EN3160 Fundamentals of Image Processing and Machine Vision: Spatial Filtering

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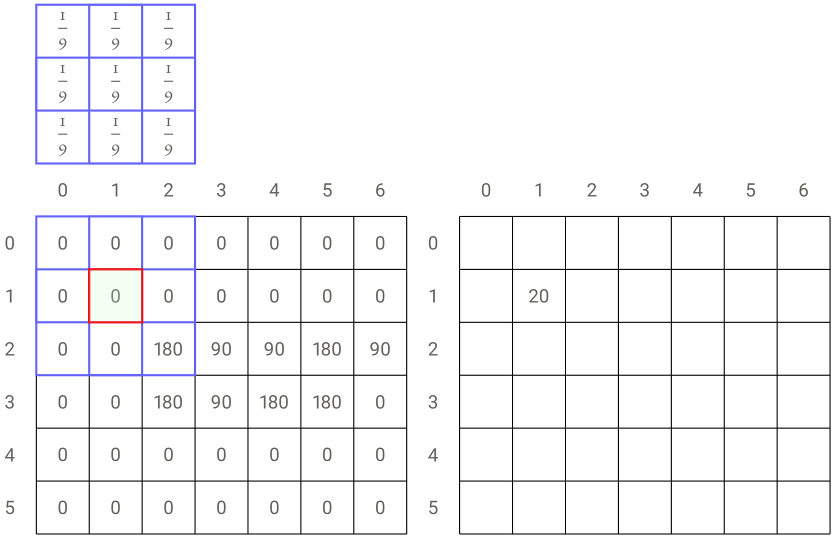
Contents

Introduction

- We use spatial filtering to enhance images, e.g., noise filtering. We can use the same technique to locate objects in images, called template matching. After completing this lesson, we would be able to process images using filters, as done in popular image processing software such as Photoshop.
- Intensity transformations affect the brightness (intensity level)of an image. The resulting value of a pixel after an intensity operation is just dependent on the original value of the same pixel.
- In contrast, the resulting value of a pixel after a spatial filtering operation depends on the neighborhood of the pixel in question. So the space around the pixel in question matters, hence, the name spatial filtering.
- In linear spatial filtering we use the 2-D convolution operation.



$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

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0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20					
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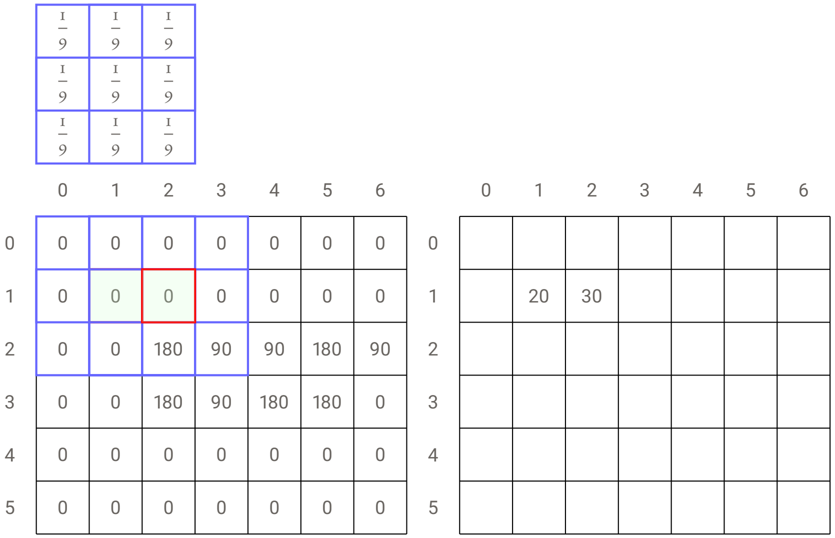
Figure: Convolution



