Baryon masses and Form factors in ξ expansion

1 Introduction

2 Lagrangian

3 Self Energy

$$\begin{aligned}\delta\Sigma &= -\left(\frac{g_A}{F_\pi}\right)^2 \frac{1}{(4\pi)^2} \sum_n G^{ia} \wp_n G^{ia} \left(-\frac{\lambda_\varepsilon}{3}(p^0-\delta m_n)(3M_a^2 - 2(p^0-\delta m_n)^2) + \right. \\ & +\frac{1}{3}(3M_a^2 - 2(p^0-\delta m_n)^2)(p^0-\delta m_n)\log\left[\frac{M_a^2}{\mu^2}\right] + \frac{1}{9}\left(16(p^0-\delta m_n)^2 - 21M_a^2\right)(p^0-\delta m_n) + \\ & \left. +\frac{2}{3}(M_a^2 - (p^0-\delta m_n)^2)^{2/3}\left(\pi + 2\arctan\left[\frac{(p^0-\delta m_n)}{\sqrt{M_a^2-(p^0-\delta m_n)^2}}\right]\right)\right) \end{aligned} \tag{1}$$

The above equation is the same as,

$$\begin{aligned}\delta\Sigma &= -\left(\frac{g_A}{F_\pi}\right)^2 \frac{1}{(4\pi)^2} \sum_n G^{ia} \wp_n G^{ia} \left(\frac{3+2\varepsilon}{9}\right)\left(\lambda_\varepsilon Q(3M_a^2-2Q^2) - Q(3M_a^2-2Q^2)\log\left[\frac{M_a^2}{\mu^2}\right] + \right. \\ & \left. +Q(5M_a^2-4Q^2) + 2\pi(M_a^2-Q^2)^{3/2} + 4(Q^2-M_a^2)^{3/2}\arctan\left[\frac{Q}{\sqrt{Q^2-M_a^2}}\right]\right) \end{aligned} \tag{2}$$

4 Partial Contributions

N	Λ	Σ	Ξ
$N \to N + \pi$	$\Lambda \to N + K$	$\Sigma \to \Sigma + \pi$	$\Xi \to \Xi + \pi$
$N \to N + \eta$	$\Lambda \to \Lambda + \eta$	$\Sigma \to \Sigma + \eta$	$\Xi \to \Xi + \eta$
$N \to \Sigma + K$	$\Lambda \to \Sigma + \pi$	$\Sigma \to N + K$	$\Xi \to \Sigma + K$
$N \to \Lambda + K$	$\Lambda \to \Xi + K$	$\Sigma \to \Lambda + \pi$	$\Xi \to \Lambda + K$
$N \to \Delta + \pi$	$\Lambda \to \Sigma^* + \pi$	$\Sigma \to \Xi + K$	$\Xi \to \Xi^* + \pi$
$N \to \Sigma^* + K$	$\Lambda \to \Xi^* + K$	$\Sigma \to \Delta + K$	$\Xi \to \Xi^* + \eta$
		$\Sigma \to \Sigma^* + \pi$	$\Xi \to \Sigma^* + K$
		$\Sigma \to \Sigma^* + \eta$	$\Xi \to \Omega + \pi$
		$\Sigma \to \Xi^* + K$	
Δ	Σ^*	Ξ^*	Ω
$\Delta \to N + \pi$	$\Sigma^* \to \Sigma + \pi$	$\Xi^* \to \Xi + \pi$	$\Omega \to \Omega + \pi$
$\Delta \to \Sigma + K$	$\Sigma^* \to \Sigma + \eta$	$\Xi^* \to \Xi + \eta$	$\Omega \to \Xi + K$
$\Delta \to \Delta + \eta$	$\Sigma^* \to N + K$	$\Xi^* \to \Sigma + K$	$\Omega \to \Xi^* + K$
$\Delta \to \Delta + \pi$	$\Sigma^* \to \Lambda + \pi$	$\Xi^* \to \Lambda + K$	
$\Delta \to \Sigma^* + K$	$\Sigma^* \to \Xi + K$	$\Xi^* \to \Xi^* + \pi$	
	$\Sigma^* \to \Delta + K$	$\Xi^* \to \Xi^* + \eta$	
	$\Sigma^* \to \Sigma^* + \pi$	$\Xi^* \to \Sigma^* + K$	
	$\Sigma^* \to \Xi^* + K$	$\Xi^* \to \Omega + K$	

