$$\begin{array}{ccc} Z & Y & X \\ \phi & \theta & \psi \end{array}$$

8

a) $${}^{E}_{B}R = \begin{bmatrix} 0.4698 & -0.882 & -0.018 \\ 0.8137 & 0.4409 & -0.378 \\ 0.342 & 0.1631 & 0.9254 \end{bmatrix}$$ //

b) $${}^{E}_{B}R(t+dt) = {}^{E}_{B}R \begin{bmatrix} 1 & -d\phi & d\theta \\ d\phi & 1 & -d\psi \\ -d\theta & d\psi & 1 \end{bmatrix}$$

$\pi = 180^{\circ}$

$\frac{15}{180}\pi$

$\omega_x = \frac{15\pi}{180} = \pi/12$

$\omega_y = \frac{-20\pi}{180} = -\pi/9$

$\omega_z = \frac{30\pi}{180} = \pi/6$

$d\phi = \pi/6 \times 0.05$

$d\theta = -\pi/9 \times 0.05$

$d\psi = \pi/12 \times 0.05$

$${}^{E}_{B}R(t+dt) = \begin{bmatrix} 0.4924 & -0.895 & -0.014 \\ 0.7958 & 0.4147 & -0.398 \\ 0.3535 & 0.1662 & 0.9173 \end{bmatrix}$$ //

c) Error $= -0.05192$ //

d) $$X_{orth} = X - \left(\frac{-0.05192}{2}\right) Y$$

$$Y_{orth} = Y - \left(\frac{-0.05192}{2}\right) X$$

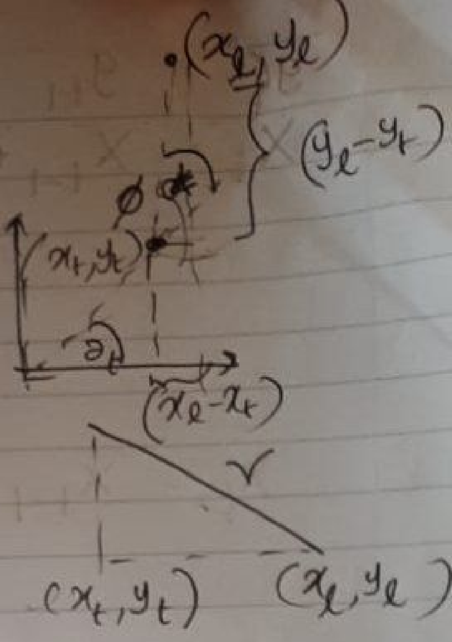
$$X_{orth} = \begin{bmatrix} 0.4691 \\ 0.8065 \\ 0.3578 \end{bmatrix} \quad // \qquad Y_{orth} = \begin{bmatrix} -0.882 \\ 0.4353 \\ 0.1753 \end{bmatrix} \quad //$$

$$Z_{orth} = \begin{vmatrix} i & j & k \\ 0.4691 & 0.8065 & 0.3578 \\ -0.882 & 0.4353 & 0.1753 \end{vmatrix} = \begin{matrix} -0.0143i \\ -j0.3978 \\ k\,0.9155 \end{matrix}$$

$$Z_{orth} = \begin{bmatrix} -0.0143 \\ -0.3978 \\ 0.9155 \end{bmatrix}$$

$g(u_t, \cancel{y_{t-1}}\, x_{t-1}) =$

e) $$\begin{bmatrix} r \\ \phi \end{bmatrix} = \begin{bmatrix} \sqrt{(x_l-x_t)^2+(y_l-y_t)^2} \\ \operatorname{atan2}\left((y_l-y_t),(x_l-x_t)\right) - \theta_t \end{bmatrix} \cancel{\begin{bmatrix} x_t \\ y_t \\ h_t \\ \theta_t \end{bmatrix}} + \delta_t$$



f) same in the slides

Question 04

a) $${}^{E}_{B}R = R_{z,60^\circ} \times R_{y,-20^\circ}, R_{x,10^\circ}$$

$$= \begin{bmatrix} C_{60^\circ} & -S_{60^\circ} & 0 \\ +S_{60} & C_{60} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{-20} & 0 & +S_{-20} \\ 0 & 1 & 0 \\ -S_{-20} & 0 & C_{-20} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_{10^\circ} & -S_{10^\circ} \\ 0 & +S_{10^\circ} & C_{10^\circ} \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 0.4698 & 0.8231 & 0.3187 \\ -0.813 & 0.5438 & -0.204 \\ 0.342 & -0.163 & 0.9254 \end{bmatrix}} //$$

