COL 864: Planning and Estimation for Autonomous Systems **II Semester 2020-2021**

Assignment 1 Report

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Contents

1 *Q1: Robot Localization on Discrete State Space*

1.1 *a: Simulation of Robot Motion*

- Simulation Time: 25 steps
- Output: Sensor observations.
- Description of output: The vector of observations at each time step, which is $[z_1, z_2, z_3, z_4]$, comprises of the observation produced by the sensors at (8, 15), (22, 15), (15, 15), (15, 22) respectively at each time step. Each sensor stochastically reports the presence/absence of the target.
- In this simulation, the initial position was chosen to be (10, 10).

Sensor Observations used in the next parts

[0, 1, 0, 0], [1, 1, 0, 0], [1, 1, 0, 0], [0, 1, 0, 0], [0, 1, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [0, 0, 0, 0], [1, 0, 0, 0], [0, 0, 0, 0], [1, 0, 0, 0], [0, 0, 0, 0], [1, 0, 0, 0], [0, 0, 0, 0], [1, 0, 1, 0], [0, 0, 0, 0]

Ground Truth Positions of the Target, used to compute error in the following predictive steps

(10, 10), (10, 11), (11, 11), (12, 11), (11, 11), (10, 11), (10, 10), (9, 10), (8, 10), (8, 9), (9, 9), (9, 10), (9, 9), (9, 8), (10, 8), (9, 8), (9, 9), (10, 9), (11, 9), (12, 9), (13, 9), (13, 10), (13, 11), (14, 11), (15, 11), (16, 11)

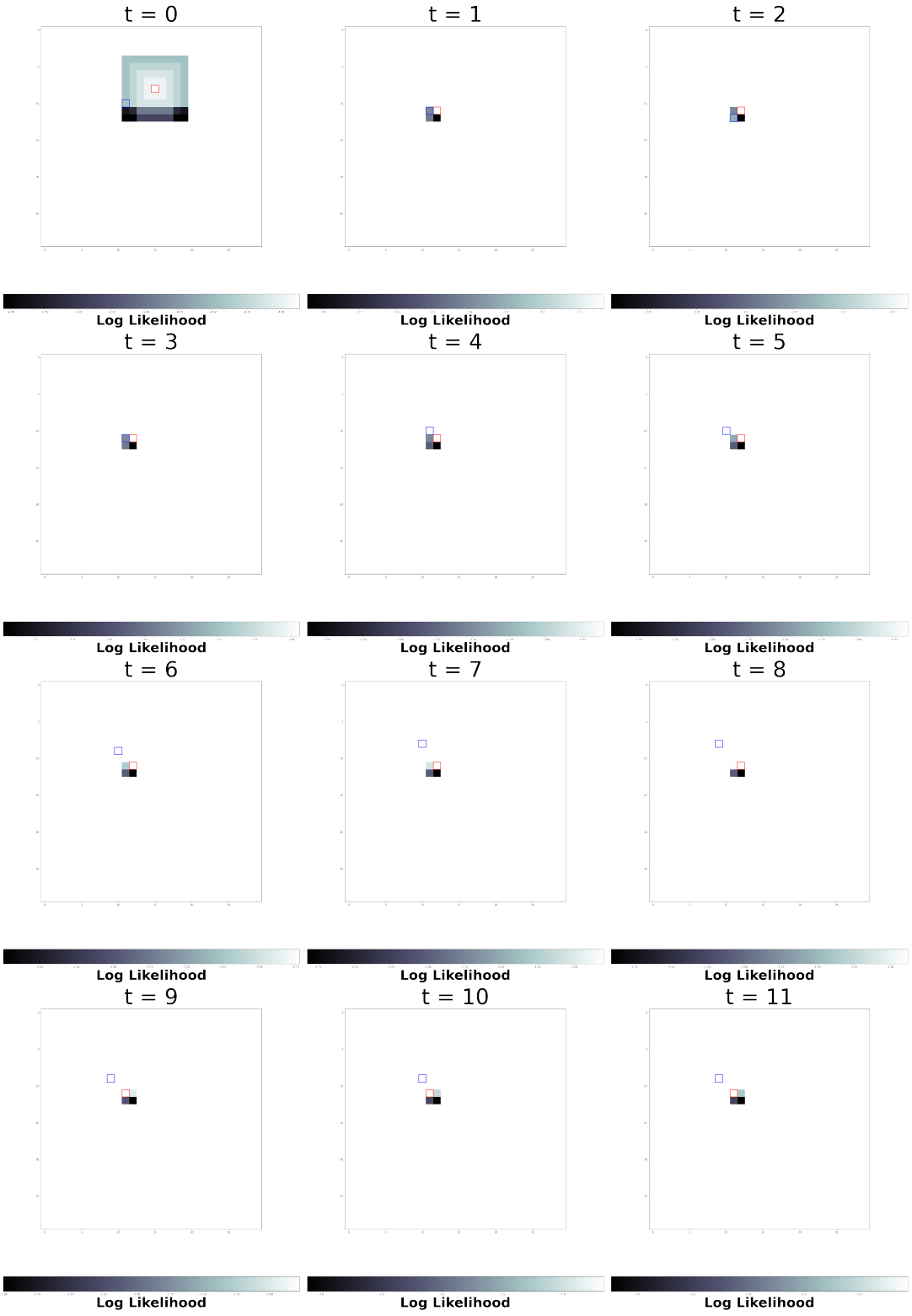
1.2 *b: Filtering to estimate the beliefs of each of the grid positions at each time step*

- Filtering Steps: 25
- Output: Log Likelihood plots (took the natural log of the belief values at each state), indicative of the Belief of the presence of the target over the various grid positions on the state space.
- The log likelihoods have been computed given the sensor observations till that time step and by using the transition model of the problem.
- The filtering operation was performed based on the following recursive operation:

$$P(\mathbf{X}_{t+1}|\mathbf{z}_{1:t+1}) = \eta P(\mathbf{Z}_{t+1}|\mathbf{X}_{t+1}) \sum_{\mathbf{x}_t} P(\mathbf{X}_{t+1}|\mathbf{x}_t) P(\mathbf{x}_t|\mathbf{z}_{1:t})$$

  where η is a normalizing constant.

