

Plotting the error bands for Off-Shell function was implemented successfully and some results are summarized as follows.

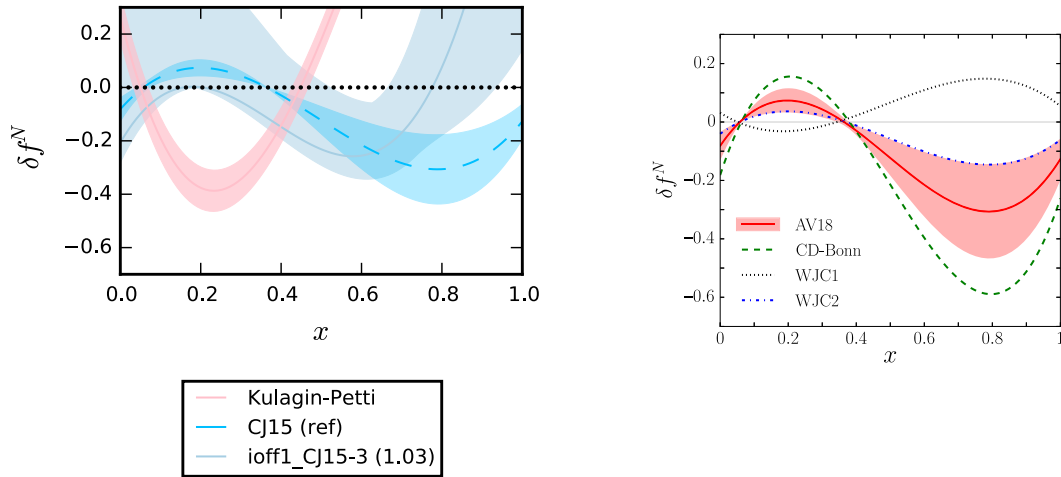
As the preliminary step, three types of fits were considered as follows.

- 1) CJ15 (In the original CJ15 work, the parameter x_1 was fixed by considering the constraint by quark sum rule)

$$\delta f = N(x - x_0)(x - x_1)(1 + x_0 - x)$$

- 2) CJ15 with ioff1 for 3-degree polynomial

$$\delta f_3 = N(x - x_0)(x - x_1)(x - x_2)$$



Observations:

- 1) CJ15 Off-Shell function's uncertainty band is reproduced
- 2) The uncertainty bands for ioff1 is not symmetrical because the parameters x_0 and x_1 are strongly correlated negatively. For example (correlation matrices for x_0 and x_1):

$$\text{ioff1} \begin{pmatrix} 1.000 & -0.976 \\ -0.976 & 1.000 \end{pmatrix}$$

The parameters for ioff1 are the following.

$$N = 8.2851 \pm 5.2739$$

$$x_0 = 0.20480 \pm 0.39590$$

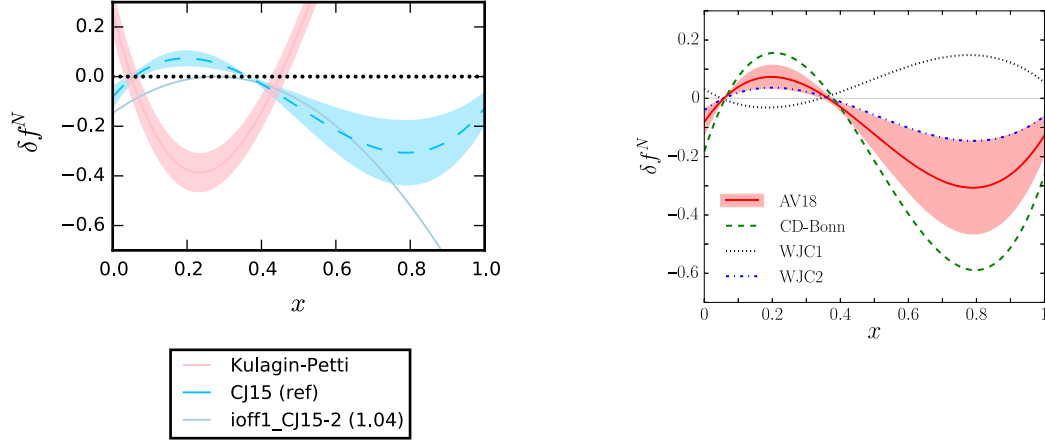
$$x_1 = 0.15685 \pm 0.37412$$

$$x_2 = 0.77609 \pm 0.11280$$

Correlation matrix for N, x_0, x_1 and x_2 :

