Homework – 3

1. Kalman filter can be used in tracking and navigation applications when the sensor observations are not sufficiently accurate. The position of an aircraft is to be tracked using noisy radar measurements together with a Kalman filter as depicted in Fig. 1.

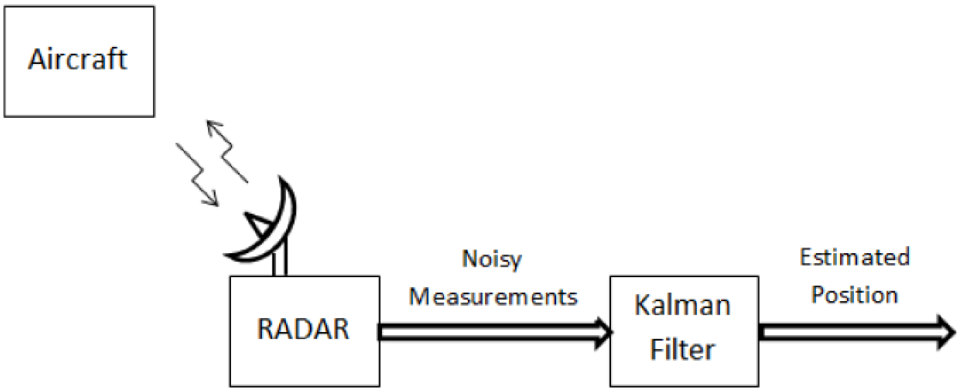

Fig. 1: Aircraft tracking using Radar measurements

The state of the aircraft is given by $X_t = [x_t \quad \dot{x}_t \quad y_t \quad \dot{y}_t \quad z_t \quad \dot{z}_t]^T$, where the frequency of radar measurements is given by f.

a. Using a constant velocity model, formulate the state transition model for the above system. What are the control inputs?
b. Assuming that the radar system outputs the 3D position of the aircraft, formulate the measurement model for the above system. The observation is given by O_t.
c. Write down the Kalman filter equations using the standard form for the prediction and update stages while mentioning the dimensions of all matrices involved. The process noise covariance is given by R_t and the measurement noise covariance is given by Q_t.

2. The 2D ground position (x_t, y_t) of a moving target is to be tracked using inaccurate measurements of an overhead camera, together with a Kalman filter. The state (Xt) of the target is given by $X_t = [x_t \quad \dot{x}_t \quad \ddot{x}_t \quad y_t \quad \dot{y}_t \quad \ddot{y}_t]^T$, and the frame rate of the camera is given by r. An image processing software system that takes in camera images outputs the 2D position of the target at a rate of $0.5r$. With the assumption that the Kalman filter runs at the same rate as the image processing system, answer the following questions:

   a. What are the conditions that must be satisfied to apply the Kalman filter to any given problem?
   b. Using a constant acceleration model, formulate the state transition equations for the above system and identify the state transition matrix A_t and the control input matrix B_t.
   c. Formulate the measurement model for the above system and identify the observation matrix C_t. Take the observation as z_t.
   d. Write down the Kalman filter equations using the standard form for the prediction and update stages. State the dimensions of all vectors and matrices involved. The process noise covariance is given by R_t and the measurement noise covariance is given by Q_t.
   e. Find the estimated 2D position of the target after one iteration of the Kalman filter using the following values with usual meanings:

$$r = 30fps$$
$$\mu_0 = [10.0 \quad 0.08 \quad 0.005 \quad 35.5 \quad 0.05 \quad 0.01]^T$$
$$K_1 = \begin{bmatrix} 0.2649 & 0.0039 & 0.0002 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2425 & 0.002 & 0.0001 \end{bmatrix}^T$$
$$z_1 = [11.2 \quad 33.2]^T$$