



## Survey of transient performance control

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### ABSTRACT

Control systems have played a key role in the advancement of technologies in almost every field. The technical competition in the fourth industrial revolution has now focused on high performance area. Transient performance control is the ultimate high-performance control and can serve applications such as flexible manufacturing, robots, UAVs, defense industry and medical systems. Rich modern control theories had been developed over the last few decades, and the majority of them are on stability related issues instead of performance. Actual applications lag the theories, causing so-called theory–practice gap. One key factor of the gap is scarcity of control design methods for transient performance. Transient performance control is to guarantee the transient performance specifications such as rise-time, overshoot and settling-time, which are truly demanded in practice. To promote transient performance control, this article reviews the control designs for transient performance, including finite-time stability and control, adaptive and prescribed performance control, and funnel control. The challenges and future research directions are addressed. The new solutions should be tested on typical industrial systems in benchmark against the existing control methods to show performance insurance and improvement of the former over the latter.

### 1. Introduction

Control systems are everywhere — manufacturing systems, chemical processes, energy plants, homes and buildings, automobiles and trains, medical devices, cellular telephones and internet, aircraft and spacecraft. Today, it is hard to point to an engineered system in modern industry or life that does not have a footprint of control. In fact, substantial, even revolutionary, advances in products performance have been achieved as a result of relevant systems & control technologies. Furthermore, the recent developments in these fields bring up the systems with the unprecedented scope, scale and complexity such as cyber–physical and human systems, and the required control task becomes more challenging than ever (Lamnabhi-Lagarrigue et al., 2017), demanding high performance on the complex systems at low costs. Nowadays, the technical competition has focused on high performance area. High performance products sell in markets in place of low ones. High performance products rely on high performance control in the end.

Transient performance control is significant as it yields incredible benefits to the end users. Transient performance of a dynamic system is commonly measured by speed (rise time, settling time) and accuracy (overshoot, settling error). For a hard disk drive in the presence of eccentricity uncertainties and external disturbances such as vibrations, the higher the track control accuracy, the more tracks/memory the disk has; the faster the control speed from track to track, the shorter time a given data transmission takes. As a comparison, if a control system is stable only, it cannot be used to carry out a job since the response accuracy and speed are unknown. For an assembling robot in face of load and environment uncertainties, the higher the motion control accuracy, higher precision job the robot does; the faster the motion control speed, shorter time the job is done, meaning higher productivity. In process industry, higher control accuracy produces higher-quality products with greater consistency, while higher control speed yields high efficiency. In addition, the costs of production can be reduced with performance control. When a plant is controlled well, there is less need for human labors to operate and maintain it. In light

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and chemical industry, higher control accuracy allows the products to be nearer the low boundary of the specifications and thus save raw materials and energy. In buildings, high performance technology may save 30% or above energy in HVAC systems, and much less energy use means much less operational cost, pollution and carbon emission. Note however that in the real world today, many systems such as renewable energy and smart grid, waste water processing plants, and techno-social networks lack effective performance control. In other systems such as UAVs in low altitude and low speed, and self-driven cars, autonomous control is still a big challenge. Thus, the quest for high performance is imperative.

The central issue in modern control theory research reported in the literature since 1960s is system stability and related topics such as asymptotic tracking, regulation, consensus, and synchronization. However, transient performance is demanded in control applications. A stable system with low speed, big transient errors or a very long settling time is obviously unacceptable in practice. In fact, only transient performance specifications matter for real control applications, while asymptotic behavior for infinite future is irrelevant. This theory–practice gap has existed for decades and is one main cause why many control theories have not yet been well adopted in industry. There were insufficient research works on control performance and those on it hardly addressed explicit transient performance specifications (rise-time, overshoot and settling-time). This could be because stability issue is of asymptotic analysis and possible to address with helps of rich mathematical tools, whereas the performance is of transient analysis and hard to do with few tools available. The importance of systems performance cannot be overstated since “performance” attributes are almost always the metrics of interests in R & D investment in any business (Samad et al., 2020). Parameters such as speed and accuracy of the system are evaluated and used to claim improvements over the state of the art. In the IFAC survey to assess the industry viewpoint on what is needed for new products and services (Samad et al., 2020), the industry respondents were asked to rank twelve “key drivers for future improvements for the next generation of product/processes and services”, and “Performance” was one top driver. Thus, transient performance control must be addressed for actual control applications in practice. This becomes the objective of the present paper, that is, to survey the control methods for transient performance, and explore the challenges and possible directions. Note further those new technologies such as medical image processing, flexible manufacturing and autonomous driving, lead to complexity levels that cannot be mastered by standard mathematical models and computer simulation tools alone. The technologies with control enhancement must be tested in physical emulators of those complex systems to gain real-time experience, detect design or implementation flaws and demonstrate functionality for end users and the public. Thus, we also encourage the case studies and filed tests of transient performance control designs on industrial and social examples in benchmark against the existing control methods.

The rest of this paper is organized as follows. Transient performance of a dynamic system and its basic control methods are introduced in Section 2. Finite-time stability and control is discussed in Section 3. The adaptive and prescribe performance control is reviewed in Section 4. The funnel and tight control presented in Section 5. The conclusions are drawn in Section 6.

## 2. Fundamentals

We begin with definition of transient performance control. Control of a dynamic system is to influence its output by manipulating its input such that it behaves as one wishes. The plant under control may not behavior as desired, due to uncertain dynamics, disturbances, measurement noise and communication errors. A controller is used to change its dynamic behavior. The dynamic behavior means the output transient response to the certain input, and its quality is called transient performance. Conventionally, it is measured based on the output

transient response to the unit step input, and typical specifications of transient performance on it include rise time, overshoot and settling time. Transient performance control is to make the control system meet transient performance specifications, while stability control or stabilization is to make the control system achieve asymptotical stability. Broadly speaking, a control design which can reshape output transient response is called transient performance control, unless only asymptotic output, or steady-state output response is addressed. Specifically speaking according to degree and type of realized transient performance, a control design may be called full transient performance control if all the given transient performance specifications are met, otherwise it is called partial transient performance control. For example, a design is full transient performance control if three specifications of rise time, overshoot and settling time are all met, while it is partial transient performance control if only one specification such as settling time is met.

Transient performance control is a very difficult problem. The root cause for this difficulty is that there exists no analytical relation between the parameters and transient performance specifications of a general dynamic system except for the special cases such as the standard 2nd-order linear system. Thus, the approximations to the transient performance specifications are made in various control designs. These designs are reviewed according to the approximation characteristics, controller types and design techniques, in this section for basic methods and in sequel sections for advanced methods.

The **proportional–integral–derivative** (PID) control is most common in the control field. The success that the PID controller has enjoyed in industry (about 90% of industrial control systems with PID Tan, Wang, & Hang, 1999) is not only because it is relatively simple to implement and easy to tune, but also because its design is judged in the end in terms of transient performance specifications (rise-time, overshoot and settling-time). Note however that the PID design procedures usually do not explicitly take time domain specifications into account but implicitly address them mostly in frequency domain. Hence, the PID design methods cannot ensure the exact performance, resulting in engineers often resorting to their experience when tuning a PID controller in site. Auto-tuning of PID controllers can largely resolve this issue and has been widely adopted in industry (Wang, Lee, & Lin, 2003). PID may be also tuned by minimizing the ITAE objective function. Note however that the PID control might fail for coupled multivariable systems (Wang, 2003). This problem is well recognized in industry. MIMO PID control becomes essential here and it deals with multiloop couplings and individual loop transient performance (Wang, Ye, Cai, & Hang, 2008). More research on MIMO PID control is required to consider transient performance and enhance applicability of PID control in complex systems.

