

Pulsar Acceleration Shifts from nearby Supernova Explosion

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We show that when a supernova explodes, a nearby pulsar signal goes through a very specific change. The observed period first changes smoothly, then is followed by a sudden change in the time derivative. A stable millisecond pulsar can allow us to measure such an effect. This may improve our measurement of the total energy released in neutrinos and also the orientation of the supernova-pulsar system.

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I. INTRODUCTION AND SUMMARY

When a core collapse supernova (SN) explodes, it releases approximately a fraction of the solar-masses worth of energy in a shell of relativistic neutrinos. Although neutrinos barely interact with other matter directly, relocating such amount of mass inevitably affects the background geometry. If an accurate timing apparatus is nearby, such as a millisecond pulsar, then one can hope to observe an effect. The purpose of this paper is to calculate such an effect and provide a better idea of how useful such an observation can be.

Since this neutrino shell is very thin—passing through either the Earth or the pulsar in a time much shorter than the typical duration for timing observations—we can treat it as a co-dimension-1 delta-function and describe the spacetime by Israel Junction Conditions (IJC) [1]. Furthermore, we will treat both the neutrinos and the pulsar signal as propagating at the speed of light. In reality that is not exactly true, with photons moving even slower than neutrinos. Nevertheless, the time delay is again much shorter compared to the duration of timing observations, thus it does not invalidate our result. The analytical model then becomes quite simple, since it involves just tracking geodesics across a thin shell [2–4].

In Sec.II, we present the general setup of geometry with junctions. In Sec.III we study the scenario in which the SN and the pulsar are exactly aligned, as shown in Fig.1. On one hand, this is the simplest scenario as it reduces to a (1+1) dimensional problem. On the other hand, this might have the strongest observable effect. After all, the signals that arrive right before and right after the neutrino shell travelled in two dramatically different geometries. We show that the observed pulsar period does not jump across the neutrino shell. In hindsight, these should not be too surprising, as they follow from the physical continuation of spacetime. Although the metrics on either side of the neutrino shell look very different, the physical distance between two points, and the relative velocity between two points, cannot change discontinuously. Thus the leading observable effect is in the derivative of the pulsar period, due to the change in its gravitational acceleration.

$$\delta(\dot{P}/P) \sim -\delta M/r_p^2, \quad (1)$$

where P is the observed pulse period, G and c are set to 1, δM is the total neutrino shell mass thus also its effective Schwarzschild radius, and r_p is the distance between the SN and the pulsar.

In Sec.IV, we study the general case in which the pulsar and SN are not aligned. The above acceleration change happens when the shell hits the pulsar. However, before seeing that, we will first see the SN explosion, and thereafter the pulses must cross the neutrino shell before reaching us. Such shell-crossing also induces observable changes in the pulsar period at similar order of magnitudes [4]. Combining these two effects, we derive a unique signature in pulsar observation accompanying the SN explosion.

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In Sec.V, we show that the required timing accuracy to measure such a signature is the time it takes light to cross the effective Schwarzschild radius of the neutrino shell. That is about 10^{-6} seconds, which is achievable by millisecond pulsars. The particular feature in the time-shifts can determine the total energy released in neutrinos and the orientation of the SN-pulsar system.

II. GEOMETRY

We will calculate the observable effect by treating the spacetime around the SN as roughly a Schwarzschild geometry, while the pulsar is like a test particle in this background.

$$ds_i^2 = - \left(1 - \frac{2M_i}{r}\right) dt_i^2 + \left(1 - \frac{2M_i}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2. \quad (2)$$

Here the i index on quantities stands for the “initial” spacetime, thus M_i is the total mass of the SN (before it loses the neutrino shell). Similarly, replacing i by f means the final spacetime where $\delta M = (M_i - M_f)$ is the total energy carried away by the neutrino shell. In principle every coordinate should have a subscript i or f . However, since we assume spherical symmetry, we can identify their angular coordinates and declare the radial coordinate as the area radius of the two-sphere. Thus, the only coordinate difference appears in t_i and t_f .

