

# Search

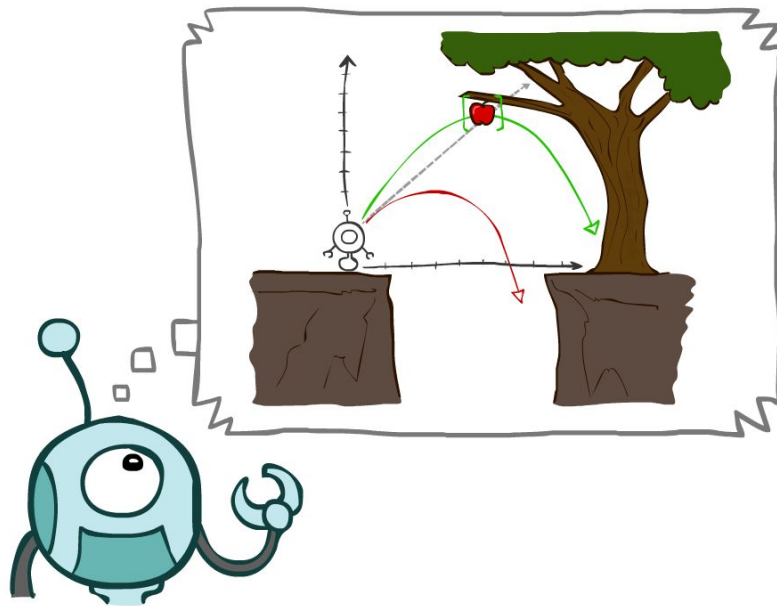
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Weicheng Ma

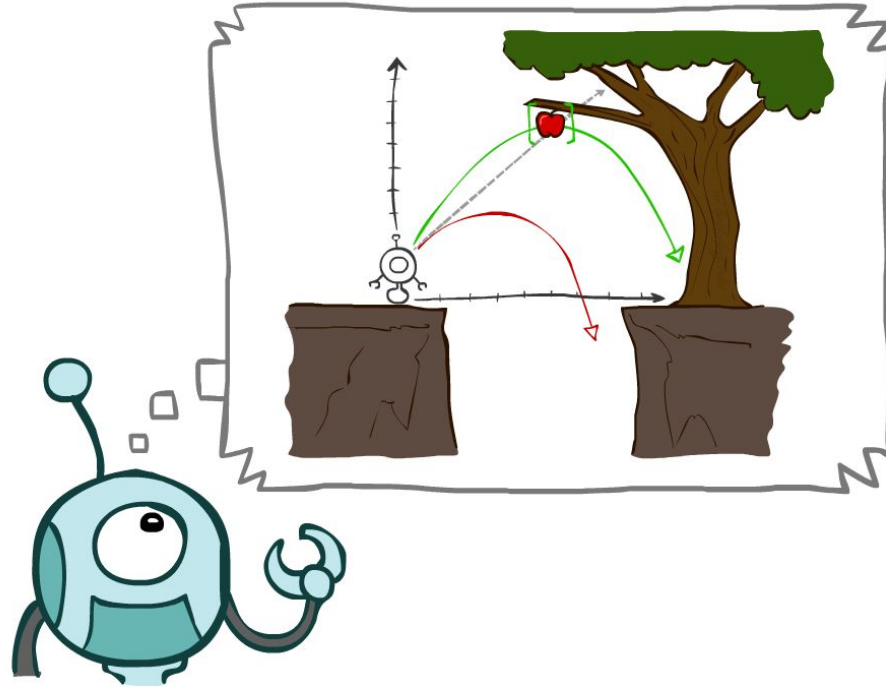
(Slides credit to Dan Klein)

# Today

- Agents that Plan Ahead
- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

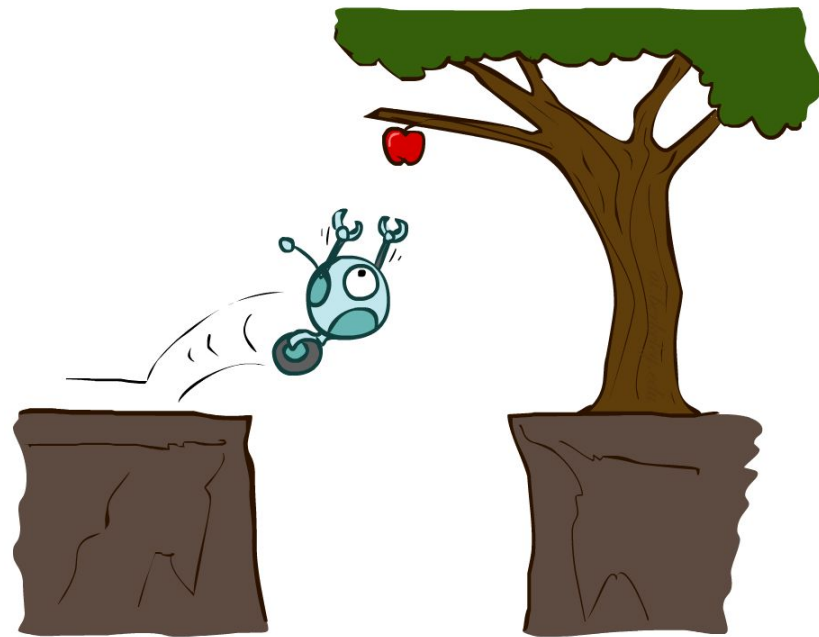


# Agents that Plan



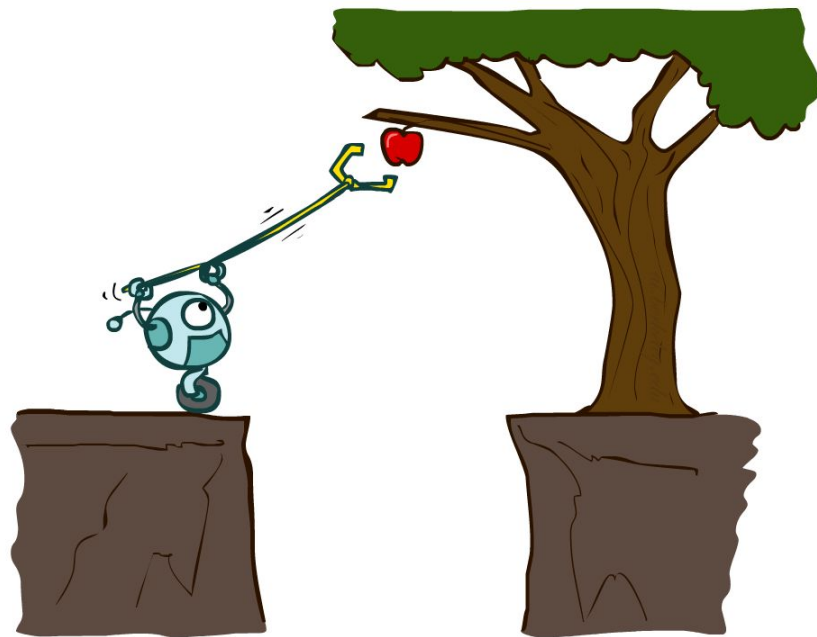
# Reflex Agents

- Reflex agents:
  - Choose action based on current percept (and maybe memory)
  - May have memory or a model of the world's current state
  - Do not consider the future consequences of their actions
  - Consider how the world IS
- Can a reflex agent be rational?



# Planning Agents

- Planning agents:
  - Ask “what if”
  - Decisions based on (hypothesized) consequences of actions
  - Must have a model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world **WOULD BE**
- Optimal vs. complete planning
- Planning vs. replanning

