Predicting the gravitational polarization rotation of the binary pulsar

Ue-Li Pen,[1,2,3,4,*] Xin Wang,[1,†] and I-Sheng Yang[1,4,‡]

[1] *Canadian Institute of Theoretical Astrophysics, 60 St George St, Toronto, ON M5S 3H8, Canada.*
[2] *Canadian Institute for Advanced Research, CIFAR program in Gravitation and Cosmology.*
[3] *Dunlap Institute for Astronomy & Astrophysics, University of Toronto, AB 120-50 St. George Street, Toronto, ON M5S 3H4, Canada.*
[4] *Perimeter Institute of Theoretical Physics, 31 Caroline Street North, Waterloo, ON N2L 2Y5, Canada.*

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I. INTRODUCTION

Einstein's theory of General Relativity has been the dominant theory of gravity for a century. Many of its signature outcomes, such as light-bending, orbital precession, and gravitational waves, have been confirmed by precision tests from both astronomy and cosmology. One last pending test is the gravitational rotation of polarizations. Indeed, since gravity directly affects the spacetime geometry, not only the paths of light can be bent. Polarizations, often thought as "internal" degrees of freedom of the light ray, cannot escape the influence of gravity either.

Such gravitational rotation of polarization has not been measured so far[1]. One obvious reason is that it is usually very small. The rotation angle of a linearly polarized light ray, caused by a gravitational lens, is suppressed by two small numbers.

$$\Delta\phi \propto \frac{2GM}{r} \cdot v \ . \tag{1}$$

Here M is the mass of the lens, r is the impact parameter—the shortest distance when the light ray pass near the lens, and v is the velocity of the lens. Unless the light goes through somewhere comparable to the Schwarzschild radius, the first factor is small. Unless the velocity is almost relativistic, the second factor is small.

Luckily, an almost edge-on, compact pulsar binary system, such as the double pulsar PSR J0737-3039, can be a very strong candidate to measure such effect. Pulsar signals are often highly polarized, allowing precise measurements of its rotation; an almost edge-on orbit allows the impact parameter to be very small at the superior conjunction; and its compact orbit means a large velocity. One main point of this paper is to show that one can expect to have $\Delta\phi \sim 10^{-7}$ from double pulsar, which has a good chance to be observable given a realistic observation campaign.

Gravitational rotation of polarization from double pulsar was previously studied in [1]. They however derived a much smaller number which is incorrect. Such mistake is not difficult to understand. Although Eq. (1) has been derived by some authors, such as in [2], a significant collection of literature is still filled with confusions. For example, **cite XXX** summarized how three different values of $\Delta\phi$ can be derived from the same physical sitution, but it provided no explicit judgement, nor convincing reason, of which one is actually correct. In fact, one may directly question the dependence of v in Eq. (1), which makes it manifestly not gauge-invariant at the leading order.

In this paper, we will resolve such confusion by starting from the operational definition of how to actually measure such rotation. It turns out that there is a unique $SO(3,1)$ gravitational rotation of the tangent space along any light ray that starts and ends in asymptotic Minkowski space. However the rotation of polarization is an $SO(2)$ projection of that. The null vector of the light ray and the timelike vector of the observer together determines which $SO(2)$ to project to, *thus the actual rotation of polarization depends on the relative velocity between the lens and the observer,* which is exactly the v dependence in Eq. (1).

In Sec.II, we will derive Eq. (1) in the Born approximation and the geometric limit. In Sec.IV, we will describe the observable effects on double pulsar and discuss the appropriate observation campaign to detect it.

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[*] Electronic address: pen@cita.utoronto.ca
[†] Electronic address: xwang@cita.utoronto.ca
[‡] Electronic address: isheng.yang@gmail.com

[1] The E-mode and B-mode in CMB are defined as the relative angle between polarization and gradient. The observed rotation is a consequence of a rotated gradient but a fixed polarization, thus it does not count.

II. DEFINITION FROM SCRATCH

A. Operational Definition

Intuitively, one can imagine two linear polarizations as two vectors attached to a light ray. Let **k** be the null vector of the light ray and **e** be a polarization vector, a parallel transport of **e** should be valid in the geometric optic limit.

$$k^a \nabla_a e^c = k^a \partial_a e^c + k^a e^b \Gamma^c_{ab} = 0 \ . \tag{2}$$

Thus when the rotation is small, using the Born approximation, integrating along a light ray from point A to point B leads to

$$\Delta e^c = \int_A^B \hat{k}^a e_0^b \Gamma^c_{ab} \ dl \ , \tag{3}$$

where $\mathbf{e_0}$ is the original vector and $\Delta\mathbf{e}$ is the change. Naturally, $\Delta\phi \equiv |\Delta\mathbf{e}|/|\mathbf{e}|$ is the straightforward definition of how much a polarization vector has been rotated.