The initial and final geometries are connected through a junction, which is the neutrino flow that we treat as a thin, null shell. The radial 4-vector of this shell is thus a null vector, which can be represented in either coordinate.

$$j_\mu \equiv \left(\left(1 - \frac{2M_i}{r}\right)^{-1}, 1, 0, 0 \right)_{\text{in metric i}}. \quad (3)$$

Throughout this paper, we will use “ \equiv ” to represent a physical quantity in a coordinate system. In the “final” metric, one replaces the “ i ” in the above expression by “ f ”. That leads to a mathematically different component form, but it represents the same physical 4-vectors.

We will treat our signal as photons and talk about their change of frequencies. In reality, one just relates this to the pulsar period by $w = (2\pi/P)$. A photon can be described by a null-4-vector with a fixed normalization as shown in section 25.6 of [5]. In a fixed Schwarzschild geometry, the general form is

$$k^\mu(w_\infty, r) \equiv w_\infty \left(\frac{1}{\left(1 - \frac{2M}{r}\right)}, \sqrt{1 - \frac{b^2}{r^2} \left(1 - \frac{2M}{r}\right)}, \frac{b}{r^2}, 0 \right). \quad (4)$$

The overall normalization w_∞ is defined as the frequency observed by an asymptotic observer at rest, and the fixed angular momentum is parametrized by the impact parameter b . For convenience, we can always assume that the motion is confined to the equator of the two sphere.

III. THE ALIGNED CASE

First we will consider the case that $b = 0$ in Eq. (4), namely the SN, the pulsar, and the Earth sit on one straight line, and the pulsar is in the middle. This reduces to a (1+1)-dimensional problem as depicted in Fig.1. In this scenario, the earth will receive two photons from the SN, one right after the other, but they have traveled through two distinctly different geometries. One might suspect a sudden and dramatic change in the signal, but it is easy to show otherwise.

Let p_μ and e_μ be the timelike 4-vectors of the pulsar and the earth, r_p and r_e are their distances to SN, while emitting/receiving these pair of photons. Continuity across the junction implies the following constraint on how they are represented in either geometry.

$$p_\mu j^\mu(r_p)_{\text{in metric i}} = p_\mu j^\mu(r_p)_{\text{in metric f}}, \quad (5)$$

$$e_\mu j^\mu(r_e)_{\text{in metric i}} = e_\mu j^\mu(r_e)_{\text{in metric f}}. \quad (6)$$

The two photons are defined by the physical fact that in the rest frame of the pulsar, they have the same frequency.

$$w_p = -p_\mu k^\mu(w_\infty^i, r_p)_i = -p_\mu k^\mu(w_\infty^f, r_p)_f. \quad (7)$$

It is then straightforward to combine these 3 equations to show that the frequencies received on earth are identical.

$$w_e = -e_\mu k^\mu(w_\infty^i, r_e)_i = -e_\mu k^\mu(w_\infty^f, r_e)_f. \quad (8)$$

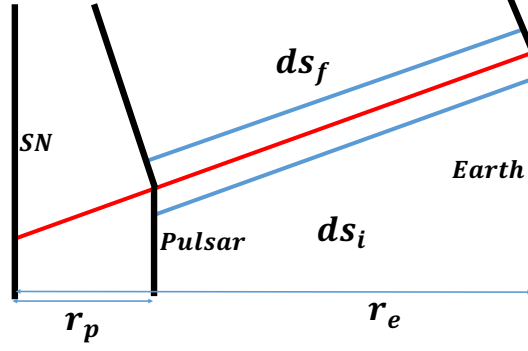


FIG. 1: Spacetime diagram showing the relative distances between the SN and earth, r_e , and SN and pulsar, r_p . The red line represents the neutrino null shell. Above this line, we are in Schwarzschild metric f , below this line, we are in Schwarzschild metric i . The two blue lines represent the photon trajectories that we receive from the pulsar right before and after we see the SN explosion.