5 Baryon masses

6 Fits

6.1 Physical

χ^2=0.42	M_π	M_K	M_N	M_Λ	M_Σ	M_Ξ	M_Δ	M_{Σ^*}	M_{Ξ^*}	M_Ω
Physical Mass (MeV)	139	497	938±5	1116±5	1189±5	1315±5	1228±5	1383±5	1532±5	1672±5
Fitted Mass (MeV)			936.42	1118.38	1189.79	1313.42	1226.31	1384.99	1533.09	1670.61

M_0	C_{HF}	μ_1	μ_{20}	μ_2	μ_3
866.81±0.59	290.70±0.78	$(-1.478 \pm 0.002) \times 10^{-3}$	$(1.783 \pm 0.006) \times 10^{-3}$	$(4.44 \pm 0.18) \times 10^{-5}$	$(1.137 \pm 0.003) \times 10^{-3}$

There are couple of constraints I have made in these fits. When the χ^2 is minimized, I required that $C_{HF} > 0$ and $300 < M_0 < 700$ MeV.

6.2 Lattice QCD

β	a	$a\mu_l$	m_l(MeV)	m_s(MeV)	$R(m_l, m_s, a)$	M_π	M_K
1.90	0.0936(13)	0.003	12.796	92.4	0.979	260.7	523.751
		0.004	17.079	92.4	0.98	297.5	528.076
		0.005	21.327	92.4	0.976	332.3	537.375
1.95	0.0823	0.0025	11.947	92.4	0.972	255.8	527.901
		0.0035	16.726	92.4	0.982	301.8	540.918
		0.0055	26.248	92.4	0.984	371.6	555.24
		0.0075	35.769	92.4	0.994	431.6	576.349
2.10	0.0646	0.0015	9.327	92.4	0.965	212.8	488.953
		0.0020	12.407	92.4	0.976	245.5	499.152
		0.0030	18.602	92.4	0.979	298.4	511.071

M_π	M_K	M_N	M_Λ	M_Σ	M_Ξ	M_Δ	M_{Σ^*}	M_{Ξ^*}	M_Ω
260.7	523.751	1102.0(18.3)	1232.9(39.4)	1310.3(43.5)	1333.1(35.6)	1490.9(83.4)	1566.9(67.8)	1613.9(53.9)	1657.5(60.9)
297.5	528.076	1092.1(23.5)	1234.3(39.4)	1292.4(43.1)	1329.4(34.9)	1456.0(86.3)	1537.2(67.4)	1586.9(54.5)	1656.2(64.8)
332.3	537.375	1140.7(13.0)	1249.6(40.2)	1338.1(43.1)	1342.2(36.9)	1492.3(83.5)	1592.0(67.5)	1613.3(60.4)	1680.8(63.7)
255.8	527.901	1070.6(14.1)	1231.4(36.4)	1306.7(39.9)	1363.2(31.2)	1517.8(74.9)	1593.8(60.8)	1637.9(50.4)	1711.1(53.5)
301.8	540.918	1145.8(11.4)	1261.0(35.6)	1318.0(38.5)	1366.2(32.0)	1515.2(75.0)	1578.7(61.0)	1612.6(50.2)	1692.4(54.4)
371.6	555.24	1204.9(3.9)	1306.3(35.1)	1358.0(39.3)	1374.8(32.2)	1562.1(74.0)	1633.2(59.8)	1660.9(48.7)	1709.3(54.0)
431.6	576.349	1276.4(10.0)	1332.8(35.8)	1390.9(37.8)	1374.6(34.5)	1601.9(73.1)	1638.2(60.0)	1663.3(48.3)	1693.1(53.6)
212.8	488.953	1031.0(12.5)	1179.8(28.7)	1252.2(30.8)	1327.2(25.0)	1407.4(59.8)	1522.2(47.0)	1597.3(38.0)	1681.6(41.8)
245.5	499.152	1072.1(21.5)	1215.7(29.4)	1277.5(32.4)	1328.2(25.8)	1448.4(63.2)	1538.0(48.6)	1581.9(39.5)	1648.4(43.7)
298.4	511.071	1103.8(20.8)	1221.6(29.1)	1278.3(32.0)	1329.0(25.3)	1448.4(60.9)	1529.4(49.7)	1588.5(40.2)	1674.0(43.4)

