State Variable Models of Physical Systems

Akshya Swain

Department of Electrical,Computer & Software Engineering,
The University of Auckland.

Learning Outcomes

After completion of this module, the students should have learned the following.

1. Limitations of transfer function models
2. Philosophy of State Variable Modelling
3. Derive the state variable models of several physical systems.
4. Advantages and disadvantages of state variable models.

What is a Model and Why do we need it?

What is a Mathematical Model?

- Loosely put, a model of a system is a tool we use to answer questions about the system without having to do an experiment.
- A collection of mathematical relationships between system/process variables which purports to describe the behavior of a physical system.
- This is a convenient surrogate of the physical system.

What are its main uses?

1. For Analysis : To investigate system response under various input conditions both rapidly, and inexpensively, without tampering with the actual physical entity.
2. For Synthesis: Analytically design controllers

DON'T CONFUSE MODELS WITH PHENOMENA!

- Model is an imaginary universe. They are not photographs of reality; they capture those aspects of the system which the designer decides to be important.

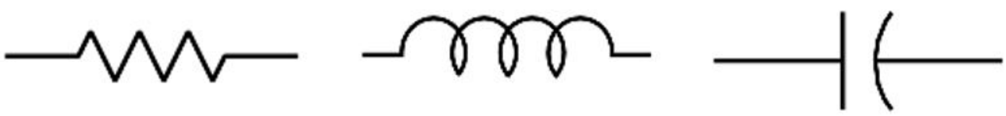
MAP IS NOT THE TERITORY; IT IS THE MODEL OF THE TERITORY

Philosophy of Model Building

Physical Phenomena can be categorized essentially into two types:

1. **Energy dissipation**
2. **Energy absorption or storage**

Trinity of Electrical Engineering: the resistor, capacitor, and inductor.



1. **Resistance** : Models the phenomenon of **energy dissipation.**
2. **Capacitance and Inductance** model the phenomenon of **energy absorption or storage.**

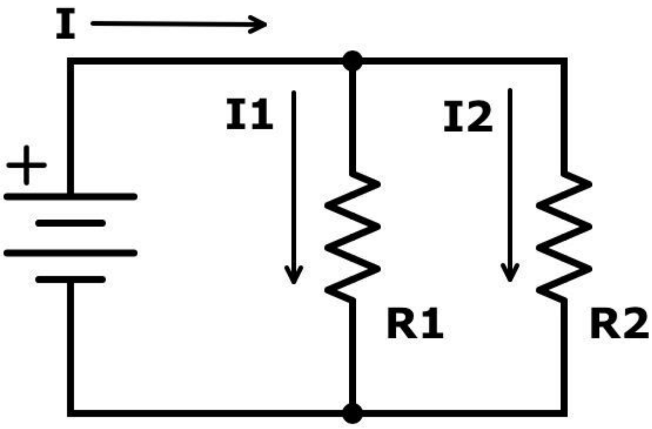
- Inductance stores energy in its **magnetic field** (Kinetic energy) and Capacitance stores energy in its **electric field** (Potential energy).

Concept of Static/Memory-less Systems

What is a static system?

1. A static system does not have any energy storing element.
2. It is also referred to as a system without memory.
3. The response **at a particular time is dependent only on the input at that time**.

Example: A Purely Resistive Circuit



Concept of Dynamic System or Systems with Memory

What is a Dynamic System?

1. A dynamic system is one which has at-least one energy storing element.
2. The dynamics system is also known as system with memory.
3. The response **at a particular time depends both on the input as well as the energy stored in the energy storing elements (initial conditions) (past values)**.

