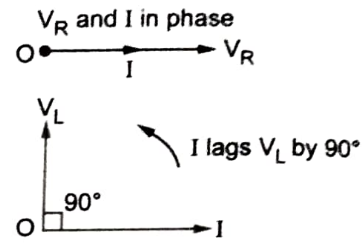


Fig. 4.5.1 (b)

- Circuit draws a current I then there are two voltage drops,

a) Drop across pure resistance, $V_R = I \times R$

... I and V_R in phase

b) Drop across pure inductance, $V_L = I \times X_L$

... I lags V_L by 90°

where $X_L = 2\pi f L$

I = r.m.s. value of current drawn

V_R , V_L = r.m.s. values of the voltage drops

- The Kirchhoff's voltage law can be applied to the a.c. circuit but only the point to remember is the addition of voltages should be a phasor (vector) addition and no longer algebraic as in case of d.c.

$$\therefore \quad \bar{V} = \bar{V}_R + \bar{V}_L = \bar{I}R + \bar{I}X_L \quad \text{(phasor addition)}$$

Phasor Diagram :

- Let us draw the phasor diagram for the above case.

Key Point *For series a.c. circuits, generally, current is taken as the reference phasor as it is common to both the elements.*

Following are the **steps to draw the phasor diagram :**

1) Take current as a reference phasor.
2) In case of resistance, voltage and current are in phase, so V_R will be along current phasor.
3) In case of inductance, current lags voltage by 90°. But, as current is reference, V_L must be shown leading with respect to current by 90°.
4) The supply voltage being vector sum of these two vectors V_L and V_R obtained by law of parallelogram.

- From the voltage triangle, shown in the Fig. 4.5.1 (d) we can write,

$$V = \sqrt{(V_R)^2 + (V_L)^2}$$

$$= \sqrt{(IR)^2 + (I \times X_L)^2}$$

$$= I\sqrt{(R)^2 + (X_L)^2}$$

$$V = I\,Z$$

Where $\boxed{Z = \sqrt{(R)^2 + (X_L)^2}}$ ... **Impedance** of the circuit.

- The impedance Z is measured in ohms.

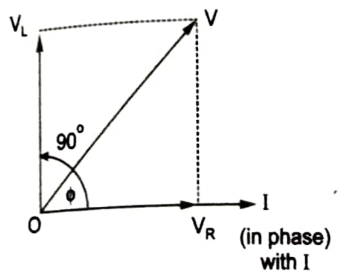


Fig. 4.5.1 (c) Phasor diagram

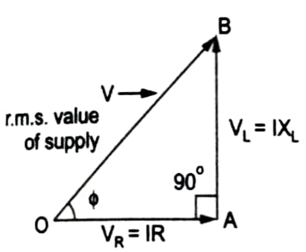


Fig. 4.5.1 (d) Voltage triangle

- It can be seen that current lags voltage by angle ϕ

$\therefore$ $v(t) = V_m \sin \omega t$ and

$i(t) = I_m \sin(\omega t - \phi)$

4.5.1 Impedance and Impedance Triangle

- **Impedance** is defined as the opposition of circuit to flow of alternating current. It is denoted by Z and its unit is ohms.
- For the R-L series circuit, it can be observed from the phasor diagram that the current lags behind the applied voltage by an angle ϕ. From the voltage triangle, we can write,

$$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z},$$

$$\sin\phi = \frac{V_L}{V} = \frac{X_L}{Z}$$

- If all the sides of the voltage triangle are divided by current, we get a triangle called **impedance triangle** as shown in the Fig. 4.5.2.

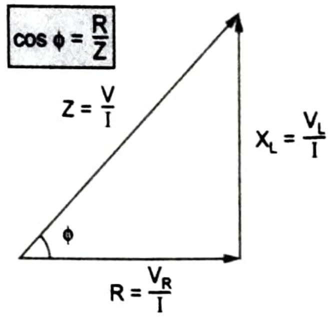


Fig. 4.5.2 Impedance triangle

- Sides of this triangle are resistance R, inductive reactance X_L and an impedance Z.
- From this impedance triangle, we can see that the X component of impedance is R and is given by, $R = Z\cos\phi$ and Y component of impedance is X_L and is given by, $X_L = Z\sin\phi$
- In rectangular form the impedance is denoted as,

$$\boxed{Z = R + j\,X_L \quad \Omega}$$

- While in polar form, it is denoted as,

$$Z = |Z| \angle\phi \quad \Omega$$

$$|Z| = \sqrt{R^2 + X_L^2}, \qquad \phi = \tan^{-1}\left[\frac{X_L}{R}\right]$$

Key Point *Thus ϕ is positive for inductive impedance.*

4.5.2 Power and Power Triangle

The expression for the current in the series R-L circuit is,

$i = I_m \sin(\omega t - \phi)$ as current lags voltage.

- The power is product of instantaneous values of voltage and current,

$\therefore$ $P = v \times i$

$$= V_m \sin \omega t \times I_m \sin (\omega t - \phi)$$

$$= V_m I_m [\sin (\omega t) \cdot \sin (\omega t - \phi)]$$

$$= V_m I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right]$$

$$= \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos (2\omega t - \phi)$$

- Now, the second term is cosine term whose average value over a cycle is zero. Hence, average power consumed is,

$$P_{av} = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi$$

$\therefore$ $\boxed{P = V\,I \cos \phi \quad \text{watts}}$ where V and I are r.m.s. values

- If we multiply voltage equation by current I, we get the power equation.

$$\overline{V}\overline{I} = \overline{V}_R \overline{I} + \overline{V}_L \overline{I} = \overline{V}\cos\phi \overline{I} + \overline{V}\sin\phi \overline{I}$$

- From this equation, power triangle can be obtained as shown in the Fig. 4.5.3.

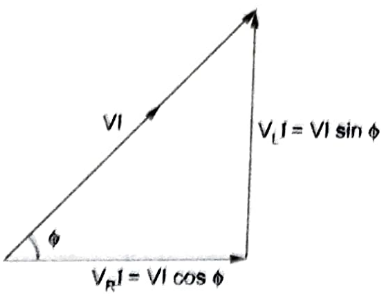


Fig. 4.5.3 Power triangle

So, three sides of this triangle are,

1) VI, 2) VI cos ϕ 3) VI sin ϕ

These three terms can be defined as below.

1. Apparent Power (S)

- It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S.

$\therefore$ $\boxed{S = V\,I \quad \text{VA}}$

It is measured in unit volt-amp (VA) or kilo volt-amp (kVA).

2. Real or True Power (P)

- It is defined as the product of the applied voltage and the active component of the current.
- It is real component of the apparent power. It is measured in unit watts (W) or kilowatts (kW).

$$\boxed{P = V\,I \cos \phi \quad \text{watts}}$$

3. Reactive Power (Q)

- It is defined as product of the applied voltage and the reactive component of the current.
- It is also defined as imaginary component of the apparent power. It is represented by 'Q' and it is measured in unit volt-amp reactive (VAR) or kilovolt-amp reactive (kVAR).

$$\boxed{Q = V\,I \sin \phi \quad \text{VAR}}$$

4.5.3 Power Factor (cos ϕ)

- It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.
- It is the ratio of true power to apparent power.

$$\text{Power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{V\,I \cos \phi}{V\,I} = \cos \phi$$

- **The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor.** It cannot be greater than 1.
- It is also defined as the ratio of resistance to the impedance.

$$\boxed{\cos \phi = \frac{R}{Z}}$$

Key Point *The nature of power factor is always determined by position of current with respect to the voltage.*

- If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.
- So, for pure inductance, the power factor is cos (90°) i.e. zero lagging while for pure capacitance, the power factor is cos (90°) i.e. zero but leading. For purely resistive circuit voltage and current are in phase i.e. $\phi = 0$. Therefore, power factor is

cos (0°) = 1. Such circuit is called unity power factor circuit.

Power factor = cos ϕ

ϕ is the angle between supply voltage and current.

Key Point *Nature of power factor always tells position of current with respect to voltage.*

4.5.4 Waveforms of Voltage, Current and Power

- The waveforms are shown in the Fig. 4.5.4.

$$P = V I \cos\phi \text{ W}$$

Ex. 4.5.1 : *A coil having resistance of 7 Ω and an inductance of 31.8 mH is connected to 230 V, 50 Hz supply. Calculate : i) The circuit current ii) Phase angle iii) Power factor iv) Power consumed v) Voltage drop across resistance and inductor.*

SPPU : Dec.-16, Marks 6

Sol. : Given : R = 7 Ω, L = 31.8 mH, V = 230 V, f = 50 Hz

To find : I, ϕ, cos ϕ, P, V_R, V_L

$$X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$$

$$Z = R + jX_L = 7 + j10 = 12.206 \angle 50°\ \Omega$$

i) $$I = \frac{V}{Z} = \frac{230 \angle 0°}{12.206 \angle 55°}$$

$$= 18.843 \angle -55°\ \text{A}$$

Circuit current = **18.843 A**

ii) ϕ = Phase angle

= – 55° (Negative i.e. lagging)

iii) $\cos\phi = \cos(-55°) = $ **0.5735 lagging**

iv) $P = V I \cos\phi = 230 \times 18.843 \times 0.5735$

= **2485.486 W**

v) $V_R = |I| \times R = 18.843 \times 7$

= **131.901 V** (magnitude)

$V_L = |I| \times |X_L| = 18.843 \times 10$

= **188.43 V** (magnitude)

Ex. 4.5.2 : *A series circuit, consist of resistance of 10 ohm and inductance of 0.1 Henry, connected across one phase 50 Hz A.C. supply. If the voltage across resistance is 50 volt, calculate*

i) Voltage drop across inductance and ii) Supply voltage.

SPPU : Dec.-14, Marks 7

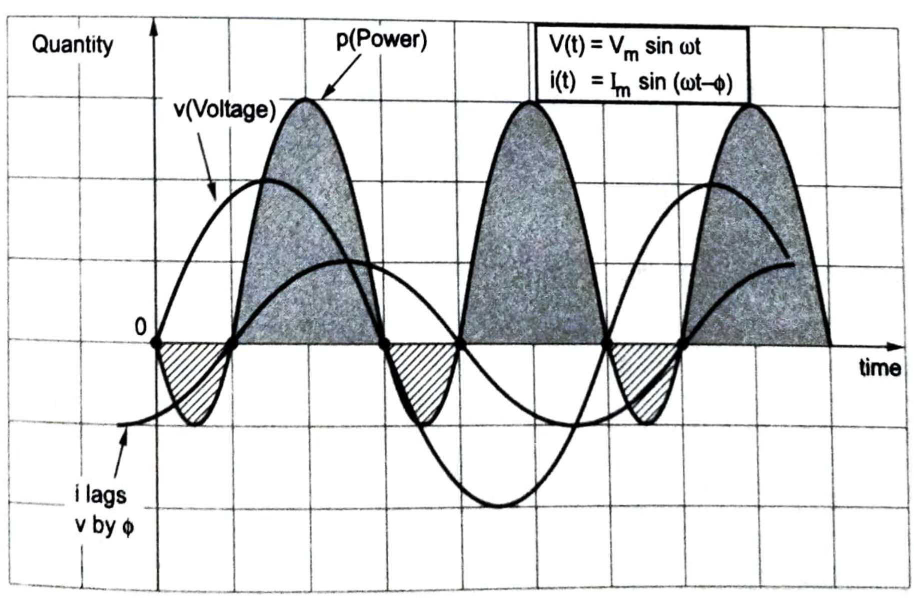


Fig. 4.5.4

Sol. : Given : $X_L = 2\pi f L = 31.4159\ \Omega$

$$\therefore \quad Z = R + jX_L = 10 + j\,31.4159\ \Omega$$

To find : V_L, V

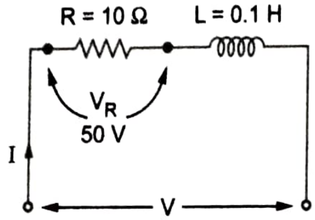


Fig. 4.5.5

$$|V_R| = |I| \times R$$

$$\therefore \quad 50 = |I| \times 10 \text{ i.e. } |I| = 5\text{ A}$$

$$\therefore \quad |I| = \frac{|V|}{|Z|} \text{ and } |Z| = 32.969$$

$$\therefore \quad 5 = \frac{|V|}{32.969}$$

$$\therefore \quad |V| = \mathbf{164.845\ V} \quad \text{... Supply voltage}$$

$$|V_L| = |I| \times X_L = 5 \times 31.4159 = \mathbf{157.0795\ V}$$

$|V_L|$ is the magnitude of voltage across inductance.

Ex. 4.5.3 : ***Find the expression for current when $v = 282.84 \sin (314\ t)$ V is applied to coil having resistance 10 ohm and inductance 0.1 H. Also calculate the power consumed.*** **SPPU : May-16, Dec.-10 Marks 7**

Sol. : Given : R = 10 Ω, L = 0.1 H,

$$V = 282.84 \sin (314\ t)\ \text{V}$$

To find : i(t), P

Compare voltage with $v(t) = V_m \sin (\omega t)$

$$\therefore \quad V_m = 282.84 \text{ V i.e. } V = \frac{V_m}{\sqrt{2}} = 200 \text{ V (RMS)}$$

$$\omega = 314 \text{ rad/s i.e. } X_L = \omega L = 31.4\ \Omega$$

$$\therefore \quad Z = R + jX_L = 10 + j\,31.4\ \Omega$$

$$= 32.954 \angle 72.334°\ \Omega$$

$$\therefore \quad I = \frac{V}{Z} = \frac{200 \angle 0°}{32.954 \angle 72.334°}$$

$$= 6.069 \angle -72.334° \text{ A}$$

$$\therefore \quad I_m = \sqrt{2} I = \sqrt{2} \times 6.069$$

$$= 8.583 \text{ A}, \phi = -72.334°$$

$$\therefore \quad i(t) = I_m \sin(\omega t + \phi)$$

$$= \mathbf{8.583 \sin (314\ t - 72.334°)\ A}$$

$$P = VI \cos \phi$$

$$= 200 \times 6.069 \times \cos (-72.334°)$$

$$= \mathbf{368.35\ W}$$

Ex. 4.5.4 : ***The resistance of 5 Ω is connected across 200 V, 50 Hz 1-phase a.c. supply. The voltage across the resistance is 50 V. Calculate :***

i) Voltage across inductance

ii) Value of inductance

iii) Power and

iv) Power factor **SPPU : May-17, Marks 7**

Sol. : Given : The circuit is shown in the Fig. 4.5.6.