(04)

$g\xi \quad g_{t-1}$

$$X_t = X_{t-1} + \begin{bmatrix} v_t \Delta t_v \cos(\omega_t \Delta t_\omega + \theta_t) \\ v_t \Delta t_v \sin(\omega_t \Delta t_\omega + \theta_t) \\ \gamma_t \Delta t_\gamma \\ \omega_t \Delta t_\omega \end{bmatrix}$$

$$X_t = X_{t-1} + \begin{bmatrix} v_t \Delta t_v \cos(\omega_t \Delta t_\omega + \theta_t) \\ \vdots \\ \omega_t \Delta t_\omega \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$X_t = X_{t-1} + F_x^T \begin{bmatrix} v_t \Delta t_v \cos(\omega_t \Delta t_\omega + \theta_t) \\ \vdots \\ \omega_t \Delta t_\omega \end{bmatrix}$$

$$F_x = \begin{bmatrix} 1 & 0 & 0 & 0 & \underbrace{0 & 0 & 0}_{\text{for landmark}} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

d) $g(u_t, X_{t-1}) \simeq g(u_t, \mu_{t-1}) + G_t(X_{t-1} - \mu_{t-1}) \leftarrow$

$$G_t = \frac{\partial}{\partial X_{t-1}}[X_t]$$

$$= I_{4\times4} + F_x^T \begin{bmatrix} 0 & 0 & 0 & -v_t \Delta t_v \sin(\omega_t \Delta t_\omega + \theta_t) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cancel{\Delta t_v \cos(\omega_t \Delta t_\omega + \theta_t)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \cancel{\Delta t_v \sin(\omega_t \Delta t_\omega + \theta_t)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & v_t \Delta t_v \cos(\omega_t \Delta t_\omega + \theta_t) & & & \end{bmatrix}$$

$$= I_{4\times4} + F_x^T P F_x \;//$$

e) $z = \begin{bmatrix} z_x \\ z_y \\ z_z \end{bmatrix}$

$$Z_t = C_t x_t + \delta_t$$

$$\begin{bmatrix} z_x \\ z_y \\ z_z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}}_{C_t} \begin{bmatrix} x_t \\ y_t \\ z_t \\ \dot{x}_t \\ \dot{y}_t \\ \dot{z}_t \end{bmatrix} + \delta_t$$

$$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

constant velocity model for state transition

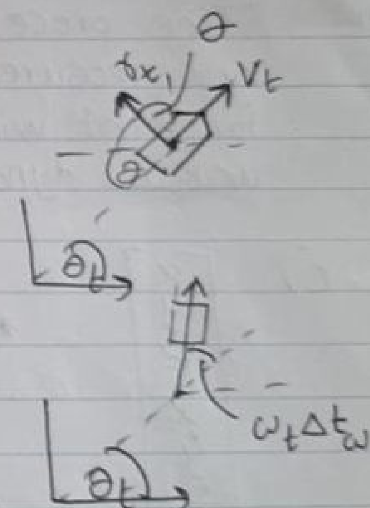
ε_t ~~Q_t~~ and ~~R_t~~ δ_t are gaussian distributed noises

$\mathcal{N}(0, \cancel{Q}R_t)$ $\mathcal{N}(0, R)$

Question 03

a)