$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
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1		20	30				
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Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40			
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Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
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1		20	30	40	40		
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Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
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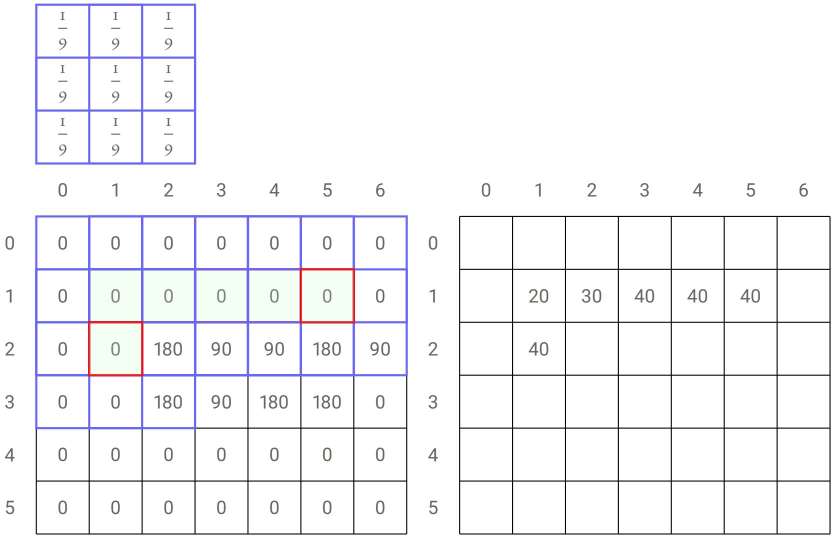
Figure: Convolution



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	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40					
3							
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5							

Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60				
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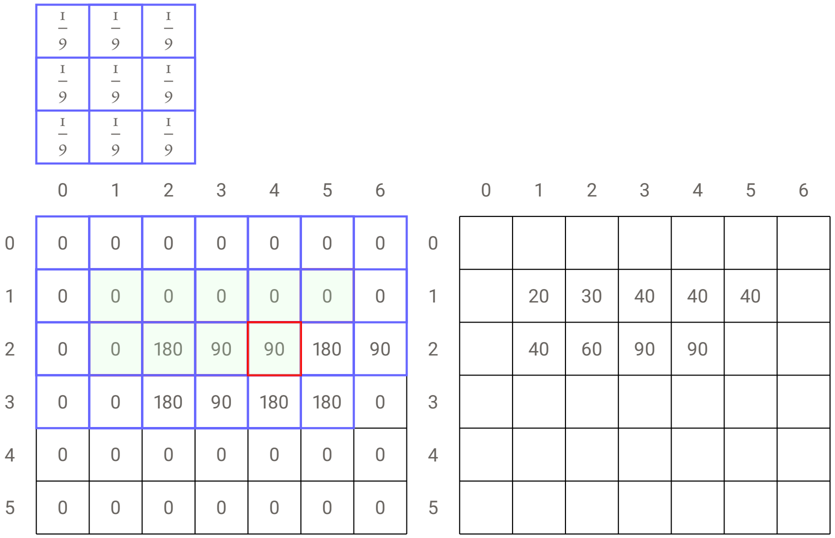
Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90			
3							
4							
5							

Figure: Convolution



$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
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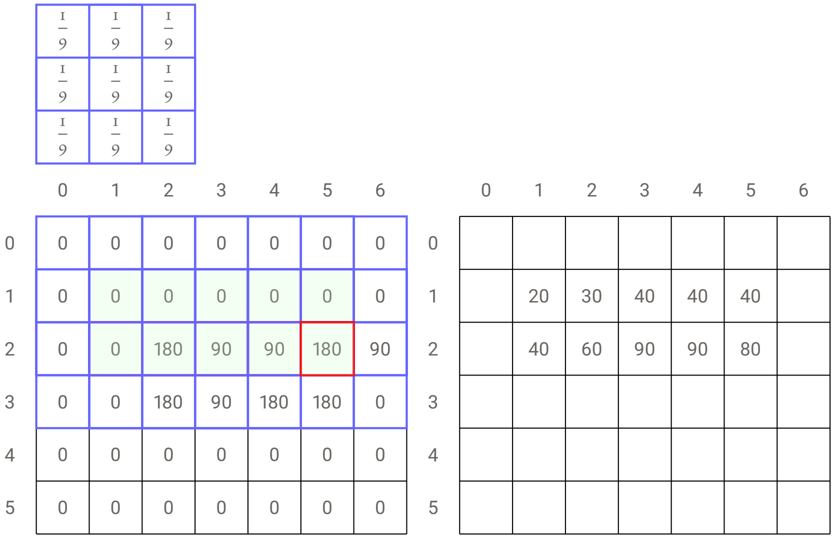
1		20	30	40	40	40	
2		40	60	90	90		
3							
4							
5							