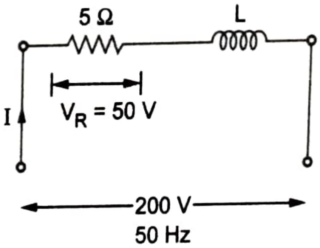


Fig. 4.5.6

To find : V_L, L, P and cos φ

$$V_R = I R \text{ i.e. } 50 = 5 \times I$$

$$\therefore \quad I = 10 \text{ A} \quad \text{...Magnitude}$$

$$Z = R + jX_L$$

$$\text{and} \quad |I| = \frac{|V|}{|Z|} = \frac{|V|}{\sqrt{R^2 + X_L^2}}$$

$$\therefore \quad 10 = \frac{200}{\sqrt{5^2 + X_L^2}} \text{ i.e. } X_L = 19.365\ \Omega$$

i) $$V_L = I \times X_L = 10 \times 19.365 = \mathbf{193.65\ V}$$

... Magnitude

ii) $$X_L = 2\pi f L \text{ i.e. } L = \frac{19.365}{2\pi \times 50} = \mathbf{0.0616\ H}$$

iii) $$P = |I|^2 R = 10^2 \times 5 = \mathbf{500\ W}$$

... Pure L does not consume power

$$\cos\phi = \frac{R}{Z} = \frac{5}{\sqrt{R^2+X_L^2}}$$

iv)

$$= 0.25 \text{ lagging} \qquad \ldots\text{p.f.}$$

Ex. 4.5.5 : ***A voltage $e = 200 \sin 100\,\pi\, t$ is applied to a load having $R = 200\ \Omega$ in series with $L = 638$ mH. Estimate :-***
i) Expression for current in $i = I_m \sin(\omega t \pm \phi)$ form
ii) Power consumed by the load iii) Reactive power of the load iv) Voltage across R and L.

SPPU : May-01, Marks 6

Sol. : Given : The circuit is shown in the Fig. 4.5.7.

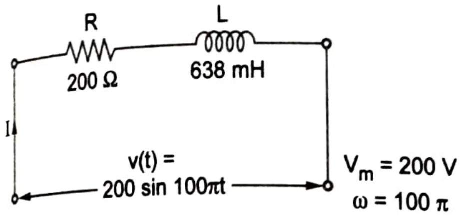


Fig. 4.5.7

To find : i(t), P, Q, V_R, V_L

$$V = \frac{200}{\sqrt{2}} = 141.421 \text{ V (r.m.s.)},\ f = \frac{\omega}{2\pi}$$

$$= 50 \text{ Hz}$$

$$X_L = \omega L = 100\,\pi \times 638 \times 10^{-3}$$

$$= 200.433\ \Omega$$

$$Z = R + j X_L = 200 + j\,200.433\ \Omega$$

$$= 283.149 \angle 45.06°\ \Omega$$

$$I = \frac{V}{Z} = \frac{141.421\angle 0°}{283.149\angle 45.06°}$$

$$= 0.5 \angle -45.06° \text{ A},$$

Current lags voltage by 45.06°.

$$I_m = \sqrt{2} \times 0.5 = 0.7071 \text{ A},\ \phi = -45.06°$$

$$i = I_m \sin(\omega t - \phi)$$

$$= \mathbf{0.7071 \sin(100\,\pi\, t - 45.06°)\ A}$$

$$P = VI\cos\phi$$

$$= 141.421 \times 0.5 \times \cos(45.06°)$$

$$= 49.9474 \approx \mathbf{50\ W}$$

iii)

$$Q = VI\sin\phi$$

$$= 141.421 \times 0.5 \times \sin(45.06°)$$

$$= \mathbf{50\ VAR}$$

iv)

$$V_R = IR = 0.5 \times 200 = \mathbf{100\ V}$$

$$V_L = I\,X_L = 0.5 \times 200.433 = \mathbf{100.21\ V}$$

Ex. 4.5.6 : ***A choke coil and pure resistance are connected in series across 230 V, 50 Hz, a.c. supply. If the voltage drop across coil is 190 V and across resistance is 80 V while current drawn by the circuit is 5 A. Calculate, i) Internal resistance of coil ii) Inductance of coil iii) Resistance R, iv) Power factor of the circuit v) Power consumed by the circuit.***

SPPU : Dec.-07, Marks 10

Sol. :

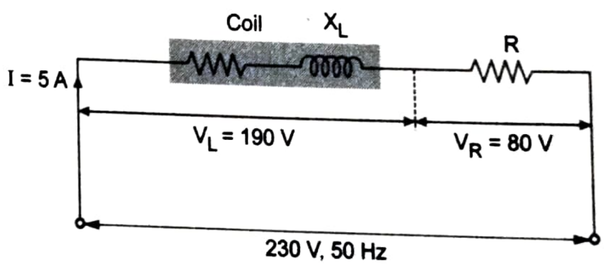


Fig. 4.5.8

$$V = 230 \text{ V},\quad I = 5 \text{ A}$$

$$Z_T = \frac{V}{I} = \frac{230}{5} = 46\ \Omega$$

$$Z_T = \sqrt{(R+r)^2 + (X_L)^2} = 46 \qquad \ldots(1)$$

Impedance of coil,

$$Z_L = \frac{V_L}{I} = \frac{190}{5} = 38\ \Omega$$

$$Z_L = r + j X_L$$

i.e. $$38 = \sqrt{r^2 + (X_L)^2} \qquad \ldots \text{Magnitude}$$

From (1)

$$(46)^2 = (R+r)^2 + (X_L)^2 \qquad \ldots(2)$$

From equation (2)

$$(38)^2 = r^2 + (X_L)^2 \quad \dots (3)$$

Now $\quad V_R = 80 \text{ V} = I R \quad \dots (4)$

$$\therefore \quad R = \frac{V_R}{I} = \frac{80}{5} = \mathbf{16\ \Omega}$$

∴ From equation (1)

$$2116 = (R)^2 + 2 R r + r^2 + (X_L)^2$$

Substituting equation (4) in equation (3),

$$2116 = (16)^2 + 2 \times 16 \times r + (38)^2$$

$$\therefore \quad \mathbf{r = 13\ \Omega}$$

From equation (2),

$$(38)^2 = (13)^2 + (X_L)^2$$

i.e. $\quad X_L = 35.707\ \Omega$

Now $\quad X_L = 2 \pi f L$

hence $\quad L = \dfrac{X_L}{2\pi f} = \dfrac{35.707}{2\pi \times 50}$

$$\therefore \quad \mathbf{L = 0.1136\ H}$$

$$Z_T = (R + r) + j (X_L)$$
$$= (16 + 13) + j (35.707)$$
$$= 29 + j\, 35.707\ \Omega$$

$$\therefore \quad \cos \phi = \frac{(R+r)}{Z_T} = \frac{29}{46} = \mathbf{0.6304\ lagging}$$

Power consumed

$$P = V I \cos \phi = 230 \times 5 \times 0.6304$$
$$= \mathbf{724.96\ W}$$

Expected Questions

1. *Derive and show the waveforms of voltage, current and power for R-L series circuit when supplied by a voltage $v(t) = V_m \sin \omega t$. Draw phasor diagram.*
SPPU : Dec.-09, 11, May-10, 12, Marks 8
2. *Define impedance and sketch impedance triangle.*
SPPU : May-08, Dec.-10, Marks 4
3. *Draw power triangle and define active power, reactive power and apparent power.*
SPPU : Dec.-06, 10, May-08, Marks 4
4. *Define power factor.*
SPPU : Dec.-03, 08, May-04, Marks 3
5. *Obtain the expression for power, when voltage $v = V_m \sin \omega t$ is applied across R-L series circuit. Draw the circuit diagram and phasor diagram.*
SPPU : May-19, Marks 6
6. *For a single-phase a.c circuit, the applied voltage is $v = V_m \sin \omega t$ and current drawn is $i = I_m \sin (\omega t - \phi)$. Derive expression for average power. Draw waveforms of voltage, current and instantaneous power over one cycle of voltage.*
SPPU : Dec.-11, 14, 15, 16, 17, May- 12, 14, Marks 6

4.6 : A.C. through Series R-C Circuit

SPPU : Dec.-97,02,05,07,10,12
May-99,01,03,04,06,08,17,18,19

- Consider a circuit consisting of pure resistance R-ohms and connected in series with a pure capacitor of C-farads as shown in the Fig. 4.6.1.

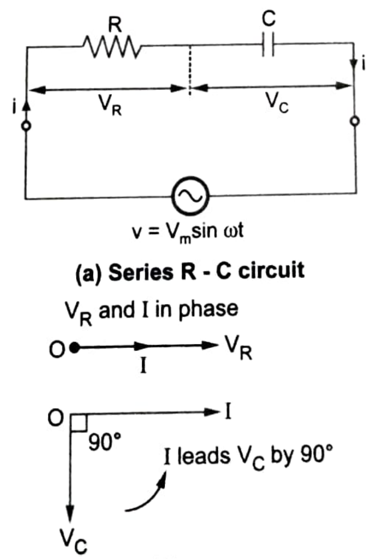


Fig. 4.6.1

- The series combination is connected across a.c supply given by

$$v = V_m \sin \omega t$$

Circuit draws a current I, then there are two voltage drops,

a) Drop across pure resistance $\quad V_R = I \times R$

b) Drop across pure capacitance $V_C = I \times X_C$

Where $X_C = \frac{1}{2\pi f C}$

and I, V_R, V_C are the r.m.s. values

The Kirchhoff's voltage law can be applied to get,

$$V = \overline{V_R} + \overline{V_C}$$

$$= \overline{IR} + \overline{IX_C} \quad \text{... (Phasor Addition)}$$

Phasor Diagram :

- Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.

Following are the **steps to draw the phasor diagram :**

1) Take current as reference phasor.
2) In case of resistance, voltage and current are in phase. So, V_R will be along current phasor.
3) In case of pure capacitance, current leads voltage by 90° i.e. voltage lags current by 90° so V_C is shown downwards i.e. lagging current by 90°.
4) The supply voltage being vector sum of these two voltages V_C and V_R obtained by completing parallelogram.

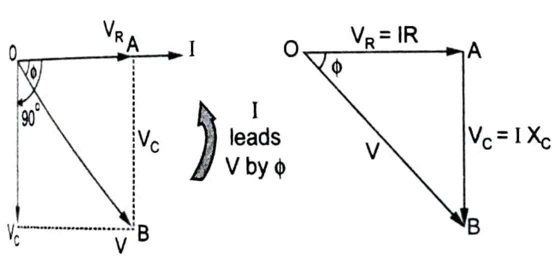


Fig. 4.6.2 Phasor diagram and voltage triangle

From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$= \sqrt{(IR)^2 + (I X_C)^2}$$

$$= I \sqrt{(R)^2 + (X_C)^2}$$

$$\therefore \quad V = I Z$$

Where $Z = \sqrt{(R)^2 + (X_C)^2}$ is the impedance of the circuit.

It can be seen that current leads voltage by angle ϕ hence

$$v(t) = V_m \sin \omega t \quad \text{and} \quad i(t) = I_m \sin(\omega t + \phi)$$

4.6.1 Impedance and Impedance Triangle

- Similar to R-L series circuit, in this case also, the impedance is nothing but opposition to the flow of alternating current. It is measured in ohms given by $Z = \sqrt{(R)^2 + (X_C)^2}$ where

  $X_C = \frac{1}{2\pi f C}\ \Omega$ called **capacitive reactance.**

- In R-C series circuit, current leads voltage by angle ϕ or supply voltage V lags current I by angle ϕ as shown in the phasor diagram in Fig. 4.6.2.
- From voltage triangle, we can write,

$$\tan \phi = \frac{V_C}{V_R} = \frac{X_C}{R},$$

$$\cos \phi = \frac{V_R}{V} = \frac{R}{Z}, \qquad \sin \phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

- If all the sides of the voltage triangle are divided by the current, we get a triangle called **impedance triangle.**
- Two sides of the triangle are 'R' and 'X_C' and the third side is impedance 'Z'.
- The X component of impedance is R and is given by, $R = Z \cos \phi$
- And Y component of impedance is X_C and is given by, $X_C = Z \sin \phi$

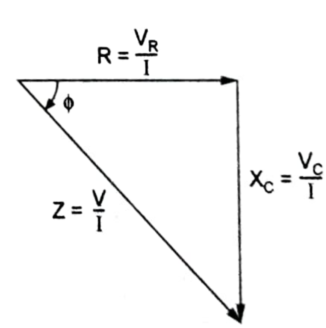


Fig. 4.6.3 Impedance triangle

But, as direction of the X_C is the negative Y direction, the rectangular form of the impedance is denoted as,

$$Z = R - j X_C \ \Omega$$

While in polar form, it is denoted as,

$$Z = |Z| \angle -\phi \ \Omega$$

$$|Z| = \sqrt{R^2 + X_C^2}, \quad \phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$$

Key Point *Thus ϕ is negative for capacitive impedance.*

4.6.2 Power and Power Triangle

- The current leads voltage by angle ϕ hence its expression is,

$$i = I_m \sin(\omega t + \phi) \text{ as current leads voltage}$$

- The power is the product of instantaneous values of voltage and current.

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t) . \sin(\omega t + \phi)]$$

$$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2} \cos(2\omega t + \phi)$$

as $\cos(-\phi) = \cos\phi$

- Now, second term is cosine term whose average value over a cycle is zero. Hence, average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} \cos\phi$$

$\therefore$ $\boxed{P = V I \cos\phi \text{ watts}}$ where V and I are r.m.s. values

- If we multiply voltage equation by current I, we get the power equation,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I} = \overline{VI\cos\phi} + \overline{VI\sin\phi}$$

- Hence, the power triangle can be shown as in the Fig. 4.6.4.

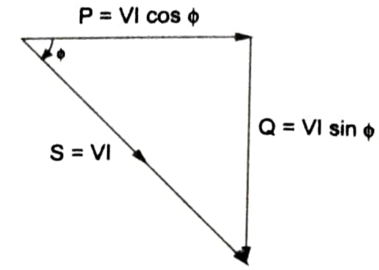


Fig. 4.6.4

Thus, the various powers are,

Apparent power,	S = V I	VA
True or average power,	P = V I cos ϕ	W
Reactive power,	Q = V I sin ϕ	VAR

For any single phase a.c. circuit, the average power is given by,

$$\mathbf{P = V I \cos\phi \quad watts}$$

Where **V, I are r.m.s. values**

$$\cos\phi = \text{Power factor of circuit}$$

cos ϕ is **lagging** for **inductive** circuit and cos ϕ is **leading** for **capacitive** circuit.

4.6.3 Waveforms of Voltage, Current and Power

The waveforms are shown in the Fig. 4.6.5.