A. Leading order expansion and physical intuition

Our conclusion above is exactly correct in full nonlinear general relativity, although it might be surprising to some readers. Here we will expand the two steps separately to leading order for better physical intuitions. First of all, it is quite realistic to set $r_e \gg r_p$ and treat it as being infinite, so the only physical change happens at the pulsar.

Instead of using Eq. (5), if we had assumed that the pulsar is at rest in both coordinates before and after the neutrino shell crosses, then we would have derived a frequency ratio of $[1 - \delta M/r_p]$ between these two photons¹. This would have reflected the fact that they climbed out of two different gravitational potentials, but it cannot be the full story. If the pulsar was at rest before the neutrino shell hits it, Eq. (5) demands that it picks up a velocity in the new coordinate

$$\begin{aligned} p_\mu &\equiv \left(\left(1 - \frac{2M_i}{r} \right)^{-\frac{1}{2}}, 0, 0, 0 \right)_i \\ &\equiv \left(\left(1 - \frac{2M_f}{r} \right)^{-\frac{1}{2}}, -\frac{\delta M}{r_p}, 0, 0 \right)_f. \end{aligned} \quad (9)$$

The new velocity, $v = \delta M/r_p$, is falling toward the SN remnant. This then creates a Doppler shift that exactly cancels the difference in gravitational potential, thus netting no frequency change.²

$$(w_e)_i \equiv w_p \left(1 - \frac{M_i}{r_p} \right) = w_p (1 - v) \left(1 - \frac{M_f}{r_p} \right) \equiv (w_e)_f. \quad (10)$$

¹ In fact, any non-relativistic initial velocity leads to this result.

² This change in coordinate velocity is not due to any direct interaction with the neutrino shell, but simply an artifact of coordinate change across the junction.

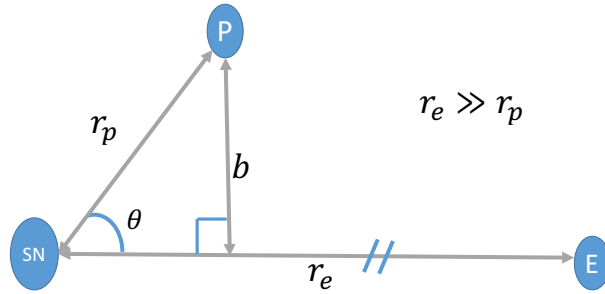


FIG. 2: General geometry of the three objects. Where b is the impact parameter of the photons from pulsar to earth.

B. Observable change in acceleration

Let us continue the approximation in the previous section and take a time derivative of Eq. (10). Realistically, $v \ll 1$ and $r_p \gg M$, then the dominant change in time comes from the velocity.

$$\frac{\dot{w}_e}{w_e} = -\dot{v} = -\frac{M}{r_p^2}. \quad (11)$$

When the pulsar crosses the neutrino shell, the enclosed mass changes from M_i to M_f , and w_e indeed changes suddenly. It is not a suddenly observable change, since it takes some time to build up the change in w_e and then in the phase before it is observable. Nevertheless, this is the leading order observable for us.

IV. GENERAL CASES

In general, the three bodies in this problem will not exactly align. Let θ be the angle between r_p and r_e as shown in Fig.2, we will calculate the general change in acceleration under the same assumption that has a convenient 1st order expansion: $r_e \gg r_p \gg M$ and a non-relativistic initial velocity.

Recall that in the aligned case, there was one special “time” (a particular photon in the train of signals from the pulsar), before which both the earth and the pulsar are outside the neutrino shell, and after which they are both inside it. In the general case, there are two such special “time”s, and this is easiest to understand from the special case in which $\theta \approx \pi$, as shown in Fig.3. t_1 is when the neutrino shell passes through the earth, which is also when we observe the SN explosion. After that, every pulse must cross the neutrino shell somewhere in its path before reaching us. Such crossing lasts until t_2 , when the neutrino shell hits the pulsar itself. From the earth’s perspective, that is when we see the first photon from the pulsar after it is hit by the neutrino shell.