χ^2	M_π	M_K	M_N	M_Λ	M_Σ	M_Ξ	M_Δ	M_{Σ^*}	M_{Ξ^*}	M_Ω
0.716	260.7	523.751	1105.28	1210.04	1301.03	1345.55	1473.64	1572.23	1628.81	1643.39
0.962	297.5	528.076	1098.26	1208.32	1282.05	1342.99	1437.81	1539.30	1605.91	1637.64
0.695	332.3	537.375	1142.20	1228.03	1329.86	1354.33	1490.30	1578.80	1637.60	1666.71
1.23	255.8	527.901	1072.79	1209.12	1297.58	1373.98	1490.90	1601.01	1666.71	1688.01
0.936	301.8	540.918	1146.54	1250.10	1313.79	1372.09	1486.0	1584.25	1644.29	1666.11
0.743	371.6	555.24	1205.08	1284.51	1349.10	1387.14	1555.08	1628.57	1676.19	1697.96
0.384	431.6	576.349	1277.22	1316.89	1385.02	1384.47	1592.39	1641.10	1672.03	1685.18
0.606	212.8	488.953	1032.25	1169.67	1248.19	1332.24	1392.01	1523.65	1615.30	1666.96
1.198	245.5	499.152	1078.89	1196.63	1269.79	1337.99	1432.80	1536.17	1602.58	1632.03
0.808	298.4	511.071	1107.6	1210.42	1273.81	1334.64	1428.05	1533.53	1609.68	1656.5

checked

χ^2	M_π	M_K	M_0	C_{HF}	μ_1	μ_{20}	μ_2	μ_3
0.716	260.7	523.751	597.229	591.227	-1.426$\times10^{-3}$	-	1.665$\times10^{-4}$	1.045$\times10^{-3}$
			$\pm$4.331	$\pm$3.19	$\pm1.66\times10^{-5}$	-	$\pm2.34\times10^{-5}$	$\pm3.13\times10^{-5}$
0.962	297.5	528.076	598.94	605.56	-1.457$\times10^{-3}$	-	1.811$\times10^{-4}$	1.025$\times10^{-3}$
			$\pm$4.839	$\pm$3.494	$\pm2.062\times10^{-5}$	-	$\pm2.687\times10^{-5}$	$\pm3.541\times10^{-5}$
0.695	332.3	537.375	632.075	619.998	-1.485$\times10^{-3}$	-	1.491$\times10^{-4}$	1.097$\times10^{-3}$
			$\pm$3.575	$\pm$2.463	$\pm1.562\times10^{-5}$	-	$\pm2.745\times10^{-5}$	$\pm3.680\times10^{-5}$
1.23	255.8	527.901	610.352	588.953	-1.464$\times10^{-3}$	-	1.940$\times10^{-4}$	1.039$\times10^{-3}$
			$\pm$3.587	$\pm$2.644	$\pm1.309\times10^{-5}$	-	$\pm1.99\times10^{-5}$	$\pm2.677\times10^{-5}$
0.935	301.8	540.918	631.504	617.761	-1.440$\times10^{-3}$	-	1.905$\times10^{-4}$	9.961$\times10^{-4}$
			$\pm$3.133	$\pm$2.192	$\pm1.206\times10^{-5}$	-	$\pm2.082\times10^{-5}$	$\pm2.804\times10^{-5}$
0.743	371.6	555.24	684.838	643.244	-1.491$\times10^{-3}$	-	1.949$\times10^{-4}$	1.026$\times10^{-3}$
			$\pm$1.266	$\pm$0.822	$\pm5.61\times10^{-6}$	-	$\pm1.725\times10^{-5}$	$\pm2.448\times10^{-5}$
0.384	431.6	576.349	520.517	777.464	-1.391$\times10^{-3}$	-1.066$\times10^{-3}$	1.719$\times10^{-4}$	9.130$\times10^{-4}$
			$\pm$2.86	$\pm$1.81	$\pm1.512\times10^{-5}$	$\pm3.388\times10^{-5}$	$\pm2.775\times10^{-5}$	$\pm3.788\times10^{-5}$
0.606	212.8	488.953	535.831	548.368	-1.419$\times10^{-3}$	-	1.544$\times10^{-4}$	1.032$\times10^{-3}$
			$\pm$3.009	$\pm$2.39	$\pm1.216\times10^{-5}$	-	$\pm1.725\times10^{-5}$	$\pm2.319\times10^{-5}$
1.198	245.5	499.152	554.677	566.036	-1.422$\times10^{-3}$	-	1.745$\times10^{-4}$	1.026$\times10^{-3}$
			$\pm$3.820	$\pm$2.999	$\pm1.623\times10^{-5}$	-	$\pm1.892\times10^{-5}$	$\pm2.527\times10^{-5}$
0.808	298.4	511.071	576.307	589.076	-1.445$\times10^{-3}$	-	1.621$\times10^{-4}$	1.022$\times10^{-3}$
			$\pm$3.761	$\pm$2.817	$\pm1.751\times10^{-5}$	-	$\pm2.065\times10^{-5}$	$\pm2.752\times10^{-5}$