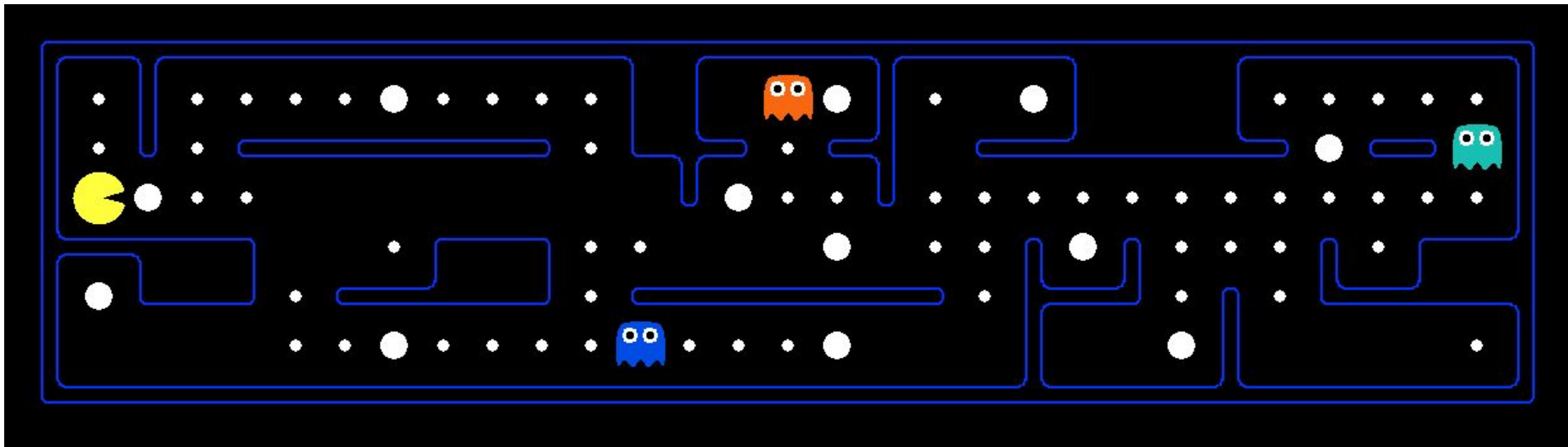


# Search Problems

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# Eat-All-Dots

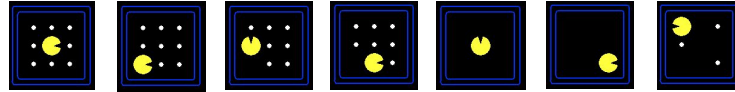


- Problem: eat all dots while keeping the ghosts perma-scared

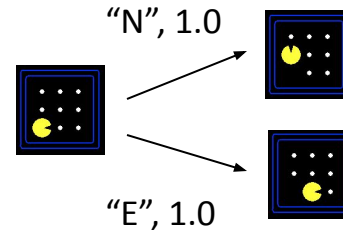
# Search Problems

- A **search problem** consists of:

- A state space



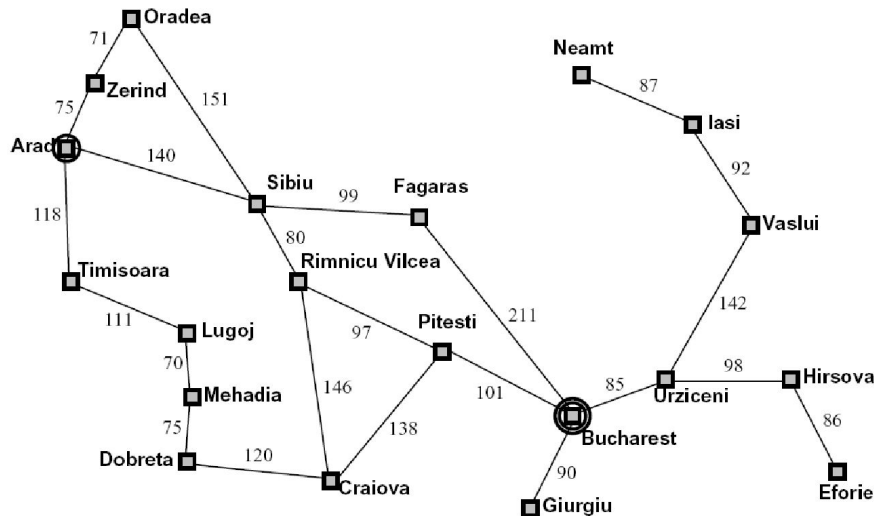
- A successor function  
(with actions, costs)



- A start state and a goal test
- A **solution** is a sequence of actions (a plan) which transforms the start state to a goal state



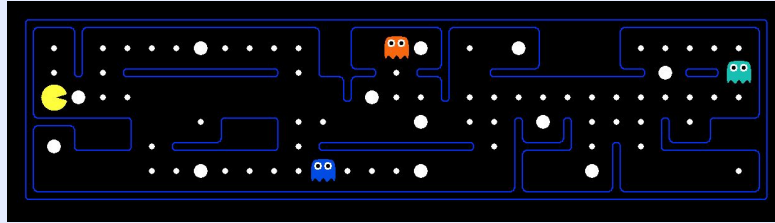
# Example: Traveling in Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

# What's in a State Space?

The **world state** includes every last detail of the environment



A **search state** keeps only the details needed for planning (abstraction)

## ■ Problem: Pathing

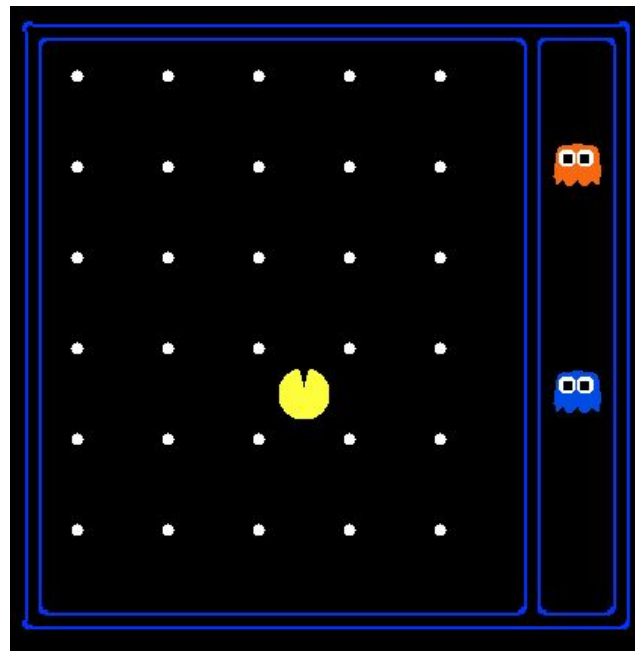
- States:  $(x,y)$  location
- Actions: NSEW
- Successor: update location only
- Goal test: is  $(x,y)=END$

## ■ Problem: Eat-All-Dots

- States:  $\{(x,y), \text{dot booleans}\}$
- Actions: NSEW
- Successor: update location and possibly a dot boolean
- Goal test: dots all false

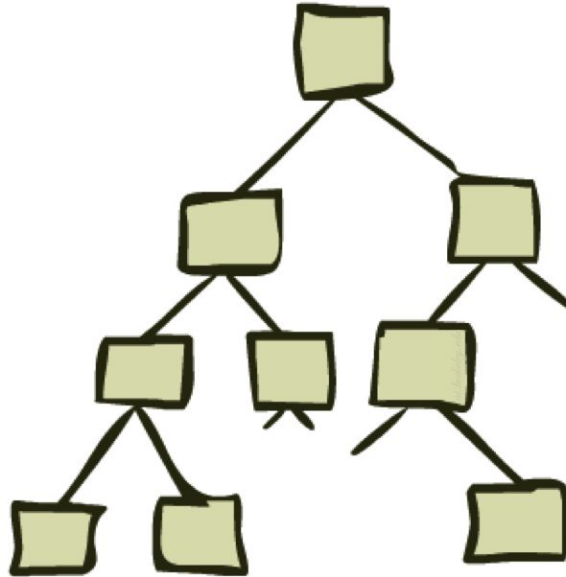
# State Space Sizes?

- World state:
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW
- How many
  - World states?  
 $120 \times (2^{30}) \times (12^2)$
  - States for pathing?  
120
  - States for eat-all-dots?  
 $120 \times (2^{30})$



