$\frac{5}{1000} \cdot 1{,}00$

**Probabilities** Determinism v/s Non Determinism 09/24/2019

Sets, Conditional Probabilities.

$$P(A|B) = \frac{P(A\cap B)}{P(B)}, \quad P(B) \neq 0$$

$$P(A\cap B) = P(A|B)\,P(B) \quad \text{— (1)} \qquad (1) = (2)$$

$$P(B|A) = \frac{P(B\cap A)}{P(A)}, \quad P(B\cap A) = P(B|A)\,P(A) \quad \text{— (2)}$$

- Dependent & Independent Events.

Probability Tree Diagrams

Bayes Theorem.

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Law of Total Probability (Marginal Probabilities)

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Q - ~~P(+|U)~~ ~~P(S|U) = .99~~ ~~P(N|+) = .99~~

No. of people = 1000

Users of drug = $\frac{5}{1000} \times 1000 = 5$

$\frac{5}{1000} \times$

sensitivity $P(+|\text{user}) = 99\%$.

$P(\text{user}) = 0.5\%$

specific $P(-|\text{non-user}) = 99\%$

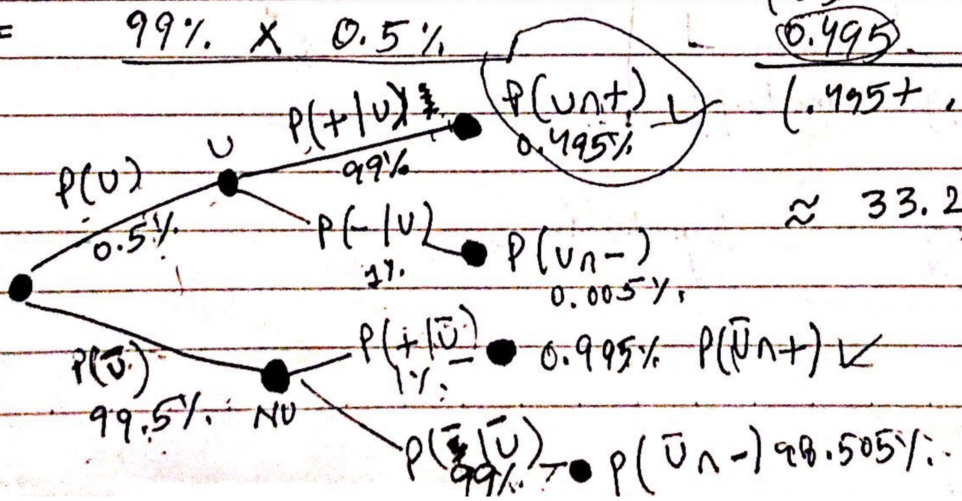
$\frac{99}{100}$ $\frac{5}{100}$

$$P(\text{user}|+) = \frac{P(+|\text{user})\,P(\text{user})}{P(+)}$$

$$= \frac{99\% \times 0.5\%}{} \qquad \frac{(0.5\% \times 99\%)}{} \qquad \frac{0.495}{(.495 + .995)} \times 100$$

$\approx 33.2\%$

$P(+) =$



# UCI M/c learning dataset

(Iris)

09/26/2019

**Bayes Classifier**

Random Variables

Mean?

Continous

Discrete

10/01/2019

→ Normal Distribution → Monte Carlo Integration

→ Pseudorandom number generators

Linear congruential generator.

$M = 9$, $c = 4$

$a = 7$, $x_0 = 3$

modulus $M$

multiplier $a$ ($2 \le a < M$)

increment $c$ ($0 \le c \le M$)

seed $x_0$

$x_{n+1} = (a x_0 + c) \bmod M$

random()

seed(a) – initializes

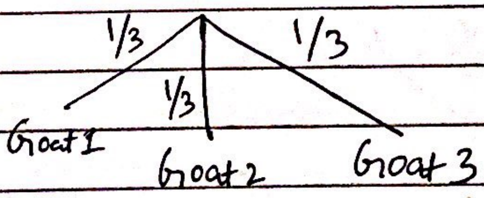
randint(a,b)

random as rand

matplotlib.pyplot as plt

rand.seed(0)

→ Monte Carlo Algorithms




→ Monty Hall Problem

| Car door | Player choice |
| --- | --- |
| random | random |
| 1, 2, 3 | 1, 2, 3 |

