CS 6643 Computer Vision, Spring 2018, **G. Gerig**

Assignment 2 Theoretical Part

Out: 3/02/2018

Due: 3/11/2018 (deadline: midnight)

Late submissions: Late submissions result in 10% deduction for each day. The assignment will no longer be accepted 3 days after the deadline.

Office hours:

		Monday	Wed	Thu	Fri
Guido Gerig	office 10.094	2 - 4pm			
Yida Zhou	yz4499@nyu.edu			1-3pm	
Zebin Xu	zebinxu@nyu.edu		2 - 4pm		
Andrew Dempsey	ad4338@nyu.edu				10 - noon
Monil D. Shah	mds747@nyu.edu	4 - 6pm			

Location: Cubicle spaces in front of my office named 10.098 A,B,D,E,H.

A) Theoretical questions:

A1) Linear Shift Invariant System (LSI)

Given the LSI criteria:

- shift invariant: if g(x) = w(x) * f(x), then w(x) * f(x-x0) = g(x-x0), where w(x) is the filter and f(x) the original signal.
- linearity would require that Op(A+B) = Op(A) + Op(B), where Op is the filter operation (smoothing or median) and A and B the sets of pixels to be processed.

Discuss/show that a smoothing filter with a $\frac{1}{3}[1,1,1]$ filter kernel is linear and shift invariant.

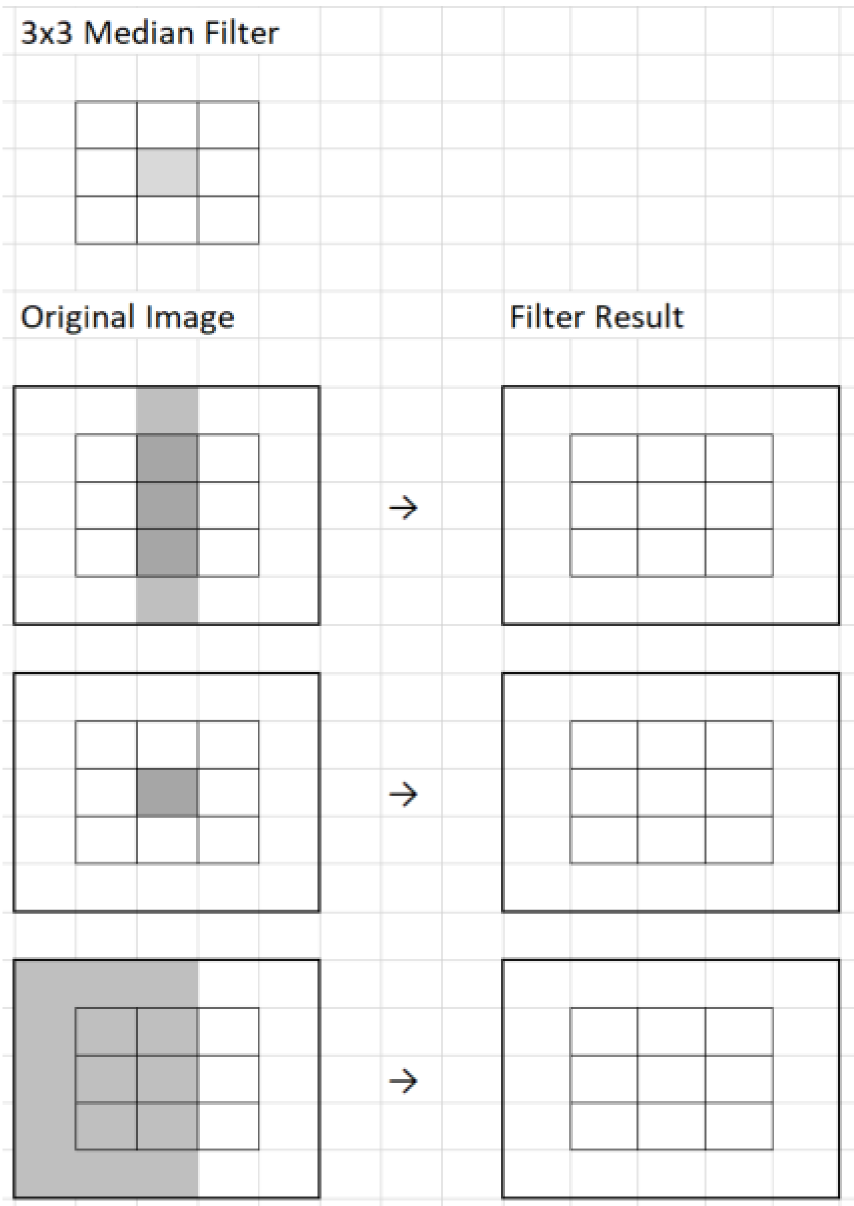
Now apply a Median filter using a 3 neighborhood. Discuss "why" or "why not" Median filtering is an LSI system. You can make a numerical example of sets A and B to demonstrate.