Before t_1 , both the pulsar and the earth are in metric i . After t_2 , both are in metric f . Thus we simply have

$$\begin{aligned} \frac{\dot{w}_e}{w_e} &= -\frac{M_i}{r_p^2} \cos \theta, \quad \text{for } t < t_1, \\ &= -\frac{M_f}{r_p^2} \cos \theta, \quad \text{for } t > t_2 = t_1 + r_p(1 - \cos \theta), \end{aligned} \quad (12)$$

which comes from the projection of gravitational acceleration in our direction. Between t_1 and t_2 , a pulse originates in metric i , but must cross the neutrino shell (being caught-up by it) and enter metric f before reaching the earth. This shell-crossing leads to a continuous change in the observed frequency. The change in photon frequency when the pulsar remains stationary was calculated in [4]. Here we include the fact that before t_2 , the pulsar is still feeling the same acceleration which lead to Eq. (12). Without loss of generality, we set $t_1 = 0$, then we can combine with the

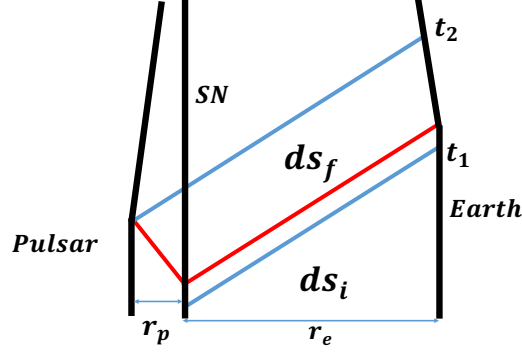


FIG. 3: Spacetime diagram showing the pulsar at $\theta \approx \pi$ with the same color-code as Fig.1. The two photons are right before the first change and right after the second change of \dot{w}_e .

result of [4] to get the full change in frequency through out the process.

$$\begin{aligned}
 w_e(t) &= w_0 \left[1 - t \frac{M_i}{r_p^2} \cos \theta \right], \quad \text{for } t < 0, \\
 &= w_0 \left[1 - t \frac{M_i}{r_p^2} \cos \theta + \frac{4\delta M t^3}{(t^2 + r_p^2 \sin^2 \theta)^2} \right], \quad \text{for } 0 < t < r_p(1 - \cos \theta), \\
 &= w_0 \left[1 - \frac{M_i}{r_p} \cos \theta (1 - \cos \theta) + \frac{\delta M}{r_p} (1 - \cos \theta) - \left(t - r_p(1 - \cos \theta) \right) \frac{M_f}{r_p^2} \cos \theta \right] \\
 &\quad \text{for } t > r_p(1 - \cos \theta).
 \end{aligned} \tag{13}$$

Here $w_0 = w_e(0)$ is the frequency observed on earth at the moment when we see the SN explosion. The first line is before we observe the SN explosion, so the only contribution to the change in the observed frequency comes from the gravitational pull to the SN, which is a term linear in t . The same gravitational pull is still present in the second line, but we need to include the effect from photons crossing the neutrino shell derived in [4]³, which is the last term. Finally in the last line, the pulsar also enters the geometry inside the neutrino shell, thus no more time-dependent effect from shell-crossing. The accumulated effect remains which is expressed as the two time-independent terms. The last term is still the gravitational pull, but with a slightly smaller mass M_f and with t set to $(t - r_p(1 - \cos \theta))$.

V. OBSERVATIONAL PRACTICALITY

A. Signal strength and shape

The observed value of w_e cannot be exactly what we calculated, because in practice, there are many other contributions to the acceleration. For example, both the SN and the pulsar may be in the galactic center which has a relative acceleration to our spiral arm. However, if the pulsar is close enough to the SN, then we can be pretty certain that in the time scale comparable to $\Delta t = r_p(1 - \cos \theta)$, the SN explosion is the dominant cause of the change which starts at $t = 0$. Namely, every line in Eq. (13) may have an additional constant, but they are all the same. Thus we can