$$\begin{pmatrix} 1.000 & 0.460 & -0.503 & -0.816 \\ 0.460 & 1.000 & -0.976 & -0.235 \\ -0.503 & -0.976 & 1.000 & 0.307 \\ -0.816 & -0.235 & 0.307 & 1.000 \end{pmatrix}$$

3) CJ15 with ioff1 2-degree polynomial $\delta f = N(x - x_0)(x - x_1)$



Observations:

Correlation matrices for N, x_0 and x_1 in ioff1 2-degree polynomial is the following.

$$\text{ioff1} \begin{pmatrix} 1.000 & 0.602 & -0.608 \\ 0.602 & 1.000 & -0.999 \\ -0.608 & -0.999 & 1.000 \end{pmatrix}$$

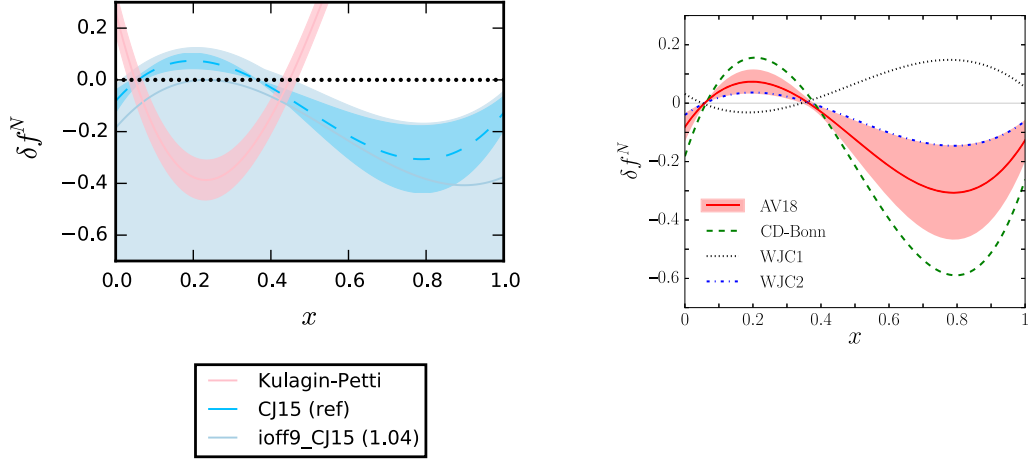
The parameters for ioff1 2D are the following. $N = -1.9052 \pm 1.0349$

$$x_0 = 0.27550 \pm 9.0427$$

$$x_1 = 0.28029 \pm 9.0625$$

4) CJ15 with ioff9 (In this case, the parameter x_1 was kept free and purely determined by the fit) with same Off-Shell function as CJ15

$$\delta f = N(x - x_0)(x - x_1)(1 + x_0 - x)$$



Observations:

Correlation matrix for N, x_0 and x_1 in ioff9 is the following.

$$\text{ioff9} \begin{pmatrix} 1.000 & 0.983 & -0.984 \\ 0.983 & 1.000 & -0.997 \\ -0.984 & -0.997 & 1.000 \end{pmatrix}$$