This however, cannot be the full story. Eq. (3) literally compares two vectors on the tangent spaces of two different points, which is meaningless. The two vectors must be in the same tangent space to provide a physically meaningful rotation. It turns out that Eq. (3) does give the correct value, but some extra care is required to give it a clean definition.

First of all, parallel transport is not limited to null rays. We can have an integral similar to Eq. (3) along any path. In particular, one can perform a loop integral, and the answer will be a meaningful comparison between two vectors on the same point. Secondly, any such loop integral gives zero in Minkowski space, thus one can define that any segment in Minkowski space contributes exactly zero to the rotation. Now if we have a light ray starts from point A and reaches point B, both in asymptotic Minkowski space, one can validate Eq. (3) by adding a term to it.

$$\Delta e^c = \int_A^B \hat{k}^a e_0^b \Gamma^c_{ab} \ dl + \int_B^A \hat{p}^a e_0^b \Gamma^c_{ab} dl \ . \tag{4}$$

The second term is a line integral from B back to A which stays in the asymptotic Minkowski part of the spacetime. It contributes exactly zero value, therefore it allows a well-defined loop integral to be assigned as the physical rotation of a line integral from A to B.

An actual observation works very similarly to Eq. (4). What we have is a source (pulsar) which constantly emits a fixed (albeit unknown) polarization. We measure the polarization during a usual time, which is a light ray from A_1 to B_1. And then we compare it with the polarization measured when its binary companion pass very close to the light of sight, which is another light ray from A_2 to B_2. We take the difference between these two measurements, which is exactly a loop integral.

$$\Delta e^c = \int_{A_2}^{B_2} \hat{k}^a e_0^b \Gamma^c_{ab} \ dl + \int_{B_2}^{B_1} \hat{p}^a e_0^b \Gamma^c_{ab} dl + \int_{B_1}^{A_1} \hat{k}^a e_0^b \Gamma^c_{ab} \ dl + \int_{A_1}^{A_2} \hat{p}^a e_0^b \Gamma^c_{ab} dl \ . \tag{5}$$

The integral B_1B_2 and A_1A_2 are along timelike trajectory of the pulsar and the earth, which are effectively in the asymptotic region and nothing happens. The integral A_1B_1 is along a light ray without the influence of the companion. Thus the above loop integral is indeed calculating the rotation of polarization caused by the passage of the companion.

XXX need a figure here

B. Observer Dependence

Eq. (3) is derived from a basic form of a vector multiplied by a small rotation matrix.

$$e^c = e_0^c + \Delta e^c = R^c{}_b e_0^b = (g^c{}_b + \Delta^c{}_b)\, e_0^b \ , \tag{6}$$

where

$$\Delta_{cb} = \int_A^B \hat{k}^a \Gamma^d_{ab} g_{cd} dl \ . \tag{7}$$

By definition of a rotation matrix, Δ_{cd} has to be anti-symmetric, which can be verified explicitly.

$$\begin{aligned} \Delta_{ac} = \int k^b \Gamma^d_{ab} g_{cd} d\lambda \; &= \; \frac{1}{2} \int k^b \left(\partial_a g_{bc} + \partial_b g_{ac} - \partial_c g_{ab} \right) d\lambda \\ &= \; \frac{1}{2} \int k^b \left(\partial_a g_{bc} - \partial_c g_{ab} \right) d\lambda \; . \end{aligned} \tag{8}$$

Note that we have to drop the boundary term for this anti-symmetry, which is allowed because this is effective a loop integral as we explained in the previous section.

This tells us that there is actually a full $SO(3,1)$ rotation, $R_{ab} \approx (g_{ab} + \Delta_{ab})$, that is associated with a light ray. This does not uniquely determine the polarization rotation, which is an $SO(2)$. It also contains extra information such as the deflection of the light ray itself. One needs to specify two polarization vectors to determine which $SO(2)$ to project to. For any observer, the polarization vectors are orthogonal to both the incoming light ray and its own worldline. Thus a projection to the co-dimension-two surface orthogonal to the light ray and the observer 4-velocity is the desired $SO(2)$ rotation of polarization. **Therefore, it is natural and necessary that polarization rotation depends on the observer velocity**, which explains the v dependence in Eq. (1).

One last possible confusion is why such dependence is on the velocity of the observer instead of the source, since they seem to play equivalent roles in the integral of Eq. (3). We remind the reader again that the apparent line integral in Eq. (3) is a convenient illusion. The physically meaningful is always along a loop, where one sends out a polarization vector and waits for it to come back to see the difference. Thus there is one unique point at which the rotation is defined. In practice, we will have no idea about the actual polarization when the signal is emitted at the pulsar. All we know are the polarizations we received on earth, so that is the unique 4-velocity we care about.

III. EXPLICIT CALCULATION

IV. EXAMPLE: DOUBLE PULSAR

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