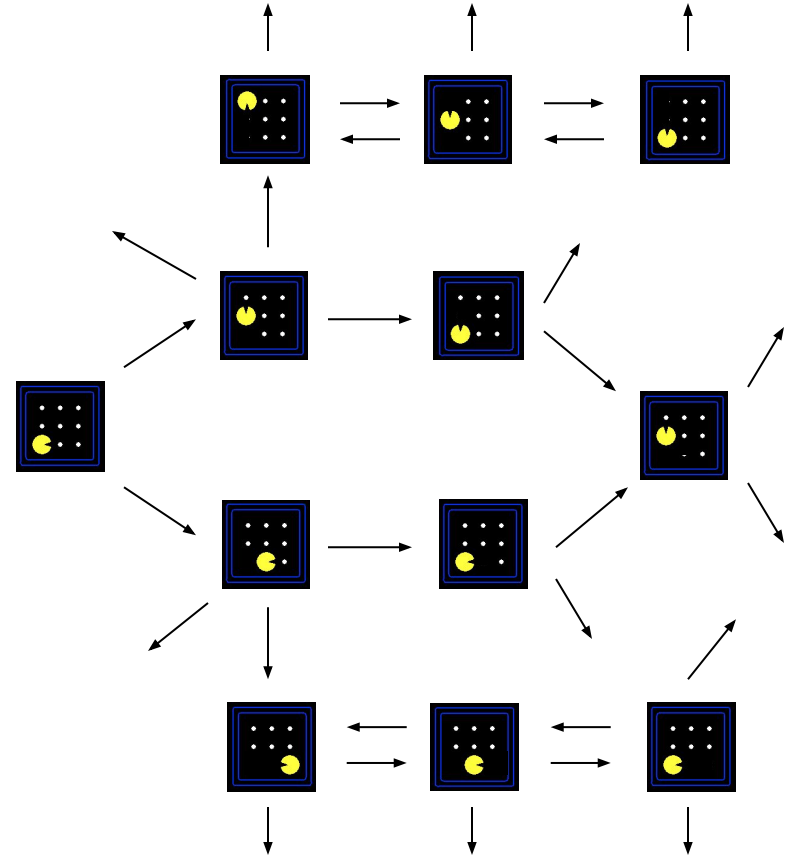
# State Space Graphs and Search Trees

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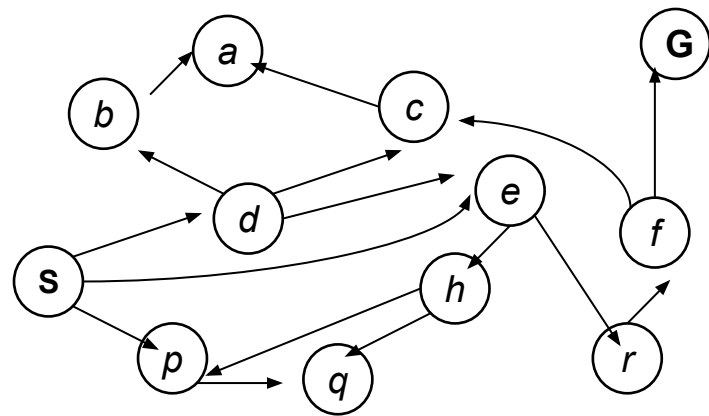
# State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea



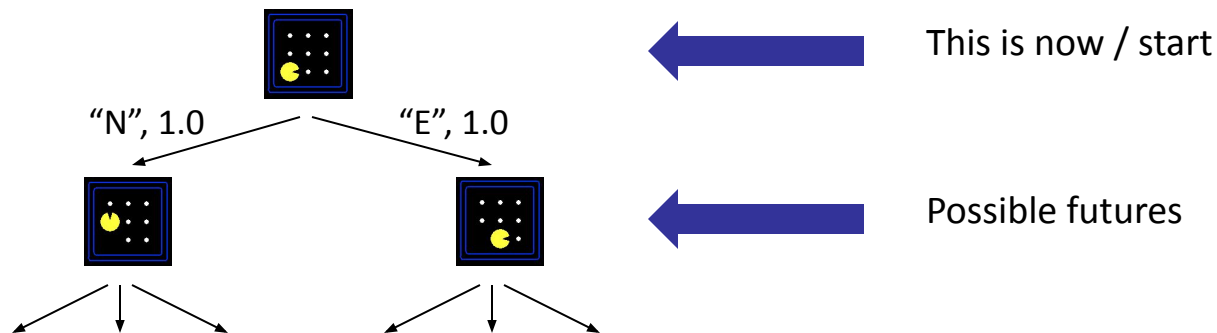
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*Tiny search graph for a tiny search problem*

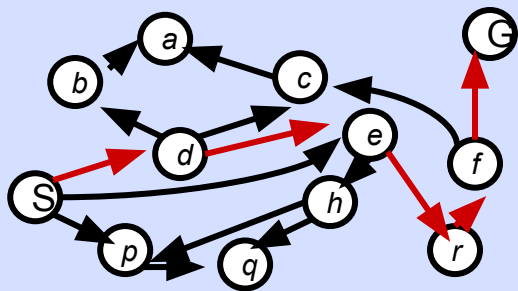
# Search Trees



- A search tree:
  - A “what if” tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree

# State Space Graphs vs. Search Trees

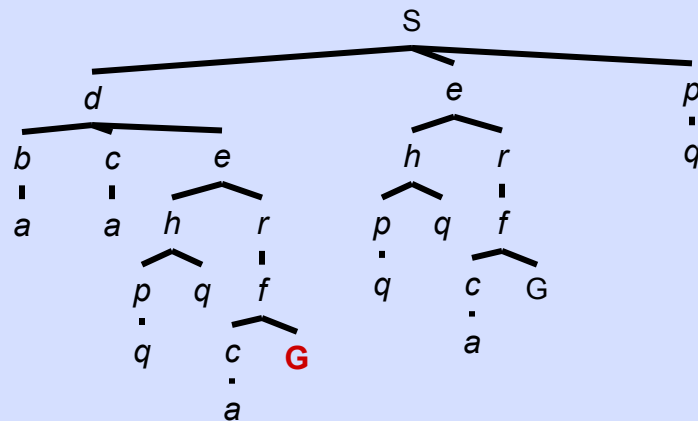
## State Space Graph



*Each NODE in in the search tree is an entire PATH in the state space graph.*

*We construct both on demand – and we construct as little as possible.*

## Search Tree

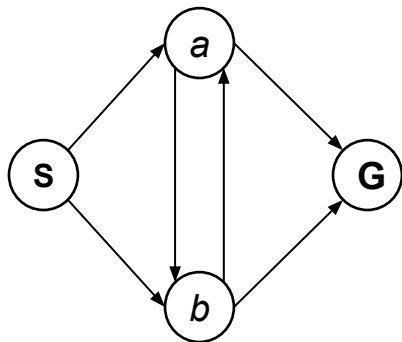




# State Space Graphs vs. Search Trees

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Consider this 4-state graph:



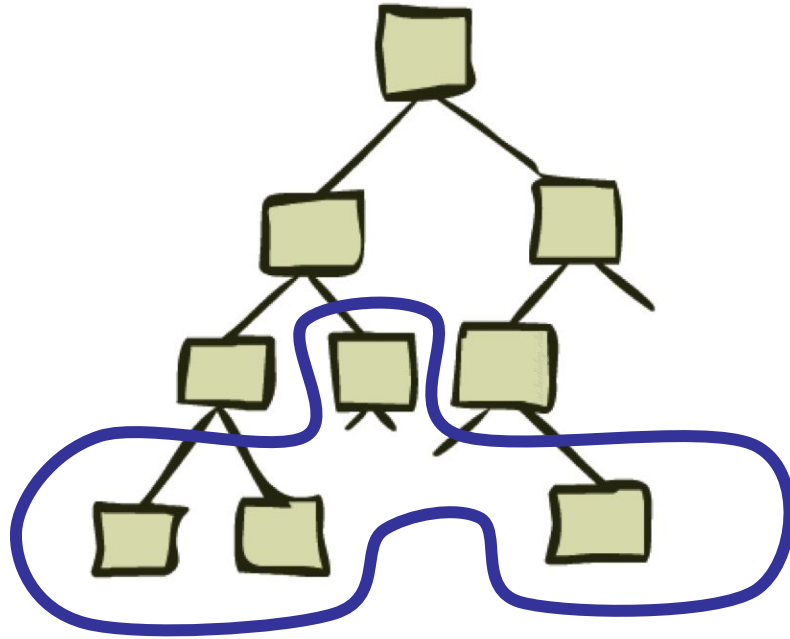
How big is its search tree (from S)?



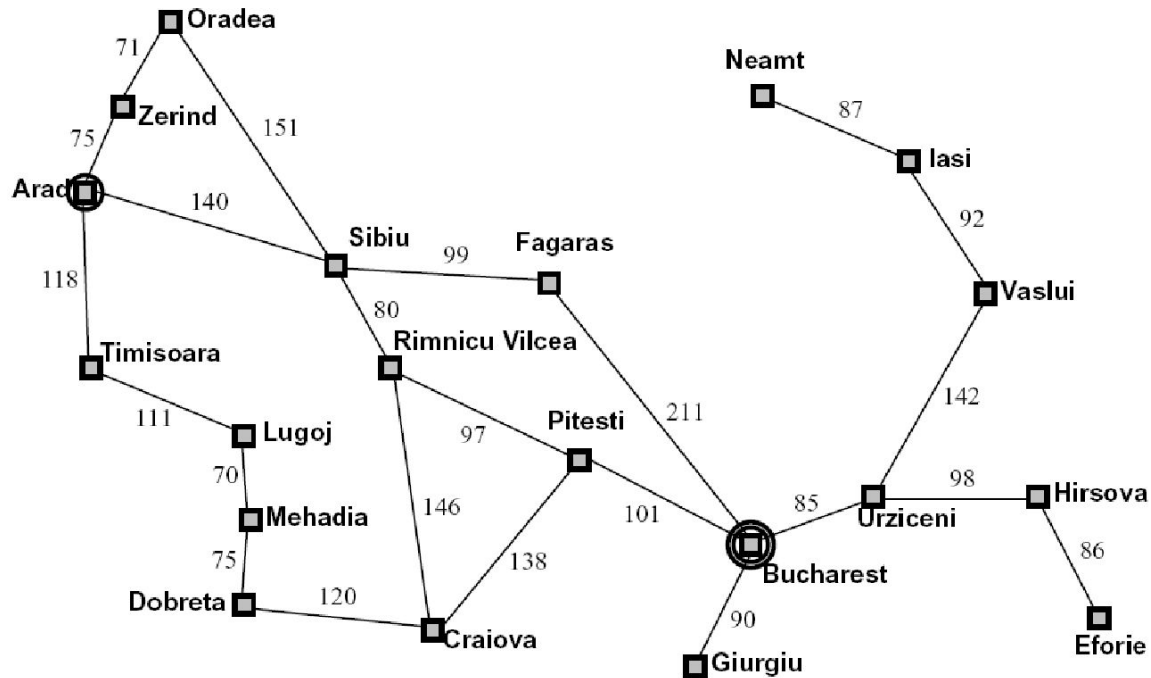
Important: Lots of repeated structure in the search tree!