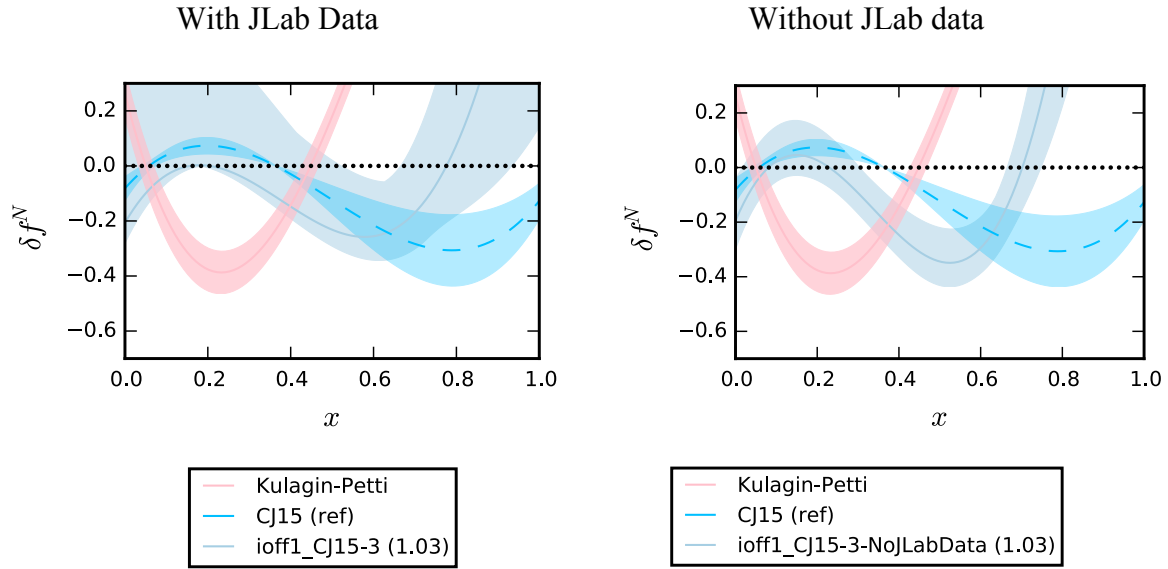
The parameters for ioff9 are the following. $N = -2.7758 \pm 5.3158$

$$x_0 = 0.22976 \pm 1.3858$$

$$x_1 = 0.23625 \pm 1.3608$$

5) CJ15 ioff1 with Vs without JLab data (Simona + BoNuS)

$$\delta f3 = N(x - x_0)(x - x_1)(x - x_2)$$



$$N = 8.2851 \pm 5.2739$$

$$x_0 = 0.20480 \pm 0.39590$$

$$x_1 = 0.15685 \pm 0.37412$$

$$x_2 = 0.77609 \pm 0.11280$$

$$N = 15.027 \pm 5.773$$

$$x_0 = 0.080580 \pm 0.071163$$

$$x_1 = 0.229570 \pm 0.086896$$

$$x_2 = 0.701960 \pm 0.039769$$

Correlation matrix for N, x_0, x_1 and x_2 :

N	x_0	x_1	x_2
1.000	0.460	-0.503	-0.816
0.460	1.000	-0.976	-0.235
-0.503	-0.976	1.000	0.307
-0.816	-0.235	0.307	1.000

The next step: Off-Shell parameterization is changed to,

$$\begin{aligned} \delta f2(new) &= a_1 + a_2x + a_3x^2 \\ \delta f3(new) &= a_1 + a_2x + a_3x^2 + a_4x^3 \end{aligned}$$

The transformation from the previous parameterization to the new is the following.

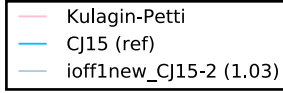
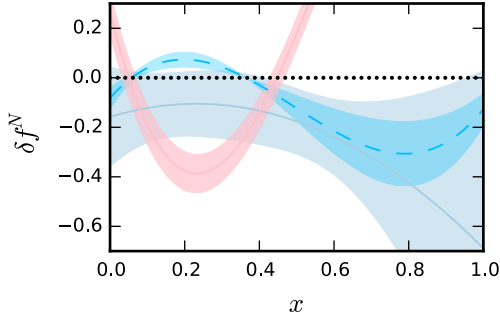
$$\begin{aligned} a_1 &= -Nx_0x_1x_2 \\ a_2 &= N(x_0x_1 + x_0x_2 + x_1x_2) \\ a_3 &= -N(x_0 + x_1 + x_2) \\ a_4 &= N \end{aligned}$$

Cross-check: The parameters from the previous ioff1-CJ15-3D was translated into the new format and calculated the .out file from the updated CJ-code and then compared it with the previous ioff1-CJ15-3.out file. Both files are similar and consistent.