Simulation

Car – 1

Player – 2

× ×

1, 2, 3 → open

→ Monty Fall Problem

10/03/2019

K-armed bandit problem

$\frac{.001}{1000} \times 100 + 0.999 \times 0 - 1$

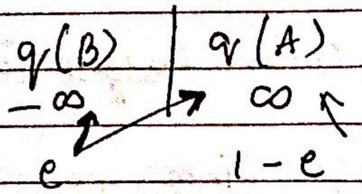
$.1 + (-1) = -.9$

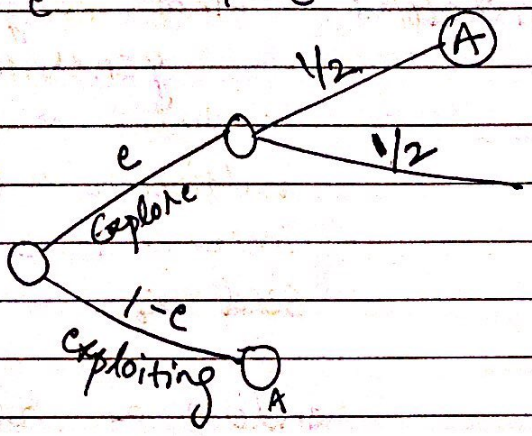
7 - 7
6 - 8

Constant decay
Constant Rate decay
Exponential decay

Incremental Update Rule

A simple Bandit Algo.




$$\boxed{\begin{array}{c} 1\times(1-e) + e\times\tfrac{1}{2} \\ (1-e) + e/2 \end{array}}$$

$(1 - 0.1) + .1/10$

$.9 + .01$

$.91$

91%

$$\left(1 - 0.01\right) + \frac{0.01}{10}$$

$.99 + .001$

$.991$ 99.1%

## Abstract Bandit Algo.

| $a$ | 1 | 2 |
|---|---|---|
| $Q_t(a)$ | 1 | 2 |

$\tau = 1$ $$P(a) = \frac{e^{Q_t(a)/\tau}}{\sum_{b=1}^{n} e^{Q_t(b)/\tau}}$$

$$P(1) = \frac{e^{Q_t(1)/\tau}}{e + e^2} = \frac{e}{e+e^2} = 0.27$$

14
22

$$P(2) = \frac{e^{Q_t(2)/1}}{e+e^2} = \frac{e^2}{e+e^2} = 0.73$$

$\tau = 1/20$

$$P(1) = \frac{e^{20}}{e^{20}+e^{40}} \simeq 0 \qquad P(2) = \frac{e^{40}}{e^{20}+e^{40}} \simeq 1$$

Upper Confidence Bound (UCB) action selection 10/08/2019

Update Rule  Stationarity vs non-Stationarity

$$Q_{n+1} = Q_n + a[R_n - Q_n]$$
$$= aR_n + (1-a)Q_n$$

Stochastic Approximation Theorem

$$= (1-a)^n Q_1 + \sum_{i=1}^{n} a(1-a)^{n-i} R_i$$

Gradient Bandit Algorithms * Gradient Descent Tutorial.

→ Optimization - Selection of best element
1st order iterative optimization algo. for finding the min. or max.

→ Gradient Vector  $z = f(x,y) = x^2 + xy + y^2$

$$\frac{df}{dx} = 2x+y, \quad \frac{df}{dy} = 2y+x$$

$$\nabla f = \left(\frac{df}{dx}, \frac{df}{dy}\right) = (2x+y, 2y+x)$$

cur_x = 3

prev_x = 3

cur_x = 3 - 0.01 × df$(4\times 3^3 - 9\times 3^2)$

Ascent/
## Gradient Descent Variants:

10/10/2019

Batch
Stochastic   http://ruder.io/optimizing-gradient-descent/
Mini Batch

→ Initialization Rule   10/15/2019

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## Probabilities

Set - Collection of objects - No repetition or ordering
set Cardinality $L$ - no. of elements in set.
Power set - $2^L$ (no. of elements)

$0 \leq P(A) \leq 1$

$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$, $P(A') = 1 - P(A)$.

- Conditional Probabilities $\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$

$P(A|B)$ Prob. of event A given that the event B has already occurred.

$$P(A \cap B) = P(A|B) \cdot P(B)$$
$$P(B \cap A) = P(B|A) \cdot P(A)$$
chain rule.

$$\boxed{\begin{aligned} P(A \cap B) &= P(A|B) \cdot P(B) \\ &= P(B|A) \cdot P(A) \end{aligned}}$$

Independent Events - If occurrence of one does not affect the prob. of occurence of the other