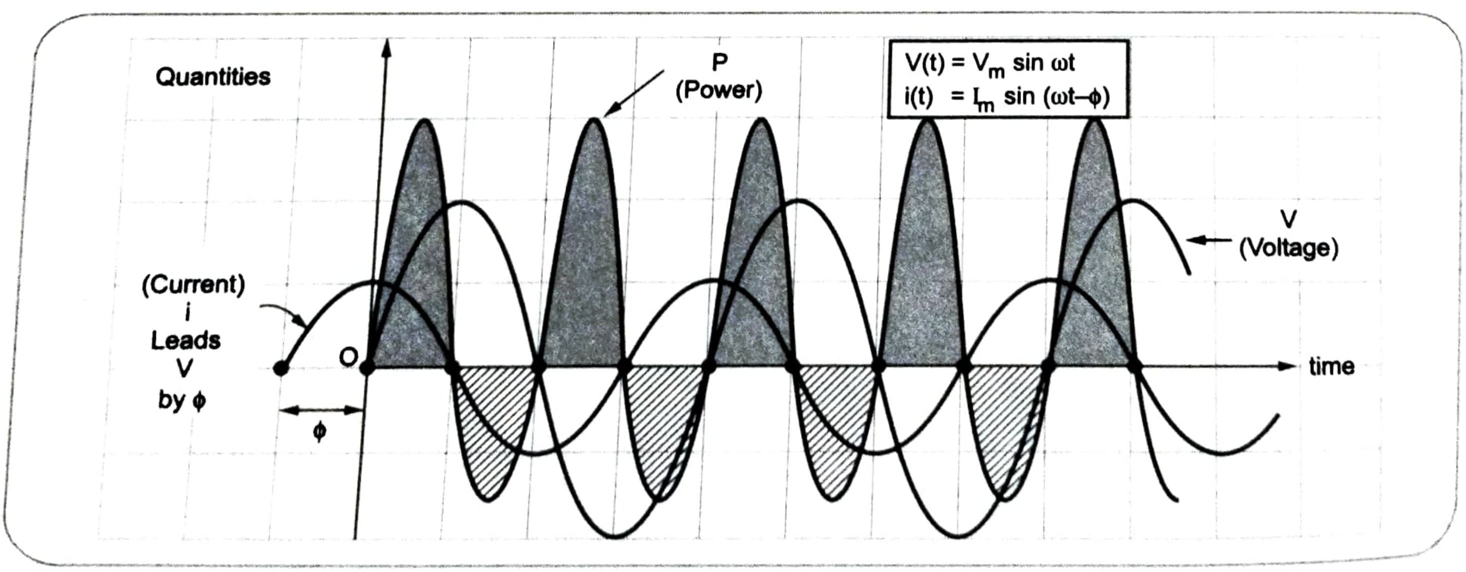


Fig. 4.6.5

Ex. 4.6.1 : *The series circuit having resistance 5 Ω and capacitance 150 µF is connected to 1-phase, 200 V, 50 Hz AC supply. Calculate - [i] Capacitive reactance X_C [ii] Impedance [iii] Current drawn by the circuit [iv] Power factor [v] Active power and [vi] Reactive power.*

SPPU : May-19, Marks 6

Sol. : Given : The circuit is shown in the Fig. 4.6.6.

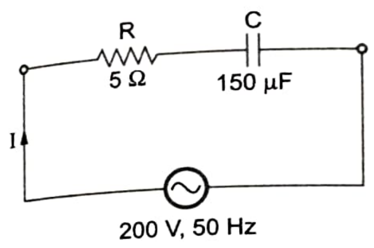


Fig. 4.6.6

To find : X_C, Z, I, cos ϕ, P, Q

i) $$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$$

$$= \mathbf{21.22\ \Omega}$$

ii) $$Z = R - jX_C = 5 - j\,21.22\ \Omega$$

$$= \mathbf{21.801 \angle -76.74^\circ\ \Omega}$$

iii) $$I = \frac{V}{Z} = \frac{200 \angle 0^\circ}{21.801 \angle -76.74^\circ}$$

$$= \mathbf{9.174 \angle +76.74^\circ\ A}$$

iv) $$\cos\phi = \cos(-76.74^\circ)$$

$$= \mathbf{0.229\ leading}$$

v) $$P = VI\cos\phi = 200 \times 9.174 \times 0.229$$

$$= \mathbf{420.85\ W}$$

vi) $$Q = VI\sin\phi$$

$$= 200 \times 9.174 \times \sin(-76.74)^\circ$$

$$= \mathbf{-1785.88\ VAR}$$

Negative sign indicates **leading nature** of reactive power.

Ex. 4.6.2 : *A resistance of 120 ohms and a capacitive reactance of 250 ohms are connected in series across a A.C. voltage source. If a current of 0.9 A is flowing in the circuit find out i) Power factor, ii) Supply voltage iii) Voltages across resistance and capacitance iv) Active power and reactive power.*

SPPU : May-03, Marks 8

Sol. : Given : The circuit is shown in the Fig. 4.6.7.

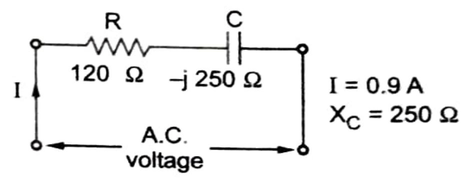


Fig. 4.6.7

To find : cos ϕ, V, V_R, V_C, P and Q.

$R = 120\ \Omega,\ X_C = 250\ \Omega,\ I = 0.9\ A$

$Z = R - jX_C = 120 - j250\ \Omega = 277.308 \angle -64.358^\circ$

Take current as reference.

$\therefore\ I = 0.9 \angle 0^\circ\ A$

i) Power factor $\cos\phi = \cos(-64.358^\circ)$

$= \mathbf{0.4327\ leading}$

ii) Supply voltage

$$V = I \times Z$$

$$= [0.9 \angle 0^\circ] \times [277.308 \angle -64.358^\circ]$$

$$\therefore\ V = \mathbf{249.5772 \angle -64.358^\circ\ V}$$

iii) $$V_R = I \times R = 0.9 \times 120$$

$$= \mathbf{108\ V}\ \text{(magnitude)}$$

$$V_C = I \times X_C = 0.9 \times 250 = \mathbf{225\ V}\ \text{(magnitude)}$$

iv) $$P = \text{Active power} = V\,I\cos\phi$$

$$= 249.5772 \times 0.9 \times 0.4327$$

$$= \mathbf{97.1928\ W}$$

$$Q = \text{Reactive power} = VI\sin\phi$$

$$= 249.5772 \times 0.9 \times \sin(-64.358°)$$

$$= \mathbf{-202.498\ VAR}$$

The negative sign indicates leading nature of reactive volt-amperes.

Ex. 4.6.3 : ***A series R-C circuit is connected across 200 V, 50 Hz a.c. supply draws a current of 20 A. When the frequency of the supply is increased to 100 Hz, the current increases to 23.4082 A. Calculate the value of resistance and capacitance of the circuit.***

Sol. : When frequency is 50 Hz i.e. $f_1 = 50$ Hz

$$|Z_1| = \frac{V}{I_1} = \frac{200}{20} = 10\ \Omega$$

When frequency is 100 Hz i.e. $f_2 = 100$ Hz

$$|Z_2| = \frac{V}{I_2} = \frac{200}{23.4082} = 8.544\ \Omega$$

$$Z_1 = \sqrt{R^2 + (X_{C_1})^2} \quad ...(1)$$

and $$Z_2 = \sqrt{R^2 + (X_{C_2})^2} \quad ...(2)$$

$\therefore$ $$(10)^2 = R^2 + (X_{C_1})^2 \quad ...(3)$$

and $$(8.544)^2 = R^2 + (X_{C_2})^2 \quad ...(4)$$

Subtracting equations (3) and (4),

$$\therefore \quad (X_{C_1})^2 - (X_{C_2})^2 = 27$$

i.e. $$\left(\frac{1}{2\pi f_1 C}\right)^2 - \left(\frac{1}{2\pi f_2 C}\right)^2 = 27$$

$$\therefore \frac{1}{4\pi^2 \times 50^2 \times C^2} - \frac{1}{4\pi^2 \times 100^2 \times C^2} = 27$$

$$\frac{1}{C^2}(7.6 \times 10^{-6}) = 27$$

i.e. $\therefore$ $$C^2 = 2.814 \times 10^{-7}$$

$\therefore$ $$C = \mathbf{5.305 \times 10^{-4}\ F}$$

Substituting in equation (3), $R = \mathbf{8\ \Omega}$

Ex. 4.6.4 : ***The waveforms of the voltage and current of a circuit are given by, e = 120 sin (314 t), i = 10 sin (314 t + π/6). Calculate the values of the resistance, capacitance which are connected in series to form the circuit.***

SPPU : Dec.-97, 02, ...

Sol. : **Given :** $e = 120 \sin(314\,t)$,

$$i = 10 \sin\left(314\,t + \frac{\pi}{6}\right)$$

To find : R and C.

Comparing with, $v = V_m \sin(\omega t)$,

$\therefore$ $V_m = 120$ and $\omega = 314$

Now $\omega = 2\pi f$ i.e. $314 = 2\pi f$

i.e. $f = 50$ Hz

$$V = \frac{V_m}{\sqrt{2}} = \frac{120}{\sqrt{2}} = 84.85\ V$$

Similarly, $i = 10 \sin(314\,t + \pi/6)$

Comparing with, $i = I_m \sin(\omega t + \phi)$,

$$I_m = 10\ A, \quad \phi = \frac{\pi}{6} = \frac{180°}{\pi} \times \frac{\pi}{6} = 30°$$

$\therefore$ $$I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07\ A$$

$$|Z| = \frac{V}{I} = \frac{84.85}{7.07} = 12\ \Omega$$

As current leads voltage by 30°, the circuit is series circuit, capacitive in nature.

As impedance is capacitive, ϕ must be negative.

$\therefore Z = 12\angle{-30°}\ \Omega = 10.393 - j\,6\ \Omega$ ... use P

Comparing with,

$Z = R - j\,X_C$, $R = \mathbf{10.393\ \Omega}$ and $X_C = 6\ \Omega$

Now $$X_C = \frac{1}{2\pi f C}$$ i.e. $6 = \frac{1}{2\pi \times ...}$

$\therefore$ $$C = \mathbf{530.45\ \mu F}$$

Expected Question

1. *Derive and show the waveforms of voltage, current and power for R-C series circuit ... supplied by a voltage v(t) = V_m sin ωt. Draw phasor diagram.* **SPPU : Dec.-05, 10, May-06, 08, ...**
2. *Derive an expression for current drawn and power consumed by a circuit consisting of 'R' and ... connected in series across v = V_m sin ωt supply.* **SPPU : Dec.-05, 10, 12, May-06, 08, 17, 18, ...**

4.7 : A.C. through Series R-L-C Circuit

SPPU : May-2000,02,04,06,09,10,11,12,13,14,15
Dec.-97,99,03,05,06,07,09,11

- Consider a circuit consisting of resistance R ohms pure inductance L henries and capacitance C farads connected in series with each other across a.c. supply. The circuit is shown in the Fig. 4.7.1.

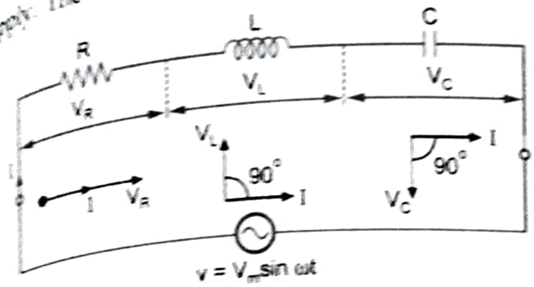


Fig. 4.7.1 R-L-C series circuit

- The a.c. supply is given by,

$$v = V_m \sin \omega t.$$

- The circuit draws a current I.
- Due to current I, there are different voltage drops across R, L and C which are given by,

a) Drop across resistance R is $V_R = I R$

b) Drop across inductance L is $V_L = I X_L$

c) Drop across capacitance C is $V_C = I X_C$

- The values of I, V_R, V_L and V_C are r.m.s. values
- The characteristics of three drops are,

a) V_R is in phase with current I.

b) V_L leads current I by 90°.

c) V_C lags current I by 90°.

- According to Kirchhoff's laws, we can write,

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{... Phasor addition}$$

Phasor Diagram

- Let us see the phasor diagram. Current I is taken as reference as it is common to all the elements.

Following are the steps to draw the phasor diagram :

1) Take current as reference.
2) V_R is in phase with I.
3) V_L leads current I by 90°.
4) V_C lags current I by 90°.
5) Obtain the resultant of V_L and V_C. Both V_L and V_C are in phase opposition(180° out of phase).
6) Add that with V_R by law of parallelogram to get the supply voltage.

- The phasor diagram depends on the conditions of the magnitudes of V_L and V_C which ultimately depends on the values of X_L and X_C. Let us consider the different cases.

4.7.1 $X_L > X_C$

- When $X_L > X_C$, obviously, I X_L i.e. V_L is greater than I X_C i.e. V_C. So, resultant of V_L and V_C will be directed towards V_L i.e. leading current I. Current I will lag the resultant of V_L and V_C i.e. $(V_L - V_C)$.
- The circuit is said to be **inductive** in nature. The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage, V. This is shown in the Fig. 4.7.2.
- From the voltage triangle,

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

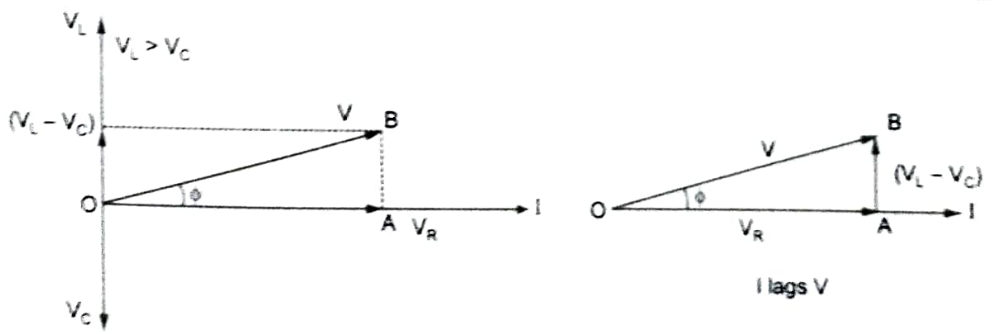


Fig. 4.7.2 Phasor diagram and voltage triangle for $X_L > X_C$

$$= \sqrt{(I\,R)^2 + (I\,X_L - I\,X_C)^2}$$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$\therefore$ $V = I\,Z$

where $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$

- So, if $v = V_m \sin \omega t$, then $i = I_m \sin(\omega t - \phi)$ as current lags voltage by angle ϕ for $X_L > X_C$.

4.7.2 $X_L < X_C$

- When $X_L < X_C$, obviously, $I\,X_L$ i.e. V_L is less than $I\,X_C$ i.e. V_C. So, the resultant of V_L and V_C will be directed towards V_C. Current I will lead $(V_C - V_L)$.
- The circuit is said to be **capacitive** in nature. The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. This is shown in the Fig. 4.7.3.
- From the voltage triangle,

$$V = \sqrt{(V_R)^2 + (V_C - V_L)^2}$$

$$= \sqrt{(I\,R)^2 + (I\,X_C - I\,X_L)^2}$$

$$= I\sqrt{(R)^2 + (X_C - X_L)^2}$$

$\therefore$ $V = I\,Z$

Where $Z = \sqrt{(R)^2 + (X_C - X_L)^2}$

- So, if $v = V_m \sin \omega t$, then $i = I_m \sin(\omega t + \phi)$ as current leads voltage by angle ϕ for $X_L < X_C$.

4.7.3 $X_L = X_C$

- When $X_L = X_C$, obviously, $V_L = V_C$. So, V_L and V_C will cancel each other and their resultant is zero.
- So, $V_R = V$ in such case and overall circuit is purely resistive in nature. The phasor diagram is shown in the Fig. 4.7.4.

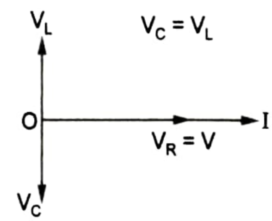


Fig. 4.7.4 Phasor diagram for $X_L = X_C$

From phasor diagram,

$$V = V_R = I\,R$$

$\therefore$ $V = I\,Z$ Where $Z = R$

4.7.4 Impedance Triangle

- The impedance is expressed as,

$\therefore$ $\boxed{Z = R + j\,X \quad \text{where} \quad X = X_L - X_C}$

- For $X_L > X_C$, ϕ is positive and the impedance triangle is as shown in the Fig. 4.7.5 (a).
- For $X_L < X_C$, $X_L - X_C$ is negative, so ϕ is negative and the impedance triangle is as shown in Fig. 4.7.5 (b).

