Plotting the error bands for Off-Shell function was implemented successfully and some results are summarized as follows.

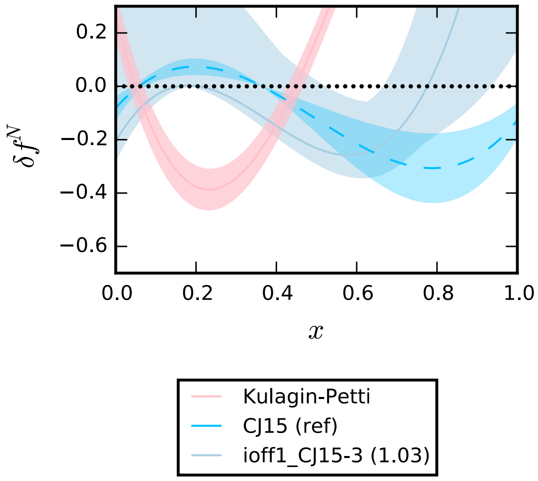
As the preliminary step, three types of fits were considered as follows.

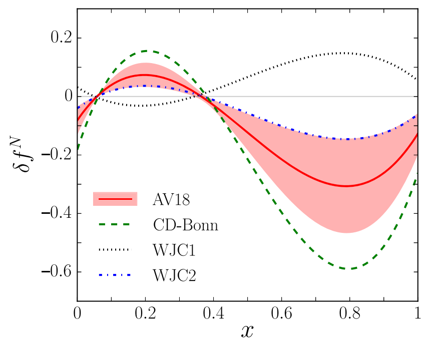
1) CJ15 (In the original CJ15 work, the parameter x_1 was fixed by considering the constraint by quark sum rule)

$$\delta f = N(x - x_0)(x - x_1)(1 + x_0 - x)$$

2) CJ15 with ioff1 for 3-degree polynomial

$$\delta f3 = N(x - x_0)(x - x_1)(x - x_2)$$





Observations:

1) CJ15 Off-Shell function's uncertainty band is reproduced
2) The uncertainty bands for ioff1 is not symmetrical because the parameters x_0 and x_1 are strongly correlated negatively. For example (correlation matrices for x_0 and x_1):

$$\text{ioff1} \quad \begin{pmatrix} 1.000 & -0.976 \\ -0.976 & 1.000 \end{pmatrix}$$

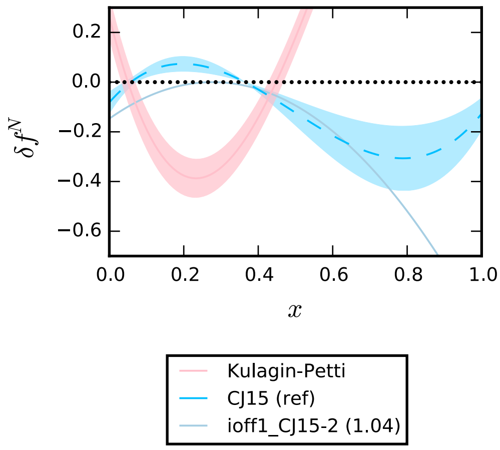
The parameters for ioff1 are the following.

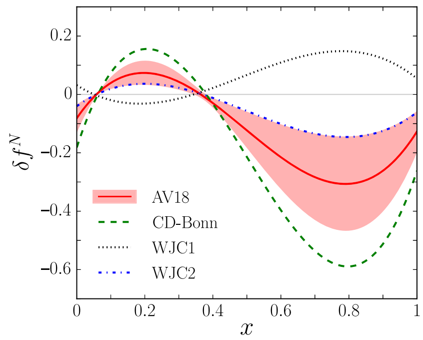
$$N = 8.2851 \pm 5.2739$$
$$x_0 = 0.20480 \pm 0.39590$$
$$x_1 = 0.15685 \pm 0.37412$$
$$x_2 = 0.77609 \pm 0.11280$$

Correlation matrix for N, x_0, x_1 and x_2 :

$$\begin{pmatrix} 1.000 & 0.460 & -0.503 & -0.816 \\ 0.460 & 1.000 & -0.976 & -0.235 \\ -0.503 & -0.976 & 1.000 & 0.307 \\ -0.816 & -0.235 & 0.307 & 1.000 \end{pmatrix}$$

3) CJ15 with ioff1 2-degree polynomial $\delta f = N(x - x_0)(x - x_1)$





Observations:

Correlation matrices for N, x_0 and x_1 in ioff1 2-degree polynomial is the following.