6) CJ15 ioff1-2D with Vs without Simona + BoNuS data, compared with CJ15+JLab HallA,C data

$$\delta f_2(new) = a_1 + a_2x + a_3x^2$$

With JLab Data



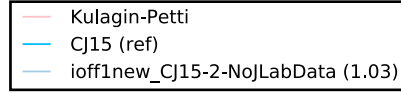
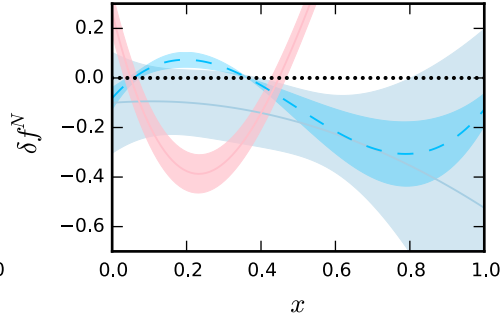
$$a_1 = -0.15805 \pm 0.12263$$

$$a_2 = 0.45820 \pm 0.74883$$

$$a_3 = -0.98925 \pm 1.0430$$

$$\begin{pmatrix} 1.000 & -0.729 & 0.552 \\ -0.729 & 1.000 & -0.947 \\ 0.552 & -0.947 & 1.000 \end{pmatrix}$$

Without JLab data



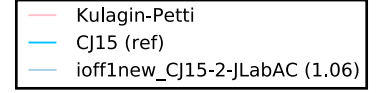
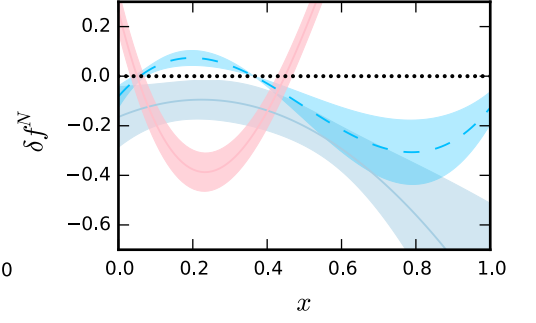
$$a_1 = -0.10033 \pm 0.12375$$

$$a_2 = 0.11336 \pm 0.74907$$

$$a_3 = -0.53736 \pm 1.0420$$

$$\begin{pmatrix} 1.000 & -0.723 & 0.536 \\ -0.723 & 1.000 & -0.944 \\ 0.536 & -0.944 & 1.000 \end{pmatrix}$$

CJ15+ JLab Halls A,C data



$$a_1 = -0.16453 \pm 0.12206$$

$$a_2 = 0.62467 \pm 0.73847$$

$$a_3 = -1.3983 \pm 1.0232$$

$$\begin{pmatrix} 1.000 & -0.734 & 0.556 \\ -0.734 & 1.000 & -0.948 \\ 0.556 & -0.948 & 1.000 \end{pmatrix}$$