- The transition model is as follows:
  1. $\mathrm{P}(\mathbf{X}_{t+1} = (a-1, b)|\mathbf{X}_t = (a, b)) = 0.4$ which corresponds to going *up* on the given grid.
  2. $\mathrm{P}(\mathbf{X}_{t+1} = (a+1, b)|\mathbf{X}_t = (a, b)) = 0.1$ which corresponds to going *down* on the given grid.
  3. $\mathrm{P}(\mathbf{X}_{t+1} = (a, b+1)|\mathbf{X}_t = (a, b)) = 0.3$ which corresponds to going *right* on the given grid.
  4. $\mathrm{P}(\mathbf{X}_{t+1} = (a, b-1)|\mathbf{X}_t = (a, b)) = 0.2$ which corresponds to going *left* on the given grid.
- The initial prior corresponded to a uniform distribution.
- The Log Likelihood observations are given in the following Fig 1.
- **Estimated and Ground Locations**: The Estimated locations are the ones which have the maximum likelihood over all the states in the state space. The estimated positions at each time step are marked with *red* boxes in the figure. The Ground truth positions at each time step, obtained from the simulation are marked with *blue* boxes at each time step.
- The time steps starting from zero, start from the instant of getting the first observation.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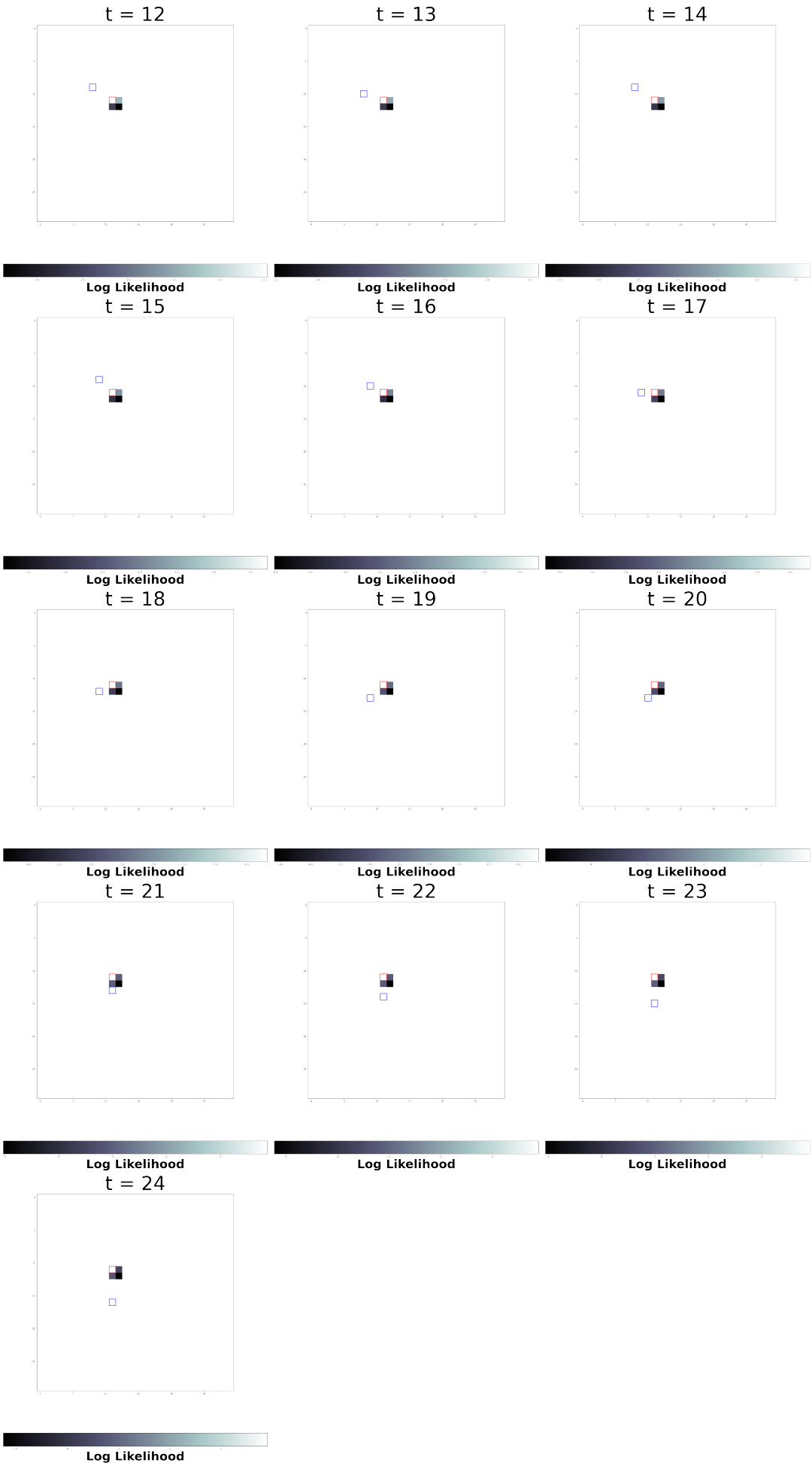


Figure 1: Variation of Log likelihood of Belief maps over time in Filtering Operation (Red: Estimated positions and Blue: Ground Truth positions)