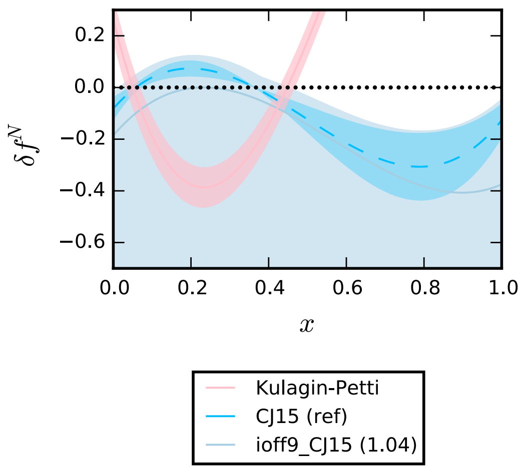
$$\text{ioff1} \quad \begin{pmatrix} 1.000 & 0.602 & -0.608 \\ 0.602 & 1.000 & -0.999 \\ -0.608 & -0.999 & 1.000 \end{pmatrix}$$

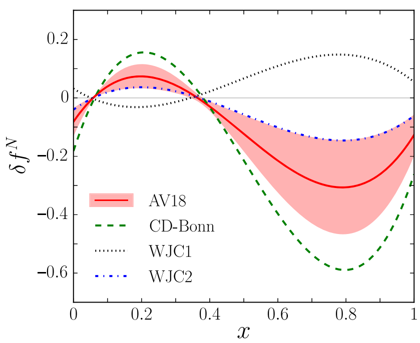
The parameters for ioff1 2D are the following.

$$N = -1.9052 \pm 1.0349$$
$$x_0 = 0.27550 \pm 9.0427$$
$$x_1 = 0.28029 \pm 9.0625$$

4) CJ15 with ioff9 (In this case, the parameter x_1 was kept free and purely determined by the fit) with same Off-Shell function as CJ15

$$\delta f = N(x - x_0)(x - x_1)(1 + x_0 - x)$$





Observations:

Correlation matrix for N, x_0 and x_1 in ioff9 is the following.

$$\text{ioff9} \quad \begin{pmatrix} 1.000 & 0.983 & -0.984 \\ 0.983 & 1.000 & -0.997 \\ -0.984 & -0.997 & 1.000 \end{pmatrix}$$

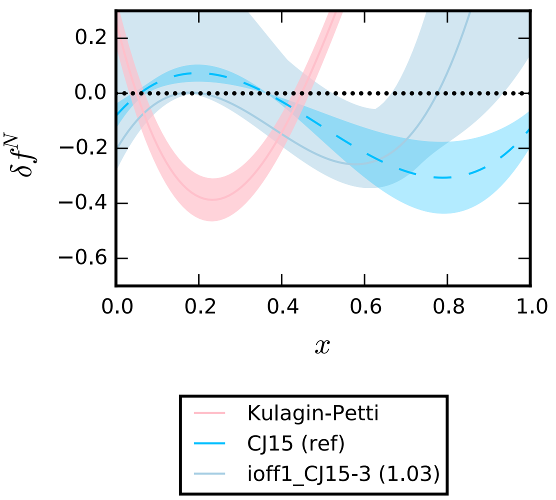
The parameters for ioff9 are the following.

$$N = -2.7758 \pm 5.3158$$
$$x_0 = 0.22976 \pm 1.3858$$
$$x_1 = 0.23625 \pm 1.3608$$

5) CJ15 ioff1 with Vs without JLab data (Simona + BoNuS)