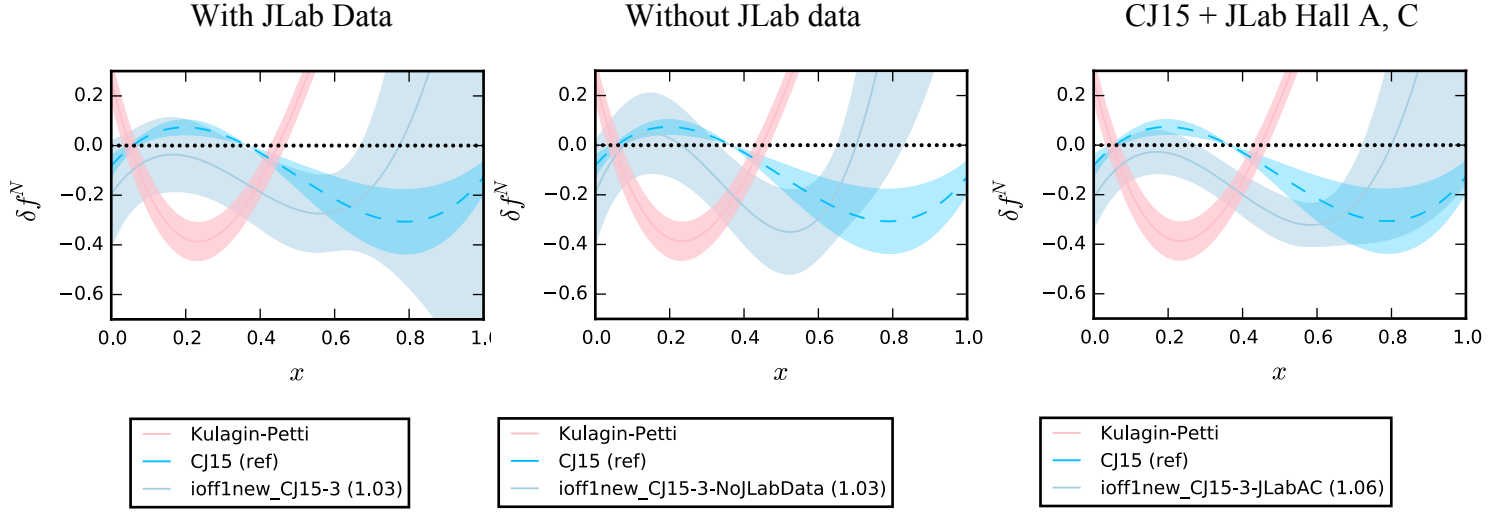
Total normalized chi2:

4700.2

4188.9

5465.5

7) CJ15 ioff1-3D with Vs without Simona + BoNuS data, compared with CJ15+JLab HallA,C data
 $\delta f3(new) = a_1 + a_2x + a_3x^2 + a_4x^3$



$$\begin{aligned} a_1 &= -0.19426 \pm 0.13027 \\ a_2 &= 2.1181 \pm 1.4585 \\ a_3 &= -8.2882 \pm 5.2752 \\ a_4 &= 7.5910 \pm 5.2540 \end{aligned}$$

$$\begin{aligned} a_1 &= -0.19194 \pm 0.12816 \\ a_2 &= 3.5517 \pm 1.5197 \\ a_3 &= -15.259 \pm 5.7363 \\ a_4 &= 15.087 \pm 5.7589 \end{aligned}$$

$$\begin{aligned} a_1 &= -0.21216 \pm 0.13039 \\ a_2 &= 2.4343 \pm 1.4408 \\ a_3 &= -9.3110 \pm 5.1906 \\ a_4 &= 8.2523 \pm 5.1806 \end{aligned}$$