# Tree Search

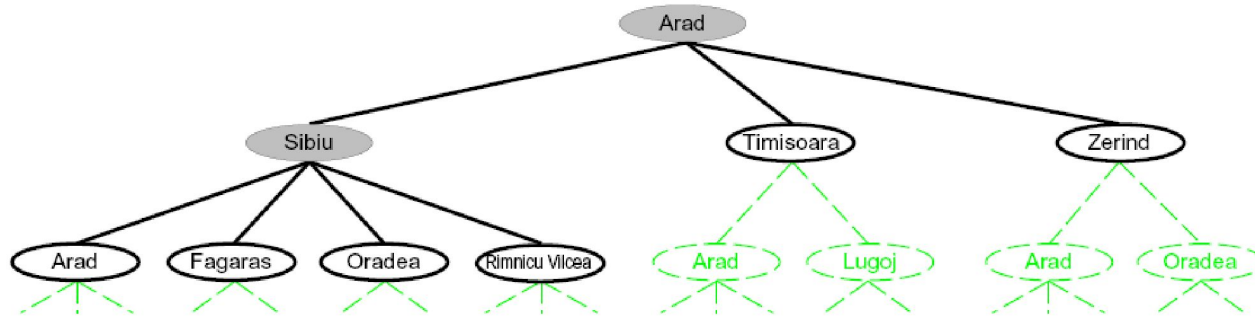
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# Search Example: Romania



# Searching with a Search Tree



## ▪ Search:

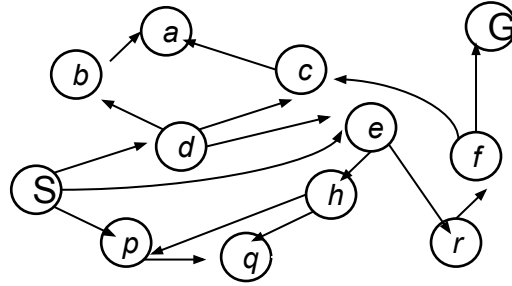
- Expand out potential plans (tree nodes)
- Maintain a **fringe** of partial plans under consideration
- Try to expand as few tree nodes as possible

# General Tree Search

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

# Example: Tree Search



# Depth-First Search

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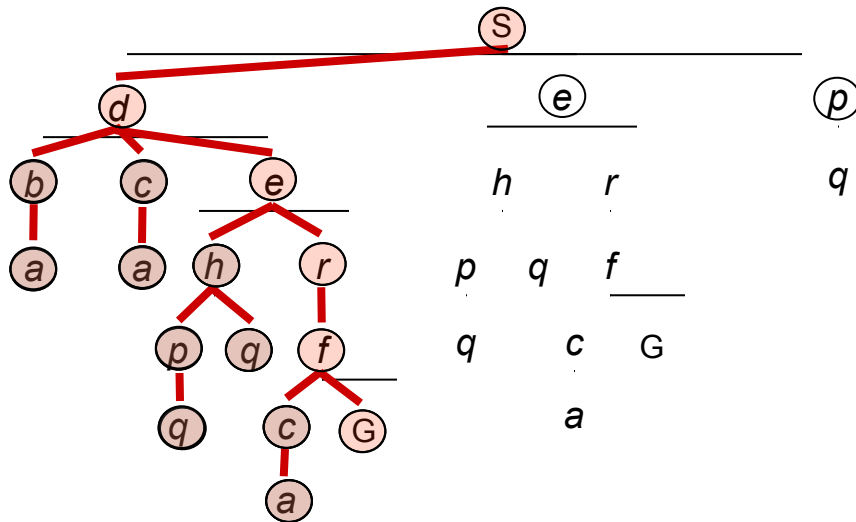
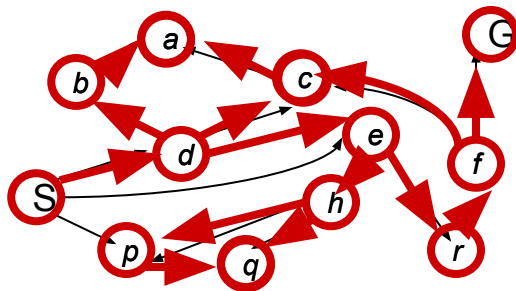