A2) 1-D Convolution

Convolve a [1,1,1] filter with a [0,0,0,1,1,1,0,0,0] signal and plot/graph the result.

A3) Median Filtering

See the following synthetic images, where empty pixels are 0 and the shaded pixels are 1's. Apply a 3x3 median filter to these 3 images, using the sliding window filtering operation as discussed in the course. Note that the border cannot be filtered, so that you can move the window only within the outer, dark boundary as shown. Mark the results for the center 3x3 region to the right.



Discuss what you get for the 3 images of a bar, a point and an intensity edge.

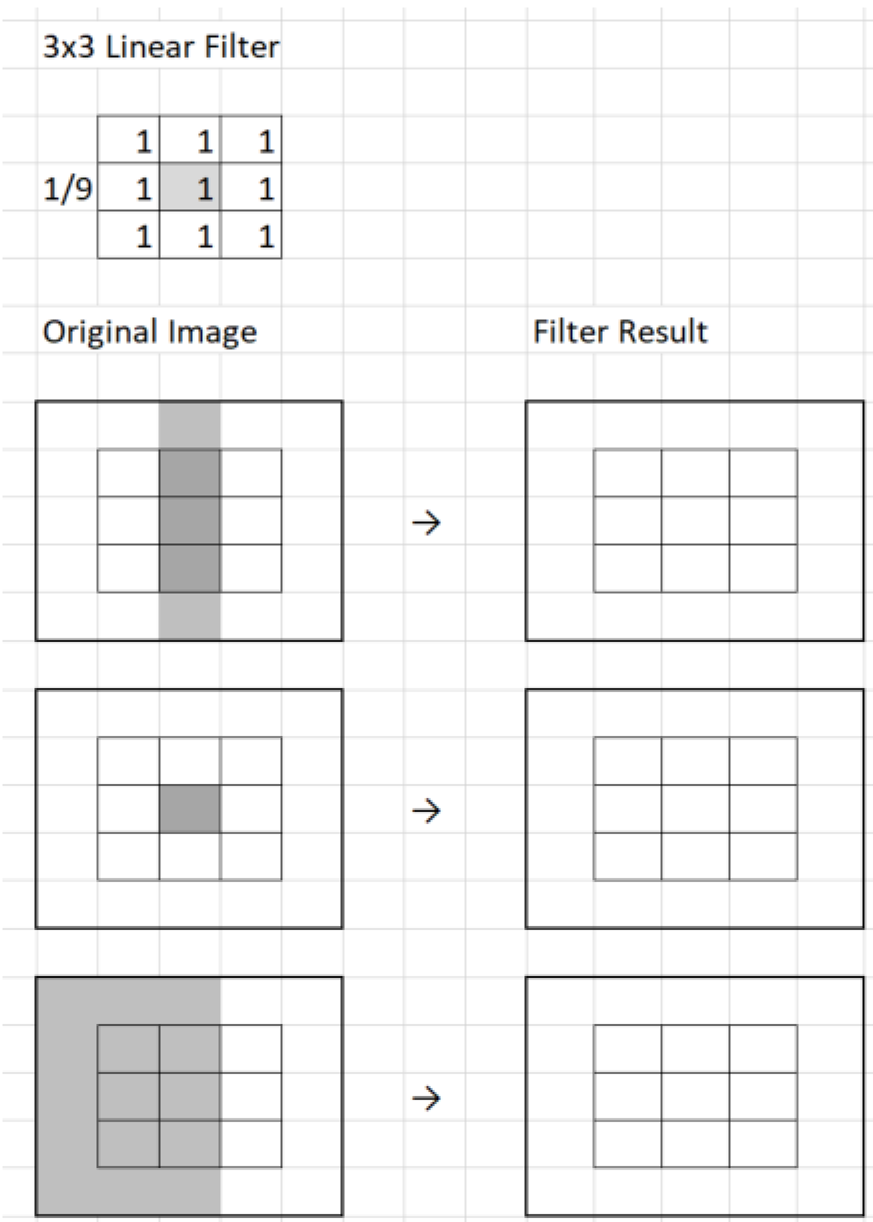
The median filter eliminates small, thin structures but preserves step edge structures.