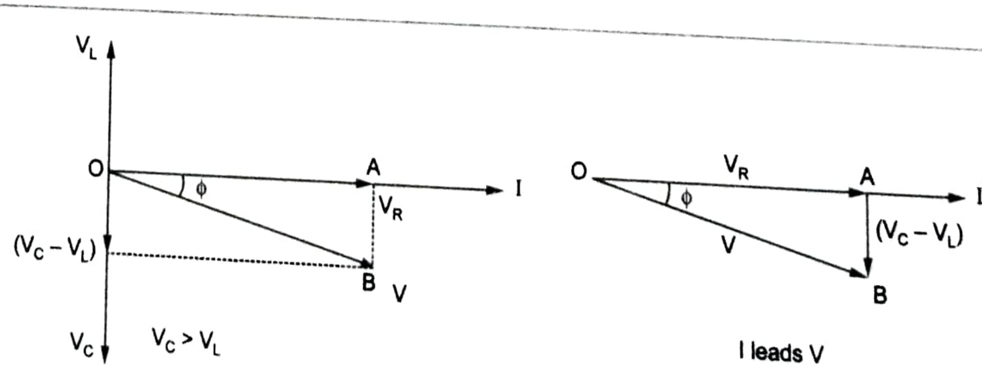


Fig. 4.7.3 Phasor diagram and voltage triangle for $X_L < X_C$

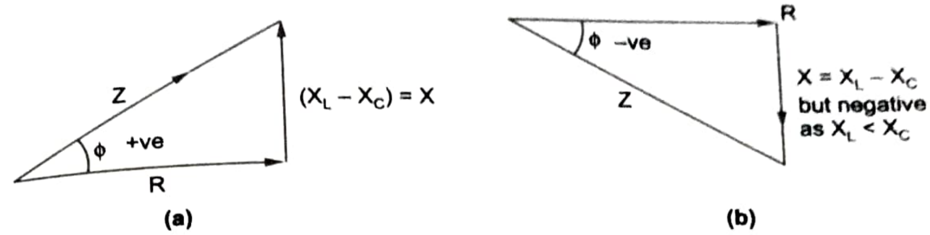


Fig. 4.7.5 Impedance triangles

- In both the cases, $R = Z \cos \phi$ and $X = Z \sin \phi$

4.7.5 Power and Power Triangle

- The average power consumed by the circuit is,

$$P_{av} = \text{Average power consumed by R + Average power consumed by L + Average power consumed by C}$$

- But, pure L and C never consume any power.

$$\therefore \quad \boxed{P_{av} = \text{Power taken by R} = I^2 R = I (I R) = I V_R}$$

But, $V_R = V \cos \phi$ in both the cases

$$\therefore \quad \boxed{P = V I \cos \phi \text{ W}}$$

- Thus, for any condition, $X_L > X_C$ or $X_L < X_C$, the power can be expressed as,

$$\boxed{P = \text{Voltage} \times \text{Component of current in phase with voltage}}$$

Key Point *The power triangle can be obtained by multiplying each side of impedance triangle by I^2.*

- The power triangles are shown in the Fig. 4.7.6.

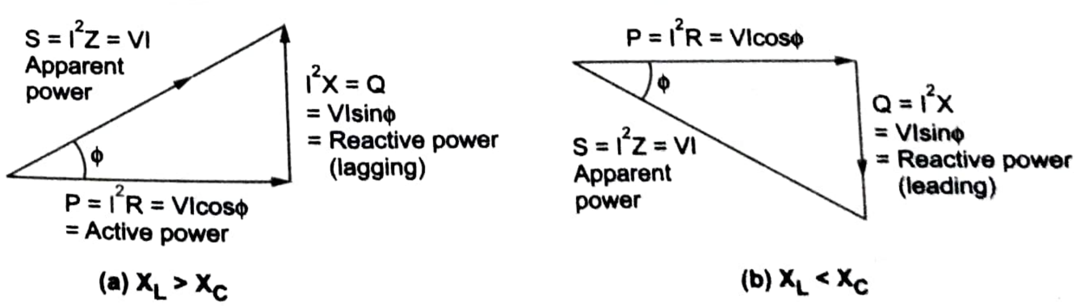


Fig. 4.7.6

Summary of RLC Circuit

Sr. No.	Circuit	Impedance (Z)		ϕ	p.f. cos ϕ	Remark
		Polar	Rectangular			
1.	Pure R	$R \angle 0° \Omega$	$R + j0 \Omega$	0°	1	Unity p.f.
2.	Pure L	$X_L \angle 90° \Omega$	$0 + j X_L \Omega$	90°	0	Zero lagging
3.	Pure C	$X_C \angle -90° \Omega$	$0 - j X_C \Omega$	- 90°	0	Zero leading
4.	Series RL	$\lvert Z\rvert \angle +\phi° \Omega$	$R + j X_L \Omega$	$0° \angle \phi \angle 90°$	cos ϕ	Lagging
5.	Series RC	$\lvert Z\rvert \angle -\phi° \Omega$	$R - jX_C \Omega$	$-90° \angle \phi \angle 0°$	cos ϕ	Leading
6.	Series RLC	$\lvert Z\rvert \angle \pm \phi° \Omega$	$R + j X \Omega$ $X = X_L - X_C$	ϕ	cos ϕ	$X_L > X_C$ Lagging $X_L < X_C$ Leading $X_L = X_C$ Unity

Table 4.7.1 Summary of R, L and C circuits

Ex. 4.7.1 : *An e.m.f given by v = 100 sin 100 πt is impressed across a circuit consisting of resistance of 40 Ω in series with 100 μF capacitor and 0.25 H inductor. Determine : i) r.m.s value of the current, ii) Power consumed iii) Power factor.*

SPPU : May-14, Marks 7

Sol. : **Given :** $V = 100 \sin 100\pi t = V_m \sin \omega t$, $R = 40\ \Omega$, $C = 100\ \mu F$, $L = 0.25$ H

To find : I, P, cos ϕ

$$V_m = 100 \text{ V}, \ \omega = 100\ \pi \text{ rad/s}$$

$$X_L = \omega L = 78.54\ \Omega$$

$$X_C = \frac{1}{\omega C} = 31.831\ \Omega$$

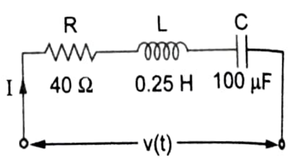


Fig. 4.7.7

$$Z = R + j X_L - j X_C = 40 + j\ 46.71\ \Omega = 61.496 \angle 49.425°\ \Omega$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 70.7106 \text{ V}$$

i) $$I \text{ (RMS)} = \frac{V_{rms}}{Z} = \frac{70.7106 \angle 0°}{61.496 \angle 49.425°} = \mathbf{1.15 \angle -49.425° \ A}$$

ii) $$P = V\ I \cos\phi = 70.7106 \times 1.15 \times \cos(-49.425°) = \mathbf{52.8922\ W}$$

iii) $$\cos\phi = \text{Power factor} = \mathbf{0.6504 \ lagging}$$

Ex. 4.7.2 : *An impedance Z_1 = (100 + j0) Ω is connected in series with another impedance Z_2 = (50 + j80) Ω. The circuit is connected to a single phase 230 V, 50 Hz supply. Calculate :*
i) Current drawn by the circuit. ii) Power consumed by whole circuit iii) Circuit power factor.

SPPU : May-15, Marks 7

Sol. : **Given :** $Z_1 = 100 + j0\ \Omega = 100 \angle 0° \Omega$, $Z_2 = 50 + j\ 80\ \Omega = 94.34 \angle 58° \Omega$,

To find : I, P and cos ϕ

$$Z_T = Z_1 + Z_2 = 150 + j\,80\ \Omega$$

$$= 170 \angle 28.072^\circ\ \Omega,\ V = 230 \angle 0^\circ\ V$$

$$I = \frac{V}{Z_T} = \frac{230 \angle 0^\circ}{170 \angle 28.072^\circ}$$

$$= \mathbf{1.353 \angle -28.072^\circ\ A} \quad \text{... Current}$$

ii) $\phi = 28.072^\circ$, $\cos\phi = 0.8823$ lagging

$$P = VI\cos\phi = 230 \times 1.353 \times 0.8823$$

$$= \mathbf{274.58\ W}$$

iii) Power factor = $\cos\phi$ = **0.8823 lagging**

Ex. 4.7.3 : *A series R-L-C circuit has resistance of 50 Ω, inductance of 0.1 H and capacitance of 50 μF connected in series across single phase 230 V, 50 Hz supply. Calculate :*

i) Current drawn by circuit

ii) Power factor of the circuit

iii) Active and rective power consumed by circuit

iv) Draw the phasor diagram.

SPPU : Dec.-07, May-09, 12, Marks 8

Sol. : Given : The arrangement is shown in the Fig. 4.7.8 (a).

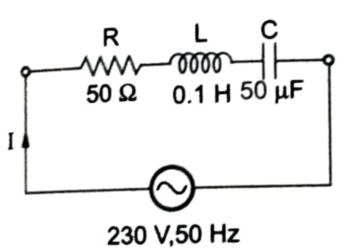


Fig. 4.7.8 (a)

To find : I, cos φ, P and Q

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1$$

$$= 31.4159\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$$

$$= 63.662\ \Omega$$

$$\therefore \quad Z = R + jX_L - jX_C$$

$$= 50 + j\,31.4159 - j63.662.$$

$$= 50 - j\,32.2461\ \Omega$$

$$= 59.4963 \angle -32.82^\circ\ \Omega$$

Let voltage be reference i.e. $V = 230 \angle 0^\circ$ V

i) $$I = \frac{V}{Z} = \frac{230\angle 0^\circ}{59.4963\angle -32.82^\circ}$$

$$= \mathbf{3.8657 \angle +32.82^\circ A}$$

ii) $$\cos\phi = \cos(32.82^\circ)$$

$$= \mathbf{0.8403\ leading}$$

iii) $$P = VI\cos\phi = 230 \times 3.8657 \times 0.8403$$

$$= \mathbf{747.1888\ W}$$

$$Q = VI\sin\phi$$

$$= 230 \times 3.8657 \times \sin(32.82^\circ)$$

$$= \mathbf{481.899\ VAR}$$

iv) The phasor diagram is shown in the Fig. 4.7.8 (b).

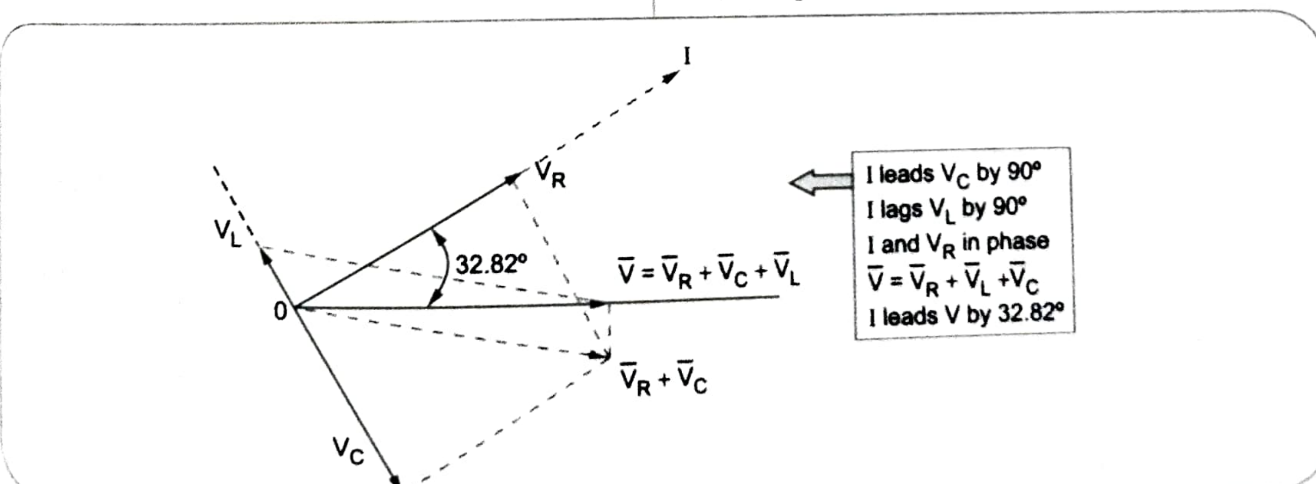


Fig. 4.7.8 (b)

Ex. 4.7.4 : ***A circuit consist of a pure inductor, a pure resistor and a capacitor connected in series. When the circuit is supplied with 100 volt, 50 Hz supply, the voltage across inductor and resistor are 240 volt and 90 volt respectively. If the circuit takes 10 A leading current calculate :***

1) Values of inductance, resistance and capacitance

2) Power factor of circuit

3) Voltage across capacitor. **SPPU : Dec.-09, Marks 8**

Sol. : Given : The circuit is shown in the Fig. 4.7.9.

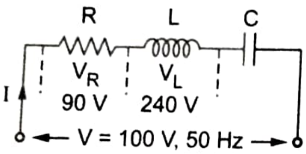


Fig. 4.7.9

To find : R, L, C and V_C

Let applied voltage be reference.

1) $V = 100\angle 0°A, \quad |I| = 10$ A

$$|V_R| = |I|\,R \quad \text{i.e.} \quad \therefore \quad 90 = 10\,R$$

$$\therefore \quad R = \mathbf{9\,\Omega}$$

$$|V_L| = |I|\,X_L \quad \text{i.e. } 240 = 10 \times X_L$$

$$\therefore \quad X_L = 24\,\Omega = 2\pi fL$$

$$\text{i.e.} \quad L = \frac{24}{2\pi \times 50} = \mathbf{0.0764\ H}$$

$$Z = R + j\,X_L - j\,X_C$$

but $X_C > X_L$ as current is leading.

$$\therefore \quad Z = R - j\,(X_C - X_L)$$

$$\text{i.e.} \quad |Z| = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\text{But} \quad |Z| = \frac{|V|}{|I|} = \frac{100}{10} = 10\,\Omega$$

$$\therefore \quad 10 = \sqrt{9^2 + (X_C - 24)^2}$$

$$\text{i.e.} \quad 100 = 81 + (X_C - 24)^2$$

$$\therefore \quad 19 = (X_C - 24)^2 \quad \text{i.e.} \quad X_C - 24 = \sqrt{19}$$

$$\therefore \quad X_C = 28.3589\,\Omega = \frac{1}{2\pi fC}$$

$$\therefore \quad C = \frac{1}{2\pi \times 50 \times 28.3589} = \mathbf{112.243\ \mu F}$$

2) $$Z = 9 + j\,24 - j\,28.3589\,\Omega$$

$$= 9 - j\,4.3589\,\Omega = 10\angle -25.8419°\,\Omega$$

$$\therefore \quad \text{Power factor} = \cos\phi = \cos(-25.8419°)$$

$$= \mathbf{0.9\ leading}$$

3) $$|V_C| = |I| \times |X_C| = 10 \times 28.3589$$

$$= \mathbf{283.589\ V}$$

Ex. 4.7.5 : ***Two impedances*** $Z_1 = 6 + j\,8$ ***ohm and*** $Z_2 = 5 + j\,12$ ***ohm are connected in series across a 100 V, 50 Hz supply. Calculate i) P. f. of the circuit and ii) Total active, reactive and apparant power consumed. Draw relevant phasor diagram.***

SPPU : May-11, Marks 10

Sol. : $$Z_{eq} = Z_1 + Z_2$$

$$= 11 + j\,20 = 22.825\angle 61.19°\,\Omega$$

i) $$\cos\phi = \cos(61.19°) = \mathbf{0.4819\ lagging}$$

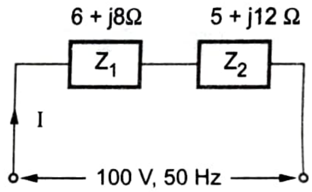


Fig. 4.7.10

ii) $$I = \frac{V}{Z_{eq}} = \frac{100\angle 0°}{22.825\angle 61.19°}$$

$$= 4.3811\angle -61.19°\ \text{A}$$

$$\therefore \quad P = V\,I\cos\phi = \mathbf{211.125\ W}$$

$$Q = V\,I\sin\phi = \mathbf{383.8818\ VAR}$$

$$S = V\,I = \mathbf{438.1085\ VA}$$

The phasor diagram is shown in the Fig. 4.7.10 (a).