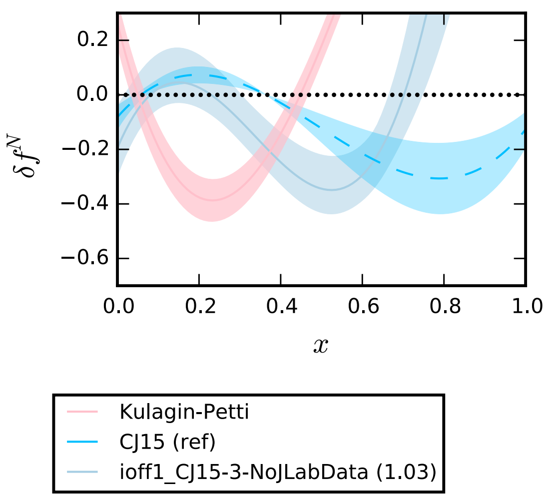
$$\delta f3 = N(x - x_0)(x - x_1)(x - x_2)$$

With JLab Data



$$N = 8.2851 \pm 5.2739$$
$$x_0 = 0.20480 \pm 0.39590$$
$$x_1 = 0.15685 \pm 0.37412$$
$$x_2 = 0.77609 \pm 0.11280$$

Without JLab data



$$N = 15.027 \pm 5.773$$
$$x_0 = 0.080580 \pm 0.071163$$
$$x_1 = 0.229570 \pm 0.086896$$
$$x_2 = 0.701960 \pm 0.039769$$

Correlation matrix for N, x_0, x_1 and x_2 :

$$\begin{array}{c} \begin{array}{cccc} N & x_0 & x_1 & x_2 \end{array} \\ \begin{pmatrix} 1.000 & 0.460 & -0.503 & -0.816 \\ 0.460 & 1.000 & -0.976 & -0.235 \\ -0.503 & -0.976 & 1.000 & 0.307 \\ -0.816 & -0.235 & 0.307 & 1.000 \end{pmatrix} \end{array} \quad \begin{array}{c} \begin{array}{cccc} N & x_0 & x_1 & x_2 \end{array} \\ \begin{pmatrix} 1.000 & -0.391 & 0.452 & -0.606 \\ -0.391 & 1.000 & -0.753 & 0.225 \\ 0.452 & -0.753 & 1.000 & -0.178 \\ -0.606 & 0.225 & -0.178 & 1.000 \end{pmatrix} \end{array}$$

The next step: Off-Shell parameterization is changed to,

$$\delta f2(new) = a_1 + a_2 x + a_3 x^2$$
$$\delta f3(new) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

The transformation from the previous parameterization to the new is the following.

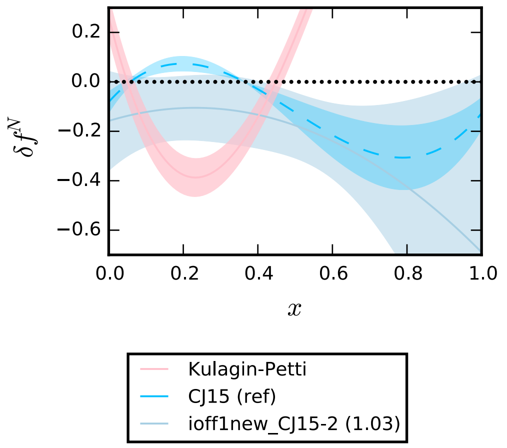
$$a_1 = -N x_0 x_1 x_2$$
$$a_2 = N(x_0 x_1 + x_0 x_2 + x_1 x_2)$$
$$a_3 = -N(x_0 + x_1 + x_2)$$
$$a_4 = N$$

Cross-check: The parameters from the previous ioff1-CJ15-3D was translated into the new format and calculated the .out file from the updated CJ-code and then compared it with the previous ioff1-CJ15-3.out file. Both files are similar and consistent.