χ^2	M_π	M_K	M_0	C_{HF}	μ_1	μ_{20}	μ_2	μ_3
0.599	212.8	488.953	534.543	547.106	-1.420$\times10^{-3}$	-	1.541$\times10^{-4}$	1.033$\times10^{-3}$
			$\pm$3.009	$\pm$2.401	$\pm1.223\times10^{-5}$	-	$\pm1.735\times10^{-5}$	$\pm2.333\times10^{-5}$
1.198	245.5	499.152	554.678	566.037	-1.422$\times10^{-3}$	-	1.745$\times10^{-4}$	1.026$\times10^{-3}$
			$\pm$3.820	$\pm$2.999	$\pm1.623\times10^{-5}$	-	$\pm1.892\times10^{-5}$	$\pm2.527\times10^{-5}$
1.22	255.8	527.901	590.135	572.568	-1.474$\times10^{-3}$	-	1.919$\times10^{-4}$	1.058$\times10^{-3}$
			$\pm$3.587	$\pm$2.697	$\pm1.418\times10^{-5}$	-	$\pm2.162\times10^{-5}$	$\pm2.902\times10^{-5}$
0.716	260.7	597.229	590.882	586.101	-1.427$\times10^{-3}$	-	1.653$\times10^{-4}$	1.051$\times10^{-3}$
			$\pm$4.330	$\pm$3.22	$\pm1.7\times10^{-5}$	-	$\pm2.39\times10^{-5}$	$\pm3.78\times10^{-5}$
0.962	297.5	528.076	598.94	605.56	-1.457$\times10^{-3}$	-	1.811$\times10^{-4}$	1.025$\times10^{-3}$
			$\pm$4.839	$\pm$3.494	$\pm2.062\times10^{-5}$	-	$\pm2.687\times10^{-5}$	$\pm3.541\times10^{-5}$

M_π	M_K	GMO	GR	ES1	ES2	ES3
260.7	523.751	138.8,(29.5)	24.2,(12.1)	76.0,(98.6)	47.0,(56.6)	43.6,(14.6)
297.5	528.076	152.3,(24.5)	12.7,(5.7)	81.2,(101.5)	49.7,(66.7)	69.3,(31.7)
332.3	537.375	121.1,(20.9)	17.2,(34.3)	99.7,(88.5)	21.3,(58.8)	67.5,(29.1)
255.8	527.901	133.3,(31.4)	-12.4,(-10.7)	76.0,(110.1)	44.1,(65.7)	73.2,(21.3)
301.8	540.918	77.0,(26.8)	-14.3,(1.7)	63.5,(98.3)	33.9,(60.0)	79.8,(21.8)
371.6	555.24	117.5,(18.2)	10.9,(9.6)	71.1,(73.5)	27.7,(47.6)	48.4,(21.8)
431.6	576.349	87.3,(12.3)	41.4,(31.5)	36.3,(48.7)	25.1,(30.9)	29.8,(13.1)
212.8	488.953	75.2,(28.2)	0.1,(7.6)	114.8,(131.6)	75.1,(91.6)	84.3,(51.7)
245.5	499.152	124.0,(25.9)	-6.8,(-1.8)	89.6,(103.4)	43.9,(66.4)	66.5,(29.4)
298.4	511.071	77.5,(20.6)	8.4,(15.3)	81.0,(105.5)	59.1,(76.2)	85.5,(46.8)