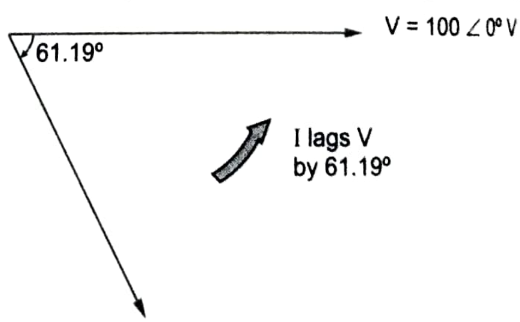


Fig. 4.7.10 (a)

Expected Question

1. *Sketch and explain the phasor diagram of RLC series circuit for*
i) $X_C > X_L$ ii) $X_C < X_L$ iii) $X_C = X_L$.

SPPU : May-04, 10, 13, 15, Dec.-03, 06, Marks 6

4.8 : Complex Power

- As seen earlier in a.c. circuits there are three types of powers exist. These are apparent power (S), active power (P) and reactive power (Q). The P and Q are the components of apparent power (S) such that

$$\boxed{|S| = \sqrt{P^2 + Q^2}}$$

While

$$\phi = \tan^{-1}\left[\frac{\sin\phi}{\cos\phi}\right]$$

$$= \tan^{-1}\left[\frac{VI\sin\phi}{VI\cos\phi}\right]$$

$$= \tan^{-1}\left[\frac{Q}{P}\right]$$

Where $P = VI\cos\phi$ and $Q = VI\sin\phi$

- Thus the apparent power can be expressed in the rectangular form as,

$$\boxed{S = P \pm jQ}$$

- This is called **complex power** where,

Real part = Active, true or real power in watts (W)

Imaginary part = Reactive power in reactive volt-amp (VAR)

Key Point *The reactive power Q may be positive or negative, depending upon nature of the circuit.*

- **The positive sign indicates lagging nature of reactive power while negative sign indicates leading nature of reactive power.**

In general if $V = V_1 \angle \theta_1$ and $I = I_1 \angle \theta_2$

Then $\phi = \theta_1 - \theta_2$

And $P = VI\cos(\theta_1 - \theta_2)$ W ... Active power

$Q = VI\sin(\theta_1 - \theta_2)$ VAR ... Reactive power

$S = P + jQ$ VA ...Complex power

If $\theta_1 - \theta_2 > 0$, Q is positive indicating lagging p.f. while

If $\theta_1 - \theta_2 < 0$, Q is negative indicating leading p.f.

Sign of reactive power	Nature of power factor	Nature of load
Q is positive	Lagging	Inductive
Q is negative	Leading	Capacitive

Table 4.8.1

Physical significance of reactive power :

The reative power is that component of power which is supplied to the reactive components of the load from the source during positive half cycle while it is returned back to supply from the components to the source during negative half cycle. It is rate of change of energy with time which keeps on flowing from the source to reative components and back from the components to the source, alternately. The reactive power charges and discharges the reactive components alternately.

Key Point *Thus reactive power never gets consumed by the circuit but flows alternately back and forth from the source to the reactive components and vice-versa.*

Expected Question

1. *What is complex power ? Explain its physical significance.*

4.9 A.C. Parallel Circuit

SPPU : Dec.-99,02,03,05,06,07,08,09,10,13,14,15
May-02,04,06,07,09,10,11,12,15,16,18

- **A parallel circuit is one in which two or more impedances are connected in parallel across the supply voltage.** Each impedance may be a separate series circuit. Each impedance is called branch of the parallel circuit.
- The Fig. 4.9.1 shows a parallel circuit consisting of three impedances connected in parallel across an a.c. supply of V volts.

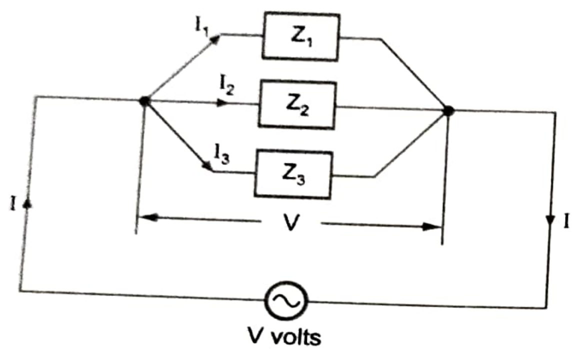


Fig. 4.9.1 A.C. parallel circuit

Key Point *The voltage across all the impedances is same as supply voltage of V volts.*

The current taken by each impedance is different.

Applying Kirchhoff's law,

$$\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3} \quad \text{... (Phasor addition)}$$

$$\therefore \quad \frac{\bar{V}}{\bar{Z}} = \frac{\bar{V}}{\bar{Z_1}} + \frac{\bar{V}}{\bar{Z_2}} + \frac{\bar{V}}{\bar{Z_3}}$$

$$\therefore \quad \frac{1}{\bar{Z}} = \frac{1}{\bar{Z_1}} + \frac{1}{\bar{Z_2}} + \frac{1}{\bar{Z_3}}$$

Where Z is called **equivalent impedance.** This result is applicable for 'n' such impedances connected in parallel.

4.9.1 Two Impedances in Parallel

- The Fig. 4.9.2 shows the impedances connected in parallel.
- The voltage across both Z_1 and Z_2 is same as V.

$$\therefore \quad \bar{I_1} = \frac{\bar{V}}{\bar{Z_1}} \text{ and } \bar{I_2} = \frac{\bar{V}}{\bar{Z_2}}$$

$$\therefore \quad \bar{I_T} = \bar{I_1} + \bar{I_2} \quad \text{...Total current}$$

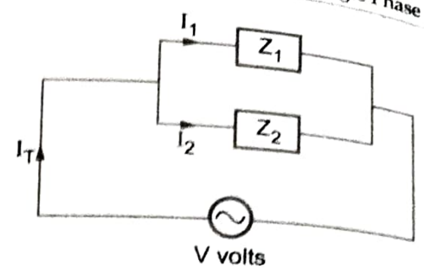


Fig. 4.9.2

If total current I_T is known, than the two branch currents can be obtained by using current division rule as,

$$\bar{I_1} = \bar{I_T} \times \frac{\bar{Z_2}}{\bar{Z_1} + \bar{Z_2}} \text{ and } \bar{I_2} = \bar{I_T} \times \frac{\bar{Z_1}}{\bar{Z_1} + \bar{Z_2}}$$

Following are the steps to solve parallel a.c. circuit :

1) The currents in the individual branches are to be calculated by using the relation

$$\bar{I_1} = \frac{\bar{V}}{\bar{Z_1}}, \quad \bar{I_2} = \frac{\bar{V}}{\bar{Z_2}}, \ldots\ldots, \quad \bar{I_n} = \frac{\bar{V}}{\bar{Z_n}}$$

While the individual phase angles can be calculated by the relation,

$$\tan \phi_1 = \frac{X_1}{R_1}, \tan \phi_2 = \frac{X_2}{R_2}, \ldots, \tan \phi_n = \frac{X_n}{R_n}$$

2) Voltage must be taken as reference phasor as it is common to all branches.
3) Represent all the currents on the phasor diagram and add them graphically or mathematically by expressing them in **rectangular form.** This is the resultant current drawn from the supply.
4) The phase angle of resultant current I is power factor angle. Cosine of this angle is the power factor of the circuit.

4.9.2 Concept of Admittance

Admittance is defined as the reciprocal of the impedance.** It is denoted by Y and is measured in unit **siemens** or **mho.

- Now, current equation for the circuit shown in the Fig. 4.9.3 is,

$$\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$$

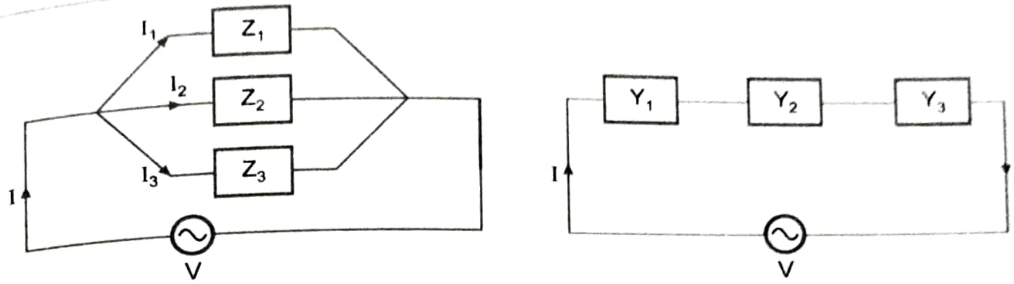


Fig. 4.9.3 Equivalent parallel circuit using admittances

$$\bar{I} = \bar{V} \times \left(\frac{1}{Z_1}\right) + \bar{V} \times \left(\frac{1}{Z_2}\right) + \bar{V} \times \left(\frac{1}{Z_3}\right)$$

$$\overline{VY} = \overline{VY_1} + \overline{VY_2} + \overline{VY_3}$$

$$\therefore \quad \bar{Y} = \bar{Y_1} + \bar{Y_2} + \bar{Y_3}$$

where Y is the **admittance** of the total circuit.

- **The three impedances connected in parallel can be replaced by an equivalent circuit, where three admittances are connected in series,** as shown in the Fig. 4.9.3.

4.9.3 Components of Admittance

- Consider an impedance given as, $Z = R \pm j X$
- Positive sign for inductive and negative for capacitive circuit.

Admittance $Y = \frac{1}{Z} = \frac{1}{R \pm j X}$

- Rationalising the above expression,

$$Y = \frac{R \mp j X}{(R \pm j X)(R \mp j X)} = \frac{R \mp j X}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2 + X^2}\right) \mp j\left(\frac{X}{R^2 + X^2}\right) = \frac{R}{Z^2} \mp j \frac{X}{Z^2}$$

$$\therefore \quad Y = G \mp j B$$

In the above expression, $G = \text{Conductance} = \frac{R}{Z^2}$

and $B = \text{Susceptance} = \frac{X}{Z^2}$

Conductance (G) :

It is defined as the ratio of the resistance to the square of the impedance. It is measured in the unit **siemens.**

Susceptance (B)

It is defined as the ratio of the reactance to the square of the impedance. It is measured in the unit **siemens.**

The susceptance is said to be inductive (B_L) if its sign is negative. The susceptance is said to be capacitive (B_C) if its sign is positive.

Note The sign convention for the reactance and the susceptance are opposite to each other.

$$Y = G + j B = |Y| \angle \phi \text{ siemens or mho}$$

$$|Y| = \sqrt{G^2 + B^2}, \ \phi = \tan^{-1} \frac{B}{G}$$

B is negative if inductive and B is positive if capacitive.

Key Point *Impedances in parallel get converted to admittances in series while impedances in series get converted to admittances in parallel.*

4.9.4 Admittance Triangles

- The sides of the triangle representing the conductance, susceptance and admittance of the circuit, it is known as **admittance triangle.**
- The Fig. 4.9.4 shows such admittance triangles.

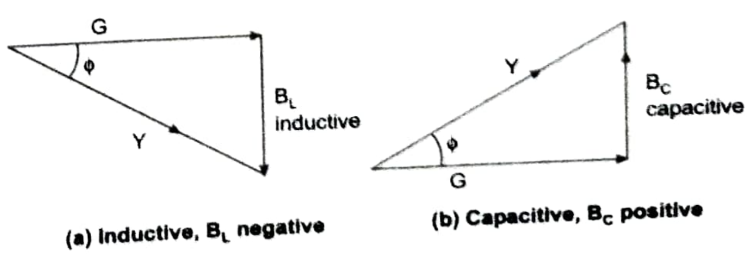


Fig. 4.9.4 Admittance triangles

4.9.5 Admittance Method to Solve Parallel Circuit

- The various steps to solve the parallel circuit by admittance method are,

Step 1 : Calculate the admittance of each branch from the respective impedance.

$$Y_1 = \frac{1}{Z_1},\ Y_2 = \frac{1}{Z_2},\ Y_3 = \frac{1}{Z_3}\ ......$$

Step 2 : Convert all the admittances to the respective rectangular form.

Step 3 : Calculate the equivalent admittance of the circuit by adding the individual admittances of the branches.

$$Y_{eq} = Y_1 + Y_2 + Y_3..... = G_{eq} + B_{eq}$$

Step 4 : The total current drawn from the supply is then given by,

$$I_T = V \times Y_{eq}$$

Step 5 : The individual branch currents can be obtained as,

$$I_1 = V \times Y_1,\quad I_2 = V \times Y_2,\quad I_3 = V \times Y_3\ ......$$

- It can be crosschecked that the vector addition of all the above currents gives the total current calculated in step 4.

Step 6 : The angle between V and I_T is the power factor angle ϕ. The cosine of this angle is the power factor of the circuit. The power factor of the circuit can also be obtained as,

$$\cos\phi = \frac{G_{eq}}{Y_{eq}}$$

- The nature of the power factor is to be decided from the sign of B_{eq}. If it is negative power factor is lagging while if it is positive the power factor is leading.

Step 7 : Voltage must be taken as reference phasor as it is common to all branches to draw the phasor diagram..