Example: R-L, R-L-C circuits

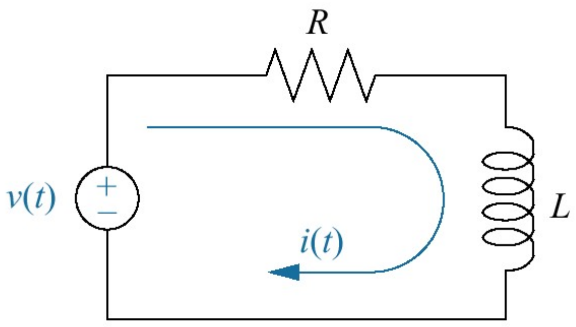


Figure: R-L Circuit

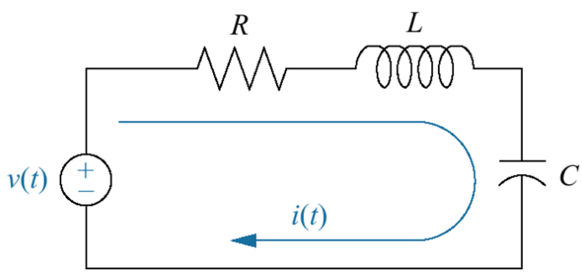


Figure: R-L-C Circuit

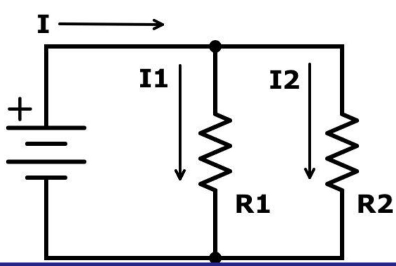
Concept of System Order

What do you understand by order of a system?

The order of a system equals to the number of independent energy storing elements of the system.

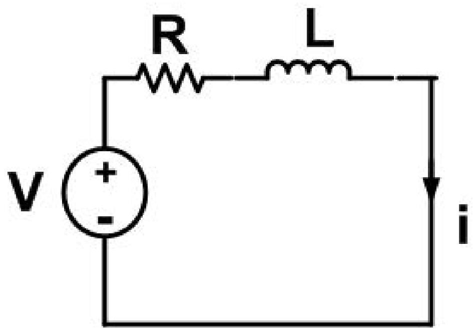
- Commonly known storing elements: **(Inductance and Capacitance)** and dissipating element **Resistance.**
- Inductance stores energy in its magnetic field and capacitance stores energy in its electric field.

Example-1:Purely Resistive Circuit: The number of energy storing elements=0. Hence its order is 0 (zero).

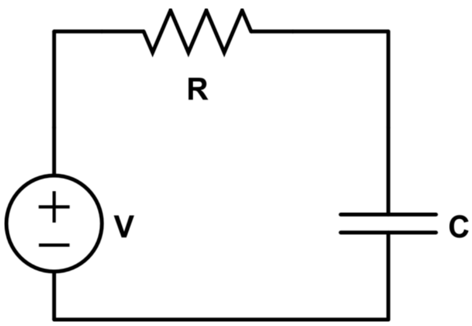


Examples: Concept of System Order

Exampl-2: RL Circuit: The number of energy storing elements equals to 1. Hence its order 1 (one).

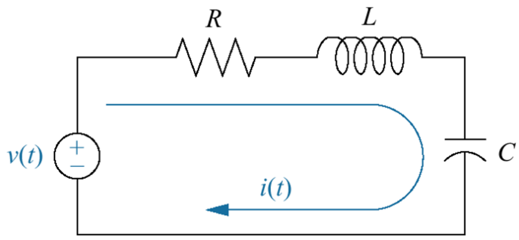


Exampl-3: RC Circuit: The number of energy storing elements equals to 1. Hence its order 1 (one).



Examples: Concept of System Order

Exampl-4: RLC Circuit: The number of energy storing elements equals to 2. Hence its order equals to 2(two).

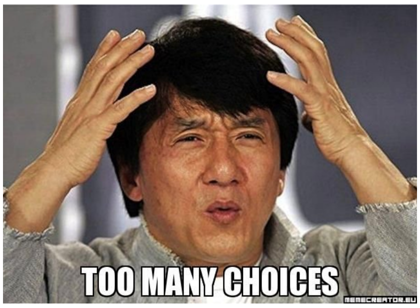


Mathematical Models

Various Types of Models:

1. Differential equation models
2. Transfer function models
3. State variable models
4. Discrete time ARMA, ARMAX models
5. Neural network models
6 Fuzzy models and so on .