Forward	$\rightarrow$	RP $\uparrow$	LP $\uparrow$
Backward	$\rightarrow$	RP $\downarrow$	~~R~~ LP $\downarrow$
Rotate left	$\rightarrow$	RP $\uparrow$	LP $\downarrow$
right	$\rightarrow$	LP $\uparrow$	RP $\downarrow$
Move up	$\rightarrow$	FP $\uparrow$	BP $\uparrow$
down	$\rightarrow$	FP $\downarrow$	BP $\downarrow$



b) $X_t = A_t X_{t-1} + B_t u_t + \varepsilon_t$

$\cancel{x_t = x_{t-1} + V_t \Delta t_v \cos\theta_t - \gamma_t \Delta t_\gamma \cos\theta_t \sin\theta_t}$

$\cancel{y_t = y_{t-1} + \gamma_t \Delta t_\gamma \cos\theta_t + V_t \Delta t_v \sin\theta_t}$

$\cancel{h_t = h_{t-1} +}$

$$x_t = x_{t-1} + V_t \Delta t_v \cos(\omega_t \Delta t_\omega + \theta_t) + \varepsilon_x$$

$$y_t = y_{t-1} + V_t \Delta t_v \sin(\omega_t \Delta t_\omega + \theta_t) + \varepsilon_y$$

$$h_t = h_{t-1} + \gamma_t \Delta t_\gamma$$

$$\theta_t = \theta_{t-1} + \omega_t \Delta t_\omega$$

Question 02

a) ~~the~~ Model predictions and measurements should have gaussian distributions.
linear system dynamics

b)

Pros	Cons
Estimate joints even in occluded areas by model's predictions	Higher computational cost.
Tune process noise covariance and measurement noise covariances for joint wise to match with unique dynamics	Hard to achieve real time performance with many KFs

c) $\begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$ * Assuming constant velocity model.

d) $$x_t = A x_{t-1} + \varepsilon_t \qquad \because u_t = 0$$

$$x_t = x_{t-1} + \Delta t \, \dot{x}_{t-1}$$

$\vdots$

similarly

$$z_t = z_{t-1} + \Delta t \, \dot{z}_{t-1}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{x}_t = A x_{t-1} + \varepsilon_t$$

$$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$$

β) $\overline{bel}(x_t = unclean) = 0.63$

$bel(x_t) = \eta \, Pr(\cancel{x} z_t | x_t) \, \overline{bel}(x_t)$

$bel(x_t = clean) = \eta \, \cancel{\epsilon} \, Pr(z_t = unclean | x_t = clean) \, \overline{bel}(x_t = clean)$

$= \eta \; 0.32 \times 0.37$

$bel(x_t = unclean) = \eta \, Pr(z_t = unclean | x_t = \cancel{\epsilon} unclean) \, \overline{bel}(x_t = unclean)$

$= \eta \; 0.77 \times 0.63$

$1 = \eta (0.32 \times 0.37 + 0.77 \times 0.63)$

$\eta = 1.657$

$bel(x_t = clean) = 1.657 \times 0.32 \times 0.37$
$= 0.196$ //

γ) yes. $bel(x_t = unclean)$ has the higher probability

Autonomous Systems

$\boxed{2023}$

Question 01

a) we need to consider the ~~nois~~ effects due to noise components which could emerge if we only rely on model based approach as they cannot cover all the aspects ~~and~~ or measurement based approaches due sensor noises. So it would be better not to trust model nor measurement but incoperate both for better estimation through probability

b) $x_t = \begin{cases} clean \\ unclean \end{cases}$

$Pr(x_t = clean) = 0.37$

i) $Pr(z_t = clean \mid x_t = clean) = 0.68$
$Pr(z_t = unclean \mid x_t = unclean) = 0.77$

More Capable to detect unclean window

ii) $\overline{bel}(x_t) = \sum Pr(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})$

d) $\overline{bel}(x_t = clean) = Pr(x_t = clean \mid u_t = \text{do nothing } \cancel{\text{unclean}}, x_{t-1} = unclean)\, bel(x_{t-1} = unclean)$

$\times$ u_t is wrong

$+ Pr(x_t = clean \mid u_t = \text{do nothing } \cancel{\text{unclean}}, x_{t-1} = clean)\, bel(x_{t-1} = clean)$