The **pole placement** is the most popular design method in modern control. It assigns the poles of the closed-loop system by static state feedback and/or dynamic output feedback. But the system zeros affect the transient response greatly. Two SISO systems with the same poles exhibits drastic different dynamics if one is of minimum phase while other is not, see Fig. 1. The same poles are not suitable for two loops of a MIMO system if one loop is fast while other is slow.

One might approach the transient problem from the optimization perspective such as the **linear quadratic regulator** (LQR) and **model predictive control** (MPC) by tuning the weights of the cost function. But this would lead to a trail-and-error with no guarantee of satisfying hard performance constraints.

## 3. Finite-time stability and control

The **Finite-time stability** (FTS) (Dorato, 2006) differs from the classical asymptotic stability in that the system state remains in a bounded set over a given finite time horizon, which could be desirable for the state not to exceed certain bounds during transients. In 1990's, computationally tractable conditions guaranteeing FTS were derived

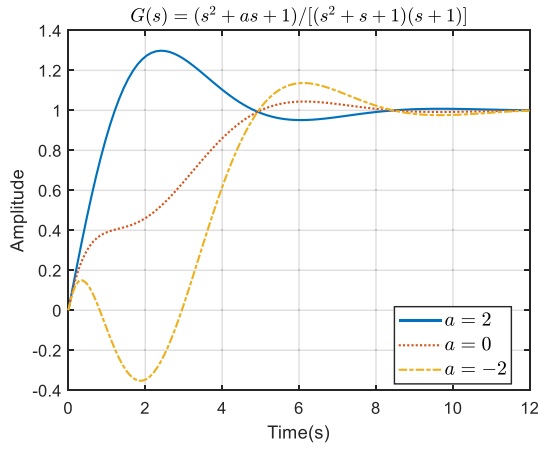


Fig. 1. Output step response of  $G(s) = \frac{s^2 + as + 1}{(s^2 + s + 1)(s + 1)}$ .

in the form of linear matrix inequalities (LMIs). Zhang, Wang, and Sun (2020) refined the definition of finite-time stability to allow the time varying confining set for the state, which reflects the common requirement of decaying error transient. And they gave some sufficient conditions on FTS for non-autonomous nonlinear systems, which become necessary and sufficient for linear time-varying systems. This method is briefly described as follows.

Consider a nonlinear nonautonomous system,

$$\dot{x}(t) = f(t, x), \quad x(t_0) = x_0, \quad (1)$$

where  $t \in [t_0, t_0 + T]$ ,  $x(t) \in \mathbb{R}^n$ ,  $f: [t_0, t_0 + T] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ .

**Definition 3.1** (Zhang et al., 2020). Given an initial time  $t_0$ , a positive scalar  $T$ , a positive definite matrix  $R$ , and a positive definite matrix-valued function  $\Gamma(\cdot)$ , defined over  $[t_0, t_0 + T]$ , with  $\Gamma(t_0) < R$ , a system is said to be finite-time stable with respect to  $(t_0, T, R, \Gamma(\cdot))$  if

$$\|x(t_0)\|_R \leq 1 \Rightarrow \|x(t)\|_{\Gamma(t)} < 1, \quad \forall t \in [t_0, t_0 + T]$$

where  $\|x(t)\|_R = x^T(t)Rx(t)$ ,  $\|x(t)\|_{\Gamma(t)} = x^T(t)\Gamma(t)x(t)$ .

This definition can be interpreted in terms of ellipsoidal domains. The set defined by  $\|x(t_0)\|_R \leq 1$  contains all the admissible initial states. The inequality  $\|x(t)\|_{\Gamma(t)} < 1$  defines a time-varying ellipsoid that bounds the state trajectory over the interval  $[t_0, t_0 + T]$ . Due to the time-varying property of  $\Gamma(t)$ , this definition is more general than the early FTS definition (Wang et al., 2008) requiring

$$x^T(t_0)Rx(t_0) \leq c_1 \Rightarrow x^T(t)Rx(t) < c_2, \quad \forall t \in [t_0, t_0 + T], \quad (2)$$

where  $c_1 < c_2$  and the state can grow larger. Note that Definition 3.1 can be used to force the state to stay in a smaller and smaller set, while one in (2) is impossible.

**Theorem 3.1** (Zhang et al., 2020). The system (1) is FTS with respect to  $(t_0, T, R, \Gamma(\cdot))$ , if there exist a positive scalar  $\eta$ ,  $0 < \eta < 1$ , and a positive definite differentiable function  $V(t, x)$ , such that the following conditions hold:

- (i)  $V(t_0, x_0) < \|x(t_0)\|_R$ ,  $V(t, x(t)) \geq \|x(t)\|_{\Gamma(t)} \forall t \in [t_0, t_0 + T]$ ;
- (ii)  $\frac{dV}{dt} \leq 0$  on the set  $D = \{x | 0 < \eta \leq V(t, x(t)) \leq 1\}$ .

In the early literature,  $V(t, x(t))$  is nonincreasing along all trajectories of (1). Theorem 3.1 here only requires that  $V(t, x(t))$  is nonincreasing on one subregion  $D$ , and thus it is less conservative.

**Example 3.1.** Consider a nonlinear nonautonomous system:

$$\begin{cases} \dot{x}_1(t) = -\exp(t)x_1(t) \sin \frac{1}{1 - x_1(t)^2 - x_2(t)^2} \\ \dot{x}_2(t) = -(t+1)x_2(t) \sin \frac{1}{1 - x_1(t)^2 - x_2(t)^2} \end{cases}$$

Let  $t_0 = 0$ ,  $T = 10$ ,  $R = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$  and  $\Gamma(t) = \begin{pmatrix} t+1 & 0 \\ 0 & t+1 \end{pmatrix}$ . We take  $V(t, x(t)) = x^T(t)P(t)x(t)$ , where

$$P(t) = \begin{pmatrix} t+1.5 & 0 \\ 0 & t+1.5 \end{pmatrix}.$$

$\eta_k = 1 - \frac{1}{2k\pi + \frac{\pi}{4}}$ ,  $\gamma_k = 1 - \frac{1}{2k\pi + \frac{3\pi}{4}}$ ,  $k = 1, 2, \dots$ . From Theorem 3.1, this system is FTS with respect to  $(t_0, T, R, \Gamma(\cdot))$ . Note that  $\Gamma(t)$  meets  $\Gamma(t_1) < \Gamma(t_2)$  for  $t_1 < t_2$ , which results in the monotonously decreasing state trajectory, while the other FTS definitions for nonlinear systems in the earlier literature cannot produce such a case.

In the new framework of Definition 3.1, FTS is studied for nonlinear impulsive systems with switching, logic choice and time delay (Saranan, Ali, & Rajchakit, 2021; Xu & Sun, 2013; Zhang, Sun, & Wang, 2018).

The majority of FT stability and stabilization (Amato, Ambrosino, Ariola, Cosentino, & Tommasi, 2014) in the literature had been under the assumption of ellipsoidal domains due to their relationship with quadratic functions and LMIs which are easy to solve. Note that it is the most common case that practical systems are subjected to polyhedral constraints, for example, upper and lower limits on state or input variables, which motivates use of polyhedral domains. Moreover, polyhedral sets give independence of variable constraints, whereas ellipsoidal sets introduce conservatism due to variables' couplings. Esterhuizen and Wang (2020b) dealt with finite-time stability and stabilization for discrete-time linear systems subject to a bounded disturbance with regard to polyhedral domains, where the convex conditions are given for their solutions which can be obtained by linear programming.

It should be pointed out that the existing works on FTS have been on analysis of a nominal system with unbounded input. The attention should be paid to control design for finite-time stabilization of an uncertain system under input saturation. Finite-time stability for discrete-time nonlinear systems is not seen yet.

Bhat and Bernstein (2000) introduced the **finite-time control** which drives the system state to the origin in finite time. The design methods of finite-time control include Lyapunov differential inequalities-based approach, homogeneous systems theory, terminal sliding mode method. Recently, some smooth finite-time control methods, such as prescribed-time control approach, and finite-time prescribed performance control method, have attracted a lot of attention.