Figure: Convolution



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	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3							
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5							

Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40					
4							
5							

Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60				
4							
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Figure: Convolution

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
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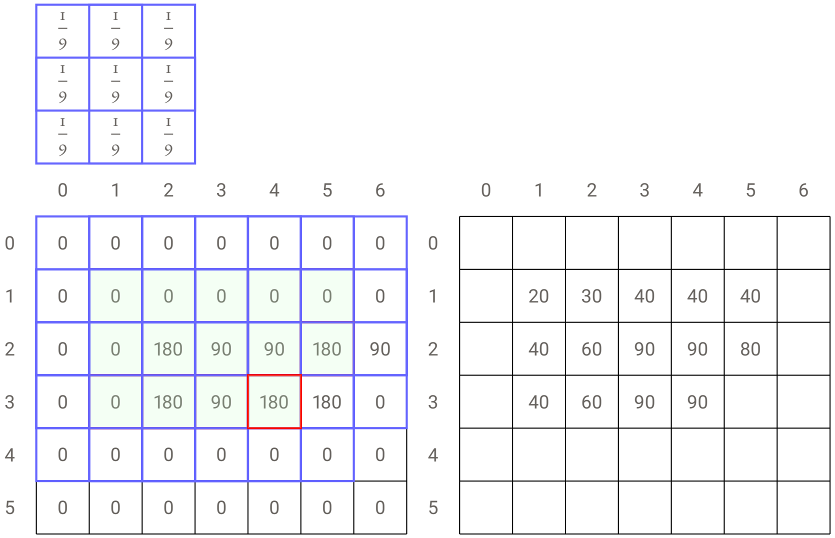
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90			
4							
5							

Figure: Convolution



$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90	90		
4							
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Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90	90	80	
4							
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Figure: Convolution

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90	90	80	
4		20					
5							

Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90	90	80	
4		20	30				
5							

Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90	90	80	
4		20	30	50			
5							

Figure: Convolution

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
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4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90	90	80	
4		20	30	50	50		
5							

Figure: Convolution

$$\begin{array}{|c|c|c|}\hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \hline \end{array}$$

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90	90	80	
4		20	30	50	50	40	
5							

Figure: Convolution

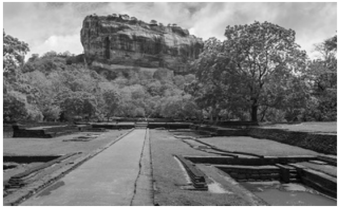
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

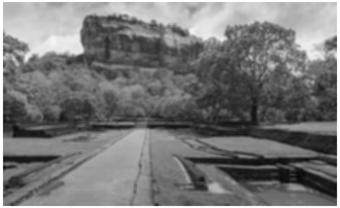
	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	180	90	90	180	90
3	0	0	180	90	180	180	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0

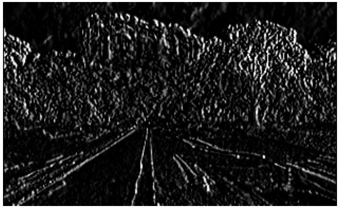
	0	1	2	3	4	5	6
0							
1		20	30	40	40	40	
2		40	60	90	90	80	
3		40	60	90	90	80	
4		20	30	50	50	40	
5							

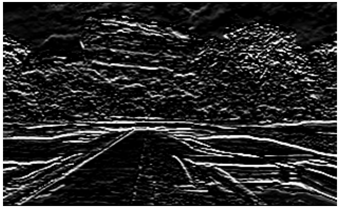
Figure: Convolution

Examples of Effect of Kernel Choices


(a) Original


(b) Avegaging


(c) Sobel horizontal


(d) Sobel vertical

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Averaging

-1	-2	-1
0	0	0
1	2	1

Sobel vertical

-1	0	1
-2	0	2
-1	0	1

Sobel horizontal

Figure: 3×3 kernels

Spatial Filtering (Averaging) Using filter2D

Listing: Spatial Filtering (Averaging)

```
%matplotlib inline
import cv2 as cv
import numpy as np
from matplotlib import pyplot as plt

img = cv.imread('../images/sigiriya.jpg', cv.IMREAD_REDUCED_GRAYSCALE_2)

kernel = np.ones((3,3),np.float32)/9
imgc = cv.filter2D(img,-1,kernel)

fig, axes  = plt.subplots(1,2, sharex='all', sharey='all', figsize=(18,18))
axes[0].imshow(img, cmap='gray')
axes[0].set_title('Original')
axes[0].set_xticks([]), axes[0].set_yticks([])
axes[1].imshow(imgc, cmap='gray')
axes[1].set_title('Averaging')
axes[1].set_xticks([]), axes[1].set_yticks([])
plt.show()
```