$= 0 \times 0.63 + \cancel{0.81}\ 1 \times 0.37$

$\cancel{= 0.2997}$ $= 0.37$

$\cancel{= 0.30}$ //

c) $x = (A^TA)^{-1}(A^Tb)$

$$A = \begin{pmatrix} 1 & 2.3 \\ 1 & 4.5 \\ 1 & 6.6 \\ 1 & 8.2 \\ 1 & 10.7 \end{pmatrix} \qquad b = \begin{pmatrix} 2.0 \\ 4.0 \\ 6.0 \\ 8.0 \\ 10.0 \end{pmatrix}$$

$$(A^TA)^{-1} = \left(\begin{pmatrix} 2.3 & 4.5 & 6.6 & 8.2 & 10.7 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} A \right)^{-1}$$

wrong transpose

$$= \begin{pmatrix} 32.3 & 250.83 \\ 5 & 32.3 \end{pmatrix}^{-1}$$

$$= \begin{bmatrix} -0.153 & 1.1895 \\ 0.0237 & -0.153 \end{bmatrix}$$

$$(A^Tb) = \begin{pmatrix} \cancel{234.8} \\ \cancel{30} \end{pmatrix} \begin{pmatrix} 30 \\ 234.8 \end{pmatrix}$$

$$x = (A^TA)^{-1}(A^Tb)$$

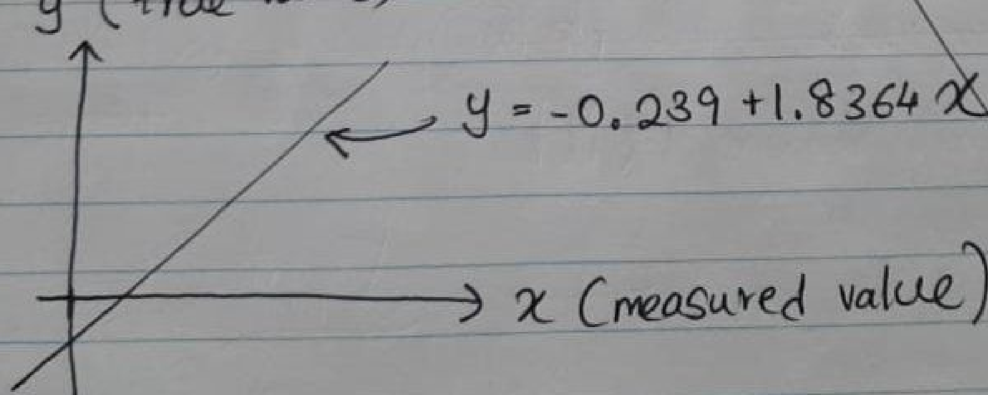
$$= \begin{bmatrix} -0.239 \\ 1.8364 \end{bmatrix}$$

$\alpha = -0.239$

$\beta = 1.8364$

d) Generalized reading based on reducing total err

e)



Question 04

a) $\varepsilon_i = \sum error^2 = \sum_{i=1}^{m} \left(f(md_i) - td_i \right)^2$

$f(md) = \alpha + \beta(md) \leftarrow$ straight line

md → measured distance

td → true distance

Minimum error $\Rightarrow \frac{d}{d\alpha}\left[\sum error^2\right] = 0$

$\frac{d}{d\beta}\left[\sum error^2\right] = 0$

$$\sum_{i=1}^{m} 2\left(\alpha + \beta(m_{d_i}) - t_{d_i}\right) \times 1 = 0$$

$$\alpha m + \beta \sum_{i=1}^{m} m_{d_i} = \sum_{i=1}^{m} t_{d_i} \quad \text{— (1)}$$

$$\sum_{i=1}^{m} 2\left(\alpha + \beta(m_{d_i}) - t_{d_i}\right) m_{d_i} = 0$$

$$\alpha \sum m_{d_i} + \beta \sum m_{d_i}^2 = \sum m_{d_i} t_{d_i} \quad \text{— (2)}$$