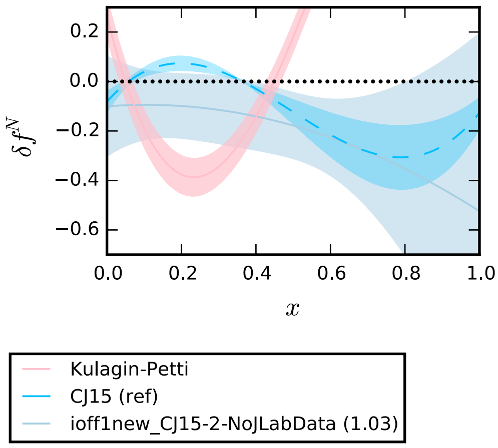
6) CJ15 ioff1-2D with Vs without JLab data (Simona + BoNuS)

$$\delta f2(new) = a_1 + a_2 x + a_3 x^2$$

With JLab Data



Without JLab data



$$a_1 = -0.15805 \pm 0.12263$$
$$a_2 = 0.45820 \pm 0.74883$$
$$a_3 = -0.98925 \pm 1.0430$$

$$a_1 = -0.10033 \pm 0.12375$$
$$a_2 = 0.11336 \pm 0.74907$$
$$a_3 = -0.53736 \pm 1.0420$$

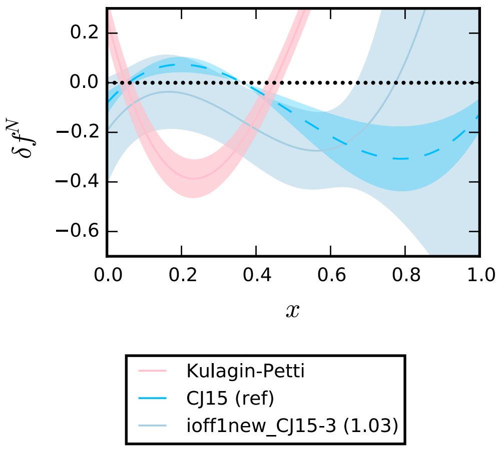
$$\begin{pmatrix} 1.000 & -0.729 & 0.552 \\ -0.729 & 1.000 & -0.947 \\ 0.552 & -0.947 & 1.000 \end{pmatrix}$$

$$\begin{pmatrix} 1.000 & -0.723 & 0.536 \\ -0.723 & 1.000 & -0.944 \\ 0.536 & -0.944 & 1.000 \end{pmatrix}$$

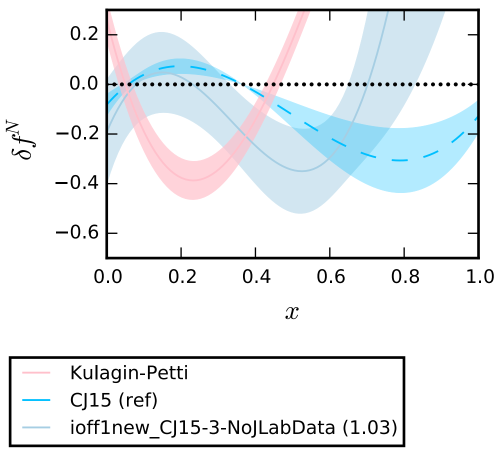
7) CJ15 ioff1-3D with Vs without JLab data (Simona + BoNuS)

$$\delta f3(new) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

With JLab Data



Without JLab data



$$a_1 = -0.19426 \pm 0.13027$$
$$a_2 = 2.1181 \pm 1.4585$$
$$a_3 = -8.2882 \pm 5.2752$$
$$a_4 = 7.5910 \pm 5.2540$$

$$a_1 = -0.19194 \pm 0.12816$$
$$a_2 = 3.5517 \pm 1.5197$$
$$a_3 = -15.259 \pm 5.7363$$
$$a_4 = 15.087 \pm 5.7589$$

$$\begin{pmatrix} 1.000 & -0.645 & 0.439 & -0.345 \\ -0.645 & 1.000 & -0.938 & 0.859 \\ 0.439 & -0.938 & 1.000 & -0.980 \\ -0.345 & 0.859 & -0.980 & 1.000 \end{pmatrix}$$

$$\begin{pmatrix} 1.000 & -0.578 & 0.363 & -0.276 \\ -0.578 & 1.000 & -0.941 & 0.871 \\ 0.363 & -0.941 & 1.000 & -0.983 \\ -0.276 & 0.871 & -0.983 & 1.000 \end{pmatrix}$$