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Home Work 3 Theoretical:

A) Theoretical Questions:

A1) Hough Transform : Parametrization

The Standard Parametrization of a line $y = m_0 x + b_0$, with m_0 and b_0 slope and intercept, has not become the standard parametrization for the Hough transform for finding lines.

- Explain why this option did become a popular choice
- Would you still use it, what you can about the discrete grid of the Hough space with axes m_0 and b_0 in regard to Hough space cell spacing and its representation of lines.

Answer) $y = m_0 x + b_0$

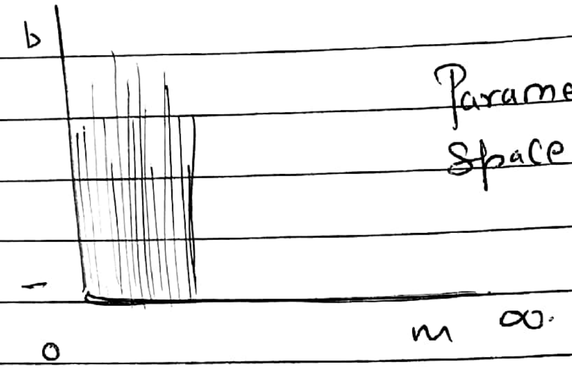
m_0 and b_0 are the parameters.

$m_0 \rightarrow$ slope, $b_0 \rightarrow$ Intercept.

The reason, why this option did not become a popular choice is because of slope (m_0) parameter. The range of slope can be from $[0, \infty]$. How to even divide this range was a big problem.

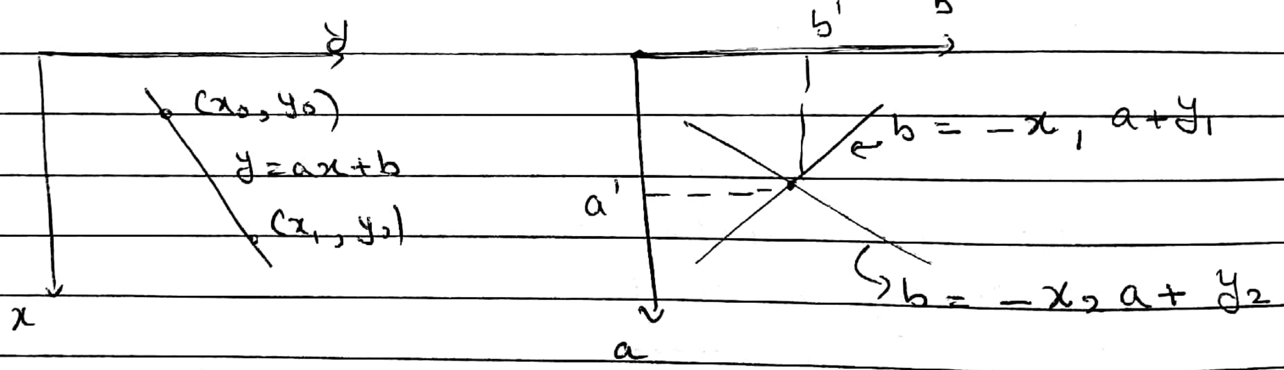
Problems with the (m, b) space:

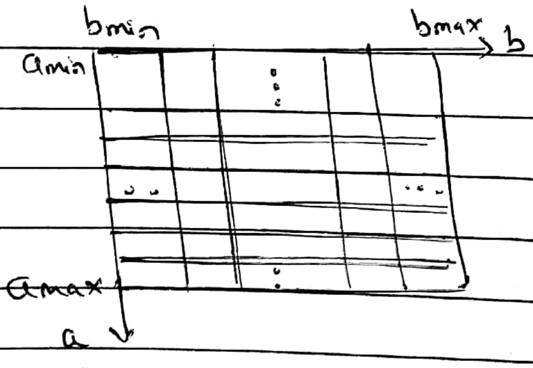
- Unbounded Parameter domains
- Vertical lines require infinite m.



→ Dividing (Samping) this range is very difficult.

Yes, we can still use it for detecting lines that are not vertical.