Dilemma:

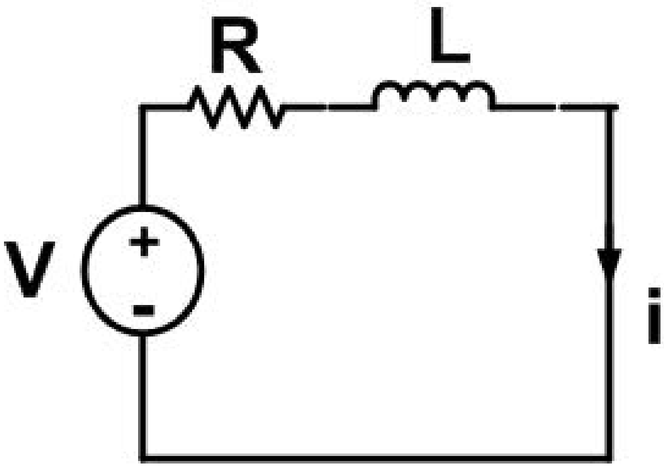


- Which types of model to be selected ?

The man with one watch is sure what time it is; the man with multiple watches is never sure!

Type of model one wants to use depends on the application.

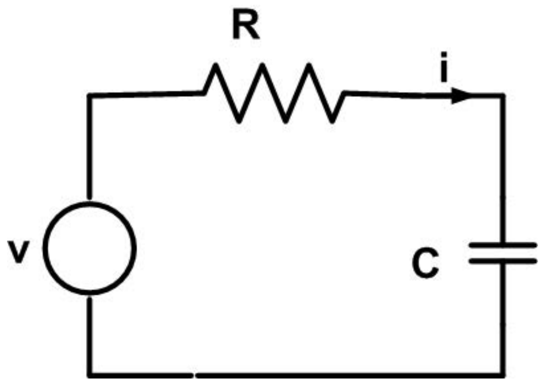
Example-1: R-L Circuit



- The dynamics of R-L circuit is given by

$$Ri(t) + L\frac{di(t)}{dt} = v(t)$$

Example-2: R-C Circuit



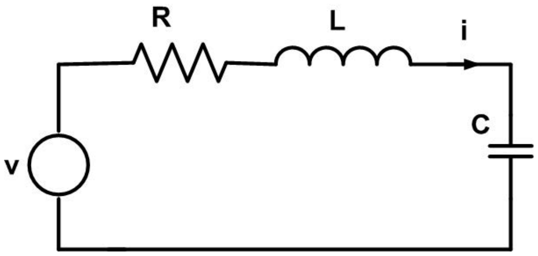
- The dynamics of R-C circuit is given by

$$Ri(t) + \frac{1}{C}\int i(t)dt = v(t)$$

$$R\frac{dq(t)}{dt} + \frac{q(t)}{C} = \frac{dv(t)}{dt}$$

Differential Equation Models

Example-3: R-L-C Circuit



- The dynamics of R-L-C circuit is given by

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = v(t)$$

$$R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}i(t) = \frac{dv(t)}{dt}$$

Transfer Function Models

Given differential equation model of the system, how do we get the transfer function model ?

1. Decide (amongst several variables), which variables will be considered as output and which variables as input
2. Assume all initial conditions to be zero
3. Take the ratio of Laplace transform of output to the Laplace transform of the input to get the transfer function

1. For zero initial conditions

$$\mathcal{L}[\dot{x}] = sX(s), \quad \mathcal{L}[\ddot{x}] = s^2X(s), \quad \mathcal{L}\left[\frac{d^nx}{dt^n}\right] = s^nX(s)$$

Example-1: Computation of Transfer Function from Differential Equation Model of a System

The differential equation model of a system is described as:

$$\ddot{y} + \dot{y} + 9y = 2\dot{u} + u$$

where y and u represent respectively the input and the the output. Compute the transfer function of the system.

Solution:

- Taking the Laplace transform the above equation with zero initial conditions give

$$s^2 Y(s) + 7sY(s) + 9Y(s) = 2sU(s) + U(s)$$
$$or, \left[s^2 + 7s + 9\right] Y(s) = [2s + 1] U(s)$$

Thus, the transfer function of the system

$$\boxed{G(s) = \frac{Y(s)}{U(s} = \frac{2s+1}{s^2+7s+9}}$$

SOME BASIC MATLAB COMMANDS

Some Basic MATLAB Commands

1. How to create a polynomial ?

The polynomial

$$p(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0$$

can be defined by a vector whose successive elements are the coefficients of this polynomial in descending powers of s.