The Lyapunov differential inequalities-based approach includes adding a power integrator technique (AAPI), finite-time dynamic surface control method (FDSC) and finite-time command filtered backstepping (FCFB) method. The AAPI was proposed by Lin and Qian (2000), which takes advantage of the characteristics of the system in the feedback design, particularly, using feedback to dominate nonlinearity rather than to cancel it. In Huang, Lin, and Yang (2005), AAPI method was used for the global finite-time stabilization of the uncertain nonlinear system with lower-triangular structure. Although AAPI method is popular in finite-time control design of various nonlinear systems, the "explosion of complexity" problem cannot be avoided. To handle this, FDSC method was proposed. The FDSC method combining with neural networks or fuzzy systems was reported for strict feedback nonlinear systems (Li & Li, 2021), non-strict feedback systems (Li, Li, & Tong, 2019) and non-affine nonlinear systems with dead zone (Chen, Wang, Liu, et al., 2019; Fu, Wang, Yu, & Lin, 2021). However, the filtered errors are not considered in the FDSC method. Another modified finite-time control method is the FCFB method developed by Yu, Shi, and Zhao (2018), in which filtered errors are reduced by introducing the error compensation mechanism. FCFB method has been widely used in various control problems for different kinds of systems, such as finite-time state feedback control for strict-feedback nonlinear systems (Zhao, Yu, & Wang, 2021), multi-agent systems (Zhao, Yu, Lin, & Ma, 2018), MIMO nonlinear systems (Cui, Yu, & Wang, 2022), switched nonlinear systems (Li, Ahn, & Xiang, 2021), Markov jumping nonlinear

systems (Zhao, Yu, & Shi, 2022) and stochastic nonlinear systems (Gao & Guo, 2019).

The **terminal sliding mode control** (TSMC) is a discontinuous finite-time control method, which was proposed by Venkataraman and Gulati in 1991 (Venkataraman & Gulati, 1991). It is based on the notion of terminal attractors to ensure finite-time convergence of the state while preserving robustness of the sliding mode control against the model error and disturbance. Man, P. Paplinski and Wu (1994) provided a TSMC scheme for rigid robotic manipulators. However, when the system state is far away from the equilibrium, the TSMC may not deliver the same convergence performance as the SMC based on linear switching hyperplanes. Yu and Man (2002) proposed a fast TSMC model that combines the advantages of TSMC and SMC, where fast (finite time) transient convergence of the state is achieved both at a distance from and at a close range of the equilibrium. Another problem in TSMC is the singularity, and the efforts have been done to solve this problem (Feng, Yu, & Man, 2002). Since the controllers designed in the aforementioned works were discontinuous, the chattering was inevitable, which is not allowed in practice. To reduce or eliminate chattering, several solutions have been proposed. For example, the boundary layer approach was applied in order to alleviate the chattering. However, the finite-time stability was lost because of the asymptotic stability in the boundary layer. Another solution was given in Yu, Yu, Shirinzadeh, and Man (2005), where a continuous finite-time control strategy for robotic manipulators using TSMC was proposed without losing finite-time stability.

The aforementioned finite-time control approaches are continuous at best but non-smooth. Song, Wang, and Holloway (2017) presented the **prescribed-time control**, which is smooth control and allows users to preset the settling time exactly and arbitrarily. Moreover, even in the presence of non-vanishing (though matched) uncertain nonlinearities, the regulation can still be achieved in prescribed finite time. The prescribed-time control approach employs a scaling of the state by a function of time that grows unbounded towards the terminal time and is followed by a design of a controller that stabilizes the system in the scaled state representation, yielding regulation in prescribed finite time for the original state. In Song, Wang, and Krstic (2019), Song et al. summarized several advantages of prescribed-time control over traditional finite-time control methods. (i) prescribed-time control is smooth control, (ii) convergence time can be preset as needed within the physically allowable range and (iii) compared with traditional finite-time control that uses fractional Lyapunov differential inequality for stability analysis, prescribed-time control avoids the difficulty in controller design and stability analysis encountered in the finite-time control for high-order systems. In Chen and Zhang (2019), on the basis of prescribed-time control, output-feedback control strategies were proposed for lower-triangular nonlinear nonholonomic systems. In Krishnamurthy, Khorrami, and Krstic (2020), prescribed-time output-feedback control for a class of nonlinear strict-feedback-like systems with state-dependent uncertainties was considered. Subsequently, prescribed-time control is applied to the consensus of multi-agent systems. The concept of practically prescribed-time stability was proposed in Wang, Liang, and Sun (2020), and an adaptive prescribed-time control scheme for a class of nonlinear tele robotic systems with actuator faults and position error constraints was presented. Recently, another prescribed-time control method in the form of linear time-varying high-gain feedback designed from solutions to a class of parametric Lyapunov equations (PLEs) was proposed by Zhou in Zhou (2020a, 2020b). In Zhou and Shi (2021), this method was used for the prescribed-time global stabilization for a class of uncertain nonlinear system.

Finite-time prescribed performance control (FPPC) is another smooth finite-time control method and was inspired by prescribed performance control. With the help of a finite-time performance function (FTPF) which can converge to a predefined arbitrarily small set in a finite time, and a monotonically increasing and bounded error transformation function (ETF) or barrier Lyapunov function, the control system

can achieve the prescribed transient and steady-state performance in a specified time. Based on the FTPF and the ETF proposed in Liu, Liu, and Jing (2019), some FPPC results have been obtained in Qiu, Wang, and Sun (2021), Sun, Wu, and Sun (2020), Wang, Bai, and Zhao (2021) and Zhou, Gao, and Li (2021). The adaptive FPPC for stochastic nonlinear systems with unknown virtual control coefficients was addressed in Liu, Gao, and Liu (2021), and to remove the condition of the initial tracking error being smaller than the initial value of FTPF, a smooth shifting function was introduced to reduce the tracking error before the settling time. In order to accelerate the convergence rate, an FTPF with parameters that can be adjusted in real time on the basis of the tracking error was introduced in Liu, Liu, Wang, et al. (2020).

In the past three decades, finite-time control had developed rapidly. But there are many challenges which offer us potential research directions as follows.

(1) Lyapunov differential inequalities-based approach, including AAPI, FDSC and FCFB, is more suitable for control design for uncertain and high-order nonlinear systems. However, the design process is rather complex. The prescribed-time control and FPPC methods as smooth finite-time control allow the users to predefine the settling-time as needed within any physically allowable range, and the design process are simpler. Smooth finite-time control method needs further development. At now, there are few forms of time-varying scaling function in prescribed-time control, and FTPF in FPPC. More forms should be developed and the impact of different forms of functions on system transient performance should be analyzed. The Lyapunov stability theory for prescribed-time control is to be further studied.

(2) At now, TSMC faces chattering and singularity, unknown relation between the controller parameters and system performance, lack of discrete-time version, and difficulty in implementation.

(3) The finite-time control for time-delay systems should be addressed. One key issue is how to construct a Lyapunov function for finite-time stability analysis of the closed-loop system with delay.

(4) The practical application of finite-time control theory is lacking. More research energy is needed to bring the theory to practice.

#### 4. Adaptive and prescribe performance control

**Adaptive control** incorporates on line control updating to adapt to the plant change or uncertainty. Self-tuning control combines an on line parameter estimator with control law update. **Model reference adaptive control** (MRAC) designs a reference model which represents the desired closed-loop performance and makes the system behavior close to the model. Numerous simulations indicate that the transient response of adaptive systems may be unacceptable due to large initial swings. An example was presented in Zang and Bitmead (1994), where an extremely poor transient behavior occurs together with ideal asymptotic performance. Therefore, it is necessary that in the adaptive control systems both transient performance analysis and asymptotic behavior are addressed.

