Tutorial - 4

Sol ①

$$T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$a = 3$, $b = 2$, $f(n) = n^2$

∵ a, b are Constant and $f(n)$ is +ve function

∴ Master's theorem is applicable

$$c = \log_b a$$
$$= \log_2 3 = 1.58$$

$$n^c = n^{1.58}$$

which is $n^2 > n^{1.58}$

Case 3 is applied here

$$\boxed{T(n) = \Theta(n^2)}$$

Sol ②

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$a = 4$, $b = 2$, $f(n) = n^2$

∵ a & b are Constant and $f(n)$ is +ve function

∴ Master's theorem is applicable

$$c = \log_b a$$

$$= \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2$$

Page :
Date: / /

$\therefore \quad n^2 = n^2$

$n^c = f(n)$

$\therefore$ Case 2 is applied

$\boxed{T(n) = \theta(n^2 \log n)}$

Sol ③ $\quad T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a = 1, \; b = 2, \; f(n) = 2^n$

a and b are Constant & f(n) is +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 1$

$n^c = n^0 = 1$

$f(n) > n^c$

$\therefore$ Case 3 is applied

$\boxed{T(n) = \theta(2^n)}$

Sol ④ $\quad T(n) = 2^n \, T(n/2) + n^n$

$a = 2^n, \; b = 2, \; f(n) = n^n$

$\because$ a is not constant, its value depends on n

$\therefore$ Master's theorem is not applicable here.

Page:
Date: / /

Sol (5)

$T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16$, $b = 4$, $f(n) = n$

$\because$ a & b are Constant and f(n) is +ve function

$c = \log_b a$

$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2$

$\because f(n) < n^c$

Case 1 is applied here

$[T(n) = \theta(n^2)]$

Sol (6)

$T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2$, $b = 2$, $f(n) = n\log n$

$\because$ a, b are Constant and f(n) is +ve function

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

$\because f(n) > n^c$

Case 3 is applied

$[T(n) = \theta(n\log n)]$

Sol (7)

$$T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$$

$a=2, b=2, \quad f(n) = n/\log n$

$\because$ a & b are constant and f(n) is +ve function

$$c = \log_b a = \log_2 2 = 1$$

$$n^c = n^1 = n$$

$\because$ non-polynomial difference b/w f(n) and n^c

$\therefore$ Master's theorem is not applicable

Sol (8)

$$T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$$

$a=2, b=4, \quad f(n) = n^{0.51}$

$\because$ a & b are constant and f(n) is +ve function

$\therefore$ Master's theorem is applicable

$$c = \log_b a = \log_4 2 = 0.50$$

$$n^c = n^{0.50}$$

$$f(n) > n^c$$

Case 3 is applied

$$\boxed{T(n) = \Theta(n^{0.5})}$$

Page :
Date: / /

Sol (9)

$T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

$a = 0.5,\ b = 2,\ f(n) = \frac{1}{n}$

$\because a < 1$

Master's theorem is not applicable

Sol (10)

$T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$\because$ a and b are Constant and f(n) is +ve function

Master's theorem is applicable

$c = \log_b a = \log_4 16 = 2\log_4 4 = 2$

$n^c = n^2$

$f(n) > n^c$

Case (3) applied $\boxed{T(n) = \theta(n!)}$

Sol (11)

$T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

a & b are Constant, f(n) is +ve function

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) < n^c$

Case 1 applied

$\boxed{T(n) = \theta(n^2)}$

Page:
Date: / /

Sol (12) $T(n) = \sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}$, $b = 2$, $f(n) = \log n$

∴ a is not constant

Master's theorem is not applicable

Sol (13) $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3$, $b = 2$, $f(n) = n$

a and b are constant and f(n) is +ve.

∵ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 0.158$

$n^c = n^{0.158}$

$f(n) < n^2$

Case 1 is applied

$\boxed{T(n) = \Theta(n^{1.58})}$

Sol (14) $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3, b = 3,$ $f(n) = \sqrt{n}$

a and b are constant, f(n) is +ve function

Master's theorem is applicable

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$f(n) < n^c$

Case 1 applied

$\boxed{T(n) = \Theta(n)}$

Page :
Date: / /

Sol (15)

$T(n) = 4T(n/2) + cn$

$a = 4, b = 2, f(n) = cn$

a and b are Constants, $f(n)$ is +ve function

Master's theorem is applicable

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = n^c$

$\boxed{T(n) = \theta(n^2)}$

Sol (16)

$T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4, f(n) = n\log n$

a and b are Constants, $f(n)$ is +ve function

Master's Theorem is applicable

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

$f(n) > n^c$

$\boxed{T(n) = \theta(n\log n)}$

Sol (17)

$T(n) = 3T(n/3) + n/2$

$a = 3, b = 3, f(n) = n/2$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$f(n) = n^c$

$\boxed{T(n) = \theta(n\log n)}$

Scanned by CamScanner

Page :
Date: / /

Sol (18)

$$T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$$

$a = 6,\ b = 3, \quad f(n) = n^2 \log n$

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$f(n) > n^c$

Case 3

$$\boxed{T(n) = \Theta(n^2 \log n)}$$

Sol (19)

$$T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

$a = 4,\ b = 2, \quad f(n) = n/\log n$

$c = \log_b a = \log_2 4 = 2$

$f(n) < n^c$

$$\boxed{T(n) = \Theta(n^2)}$$

Sol (20)

$$T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$$

$\because$ a and b are Constant but function is -ve
Master's theorem is not applied

Page :
Date: / /

Sol (21)

$T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$C = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

$f(n) > n^c$

$\boxed{T(n) = \Theta(n^2)}$

Sol (22)

$T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

∵ $f(n)$ is not regular function

∴ Master's theorem is not applicable