Dependent Events - dependent on one another

$P(\text{Type 1}) = 0.5$ $\qquad$ $P(\overset{70}{W} \mid \text{Type 1}) = 0.3$

$P(\text{Type 2}) = 0.25$ $\qquad$ $P(W \mid \text{Type 2}) = 0.4$

$P(\text{Type 3}) = 0.25$ $\qquad$ $P(W \mid \text{Type 3}) = 0.5$

$$P(W \cap \text{Type 1}) = \underbrace{0.3 \times 0.5}$$

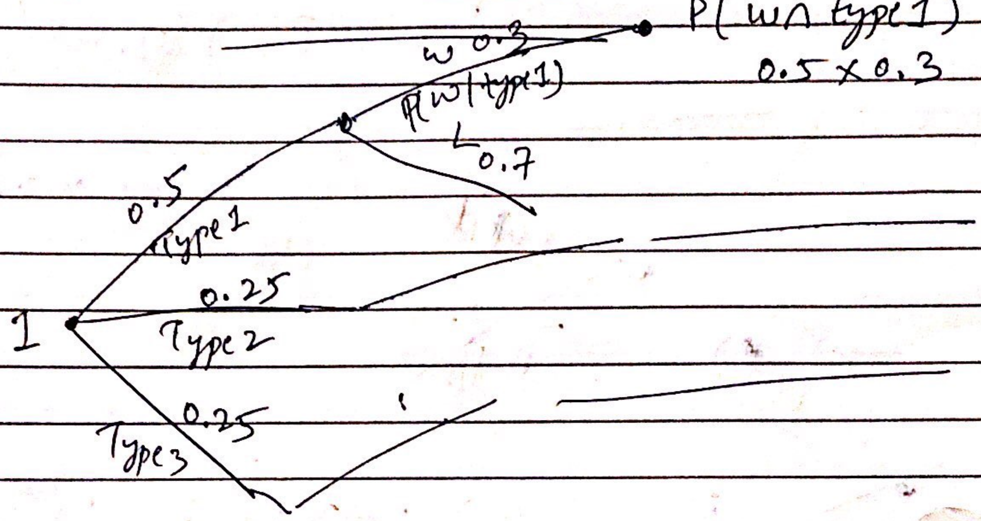
$$+$$

$$P(W \cap \text{Type 2}) = \underbrace{0.4 \times 0.25}$$

$$+$$

$$P(W \cap \text{Type 3}) = \underbrace{0.5 \times 0.25}$$

1




<u>Independent</u> $\qquad P(A \mid B) = P(A)$

$\cancel{P(B \mid A) = P(B)}$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$

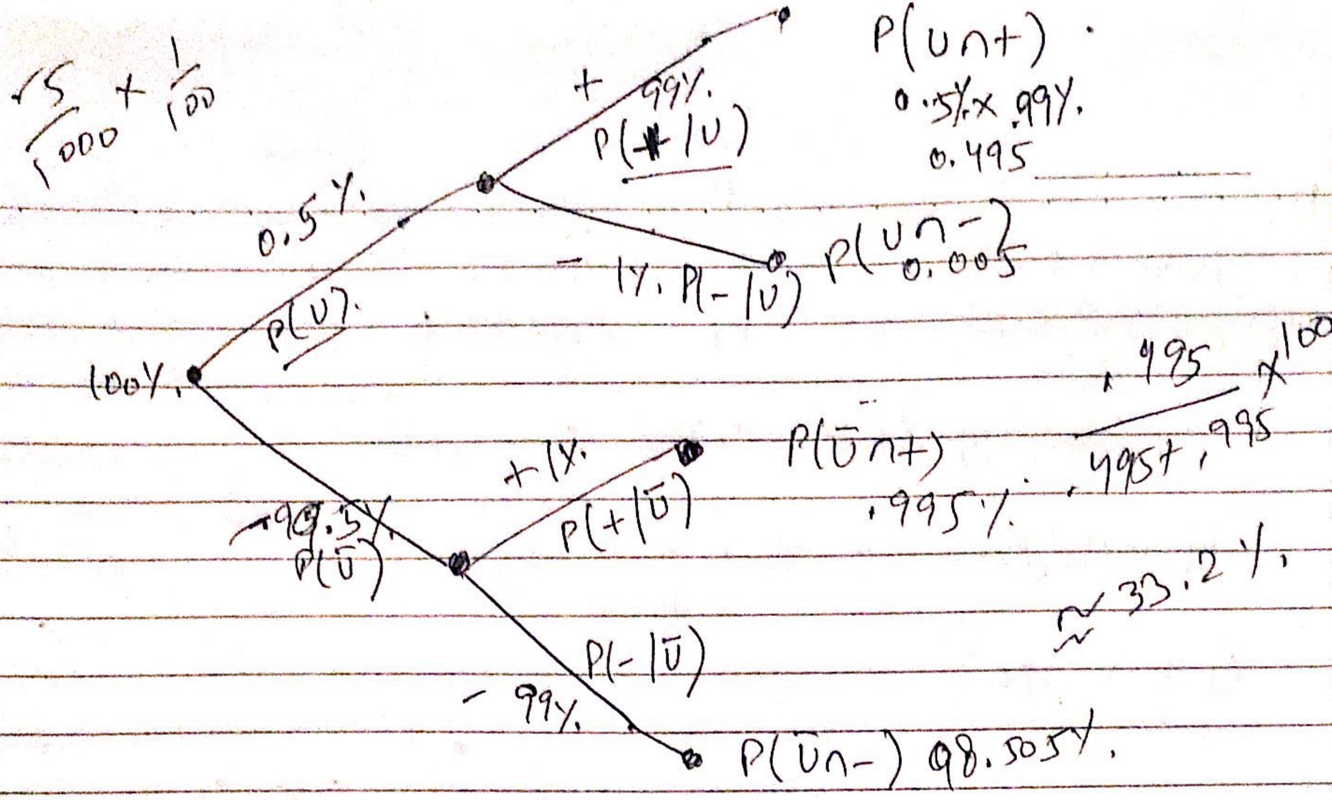
$\boxed{P(A \cap B) = P(A) \cdot P(B)}$ — independent