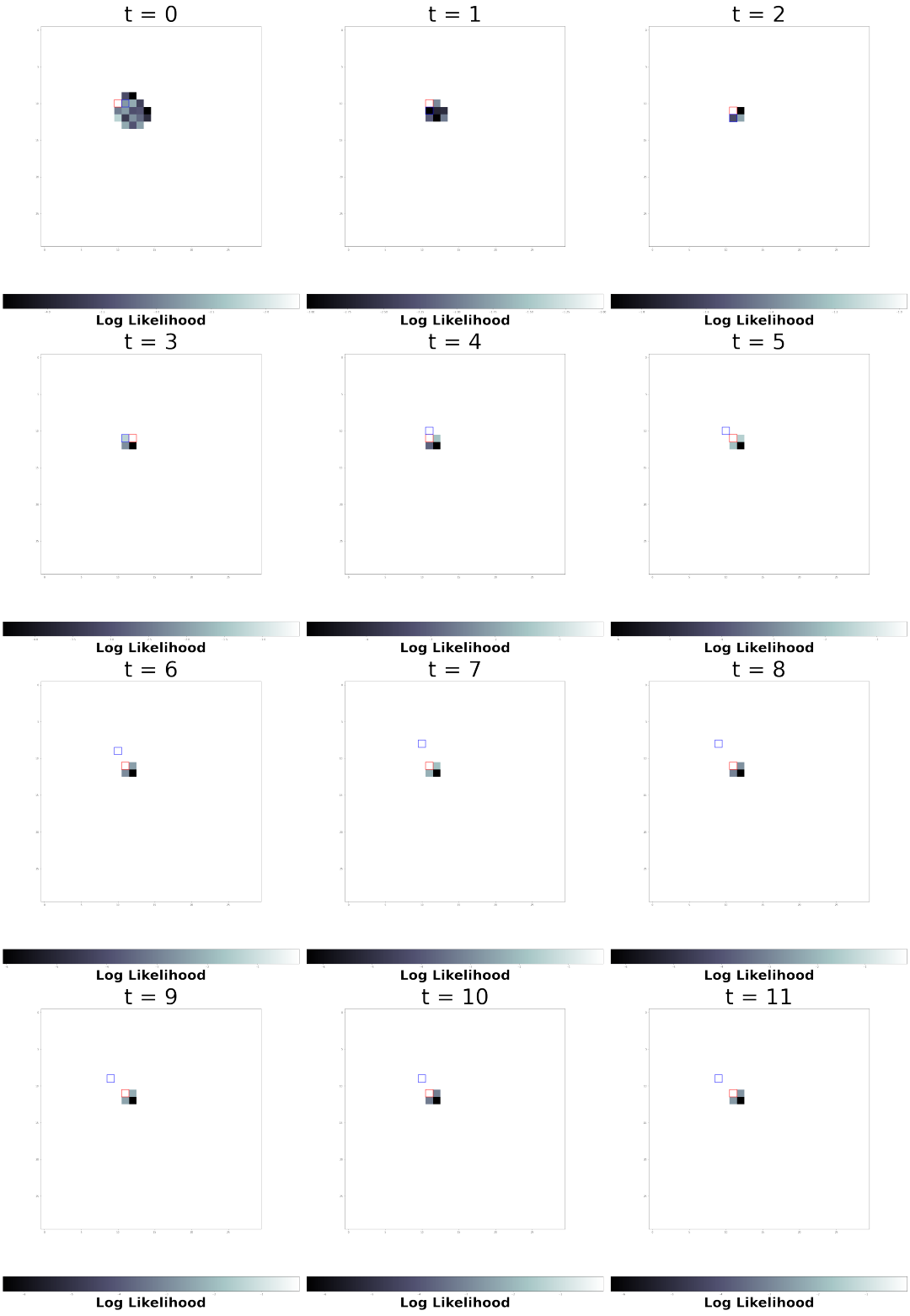
1.3 *c: Smoothing to estimate the beliefs of each of the grid positions at each time step*

- Smoothing Steps: 25
- Output: The log likelihood of all states over 25 time steps was recorded. It represents the beliefs of the presence of the target robot each of states in the grid state space. The belief values have been derived considering all of the sensor observations available to the model (till 25 time steps) and the transition model. Thus, the likelihood maps obtained for each time step are after *smoothing* it by the future observations and making the predictions better.
- The smoothing operation performed here involves a forward pass and a backward procedure, to incorporate the future sensor observations.
- The basic equation governing the algorithm for the Smoothing operation is:

$$\begin{aligned} P(\mathbf{X}_k|\mathbf{z}_{1:t}) &= \alpha P(\mathbf{X}_k|\mathbf{z}_{1:k})P(\mathbf{z}_{k+1:t}|\mathbf{X}_k) \\ &= \alpha f_{1:k} b_{k+1:t} \end{aligned}$$

$$P(\mathbf{z}_{k+1:t}|\mathbf{X}_k) = \sum_{\mathbf{x}_{k+1}} P(\mathbf{z}_{k+1}|\mathbf{x}_{k+1})P(\mathbf{z}_{k+2:t}|\mathbf{x}_{k+1})P(\mathbf{x}_{k+1}|\mathbf{X}_k)$$