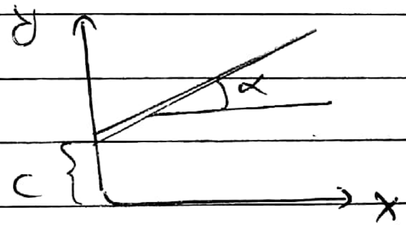


Plot the line in Parameter space $b = x_1 a + y_1$. All Points (x_1, y_1) on the same line will pass the same Parameter space (a, b). Quantize the parameter space and tally # of times each points fall into the same accumulator cell. The cell count = # of Points in the same line.

A2) Show that the polar representation of a line, $x\cos\theta + y\sin\theta = \rho$, represents a cosine function in parameter space with axes θ and ρ. (Remember that general cosine function is given as $y = a\cos(\alpha - \delta)$, with a = amplitude and δ = phase shift.

Answer)

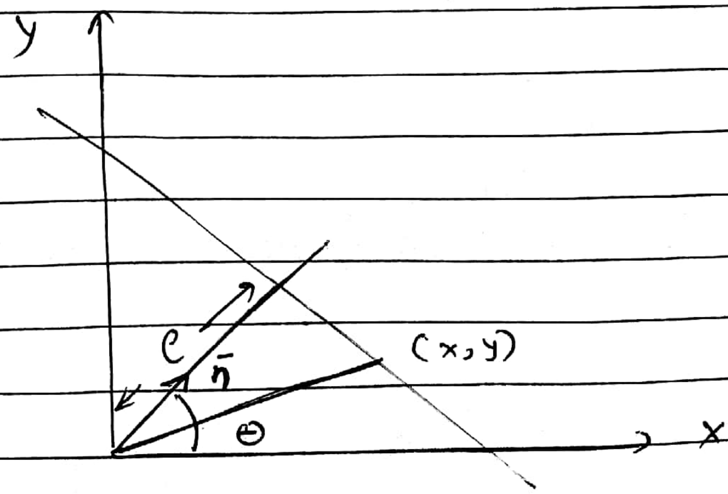
$$y = mx + c$$
$$m = \tan\alpha.$$



Normal form:

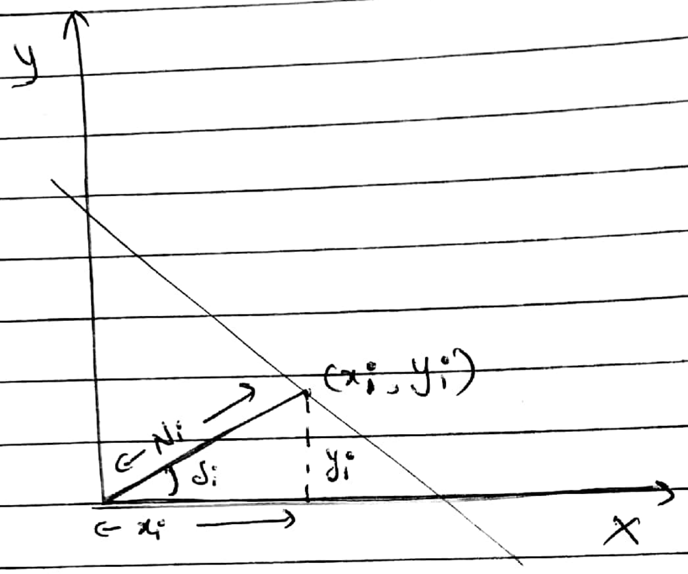
$$\rho = x\cos\theta + y\sin\theta$$

$(x, y) = \bar{x}$: Point Co-ordinate.
$(\rho, \theta) = \bar{a}$: Parameter Vector.



$$\rho = \begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \Rightarrow \text{Dot product}$$

Each point (x, y), will add a Sinusoid in the (θ, ρ) parameter space.



$$\cos \delta_i = \frac{x_i}{N_i}, \quad \sin \delta_i = \frac{y_i}{N_i}$$

$$N_i = \sqrt{x_i^2 + y_i^2}$$

$$\delta = \tan^{-1}\left(\frac{y_i}{x_i}\right)$$

$$\rho = x_i \cos\theta + y_i \sin\theta$$

$$\frac{\rho}{N_i} = \frac{x_i}{N_i}\cos\theta + \frac{y_i}{N_i}\sin\theta$$ (Dividing both sides by N_i)

$$\frac{\rho}{N_i} = \cos\delta_i \cos\theta + \sin\delta_i \sin\theta$$

we have $\cos A \cos B + \sin A \sin B = \cos(A - B)$.