Consider the following uncertain dynamic system,

$$\dot{x} = Ax + B[f(x) + u] \quad (3)$$

where  $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$  is the system state,  $u \in \mathbb{R}^m$  is the control input;  $A \in \mathbb{R}^{n \times n}$  and  $B \in \mathbb{R}^{n \times m}$  are system matrices, and the pair  $(A, B)$  is controllable and  $\det(B^T B) \neq 0$ ;  $f(x) \in \mathbb{R}^m$  is an unknown linear/nonlinear function. The purpose of the control design is to obtain an adaptive control to make the state  $x$  of system (3) track the state  $x_r$  of the following desired reference model

$$\dot{x}_r = A_r x_r(t) + B_r r(t).$$

where  $x_r \in \mathbb{R}^n$  is the state of the reference model,  $r \in \mathbb{R}^m$  is a given bounded command,  $A_r \in \mathbb{R}^{n \times n}$  is a Hurwitz system matrix and  $B_r \in \mathbb{R}^{n \times m}$  is the input matrix. The control objective of the model reference adaptive control with improved transient performance is to



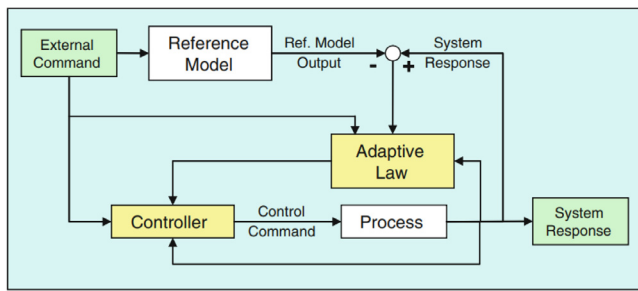


Fig. 2. Model reference adaptive control.

design a  $u(t)$  such that the convergence rate of  $e = x - x_r$  could become faster, i.e., faster than a given exponential signal, see Fig. 2.

To guarantee the transient performance of adaptive control systems is a challenging issue. The major difficulty is that in adaptive control design, either the backstepping approach or the gradient-descent certainty-equivalence method, the transient behavior of the control system could not be quantified directly. The slow estimation of the unknown parameters will also make the transient performance of control system poor. However, large gains in the learning rate of the adaptive laws can enlarge the measure noise and errors, violate the actuator rate and saturation constraints, and excite unmodeled system dynamics. Therefore, a critical trade-off between system stability and control adaptation rate exists in most adaptive control approaches. With helps of stability analysis of adaptive control systems and the robustness enhancement, several attempts have been made to analyze the transient performance of adaptive control systems.

In Miller and Davison (1991), a model reference adaptive control with an LTI compensator together with a switching mechanism is proposed, instead of forcing the error between the output of the plant and the reference model output converging to zero, the final error will be confined within an arbitrarily small constant after an arbitrarily short period of time. In Sun (1993), a modified MRAC scheme was proposed to improve the transient performance of adaptive systems while maintaining ideal asymptotic properties possessed by a standard MRAC. The estimation error, which is generated by the identification scheme, is used directly as a control signal to counteract errors resulting from the certainty equivalence design. In Krstic, Kokotovi, and Kanellakopoulos (1993), the  $\mathcal{L}_2$  and  $\mathcal{L}_\infty$  bounds of a new class of adaptive controllers are analyzed and the error state of the adaptive system can be made arbitrarily small, except for the parameter estimate error  $\tilde{\theta}(t)$ , which is bounded by a constant proportional to  $\tilde{\theta}(0)$ . The performance bounds are computable and are explicit functions of initial conditions and design parameters. In Chien and Fu (1992), a new robust MRAC scheme for a class of multivariable unknown plants is presented using the concept of variable structure design to improve the performance of the output-tracking property by a Lyapunov approach without persistent excitation. In Ydstie (1992), it was shown that a discrete adaptive controller and a fixed gain gradient estimator with modifications can be applied to linear systems with mismatched uncertainties so that the transient performance of the adaptive system is related to the size of the external perturbations. In Ortega (1993), it was shown that the  $\mathcal{L}_2$  norm of the tracking error is uniformly bounded by the initial parameter estimation error if the normalization of parameter adjustment is abandoned, and if the initial conditions are sufficiently small. In Datta and Ioannou (1994), two criteria to assess transient performance in MRAC under the ideal and non-ideal situations were proposed, i.e., the mean square tracking error criterion and the  $\mathcal{L}_\infty$  tracking error bound criterion, and a modified scheme was given which can have an arbitrarily improved nominal performance in the ideal case and in the presence of bounded input disturbances. In Datta (1994), it was shown that the similar control scheme in continuous control systems does not yield a similar transient behavior for discrete-time

control systems. In Narendra and Balakrishnan (1994), a stable control strategy is proposed for improving the transient response by using multiple models of the plant where the models are identical except for initial estimates of the unknown plant parameters.

The backstepping was proposed as an alternative for designing adaptive controllers. It is a Lyapunov-based design that makes the main loop strictly passive. The resulting controller is nonlinear and, in most cases, has no equivalent certainty-equivalence based regulator. In Ikhouane, Rabeh, and Giri (1997), an adaptive controller that ensures the closed-loop global stability and asymptotic performances are designed with backstepping approach and the transient behavior is quantified for the backstepping adaptive controller. A  $\mathcal{L}_\infty$  bound on the tracking error is explicitly given as a function of the design parameters with skew symmetric structure of the system matrix  $A_z(z, t)$  and the negativity of its diagonal terms.

In Yucelen and Haddad (2013), an adaptive control architecture for nonlinear uncertain dynamical systems was presented to achieve fast adaptation using high-gain learning rates, which involves a modification term in the update law. This term filters out the high-frequency content contained in the update law while preserving asymptotic stability of the system error dynamics. In Ichmann E. P. Ryan and Sangwin (2002), the transient response of MRAC for nonlinear aircraft systems with unmatched dynamics is enhanced by developing a new compensator containing the undesired transient residual error to modify the external command. An error feedback term in the reference model to accelerate the convergence of the tracking error is added to diminish the potential high-frequency oscillations induced by the high-gain learning in the adaptive law. In Lavretsky (2011) and Stepanyan and Krishnakumar (2012), new MRAC architectures were developed by introducing an observer-like feedback term in the reference model, which was subsequently named as the closed-loop reference model (CRM). The key idea is to add an observer-like term containing the state error between the reference model and the controlled system in the reference model. Then, the associated feedback coefficient can change the eigenvalues of the closed-loop system to shape the error convergence response. In Na, Herrmann, and Zhang (2017), transient performance is improved by introducing a nonlinear compensator to reshape the closed-loop system transient and a new adaptive law to guarantee convergence. In Yang, Na, and Gao (2020), a MRAC design scheme to improve its transient control response was proposed with a compensator to on line extract the undesired dynamics in the online learning, which is incorporated into the reference model and control simultaneously. An error feedback term is incorporated into the reference model to speed up the convergence of both the compensator and tracking error. Moreover, a leakage term containing the estimation error is constructed and then added to the adaptive law to guarantee the convergence of both the estimation error and tracking error. To reveal the mechanisms behind these proposed methods, a new methodology to analyze the transient error bounds based on  $\mathcal{L}_2$ -norm and Cauchy-Schwartz inequality is also developed.

Based on the literature reviews, the future research work on improved transient performance of adaptive control systems lies in several directions. The first one is to improve the control architecture of the model reference adaptive control to allow faster convergence rate of both tracking error and parameter estimation. On the other hand, the control systems are often with external disturbances, which deteriorate the transient performance. Therefore, how to attenuate and quantify the effects of disturbances on transient performance as well as the tracking errors in sense of  $\mathcal{L}_2$  and  $\mathcal{L}_\infty$  is another research direction in robust adaptive control. The funnel control with guaranteed transient performance is also worthy of further exploration, which gives inspiration to the Prescribed Performance Bounds (PPB) control directly.