b)

$$\left(\sum^{m} 1\right)\alpha + \left(\sum^{m} m_{d_i}\right)\beta = \sum^{m} t_{d_i}$$

$$\left(\sum^{m} m_{d_i}\right)\alpha + \left(\sum^{m} m_{d_i}^2\right)\beta = \sum^{m} m_{d_i} t_{d_i}$$

$$A^T \underbrace{\begin{pmatrix} 1 & m_{d1} \\ 1 & m_{d2} \\ \vdots & \vdots \\ 1 & m_{dm} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 1 & m_{d1} \\ 1 & m_{d2} \\ \vdots & \vdots \\ 1 & m_{dm} \end{pmatrix}^T}_{A^T} \underbrace{\begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ \vdots \\ t_m \end{pmatrix}}_{b}$$

c) $P_1 = 2$, $P_2 = 3.5$, $P_3 = 4.0$, $P_4 = 5.5$

$$Pr(z_t|x_t) = \frac{1}{\sqrt{2\pi(0.5)^2}} \exp\left(-\frac{(z_t - x_t)^2}{2\times 0.5^2}\right)$$

$$Pr(z|P_1) = 4.9885\times10^{-5}$$
$$Pr(z|P_2) = 0.2995$$
$$Pr(z|P_3) = 0.7365$$
$$Pr(z|P_4) = 0.0272$$

$$w_1 = 4.692\times10^{-5}$$
$$w_2 = 0.028$$
$$w_3 = 0.693$$
$$w_4 = 0.026$$

d) $$\hat{N}_{eff} = \frac{1}{\sum_{m=1}^{4}\left(w_t^{[m]}\right)^2} = 2.076 \simeq 2 \text{ particles}$$

Have severe degenarcy. Only 2 particles contribute to the state.

e) we can introduce different measurement models based on IMU data, until gps readings are availab ~~Also by incorporating lidar and wheel encoders for System~~ wheel encoder, lidar data

d) $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$

$= \begin{bmatrix} 2 \\ 2.2 \end{bmatrix} \begin{bmatrix} 4.4 + 0.2 \end{bmatrix}^{-1}$ $\quad H_t = [0, 2]$

$= \begin{bmatrix} 2 \\ 2.2 \end{bmatrix} \frac{1}{4.6} = \begin{bmatrix} \cancel{9.2} \\ \cancel{10.12} \end{bmatrix} \begin{bmatrix} 0.4347 \\ 0.4782 \end{bmatrix}$

$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$

$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.4347 \\ \cancel{9.2} \\ \cancel{10.12} \\ 0.4782 \end{bmatrix} \begin{bmatrix} 1.22 - 1 \end{bmatrix}$

$= \cancel{\begin{bmatrix} 2.024 \\ 3.2264 \end{bmatrix}}$ // $= \begin{bmatrix} 0.0956 \\ 1.1052 \end{bmatrix}$ //

Question 03

a) In PFs sampling process is based on another known pdf and then weighted to get the desired pdf. But this does not ensure it captures important features (sudden spikes) in desired pdf due to its distribution is sampling distribution. So we do resampling to get higher probablilities in desired pdf removing very low probabilities //

b) $\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$

~~c)~~

03

$$g(u_t, x_{t-1}) = \begin{bmatrix} x_{t-1} + u_t \sin[(y_{t-1})|_{\mu_{t-1}}] \\ y_{t-1} + u_t \cos[(x_{t-1})|_{\mu_{t-1}}] \end{bmatrix} + \begin{bmatrix} 1 & u_t\cos(y_{t-1}) \\ -u_t\sin(x_{t-1}) & 1 \end{bmatrix}_{\mu_{t-1}} \left[X_{t-1} - \mu_{t-1} \right] //$$

substitute

b) $h(X_t) = x_t^2 + y_t^2$

$$h(x_t) \simeq h(\bar{\mu}_t) + H(\bar{\mu}_t)(X_t - \bar{\mu}_t)$$

$$h(x_t) = H(\bar{\mu}_t) X_t + \{ h(\bar{\mu}_t) - H(\bar{\mu}_t)\bar{\mu}_t \} //$$