M_π	M_K	GMO	GR	ES1	ES2	ES3
212.8	488.953	75.2,(28.21)	0.1,(7.60)	114.8,(131.63)	75.1,(91.65)	84.3,(51.66)
245.5	499.152	124.0,(25.91)	-6.8,(-1.78)	89.6,(103.37)	43.9,(66.40)	66.5,(29.44)
255.8	527.901	133.3,(31.38)	-12.4,(-10.69)	76.0,(110.10)	44.1,(65.70)	73.2,(21.30)
260.7	523.751	138.8,(29.47)	24.2,(12.06)	76,(98.58)	47,(56.58)	43.6,(14.58)
297.5	528.076	152.3,(24.49)	12.7,(5.66)	81.2,(101.49)	49.7,(66.61)	69.3,(31.73)
298.4	511.071	77.5,(20.59)	8.4,(15.32)	81.0,(105.47)	59.1,(76.15)	85.5,(46.82)

7 Sigma Terms

1.

$$\begin{aligned} M_\pi^2 &= 2m_uB \\ M_K^2 &= (m_u+m_s)B \\ m_u &= \frac{M_\pi^2}{2B} \end{aligned} \tag{3}$$

$2M_K^2 - M^2$

2.

$$
\begin{aligned}
\sigma_{M_B,m_u} &= m_u \frac{\partial}{\partial m_u} M_B \\
&= \left(\frac{M_\pi^2}{2B}\right)\left(\frac{\partial M_B}{\partial M_\pi^2}\frac{\partial M_\pi^2}{\partial m_u} + \frac{\partial M_B}{\partial M_K^2}\frac{\partial M_K^2}{\partial m_u}\right) \\
&= \left(\frac{M_\pi^2}{2B}\right)\left(\frac{\partial M_B}{\partial M_\pi^2}2B + \frac{\partial M_B}{\partial M_K^2}B\right) = M_\pi^2\left(\frac{\partial M_B}{\partial M_\pi^2} + \frac{1}{2}\frac{\partial M_B}{\partial M_K^2}\right)
\end{aligned}
\tag{4}
$$

3.

$$
\begin{aligned}
\sigma_{M_B,m_s} &= m_s \frac{\partial}{\partial m_s} M_B \\
&= \left(\frac{2M_K^2 - M_\pi^2}{2B}\right)\left(\frac{\partial M_B}{\partial M_\pi^2}\frac{\partial M_\pi^2}{\partial m_s} + \frac{\partial M_B}{\partial M_K^2}\frac{\partial M_K^2}{\partial m_s}\right) \\
&= \left(\frac{2M_K^2 - M_\pi^2}{2B}\right)\left(\frac{\partial M_B}{\partial M_\pi^2}0 + \frac{\partial M_B}{\partial M_K^2}B\right) = \left(\frac{2M_K^2 - M_\pi^2}{2}\right)\frac{\partial M_B}{\partial M_K^2}
\end{aligned}
\tag{5}
$$

So to calculate sigma terms, I have picked all cases where $M_\pi < 300$MeV, including the physical case. Then I obtained the low energy constants for all masses. Using that, I have obtained $\sigma_{M_B,\hat{m}}$ values at each M_π.

7.1 Data Set

I used the following data set to obtain the low energy constants which depend on M_π and M_K. The low energy constants

M_π	M_K	M_N	M_Λ	M_Σ	M_Ξ	M_Δ	M_{Σ^*}	M_{Ξ^*}	M_Ω
139	497	938(25)	1116(25)	1189(25)	1315(25)	1228(25)	1383(25)	1532(25)	1672(25)
212.8	488.953	1031.0(12.5)	1179.8(28.7)	1252.2(30.8)	1327.2(25.0)	1407.4(59.8)	1522.2(47.0)	1597.3(38.0)	1681.6(41.8)
245.5	499.152	1072.1(21.5)	1215.7(29.4)	1277.5(32.4)	1328.2(25.8)	1448.4(63.2)	1538.0(48.6)	1581.9(39.5)	1648.4(43.7)
260.7	523.751	1102.0(18.3)	1232.9(39.4)	1310.3(43.5)	1333.1(35.6)	1490.9(83.4)	1566.9(67.8)	1613.9(53.9)	1657.5(60.9)

are,

χ^2	M_π	M_K	M_0	C_{HF}	μ_1	μ_0	μ_{20}	μ_2	μ_3
9.50	139	497	134.449	254.749	-1.509×10^{-3}	4.396×10^{-3}	1.717×10^{-3}	3.302×10^{-5}	1.183×10^{-3}

