UNIVERSITY OF MORATUWA
Faculty of Engineering
Department of Electronic and Telecommunication Engineering
B. Sc. Engineering
Semester 8 Examination

EN4593 AUTONOMOUS SYSTEMS

Time allowed: *Two (2)* hours June 2024

INSTRUCTIONS TO CANDIDATES:

- This paper contains **4** questions on **5** pages.
- Answer **ALL** questions.
- All questions carry equal marks.
- This is an **open**-book examination.
- This examination accounts for **60**% of the module assessment. The total maximum mark attainable is **100**. The marks assigned for each question and sections are indicated in square brackets.
- Electronic/communication devices are not permitted. Only equipment allowed is a calculator approved and labeled by the Faculty of Engineering.
- Derivations are not required if they are not explicitly requested for in the question.
- Assume reasonable values for any data not given in or with the examination paper. Clearly state such assumptions.
- If you have any doubt as to the interpretation of the wording of a question, make your own decision, and clearly state it.

ADDITIONAL MATERIAL:

- No additional material is provided.

Continued ...

Question 1.

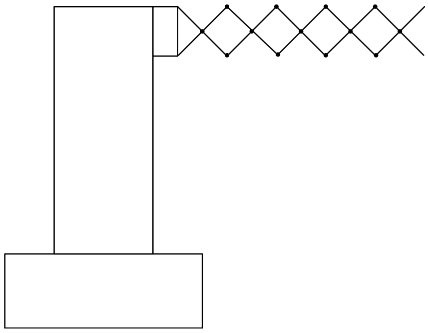

Figure Q1: Extensible robotic scissor arm

An autonomous robotic arm in a factory (Fig. Q1) can have its actuator in one of two discrete states: $s = 0$ (retracted), $s = 1$ (extended). The robot uses a Bayes filter to estimate the actuator's state based on noisy sensor measurements and control inputs. The control input u_t attempts to toggle the state, and the actual change might be affected by noise. The sensor provides noisy measurements z_t of the actuator's state. The following information is provided:

- Initial Belief
  The initial state of the actuator is unknown because the system has just been powered on and the actuator hasn't been calibrated yet, leading to a uniform initial belief about the actuator's state.
- Transition Model
  The control input $u_t = 1$ attempts to toggle the state with a probability of 0.8, or keeps the state unchanged with a probability of 0.2.
- Sensor Model
  The sensor provides a measurement z_t which corresponds to the actual state plus Gaussian noise with 0 mean and 0.4 standard deviation. The Gaussian likelihood of a measurement z_t conditioned on the state s_t is given by,

$$P(z_t|s_t) = \frac{1}{\sqrt{2\pi}(0.4)} \exp\left[-\frac{1}{2}\left(\frac{z_t - s_t}{0.4}\right)^2\right].$$

(a) Using the transition model, update the belief for all states at time $t = 1$ given the initial belief and the control input $u_1 = 1$. [6 marks]

(b) Given a sensor measurement $z_1 = 0.7$, update the belief using the sensor model. Normalize the updated belief. [10 marks]

(c) What is the most likely state of the actuator? [3 marks]

(d) Discuss the impact of sensor noise on the belief distribution. Explain how the Bayes filter helps in mitigating the uncertainties in both the transition model and the sensor model. [6 marks]

. .

Continued ...

Question 2.

Consider the following nonlinear system described by the equations:

$$X_t = g(u_t, X_{t-1}) + \epsilon_t$$
$$Z_t = h(X_t) + \delta_t$$

where,

$$g(u_t, X_{t-1}) = \begin{cases} x_{t-1} + u_t \sin(y_{t-1}) \\ y_{t-1} + u_t \cos(x_{t-1}) \end{cases} \quad , \quad h(X_t) = {x_t}^2 + {y_t}^2.$$

X_t is the state of the system with x_t, y_t state variables, Z_t is the observation, and u_t is the control input. ϵ_t and δ_t represent noise components. To estimate the state of the system, the extended Kalman filter (EKF) is to be used.