- The computation for smoothing is seeded by uniform initial belief values for the forward filtering term in the smoothing probability and $b_{t+1:t} = \mathbf{1}$, which is a matrix of 1.
- The Log Likelihood observations are given in the following Fig 2.
- **Estimated and Ground Locations**: The Estimated locations are the ones which have the maximum likelihood over all the states in the state space. The estimated positions at each time step are marked with *red* boxes in the figure. The Ground truth positions at each time step, obtained from the simulation are marked with *blue* boxes at each time step.
- The time steps starting from zero, start from the instant of getting the first observation.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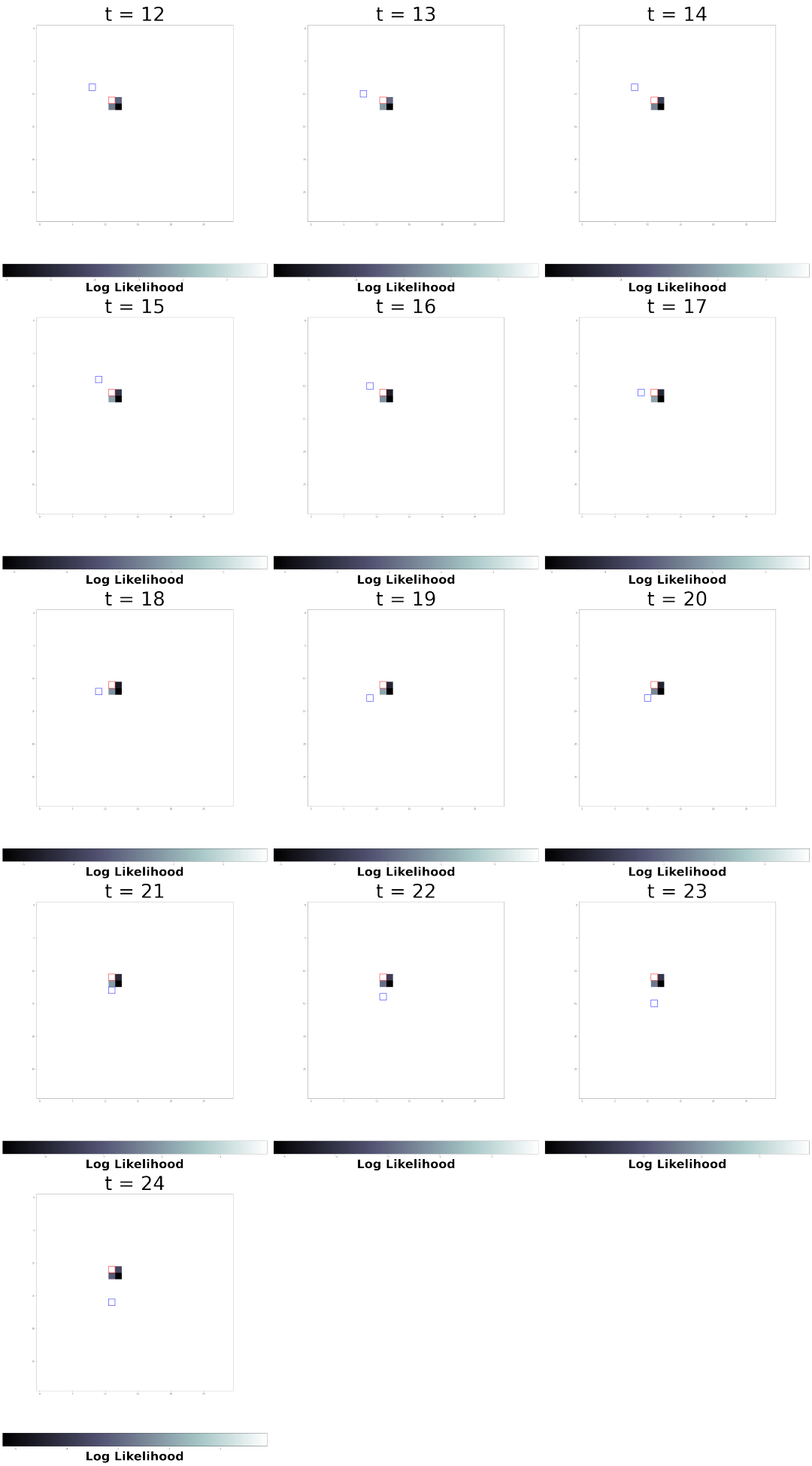


Figure 2: Variation of Log likelihood of Belief maps over time in Smoothing operation (Red: Estimated positions and Blue: Ground Truth positions)

1.4 *d: Comparing Filtering and Smoothing with the Ground Truth Path using the Manhattan Distance Error Metric*

- Number of Time Steps: 25
- Procedure: The Manhattan Error has been computed between the estimated and ground truth positions at all the time steps and displayed in Fig 3. The Manhattan Error has also been computed between the estimated and ground truth paths, which are generated by the Filtering and Smoothing Procedures till each time step. They are displayed in Fig 4.
- The estimated positions are the states which have the maximum likelihood over the state space.
- The plots indicate the following:

  1. The Path error plot clearly indicates that the *Filtering* procedure gives higher path error than Smoothing procedure at each time step.
  2. This is because, Filtering takes into account only the observations which were generated till the time step of estimating the state beliefs. Smoothing, on the other hand, considers all the sensor observations available. Thus, it is expected that Smoothing will produce better results and outperform Filtering.
  3. When high accuracy of the prediction is the main objective, then Smoothing should be used. In other cases, where the time taken by the algorithm is of concern, Filtering is preferred.
  4. The Manhattan error at each time step, between the estimated and ground truth positions indicates that due to Smoothing operation, the error in the initial positions of the trajectory falls drastically. This is because, Smoothing is using the entire set of Sensor observations to generate the beliefs for the states in the initial time steps.
  5. The Manhattan error is same for Smoothing and Filtering operations both at the last time step, as both the models are using the entire set of sensor observations to compute the likelihoods.
  6. The error due to Smoothing is seen to be lesser than or equal to that produced by filtering at each time step.