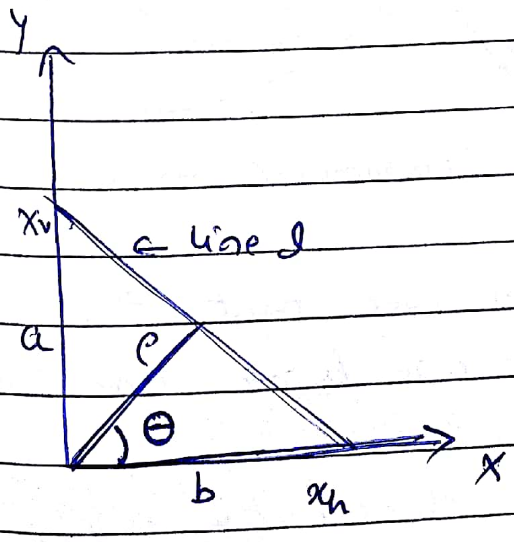
So, $\frac{\rho}{N_i} = \cos(\theta - \delta_i)$

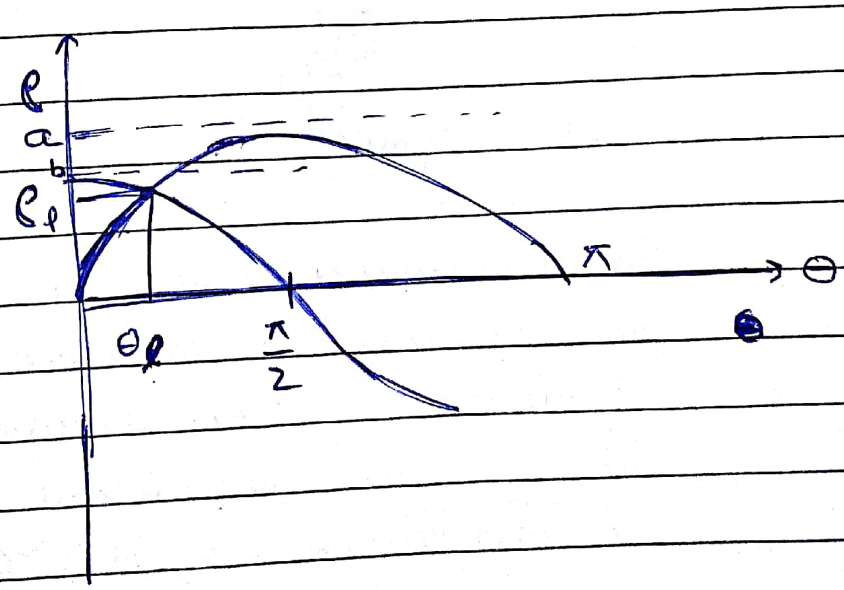
$\rho = N_i \cos(\theta - \delta_i)$

where, $N_i = \sqrt{x_i^2 + y_i^2}$
and $\delta_i = \tan^{-1}\left(\frac{y_i}{x_i}\right)$.

Therefore, $x\cos\theta + y\sin\theta = \rho$, represents a Cosine function in Parameter space with axes θ and ρ.

A3) Given a Scenario with a line (x, y) - space intersecting with the horizontal axis at $\bar{x}_h$ and with the Vertical axis at $\bar{x}_v$, Calculate and plot the Corresponding Cosine curves in the (θ, ρ) parameter space.
Calculate the intersection of the two parameter curves and discuss how its Co-ordinate (θ, ρ) represent the line l in image space.





Answer)

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$x_v \Rightarrow \rho_1 = a\cos\left(\theta - \frac{\pi}{2}\right)$

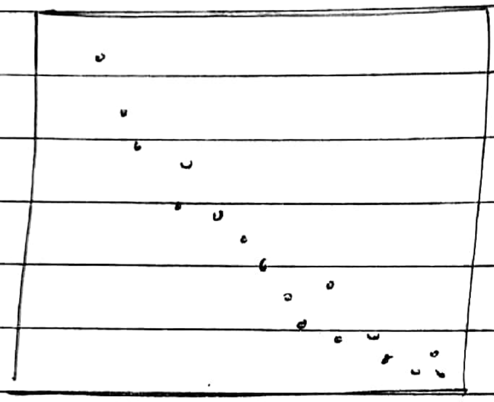
$x_h \Rightarrow \rho_2 = b\cos(\theta - 0)$

↳ Representation of line ~~eqn~~

Intersection: (ρ_l, θ_l) : Parametrisation of line through x_v and x_h.