Bayes

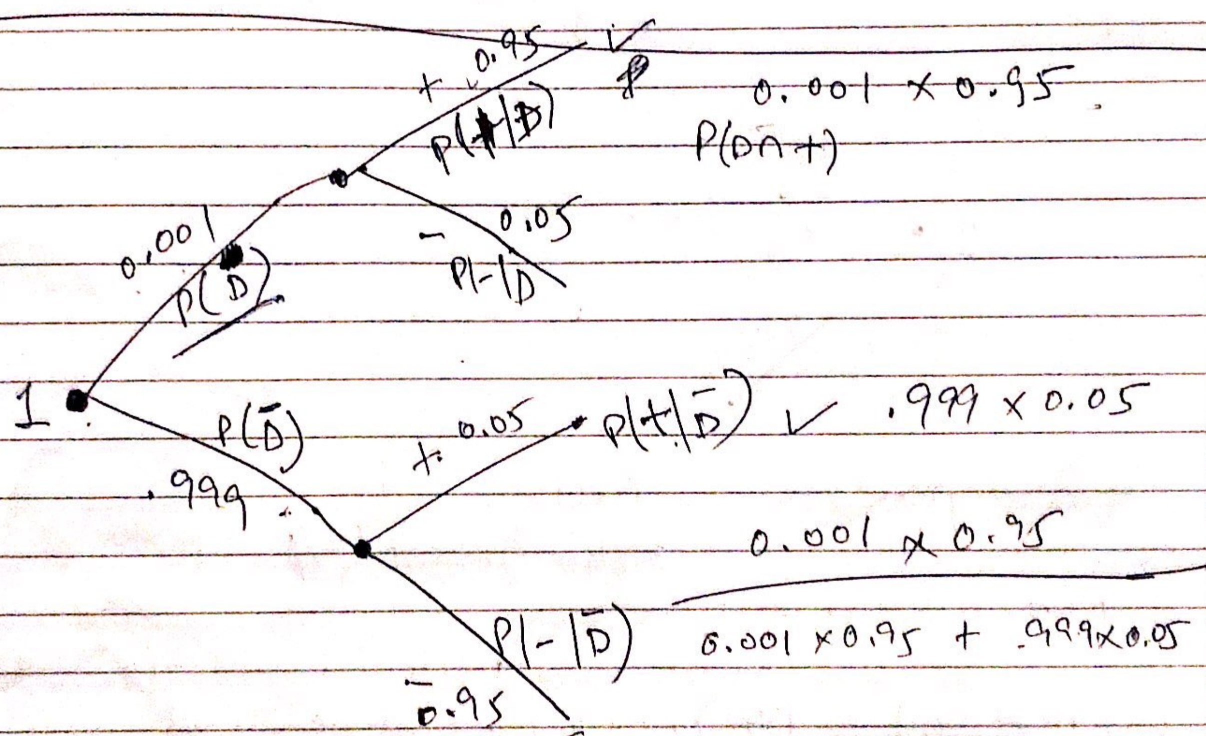
$$P(A \mid B) \cdot P(B) = P(B \mid A) \cdot P(A)$$