Spatial Filtering (Sobel Vertical) Using filter2D

Listing: Spatial Filtering (Sobel Vertical)

```
%matplotlib inline
import cv2 as cv
import numpy as np
from matplotlib import pyplot as plt

img = cv.imread('../images/sigiriya.jpg', cv.IMREAD_REDUCED_GRAYSCALE_2)

kernel = np.array([(-1, -2, -1), (0, 0, 0), (1, 2, 1)], dtype='float')
imgc = cv.filter2D(img,-1,kernel)

fig, axes  = plt.subplots(1,2, sharex='all', sharey='all', figsize=(18,18))
axes[0].imshow(img, cmap='gray')
axes[0].set_title('Original')
axes[0].set_xticks([]), axes[0].set_yticks([])
axes[1].imshow(imgc, cmap='gray')
axes[1].set_title('Averaging')
axes[1].set_xticks([]), axes[1].set_yticks([])
plt.show()
```

Convolution and Correlation

1. Spatial filtering is, in fact, convolution.
2. As the filters are typically symmetric—i.e., a 180° rotation results in the same kernel—correlation is equivalent to convolution.
3. Correlation is also the scalar product between the kernel and the underlying image patch. Therefore, it is a measure of similarity between the kernel and the underlying image patch.
4. As a result, when the kernel and the patch are “similar”, the output is high. In view of this, spatial filtering seeks for patches in the image that are similar to the kernel.
5. Implementing filtering using loops (four nested for loops) in a non-C fashion is inefficient. Instead, use `filter2D`.

Convolution and Correlation

The convolution sum expression that we leaned in signal and systems is

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

In 2-D, as applicable in image processing, collation sum with a kernel $w[m,n]$ with non-zero values in $(m,n) \in ([-a,a],[-b,b])$ is

$$(w * f)[m,n] = w[m,n] * f[m,n] = \sum_{s=-a}^{a} \sum_{k=-b}^{b} w[s,t]f[m-s,n-t].$$

Correlation:

$$(w \circledast f)[m,n] = w[m,n] \circledast f[m,n] = \sum_{s=-a}^{a} \sum_{k=-b}^{b} w[s,t]f[m+s,n+t].$$

Listing: Filtering Using Loops

```python
%matplotlib inline
import cv2 as cv
import matplotlib.pyplot as plt
import numpy as np
import math

def filter(image, kernel):
    assert kernel.shape[0]%2 == 1 and kernel.shape[1]%2 == 1
    k_hh, k_hw = math.floor(kernel.shape[0]/2), math.floor(kernel.shape[1]/2)
    h, w = image.shape
    image_float = cv.normalize(image.astype('float'), None, 0.0, 1.0, cv.
        NORM_MINMAX)
    result = np.zeros(image.shape, 'float')

    for m in range(k_hh, h - k_hh):
        for n in range(k_hw, w - k_hw):
            result[i,j] = np.dot(image_float[m-k_hh:m + k_hh + 1, n - k_hw : n
                 + k_hw + 1].flatten(), kernel.flatten())
    return result

img = cv.imread('../images/keira.jpg', cv.IMREAD_REDUCED_GRAYSCALE_8)
```

```
f, axarr = plt.subplots(1,2)
axarr[0].imshow(img, cmap="gray")
axarr[0].set_title('Original')
kernel = np.array([(1/9, 1/9, 1/9), (1/9, 1/9, 1/9), (1/9, 1/9, 1/9)], dtype='
    float')
imgb = filter(img, kernel)
imgb = imgb*255.0
imgb = imgb.astype(np.uint8)

axarr[1].imshow(imgb, cmap="gray")
axarr[1].set_title('Filtered')
```

Example

Consider the image

$$f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and the filtering kernel

$$w = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

. Appropriately pad the image. Carry out a. correlation b. convolution.