Ex. 4.9.1 : *A circuit is shown in Fig. 4.9.5. Draw its equivalent admittance circuit. Also calculate admittance, conductance and susceptance.*

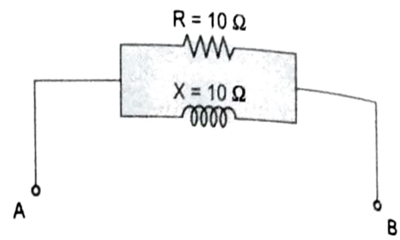


Fig. 4.9.5

Sol. : The impedance of branch 1, $Z_1 = R + j\,0$ where $R = 10\ \Omega$

$$\therefore \quad Z_1 = 10 + j\,0 = 10\angle 0°\ \Omega$$

The impedance of branch 2, $Z_2 = 0 + j\,X$

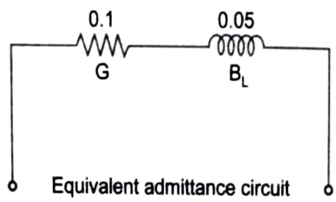


Fig. 4.9.5 (a)

Where $X = 20\ \Omega$

$$\therefore \quad Z_2 = 0 + j\,20 = 20\angle 90°\ \Omega$$

$$\text{Admittance } Y_1 = \frac{1}{Z_1} = \frac{1}{10\angle 0°}$$

$$= 0.1\angle 0°\ \text{siemens}$$

$$\text{Admittance } Y_2 = \frac{1}{Z_2} = \frac{1}{20\angle 90°}$$

$$= 0.05\angle -90°\ \text{siemens}$$

$$Y_1 = 0.1\angle 0° = 0.1 + j\,0\ \text{siemens}$$

$$Y_2 = 0.05\angle -90° = 0 - j\,0.05\ \text{siemens}$$

$$\overline{Y} = \overline{Y}_1 + \overline{Y}_2 = 0.1 + j\,0 + 0 - j\,0.05$$

$$= 0.1118\angle -26.56°$$

$\therefore$ Conductance G = **0.1**

Susceptance B = **0.05**

and admittance Y = **0.1118 siemens**

Ex. 4.9.2 : ***A coil having resistance of 50 ohm and inductance of 0.02 H is connected in parallel with a capacitor of 35 μF across a single phase 200 V, 50 Hz supply. Calculate branch current and total current drawn by the circuit.*** **SPPU : Dec.-14, Marks 6**

Sol. : The circuit is shown in the Fig. 4.9.6.

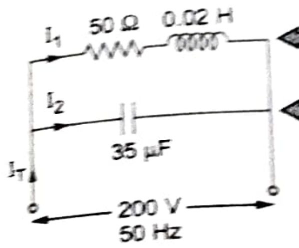


Fig. 4.9.6

To find : I_T, I_1 and I_2

$$Z_1 = R + j2\pi fL = 50 + j\,6.2831\ \Omega$$

$$= 50.3932\ \angle 7.162°\ \Omega$$

$$Z_2 = 0 - j\frac{1}{2\pi fC} = 0 - j\,90.9456\ \Omega$$

$$= 90.9456\ \angle -90°\ \Omega$$

Let V be reference i.e. $200\ \angle 0°$ V

$$\bar{I}_1 = \frac{V}{Z_1} = \frac{200\ \angle 0°}{50.3932\ \angle 7.162°}$$

$$= \mathbf{3.9687\ \angle -7.162°\ A}$$

$$\bar{I}_2 = \frac{V}{Z_2} = \frac{200\ \angle 0°}{90.9456\ \angle -90°}$$

$$= \mathbf{2.2\ \angle +90°\ A}$$

$$\bar{I}_T = \bar{I}_1 + \bar{I}_2$$

$$= 3.9377 - j0.4947 + 0 + j\,2.2$$

$$= 3.9377 + j1.7053\ A$$

$$= \mathbf{4.291\ \angle 23.414°\ A}$$

Ex. 4.9.3 : ***Two circuits having impedances $Z_1 = 8 + j6\ \Omega$ and $Z_2 = 5 + j10\ \Omega$ are connected in parallel across 200 V, 50 Hz, 1-ph a.c. supply. Calculate :***
i) Current drawn by each circuit ii) Total current iii) p.f. of whole circuit.

SPPU : May-11, Dec.-15, Marks 7

Sol. : Given : $Z_1 = 8 + j6\ \Omega = 10\ \angle 36.869°\ \Omega$

$Z_2 = 5 + j10\ \Omega = 11.18\ \angle 63.435°\ \Omega$

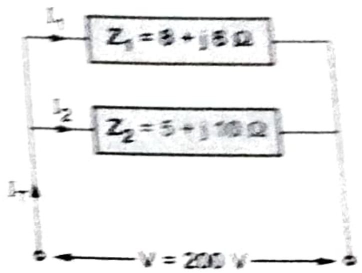


Fig. 4.9.7

To find : I_1, I_2, I_T, cos ϕ

Let V = 200 V is reference.

i)

$$I_1 = \frac{V}{Z_1} = \frac{200\ \angle 0°}{10\ \angle 36.869°}$$

$$= \mathbf{20\ \angle -36.869°\ A = 16 - j\ 12\ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{200\ \angle 0°}{11.18\ \angle 63.435}$$

$$= \mathbf{17.89\ \angle -63.435°A = 8 - j\ 16\ \Omega}$$

ii)

$$I_T = \bar{I}_1 + \bar{I}_2 = 16 - j\ 12 + 8 - j\ 16$$

$$= 24 - j\ 28\ A$$

$$= \mathbf{36.878\ \angle -49.398°\ A}$$

iii) p.f. of whole circuit = cos (– 49.398°) = **0.6508 lagging**

Ex. 4.9.4 : ***Two impedance $Z_1 = 30\ \angle 45°\ \Omega$ and $Z_2 = 45\ \angle 30°\ \Omega$ are connected in parallel across a single phases 230 V, 50 Hz supply. Calculate the i) Current drawn ii) Power factor and iii) Power consumed by the circuit.*** **SPPU : May-18, Marks 7**

Sol. : The circuit is shown in the Fig. 4.9.8.

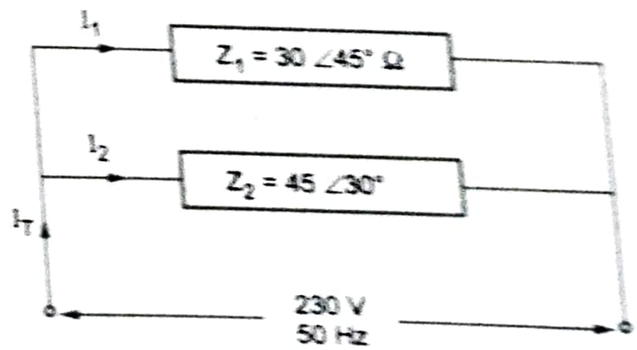


Fig. 4.9.8

Let $\quad V = 230 \angle 0° \text{ V}$ is reference

i) $$I_1 = \frac{V}{Z_1} = \frac{230\angle 0°}{30\angle 45°}$$

$$= \mathbf{7.667\angle -45° \text{ A}}$$

$$= \mathbf{5.421 - j5.421 \text{ A}}$$

$$I_2 = \frac{V}{Z_2} = \frac{230\angle 0°}{45\angle 30°}$$

$$= \mathbf{5.111\angle -30° \text{ A}}$$

$$= \mathbf{4.426 - j2.555 \text{ A}}$$

$\therefore$ $$I_T = \bar{I}_1 + \bar{I}_2$$

$$= 5.421 - j\,5.421 + 4.426 - j\,2.555$$

$$= 9.847 - j\,7.976 \text{ A}$$

$$= \mathbf{12.672\angle -39° \text{ A}}$$

ii) $$\cos\phi = \cos(-39°) = \mathbf{0.777 \text{ lagging}}$$

iii) $$P = VI_T \cos\phi = 230 \times 12.672 \times 0.777$$

$$= \mathbf{2264.613 \text{ W}}$$

Expected Questions

1. *Define admittance.*

SPPU : May-04, 06, 07, 09, Dec.-08, 09, 10, Marks 2

2. *What is admittance ? Which are its two components ? State its unit. How the admittance is expressed in rectangular and polar form.*

SPPU : Dec.-10, May-10, 12,15,16, Marks 5

3. *Define conductance and susceptance.*

SPPU : May-04, 06, 07, 08, 09, Dec.-06, 08, 09, Marks 2

4. *Draw the admittance triangle and explain.*

SPPU : Dec.-05,13, May-08, 09, Marks 3

5. *Explain the admittance method to solve parallel a.c. circuit.*

4.10 : Resonance in Series R-L-C Circuit

**SPPU : Dec.-97,2000,01,04,05,06, 09,11,17,18
May-99,2000,01,02,04,05,07,08,10,19**

- We know that both X_L and X_C are the functions of frequency f. When f is varied both X_L and X_C also get varied.
- At a certain frequency, X_L becomes equal to X_C. Such a condition when $X_L = X_C$ for a certain frequency is called **series resonance**.
- At resonance the reactive part in the impedance of RLC series circuit is zero.
- The frequency at which the resonance occurs is called **resonant frequency** denoted as ω_r rad/sec or f_r Hz.

4.10.1 Characteristics of Series Resonance

- In a series resonance, the voltage applied is constant and frequency is variable. Hence following parameters of series RLC circuit get affected due to change in frequency :

1) X_L	2) X_C	3) Total reactance X
4) Impedance Z	5) I	6) $\cos\phi$

- As $X_L = 2\pi fL$. as frequency is changed from 0 to ∞, X_L increases linearily and graph of X_L against f is straight line passing through origin.
- As $X_C = \dfrac{1}{2\pi fC}$, as frequency is changed from 0 to ∞, X_C reduces and the graph of X_C against f is rectangular hyperbola. Mathematically sign of X_C is opposite to X_L hence graph of X_L Vs f is shown in the first quadrant while X_C Vs f is shown in the third quadrant.
- At $f = f_r$, the value of $X_L = X_C$ at this frequency.
- As $X = X_L - X_C$, the graph of X against f is shown in the Fig. 4.10.1.
- For $f < f_r$, the $X_C > X_L$ and net reactance X is **capacitive** while for $f > f_r$, the $X_L > X_C$ and net reactance X is **inductive**.
- $Z = R + jX = R + j(X_L - X_C)$ but at $f = f_r$, $X_L = X_C$ and $X = 0$ hence the net impedance $Z = R$ which is purely resistive. So **impedance** is **minimum** and **purely resistive** at series resonance. The graph of Z against f is also shown in the Fig. 4.10.1. (See Fig. 4.10.1 on next page)

Key Point *As impedance is minimum, the **current** $I = V/Z$ is **maximum** at series resonance.*

- The power factor $\cos\phi = R/Z$ and at $f = f_r$ as $Z = R$ the **power factor** is **unity** and at its **maximum** at series resonance. For $f < f_r$ it is **leading** in nature while for $f > f_r$ it is **lagging** in nature.

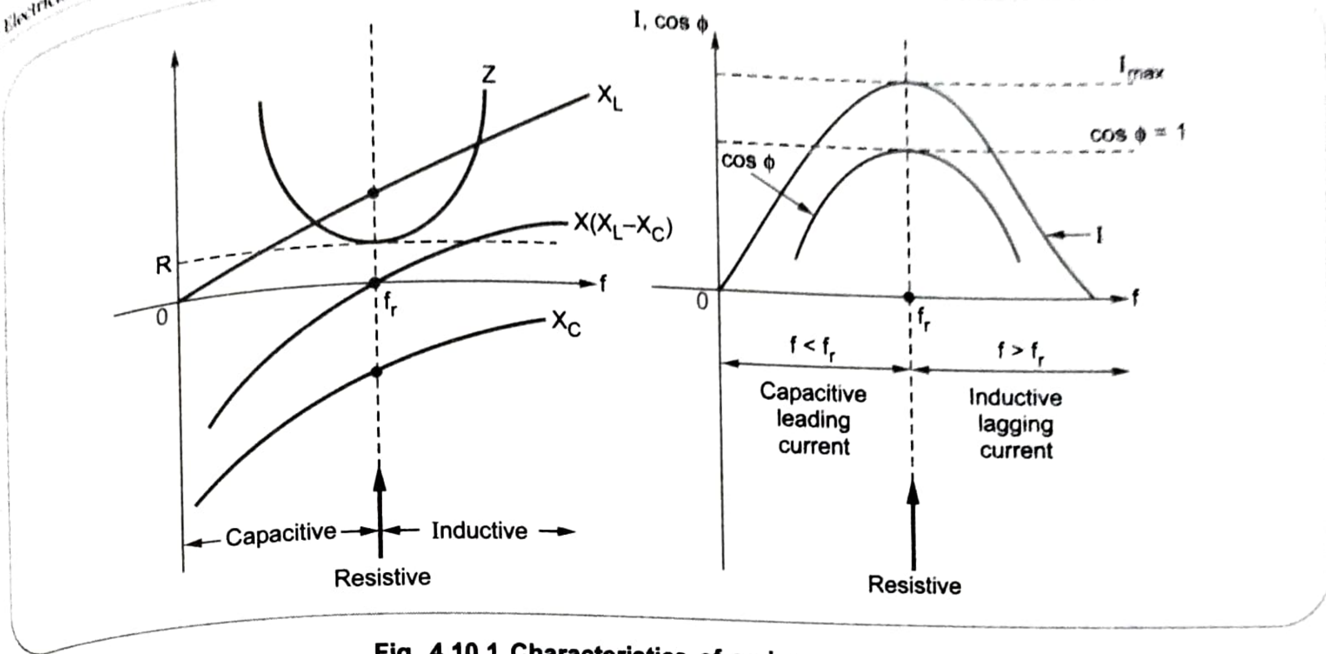


Fig. 4.10.1 Characteristics of series resonance

4.10.2 Expression for Resonant Frequency

Let f_r be the resonant frequency in Hz at which,

$$X_L = X_C$$

i.e. $$2\pi f_r L = \frac{1}{2\pi f_r C} \quad \text{... Series resonance}$$

i.e. $$(f_r)^2 = \frac{1}{4\pi^2 LC}$$

i.e. $$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$$

∴ $$\boxed{\omega_r = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$$

4.10.3 Bandwidth of Series R-L-C Circuit

- At series resonance, current is maximum and impedance Z is minimum.
- Now power consumed in a circuit is proportional to square of the current as $P = I^2R$.
- So at series resonance as current is maximum, power is also at its maximum i.e. P_m.
- The Fig. 4.10.2 shows the graph of current and power against frequency.
- It can be observed that at two frequencies f_1 and f_2 the power is half of its maximum value. These frequencies are called **half power frequencies**.

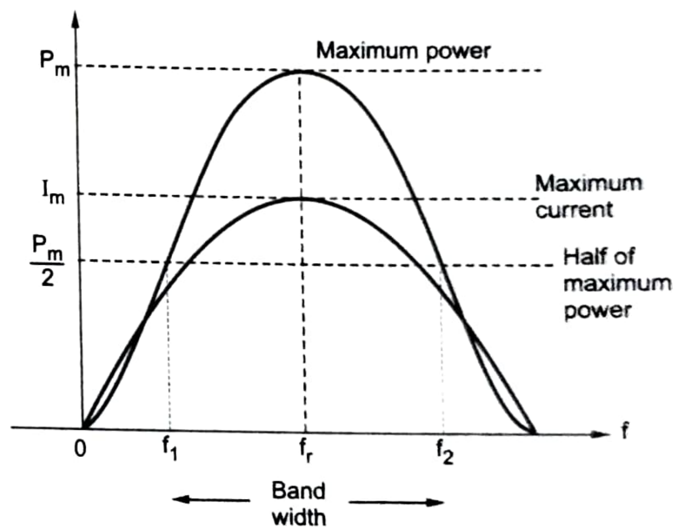


Fig. 4.10.2 Bandwidth

Definition of Bandwidth :

- The difference between the half power frequencies f_1 and f_2 at which power is half of its maximum is called bandwidth of the series R-L-C circuit.