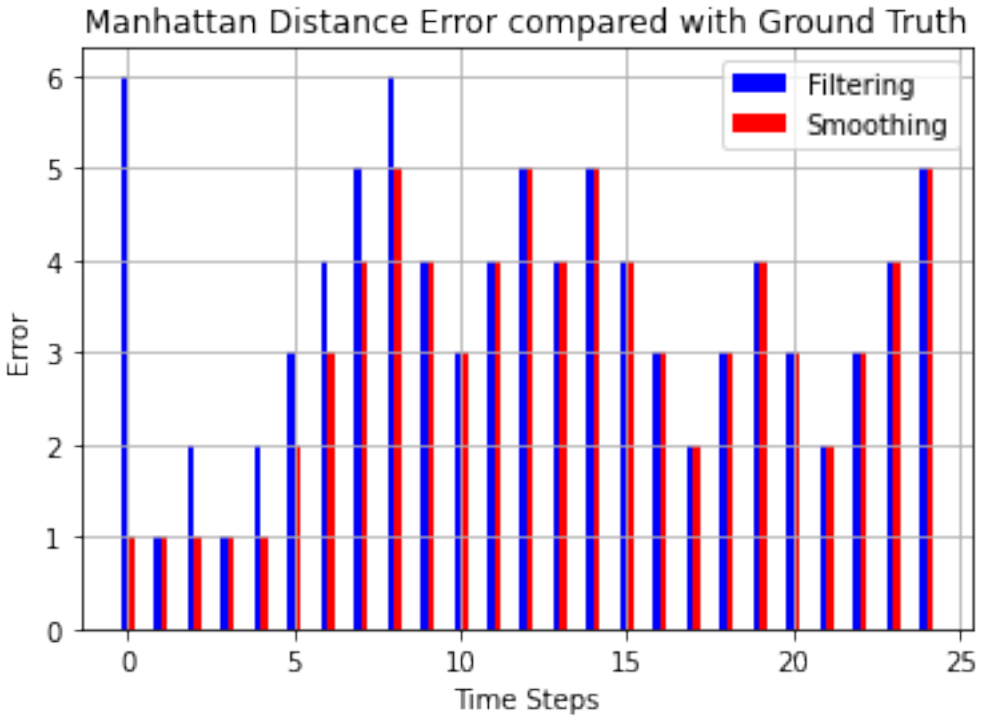


Figure 3: Manhattan error obtained in Filtering and Smoothing Steps

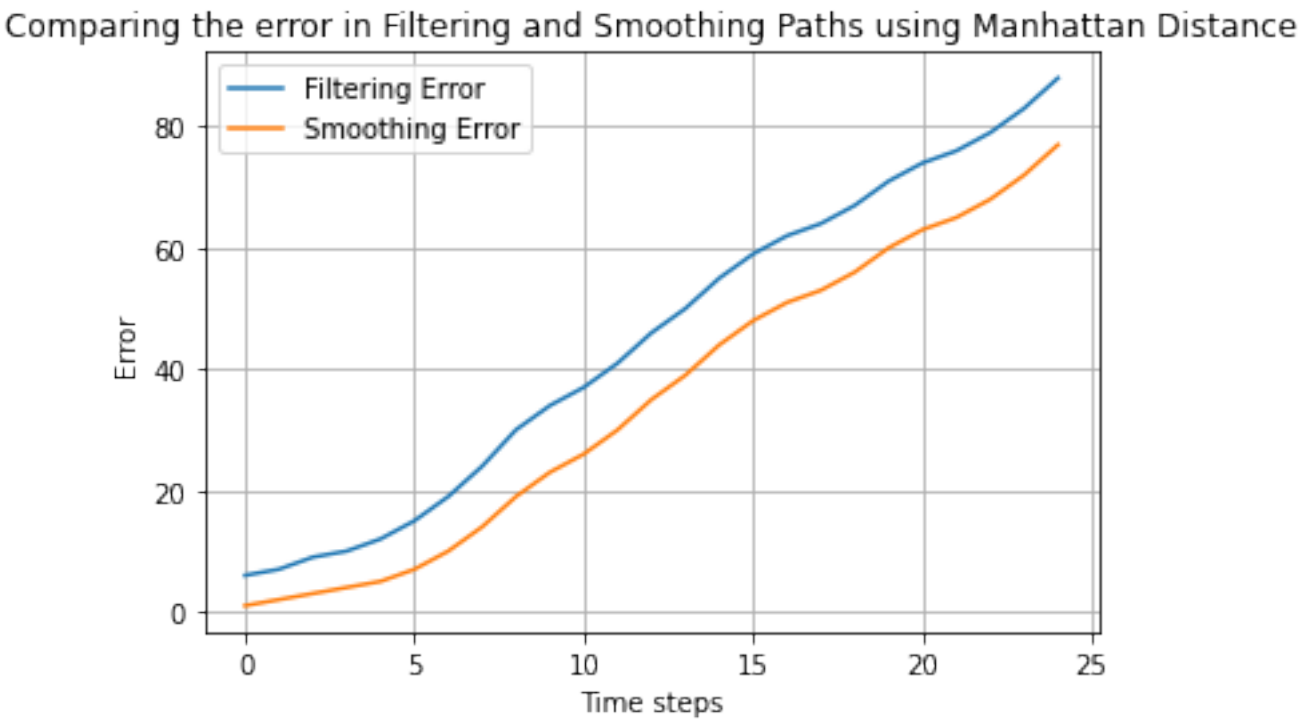


Figure 4: Manhattan error of Filtering and Smoothing Paths

1.5 e: Predictive Likelihood

- Output: Log Likelihood plots (took the natural log of the belief values at each state), indicative of the Belief of the presence of the target over the various grid positions on the state space.
- The log likelihoods have been computed using the transition model of the problem only. This is because no observations exist for future states.
- The Prediction operation was performed based on the following recursive operation:

$$P(X_{t+k+1}|z_{1:t}) = \sum_{x_{t+k}} P(X_{t+k+1}|x_{t+k})P(x_{t+k}|z_{1:t})$$

- The transition model is as follows:
  1. $\mathrm{P}(X_{t+1} = (a-1, b)|X_t = (a, b)) = 0.4$ which corresponds to going *up* on the given grid.
  2. $\mathrm{P}(X_{t+1} = (a+1, b)|X_t = (a, b)) = 0.1$ which corresponds to going *down* on the given grid.
  3. $\mathrm{P}(X_{t+1} = (a, b+1)|X_t = (a, b)) = 0.3$ which corresponds to going *right* on the given grid.
  4. $\mathrm{P}(X_{t+1} = (a, b-1)|X_t = (a, b)) = 0.2$ which corresponds to going *left* on the given grid.
- The initial prior for the prediction operation is the belief matrix generated at the end of the filtering procedure on the same problem.
- The Log Likelihood observations are given in the following figure.
  1. **With 10 steps of Future** Prediction Steps: 10: Fig 5
  2. **With 25 steps of Future** Prediction Steps: 25: Fig 6
- **Estimated Locations**: The Estimated locations are the ones which have the maximum likelihood over all the states in the state space. The estimated positions at each time step are marked with *red* boxes in the figure.
- The time steps starting from zero, start from the first instant of entering into the future, that is the first time instant from which we have no sensor observations, in both the figures.