# Depth-First Search

*Strategy: expand a  
deepest node first*

*Implementation:*

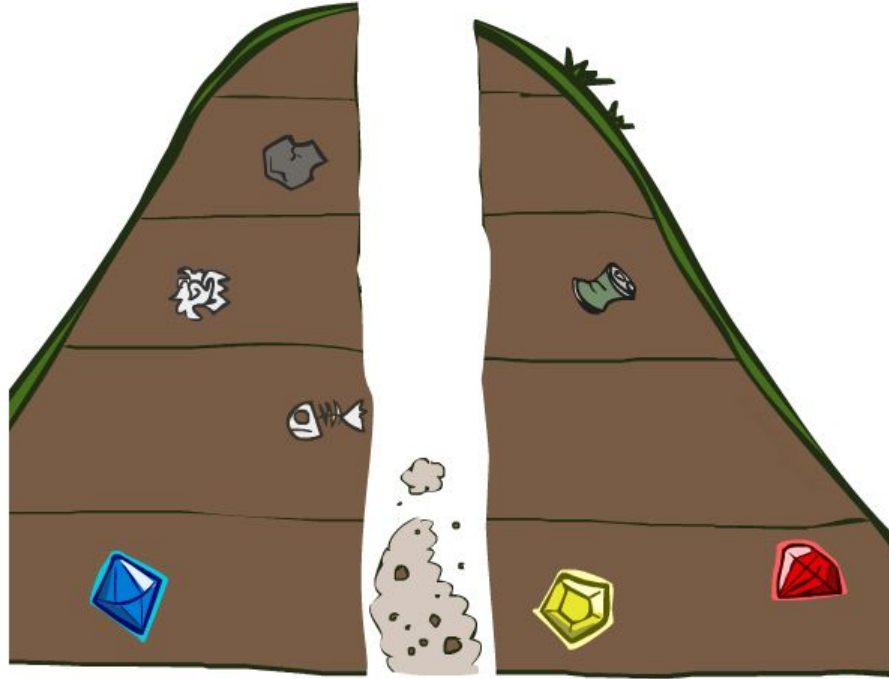
*Fringe is a LIFO stack*





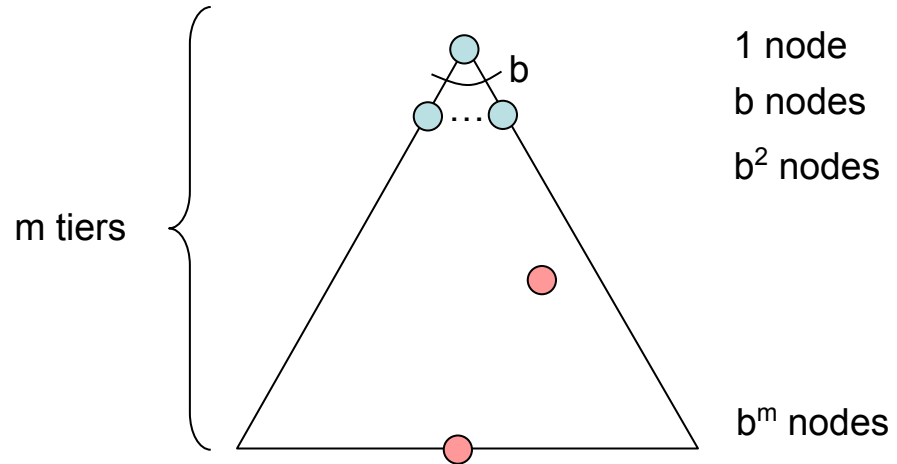
# Search Algorithm Properties

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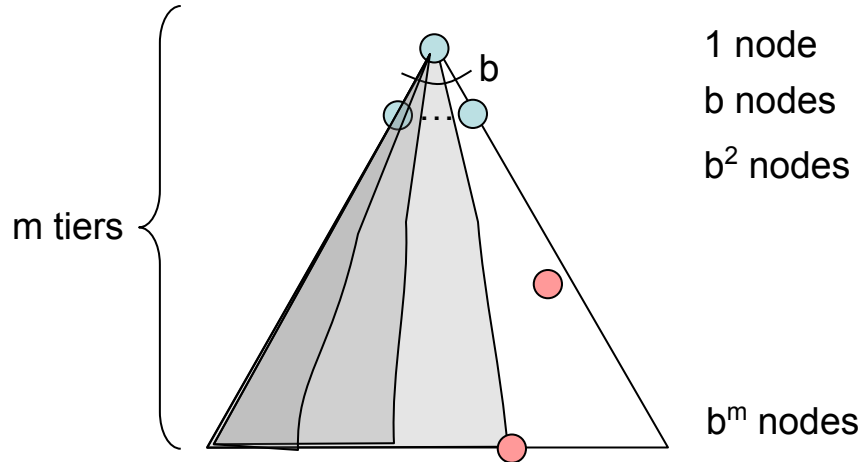
# Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - $b$  is the branching factor
  - $m$  is the maximum depth
  - solutions at various depths
- Number of nodes in entire tree?
  - $1 + b + b^2 + \dots + b^m = O(b^m)$



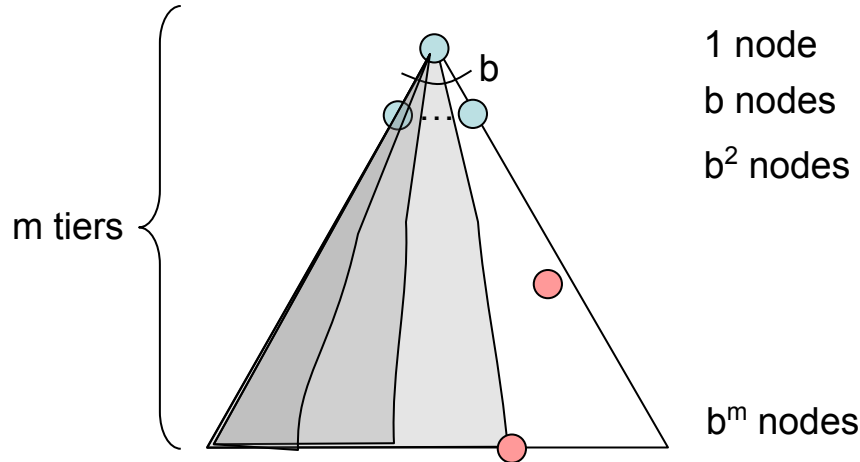
# Depth-First Search (DFS) Properties

- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If  $m$  is finite, takes time  $O(b^m)$
- How much space does the fringe take?
  - Only has siblings on path to root, so  $O(bm)$
- Is it complete?
- Is it optimal?



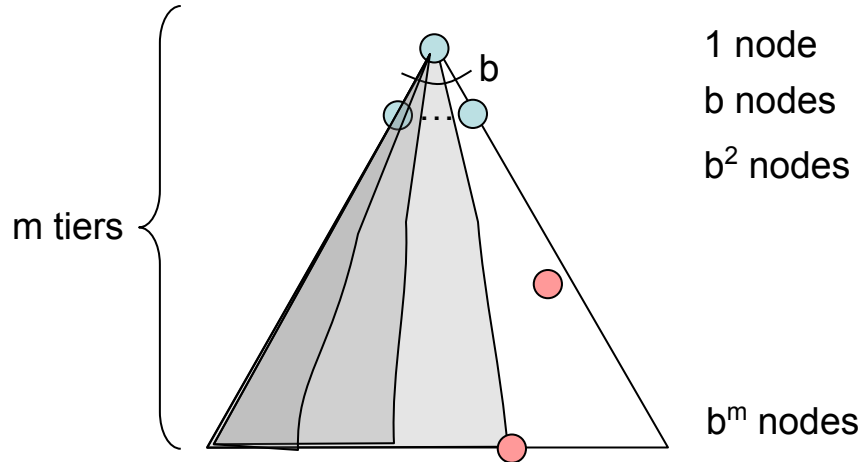
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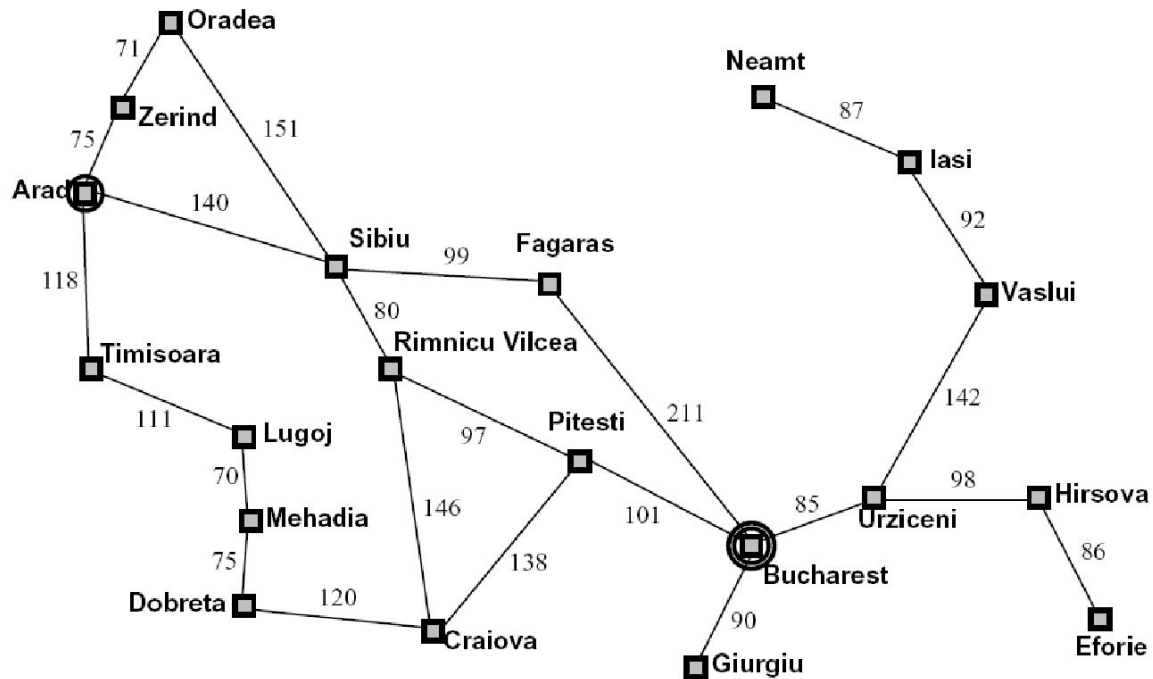


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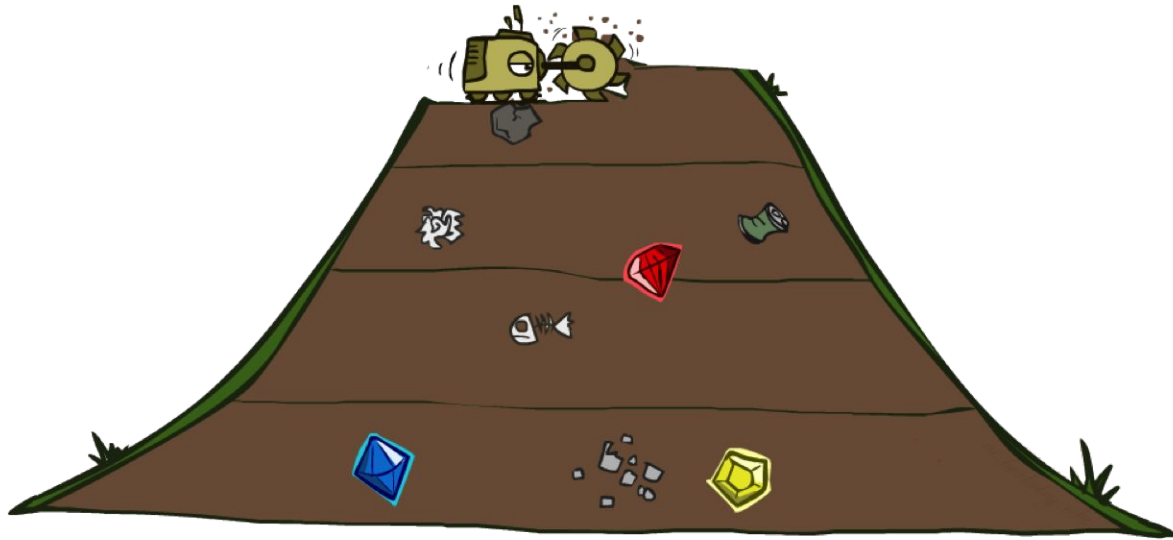