∴ $$\boxed{\text{B.W.} = f_2 - f_1}$$

- In the bandwidth, the power is more than half the maximum value.
- The bandwidth decides the selectivity. The **selectivity** is defined as the ratio of the resonant frequency to the bandwidth.

$$\boxed{\text{Selectivity} = \frac{f_r}{\text{B.W.}} = \frac{f_2 - f_1}{\text{B.W.}}}$$

Key Point *Thus if the bandwidth is more, the selectivity of the circuit is less.*

- Out of the two half power frequencies, the frequency f_2 is called **upper cut-off frequency** while the frequency f_1 is called **lower cut-off frequency**.

4.10.4 Expressions for Lower and Upper Cut-off Frequencies

- The current in a series RLC circuit is given by the equation,

$I = \frac{V}{Z}$ but $Z = R + j(X_L - X_C)$

$$\therefore I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

... (4.10.1)

At resonance,

$$I_m = \frac{V}{R} \quad \text{(maximum value)} \quad \text{... (4.10.2)}$$

And $P_m = I_m^2 R$

At half power point,

$$P = \frac{P_m}{2} = \frac{I_m^2}{2} R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R$$

$$\therefore \quad I = \frac{I_m}{\sqrt{2}} \quad \text{at half power frequency}$$

Equating equations (4.10.1) and (4.10.2),

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2} \cdot R}$$

i.e. $$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}\, R$$

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2 R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R \quad \text{...(4.10.3)}$$

- From the equation (4.10.3) we can find two values of half power frequencies which are ω_1 and ω_2 corresponding to f_1 and f_2.

$$\therefore \quad \omega_2 L - \frac{1}{\omega_2 C} = +R \quad \text{...(4.10.4)}$$

$$\text{And} \quad \omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{... (4.10.5)}$$

$$\therefore (\omega_1 + \omega_2) L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = 0$$

... adding (4.10.4) and (4.10.5)

$$\therefore \quad (\omega_1 + \omega_2) L = \frac{(\omega_1 + \omega_2)}{\omega_1 \omega_2} \cdot \frac{1}{C}$$

$$\text{i.e.} \quad \omega_1 \omega_2 = \frac{1}{LC} \quad \text{... (4.10.6)}$$

$$\text{But} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \boxed{\omega_1 \omega_2 = (\omega_r)^2 \text{ i.e. } f_1 f_2 = (f_r)^2} \quad \text{... (4.10.7)}$$

- The equation (4.10.7) shows that **the resonant frequency is the geometric mean of the two half power frequencies.**

$$\therefore \quad \boxed{f_r = \sqrt{f_1 f_2}} \quad \text{... (4.10.8)}$$

Subtracting (4.10.5) from (4.10.4) we get,

$$(\omega_2 - \omega_1) L - \left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right)\frac{1}{C} = 2R$$

$$\therefore (\omega_2 - \omega_1) + \frac{(\omega_2 - \omega_1)}{\omega_1 \omega_2} \cdot \frac{1}{LC} = \frac{2R}{L}$$

... Dividing both sides by L

$$\therefore \quad (\omega_2 - \omega_1) + (\omega_2 - \omega_1) = \frac{2R}{L} \quad \text{... As } \frac{1}{\omega_1 \omega_2} = LC$$

$$\therefore \quad (\omega_2 - \omega_1) = \frac{R}{L} \quad \text{i.e.} \quad f_2 - f_1 = \frac{R}{2\pi L} \quad \text{... (4.10.9)}$$

$$\text{Thus} \quad \boxed{\text{B.W.} = \frac{R}{2\pi L}}$$

The bandwidth is also denoted as,

B.W. = $2\Delta f$ where

$$\boxed{\Delta f = \frac{R}{4\pi L}}$$ as shown in the Fig. 4.10.3

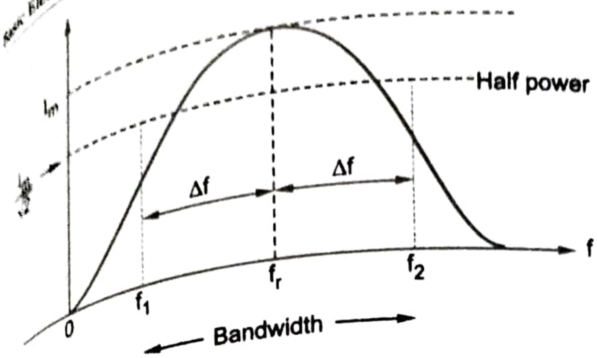


Fig. 4.10.3

From Fig. 4.10.3 we can write,

$$\boxed{f_1 = f_r - \Delta f \quad \text{and} \quad f_2 = f_r + \Delta f}$$

4.10.5 Quality Factor

- *The quality* factor of R-L-C series circuit is the *voltage magnification* in the circuit at resonance.

$$\text{Voltage magnification} = \frac{\text{Voltage across L or C}}{\text{Supply voltage}}$$

Now $\quad V_L$ = Voltage across L

$$= I_m X_L = I_m \omega_r L \text{ at resonance}$$

And at resonance, $\quad I_m = \dfrac{V}{R} \quad$ and $\quad V_L = \dfrac{V\omega_r L}{R}$

$$\therefore \text{Voltage magnification} = \frac{\frac{V\omega_r L}{R}}{V} = \frac{\omega_r L}{R}$$

This is nothing but quality factor Q.

$$\therefore \quad \boxed{Q = \frac{\omega_r L}{R}} \quad \text{but} \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} \quad \text{while} \quad \text{B.W.} = \frac{R}{2\pi L}$$

$$\therefore \quad Q \times \text{B.W.} = \frac{1}{R}\sqrt{\frac{L}{C}} \times \frac{R}{2\pi L} = \frac{1}{2\pi\sqrt{LC}} = f_r$$

$$\therefore \quad \boxed{Q = \frac{f_r}{\text{B.W.}}}$$

- The **significance of quality factor** can be stated as,
  1. It indicates the selectivity or sharpness of the tuning of a series circuit.
  2. It gives the correct indication of the selectivity of such series R-L-C circuit which are used in many radio circuits.

Key Point ***At the resonant frequency, the impedance is minimum and hence the circuit is known as acceptor circuit at resonance.***

Ex. 4.10.1 : *A coil of 15 mH is connected in series with 25 Ω resistance and a capacitor across 230 V 50 Hz supply. Find the value of capacitor so that circuit draws maximum current. What will be the power factor and power consumed ?*

SPPU : Dec.-17, 18, Marks 7

Sol. : The circuit is shown in the Fig. 4.10.4

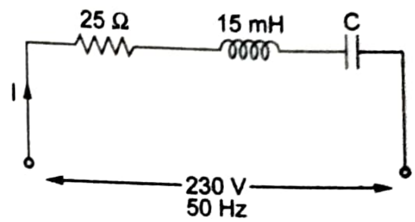


Fig. 4.10.4

For current to be maximum, the circuit must be in resonance condition

i.e. $\quad X_L = X_C$

$$\therefore \quad 2\pi f L = \frac{1}{2\pi f C} \quad \text{i.e.} \quad C = \frac{1}{(2\pi \times 50)^2 \times 15 \times 10^{-3}}$$

$$\therefore \quad C = \mathbf{675.474\,\mu F}$$

Power factor = **Unity** $\quad$ ... **Resonance**

$$\therefore \quad P = VI\cos\phi \quad \quad \ldots I = \frac{V}{R} = \frac{230}{25} = 9.2 \text{ A}$$

$$\therefore \quad P = 230 \times 9.2 \times 1 = \mathbf{2116\ W}$$

Ex. 4.10.2 : *A RLC series circuit with a resistance of 10 Ω, impedance of 0.2 H and a capacitance of 40μF is supplied with a 100 V supply at variable frequency. Find the following w.r.t the series resonant circuit :*

i) The frequency at resonance ii) The current
iii) Power iv) Power factor
v) Voltage across R, L, C at that time
vi) Quality factor of the circuit
vii) Half power points
viii) Phasor diagram.

Sol. : The given values are, $R = 10\ \Omega$, $L = 0.2$ H, $C = 40\ \mu F$ and $V = 100$ V

To find : f_r, I P, $\cos\phi$, V_R, V_L, V_C, Q, f_1, f_2

i) $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.2\times 40\times 10^{-6}}} = \mathbf{56.2697\ Hz}$

ii) $I_m = \dfrac{V}{R} = \dfrac{100}{10} = \mathbf{10\ A}$

... Current is maximum at resonance

iii) $P_m = I_m^2\ R = (10)^2 \times 10 = \mathbf{1000\ W}$

iv) Power factor is **unity**, as impedance is **purely resistive** at resonance

v) $V_R = I_m\ R = 10\ \times 10 = \mathbf{100\ V}$

$X_L = 2\ \pi\ f_r\ L = 2\pi\ \times 56.2697 \times 0.2 = 70.7105\ \Omega$

$\therefore\ V_L = I_m\ X_L = 10 \times 70.7105 = \mathbf{707.105}$ V

And $X_C = \dfrac{1}{2\pi f_r C} = \dfrac{1}{2\pi\times 56.2697\times 40\times 10^{-6}} = 70.7105\ \Omega$

$\therefore\ \boxed{V_C = I_m\ X_C = \mathbf{707.105}\ V}$

Thus $V_L = V_C$ at resonance

vi) $Q = \dfrac{\omega_r L}{R} = \dfrac{2\pi f_r L}{R} = \mathbf{7.071}$

vii) $\Delta f = \dfrac{R}{4\pi L} = \dfrac{10}{4\pi\times 0.2} = 3.9788$

$\therefore\ f_1 = f_r - \Delta f = 56.2697 - 3.9788 = \mathbf{52.2909\ Hz}$

and $f_2 = f_r + \Delta f = 56.2697 + 3.9788 = \mathbf{60.2485\ Hz}$

viii) B.W. $= f_2 - f_1 = 60.2485 - 52.2909 = \mathbf{7.9576\ Hz}$

The phasor diagram is shown in the Fig. 4.10.5.

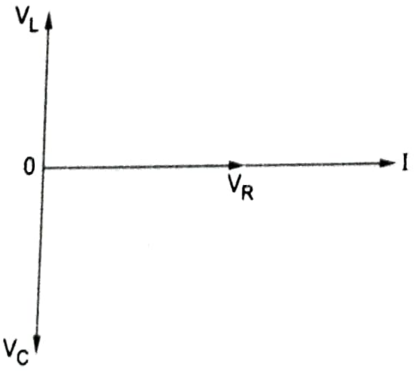


Fig. 4.10.5

Ex. 4.10.3 : *A series R-L-C circuit is connected to 230 V a.c. supply. The current drawn the circuit at the resonance is 25 A. The voltage drop across the capacitor is 4000 V, at the series resonance. Calculate the resistance, inductance if capacitance is 5 µF, also calculate the resonant frequency.*

Sol. : At resonance,

$$R = \frac{V}{I} = \frac{230}{25} = \mathbf{9.2\ \Omega}$$

Voltage drop across capacitor

$$V_C = I\ X_C \text{ i.e. } 4000 = 25 \times X_C \text{ i.e. } X_C = 160\ \Omega$$

At resonant frequency,

$$f = f_r \quad \text{And } X_C = \frac{1}{2\pi f_r C}$$

$$\therefore\ 160 = \frac{1}{2\pi f_r \times 10^{-6}} \text{ i.e. } f_r = \mathbf{198.943\ Hz}$$

At resonance $X_L = X_C = 160\ \Omega$

$$2\pi f_r L = 160$$

$$\therefore\ L = \frac{160}{2\pi f_r} = \frac{160}{2\pi\times 198.943} = \mathbf{0.128\ H}$$

Ex. 4.10.4 : *A series circuit consisting of a coil and a variable capacitance having reatance X_C. The coil has resistance of 10 Ω, inductive reactance of 20 Ω. It is observed that at certain value of capacitance current in the circuit is maximum, find 1) This value of capacitance 2) Impedance of the circuit 3) Power factor 4) Current, if applied voltage is 100 V, 50 Hz.*

SPPU : May-05, Marks 8

Sol. : The circuit diagram is shown in the Fig. 4.10.6.

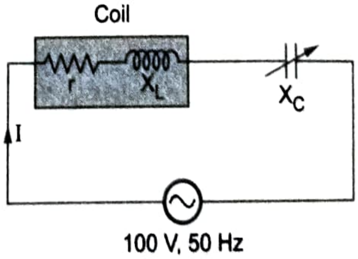


Fig. 4.10.6

$$r = 10\ \Omega,\ X_L = 20\ \Omega,\ I = I_{max}$$

This is the condition of series resonance for which $I = I_{max}$,

$$Z = r \text{ and } X_L = X_C$$

$X_C = 20\ \Omega = \frac{1}{2\pi f C}$

$C = \frac{1}{2\pi \times 50 \times 20} = \mathbf{159.1549\ \mu F}$

2) $Z = R + j X_L - jX_C = 10 + j\,20 - j\,20 = 10\ \Omega$

3) Power factor $= \cos\phi = \cos 0^\circ = \mathbf{1}$

For $X_L = X_C$, the V and I are inphase hence $\phi = 0^\circ$

4) $I = \frac{V}{Z} = \frac{100}{10} = \mathbf{10\ A}$

Ex. 4.10.5 : A series circuit consisting of a 12 Ω resistance, 0.3 henry inductance and a variable capacitor is connected across 100 V, 50 Hz A.C. supply. The capacitance value is adjusted to obtain maximum current. Find this capacitance value and the power drawn by the circuit under this condition. Now supply frequency is raised to 60 Hz, the voltage remaining same at 100 V. Find the value of inductive and capacitive reactance. **SPPU : May-10, Marks 8**

Sol. : The circuit is shown in the Fig. 4.10.7

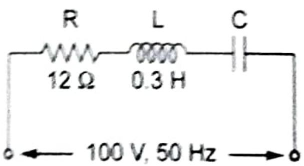


Fig. 4.10.7

For current to be maximum, there must be resonance at $f_r = f = 50$ Hz

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad 50 = \frac{1}{2\pi\sqrt{0.3 \times C}}$$

$$\text{i.e.} \quad \sqrt{0.3 \times C} = 3.1831 \times 10^{-3}$$

$$\therefore \quad C = \mathbf{33.7737\ \mu F}$$

$$P = VI = \frac{V^2}{R} \quad \text{...as } I_{max} = \frac{V}{R} \text{ for resonance}$$

$$\therefore \quad P = \frac{(100)^2}{12} = \mathbf{833.33\ W}$$

Now frequency is changed to $f_2 = 60$ Hz

$$X_L = 2\pi f_2 L = 2\pi \times 60 \times 0.3 = \mathbf{113.0973\ \Omega}$$

and $$X_C = \frac{1}{2\pi f_2 C} = \frac{1}{2\pi \times 60 \times 33.7737 \times 10^{-6}}$$

$$= 78.5398\ \Omega$$

Expected Questions

1. *What is resonance in series circuit ?* **SPPU : Dec.-04, 06, May-19, Marks 2**
2. *State the charactersitics of series resonance.*
3. *Derive the equation for the resonant frequency in series RLC circuit.* **SPPU : Dec.-04, 06, 09, May-02, 19, Marks 6**
4. *Define bandwidth of series RLC circuit.*
5. *What is selectivity ?*
6. *Define half power frequencies.*
7. *Derive the expressions for half power frequencies for series resonance.*
8. *Define quality factor of series RLC circuit and obtain expression for it.*

Formulae At a Glance

Pure Resistive Circuit :

- In purely resistive circuit, the current and the voltage applied are in phase with each other.
- $v = V_m \sin \omega t$ and $i = \left(\frac{V_m}{R}\right) \sin(\omega t)$

$\therefore \quad P_{av} = V_{rms} \times I_{rms}$ watts $= V \times I$ watts $= I^2 R$ watts

Pure Inductive Circuit :

- If $v = V_m \sin \omega t$ is applied to pure inductance then,

$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$ where $I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$

and $X_L = \omega L = 2\pi f L\ \Omega$

- In purely inductive circuit, **current lags voltage by 90°.**
- The term, X_L, is called **Inductive Reactance** and is measured in **ohms.**
- Power, $p(t) = -\frac{V_m I_m}{2} \sin(2\omega t)$
- Pure inductance never consumes power.

