Homework - I

1. Prove the following:

   a. $P(x|z) = \sum_y P(x|y,z)P(y|z)$

   b. $P(x|y,z) = \frac{P(y|x,z)P(x|z)}{P(y|z)}$

2. An autonomous robotic system is to be used to contain and extinguish bush fires. The expected procedure is to approach a tree, make an observation (z_t) about its condition (x_t) and start the extinguisher actuator system if the tree is found to be burning. The robot will approach a new tree only when the current tree is no longer burning. The probabilities associated with state transition with control action (u_t) are as follows:

   P(x_t = burning | x_{t-1}= burning, u_t = extinguish) = 0.3
   P(x_t = not-burning | x_{t-1}= not-burning, u_t = extinguish) = 1.0

   The observations are made using a thermal camera system, which is governed by the following probabilities:

   P(z_t = burning | x_t= burning) = 0.8
   P(z_t = not-burning | x_t= not-burning) = 0.7

   Assuming that the u_t = do-nothing action is 100% reliable, answer the following questions; use Bayes filter when necessary.

   a. Discuss why an autonomous system is suitable for the above application, and clearly mention the challenges associated with developing such a system.
   b. Suggest a suitable technique to obtain the above measurement model probabilities.
   c. At time instance T, the autonomous system estimates a particular tree's state to be 'burning' with a (posterior) probability of 0.2. That is, Bel (x_T = burning) = 0.2. The robot decides to continue operating the extinguisher actuator system for the next time instance (T + 1), as well. Find $\overline{Bel}$ (x_{T+1} = burning), where Bel and $\overline{Bel}$ have their usual meanings.
   d. The thermal camera system observes that the above tree is still burning at time instance T + 1. Find Bel (x_{T+1} = burning).
   e. When should the autonomous system approach a new tree? Suggest a reasonable transition method.