$$\boxed{P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}}$$

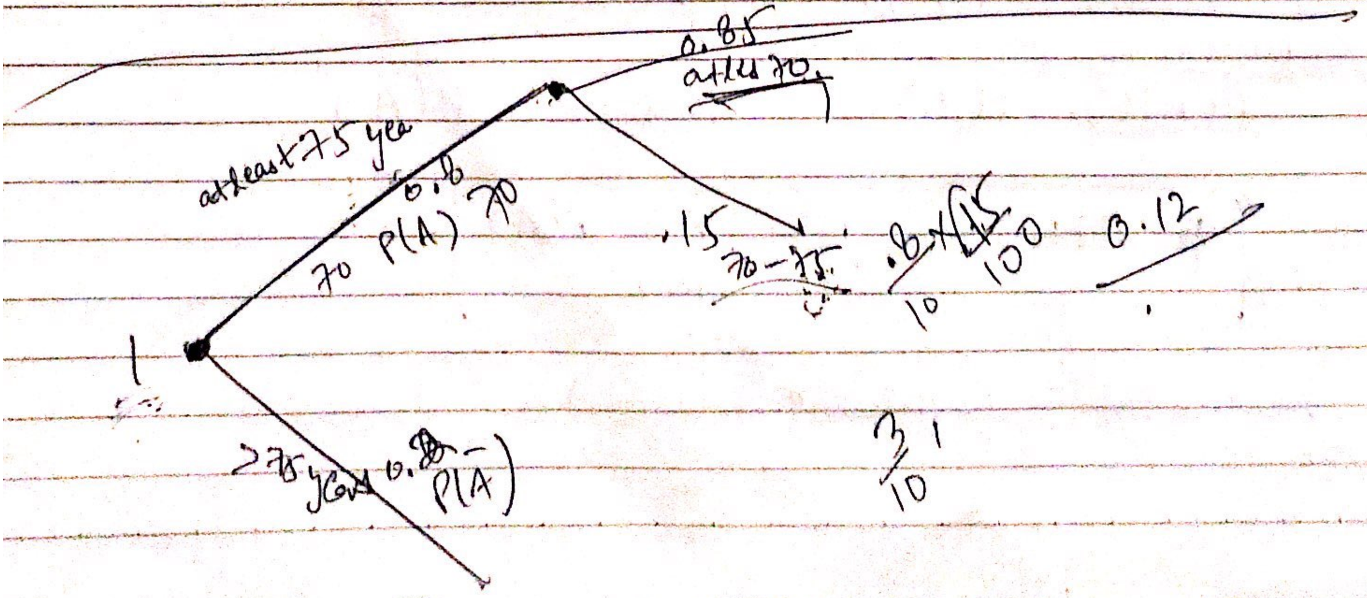
Mutually Exclusive ⇒ $P(A \cap B) = 0$



5/1000 + 1/100
P(U∩+)
0.5% × 99%
0.495
+ 99%
P(+|U)
0.5%
P(U)
- 1% P(-|U)
P(U∩-) 0.005
100%
.495 / (.495 + .995) × 100
P(Ū∩+)
.995%
≈ 33.2%
+ 1%
P(+|Ū)
99.5%
P(Ū)
P(-|Ū)
- 99%
P(Ū∩-) 98.505%

+ 0.95
P(+|D)
0.001 × 0.95
P(D∩+)
0.001
P(D)
- 0.05
P(-|D)
1
P(D̄)
.999
+ 0.05
P(+|D̄)
.999 × 0.05
0.001 × 0.95 / (0.001 × 0.95 + .999 × 0.05)
P(-|D̄)
- 0.95

0.85
at least 70
at least 75 year
0.8
70 P(A) 70
.15
70-75
.8 × 15/100
8/10
0.12
1
≥75 year 0.8
P(Ā)
3/10

P(at least 70 years old) = 0.85

P(at least 75 years old) = 0.8

$\frac{3}{10} \times \frac{3}{10}$

10 suspects.

$P(C) = 4/10$ $\frac{12}{10}$

max. of 2 rolls is 2

~~(1,1)~~, (1,2), (2,2)

(2,1) $3/16$

min. of 2 rolls is 2

(2,2), (2,3), (2,4)

(3,2), (4,2)