Pure Capacitive Circuit :

- If $v = V_m \sin \omega t$ is applied to pure capacitance then,

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \text{ where}$$

$$I_m = \frac{V_m}{X_C} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\,\Omega$$

- The current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. + 90°.
- The **current leads voltage applied by 90°**. The positive sign indicates leading nature of the current.
- Power, $p(t) = \frac{V_m I_m}{2} \sin(2\omega t)$
- Pure capacitance never consumes power.
- Average power consumption in a.c. circuits is,
- $P = V I \cos\phi$ W where V and I are r.m.s. values $\cos\phi$ is called power factor.

Sr. No.	Circuit	Impedance (Z)		ϕ	p.f. $\cos\phi$	Remark
		Polar	Rectangular			
1.	Pure R	$R \angle 0°\,\Omega$	$R + j0\,\Omega$	0°	1	Unity p.f.
2.	Pure L	$X_L \angle 90°\,\Omega$	$0 + j X_L\,\Omega$	90°	0	Zero lagging
3.	Pure C	$X_C \angle -90°\,\Omega$	$0 - j X_C\,\Omega$	- 90°	0	Zero leading
4.	Series RL	$\lvert Z\rvert \angle +\phi°\,\Omega$	$R + j X_L\,\Omega$	$0° \angle \phi \angle 90°$	$\cos\phi$	Lagging
5.	Series RC	$\lvert Z\rvert \angle -\phi°\,\Omega$	$R - jX_C\,\Omega$	$-90° \angle \phi \angle 0°$	$\cos\phi$	Leading
6.	Series RLC	$\lvert Z\rvert \angle \pm \phi°\,\Omega$	$R + j X\,\Omega$ $X = X_L - X_C$	ϕ	$\cos\phi$	$X_L > X_C$ Lagging $X_L < X_C$ Leading $X_L = X_C$ Unity

- For series R-L circuit, $Z = R + j X_L = |Z| \angle \phi\ \Omega$, $v(t) = V_m \sin \omega t$ and $i(t) = I_m \sin(\omega t - \phi)$,
$|Z| = \sqrt{R^2 + X_L^2}$, $\phi = \tan^{-1}\left[\frac{X_L}{R}\right]$, ϕ is positive for inductive impedance
- For series R-C circuit, $Z = R - j X_C = |Z| \angle -\phi\ \Omega$

$$v(t) = V_m \sin \omega t \quad \text{and} \quad i(t) = I_m \sin(\omega t + \phi)$$

$$|Z| = \sqrt{R^2 + X_C^2}, \quad \phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$$

ϕ is negative for capacitive impedance

- S = VI = Apparent power ... measured in volt-amp (VA)
- P = True or active power = $V I \cos\phi$ watts
- Q = Reactive power = $V I \sin\phi$ VAR
- Power factor = $\cos\phi = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{R}{Z}$

- Admittance Y is reciprocal of impedance Z i.e. $Y = \frac{1}{Z}$
- Admittance $Y = G + jB = \frac{R}{Z^2} \mp j\frac{X}{Z^2} = |Y| \angle \phi$ siemens or mho
- G = Conductance $= \frac{R}{Z^2}$, B = Susceptance $= \frac{X}{Z^2}$

$|Y| = \sqrt{G^2 + B^2}$, $\phi = \tan^{-1}\frac{B}{G}$

B is negative if inductive and B is positive if capacitive

- If there are two impedances connected in parallel and if I_T is the total current, then current division rule can be applied to find individual branch currents as,
- $\bar{I}_1 = \bar{I}_T \times \frac{\overline{Z_2}}{\overline{Z_1} + \overline{Z_2}}$ and $\bar{I}_2 = \bar{I}_T \times \frac{\overline{Z_1}}{\overline{Z_1} + \overline{Z_2}}$
- For series resonance, $f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz and

$\omega_r = \frac{1}{\sqrt{LC}}$ rad/sec

- B.W. = Bandwidth = $f_2 - f_1$ where f_1 and f_2 are half power frequencies
- Selectivity $= \frac{f_r}{B.W.} = \frac{f_2 - f_1}{B.W.}$
- $f_r = \sqrt{f_1 f_2}$ where f_1 and f_2 are half power frequencies
- B.W. = bandwidth $= \frac{R}{2\pi L}$
- $f_1 = f_r - \Delta f$ and $f_2 = f_r + \Delta f$ where $\Delta f = \frac{R}{4\pi L}$
- Q = Quality factor $= \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_r}{B.W.}$

Examples for Practice

Ex. 1 : *A 50 Hz, alternating voltage of 150 V (r.m.s.) is applied independently to (i) Resistance of 10 Ω (2) Inductance of 0.2 H (3) Capacitance of 50 μF Find the expression for the instantaneous current in each case. Draw the phasor diagram in each case.*

[Ans. : 21.213 sin (100 π t) A, 3.37 sin $\left(100\pi t - \frac{\pi}{2}\right)$ A, 3.33 sin $\left(100\pi t + \frac{\pi}{2}\right)$ A]

Ex. 2 : *An alternating current, $i = 414 \sin(2\pi \times 50 \times t)$ A, is passed through a series circuit consisting of a resistance of 100-ohm and an inductance of 0.31831 henry. Find the expressions for the instantaneous values of the voltages across i) The resistance, ii) The inductance and iii) The combination.*

SPPU : Dec.-2000

[Ans. : 141.4 sin (2 π × 50 t) V, 141.4 sin (2 π × 50 t + 90°) V, 200 sin (2 π × 50 t + 45°) V]

Ex. 3 : *A room heater of 2 kW, 125 V rating is to be operated on 230 V, 50 Hz, a.c., supply. Calculate the value of inductance, that must be connected in series with the heater so that heater will not get damaged due to over voltage.* **[Ans. : 0.0384 H]**

Ex. 4 : *A heater operates at 100 V, 50 Hz and takes current of 8 A and consumes 1200 W power. A choke coil is having ratio of reactance to resistance as 10, is connected in series with the heater. The series combination is connected across 230 V, 50 Hz a.c. supply. Calculate the i) Resistance of choke coil ii) Reactance of choke coiliii) Power consumed by choke coil iv) Total power consumed.*

[Ans. : 2.4552 Ω, 24.552 Ω, 157.13 W, 157.13 W, 957.1328 W]

Ex. 5 : *A coil draws 5 amps when connected to 100 volts 50 Hz supply. The resistance of the coil is 5 Ω determine*
i) Inductance of the coil
ii) Real power, reactive power, apparent power for the coil. **SPPU : May-99**

[Ans. : 61.64 mH, 125 W, 484.1229 VAR, 500 VA]

Ex. 6 : *An e.m.f. given by $v = 100 \sin \pi t$ is impressed across a circuit consists of resistance of 40 Ω in series with 100 μF capacitor and 0.25 H inductor.*
Determine - i) R.M.S. value of current
ii) Power consumed
iii) Power factor. **SPPU : Dec-04**

[Ans. : 1.1498 A, 52.8837 W, 0.6504 lagging]

Ex. 7 : A coil has inductance of 20 mH and resistance 5 ohm. It is connected across a supply voltage of $v = 48 \sin 314\ t$. Obtain the expression for current drawn by the coil.

SPPU : May-07

[Ans. : 5.9795 sin (314 t – 51.474°) A]

Ex. 8 : Calculate the resistance and inductance or capacitance in series for each of the following impedances. Assume the frequency to be 60 Hz.

i) (12 + j 30) ohms ii) – j 60 ohms

iii) 20 ∠ 60° ohms. **SPPU : May-99**

[Ans. : 79.58 mH, 44.209 μF, 45.94 mH]

Ex. 9 : A current $i = 5\sqrt{2} \sin\left(300\ t + \frac{\pi}{3}\right)$ ampere flows through a circuit. The voltage across which is $200\sqrt{2} \sin (300\ t)$ volt. Find the frequency, the rms values of voltage and current, the resistance, reactance and impedance of the circuit and the power factor.

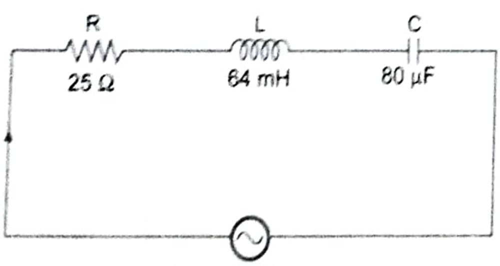
SPPU : May-04

[Ans. : 47.746 Hz, 200 V, 5 A, Ω, 20 Ω, 34.641 Ω, 0.5 leading]

Ex. 10 : Show the waveforms of voltage, current and power if $v = V_m \sin \omega t$ volt is applied across a R - C series circuit. **SPPU : Dec.-05**

Ex. 11 : A series circuit consisting of 25 Ω resistor, 64 mH inductor and 80 μF capacitor, is connected to a 110 V, 50 Hz, single phase supply as shown in Fig. 4.1. Calculate the current, voltage across individual element and the overall p.f. of the circuit. Draw a neat phasor diagram showing $\bar{I}$, $\bar{V}_R$, $\bar{V}_L$, $\bar{V}_C$ and $\bar{V}$.

SPPU : May-02



110 volt, 50 Hz supply

Fig. 4.1

[Ans. : 69.50 ∠ 128.2° volts, 134.10 ∠ – 51.9° volts, 0.7858 leading]

Ex. 12 : A series circuit having pure resistance of 40 ohms, pure inductance of 50.07 mH and a capacitor is connected across a 400 V, 50 Hz, A.C. supply. This R, L, C combination draws a current of 10 A. Calculate i) Power factor of the circuit and ii) Capacitor value.

SPPU : May-2000

[Ans. : 2.023 × 10^{-4} F]

Ex. 13 : Two coils A and B are connected in series across 200 V, 50 Hz a.c. supply. The power input to the circuit is 2.2. kW and 1.5 kVAR. If the resistance of coil A is 4 Ω and the reactance is 8 Ω. Calculate resistance and reactance of coil B.

Also calculate active power consumed by coil A and B, total impedance of the circuit.

[Ans. : 707.56 W, 1.4858 kW, 0.5 Ω]

Ex. 14 : Three impedances $Z_1 = (8 + j6)$ ohm, $Z_2 = (4 + j3)$ ohm and $Z_3 = (18 - j\,9)$ ohm are connected in series across the a.c. supply. If the voltage drop across Z_1 is $(40 + j\ 30)$ volts, calculate :

i) The current in the circuit

ii) The voltage drops across Z_2 and Z_3

iii) Total supply voltage

iv) Total power consumed by the series circuit

v) Power factor of the circuit.

Draw phasor diagram for the circuit.

SPPU : Dec.-97

Ex. 15 : A series circuit consists of resistance of 10 ohm, an inductance of $\frac{200}{\pi}$ mH and capacitance of $\frac{1000}{\pi}$ μF. Calculate

1) Current flowing in the circuit if supply voltage is 200 V, 50 Hz

2) p.f. of the circuit, 3) Power drawn from the supply. Also draw the phasor diagram.

SPPU : May-07

[Ans. : 14.1421 ∠ – 45° A, 2000 W, 0.7071 lagging]

Ex. 16 : Two impedances $Z_1 = 40 \angle 30°$ ohm and $Z_2 = 30 \angle 60°$ ohm are connected in series across single phase, 230 V, 50 Hz supply. Calculate the 1) Current drawn, 2) p.f. and 3) Power consumed by the circuit. **SPPU : May-07**

[Ans. : 3.399 $\angle -42.807°$ A, 0.7336 lagging, 573.5064 W]

Ex. 17 : A series R-L-C circuit with resistance of 50 Ω, capacitance of 25 μF and an inductance of 0.15 H is connected across 230 V, 50 Hz supply. Determine i) Impedance ii) Current iii) Power factor and iv) Power consumption of the circuit. **SPPU : Dec.-11**

[Ans. : i) 94.51 $\angle -58.05°$ Ω, ii) 2.433 A, iii) 0.529 leading v) 295.98 W]

Ex. 18 : Two impedance $Z_1 = 5 - j\ 13.1\ \Omega$ and $Z_1 = 8.57 + j\ 6.42\ \Omega$ are connected in parallel across a voltage of (100 + j200) volts. Estimate :-

i) Branch currents in complex form ii) Total power consumed, Draw a neat phasor diagram showing voltage, branch currents and all phase angles. **SPPU : May-01**

[Ans. : – 10.782 + j 11.75 A, 18.668 + j 9.3483 A, 22.5239 $\angle 69.5°$ A, 5008.212 W]

Ex. 19 : Two admittances, $Y_1 = (0.167 - j0.167)$ siemen, and $Y_2 = (0.1 + j\,0.05)$ siemen are connected in parallel across a 100 V, 50-Hz single-phase supply. Find the current in each branch and the total current. Also find the power-factor of the combination. Sketch a neat phasor diagram. **SPPU : Dec.-2000**

[Ans. : 16.699 – j 16.699 A, 10 + j 5 A, 29.15 $\angle -23.66°$ A, 0.9159 lagging]

Ex. 20 : Two impedances Z_1 and Z_2 are connected in parallel across applied voltage of (100 + j200) volts. The total power supplied to the circuit is 5 kW. The first branch takes a leading current of 16 A and has a resistance of 5 ohms while the second branch takes a lagging current at 0.8 power factor. Calculate i) Current in second branch ii) Total current iii) Circuit constants. **SPPU : Dec.-01**

[Ans. : 20.79 A, 22.48 A, 20.79 A]

Ex. 21 : A coil has a resistance of 4 Ω and inductance of 0.05 H, forms one branch of parallel circuit. The other branch has a similar coil but in series with it is R-C combination. If current in the two branches are equal in magnitude but have a phase difference of $\frac{1^{th}}{4}$ time period of 50 Hz voltage applied. Calculate values of R and C in second branch. Also find total current and total power factor if V = 200 V a.c. **[Ans. : 1.615 × 10^{-4} F, 17.45 $\angle -30.71°$ A, 0.8596 lagging]**

Ex. 22 : Two impedances (8 + j 6) Ω and (3 – 4 j) Ω are connected in parallel. If the total current drawn by the combination is 25 Amp, find the current and power taken by each impedance. **SPPU : May-08**

[Ans. : 11.1803 $\angle -63.4349°$ A, 22.3607 $\angle 26.5642°$ A, 1000 W, 1500 W]