## Homework Assignment

## Problem 1

Autonomous underwater vehicles (AUVs) are becoming increasingly popular for deep-sea exploration. AUVs can operate in dangerous and remote environments without putting human lives at risk, especially where conditions can be harsh and unpredictable.

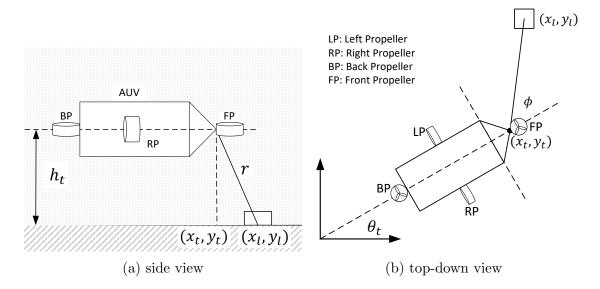


Figure 1: Autonomous underwater SLAM system

An AUV as depicted in Fig. 1 is to be utilized for the exploration of a certain shipwreck. The AUV achieves autonomous navigation using a semi 3D-SLAM system with a feature-based map. The state of the AUV is given by its 3D position  $(x_t, y_t, h_t)$  and its 2D orientation  $\theta_t$ . Distinct rocks on flat seabed are considered to be landmarks in the map. The state of a landmark is given by its 2D coordinates  $(x_l, y_l)$ . Consider the following conditions for the motion model of the AUV:

- AUV receives a control input  $u_t = [v_t \ \omega_t \ \gamma_t]^T$  at each time step.  $v_t$  is the horizontal translational velocity,  $\omega_t$  is the horizontal rotational velocity, and  $\gamma_t$  is the vertical velocity.
- During each time step  $(\Delta t)$ , the overall effect of  $u_t$  is such that the AUV, first, adjusts its height using  $\gamma_t$  for  $\Delta t_{\gamma}$ , and then rotates on the spot using  $\omega_t$  for  $\Delta t_{\omega}$ , and finally moves in a straight line using  $v_t$  for  $\Delta t_v$  ( $\Delta t = \Delta t_{\gamma} + \Delta t_{\omega} + \Delta t_v$ ).

The perception system returns sonar range-bearing measurements  $(r, \phi)$  as illustrated in Fig. 1.

- (a) Explain how the AUV achieves different types of motion using its propellers (LP, RP, BP, and FP). [6 marks]
- (b) Construct the motion model as a function of control input  $u_t$  and the AUV's previous state. [4 marks]
- (c) Augment the motion model to the full SLAM state transition model for a single landmark situation. [3 marks]
- (d) Derive the Jacobian matrix of the augmented state transition model with respect to the state variables of the augmented state. [4 marks]
- (e) Formulate the measurement model of the system. [4 marks]
- (f) Derive the Jacobian matrix of the measurement model with respect to the state variables of the augmented state. [4 marks]