# Search Example: Romania



# Breadth-First Search

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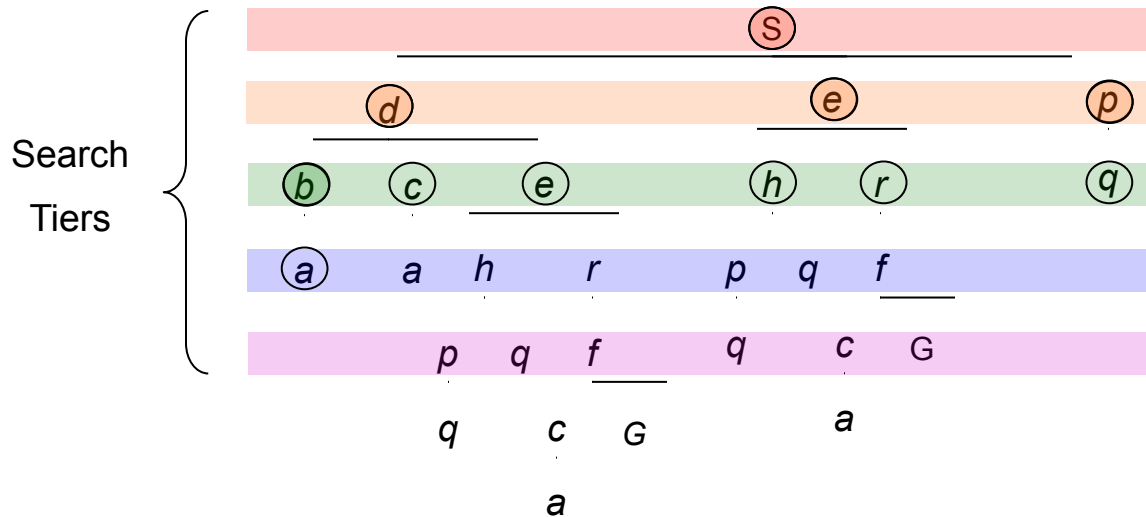
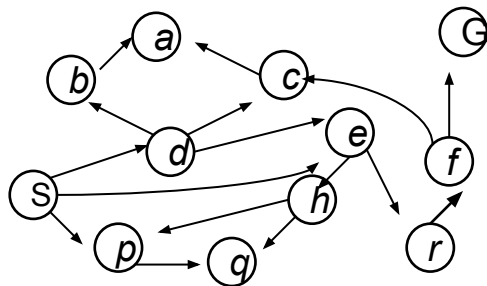


# Breadth-First Search

*Strategy: expand a shallowest node first*

*Implementation:*

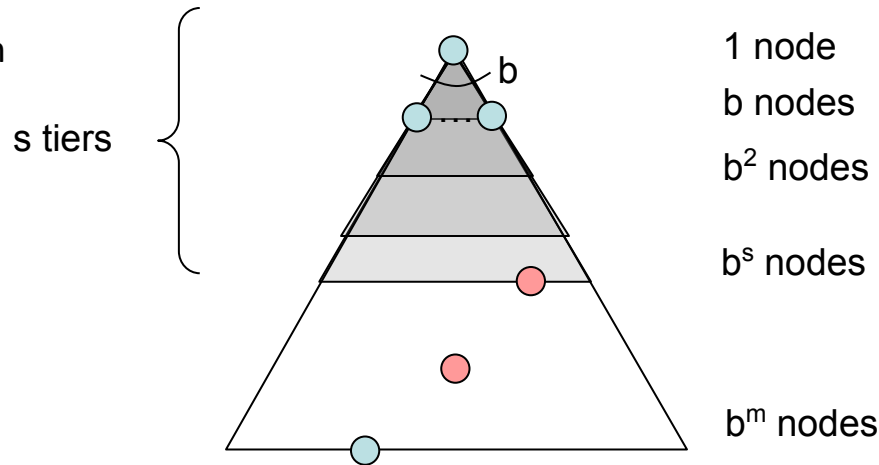
*Fringe is a FIFO queue*





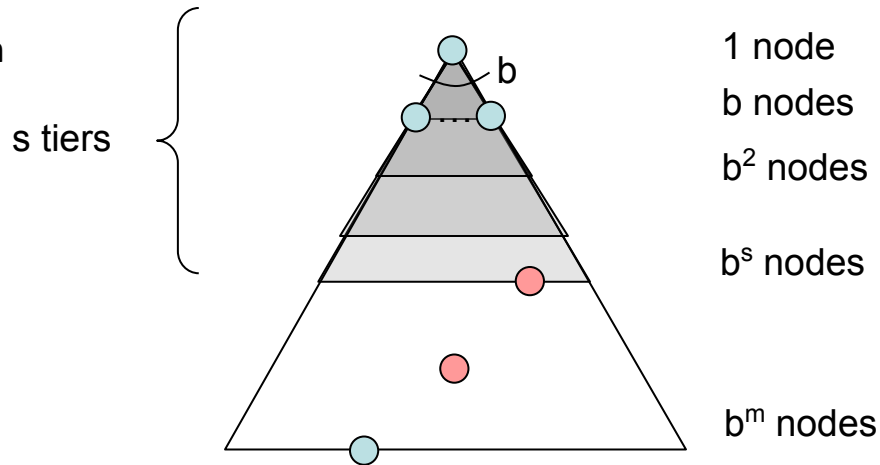
# Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be  $s$
  - Search takes time  $O(b^s)$
- How much space does the fringe take?
  - Has roughly the last tier, so  $O(b^s)$
- Is it complete?
- Is it optimal?



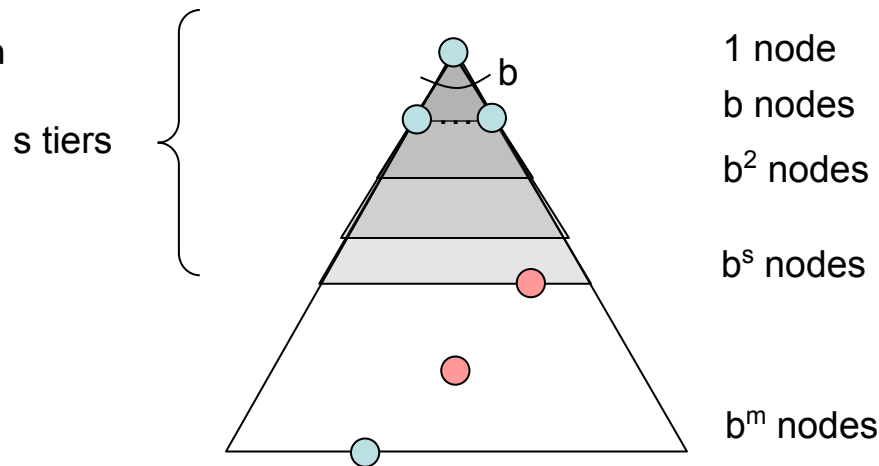
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  - $s$  must be finite if a solution exists, so yes!
- Is it optimal?