Discussion

- The plots help in verifying the fact that the applying the transition model adds to the uncertainty in the likelihood of the presence of the target.
- Sensor Observations decrease the uncertainty in the position of the target as is clear from the Filtering and Smoothing Observations.
- As it can be seen, the likelihood of each state being occupied with the target, rises by increasing the number of steps into the future. The plots with 10 steps of progress indicate a lower likelihood of the occupancy of a state with the target than the plots with 25 steps of progress.
- The randomness in the position of the target thus rises.

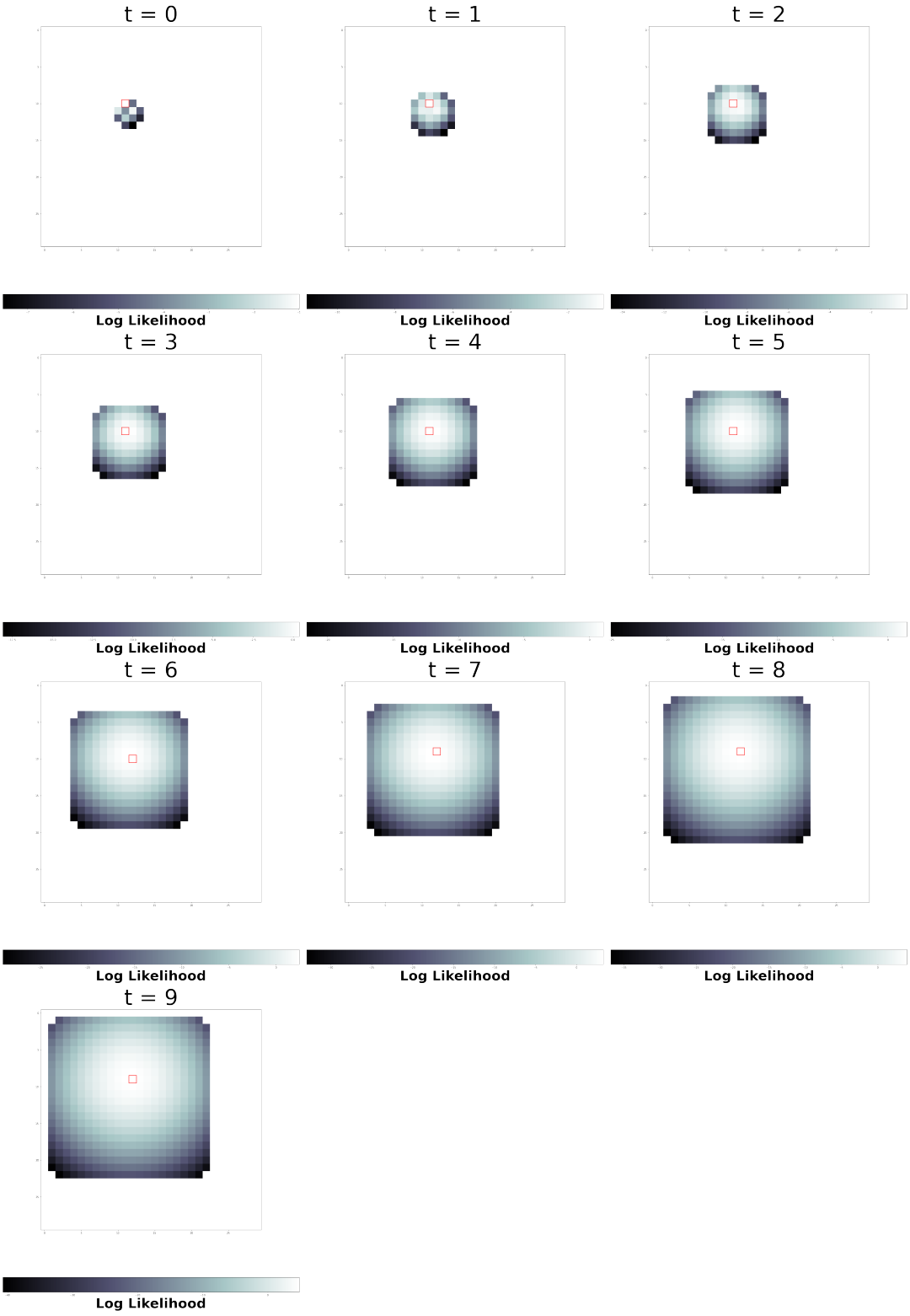
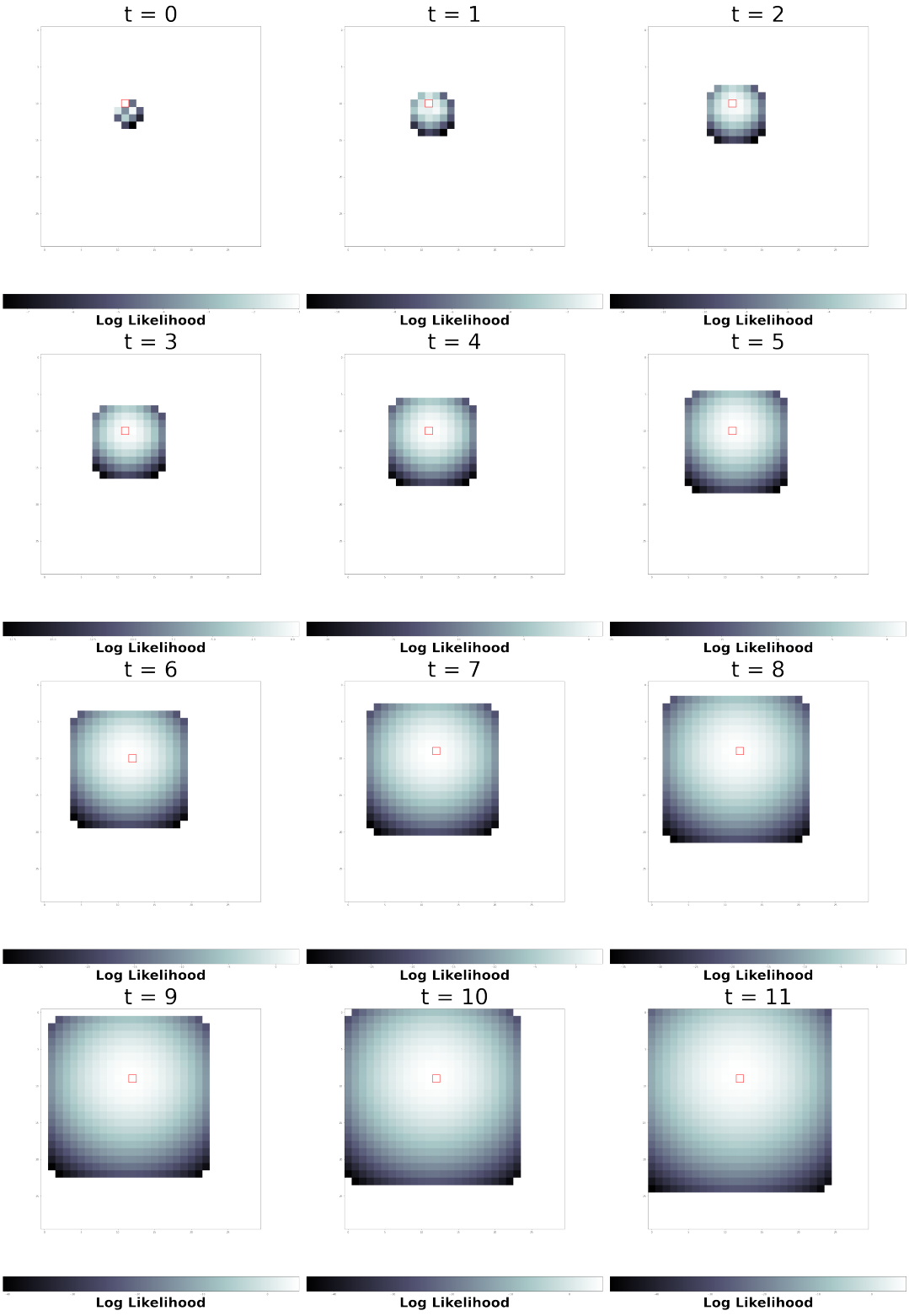


