Homework Assignment

Question

Consider the following nonlinear system described by the equations:

$$X_t = g(u_t, X_{t-1}) + \epsilon_t$$
$$Z_t = h(X_t) + \delta_t$$

where,

$$g(u_t, X_{t-1}) = \begin{cases} x_{t-1} + u_t \sin(y_{t-1}) \\ y_{t-1} + u_t \cos(x_{t-1}) \end{cases} \quad , \quad h(X_t) = {x_t}^2 + {y_t}^2.$$

X_t is the state of the system with x_t, y_t state variables, Z_t is the observation, and u_t is the control input. ϵ_t and δ_t represent noise components. To estimate the state of the system, the extended Kalman filter (EKF) is to be used.

(a) Linearize the function g around the most suitable operating point using the first-order Taylor expansion. [5 Marks]

(b) Linearize the function h around the most suitable operating point using the first-order Taylor expansion. [5 Marks]

(c) Given the initial state estimate $X_0 = [0 \quad 0]^T$ and the initial error covariance $\Sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, perform the EKF prediction step to calculate the predicted state and error covariance for the control input $u_1 = 1$ at next time step $t = 1$. Use process noise covariance $R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$. [5 Marks]

(d) If the measurement at $t = 1$ is $Z_1 = 1.22$, perform the EKF update step to compute the updated state estimate and error covariance. Assume the measurement noise covariance $Q = 0.2$. [10 marks]