Example: Represent the following polynomials in MATLAB

$$p_1(s) = s^3 + 5s^2 + 7s + 9, \quad \& \quad p_2(s) = s^4 + 3s^3 + 2s$$

MATLAB Code

```
p1=[1 5 7 9]
p2=[1 3 0 2 0]
```


Some Basic MATLAB Commands

2. How to create a transfer function?

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

i. Define two vectors **num** and **den** (say) whose successive elements are the coefficients of numerator and denominator polynomials in descending powers of s.

ii. Use **tf** function of MATLAB which creates a continuous time transfer function with numerator and denominator specified by *num* and *den*.

Example: Represent the following transfer function in MATLAB

$$G(s) = \frac{s+2}{s^3 + 5s^2 + 7s + 9}$$

MATLAB Code

```
num=[1 2]; den=[1 5 7 9];
sysG=tf(num,den);
```

Some Basic MATLAB Commands

3. How to create an array of n equally spaced numbers between a and b

   MATLAB Code

```
t=linspace(a,b,n)';     % The transpose ' creates t as a column vector.
```

4. How to find the roots of a polynomial and its value?

   Example: Consider the polynomial $p(t) = t^3 - 4t^2 + 5t + 9$. Define this polynomial in MATLAB and find its value at $t = 2$ and its roots.

   MATLAB Code

```
p=[1 -4 5 9];     % We represent this as a row vecot
r=roots(p);
pvalue=polyval(p,2);
```


Computing System Response using MATLAB

Compute the response of the system with transfer function

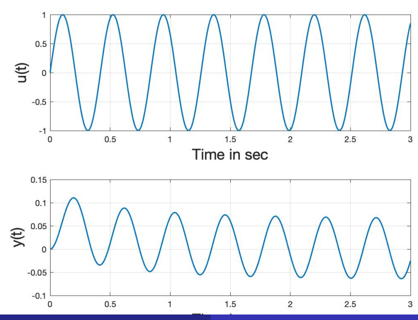
$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+2}{s^2+4s+3}$$

for an input $u(t) = sin15t$ until time $t = 3sec$ at an interval of 0.01 sec.

MATLAB Code

```
num=[1 2];
den=[1 4 3];
sys=tf(num,den);
t=0.0:0.01:3.0;
t=t';
u=sin(15*t);
y=lsim(sys,u,t);
subplot(2,1,1)
plot(t,u,'linewidth',1.5)
grid
xlabel('Time in sec','FontSize',18)
ylabel('u(t)','FontSize',18)
```

```
subplot(2,1,2)
plot(t,y,'linewidth',1.5)
grid xlabel('Time in
sec','FontSize',18)
ylabel('y(t)','FontSize',18)
```



1. Defined only for linear time-invariant system; (not defined for nonlinear systems ?) .
2. It becomes highly cumbersome for use with Multiple Input Multiple Output (MIMO) systems.
3. For a given input, it reveals information about the system output only, and provides no information regarding the internal state of the system.
4. It may be necessary and advantageous to provide a feedback proportional to some of the internal variables of the system, rather than the output alone, for the purpose of stabilizing and improving the performance of the system.
5. Independent of the input of the system
6. It is a function of the complex variable "s"; not a function of the real variable, time, or any other variable that is used as the independent variable.

WHERE DO WE GO NEXT ?

FIND AN ALTERNATIVE TO TRANSFER FUNCTION MODEL WHICH CAN ALLEVIATE SOME PROBLEMS OF TRANSFER FUNCTION MODEL

Introduce a new type of model called state variable model.

Given either

a. the differential equation models of the system or

b the transfer function models of the system

Obtain the state variable model from either of these forms of models. Why ?

- Transfer Function Model: Essentially an external description about the system.
- State Variable Model: essentially is an internal description of the system

Philosophy of State Variable Modeling

1. A system having **n- number of independent energy storing elements** can be modelled by **a single n-th order differential equation**.
2. The state variable model essentially **splits the single (one) differential equation of order "n" into n-number of first order differential equations**.
3. Thus a single n-th order differential equation becomes n-number of 1st order differential equations.

Example-1:Philosophy of State Variable Modeling

Consider the system described by

$$\ddot{y} + a_1\dot{y} + a_0 y = u$$

Represent this in state variable form i.e. represent this system with 2-number of first order differential equations.