³ See Eq (23) in [4]. We have replaced b in that equation with $r_p \sin \theta$ to be consistent with notation used in this paper.

calculate the change in frequency, caused by the SN explosion, by subtracting the first line of Eq. (13) from the rest.

$$\begin{aligned}\delta w_e(t) &= w_0 \frac{4\delta M t^3}{(t^2 + r_p^2 \sin^2 \theta)^2}, \quad \text{for } 0 < t < r_p(1 - \cos \theta), \\ &= w_0 \left[\frac{\delta M}{r_p} (1 - \cos \theta)^2 + t \frac{\delta M}{r_p^2} \cos \theta \right], \quad \text{for } t > r_p(1 - \cos \theta).\end{aligned}\tag{14}$$

This shows a change of frequency, therefore the pulse period, starting from 0 at $t = 0$. It first takes some time for this change to grow, then we also need to wait for a few pulses for the change in the period to accumulate to a visible change in the pulse arrival time. In other words, we can estimate the detectability of our effect by Eq. (11).

$$\delta T_{\text{arrival}} \sim \delta \left(\frac{\dot{w}_e}{w_e} \right) (\Delta t)^2 \sim \frac{\delta M}{r_p^2} (\Delta t)^2 \sim \delta M.\tag{15}$$

Here Δt is the duration of the observation. As explained earlier, a time derivative in the period needs to first build up to the actual change in the period, then such change again builds up through several periods to show a change in arrival time. Thus we have a $(\Delta t)^2$ dependence.

The effect has a natural time scale of $\Delta t \sim r_p$, as shown in Eq. (14). Interestingly, the $1/r_p^2$ in gravitational force exactly cancels with this double-time-accumulation factor. That means independent of how far away the pulsar is from the SN, we can use it to detect this change as long as the timing accuracy is better than the light-crossing time of the effective Schwarzschild radius of the neutrino shell⁴. We can estimate $\delta M \sim$ a fraction of solar mass $\sim 10^{-6}$ seconds. The timing accuracy for a stable, millisecond pulsar is already better [6]⁵.

Beyond this simple estimation, the detail shape of the time-dependent change of the arrival time is given by integrating Eq. (14). As already shown in [4], the first stage of such change depends only on the “impact parameter” $r_p \sin \theta$. This corresponds to the angular distance between the pulsar and SN in our sky, which can be measured on its own quite accurately. On the other hand, the discontinuous change in the derivative of frequency happens at $t = r_p(1 - \cos \theta)$ as the pulsar is now in the final metric and so feels a different acceleration. Note that $t = r_p(1 - \cos \theta)$ is also the time that corresponds to the depth difference between SN and pulsar which is notoriously hard to measure. We select a few values of θ and plot their $\delta T_{\text{arrival}}$ as a function of time in Fig.4. With a good measure, it should not be difficult to fit these curves, therefore an improved measurement of their depth difference.

B. Chance for a good target

Most of the millisecond pulsars (MSP) are distributed in the galactic centre, a region with height $\sim 500\text{pc}$ and radius $\sim 7.5\text{kpc}$. The mean distance from a pulsar to its nearest pulsar neighbour resulting from random catalogs of 11,000 pulsars generated using [8] is 200. Extrapolating to 30,000 pulsars with SKA2 [7] and drawing SNe from the same spatial distribution (which is the most likely scenario) leads to a mean distance from a SN to the nearest timed MSP of 100 light years. It might not be unreasonable, therefore, to expect $r_p \sim 30$ light years, which makes the second change in frequency observable.

This means that as soon as we see the next SNe, we can start to follow the first stage timing shift in all MSPs. Some of them will show the predicted behavior within a few decades. By about the same time, it is likely that one or two among those will go through the second stage, having a sudden change in the second derivative of timing shifts. Given a few SNe per century [9], the detection of these effects is possible in the foreseeable future.

Acknowledgments

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⁴ In practice, we still want the pulsar to be somewhere close to the SN. Without that condition, we cannot be certain that there are no other comparable changes to acceleration during δt .