The Prescribed performance control (PPC) was proposed by Bechlioulis and Rovithakis (2008) in 2008, which gives a quantitative way to achieve the reference tracking with the predefined performance. By prescribed performance, it is meant that the tracking error converges

to a predefined arbitrarily small set, with convergence rate no less than a preassigned value, which ensures settling time less than a pre-specified constant. This is accomplished by recasting the “constrained” system into an equivalent “unconstrained” one via an appropriate transformation, and stability of the latter leads to a solution for the original performance problem. The early results on PPC were based on the specific model information or the approximation or identification technique instead. This method was applied with improvements to the SISO pure-feedback systems (Bechlioulis & Rovithakis, 2014), the SISO cascaded nonlinear systems (Bechlioulis & Rovithakis, 2011), and the SISO p-normal nonlinear systems (Lv, De Schutter, Cao, & Baldi, 2022).

PPC of the SISO nonlinear systems in a lower-triangular structure was proposed in Zhang and Yang (2017), where the recasting transformation is applied to both the tracking error and the intermediate errors. In this framework, the control design necessitates only the control direction and the initial condition of the system. To disassociate the control design from the initial condition of the closed-loop system, a tuning function-based PPC approach was devised. The tuning function is adopted to adjust the tracking error and the intermediate errors. In this way, the problem of global prescribed-time prescribed-accuracy convergence of the errors is transformed to that of local constant constraint problem for the adjusted errors which can be ensured by the conventional PPC method. Therefore, global PPC of the SISO nonlinear lower-triangular systems is accomplished via off line control designs. On the other hand, taking advantage of the potential of performance monitoring of PPC, a supervisory switching-based fault-tolerant PPC approach was proposed (Zhang & Yang, 2020c) for the SISO strict-feedback systems subject to actuator failures. Therein, the performance functions serve as the monitoring functions for fault detection and control switching (to the backup actuator). To eliminate the need for the known control directions, an orientation function-based PPC approach was put forward (Zhang & Yang, 2019b). The orientation function is driven by the error transformation function and switches its sign between positive and negative with increasing or decreasing of the error transformation function. This means that the sign adaptation of the control signal is performed in the phase of the error close to the performance boundary rather than violating the boundary. Therefore, the prescribed performance especially in the transient phase is guaranteed. In this way, model-free PPC for the nonlinear lower-triangular systems is achieved. An extension to output-feedback model-free global PPC was given with the help of the tuning function and the input-driven filter (Zhang, Wang, & Ding, 2021).

By feat of the robustness of PPC to the output or state interconnections, decentralized PPC for the interconnected nonlinear systems was reported (Bikas & Rovithakis, 2019), where each subsystem is in the lower-triangular structure. For the inputs-coupled MIMO nonlinear systems in the Byrnes–Isidori canonical form, a decoupling PPC approach was proposed. Therein, a diagonal matrix composed of the partial derivatives of the error transformation functions is introduced to the PPC law to deal with the inputs coupling in the case where the system is strongly controllable. On this basis, a control elements selection strategy was introduced (Theodorakopoulos & Rovithakis, 2016) to the decoupling PPC design to guarantee zero overshoot for the tracking errors; the combination of the decoupling PPC approach and the recursive design strategy yields solutions to PPC of the MIMO nonlinear systems in a block triangular structure even in the case of nonuniform relative degrees. In view of the fact that the system may lose the strong controllability due to for example actuator faults, a novel decoupling PPC approach based on a mixed-gain compensation strategy was proposed (Zhang & Yang, 2020b). Therein, the norm of the partial derivatives matrix multiplying the error transformation vector is introduced to the decoupling PPC design to adapt the control gain. This ensures that the control gain is strengthened automatically based on the system performance to compensate for the lost effectiveness of the actuators. Therefore, the requirement for the strong controllability of the plant under decoupling PPC is relaxed to only the controllability.

In this spirit, an adaptive version of this approach was developed for the first-order inputs-coupled MIMO nonlinear systems (Zhang & Yang, 2019a), in which the mixed-gain compensation strategy is adopted in an adaptive way to compensate for the unknown nonlinearities rather than just the parametric uncertainties. And this guarantees asymptotic stability of the closed-loop system which in general fails to be achieved by most nonlinear control methods.

The aforesaid decentralized PPC and decoupling PPC methods are effective for the fully-actuated systems. In engineering applications especially for motion control systems, however, various vehicle systems such as surface vehicles, underwater vehicles and air vehicles are in underactuated modes (the system assembled with fewer actuators than its degrees-of-freedom). For the surface or underwater vehicles, the early results on PPC include the concept of the approach angle (Chen, Cui, Yang, & Yan, 2020), the azimuth angle (Dai, He, Wang and Yuan, 2019), or the transverse function (Dai, He and Lin, 2019) to cope with under-actuation. However, this may cause the singularity or discontinuity issue of the control input when the tracking error goes across zero, or weaken the robustness of PPC due to the need for the known inertial matrix of the vehicle. Instead of involving these concepts, the distance error and the orientation errors with respect to the reference trajectory are introduced to formulate the control objective and present the dynamics of the closed-loop system (Bechlioulis, Karras, Heshmati-Alamdari, & Kyriakopoulos, 2017; Zhang & Yang, 2020a). In this way, the underactuated system is rewritten by a fully-actuated system in which the under-actuation problem is transformed to the controllability problem. For the new system, the trajectory tracking and the controllability can be simultaneously guaranteed by stabilizing the distance error and the orientation errors under the predefined constraints, yielding solutions to singularity-free continuous robust PPC of underactuated surface or underwater vehicles. Following this idea, some extensions to the underactuated air vehicles were made (Marantos, Bechlioulis, & Kyriakopoulos, 2017; Verginis, Bechlioulis, Soldatos, & Tsipianitis, 2022). In this framework, however, the specific initial condition of the control system and the uniform boundedness of the un-actuated degrees of freedom are assumed. To relax these assumptions, the adaptive technique (Zhang & Chai, 2022) and a tuning function-based initialization technique (Zhang, Yang, & Chai) are introduced to the PPC designs, respectively.

Prescribed performance control can meet a given settling-time. Its solution may need arbitrary fast and large control action, which is impractical, and thus may fail for input saturation and time delay. We see the following issues with PPC.

(1) Input saturation. According to the current theory of PPC, time for the tracking error to converge to the small set can be arbitrarily chosen by the designer. In practice, however, the settling time of the control system depends on the input constraint of the plant, e.g., the maximum amplitude of the actuator output. A physical system has a finite actuator capacity or limited amplitude of the plant input. PPC may need extremely large control action which cannot be realized due to input saturation. Our simulation shows that instability can occur in such a case, indeed.

(2) Time delay. Any engineering system has some time delay. Signal transmission and/or processing can cause time delay even if the physical system is delay-free. PPC needs immediate control action which cannot be realized due to time delay in the signal flow.

(3) Complete specifications. PPC can meet a given settling-time but does not include other transient specifications (rise time & overshoot) into design. A practical control system should meet all three transient performance specifications simultaneously.

(4) Discrete-time PPC. The measurement, transmission and computation in modern control systems are in a discrete-time manner. This however challenges the PPC methodology because the continuous behavior of the constrained error close to the performance boundary has to be captured to update the control signal.

(5) Distributed fault-tolerant PPC. The mixed-gain compensation scheme in the fault-tolerant PPC method involves all the errors in the

closed-loop system. In the context of MASs, this means that all the neighborhood errors should be available for the local controller to deal with faults.

We propose to address those issues with PPC as follows.