Figure 5: Predictive Likelihood for 10 steps (Red: Estimated positions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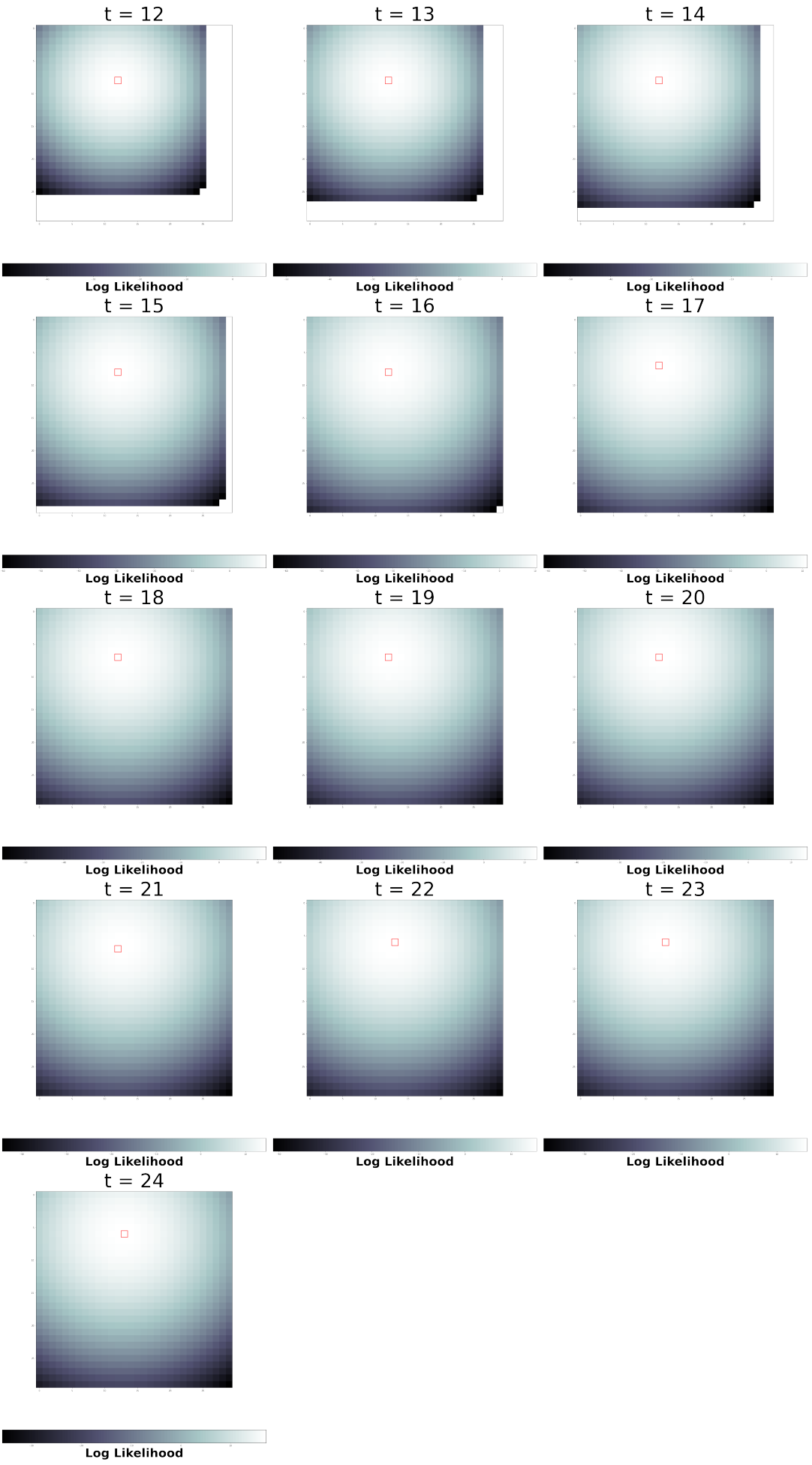