A4) Noisy line structures

Given points forming a line but it's location corrupted by noise (see below), how would noise affect the clustering of curves in Hough space? What could you do to still find a peak associated parameters that would represent the noisy line?



Answer)

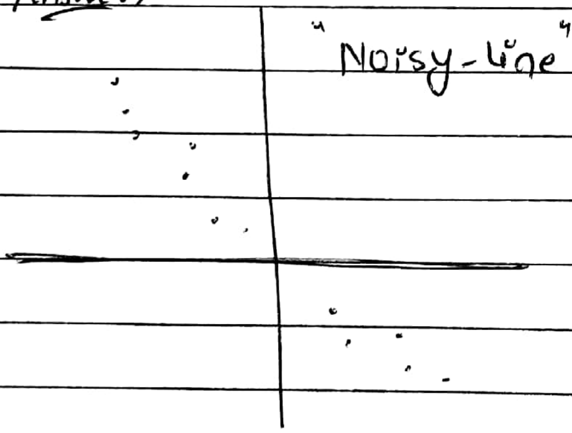
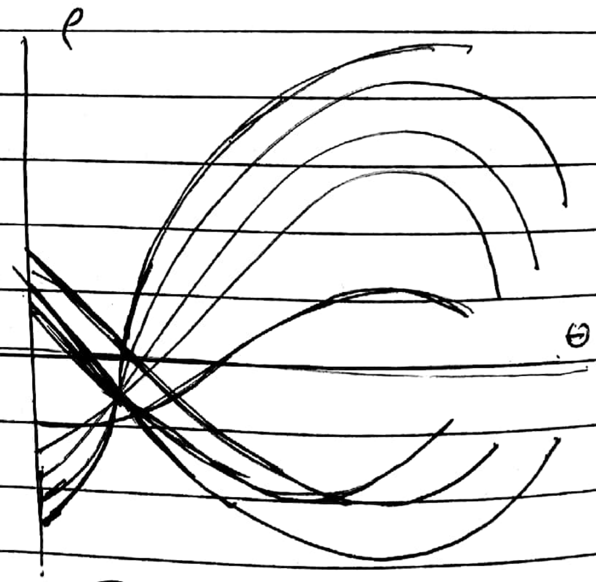


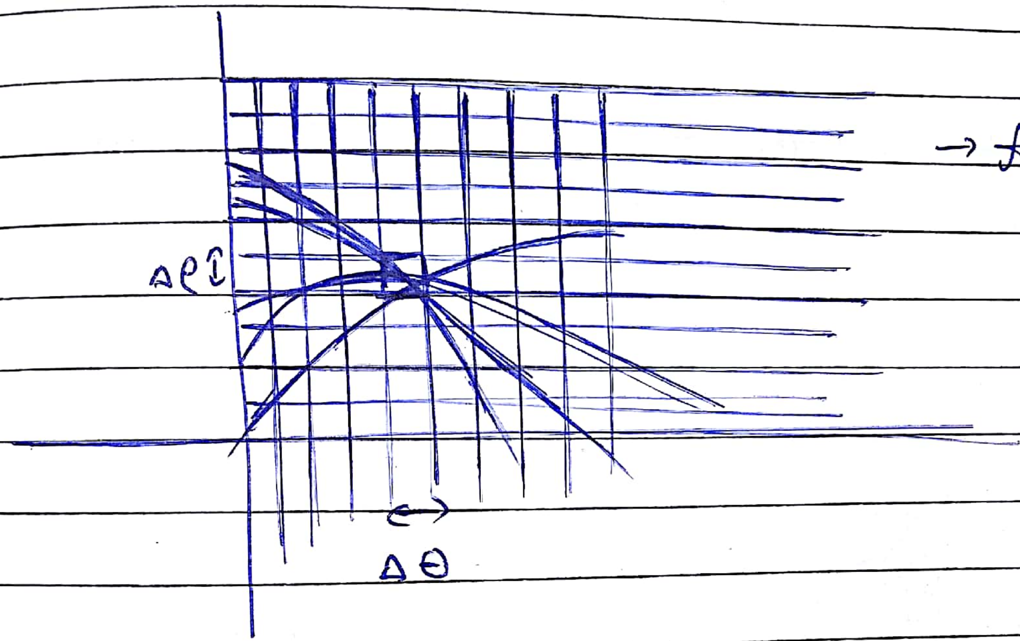
Image space.