(1) Input saturation. There were some specific methods to treat input saturation. Better methods should be sought. We use a nonlinear transformation,  $v \in \mathbb{R}$ ,  $u = g(v)$  such that  $u$  is bounded for unconstrained  $v$ . Such a function can be easily chosen and it imposes additional nonlinearity on the resulting system and it effects solvability of the problem, which is reasonable due to limited inputs. Under input saturation, the solvability problem occurs and therefore there will be the boundaries for the specifications. Finding the boundaries for the specifications is important for practical control applications.

(2) Time delay. We consider input delay as it can represent lumped delay in the process and signal transmission. We apply the Homotopy Analysis Method (HAM) (Liao, 2011) which solves complex nonlinear differential equations. In the delay case, HAM (which is based on the concept of the homotopy from topology) can generate from the known solution on the delay-free system a convergent series solution for the delay system. To be specific, consider a nonlinear system with delay,

$$\dot{x}(t) = f(x(t), u(t - \tau)) \quad (4)$$

where  $\tau > 0$  is the time delay. We construct a zeroth-order deformation equation:

$$(1 - q)(\dot{X}(t, q) - f(X(t, q), u(t))) = c_0 q (\dot{X}(t, q) - f(X(t, q), u(t - \tau))) \quad (5)$$

where the nonzero constant  $c_0$  is the convergence-control parameter and chosen by the user. When  $q = 0$ , Eq. (5) becomes the delay-free system for which the existing method has the solution  $X(t, 0) = x_0(t)$ . When  $q = 1$ , (5) reduces to (4) and  $X(t, 1) = x(t)$  is the solution of (4). Formally, expand the solution  $X(t, q)$  of (5) into a Taylor series at  $q = 0$  to get

$$X(t, q) = \sum_{k=0}^{\infty} x_k(t) q^k$$

where  $x_k = \frac{1}{k!} \frac{\partial^k X(t, q)}{\partial q^k} \Big|_{q=0}$  can be determined recursively from the equation resulting from differentiating (5) with regards to  $q$  and setting  $q = 0$ ,  $k = 1, 2, \dots$ . Choose  $c_0$  such that the above series converges at  $q = 1$ . Then we obtain the solution to (4) as

$$x(t) = X(t, 1) = \sum_{k=0}^{\infty} x_k(t)$$

(3) Complete specifications. We change the problem formulation to include all three transient specifications (rise time, overshoot and settling time) into control design. We split the performance into three stages in time. Refer to Kurzhanski (2014) where the specifications are met by upper-bounding and lower-bounding functions in each sampling period. We can use two continuous functions (upper and lower ones) to represent them. We split these functions to three stages over time: the first stage for rise time, the 2nd one for overshoot and the third one for settling time. Then, the control design is to meet these constraints with the closed-loop stability and signal boundedness. One can do it stage by stage with the same design method as one stage method.

## 5. Funnel and tight control

In Ilchmann, Ryan, and Sangwin (2002), the universal tracking control of a class of dynamical systems modeled by functional differential equations with prescribed transient behavior is investigated. The control objective is to ensure that for an arbitrary  $R^M$ -valued reference signal  $r$  of class  $W^{(1,\infty)}$  (absolutely continuous and bounded with essentially bounded derivative), the tracking error  $e$  between the plant output and reference signal evolves within an envelope or **funnel** (Fig. 3) in the sense that  $\varphi(t)\|e(t)\| < 1$  for all  $t \geq 0$ , where  $\varphi$  is a

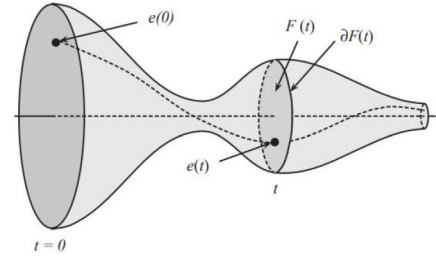


Fig. 3. Funnel control.

prescribed real-valued function of class  $W^{(1,\infty)}$  with the property that  $\varphi(s) > 0$  for all  $s > 0$  and  $\liminf_{s \rightarrow \infty} \varphi(s) > 0$ .

In Ilchmann, Ryan, and Trenn (2005), the tracking of a reference signal (bounded with essentially bounded derivative) for a class of nonlinear systems is investigated with prescribed accuracy: given  $\lambda > 0$  (arbitrarily small), determine a feedback strategy which ensures that for every admissible system and reference signal, the tracking error  $e = y - r$  is ultimately smaller than  $\lambda$  (that is,  $\|e(t)\| < \lambda$  for all  $t$  sufficiently large) and with prescribed performance: the evolution of the tracking error should be contained in a prescribed performance funnel  $F$ , where the nonlinear, memoryless feedback control gain is given by  $k(t) = \mathcal{K}_F(t, e(t))$ , where  $\mathcal{K}_F$  is any continuous function exhibiting two specific properties, the first of which ensures that if  $(t, e(t))$  approaches the funnel boundary, then the gain attains values sufficiently large to preclude boundary contact, and the second of which obviates the need for large gain values away from the funnel boundary. The theory is applicable to dynamical systems of low relative degree with the same number of inputs as outputs (Ilchmann, Ryan, & Townsend, 2007). Later works (Hopfe, Ilchmann, & Ryan, 2010a, 2010b) considered constrained inputs in the formulation. The target tube problem was introduced in Bertsekas and Rhodes (1971), where one specifies a time-varying set, called the target-tube, over a finite time horizon and then seeks a control law that keeps the state in this tube. Kurzhanskiy and Varaiya (2011) addressed the problem using “solvability set”, where at every time instant, a control value is selected such that the state evolves into the solvability set.

We would like to point out that the existing methods as surveyed above or many others in the literature treat the transient performance specifications with approximations to facilitate their solutions. They impose certain kinds of symmetric error bounds such as an ellipsoidal target tube and uniform decay rates in place of the performance specifications but cannot explicitly tell or exactly meet time domain performance specifications. There are normally only two types of control tasks: tracking and regulation. For tracking control, the system output needs to track a reference such as a unit-step function. Typically, the output will rise from zero, over-shoot and settle down to one. The uniform bound or delay rate on the output or the output error does not represent such realistic time-domain performance specifications exactly. For regulation control, the output is regulated against the disturbance to settle down at the original value. Typically, the error will deviate from zero, increase in magnitude (either positive or negative) and eventually settle down to zero. Once again, the uniform bound or delay rate on the output/errors does not represent such realistic time-domain performance specifications exactly.

The **polyhedral tubes control** (PTC) was proposed by Esterhuizen and Wang (2017, 2020a) to achieve full transient performance control. The explicit and exact conversion is made from the transient performance specifications into a series of the polyhedral state sets which vary over time with their boundaries matching the performance specifications at any time. A MIMO uncertain system is considered, and a theorem is established that gives necessary and sufficient conditions for the state to evolve from one polyhedral subset of the state-space



to another. Then, an algorithm is presented which constructs a time-varying linear output feedback law which guarantees that the state evolves within a time-varying polyhedral target tube specifying the system's desired transient performance. The generalizations are also made involving constraints on the control signal and a bounded additive disturbance. This formulation is very general and includes the reference tracking with any desired transient behavior in the face of disturbances, as specified, for example, by the most popular step response specifications. The resulting solution is demonstrated by practical examples. This method is briefly described as follows.

We consider a discrete-time linear system,

$$x(k+1) = Ax(k) + Bu(k),$$

$$y = Cx(k),$$

$k = 0, \dots, K$ , where  $x(k) \in \mathbb{R}^n$  is the state,  $u(k) \in \mathbb{R}^m$  is the control and  $y(k) \in \mathbb{R}^p$  is the output. The finite horizon length is specified by  $K \in \mathbb{N}$ . With  $M \in \mathbb{R}^{q \times r}$  and  $m \in \mathbb{R}^q$ ,  $\mathcal{P}(M, m) = \{x \in \mathbb{R}^n : Mx \leq m\}$  denotes a polyhedral set.