Solution

- Let us define the state variables as: $x_1 = y$, $x_2 = \dot{y}$.
- From these relations and system equation we get

$$\begin{aligned}\dot{x}_1 &= \dot{y} = x_2 \\ \dot{x}_2 &= \ddot{y} = -a_0 y - a_1\dot{y} + u \\ &= -a_0 x_1 - a_1 x_2 + u\end{aligned}$$

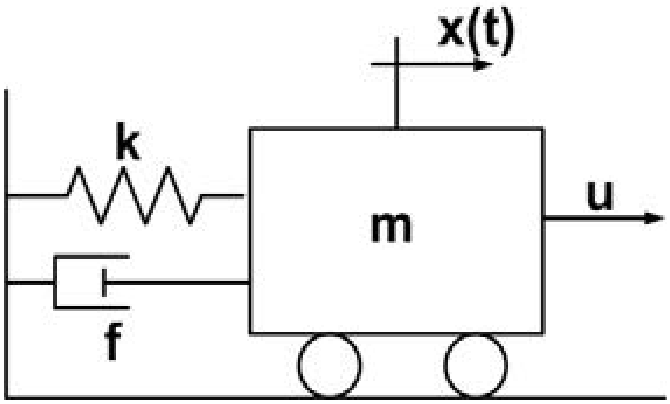
- In matrix form , this can be represented as:

$$\underbrace{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix}0 & 1\\ -a_0 & -a_1\end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{\mathbf{B}} u = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\mathbf{x}} = \mathbf{C}\mathbf{x}$$

Example-2: State Variable Model of Mass Spring Damper System

- Consider the mass spring damper system shown.



- Determine the state variable model of the system considering the output displacement and its derivative as state variables and force as input.

Solution: Example-2: State Variable Modeling of Mass Spring Damper System

- The dynamics of this system is given by

$$m\ddot{x} + f\dot{x} + kx = u$$

- Now the specified state variables are: $x_1 = x$, and $x_2 = \dot{x}$.
- From these relations and system equation we get

$$\dot{x}_1 = \dot{x} = x_2, \quad \dot{x}_2 = \ddot{x} = -\frac{k}{m}x_1 - \frac{f}{m}x_2 + \frac{1}{m}u$$

- In matrix form, it is represented as:

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{f}{m} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{\mathbf{B}} u = \mathbf{Ax} + \mathbf{B}u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} = \mathbf{Cx}$$

Computation of Poles and Eigenvalues of Mass Spring Damper System

- The various matrices in the state space model are called as:
  - $\mathbf{A}$ = System Matrix
  - $\mathbf{B}$ = Input Matrix
  - $\mathbf{C}$ = Output Matrix

  For $m = f = k = 1$,the transfer function of the system is given as:

$$G(s) = \frac{1}{s^2 + s + 1} \tag{1}$$

- Poles of system are located at : -0.5 ± 0.866
- The state equations for these parameters are

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u = \mathbf{Ax} + \mathbf{B}u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{Cx} \tag{2}$$

- The eigenvalues of the system matrix $\mathbf{A}$ is obtained by computing the roots of the following:

$$|\lambda \mathbf{I} - \mathbf{A}| = 0$$

- This gives

$$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| - \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} = 0 \implies \left| \begin{bmatrix} \lambda & -1 \\ 1 & \lambda + 1 \end{bmatrix} \right| = 0$$

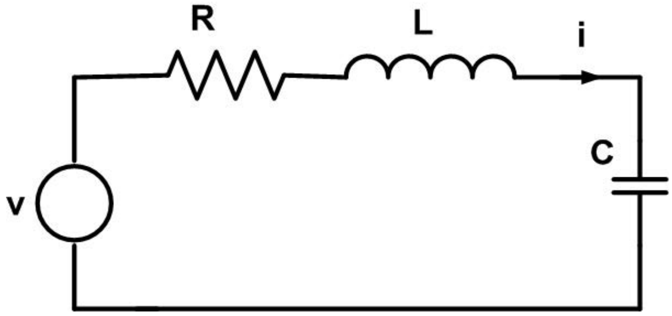
$$\implies \lambda(\lambda + 1) + 1 = 0 \implies \lambda^2 + \lambda + 1 = 0 \tag{3}$$

- This gives $\lambda_1, \lambda_2 = -0.5 \pm j0.866$

Thus the poles of the system and eigenvalues of the system matrix are same. **(provided there is no pole-zero cancellation)**(to be discussed later).

Example-3: State Variable Model of RLC Circuit

- Consider the RLC circuit shown.