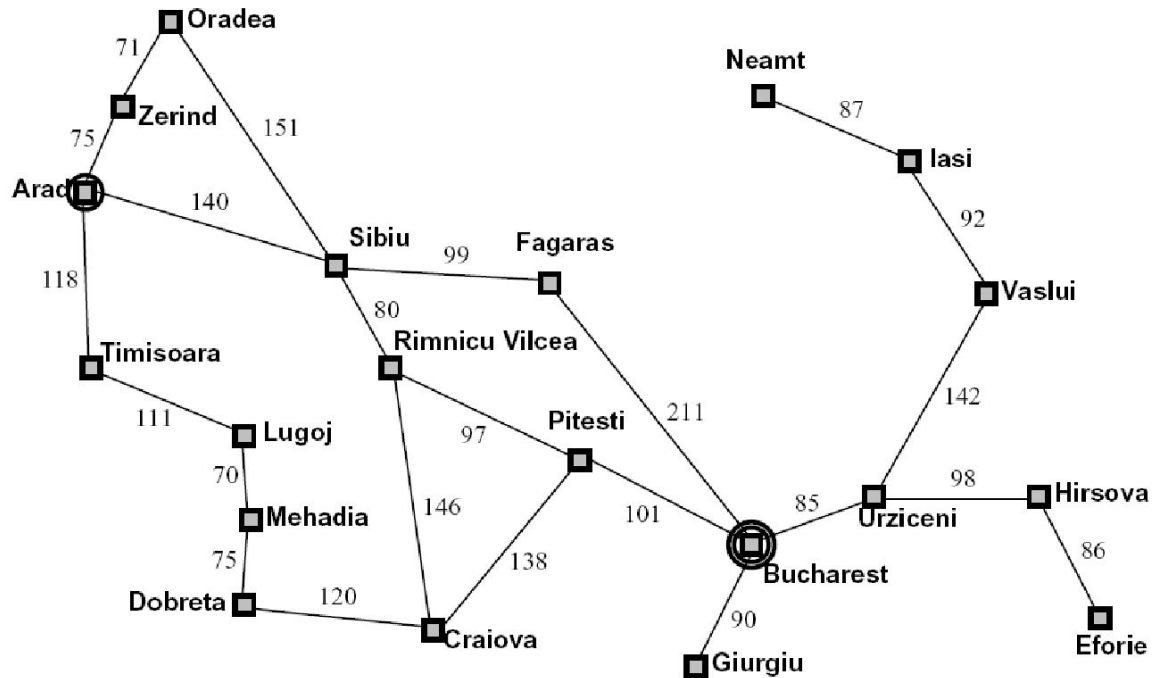


# Breadth-First Search (BFS) Properties

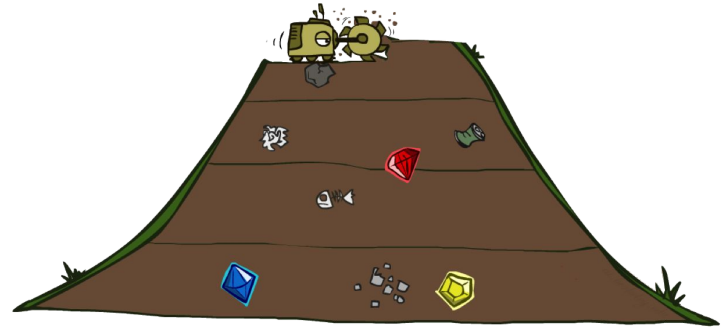
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- Is it complete?
  - $s$  must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



# Search Example: Romania



# DFS vs BFS



# Quiz: DFS vs BFS

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- When will BFS outperform DFS?
- When will DFS outperform BFS?

# Quiz: DFS vs BFS

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- When will BFS outperform DFS?

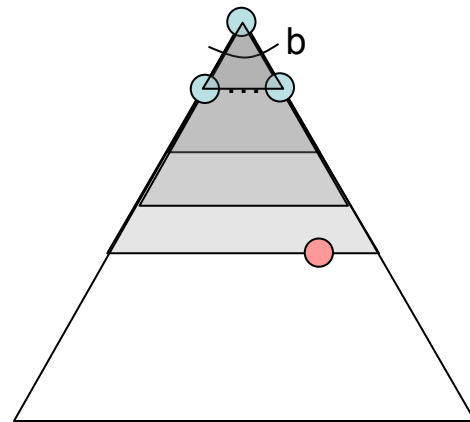
Sparse graph with low branching factors

- When will DFS outperform BFS?

High branching factors

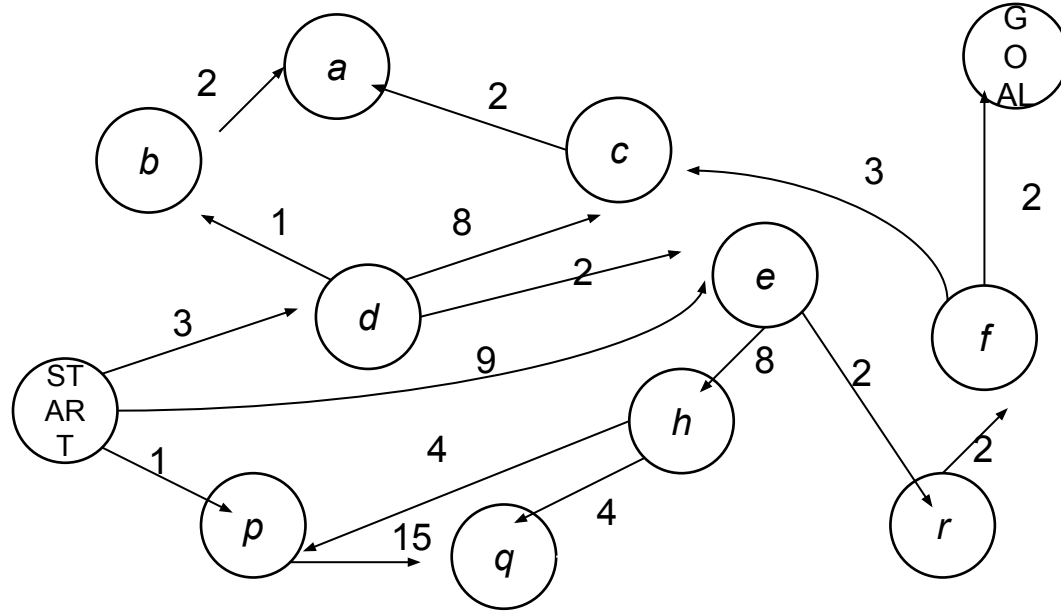
# Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution...
  - Run a DFS with depth limit 2. If no solution...
  - Run a DFS with depth limit 3. ....
- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!





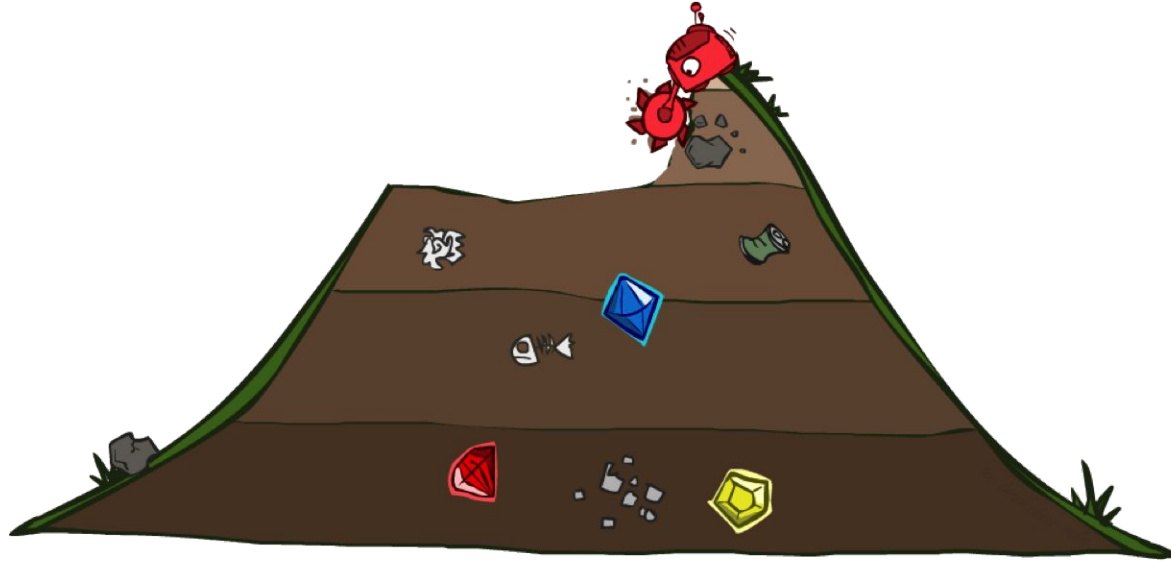
# Cost-Sensitive Search



BFS finds the shortest path in terms of number of actions.  
It does not find the least-cost path. We will now cover  
a similar algorithm which does find the least-cost path.