Suppose that the traditional transient performance specifications are that the  $i$ th output variable,  $y_i$ , initiating at  $y_i(0)$ , reaches a set-point,  $y_i^{sp}$ , within a settling-time,  $t_s$ , and with a steady-state error of  $\lambda_s$ . Moreover, suppose this should occur within a peak over-shoot of  $y_i^p$  and that the output variable rises to within  $\lambda_r$  of  $y_i^{sp}$  with a rise-time of  $t_r$ . Letting the sampling time be denoted by  $T_s$ . For each time instant  $t = kT_s$ ,  $y_i(kT_s) = C_i x$  should meet  $\underline{h}_i(kT_s) \leq C_i x \leq \bar{h}_i(kT_s)$ ,  $k = 1, 2, \dots, K$ , where  $\underline{h}_i(t)$  and  $\bar{h}_i(t)$  are the chosen lower and upper bound functions which enclose all the output transients which meet the specifications as shown in Fig. 4. The above inequalities become the constraints on the state and the constrained state is in a polyhedral set with appropriate  $M$  and  $m$  derived from  $C_i$ ,  $\underline{h}_i$  and  $\bar{h}_i$  of the system model and specifications. Thus, they are collected to represent the desired transient performance in terms of the state with a time-varying polyhedral target tube,  $\mathcal{H}(k) \in \mathbb{R}^n$ ,

$$\mathcal{H} = \mathcal{P}(Q(k), \phi(k)), \quad k = 0, 1, \dots, K,$$

The PTC goal is to find a time-varying output feedback such that the state remains in the target tube over the horizon. That is, find an  $F(k) \in \mathbb{R}^{n \times p}$  such that, with  $u(k) = F(k)y(k)$  for  $k = 0, 1, \dots, K-1$ , we have  $x(k) \in \mathcal{H}(k)$  for  $k = 0, 1, \dots, K$ .

The PTC solution approach is to start at the end of the horizon and to find the sequence of output feedback matrices  $F(k)$  backwards in time. The main tool used is the following theorem, where the one-step reachable set from an arbitrary set  $S \subset \mathbb{R}^n$  under the system  $x(k+1) = Ax(k)$  is denoted by

$$\mathcal{R}(A, S) := \{Ax(k) : x(k) \in S\}.$$

**Theorem 5.1** (Esterhuizen & Wang, 2020a). Consider the system  $x(k+1) = Ax(k)$  for an arbitrary  $k \in \mathbb{Z}$  along with two polyhedral sets,  $\mathcal{P}(M_1, \mu) \subset \mathbb{R}^n$  and  $\mathcal{P}(M_2, \nu) \subset \mathbb{R}^n$ . The following holds:  $\mathcal{R}(A, (M_1, \mu)) \subset \mathcal{R}(M_2, \nu)$  if and only if there exists a real matrix  $G$  satisfying:

$$G \geq 0, \quad (6)$$

$$GM_1 = M_2A, \quad (7)$$

$$G\mu \leq \nu. \quad (8)$$

Note that  $G \geq 0$  means that all entries in  $G$  are nonnegative. Thus, if such a  $G$  can be found, then every state in  $\mathcal{P}(M_1, \mu)$  will be driven to  $\mathcal{P}(M_2, \nu)$  under  $A$  in one time step.

We can now use this result to attempt to solve our problem, starting at the end of the horizon. We would like feedback  $F(K-1)$  such that all states in  $\mathcal{H}(K-1)$  are driven into  $\mathcal{H}(K)$ . Thus, noting that the closed-loop reads  $x(k+1) = (A + BF(k)C)x(k)$ , from (6)–(8) we seek a  $G(K-1)$  and an  $F(K-1)$  such that

$$G(K-1) \geq 0,$$

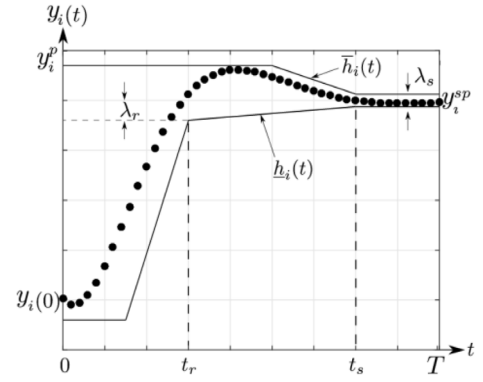


Fig. 4. Transient performance specifications for a step set-point change. The set-point is specified by  $y_i^{sp}$ , the steady-state error by  $\lambda_s$ , the  $\lambda_r$ -rise-time by  $t_r$  and the peak over-shoot by  $y_i^p$ .

$$G(K-1)Q(K-1) = Q(K)[A + BF(K-1)C],$$

$$G(K-1)\phi(K-1) \leq \phi(K).$$

This is just a linear programming problem (LP) and is easy to solve. However, there might not exist a solution to this LP, especially if the design constraints are too strict. An attempt to work around this is to instead find a subset  $\chi(K-1) \subset \mathcal{H}(K-1)$ . Once a  $G(K-1)$ ,  $F(K-1)$  and  $\chi(K-1)$  have been found, takes a step back in time to again find a  $G(K-2)$ ,  $F(K-2)$  and  $\chi(K-1) \subset \mathcal{H}(K-1)$  such that all points in  $\chi(K-2)$  are driven into  $\chi(K-1)$  with the feedback  $F(K-2)$ . This is then iterated until the beginning of the horizon. It is shown that under some mild assumptions, the algorithm is guaranteed to find a solution.

The polyhedral tubes control (PTC) is the first feedback controller design method for meeting all the transient performance specifications. This approach is novel. The new challenges are to be overcome to further this approach.

(1) Note that PTC is a mix of analytical and numerical methods. Whether or not it produces a controller solution depends on the plant dynamics, input size and specifications. For example, no controller on the room temperature can finish a transient of the 10C-degree set-point change in 5 s as the room's thermal dynamics is much slower and air-conditioner has limited capacity. It is desirable to find the existence conditions for a transient control problem without carrying out actual control computation.

(2) In PTC, the controller is computed off line by a numerical algorithm. By nature of off-line design, the controller should work for any unknown initial state which is assumed to be in a prescribed set. The reachable state set under all the possible controls at the next time instant is computed with regards to the above initial set (but not to a particular initial state in real time), and then a specific control is determined to bring the actual state into the performance set. This process is repeated for all time instants, where the reachable set could be large due to the uncertainty on the initial state. This is significant conservatism as the designed control sequence works for any initial state in the prescribed set while only one initial state occurs in real life. It is desirable to develop new algorithms with much reduced conservatism.

(3) PTC supposes a finite time horizon task such as an assembling job. Many control tasks do not have specific ending time.

We propose to resolve those challenges with PTC as follows.

(1) Solvability. Use the new advances in the reachable set theory to build the relations between the plant properties (dynamic response speed and input size) and achievable performance (rise time, over-shoot, settling time). And find the conditions under which the transient control problem has a solution.

(2) On line Algorithm. We can find a tight estimation of the state set in real time. Use the first few output observations, we can compute the



initial state with observability condition and thus use it to initialize Kalman filter with high accuracy. Run Kalman filter to estimate the state on line, and obtain the interval state with the estimated state mean and its error bound. This state set is polyhedral and should be much smaller than the one computed from evolution from the initial state region. We can do the same for disturbance, that is, we estimate the disturbance. This enables the MPC's finite time horizon mode: use the estimated state and disturbance sets at the current time step to compute the reachable state sets over a few next steps, obtain the control sequence over a few next steps using PTC, and apply the first control. The process is repeated at the next time step. This new procedure can greatly reduce the conservatism with the current PTC because of the much-reduced state set for which the control works.