- Determine the state variable model of the system considering the voltage across the capacitor and current through the inductor as state variables. The voltage across the capacitor is considered as output.

Solution: Example-3: State Variable Modeling of RLC Circuit

- The differential equation model of this system is given by

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = v, \quad \frac{1}{C}\int idt = v_c$$

- Considering the voltage across the capacitor $v_c(t)$ and current through the inductor $i(t)$ as state variables, gives $x_1 = v_c$ and $x_2 = i(t)$
- The system equations in terms of these state variables are:

$$C\frac{dv_c}{dt} = i \implies C\frac{dx_1}{dt} = x_2 \implies \dot{x}_1 = \frac{1}{C}x_2$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}idt = v \implies Rx_2 + L\dot{x}_2 + x_1 = u$$

$$\implies \dot{x}_2 = -\frac{1}{L}x_1 - \frac{R}{L}x_2 + \frac{1}{L}u$$

- In matrix form, it is represented as:

$$\underbrace{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix}0 & \frac{1}{C}\\ -\frac{1}{L} & -\frac{R}{L}\end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}}_{\mathbf{B}} u = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\mathbf{x}} = \mathbf{C}\mathbf{x} \tag{4}$$

- The eigenvalues are computed from

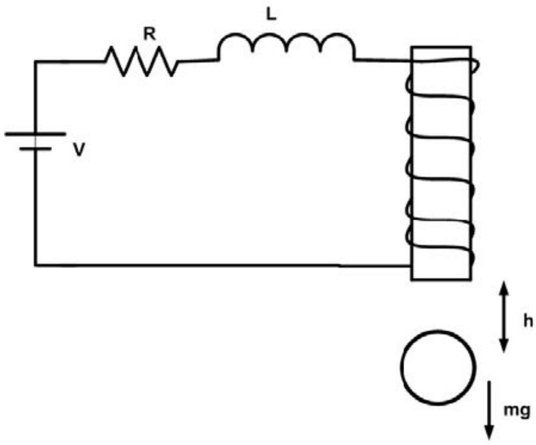
$$\left|\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\right| - \begin{bmatrix}0 & \frac{1}{C}\\ -\frac{1}{L} & -\frac{R}{L}\end{bmatrix} = 0 \implies \left|\begin{bmatrix}\lambda & -\frac{1}{C}\\ \frac{1}{L} & \lambda + \frac{R}{L}\end{bmatrix}\right| = 0$$

$$\implies \lambda(\lambda + \frac{R}{L}) + \frac{1}{LC} = 0 \implies \lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0 \tag{5}$$

- These are same as the poles of the system.

Example-4: State Variable Model of Magnetic Levitation System (contd)

- Consider the magnetic levitation system shown.



- The current through the coil induces a magnetic force which can balance the force of gravity and cause the ball to be suspended in midair.

Example-4: State Variable Model of Magnetic Levitation System (contd)

Let

h = the vertical position of the ball

i =current through the electromagnet

V=applied voltage, m=mass of the ball, g= gravity L=inductance , R=resistance

K = coefficient that determines the magnetic force exerted on the ball.

- The dynamics of magnetic levitation system is given by

Mechanical Dynamics:

$$m\frac{d^2h}{dt^2} = mg - \frac{Ki^2}{h}$$

Electrical Dynamics:

$$Ri + L\frac{di}{dt} = V$$

Example-4: State Variable Model of Magnetic Levitation System

- Considering the vertical position, displacement and current as state variables and the position as output variable gives:
  $x_1 = h$, $x_2 = \dot{h}$, $x_3 = i$ and $u = V$,$y = h$
- The dynamics of the system in terms of these variables is given by

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= g - \frac{Kx_3^2}{mx_1} \\
\dot{x}_3 &= -\frac{R}{L}x_3 + \frac{1}{L}u \\
y &= x_1
\end{aligned}$$