Solution

Consider the image

$$f_{\text{padded}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad w = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Correlation result and convolution result, respectively

$$(w \circledast f) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 8 & 7 & 0 \\ 0 & 6 & 5 & 4 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (w * f) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 4 & 5 & 6 & 0 \\ 0 & 7 & 8 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

.

Convolution: Key Properties

1. Linearity: $\text{filter}(f_1 + f_2) = \text{filter}(f_1) + \text{filter}(f_2)$
2. Shift invariance: same behavior regardless of pixel location: $\text{filter}(\text{shift}(f)) = \text{shift}(\text{filter}(f))$
3. Theoretical result: any linear shift-invariant operator can be represented as a convolution.

Other Properties

1. Commutative: $a * b = b * a$
2. Conceptually no difference between filter and signal
3. Associative: $a * (b * c) = (a * b) * c$
4. Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
5. This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
6. Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
7. Scalars factor out: $ka * b = a * kb = k(a * b)$
8. Identity: unit impulse $e = [\ldots, 0, 0, 1, 0, 0, \ldots]$, $a * e = a$

Source: *Svetlana Lazebnik*

At the Edge

for 3*3 kernel we have to remove all edge pixels

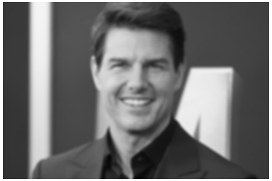
1. The filter window falls off the edge of the image.
2. Need to extrapolate the image.
3. Methods: various border types, image boundaries are denoted with '|'

BORDER_REPLICATE:	aaaaaa|abcdefgh|hhhhhhh	
BORDER_REFLECT:	fedcba|abcdefgh|hgfedcb	
BORDER_REFLECT_101:	gfedcb|abcdefgh|gfedcba	
BORDER_WRAP:	cdefgh|abcdefgh|abcdefg	
BORDER_CONSTANT:	iiiiii|abcdefgh|iiiiiii	with some specified 'i'

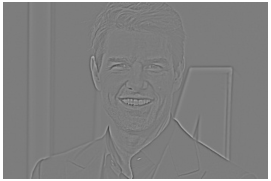
Sharpening

(a) Original

–

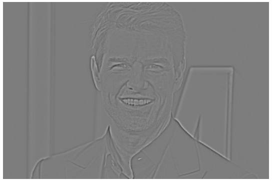

(b) Smoothed

=


(c) Original – Smoothed

(d) Original

–


(e) Original – Smoothed

=

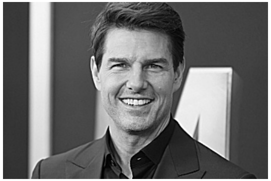

(f) Sharpened

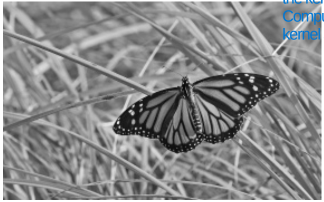
Figure: Sharpening (125 added to Original – Smoothed to display)

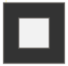
Source: *Svetlana Lazebnik*

Box Filter vs. Gaussian Filter

Box Filter (or Mean Filter):
Kernel Shape: The box filter uses a simple square or rectangular kernel. The kernel is usually of a fixed size and contains equal weights (usually 1) in all its elements.
Smoothing Effect: The box filter performs a basic averaging operation. It calculates the average pixel value within the kernel's neighborhood and assigns this average value to the central pixel. This operation effectively blurs the image by replacing each pixel's value with the average value of its neighboring pixels.
Effect on Image: The box filter results in uniform blurring across the image, which can be effective for reducing noise and making image features less pronounced. However, it doesn't preserve fine details well and can create a blocky or pixelated appearance if the kernel size is large.
Computation: Box filtering is computationally efficient because it involves simple averaging operations and is less sensitive to kernel size compared to the Gaussian filter.

- What's wrong with this filtering operation?
- What's the solution?