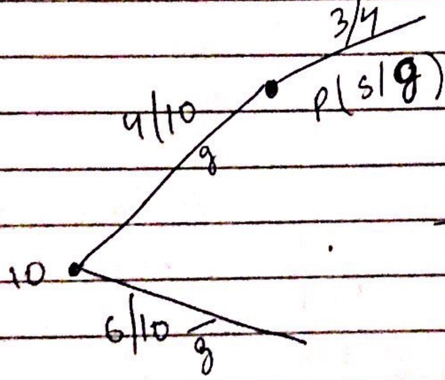
$5/16$

$P(A \cap B) = 1/16$

$\frac{1}{16} \neq \frac{3}{16} \times \frac{5}{16}$

$\frac{3}{4} + \frac{8}{6}$

$\frac{18+12}{24} = \frac{30}{24} \times 100$




g =

$\frac{3}{10} \times \frac{3}{4} + \frac{6}{10} \times \frac{3}{6}$

$\frac{9}{40} + \frac{\cancel{18}\,3}{\cancel{60}\,10} = \frac{9+12}{40}$

$\frac{105}{2}$

$\frac{21}{40} \times \frac{5}{100}{2}$

$\cancel{\frac{3}{10} + \frac{3}{4}}$

$\cancel{\frac{12+30}{40}}$

$\frac{3}{10} \times \frac{3}{4} + \frac{3}{10} \times \frac{3}{6}$

$\frac{9}{40} + \frac{9}{60}$

$P(A \cup B) = \frac{3}{4} + \frac{3}{6}$

$\frac{540+360}{240}$ $\frac{4}{10}$ $\frac{6}{10}$

$\frac{\begin{matrix}540\\360\end{matrix}}{900}$ $\frac{900}{240}$

3

3

$$\frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} + \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$$

$$\frac{24 + 120}{10 \times 9 \times 8} = \frac{144}{\cancel{10} \times 9 \times 8} \times 10\cancel{0}$$

$$= 20\%$$

1

$P(\text{Type } 1) = 0.5$  $P(w \mid \text{Type } 1) = 0.3$

$P(\text{Type } 2) = 0.25$  $P(w \mid \text{Type } 2) = 0.4$

$P(\text{Type } 3) = 0.25$  $P(w \mid \text{Type } 3) = 0.5$

$$P(w \mid \text{Type}) = \frac{P(w \cap \text{Type } 1)}{P(\text{Type } 1)}$$

$.25 \times 4 = 1.00$, $1000$

$P(w \cap \text{Type } 1) = 0.3 \times 0.5 = .15$

$P(w \cap \text{Type } 2) = 0.4 \times .25 = 0.1$

$P(w \cap \text{Type } 3) = 0.5 \times .25 = 0.125$

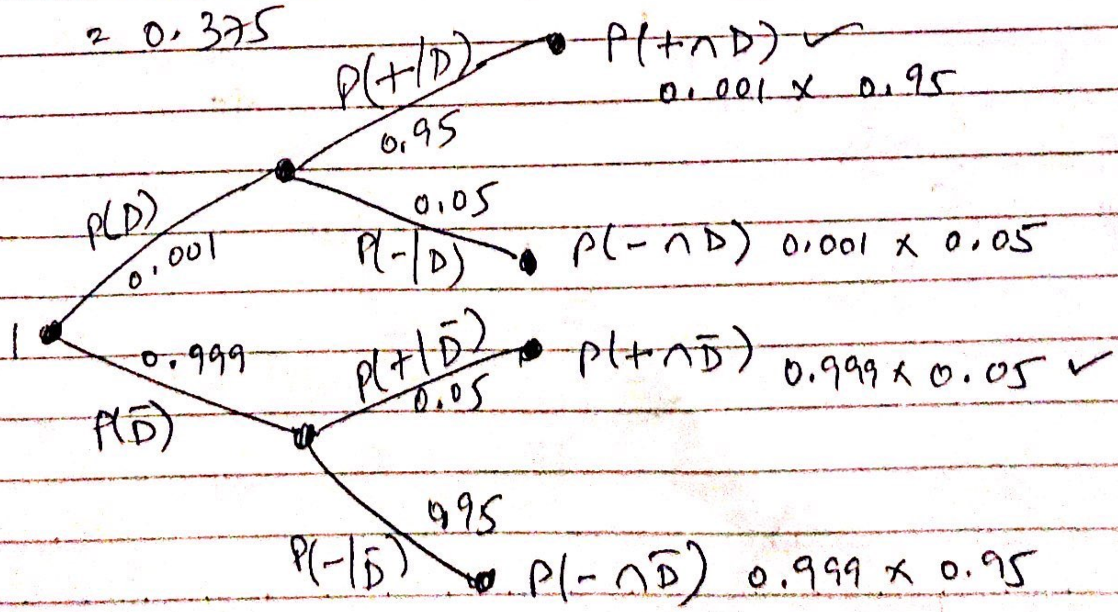
$.25 \times 5 = .125$, $1000$

$.15 + .1 + .125 = .375$

$P(w) = 0.15 + 0.1 + 0.125$

$= 0.375$

2



$0.001 \times 0.95 + 0.999 \times 0.05$ $\quad$ $0.00095 + 0.04995$

$$= \frac{0.00095}{0.0509} = 0.0187$$

③ <u>Sample Space</u> $\quad$ Equal prob. $= 1/16$

(1,1), (1,2), (1,3), (1,4)
(2,1), (2,2), (2,3), (2,4)
(3,1), (3,2), (3,3), (3,4)
(4,1), (4,2), (4,3), (4,4)