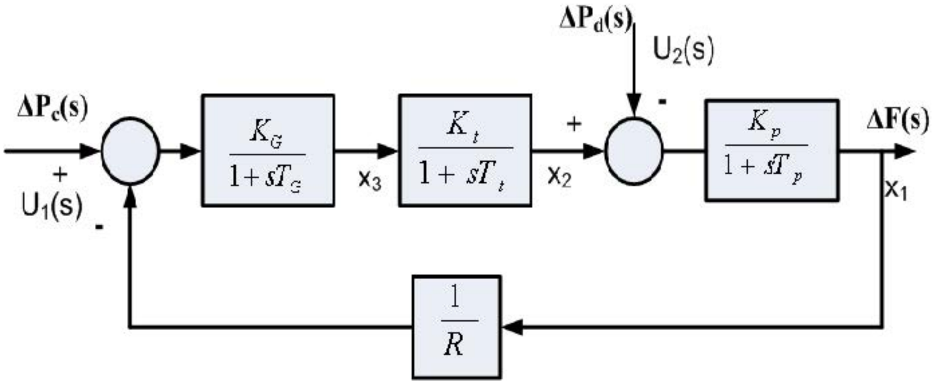
Note

In general, the nonlinear system can not be represented in the standard form of linear system representation.

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

Example-5: State space model of an isolated thermal power system

Consider the single-area thermal power system shown below.



- The objective is to represent this system in state space form considering the outputs of generator block, turbine block and governor block as state variables (as shown in the Figure).

Example-5: State space model of an isolated thermal power system (contd)

- Now for the first state x_1,

$$x_1 = (x_2 - \Delta P_d)\frac{K_p}{1 + sT_p} \quad or, \cancel{x_1}(1 + sT_p)x_1 = K_p(x_2 - \Delta P_d)$$

$$or, x_1 + sT_p x_1 = K_p x_2 - K_p \Delta P_d \quad or, \; sT_p x_1 = -x_1 + K_p x_2 - K_p \Delta P_d$$

$$or, \; T_p \dot{x}_1 = -x_1 + K_p x_2 - K_p \Delta P_d$$

Thus

$$\boxed{\dot{x}_1 = -\frac{1}{T_p}x_1 + \frac{K_p}{T_p}x_2 - \frac{K_p}{T_p}\Delta P_d}$$

Example-5: State space model of an isolated thermal power system (contd)

- For the second state x_2,

$$x_2 = x_3 \left[\frac{K_t}{1+sT_t}\right] \quad or, x_2(1+sT_t) = K_t x_3 \quad or, x_2 + sT_t x_2 = K_t x_3$$

$$or, sT_t x_2 = -x_2 + K_t x_3 \quad or, \; T_t \dot{x}_2 = -x_2 + K_t x_3$$

Thus

$$\boxed{\dot{x}_2 = -\frac{1}{T_t} x_2 + \frac{K_t}{T_t} x_3}$$

Example-5: State space model of an isolated thermal power system (contd)

- Now for the third state x_3,

$$x_3 = \left[u_1 - \frac{1}{R}x_1\right]\frac{K_g}{1+sT_g} \quad or, x_3(1+sT_g) = K_g(u_1 - \frac{1}{R}x_1) \quad or,$$

$$T_g x_3 = -x_3 - \frac{K_g}{R}x_1 + K_g u_1 \quad or,\ T_g\dot{x}_3 = -x_3 - \frac{K_g}{R}x_1 + K_g u_1$$

Thus

$$\boxed{\dot{x}_3 = -\frac{1}{T_g}x_3 - \frac{K_g}{RT_g}x_1 + \frac{K_g}{T_g}u_1}$$

Example-5: State space model of an isolated thermal power system

- Thus the state equations describing the system are

$$\dot{x}_1 = -\frac{1}{T_p}x_1 + \frac{K_p}{T_p}x_2 - \frac{K_p}{T_p}\Delta P_d = -\frac{1}{T_p}x_1 + \frac{K_p}{T_p}x_2 - \frac{K_p}{T_p}u_2$$
$$\dot{x}_2 = -\frac{1}{T_t}x_2 + \frac{K_t}{T_t}x_3$$
$$\dot{x}_3 = -\frac{1}{T_g}x_3 - \frac{K_g}{RT_g}x_1 + \frac{K_g}{T_g}u_1$$

- In matrix form, it is expressed as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_p} & \frac{K_p}{T_p} & 0 \\ 0 & -\frac{1}{T_t} & -\frac{K_t}{T_t} \\ -\frac{K_g}{T_g R} & 0 & -\frac{1}{T_g} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{K_p}{T_p} \\ 0 & 0 \\ \frac{K_g}{T_g} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