Parameter space

In parameter space, we can see that the sinusoids does not intersect at one point and some of them are even outliers.
But, if we still want to detect the lines, we can make use of discretization of Param-

eter space. If we have a voting space, we can choose the cell size, we can choose a $\Delta\rho$ and $\Delta\theta$, to accumulate with many intersecting lines, that do not intersect properly.



→ Accumulation of Votes

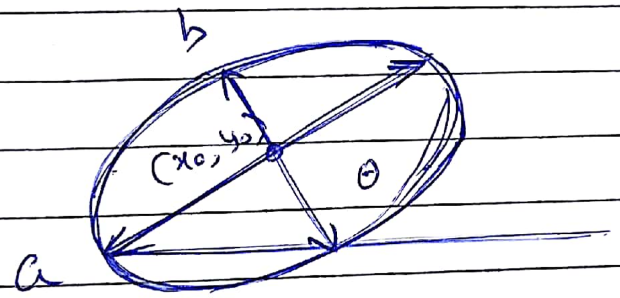
- Cos - curves in parameter space intersect $(\Delta\rho, \Delta\theta)$ cells.
- Cells $(\Delta\rho, \Delta\theta)$ collect all intersecting curves.

Each cell can be incremented by curves.

A5) Hough Transform for Ellipses:

We have learned that the Hough transform for circles requires three Parameters, two for the center and one for the radius, thus spanning a 3-D parameter space. Now let us find ellipses in standard form, would Generalized Hough Transform (GHT), as discussed in the course using R-tables for creating a template, eventually offer a solution? Sketch with drawing and short explanation. What about finding ellipses in different orientation given the GHT?

Answer)



Equation $\rightarrow$ $$\frac{(x-x_0)^2}{a} + \frac{(y-y_0)^2}{b} = 1$$

$\Rightarrow$ Here, Hough transform require four Parameters. This puts a heavy load on memory use and computational effort, but the accumulated votes will start to spread out in high-dimensional space and become sparse. It becomes more difficult to detect meaningful local maxima

In high - dimensional spaces with sparse votes.

Procedure for Hough transform (Ellipse)

for X := X_{min} Step dx to X_{max} do
for y := y_{min} step dy to y_{max} do
begin
$dx = P(x+\Delta, y) - P(x, y)$;
$dy = P(x, y+\Delta) - P(x, y)$;

for each in Parameter space (a, b, θ)
{ for a = a_{min} Step da until a_{max} do
for b = b_{min} Step db until b_{max} do
for $\theta = \theta_{min}$ step $d\theta$ until θ_{max} do

begin
$Angle = \arctan\left(\frac{dy}{dx}\right) - \theta - \frac{\pi}{2}$
$\xi = \tan(angle)$

$$dx = \text{Sign } x(dx, dy) \frac{a^2}{\sqrt{1 + \frac{b^2}{a^2 \xi^2}}}$$

$$dy = \text{Sign } y(dx, dy) \frac{b^2}{\sqrt{\left(1 + \frac{a^2}{b^2}\right)^2}}$$

Rotate by Theta (dx, dy):
$x_0 = x + dx$;
$y_0 = y + dy$;
$A(x_0, y_0, \theta, a, b) = A(x_0, y_0, \theta, a, b) + 1$
end

end
Get x_0 and y_0 and update accumulator A[a]++

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High-dimensional space can be reduced by following method:

Detection of majority votes for the ellipse center only, followed by a search for pixel clusters in the image that satisfied the ellipse equation. With multi-pass techniques, five-dimensional space can be broken in to low-dimensional subspaces. For Eg: One 2-Dimensional space for (x_0, y_0) Another 2-Dimensional space for (a, b) and a one-Dimensional space for θ.

Breaking down voting process in to subspaces, will decrease the robustness of the algorithm.

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