(a) Original


(b) Kernel (much zoomed)

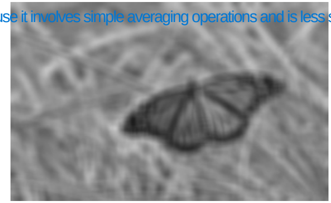
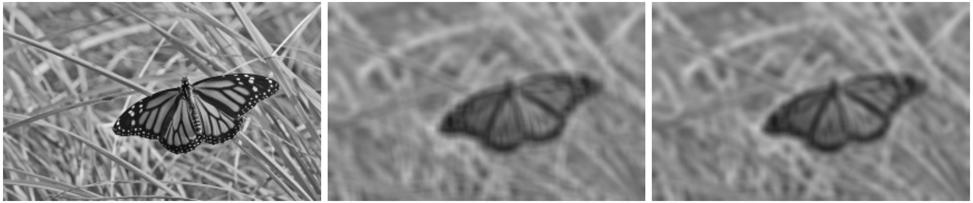

(c) Box Filtered

Figure: Smoothing with Box Filter

Source: *D. Forsyth*

Box Filter vs. Gaussian Filter



(a) Original (b) Box Filtered (c) Gaussian Filtered

Figure: Smoothing with Box and Gaussian Filter

Source: *D. Forsyth*

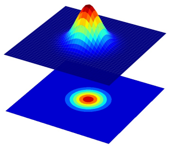
Kernel Shape: The Gaussian filter uses a Gaussian-shaped kernel. The kernel is defined by a bell-shaped curve with a central peak and tails that extend to infinity. The values in the kernel are weighted based on their distance from the central pixel.

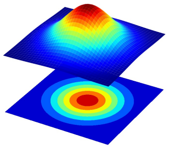
Smoothing Effect: The Gaussian filter performs a weighted averaging operation. It assigns higher weights to pixels closer to the center of the kernel and lower weights to pixels farther away. This weighting creates a smoothing effect that is stronger at the center of the kernel and gradually decreases towards the edges.

Effect on Image: The Gaussian filter provides a more natural and aesthetically pleasing smoothing effect compared to the box filter. It preserves edges and fine details better while reducing noise. The degree of smoothing can be controlled by adjusting the standard deviation of the Gaussian kernel.

Gaussian Kernel

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}.$$


(a) $\sigma = 1$


(b) $\sigma = 2$

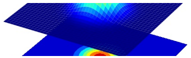

(c) $\sigma = 5$

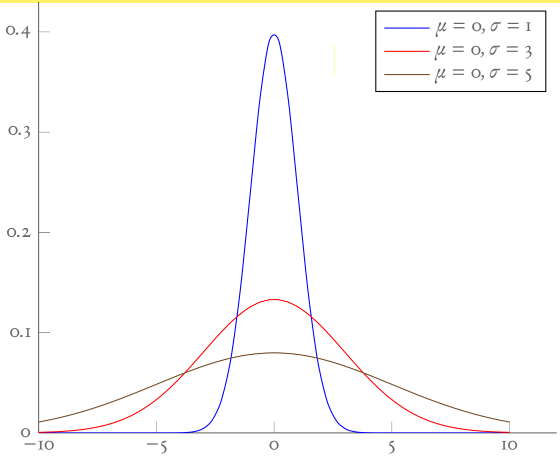
Figure: 2-D Gaussians

we should normalized in order to make sure there is no any intensity changings

- Constant factor at front makes volume sum to 1 (can be ignored when computing the filter values, as we should renormalize weights to sum to 1 in any case).
- The Gaussian function has infinite support, but discrete filters use finite kernels.

Choosing Gaussian Kernel Width

Rule of thumb: set the filter half-width to about 3σ.



half width referring to the distance from the center of the filter to the point where the filter response has dropped to half of its maximum value. This distance is not a fixed value but depends on the specific sigma value chosen for the Gaussian filter.

Separability of the Gaussian Filter

$$
\begin{aligned}
G_\sigma(x,y) &= \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}, \\
&= \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}\right] \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}}\right].
\end{aligned}
$$

- The 2-D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y.
- In this case, the two Gaussians are the (identical) 1-D Gaussians.

Separability of the Gaussian Filter

$$\begin{aligned} G_\sigma(x,y) &= \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}, \\ &= \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}\right]\left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}}\right]. \end{aligned}$$

- The 2-D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y.
- In this case, the two Gaussians are the (identical) 1-D Gaussians.
- Separability means that a 2-D convolution can be reduced to two 1-D convolutions (one among rows and one among columns).
- What is the complexity of filtering an $n \times n$ image with an $m \times m$ kernel?