SELF STUDY

Self-Study: Formal Definitions of Some Terms

- **State:** The state of a dynamic system is the smallest set of variables (called state variables) such that the knowledge of these variables at $t = t_0$, together with the knowledge of the input for $t \geq t_0$, completely determines the behavior of the system for any time $t \geq t_0$.
- **State Variables:** The state variables of a dynamic system are variables making up the smallest set of variables that determine the state of the dynamic = system. If at least n variables $x_1, x_2, \dots, x_n$ are needed to completely describe the behavior of a dynamic system, then such n variables are a set of state variables.
- **State Vector:** If n state variables are needed to completely describe the behavior of a given system, then these n-state variables can be considered as the n-components of a vector x. Such a vector is called a state vector. A state vector is thus a vector that determines uniquely the system states $x(t)$ for any time $t \geq t_0$, once the state at $t = t_0$ is given and the input $u(t)$ for $t \geq t_0$ is specified.
- **State Space:** The n-dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis,... x_n axis where $x_1, x_2, \dots, x_n$ are state variables, is called a state space. Any state can be represented by a point in the state space.

THE UNIVERSITY OF AUCKLAND
Te Whare Wānanga o Tāmaki Makaurau
NEW ZEALAND

General Structure of State Variable Models

- Consider a multi-input-multi-output (MIMO) system with
  - n-number of state variables $x_1, x_2, \dots, x_n$
  - m-number of inputs $u_1, u_2, \dots, u_m$ and
  - p-number of outputs $y_1, y_2, \dots, y_p$
- The general form of state equations are given by

$$\dot{x}_1(t) = f_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_m; t)$$
$$\vdots$$
$$\dot{x}_n(t) = f_n(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_m; t)$$

- The output equations are written as

$$y_1(t) = g_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_m; t)$$
$$\vdots$$
$$y_p = g_p(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_m; t)$$

THE UNIVERSITY OF AUCKLAND
Te Whare Wānanga o Tāmaki Makaurau
NEW ZEALAND

General Structure of State Variable Models

- The state and output equations can alternately be expressed as:

$$\dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}, \mathbf{u}, \mathbf{t}\right)$$
$$\mathbf{y}(t) = \mathbf{g}\left(\mathbf{x}, \mathbf{u}, \mathbf{t}\right)$$

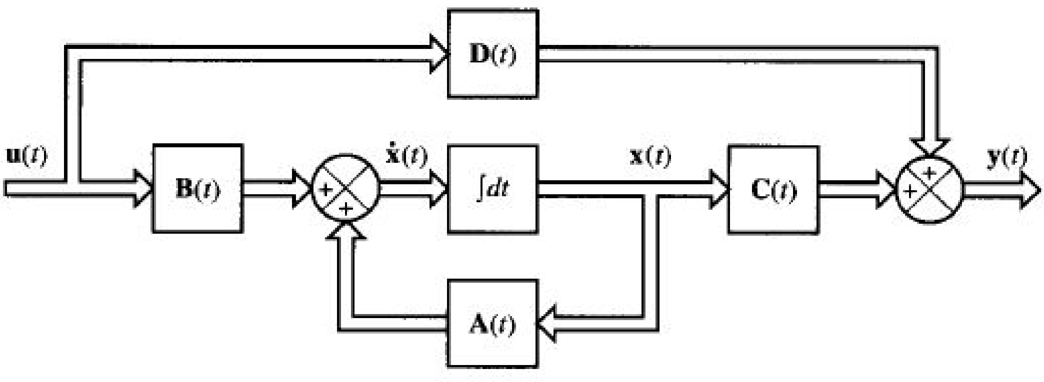
- By linearizing these equations around the operating point gives the following linearized state equations:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)$$

where
- $\mathbf{A}(t)$=System Matrix
- $\mathbf{B}(t)$=Input Matrix
- $\mathbf{C}(t)$=Output Matrix

General Structure of State Variable Models

- The block diagram of general state space model is shown below



Advantages

- These models are more convenient for dealing with multi-input/ multi-output (MIMO) systems (both analysis and design).
- The effects of initial conditions can be taken into account.
- These are also applicable for modeling nonlinear systems.

Disadvantages

- Transfer function representation is restricted to the input-output dynamics only. Therefore, it hides uncontrollable dynamics and unobservable dynamics
- It is bit more difficult to describe how performance specifications should influence the controller design than it is in classical setting
- Feedback control in state-space setting often require knowledge of the full system state, including components which may not be directly measured