$$H(X_t) = \begin{bmatrix} \frac{\partial}{\partial x_t}(x_t^2 + y_t^2) & \frac{\partial}{\partial y_t}(x_t^2 + y_t^2) \end{bmatrix}$$

$$= \begin{bmatrix} 2x_t & 2y_t \end{bmatrix}$$

$$H(\bar{\mu}_t) = \begin{bmatrix} 2x_t & 2y_t \end{bmatrix}_{X_t = \bar{\mu}_t} //$$

c) $\bar{\mu}_t = g(u_t, \mu_{t-1}) \,|_{u_t = 1}$

$$\bar{\mu}_t = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

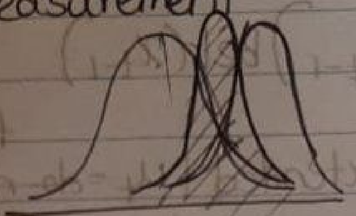
$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.1 & 1 \\ 1 & 1.1 \end{bmatrix} //$$

d) transition model $\rightarrow$ 0.8 ~~sucess rate~~ accuracy

Measurement $\rightarrow$



Question 02

a) $g(u_t, X_{t-1}) \simeq g(u_t, \mu_{t-1}) + G(u_t, \mu_{t-1})(X_{t-1} - \mu_{t-1})$

$$g(u_t, X_{t-1}) = \begin{cases} x_{t-1} + u_t \sin(y_{t-1}) \leftarrow R_1 \\ y_{t-1} + u_t \cos(x_{t-1}) \leftarrow R_2 \end{cases}$$

$$G(u_t, X_{t-1}) = \begin{bmatrix} \frac{\partial}{\partial x_{t-1}} R_1 & \frac{\partial}{\partial y_{t-1}} R_1 \\ \frac{\partial}{\partial x_{t-1}} R_2 & \frac{\partial}{\partial y_{t-1}} R_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & u_t \cos(y_{t-1}) \\ -u_t \sin(x_{t-1}) & 1 \end{bmatrix}_{X_{t-1} = \mu_{t-1}}$$

$$= \begin{bmatrix} 1 & u_t \cos(y_{t-1}) \\ -u_t \sin(x_{t-1}) & 1 \end{bmatrix}_{\mu_{t-1}}$$

Autonomous Systems (01)

$\boxed{2024}$

Question (01)

a) $\overline{Bel}(x_t) = \sum Pr(x_t | u_t, x_{t-1}) Bel(x_{t-1})$

$\overline{Bel}(x_t = extended) = Pr(x_t = extended \mid u_t = \cancel{do\ nothing}\ toggle,\ x_{t-1} = retracted)\ Bel(x_{t-1} = retr)$

$+$

$Pr(x_t = extended \mid u_t = \cancel{do\ nothing}\ toggle,\ x_{t-1} = extended)\ Bel(x_{t-1} = extended)$

$= \cancel{0 \times 0.8}\ 0.8 \times 0.5 + \cancel{1}\ 0.2 \times 0.5 = 0.5$

$\cancel{= 0.5}$

$\overline{Bel}(x_t = retracted)\ \cancel{= 0.5} = 0.5$

b) $Bel(x_t = extended) = \eta\, Pr(z_t | x_t = extended)\ \overline{Bel}(x_t = extended)$

$Pr(z_t = 0.7, S_t = 0) = 0.13641$

$Pr(z_t = 0.7, S_t = 1) = 0.47614$

$Bel(x_t = extended) = \eta\ 0.476 \times 0.5$

$Bel(x_t = retracted) = \eta \times 0.136 \times 0.5$

$\eta = \frac{1}{(0.476 + 0.136) \times 0.5} = 3.268$

$Bel(x_t = extended) = 0.778$ //

$Bel(x_t = retracted) = 0.222$ //

c) Extended actuator //