(a) $P(A \cap B) = 1/16$
$P(A) = \{1st\ roll\ results\ in\ i\} = 4/16 = 1/4$
$P(B) = \{2nd\ roll\ results\ in\ j\} = 4/16 = 1/4$
→ for independent events = $P(A)$
$P(A \cap B) = P(A|B) \cdot P(B)$
$= P(A) \cdot P(B) = 1/4 \times 1/4 = 1/16$
independent events.

(b) $P(A \cap B) = 1/16$ $\quad$ $P(A \cap B) = P(A) \cdot P(B)$
$P(A) = 4/16 = 1/4$ $\quad$ $= 1/4 \times 1/4$
$P(B) = 4/16 = 1/4$ $\quad$ independent $= 1/16$

(c) $P(A \cap B) = 1/16$ $\quad$ $P(A \cap B) = P(A) \cdot P(B)$
$P(A) = 3/16$ $\quad$ $1/16 \neq 3/16 \times 5/16$
$P(B) = 5/16$ $\quad$ Not independent.

---

<u>Assingment</u>

③ 10 suspects. $\quad \overbrace{\frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}}^{G} + \overbrace{\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}}^{NG}$

$$\frac{24 + 120}{10 \times 9 \times 8} = \frac{\cancel{144}\ \cancel{48}\ 16\ 2}{\cancel{10} \times \cancel{9} \times \cancel{8}} \times 1\cancel{0}0 = 20\%$$

Compound event - subset of sample space consisting of more than 1 outcome.

Simple event - subset with single outcome.

Random variable - Needs to be a number.
$X=1$ (Head) $X=0$ (Tail).

Discrete Random Variable - Countable no. of distinct values
Continous Random Variable - any value within a range of values

F(x) cdf - cumulative distribution funcⁿ

| W | 0 | 1 | 2 | 3 | |
|---|---|---|---|---|---|
| P(W) | 0.02 | 0.27 | 0.33 | 0.38 | pmf |
| F(W) | 0.02 | 0.29 | 0.62 | 1 | cdf |

$\mu$ (mean) $= \Sigma (x) \cdot P(x)$

$\sigma^2 = V(x)$ (variance) $= \Sigma (x-\mu)^2 \cdot P(x)$
$\sigma = \sqrt{\sigma^2}$ (standard deviation)

$$Pdf = f(x) = \frac{d}{dx} F(x)$$

$$cdf = \int_{-\infty}^{x} f(t)\,dt$$

Pseudo Random Number generators

Algo. that produce Random no.s.

Bandits  K-armed bandit problem.
Action $A_t$ from K possibilities → receive Reward $R_t$
finite sequence of time steps $t = 1, 2, 3, ---$

goal is to max. expected total reward.

$$q_*(a) = E(R_t \mid A_t = a), \quad \forall a \in \{1, 2, ---, K\}$$

$E[x_a] = -\$0.7$
$E[x_b] = \$0.5$ ← $q_*(b) = \$0.5$
$E[x_c] = \$0.4$

greedy action selection - always choose the action with best value estimate

$$A^*_t = \text{argmax}_a Q_t(a) \quad \text{exploitation}$$

Randomly choose when ties.

Random action selection -

$$A^{uni}_t = \text{uniform}(\{a_1, a_2, ---, a_K\}) \quad \text{exploration.}$$

- e-greedy action selection  $e = 0.1$

$$A_t = \begin{cases} A^*_t = \text{argmax}_a Q_t(a) & \text{with prob. } 1-e \\ A^{uni}_t = \text{uniform}(\{a_1, a_2, ---, a_K\}) & \text{with prob. } e \end{cases}$$

- Decaying e-greedy action selection
constant decay
const. rate decay
exponential decay

At some point there is no need to explore anymore
switch to greedy after some pt. in time.

$$A_t = \begin{cases} A^*_t = \text{argmax}_a Q_t(a) & \text{with prob. } 1-e_t \\ A^{uni}_t = \text{uniform}(\{a_1, a_2, ---, a_K\}) & \text{with prob. } e_t \end{cases}$$