# Uniform Cost Search

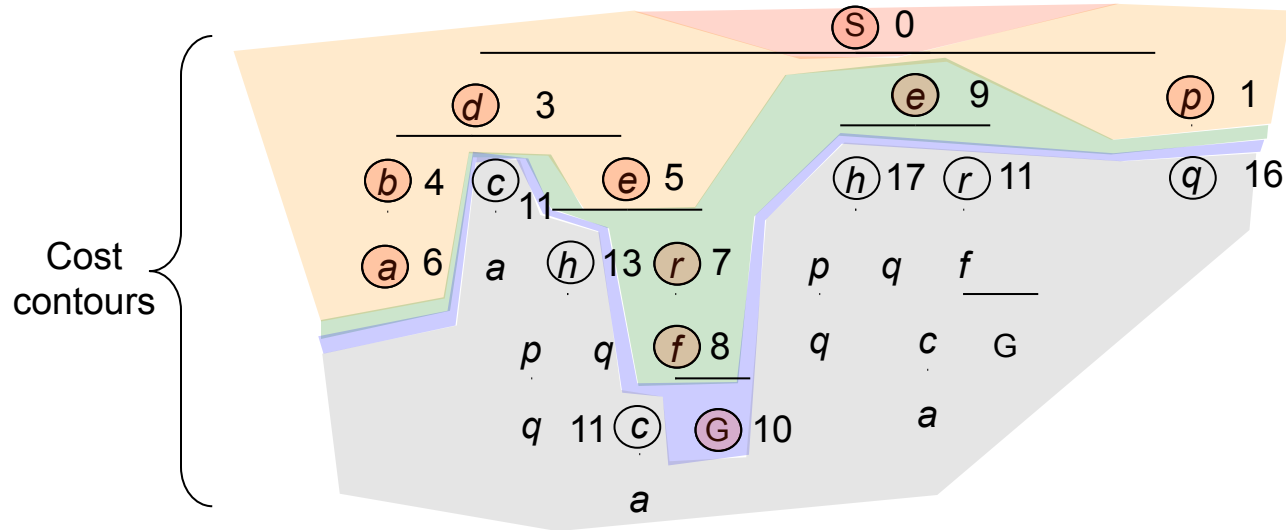
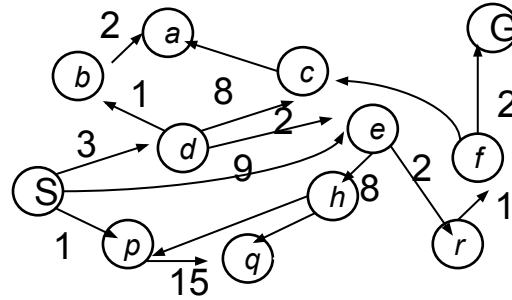
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# Uniform Cost Search

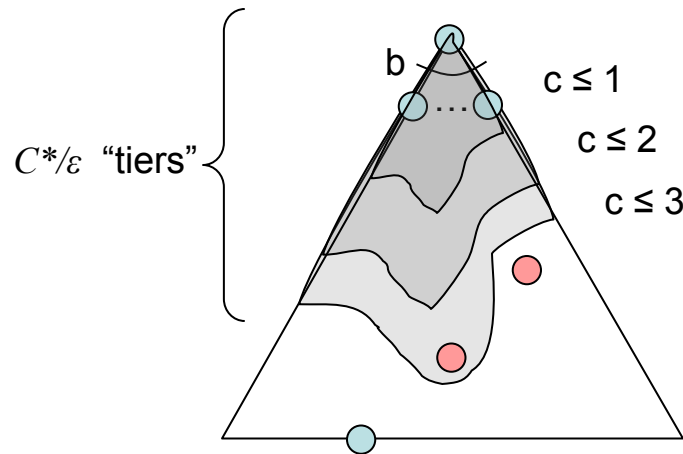
Strategy: expand a  
cheapest node first:

Fringe is a priority queue  
(priority: cumulative cost)



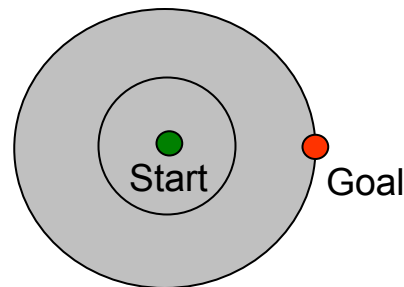
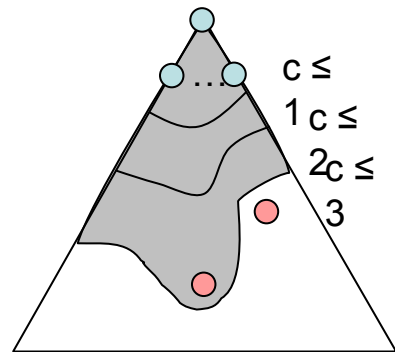
# Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs  $C^*$  and arcs cost at least  $\varepsilon$ , then the “effective depth” is roughly  $C^*/\varepsilon$
  - Takes time  $O(b^{C^*/\varepsilon})$  (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so  $O(b^{C^*/\varepsilon})$
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes! (Proof next lecture via A\*)



# Uniform Cost Issues

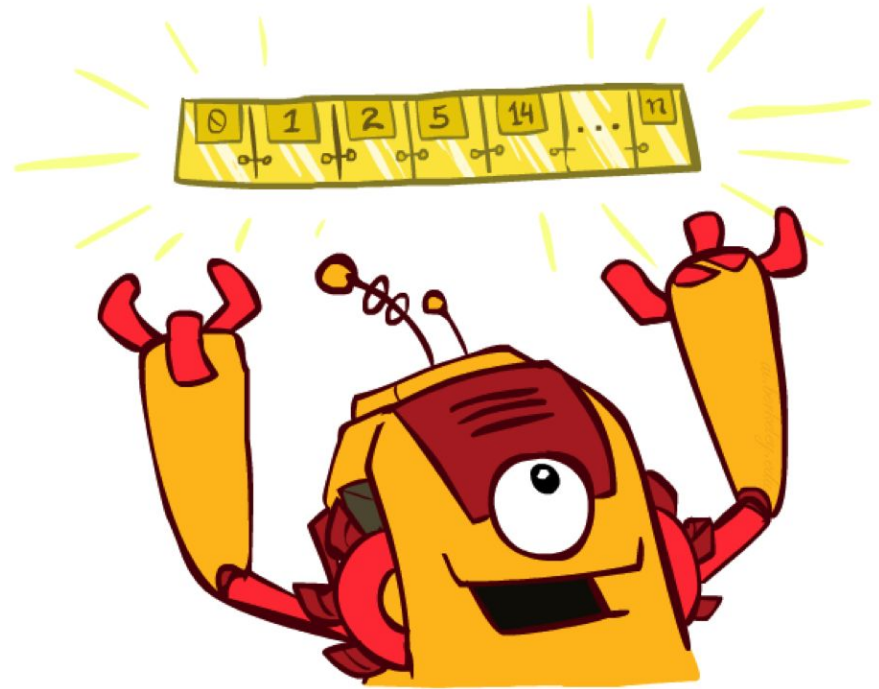
- Remember: UCS explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every “direction”
  - No information about goal location
- We'll fix that soon!



[Demo: empty grid UCS (L2D5)]  
[Demo: maze with deep/shallow  
water DFS/BFS/UCS (L2D7)]

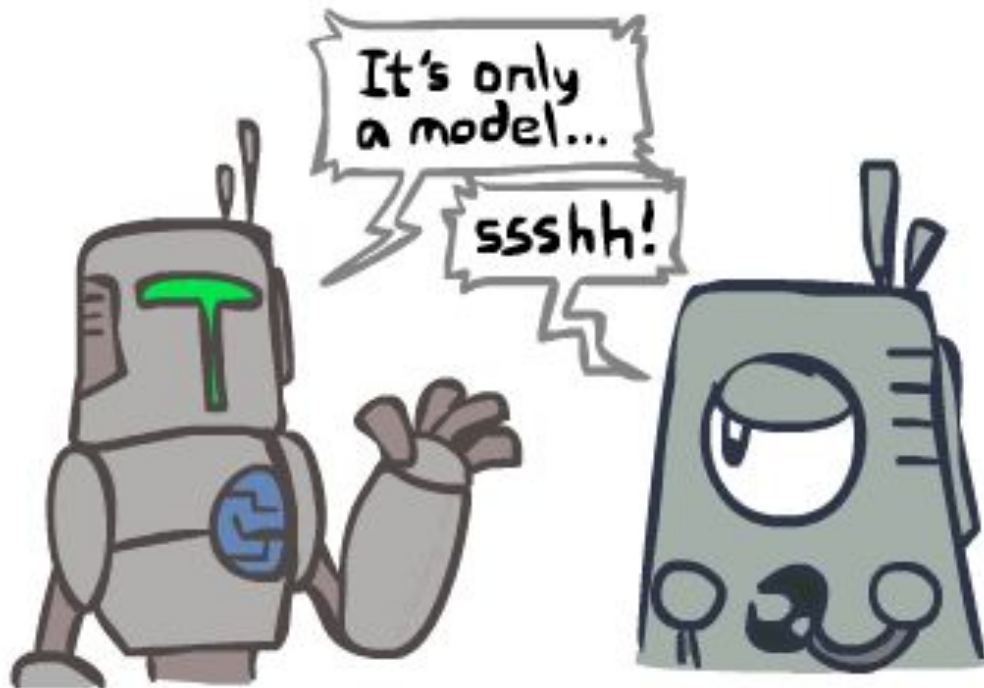
# The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the  $\log(n)$  overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object



# Search and Models

- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all “in simulation”
  - Your search is only as good as your models...



# Search Gone Wrong?