Separability of the Gaussian Filter

Non-Separable Gaussian Filter:
For a non-separable Gaussian filter, you apply the full 2D Gaussian kernel directly to the image. The computational complexity of this operation is O(n^2 * m^2), where n is the size of the image, and m is the size of the kernel. This is because for each pixel in the n × n image, you perform a convolution with the entire m × m kernel, resulting in a nested loop structure with n^2 iterations, each involving m^2 multiplications and additions.

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}},$$
$$= \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}\right]\left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}}\right].$$

Separable Gaussian Filter:
A separable Gaussian filter can be applied more efficiently by first convolving the image with a 1D Gaussian kernel in the horizontal direction and then convolving the result with the same 1D Gaussian kernel in the vertical direction. The computational complexity of this operation is O(n^2 * m) since you perform two 1D convolutions separately. The complexity is significantly reduced compared to the non-separable filter because you are applying a 1D kernel twice, which requires fewer multiplications and additions compared to the 2D convolution.

- The 2-D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y.
- In this case, the two Gaussians are the (identical) 1-D Gaussians.
- Separability means that a 2-D convolution can be reduced to two 1-D convolutions (one among rows and one among columns).
- What is the complexity of filtering an $n \times n$ image with an $m \times m$ kernel? $O(n^2 m^2)$.
- What is the complexity if the kernel is separable?

$O(n^2 m)$

Separability of the Gaussian Filter

$$
\begin{aligned}
G_\sigma(x, y) &= \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}, \\
&= \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \right] \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} \right].
\end{aligned}
$$

- The 2-D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y.
- In this case, the two Gaussians are the (identical) 1-D Gaussians.
- Separability means that a 2-D convolution can be reduced to two 1-D convolutions (one among rows and one among columns).
- What is the complexity of filtering an $n \times n$ image with an $m \times m$ kernel? $O(n^2 m^2)$.
- What is the complexity if the kernel is separable? $O(n^2 m)$.

Source: *D. Forsyth*

Noise

- Salt and pepper noise: contains random occurrences of black and white pixels
- Impulse noise: contains random occurrences of white pixels
- Gaussian noise: variations in intensity drawn from a Gaussian normal distribution

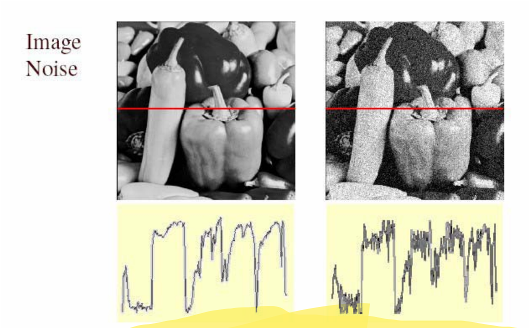
Figure

Source: *S. Sietz*

Gaussian Noise

- Mathematical model: sum of many independent factors
- Good for small standard deviations
- Assumption: independent, zero-mean noise

can seperate



$$f(x,y) = \overbrace{\hat{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise: $\eta(x,y) \sim \mathcal{N}(\mu, \sigma)$

Figure

Source: *M. Hebert*

Reducing Gaussian Noise

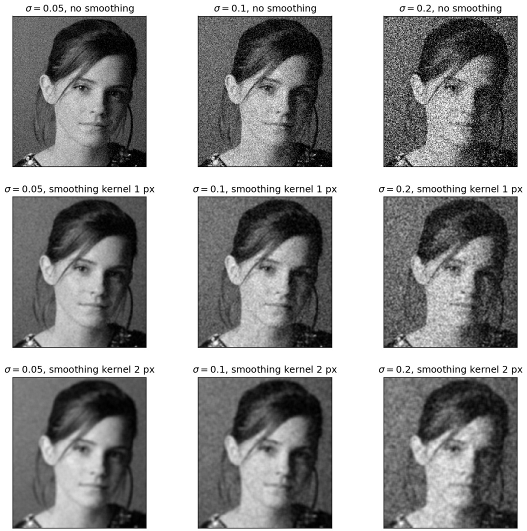

$\sigma = 0.05$, no smoothing
$\sigma = 0.1$, no smoothing
$\sigma = 0.2$, no smoothing
$\sigma = 0.05$, smoothing kernel 1 px
$\sigma = 0.1$, smoothing kernel 1 px
$\sigma = 0.2$, smoothing kernel 1 px
$\sigma = 0.05$, smoothing kernel 2 px
$\sigma = 0.1$, smoothing kernel 2 px
$\sigma = 0.2$, smoothing kernel 2 px

Figure: Reducing Gaussian noise: noise σ and smoothing kernel size

Smoothing with larger standard deviations suppresses noise, but also blurs the image.