Do the same for a 3x3 linear filter

Discuss what you get for the 3 images of a bar, a point and an intensity edge.

Discuss what you get for the 3 images of a bar, a point and an intensity edge. Then discuss observed differences between the Media and Linear filter.

A linear smoothing filter blurs line, point and edge structures – the images become unsharp.



A4) Separability

Show that the 2D Gaussian filter (G(x,y;sigma) function can be separated into a product of two 1D components G(x;σ) and G(y; σ), each only be a function of x or y, respectively.

2-D Gaussian: $$\frac{1}{2\pi\sigma^2}\, exp^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

a) decompose the 2D Gaussian into the two components

see handouts/slides for results

b) discuss how this helps to speed up the filtering operation by making an example of a m x m filter size covering the Gaussian, and then use the two 1-D filters.

The mxn filters requires mxn multiplications and the sum. Separating a filter into two 1-D only requires m+n multiplications. E.g., an 11x11 filter would require 121 multiplications, whereas running it with 1-D separable filters would be 11+11=22, more than 5 times less.

A5) Linearity and Separability

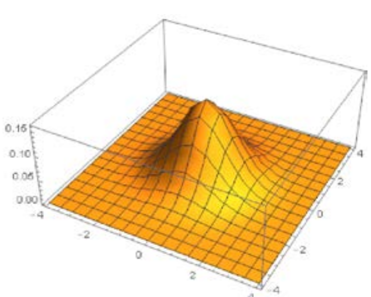
The Canny filter was described as applying the first derivative of a Gaussian as approximation of finite differences at a specific scale.

In 2-D, we will have to apply a derivatives of the Gaussian in x and y and can then calculate edge-magnitude and edge-direction. In the course, we explained that we apply linearity and choose the following options, please consider the position of the brackets:

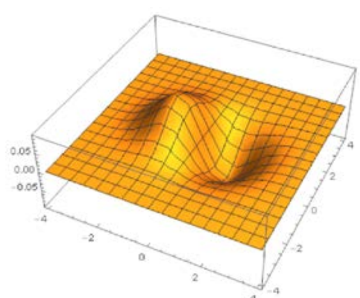
$$\left[\frac{\partial}{\partial x} \circ G(x,y;\sigma)\right] \circ f(x,y) \quad \text{or} \quad \frac{\partial}{\partial x} \circ [G(x,y;\sigma) \circ f(x,y)]$$

and the same for the y direction:

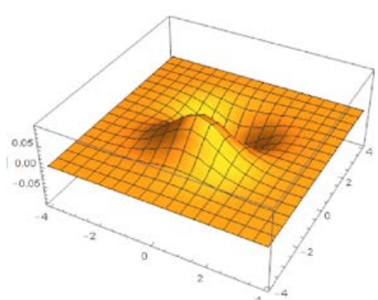
$$\left[\frac{\partial}{\partial y} \circ G(x,y;\sigma)\right] \circ f(x,y) \quad \text{or} \quad \frac{\partial}{\partial y} \circ [G(x,y;\sigma) \circ f(x,y)]$$



2D Gaussian



x partial derivative



y partial derivative

Would you implement these operations as discrete filters as discussed in the class, which of the two options would you prefer in regard to number of operations and thus efficiency?

Hint: Compare brackets indicating which operations can be combined first. Think about implementation of a separable filter such as the Gaussian (see previous section), and about the filter mask used for x- and y-derivatives. Then compare to computational costs of applying two first directional derivatives of the 2-D Gaussian to the image.

For those who want to dig somewhat deeper, you can check if the first partial derivatives of the 2-D Gaussian would be separable?

$$\frac{\partial G(x,y;sig)}{\partial x} = -\frac{e^{\frac{-x^2-y^2}{2\,\mathrm{sig}^2}}}{2\,\pi\,\mathrm{sig}^4} \qquad \frac{\partial G(x,y;sig)}{\partial y} = -\frac{y\,e^{\frac{-x^2-y^2}{2\,\mathrm{sig}^2}}}{2\,\pi\,\mathrm{sig}^4}$$

A6) Canny non-maximum suppression

Please review the slides and materials on the Canny filter and explain in a short text what we mean by “Non-Maximum Suppression” as the last processing step in edge filtering, and how this is “roughly” done on a discrete grid.