(3) Infinite time horizon. Extend the PTC formulation for time beyond the terminal time: keep the state in the terminal target set forever after the terminal time. We can use MPC again: design a control sequence with the above process in infinite way.

(4) Output reachable set. The current PTC is based on theory on reachable state sets. Develop theory and algorithms for output reachable sets and employ them to design controllers for transient performance, which will avoid estimation of the state and overcome conservativeness of the PTC.

A general and effective approach is sought after in addition to PTC. We think of it from the bottom up view. Obviously, full transient performance control is very challenging due to lack of analytical relation between the system parameters and its transient response in general. Essentially, transient performance involves the characteristics of I-O mapping of the control system in general. The key issues are how much of the plant mapping can be changed by a realizable controller to meet the desired closed-loop mapping and how this controller can be designed effectively and efficiently. For a linear SISO system, a transfer function can represent a I-O mapping. Its denominator is determined by all its poles and can be assigned by feedback control, but it is only a part of the mapping. The full mapping is uniquely determined by all its poles and zeros. But the zeros cannot be altered by state feedback and they can be partially affected by dynamic output feedback. Internal model control reveals that the non-minimum phase part of the plant must remain in any stable feedback control system. The relation between this unchangeable part and transient behavior is unclear as it depends on other part of the system. For MIMO linear systems and nonlinear systems, much less is known so far. Therefore, analytical solution for transient performance control is unlikely to find. Numerical methods should be sought for.

Machine learning and learning control are well developed and potentially useful to develop performance control. This could open a new door. We propose a brand-new approach: Tight Control with Transient Performance Specifications. Consider the conventional feedback system in Fig. 5.

(1) Draw the desired output response  $y_d(t)$  to meet the given transient performance specifications. Feed this  $y_d(t) = r(t)$  as the reference to the control system. The ideal case is to zero the error,  $e(t) = y(t) - y_d(t)$  for all  $t$ . Practically, tight control is to find the controller  $K$  such that  $|e(t)| < \varepsilon(t)$ ,  $t = 1, 2, \dots$ ,  $\varepsilon(t) > 0$ , where  $\varepsilon$  is the small error bound function. In contrast, stability or tracking control requires  $e(\infty) = 0$  only without knowing the error at finite time.

(2) Design the initial  $K$  with the servo control method so as to obtain the stable bounded error and input sequences:  $e(t) \triangleq e^0(t)$ ,  $u(t) \triangleq u^0(t)$ ,  $t = 0, 1, \dots$ . Use the iterative learning control (Liu & Wang, 2012) to find  $u^{i+1}(t) \triangleq u^i(t) + \Delta u^i(t)$ ,  $t = 0, 1, \dots$ , such that  $u^i(t)$  satisfies the error bound for all  $t$ . Let the final  $u^{i^*}(t) \triangleq u^*(t)$ ,  $e^{i^*}(t) \triangleq e^*(t)$ ,  $t = 0, 1, 2, \dots$

(3) Given the data set of  $\{e^*(t), u^*(t), t = 0, 1, \dots\}$ , use the system identification/machine learning to find the controller  $K$ . The controller form could be practical PID,  $m$ th order proper rational function, or neural network.

Note that the above tight control adopts the iterative learn control as a tool to train the control sequence. But it has other ingredients

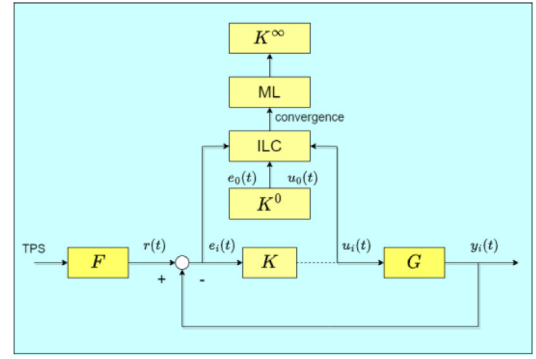


Fig. 5. Tight control. (TPS: Transient performance specifications;  $F$ : formulator;  $G$ : plant;  $K$ : controller;  $K^0$ : stabilizing controller;  $K^\infty$ : designed controller; ILC: iterative learning control; ML: machine learning;  $i$ : learning iteration).

to make differences from ILC. It serves general transient performance specifications for a general plant, which can have plant uncertainties and disturbances, while the ILC is limited to a fixed repetitive task. It transforms transient performance specifications to the reference bands (see the upper and lower bounds in Fig. 4.), while the ILC is for a single reference. It trains the control sequence to meet the error bound function, instead of convergence to zero used in ILC. More importantly, it trains the control sequence for a set of models which are samples of the plant with regards to its uncertainties and disturbances, instead of a single model used in ILC. This sampling process and performance assessment are carried out in the statistical learning framework (Chamanbaz, Dabbene R. Tempo, Venkatakrishnan, & Wang, 2014). The learning control and machine learning are well established in the literature, and there are rich tools to employ. We can handle time delay and input saturation and treat single variable or multivariable, linear or nonlinear systems in this same framework. Thus, this approach should be general and effective. It has great potentials for theoretical development and practical applications.

## 6. Conclusions

Transient performance control is necessary for real applications and it has recently attracted much research attention. But it is in its infancy, and substantial research is demanded to have complete solutions. We believe that transient performance control is well justified from application point of view and there are supporting theoretical and computational tools available for such control development, see Fig. 6. We call for effective methods and designs for transient performance control and carry out case studies to demonstrate their effectiveness with actual performance insurance. The advocated transient performance control may impact the control research and applications as follows.

(1) creation of a new branch of control theory. The new theory matches actual requirements in control applications, and fills in the control research gap mentioned in introduction. It may change the current stability-central research to the new trend of performance-central research. Thus, it can attract vast interests from the control community, become dominant in this field, and also spread to other close fields such as communication and AI.

(2) creation of a new control technology based on this new theory. The new theory can produce a new technology of control design and implementation. The new technology will be very powerful with performance insurance and improvement over the current control technology represented by the conventional controller (PID) and advanced controller (model predictive control), which cannot tell or guarantee transient performance. It may find wide applications in industry.

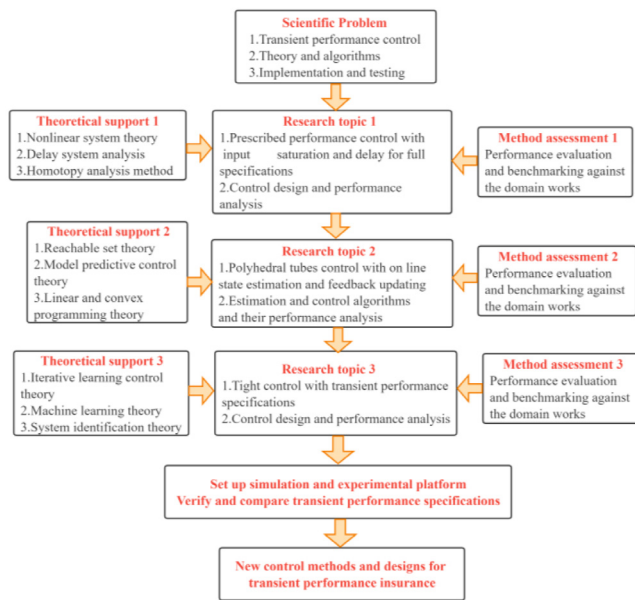


Fig. 6. Research topics and methods.

## Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Qing-Guo Wang acknowledges the financial support of BNU Talent seed fund, UIC Start-up Fund (R72021115), Guangdong Key Lab of AI and Multi-modal Data Processing (2020KSYS007), the Guangdong Provincial Key Laboratory of Interdisciplinary Research and Application for Data Science (2022B1212010006), Guangdong Higher Education Upgrading Plan 2021-2025 (R0400001-22, R0400025-21), UIC, China, which partially funded his research on this work.

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