0.1 0.005 $\frac{1+0.05}{1\alpha}$ $0 + \frac{1}{10}[1-0]$ .2

## Incremental update rule

$$Q_n = \frac{R_1 + R_2 + \dots + R_{n-1}}{n-1} =$$

$$Q_{n+1} = Q_n + \frac{1}{n}[R_n - Q_n] \quad = \text{stationary}$$

$$-0.1 + \frac{1}{3}[-0.4 + 0.1] \qquad \text{non - stationar}$$

$$Q_{n+1} = Q_n + \alpha[R_n - Q_n]$$

$$Q_2 = Q_1 + 0.2[1 - 0]$$

$$0 + \quad 0.2$$

## Simple Bandit Algo.

Initialize, for $a = 1$ to $k$:

$Q(a) \leftarrow 0$

$N(a) \leftarrow 0$

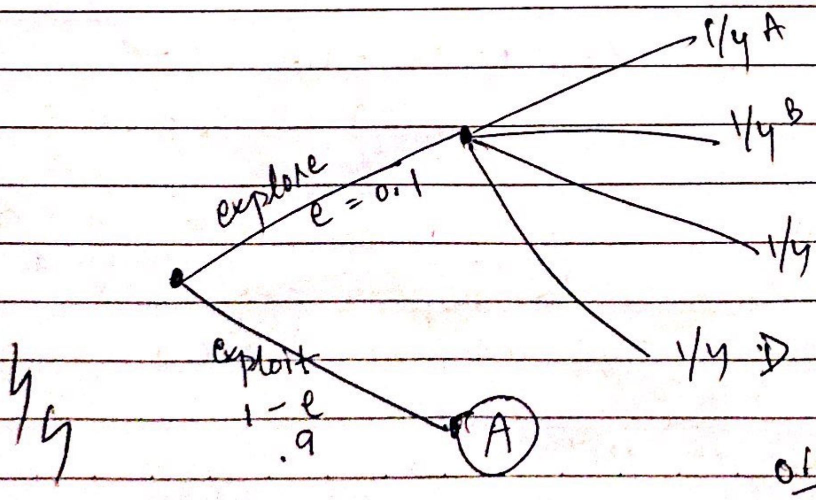
Repeat forever:

$$A \leftarrow \begin{cases} \text{argmax}_a\, Q(a) & \text{with prob. } 1-e \\ \text{random action} & \text{with prob. } e \end{cases}$$

$R \leftarrow \text{bandit}(A)$

$N(A) \leftarrow N(A) + 1$

$Q(A) \leftarrow Q(A) + \frac{1}{N(A)}[R - Q(A)]$




$0.1 \times .25$

$\frac{25}{1000}$ .025

$\left(.9 \times 1 + 0.1 \times \frac{1}{4}\right) +$

$\frac{0.1}{10} \times \frac{1}{4}$ .25 .025

$1 - .025$

$.975$

formulas.

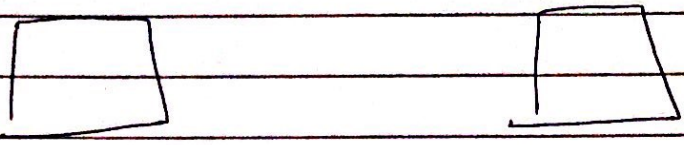
a b c d $\qquad 1(1-e) + P(e)$

$R_1 = \$1 \qquad R_p \quad Q_1 = 0$

$Q_2 = 0 + \frac{10}{1K}[1-0] \qquad 1(1-0.1) + \frac{0.1}{2} \qquad$ .apr

$= 5 \qquad\qquad .99 + .01$

$$(1-\alpha)^K Q_1 + \sum_{i=1}^{K} \alpha (1-\alpha)^{K-i} R_i$$



$q(a)$ 0.1 $\qquad q(b) = 0.2 \qquad 0.5$

$q(a) = 0.9 \qquad q(b) = 0.8 \qquad 0.5$

UCB

value of bound is max. then we play on that m/c.

$$A_t = \text{argmax}_a \left[ Q_t(a) + c \sqrt{\frac{\log t}{N_t(a)}} \right]$$

Initial value bias

$n = 1$

$$Q_{n+1} = Q_1 + \alpha [R_1 - Q_1] = \alpha R_1 + (1-\alpha) Q_1$$

Softmax.

$$P(a) = \frac{e^{Q_t(a)/\tau}}{\sum_{b=1}^{K} e^{Q_t(b)/\tau}}$$

design
performance
comparing