Reducing Salt-and-Pepper Noise

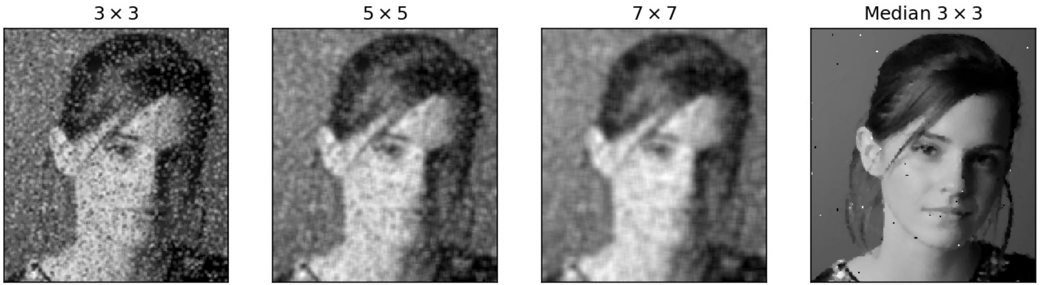
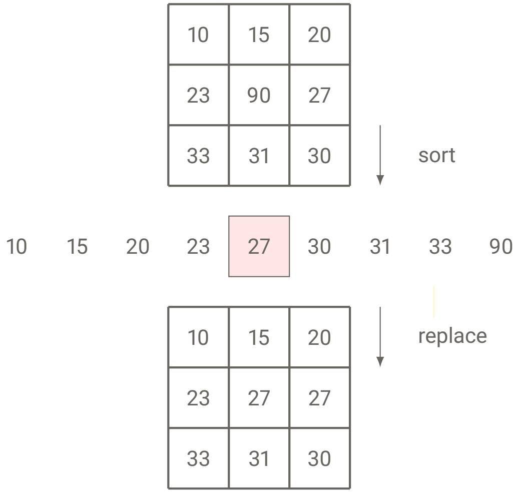


Figure: Inability to reduce salt and pepper noise with Gaussian filtering

Median Filtering



Figure

A median filter operates over a window by selecting the median intensity in the window.
Is median filtering linear?

Source: *K. Grauman*

No, median filtering is not a linear operation. Linear operations satisfy the superposition principle, which means that the output of the operation on a sum of two inputs is equal to the sum of the outputs on each individual input. In mathematical terms, if f(x) is a linear operation, it should satisfy:

f(a * x + b * y) = a * f(x) + b * f(y)

However, median filtering does not obey this principle. When you apply a median filter to an image or a signal, it selects the median value within a window (neighborhood) of data points. The median is not a linear function of the values in the window because it involves sorting the values and selecting the middle value.

For example, if you have two images, x and y, and you apply median filtering separately to each image:

median(a * x) a * median(x)

The median of the scaled image (a * x) will not equal the scaled median (a * median(x)). Median filtering operates by selecting the middle value within a set of values, and this operation is not linear because it depends on the order and distribution of the data within the window

Sharpening Revisited

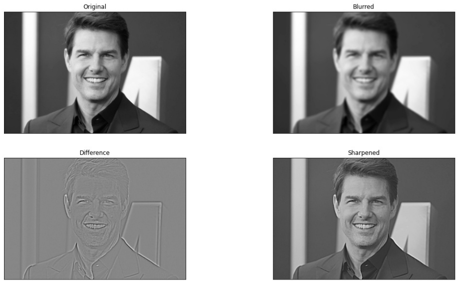


Figure: Sharpening

1. What does blurring take away?
2. We add it back.

Source: *Svetlana Lazebnik*

Unsharp Mask Filter

$$f + \alpha(f - f * g) = (1 + \alpha)f - \alpha f * g = f * ((1 + \alpha)e - g)$$