Figure 6: Predictive Likelihood over 25 steps (Red: Estimated positions)

1.6 *f: Most Likely Path*

- The most likely path for the given sensor observations and transition model is determined using the Viterbi Algorithm.
- The recursive step in the algorithm uses the following relation:

$$\max_{\mathbf{x}_1,...,\mathbf{x}_t} P(\mathbf{x}_1,...,\mathbf{x}_t,\mathbf{X}_{t+1}|\mathbf{z}_{1:t+1}) = \alpha P(\mathbf{z}_{t+1}|\mathbf{X}_{t+1}) \max_{\mathbf{x}_t}(P(\mathbf{X}_{t+1}|\mathbf{x}_t) \max_{\mathbf{x}_1,...,\mathbf{x}_{t-1}} P(\mathbf{x}_1,...,\mathbf{x}_t|\mathbf{z}_{1:t}))$$

- The initial prior involved in the algorithm corresponded to a uniform distribution.
- The Estimated and Ground truth locations over all the time steps till T = 25 are shown in the following Fig 7
- **Estimated and Ground Locations**: The Estimated locations are the ones which have the maximum likelihood path over all the paths possible, till their time step. The estimated positions at each time step are marked with *red* boxes in the figure. The Ground truth positions at each time step, obtained from the simulation are marked with *blue* boxes at each time step.
- The time steps starting from zero, start from the instant of getting the first observation.


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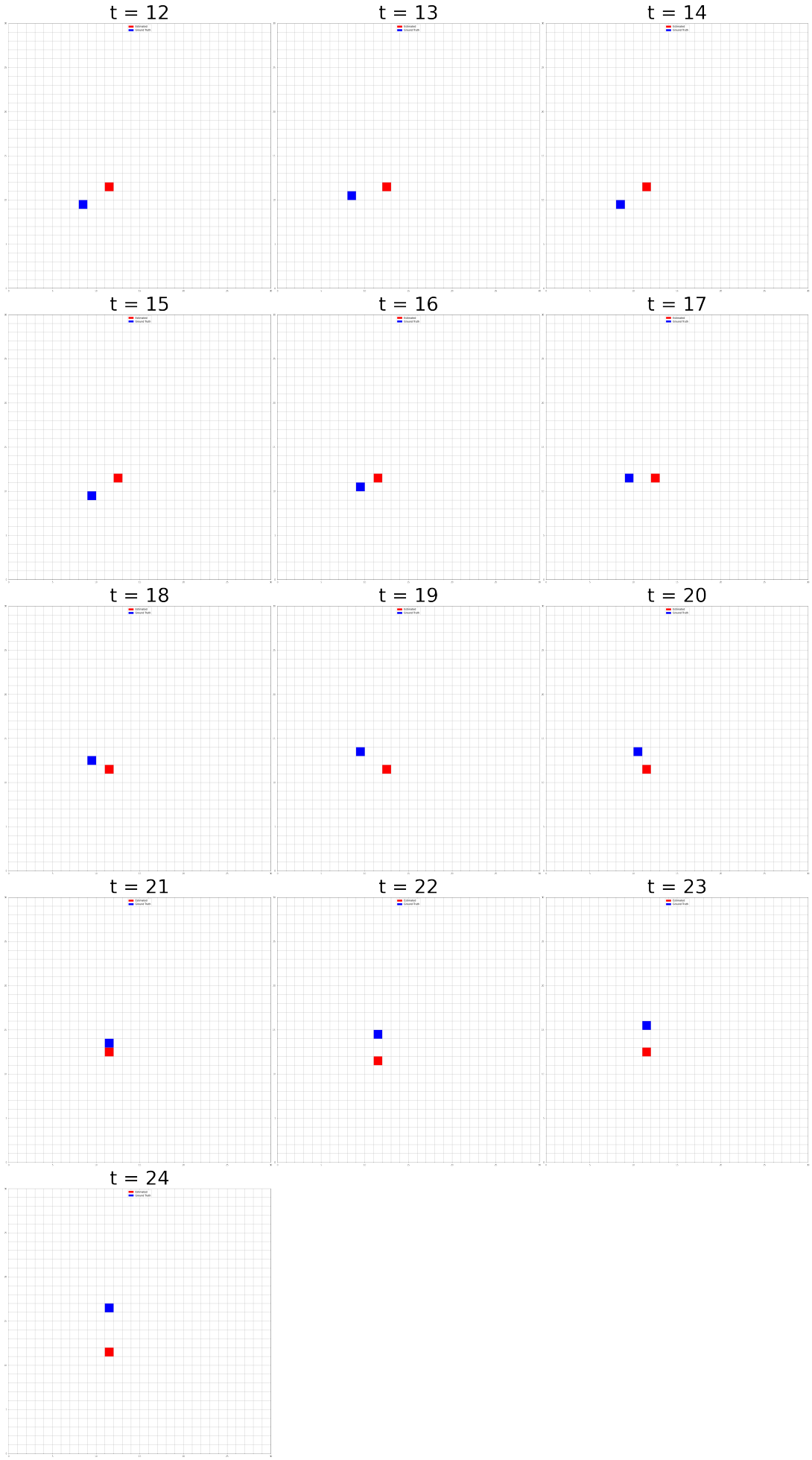

Figure 7: Most Likely Path (Red: Estimated positions and Blue: Ground Truth positions)