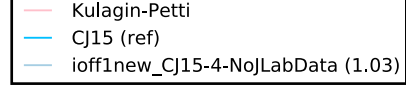
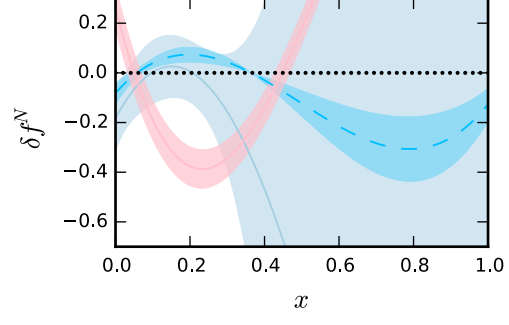
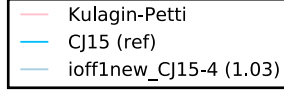
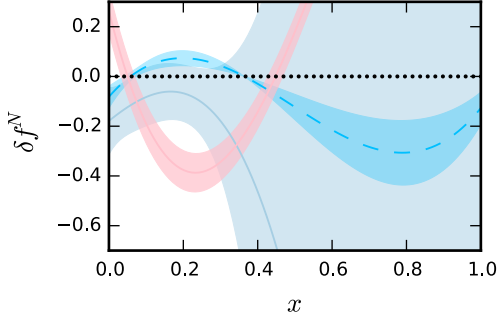
$$\begin{pmatrix} 1.000 & -0.645 & 0.439 & -0.345 \\ -0.645 & 1.000 & -0.938 & 0.859 \\ 0.439 & -0.938 & 1.000 & -0.980 \\ -0.345 & 0.859 & -0.980 & 1.000 \end{pmatrix} \begin{pmatrix} 1.000 & -0.578 & 0.363 & -0.276 \\ -0.578 & 1.000 & -0.941 & 0.871 \\ 0.363 & -0.941 & 1.000 & -0.983 \\ -0.276 & 0.871 & -0.983 & 1.000 \end{pmatrix} \begin{pmatrix} 1.000 & -0.651 & 0.444 & -0.348 \\ -0.651 & 1.000 & -0.937 & 0.858 \\ 0.444 & -0.937 & 1.000 & -0.980 \\ -0.348 & 0.858 & -0.980 & 1.000 \end{pmatrix}$$

Total normalized chi2:
4698.5

4182.7

5463.0

8) CJ15 ioff1-4D with Vs without JLab data (Simona + BoNuS)



$$a_1 = -0.17884 \pm 0.13451$$

$$a_2 = 1.2114 \pm 2.7510$$

$$a_3 = -1.7428 \pm 17.833$$

$$a_4 = -7.6538 \pm 39.980$$

$$a_5 = 11.072 \pm 28.691$$

$$a_1 = -0.18204 \pm 0.13186$$

$$a_2 = 2.8140 \pm 2.9174$$

$$a_3 = -9.9259 \pm 19.031$$

$$a_4 = 2.7507 \pm 42.270$$

$$a_5 = 8.8970 \pm 30.071$$

$$\begin{pmatrix} 1.000 & -0.535 & 0.352 & -0.278 & 0.235 \\ -0.535 & 1.000 & -0.956 & 0.899 & -0.847 \\ 0.352 & -0.956 & 1.000 & -0.985 & 0.955 \\ -0.278 & 0.899 & -0.985 & 1.000 & -0.991 \\ 0.235 & -0.847 & 0.955 & -0.991 & 1.000 \end{pmatrix}$$

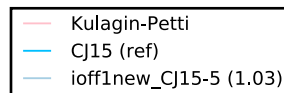
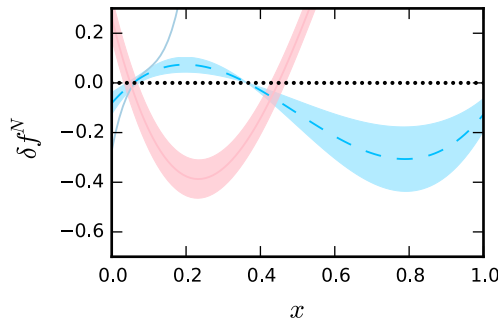
$$\begin{pmatrix} 1.000 & -0.481 & 0.313 & -0.250 & 0.214 \\ -0.481 & 1.000 & -0.961 & 0.906 & -0.853 \\ 0.313 & -0.961 & 1.000 & -0.984 & 0.953 \\ -0.250 & 0.906 & -0.984 & 1.000 & -0.990 \\ 0.214 & -0.853 & 0.953 & -0.990 & 1.000 \end{pmatrix}$$

Total normalized chi2:

4698.3

4182.7

9) CJ15 ioff1-5D



$$a_1 = -0.27333 \pm 0.14049$$

$$a_2 = 7.8061 \pm 4.7785$$

$$a_3 = -69.657 \pm 44.018$$

$$a_4 = 247.22 \pm 155.37$$

$$a_5 = -386.07 \pm 234.61$$

$$a_6 = 217.21 \pm 126.93$$

Total normalized chi2 = 4695.4