CJ NOTES

SHUJIE LI

1. σ_r From Measured Cross Section

The reduced cross section

$$\sigma_r(x,Q^2) = F_2(x,Q^2) - \frac{y^2}{Y^+}F_L(x,Q^2). \tag{1}$$

This quantity is connected to the experiment measurement by

$$\sigma_r(x,Q^2) = \frac{xQ^4}{2\pi\alpha^2(\hbar c)^2 y^+}\frac{d^2\sigma}{dxdQ^2},$$

where $y = \nu/E$, and $y^+ = 1 + (1-y)^2 + \frac{2m_p^2x^2y^2}{Q^2}$.$(\hbar c)^2 = 0.38938\ mbarn \cdot GeV^2$. $\frac{d^2\sigma}{dxdQ^2}$ is the differential cross section in $barn \cdot GeV^{-2}$. Often the experiments provide their results in $\dfrac{d^2\sigma}{dE'd\Omega}$ $(barn \cdot sr^{-1} \cdot GeV^{-1})$, which requires a unit conversion:

$$\begin{aligned}
\frac{d^2\sigma}{dxdQ^2} &= \frac{dE'd\Omega}{dxdQ^2}\frac{d^2\sigma}{dE'd\Omega} \\
&= \frac{\pi m_p(E-E')^2}{E^2E'^2(1-cos\theta)}\frac{d^2\sigma}{dE'd\Omega} \\
&= \frac{\pi y}{xE'}\frac{d^2\sigma}{dE'd\Omega}.
\end{aligned}$$

Another useful equation is

$$\frac{d^2\sigma}{dE'd\Omega} = \Gamma\big(\sigma_T(x,Q^2) + \epsilon\sigma_L(x,Q^2)\big) = \Gamma(1+\epsilon R)\sigma_T(x,Q^2),$$

where the transverse virtual photons flux $\Gamma = \dfrac{\alpha E'\kappa}{2\pi^2Q^2E(1-\epsilon)}$,$\kappa = \nu(1-x)$,

$R = \dfrac{\sigma_L(x,Q^2)}{\sigma_T(x,Q^2)} = \dfrac{F_L}{F_2 - F_L}$.

$\epsilon = \big[1 + 2(1 + \dfrac{\nu^2}{Q^2})\tan^2\dfrac{\theta}{2}\big]^{-1}$ is the ratio of the longitudinal to transverse virtual photon polarization.

2. σ_r From F_2

$$
\begin{aligned}
F_1(x,Q^2) &= \frac{\kappa M}{4\pi^2\alpha(\hbar c)^2}\sigma_T(x,Q^2),\\
F_L(x,Q^2) &= \frac{2x\kappa M}{4\pi^2\alpha(\hbar c)^2}\sigma_L(x,Q^2).
\end{aligned}
$$

$$
\begin{aligned}
\Longrightarrow F_2(x,Q^2) &= \frac{\kappa}{4\pi^2\alpha(\hbar c)^2}\frac{\nu}{1+\nu^2/Q^2}\big[\sigma_T(x,Q^2)+\sigma_L(x,Q^2)\big]\\
&= \frac{\kappa}{4\pi^2\alpha(\hbar c)^2}\frac{\nu}{1+\nu^2/Q^2}\big[\frac{\sigma_L(x,Q^2)}{R}+\sigma_L(x,Q^2)\big]\\
&= \frac{\nu}{1+\nu^2/Q^2}\frac{F_L(x,Q^2)}{2xM}\big(1+\frac{1}{R}\big) \qquad (2)
\end{aligned}
$$

Plug eq. 2 into 1, σ_r can be calculated given R and $F_2(x,Q^2)$. Similarly, $F_2(x,Q^2)$ can be pulled out from the cross section given R :

$$
F_2(x,Q^2) = \frac{\nu}{1+\nu^2/Q^2}\frac{1+R}{\Gamma(1+\epsilon R)}\frac{\kappa}{4\pi^2\alpha(\hbar c)^2}\frac{d^2\sigma}{dE'd\Omega} \qquad (3)
$$