(a) Linearize the function g around the most suitable operating point using the first-order Taylor expansion. [5 Marks]

(b) Linearize the function h around the most suitable operating point using the first-order Taylor expansion. [5 Marks]

(c) Given the initial state estimate $X_0 = [0 \quad 0]^T$ and the initial error covariance $\Sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, perform the EKF prediction step to calculate the predicted state and error covariance for the control input $u_1 = 1$ at next time step $t = 1$. Use process noise covariance $R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$. [5 Marks]

(d) If the measurement at $t = 1$ is $Z_1 = 1.22$, perform the EKF update step to compute the updated state estimate and error covariance. Assume the measurement noise covariance $Q = 0.2$. [10 marks]

. .

Continued ...

Question 3.

Assume that you are working on the development of a self-driving car (Fig. Q3) that uses a particle filter for localization. The car is equipped with a GPS sensor, a LiDAR sensor, an IMU sensor, and wheel encoders.

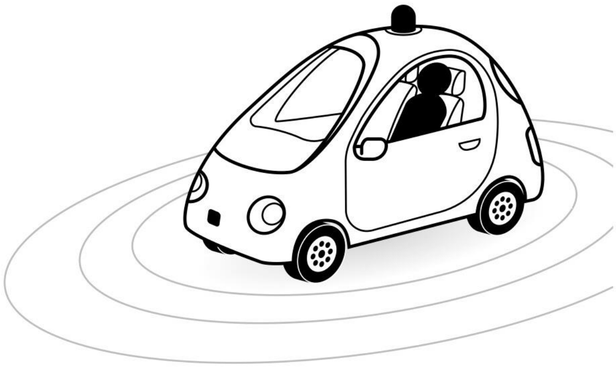

Figure Q3: Prospective self-driving car

(a) Discuss the role of resampling in a particle filter. Why is it necessary?[5 marks]

(b) Describe how you would initialize the particle filter for localization of the self-driving car. Assuming a 2D setting, specify the state variables you would use and the initial distribution of particles. [5 marks]

(c) Consider a simplified scenario where your self-driving car is localized in a 1D environment. You have four (4) particles with positions $p_1 = 2.0$, $p_2 = 3.5$, $p_3 = 4.0$, and $p_4 = 5.5$. The LiDAR sensor reports a measurement of 4.2 with 0 mean and 0.5 standard deviation Gaussian noise. Calculate the weights of each particle using a Gaussian likelihood model. Normalize the weights.
Hint: The likelihood model $p(z|x)$ should compare the actual measurement (z) to predicted mean measurement given a certain position (x) using a Gaussian distribution. [8 marks]

(d) Using the weights calculated in (c), compute the effective number of particles $\hat{N}_{eff}$. Comment on the degeneracy of the filter. [3 marks]

(e) Imagine your self-driving car is moving in an urban environment where GPS signals are occasionally lost due to tall buildings. How would the particle filter handle this situation to maintain accurate localization? [4 marks]

. .

Continued ...

Question 4.

You are provided with data from a sonar sensor used in an autonomous underwater vehicle (AUV) for navigation and object detection (Fig. Q4). Due to slight misalignments and manufacturing imperfections, the raw measurements contain systematic errors. The calibration process aims to correct these errors using a least squares approach.

Figure Q4: AUV using sonar sensor for navigation and object detection

Consider the following dataset of raw measurements obtained from the sonar sensor:

True Distance (m)	Measured Distance (m)
2.0	2.3
4.0	4.5
6.0	6.6
8.0	8.2
10.0	10.7

(a) Formulate the process of calibration as a straight line fitting problem. [5 marks]

(b) Define the model parameters and write the system of equations in matrix form suitable for the least squares method. [5 marks]

(c) Calculate the model parameters using the least squares method. [5 marks]

(d) Interpret the meaning of the estimated parameters in the context of sonar sensor calibration. [5 marks]

(e) Explain how these parameters can be used to correct future measurements from the sonar sensor using an example. [5 marks]

. .

- End of Question Paper -