Using these low energy constants we can calculate the sigma terms.
Also, we can study a set of equations (GMO & ES)using $m_0 = \frac{1}{3}(2\hat{m} + m_s)$ and $m_8 = m_s - \hat{m}$ as follows.

$$
\begin{aligned}
&\frac{\partial}{\partial m_0} M_B : \\
&\qquad N = \Lambda = \Sigma = \Xi \\
&\qquad \Delta = \Sigma^* = \Xi^* = \Omega
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial m_8} M_B : \\
&\qquad N + \Sigma + \Xi = 0 \\
&\qquad \Lambda + \Sigma = 0 \\
&\qquad 2\Delta + \Omega = 0 \\
&\qquad \Delta + \Xi^* = 0 \\
&\qquad \Sigma^* = 0
\end{aligned}
\tag{7}
$$

These two equations are exact at tree level. And violations are calculated.

$$
\begin{aligned}
\sigma_{\Lambda,m} &= \frac{1}{3}\left(2\sigma_{N,m}+2\sigma_{\Xi,m}-\sigma_{\Sigma,m}\right) && (8)\\
\sigma_{\Omega,m} &= 3\sigma_{\Sigma^*,m}-2\sigma_{\Delta,m} && (9)\\
\sigma_{\Xi^*,m} &= 2\sigma_{\Sigma^*,m}-\sigma_{\Delta,m} && (10)\\
\sigma_{\Xi,m_s} &= \frac{m_s}{2m}(\sigma_{N,m}-\sigma_{\Xi,m}+\sigma_{\Sigma,m}) && (11)\\
\sigma_{\Lambda,m_s} &= \frac{m_s}{6m}(-\sigma_{N,m}-\sigma_{\Xi,m}+5\sigma_{\Sigma,m}) && (12)\\
\sigma_{\Sigma,m_s} &= \frac{m_s}{2m}(\sigma_{N,m}+\sigma_{\Xi,m}-\sigma_{\Sigma,m}) && (13)\\
\sigma_{N,m_s} &= \frac{m_s}{2m}(-\sigma_{N,m}+\sigma_{\Xi,m}+\sigma_{\Sigma,m}) && (14)\\
\sigma_{\Omega,m_s} &= \frac{m_s}{2m}\left(4\sigma_{\Delta,m}-3\sigma_{\Sigma^*,m}\right) && (15)\\
\sigma_{\Xi^*,m_s} &= \frac{m_s}{2m}\left(2\sigma_{\Delta,m}-\sigma_{\Sigma^*,m}\right) && (16)\\
\sigma_{\Sigma^*,m_s} &= \frac{m_s}{2m}\sigma_{\Sigma^*,m} && (17)\\
\sigma_{\Delta,m_s} &= \frac{m_s}{2m}\left(-2\sigma_{\Delta,m}+3\sigma_{\Sigma^*,m}\right) && (18)
\end{aligned}
$$

Among the above relations, first three relations + last relation violations have natural values. But the rest of all relations violations are large. When we change the M_K value less than 300 MeV, we could see these large violations get smaller. This observation may tells us that how powerful these ξ expansion is.

The next step is to check the following. We have GMO, ES, GR mass relations and the violations are calculable and they are suppressed by order of $1/N_c$.
The next step is to check those additional relations, to see whether their violations also suppressed by order $1/N_c$.

Yes! We observed,

1. GMO and ES are satisfied at any N_c at tree level and at one loop they are μ independent
2. GR has a violation proportional to μ_2 at tree level for any N_c
3. The violations to GMO,ES and GR relations goes like $1/N_c$
4. The other relations are now written in terms of N_c as the general form, and they seemed to violate like order N_c
5. Now the goal is to find new set of relations with minimal violations
6. $1/N_c^2$