⁵ Our conservative estimation assumes only two measurements: in the beginning and the end of the duration δt . In practice, one has N measurements during the process, which increases the accuracy by another factor of \sqrt{N} .

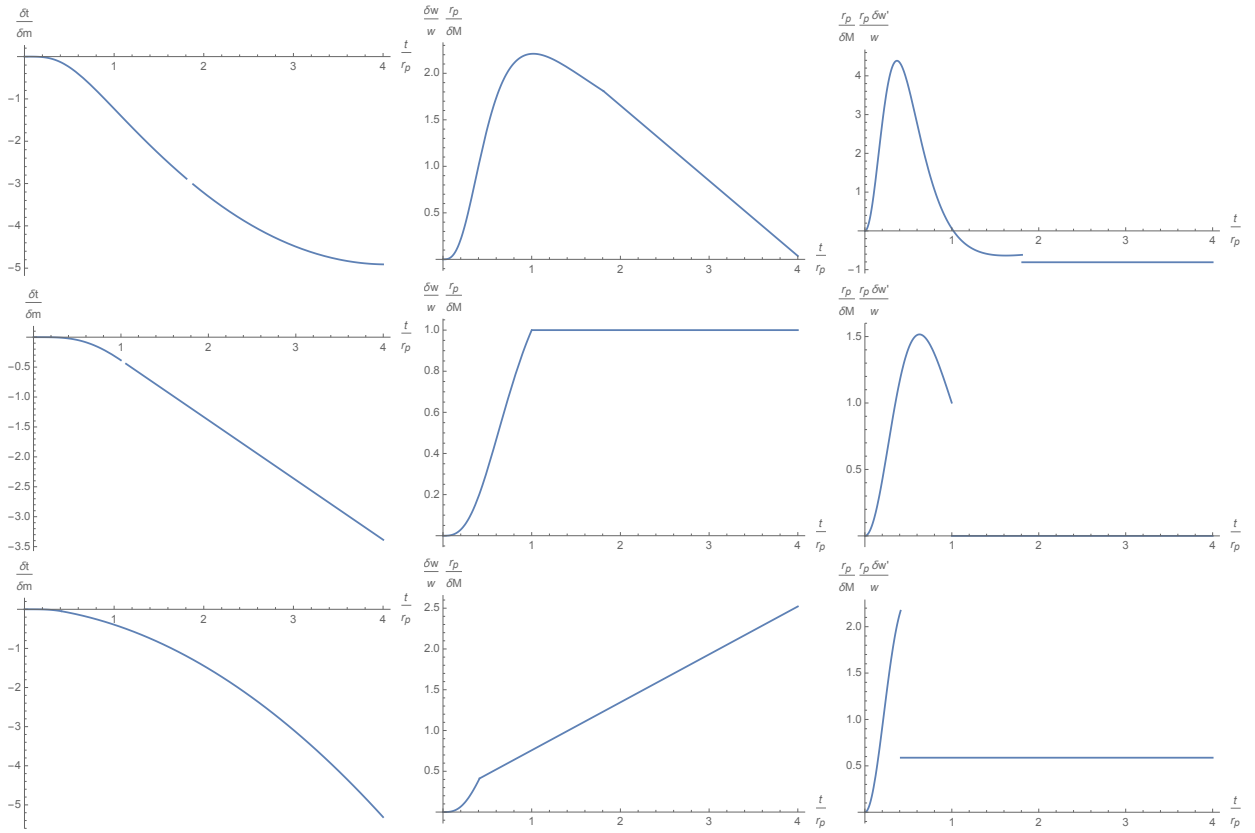


FIG. 4: From left to right, the pulse arrival time shift and its first and second derivatives. The arrival time shift is the direct observable, its first derivative is simply Eq. (14), while second derivatives highlights the discontinuity—the most distinctive feature of our effect. From top to bottom, $\theta = 0.8\pi, 0.5\pi, 0.3\pi$. The distance to supernova r_p and the neutrino shell mass (its effective Schwarzschild radius) δM are used to make all quantities dimensionless.

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